# ENPM 808M: Homework #1

Due on Wednesday, September 16, 2015

*Dr. William Levine 4:00 PM*

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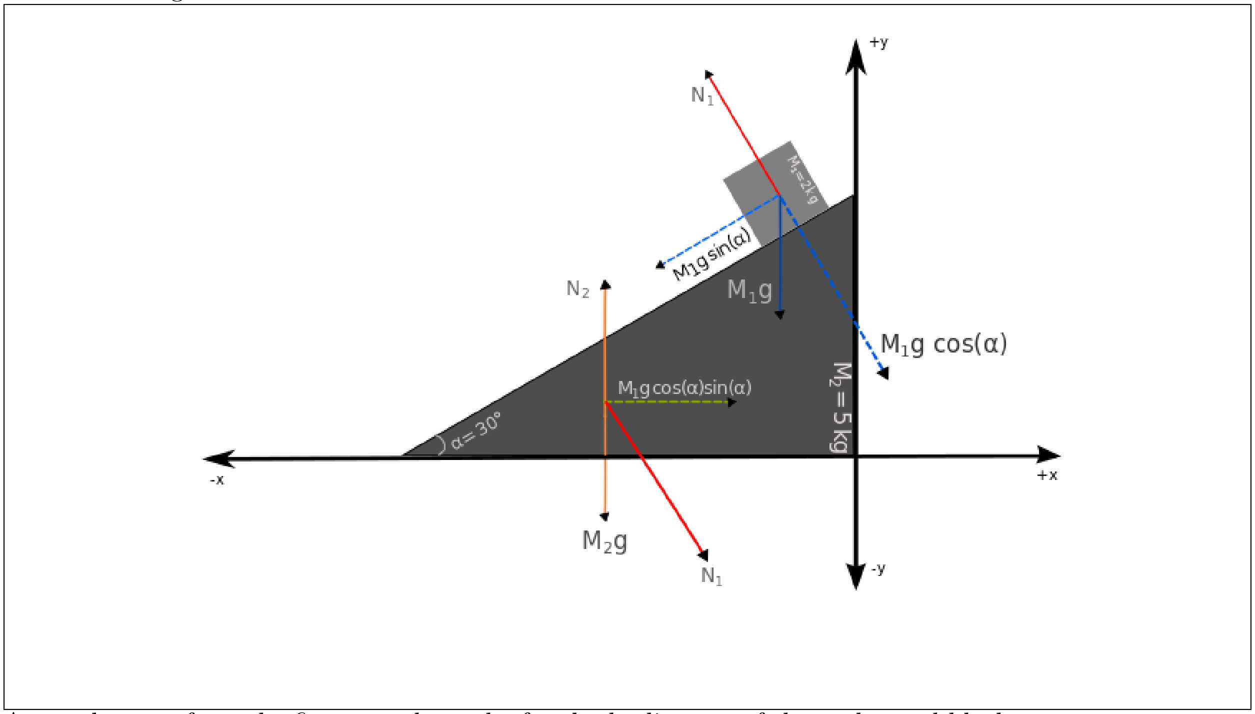
# Contents

# Problem 1

Consider the block sliding on a wedge from Lecture #1. Suppose the block has mass $M_1$ and the wedge has mass $M_2$. The angle of the block is $\alpha$ as in the lecture. When $M_1$ slides down and to the left, the wedge $M_2$ slides to the right. Find formulae for the acceleration of $M_1$ and $M_2$ in the absence of friction.




As can be seen from the figure, we have the free body diagram of the wedge and block system.

The block slides along the wedge with an acceleration in the $x$ and $y$ directions while the wedge has only an acceleration along the $x$-axis.
Now, consider the forces on the block only. We have a downward force of $M_1 g$ perpendicular to the reference frame, which is the force due to the acceleration due to gravity. We also have a normal reaction of force $N_1$ caused due to the contact forces between the block and the wedge. If we resolve these forces into their horizontal and vertical components, we have:

$$(F_1)_{\parallel} = M_1 \times g \times cos(\alpha) \tag{1}$$
$$(F_1)_{\perp} = M_1 \times g \times sin(\alpha) \tag{2}$$
$$(N_1)_{\perp} = N_1 \times sin(\alpha) \tag{3}$$

Now, consider the forces on the wedge only. We have a downward force of $M_2 g$ perpendicular to the reference frame, which is the force due to the acceleration due to gravity. We also have a normal reaction of force $N_2$ caused due to the contact forces between the wedge and the ground.

Now, consider the system of block and wedge together. It is assumed that there is negligible force due to friction. As a result, the wedge moves in the right direction due to the horizontal component of the downward force being applied by the block, which is sliding to the right.

Problem 1 continued on next page. . .

The forces $N_2$ and $M_2g$ are equal and acting in opposite directions, hence there is no vertical motion of the wedge. There is only the horizontal component of force $N_1$ from the block acting on the wedge. We have:

$$N_1 = M_1 \times g \times cos(\alpha) \tag{4}$$

and the horizontal component of Equation 4 as:

$$(N_1)_{\parallel} = M_1 \times g \times cos(\alpha) \times sin(\alpha) \tag{5}$$

From Newton's Second Law of Motion, we have:

$$F = M \times a \tag{6}$$

$$\implies a = \frac{F}{M} \tag{7}$$

where $a$ is the acceleration, $F$ is the force applied, $M$ is the mass of the object.

So, from Equation 7, we calculate the force and the resulting acceleration on the wedge as follows:

$$N_2 = Ncos(\theta) - Nsin(\theta) \tag{8}$$

$$N_2 sin(\theta) = M_2 \times A \tag{9}$$

$$\implies A = \frac{Nsin(\theta)}{M_2} \tag{10}$$

So, from Equation 7, we calculate the force and the resulting acceleration on the block as follows:

Along the vertical direction,

$$M_1 \times g - N_1 \times cos(\theta) = M_1 \times sin(\theta) \tag{11}$$

$$M_1 \times g - \left(\frac{M_2 \times A}{sin(\theta)}\right) = M_1 \times a \times sin(\theta) \tag{12}$$

$$M_1 \times g \times sin(\theta) - M_2 \times A \times cos(\theta) = M_1 \times a \times sin^2(\theta) \tag{13}$$

Along the horizontal direction,

$$N_1 \times sin(\theta) = M_1 \times (a \times cos(\theta) - A) \tag{14}$$

$$N_1 \times sin(\theta) + M_1 \times A = M_1 \times a \times cos(\theta) \tag{15}$$

$$\left(\frac{M_2 \times A}{sin(\theta)}\right) \times sin(\theta) + M_1 A = M_1 acos(\theta) \tag{16}$$

$$A(M_2 + M_1) = M_1 acos(\theta) \tag{17}$$

$$\implies A = \frac{M_1 acos(\theta)}{(M_1 + M_2)} \tag{18}$$

Substituting $A$ from Eqn.18 in Eqn.13, we have

$$M_1 gsin(\theta) - M_2\Big(\frac{M_1 acos(\theta)}{M_1 + M_2}\Big) \times cos(\theta) = M_1 asin^2(\theta) \tag{19}$$

$$M_1(M_1 + M_2)gsin(\theta) - M_1 M_2 acos^2(\theta) = M_1(M_1 + M_2)asin^2(\theta) \tag{20}$$

$$(M_1 + M_2)gsin(\theta) = M_1 asin^2(\theta) + M_2 asin^2(\theta) + M_2 acos^2(\theta) \tag{21}$$

$$(M_1 + M_2)gsin(\theta) = M_1 asin^2(\theta) + M_2 a \tag{22}$$

$$\implies a = \frac{(M_1 + M_2)gsin(\theta)}{M_1 sin^2(\theta) + M_2} \tag{23}$$

Thus, from Eqn.23 we have the acceleration of the block as:

$$\mathbf{a = \frac{7 \times 9.8 \times 0.5}{(2 \times 0.25) + 5} = 6.23 m/s^2} \tag{24}$$

$$\tag{25}$$

So, taking the components of this acceleration, we have

$$\mathbf{a_x = 6.23 \times cos(30) = 5.390 m/s^2} \tag{26}$$

$$\mathbf{a_y = 6.23 \times sin(30) = 3.115 m/s^2} \tag{27}$$

Thus, from Eqn.18 we have the acceleration of the wedge as:

$$\mathbf{A_x = \frac{(2 \times 6.23 \times cos(30))}{2 + 5}} \tag{28}$$

$$\mathbf{A_x = 1.530 m/s^2} \tag{29}$$

# Problem 2

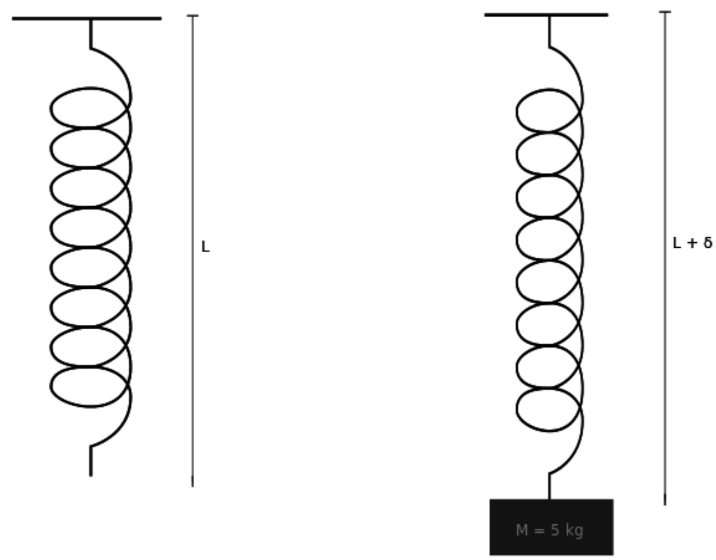
A 5 kilogram weight hangs from a spring of such stiffness that the spring stretches $\delta = 1cm$ under the weight.

a. Calculate the stiffness of the spring.

b. Calculate the natural frequency of the up and down oscillations of the weight if it starts at some length other than stretched by $\delta = 1cm$.

c. Write a formula for the frequency in terms of $\delta$ alone, in which neither the mass $m$ nor the spring constant $k$ appears.

## (a)




As can be seen in above figure, we have a spring of length $L$ when it is in its natural position, without any external mass attached to it. Once a mass $M$ of 5 kg is attached to the end of the spring, there is a change in length of $\delta = 1cm$.
Now, we know from Hooke's Law that *the force needed to extend or compress a spring by some distance is proportional to that distance.*
Or, formally, we have

$$F = -k \times \delta \tag{30}$$

where $k$ is the spring constant, or a characteristic value of the spring denoting its stiffness.

So, from Equation 30, we have:

$$F = -k \times \delta \tag{31}$$

$$\implies k = \frac{-F}{\delta} \tag{32}$$

$$\implies k = \frac{5 \times 9.8}{0.01} = 4900\text{N/m} \tag{33}$$

## (b)

Continuing with figure shown previously, the formula for the frequency of oscillations of the weight $M$ attached to the spring when released from a length other than $\delta$ is:

$$\omega = \frac{1}{2\pi}\sqrt{\frac{k}{M}} \tag{34}$$

where $\omega$ is the frequency of oscillation, $k$ is the spring constant of the spring and $M$ is the mass of the weight attached to the spring.

So, from Equation 34, we have:

$$\omega = \frac{1}{2\pi}\sqrt{\frac{4900}{5}} \tag{35}$$

$$\implies \omega = \frac{1}{2\pi}\sqrt{980} \tag{36}$$

$$\implies \omega = \mathbf{4.98s^{-1}} \tag{37}$$

## (c)

Continuing with figure shown previously, the formula for the frequency of oscillations of the weight $M$ attached to the spring when released from a length other than $\delta$ is:

$$\omega = \frac{1}{2\pi}\sqrt{\frac{k}{M}} \tag{38}$$

where $\omega$ is the frequency of oscillation, $k$ is the spring constant of the spring and $M$ is the mass of the weight attached to the spring.

Also, we know that:

$$k = \frac{M \times g}{\delta} \tag{39}$$

So, from Equation 38 and 39, we have:

$$\omega = \frac{1}{2\pi}\sqrt{\frac{\frac{M \times g}{\delta}}{M}} \tag{40}$$

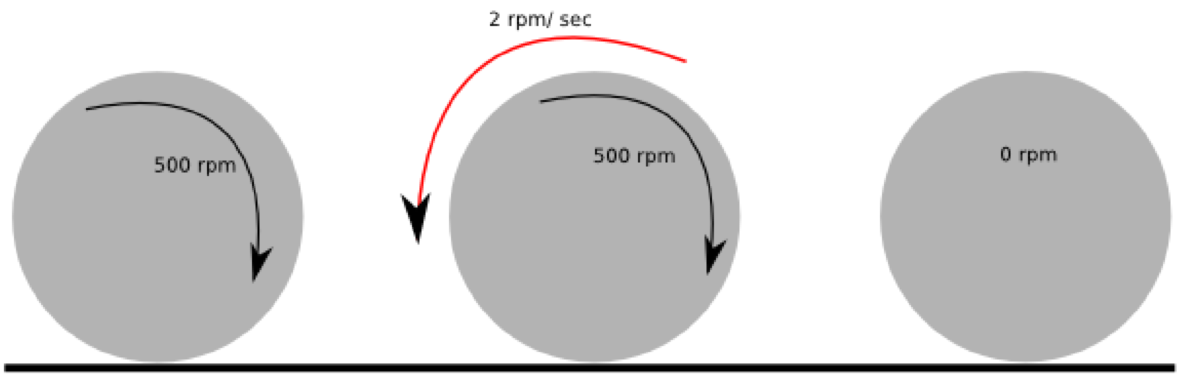
Simplifying, we have:

$$\omega = \frac{\mathbf{1}}{\mathbf{2}\pi}\sqrt{\frac{\mathbf{g}}{\delta}} \tag{41}$$

where $g$ is the acceleration due to gravity and $\delta$ is the displacement of the spring when a weight is attached to it. As can be seen, Equation 41 does not contain any terms containing the mass $M$ or the spring constant $k$.

# Problem 3

A wheel has initial angular velocity 500 rpm and is being slowed down at a rate of 2 rpm per second. How many rotations does the wheel make before it comes to a stop?

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From the figure, we have the wheel rotating at an angular velocity $\omega = 500rpm$. There is a retardation of 2 rpm per second.
We have the following formula for angular motion:

$$\theta = (\omega_0 \times t) + \left(\frac{1}{2} \times \alpha \times t^2\right) \tag{42}$$
$$\omega = \omega_0 + \alpha \times t \tag{43}$$

where $\theta$ is the angular displacement of the wheel, $\omega_0$ is the initial angular velocity of the wheel, $\omega$ is the final velocity of the wheel and $\alpha$ is the angular acceleration of the wheel and $t$ is the time.

So, we have:

$$\omega_0 = 500rpm = \frac{500}{60}r/s = 8.333r/s \tag{44}$$
$$\alpha = \textit{2 rev. per minute per second} = 0.03r/s^2 \tag{45}$$
$$\tag{46}$$

From Equation 43 we have:

$$0 = 500 - 2 \times t \tag{47}$$
$$\implies t = 250s \tag{48}$$

Using Equations 42 and 48,

$$\theta = (8.333 \times 250) + \left(\frac{1}{2} \times 0.03 \times 250^2\right) \tag{49}$$
$$\implies \theta = \frac{62500}{60} = \mathbf{1041.667\ revolutions} \tag{50}$$

Thus, the wheel takes **1041.667 revolutions** to come to a stop.