

# ENPM 808Q Mini-project 1

University of Maryland at College Park

Due on November 2, 2015

## INSTRUCTIONS

Please include all relevant calculations, simulation plots, simulation schemes and algorithms as part of your solutions. Please submit your solutions in a single .pdf file.

**Exercise 1** Consider a second order system described by the following differential equation:

$$M\ddot{x}(t) + \varrho\dot{x}(t) + Kx(t) = F(t), \quad t \geq 0 \quad (1)$$

where  $F$  and  $x$  represent the input and output of the system, respectively. Assume that the constants  $M$ ,  $\varrho$  and  $K$  are unknown. You are given samples  $\{(t_1, x_s(t_1)), (t_2, x_s(t_2)), \dots\}$ , where  $x_s$  represents the response to a unit step input, with zero initial conditions, i.e.,  $x_s(0) = 0$  and  $\dot{x}_s(0) = 0$ .

- A) Is it always possible to determine exactly (or approximately)  $M$ ,  $\varrho$  and  $K$  from the samples provided? If so then provide an algorithm, and illustrate its properties and performance in simulation.
- B) Is it possible to determine the constants exactly (or approximately) if the list is finite? If you can determine the constants exactly then determine the minimum number of samples you need. Provide an algorithm and simulations of its properties to illustrate your answer.

**Exercise 2** Consider the system in (1).

- A) Check for what combination of parameters  $M$ ,  $\varrho$  and  $K$  the system is controllable.
- B) For  $M = 1$ ,  $\varrho = 0.1$  and  $K = 1$ , compute  $e^{At}$  analytically. Compute the unit step response analytically. Plot your response and compare with simulation.
- C) Consider the parameters in B) with  $x(0) = 1$  and  $\dot{x}(0) = 0$ . Determine a control such that  $x(t) = 0$  for  $t \geq 10$ .

**Exercise 3** Consider the following system:

$$M\ddot{x}(t) + \varrho\dot{x}(t) + Kx(t) = F(t) + F_d, \quad t \geq 0 \quad (2)$$

where  $F_d$  represents a constant unknown disturbance. We wish to regulate  $x(t)$  so as to track a known constant reference  $x_r$ .

- A) Assume that  $M = K = 1$  and  $\varrho = 0.1$  and design a PID controller that will steer  $x(t)$  to track  $x_r$  with zero asymptotic error. Here, we assume that  $x(0) = \dot{x}(0) = 0$ ,  $0.1 < F_d < 0.1$  and that  $F(t)$  saturates at  $|F(t)| \geq 5$ . Provide your best design for which  $F(t)$  never saturates. Provide simulations that illustrate your design process (include one for which saturation occurs).

~~B) Ignore the saturation in A) and obtain the state space representation of the closed loop system in the linear regime (no saturation) and check stability for the chosen PID. Is there any choice of the PID parameters for which the closed loop is unstable?~~

~~**Assignment from the book** Solve problems 6.18, 6.19, 6.20 and 6.21~~