Ans 1) Problem 2.34 (a) using tabular method.

	z[0]	z[1]	z[2]	z[3]	z[4]	z[5]	z[6]	z[7]	z[8]
	1	1	1	1	2	2	2	2	2
x[-5] 1	1	1	1	1	2	2	2	2	2
x[-4] 1	1	1	1	1	2	2	2	2	2
x[-3] 1	1	1	1	1	2	2	2	2	2
x[-2] 1	1	1	1	1	2	2	2	2	2
x[-1] 1	1	1	1	1	2	2	2	2	2
x[0] 1	1	1	1	1	2	2	2	2	2
x[1] 1	1	1	1	1	2	2	2	2	2

n	m[n]
-5	1
-4	2
-3	3
-2	4
-1	6
0	8
1	10
2	11
3	12
4	11
5	10
6	8
7	6
8	4
9	2
10	0

Am 23+ (b) $x(n) * y(n)$

		y[-3]	y[-2]	y[-1]	y[0]	y[1]	y[2]	y[3]	y[4]
		1	1	1	1	-1	-1	-1	-1
x(-5)	1	1	1	1	1	-1	-1	-1	-1
x[-4]	1	1	1	1	1	-1	-1	-1	-1
x[-3]	1	1	1	1	1	-1	-1	-1	-1
x[-2]	1	1	1	1	1	-1	-1	-1	-1
x[-1]	1	1	1	1	1	-1	-1	-1	-1
x[0]	1	1	1	1	1	-1	-1	-1	-1
x[1]	1	1	1	1	1	-1	-1	-1	-1

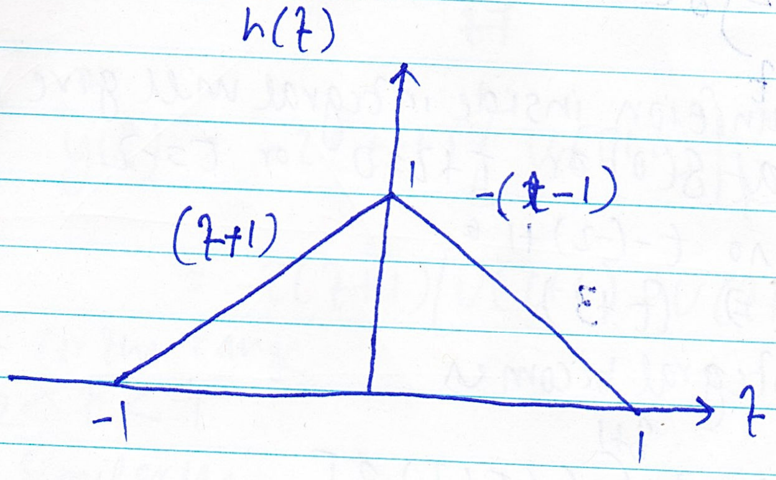
n	m(n)
-8	1
-7	2
-6	3
-5	4
-4	3
-3	2
-2	1
-1	-1
0	-2
1	-3
2	-4
3	-3
4	-2
5	-1
6	0

A.2.34(c) $x[n] * f(n)$

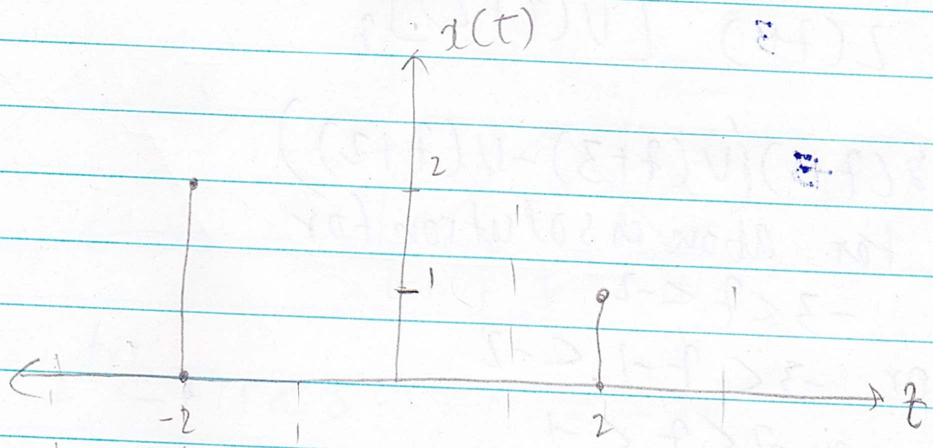
	f[-5]	f[-4]	f[-3]	f[-2]	f[-1]	f[0]	f[1]	f[2]	f[3]	f(4)	f(5)
	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5
x[-5] 1	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5
x[-4] 1	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5
x[-3] 1	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5
x[-2] 1	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5
x[-1] 1	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5
x[0] 1	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5
x[1] 1	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5

[n]	m[n]
-10	-2.5
-9	-4.5
-8	-6
-7	-7
-6	-7.5
-5	-7.5
-4	-7
-3	-3.5
-2	0
-1	3.5
0	7.0
1	7.5
2	7.5
3	7.0
4	6.0
5	4.5
6	2.5
7	0

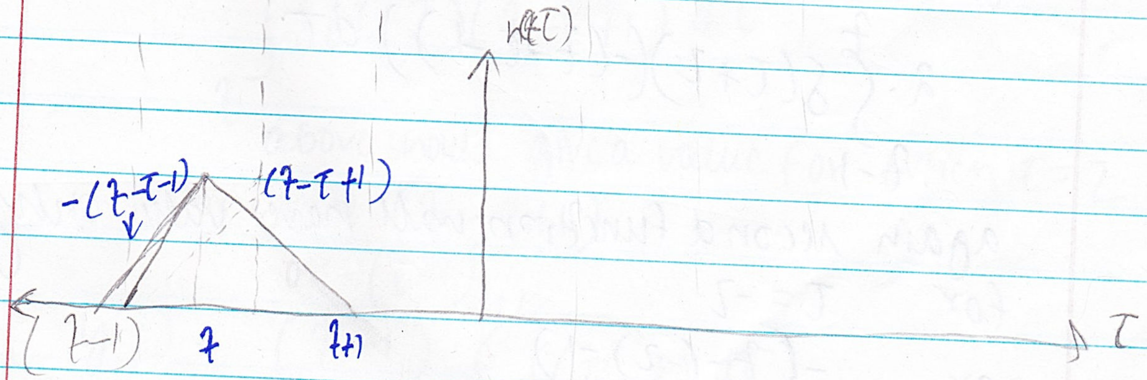
Problem 2.38 (a)



$x(t) = 2\delta(t+2) + \delta(t-2)$



Fliping and shifting h(t)



for range $-2 < t+1 < -1$

or $-3 < t < -2$

$$2\cdot\int_{t}^{t+1} \delta(\tau+2)\,(t-\tau+1)\,d\tau$$

The function inside integral will give a value only at $\delta(0)$ or $\tau+2=0$ or $\tau=-2$

so $t-(-2)+1$

$\Rightarrow (t+3)$

or integral becomes

$$2(t+3)\int_{t}^{t+1} \delta(\tau+2)\,d\tau$$

$$\Rightarrow 2(t+3)\left[u(\tau+2)\right]_{t}^{t+1}$$

$$\Rightarrow 2(t+3)\big(u(t+3)-u(t+2)\big)$$

for above is solution for

$-3<t<-2$

for $-3<t-1<-2$

or $-2<t<-1$

we will have

$$2\cdot\int_{t-1}^{t} \delta(\tau+2)\big(-(t-\tau-1)\big)\,d\tau$$

again second function will have value only for $\tau=-2$

or $-(t-(-2)-1)$

$-(t+2-1)$

or $-(t+1) \Rightarrow (-t-1)$

- $\cancel{1}$ overlap
$-t+\tau+1$ $\quad -x+1$
$(-2t-2)$ $\quad (-t-1)-2+1$

or $-2(t+1)\int_{t-1}^{t} \delta(\tau+2)\,d\tau$

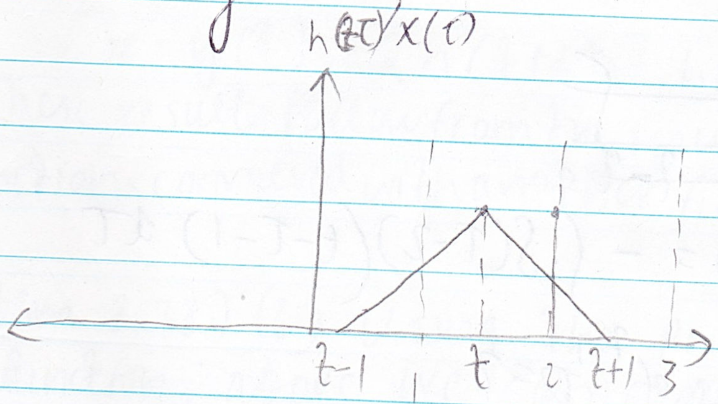
or

$$y(t) = -2(t+1)\Big[u[\tau+2]\Big]_{t-1}^{t}$$

$$= -2(t+1)\Big[u(t+2) - u(t+1)\Big]$$

for the range
$-2 < t < -1$

Similarly



for
$2 < t+1 < 3$
or $1 < t < 2$

$$\int_{t}^{t+1} \delta(\tau-2)(t-\tau+1)\,d\tau$$

above should give a value for when $\tau = 2$
or when $(t-2+1)$
or $(t-1)$

$$y(t) = (t-1)\int_{t}^{t+1} \delta(\tau-2)\,d\tau$$

$$= (t-1)\Big[u[\tau-2]\Big]_{t}^{t+1}$$

or $(t-1)\left(u[t-1] - u[t-2]\right)$

for $1 < t < 2$

similarly for

$1 < t-1 < 2$

~~or $2 < t < 3$~~

or $2 < t < 3$

we have

~~$y(t) = \int_{t-t}$~~

$$y(t) = -\int_{t-1}^{t} \delta(\tau-2)(t-\tau-1)\, d\tau$$

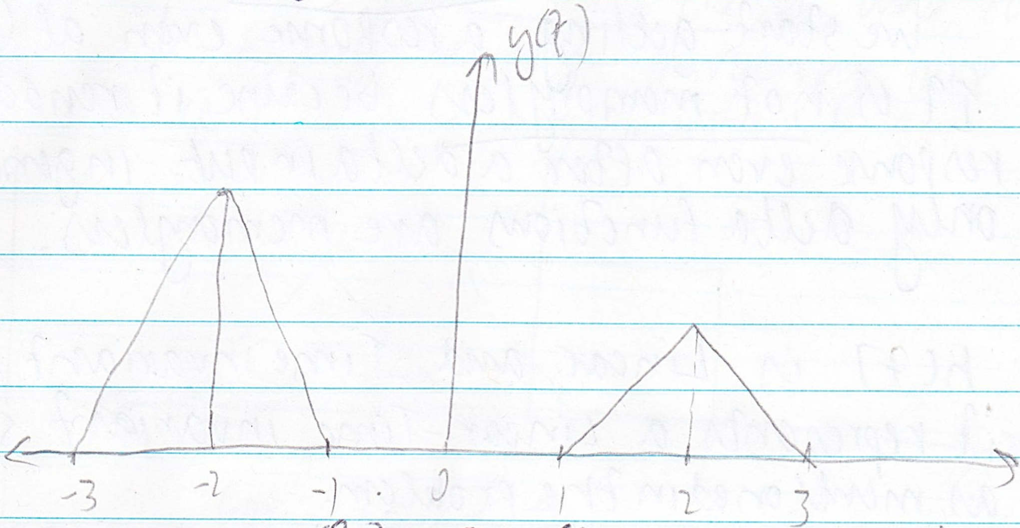
value of $\tau = 2$

$$y(t) = -(t-3)\int_{t-1}^{t} \delta(\tau-2)\, d\tau$$

$$y(t) = -(t-3)\left[u[\tau-2)\right]_{t-1}^{t}$$

$$= -(t-3)\left(u(t-2) - u(t-3)\right)$$

So $y(t)$ should look like this:



or $y(t) = 2h(t+2) + h(t-2)$

These results follow from the result that a delta function convolved with another function

Problem 2.38) (b) Using the principle of delta function above we can easily find $y(t)$

We know

$x(t) = \delta(t-1) + \delta(t-2) + \delta(t-3)$

or

$y(t) = x(t) * h(t)$

or

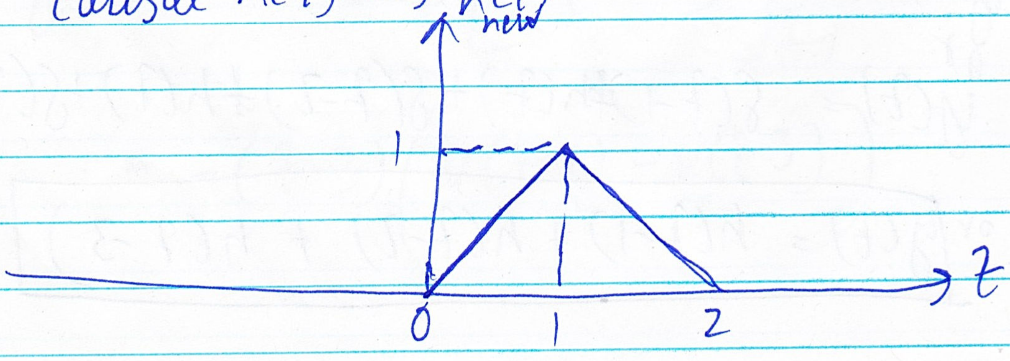
$y(t) = \delta(t-1) * h(t) + \delta(t-2) * h(t) + \delta(t-3) * h(t)$

or $\boxed{y(t) = h(t-1) + h(t-2) + h(t-3)}$

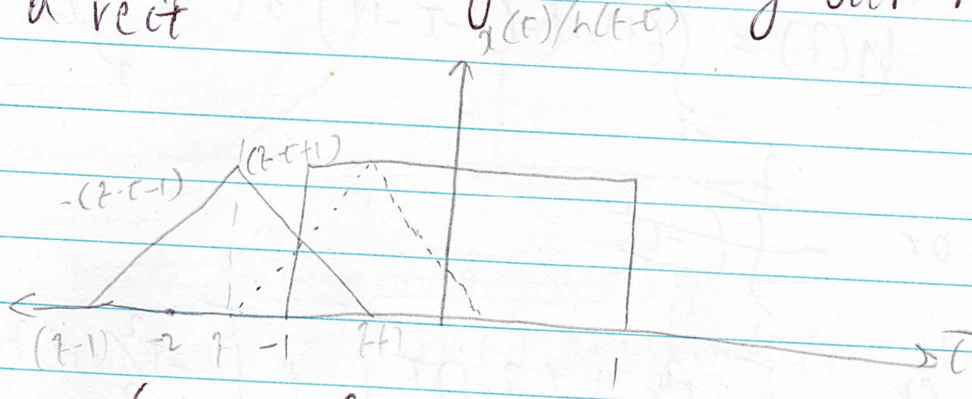
Ans 2.)(a) h(t) is not causal because we start getting a response even at ($t<0$)
It is not memoryless because it renders a response even after a delta input. in general only delta functions are memoryless.

h(t) is Linear and Time invariant because it represents a Linear-Time invariant system as mentioned in the problem.
Moreover, when scale and shifted delta functions are passed through it gives a scaled and shifted response (Although this is not sufficient evidence)

(b) We can change the causality by shifting h(t) by one unit to the right such that there is no response before $t<0$
Any shift that results in part of h(t) before ~~t<0~~ $t=0$ will make our h(t) non causal.
causal h(t) $\rightarrow h_{new}(t)$



(c) This is a low pass filter and this can be found out by convolving our $h(t)$ with a rect



for $-2 < t < -1$

$$y(t) = \int_{-1}^{t+1} (1)\cdot(t-\tau+1)\, d\tau$$

$$y(t) = \int_{-1}^{t+1} (-\tau + t+1)\, d\tau$$

$$= \left[\frac{-\tau^2}{2} + (t+1)\tau\right]_{-1}^{t+1}$$

$$= \left[\frac{-(t+1)^2}{2} + (t+1)^2 - \left(-\frac{1}{2} - (t+1)\right)\right]$$

$$= \left[\frac{(t+1)^2}{2} + t + 3/2\right]$$

for $y(t)$ $-2 < t < -1$

when $-1 < t < 0$

$$y(t) = \int_{-1}^{t} (1)\cdot(-(t-\tau-1))\, d\tau + \int_{t}^{t+1} (t-\tau+1)\, d\tau$$

or ~~$\int_{-1}^{t} (-\tau^2$~~

or

$$\Rightarrow -\left[-\frac{\tau^2}{2} + (t-1)\tau\right]_{-1}^{t} + \left[-\frac{\tau^2}{2} + (t+1)\tau\right]_{t}^{t+1}$$

$$\Rightarrow -\left[-\frac{t^2}{2} + t(t-1) - \left(-\frac{1}{2} - (t-1)\right)\right]$$

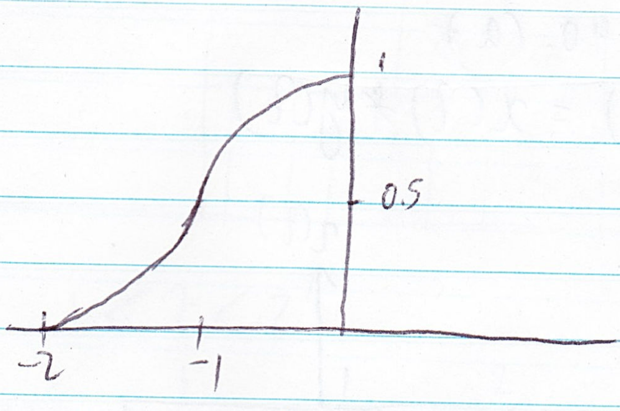
$$+$$

$$\left[-\frac{(t+1)^2}{2} + (t+1)^2 - \left(-\frac{t^2}{2} + t(t+1)\right)\right]$$

on simplifying this gives

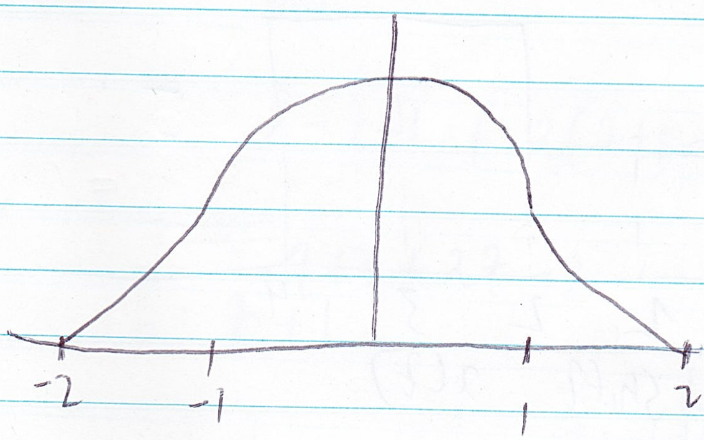
$$y(t) = \left(1 - \frac{t^2}{2}\right)$$

Plotting half the function in MATLAB shows $\rightarrow$



~~Since two functions are~~ ~~sym~~

Since $x(t)$ and $h(t)$ are symmetric the other side of $y(t)$ should be a mirror image.



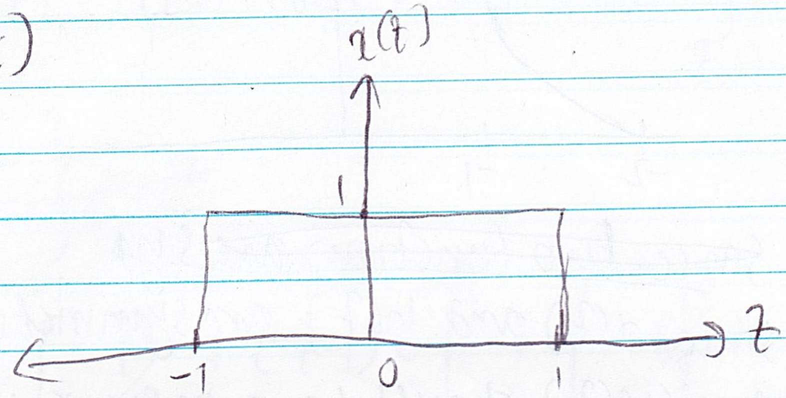
This is indeed a Low pass filter as the sharp edges have been smoothed.

P.T.O.

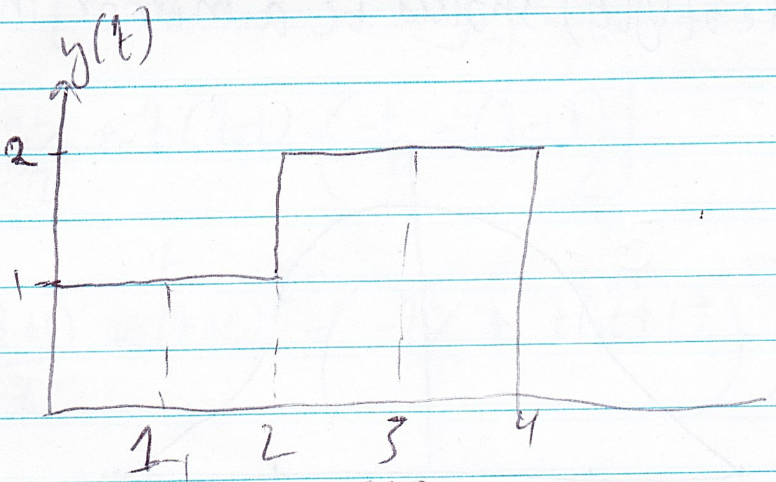
Ans 3.) 2.40. (a)

$m(t) = x(t) * y(t)$

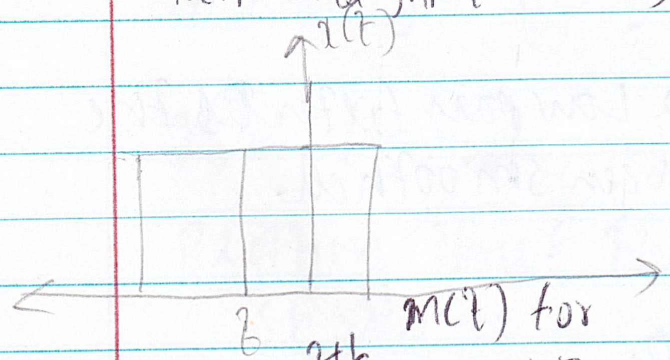
$x(t)$



$y(t)$

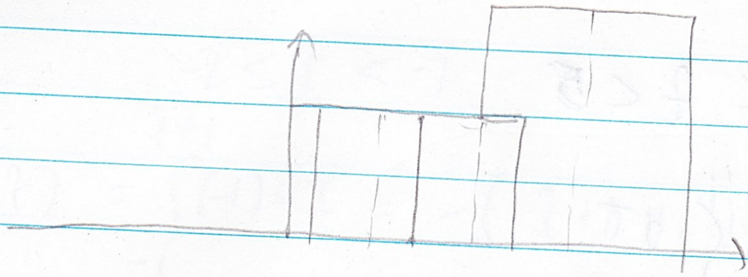


Flip and shift $x(t)$



$m(t)$ for $0 < t+1 < 2$ or $-1 < t < 1$

$$\int_0^{t+1} (1)\cdot(1)\, d\tau = (t+1)$$



$1 < t < 2$

$$\cancel{m(t) = \int_{t-1}^{1} (1)(1)\,d\tau + \int_{t}^{2} (1)(1)\,d\tau}$$

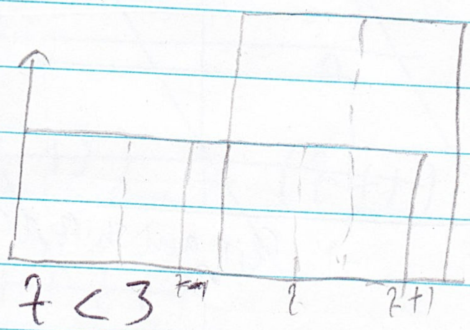
$$m(t) = \int_{t-1}^{2} (1)(1)\,d\tau + \int_{2}^{t+1} (2)\,d\tau$$

$$= 2 - t + 1 + 2(t+1-2)$$

$$\cancel{= 2}$$

$$= -t + 3 + 2t - 2$$

$$= t + 1$$



$2 < t < 3$

$$\int_{t-1}^{2} d\tau + \int_{2}^{t+1} 2\,d\tau$$

$$= 2 - t + 1 + 2(\cancel{t}+1-2) = t+1$$

$$3 < t < 5$$

$$m(t) = 2\int_{t-1}^{4} d\tau$$

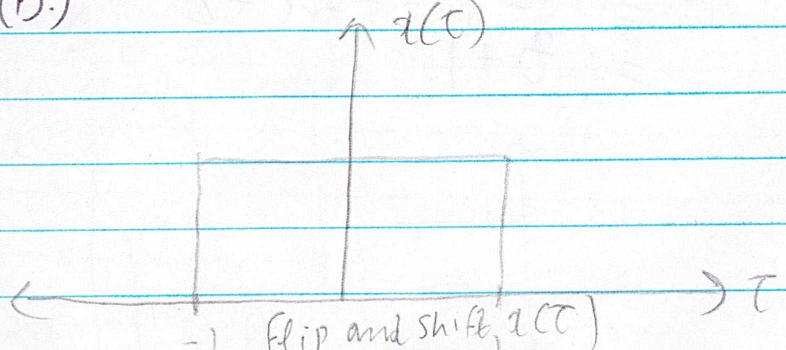
$$= 2(4 - t + 1)$$

$$= 2(5 - t)$$

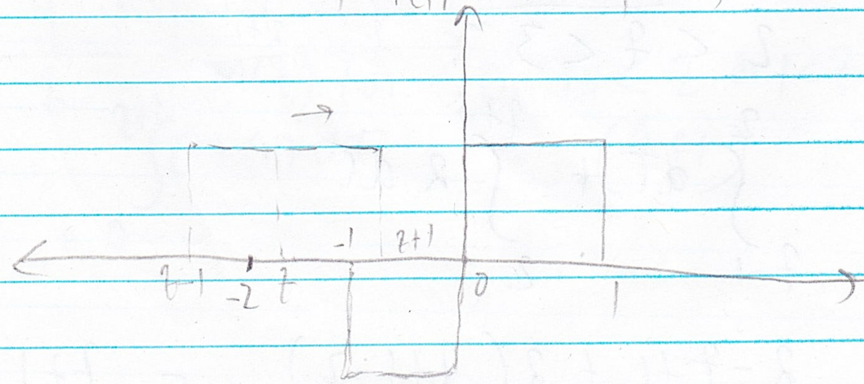
$$= 10 - 2t$$

$$m(t) = \begin{cases} t+1 & -1 \le t < 3 \\ 10 - 2t & 3 \le t < 5 \\ 0 & \text{everywhere else} \end{cases}$$

A 2.40 (B.)



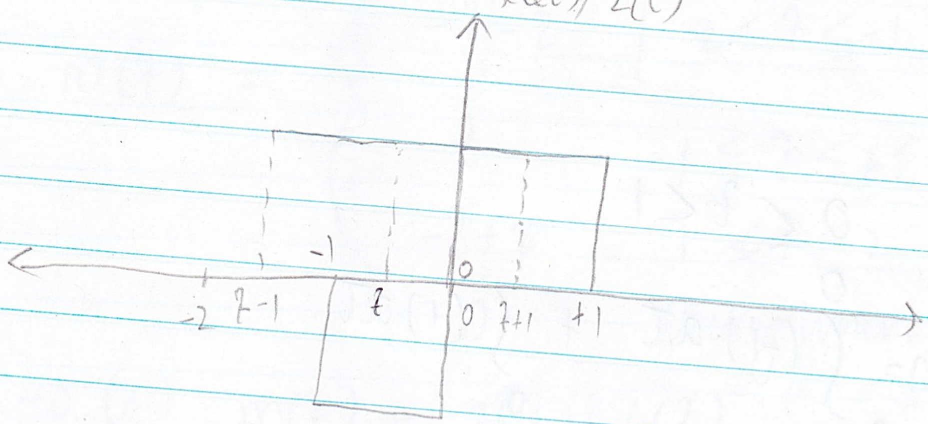
flip and shift x(τ)



$$-2 \le t < -1$$

$$m(t) = \int_{-1}^{t+1} (-1)\,d\tau = -(t+1+1)$$

$$= -t-2$$



$$-1 \le t < 0$$

$$\cancel{m(t) = \int_{t-1}^{0} (-1)\,d\tau + \int_{0}^{t+1} d\tau}$$

$$\cancel{= -(0-(t-1)) + (t+1)}$$

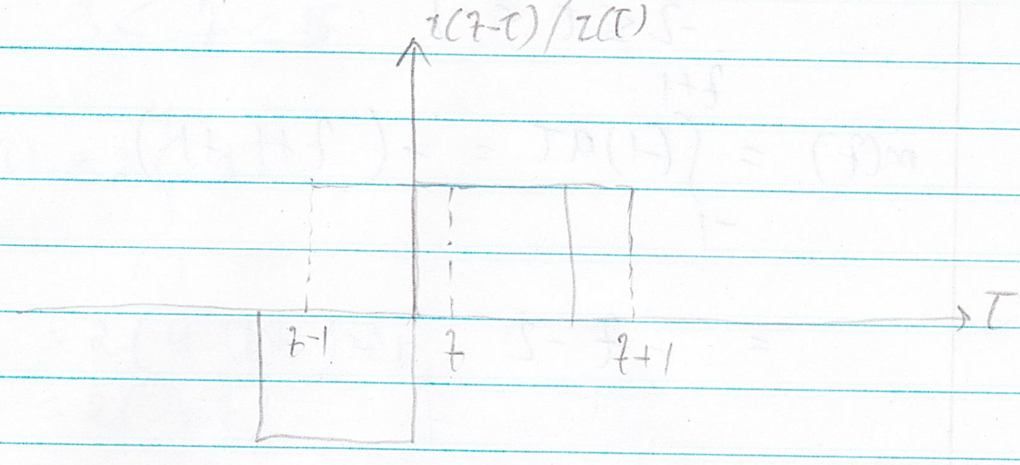
$$\cancel{= t-1+t+1}$$

$$\cancel{= 2t}$$

$$m(t) = \int_{-1}^{0} (-1)\,d\tau + \int_{0}^{t+1} (1)\,d\tau$$

$$= -(0+1) + t+1$$

$$= t+\cancel{1}-\cancel{1} = t$$



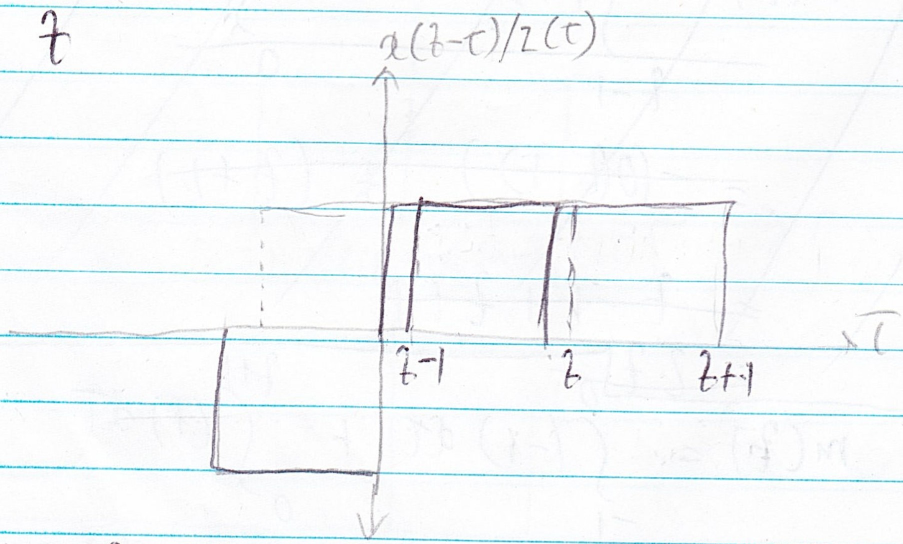
$0 \le t < 1$

$$m(t) = \int_{t-1}^{0} (-1)\, d\tau + \int_{0}^{1} (1)\, d\tau$$

$$= -(0-(t-1)) + 1$$

$$= t-1+1$$

$$= t$$



$1 \le t < 2$

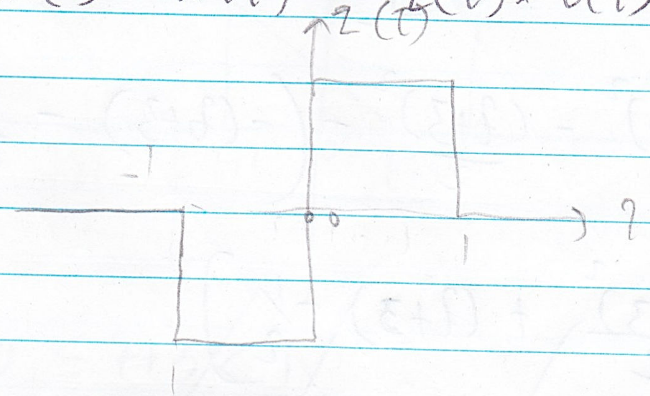
$$m(t) = \int_{t-1}^{1} d\tau$$

$$= (1 - t + 1)$$

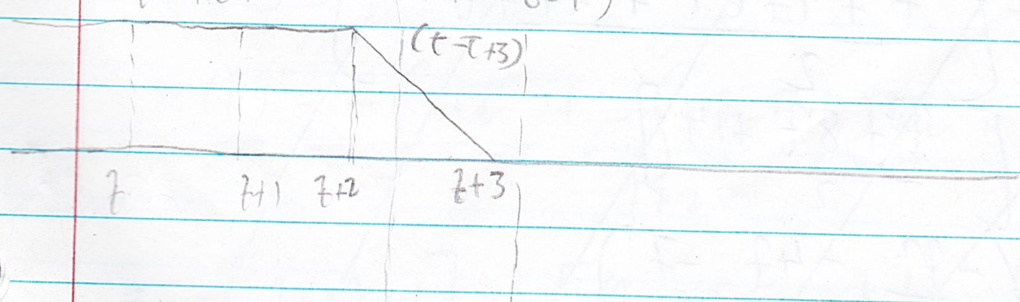
$$= -t + 2$$

$$m(t) = \begin{cases} -t-2 & -2 \le t \le -1 \\ t & -1 \le t \le +1 \\ -t+2 & 1 \le t \le 2 \\ 0 & \text{every where else} \end{cases}$$

2.40: (k) $m(t) = z(t) * b(t)$



(i) Flip and shift $b(\tau)$



for $-1 \le t+3 < 0$

$m(t) =$ or $-4 \le t < -3$

$$m(t) = \int_{-1}^{t+3} (-1)(t-\tau+3)\,d\tau$$

$$-\int_{-1}^{(t+3)} (t+3-\tau)\,d\tau$$

$$= -\left[(t+3)\tau - \frac{\tau^2}{2}\right]_{-1}^{t+3}$$

$$= -\left[(t+3)^2 - \frac{(t+3)^2}{2} - \left(-(t+3) - \frac{1}{2}\right)\right]$$

$$= -\left[\frac{(t+3)^2}{2} + (t+3) - \frac{1}{2}\right]$$

$$= -\left[\frac{t^2+9+6t}{2} + \frac{2t+6-1}{2}\right]$$

$$= -\left[\frac{t^2+8t+14}{2}\right]$$

$$\text{or} = \left(-\frac{t^2}{2} - 4t - 7\right)$$

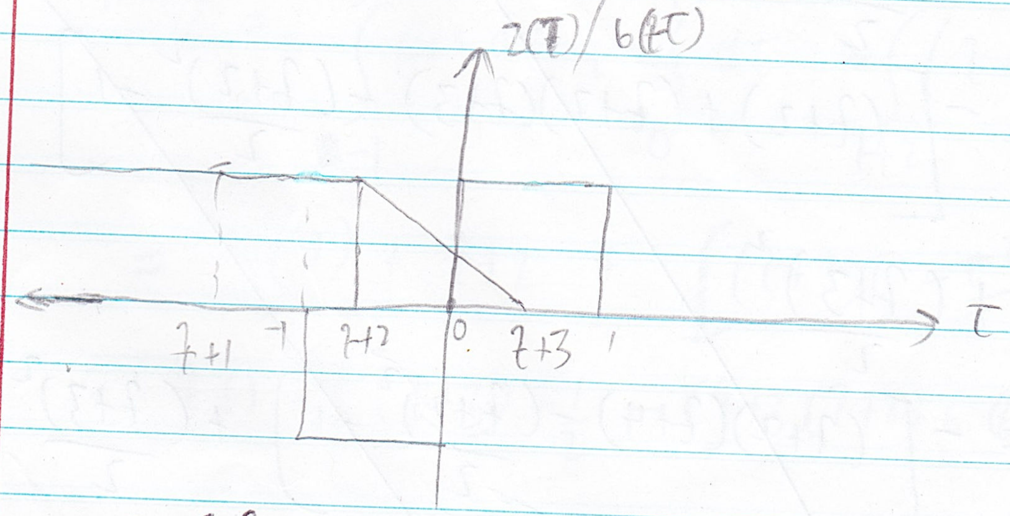
$$= -\left[\frac{(t+3)^2}{2} + (t+3) + \frac{1}{2}\right]$$

$$= -\left[\frac{(t+3)^2 + 2(t+3) + 1}{2}\right]$$

$$= -\frac{1}{2}\left((t+3+1)^2\right)$$

$$= -\frac{1}{2}(t+4)^2$$

for $-4 < t < -3$



$0 \leq t+3 < 1$

$-3 \leq t < -2$

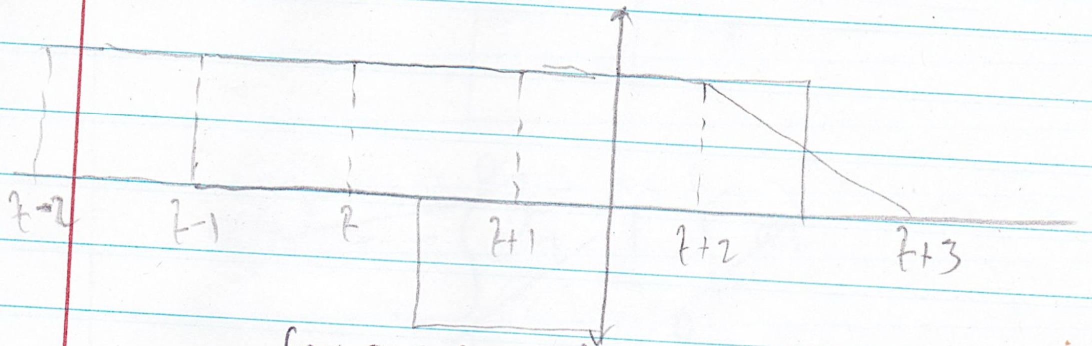
$$m(t) = -\int_{-1}^{t+2} d\tau + -\int_{t+2}^{0} (t-\tau+3)\,d\tau + \int_{0}^{t+3} (t-\tau+3)\,d\tau$$

$$= -\left[(t+2)+1\right] + -\left[(t+3)\tau - \frac{\tau^2}{2}\right]_{t+2}^{0} + \left[(t+3)\tau - \frac{\tau^2}{2}\right]_{0}^{t+3}$$

This simplifies to

$$= \frac{11}{2} + 5t + t^2$$

for $-3 \le t \le -2$



for $2 \le t \le -1$

$$m(t) = \int_{-1}^{0} (-1)\, d\tau + \int_{0}^{t+2} d\tau + \int_{t+2}^{1} (t - \tau + 3)\, d\tau$$

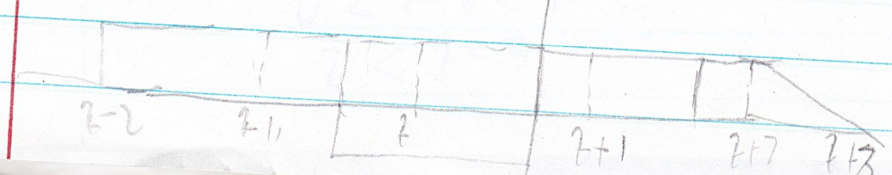
$$= -(1) + t+2 + \left[(t+3)\tau - \frac{\tau^2}{2} \right]_{t+2}^{1}$$

$$= t+1 + \left[(t+3) - \frac{1}{2} - (t+3)(t+2) + \frac{(t+2)^2}{2} \right]$$

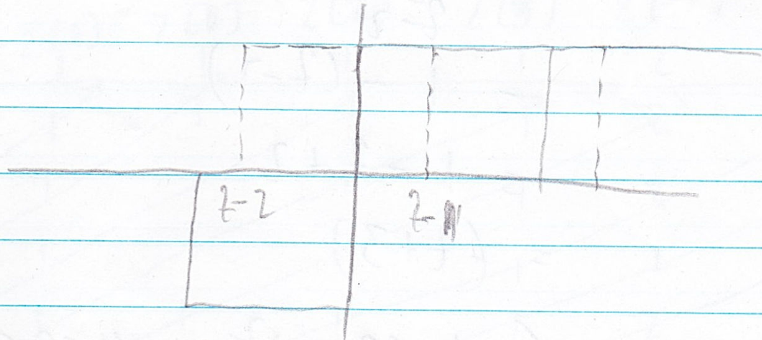
$$= -\frac{1}{2}(t+1)^2$$

$2 \le t \le -1$

Looking at figure $m(t) = 0$ ~~$1 \le t \le 1$~~ $t \in (-1, 1)$



$1 \le t \le 2$



$$\cancel{m(t) = -\int_{t-2}^{0} d\tau + \int_{0}^{1} dt}$$

$$\cancel{= -(-(t-2)) + 1}$$

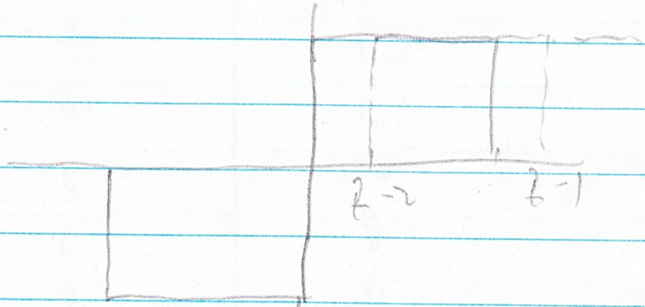
$$\cancel{= t-2+1}$$

$$\cancel{= t+1}$$

$$m(t) = -\int_{t-2}^{0} d\tau + \int_{0}^{1} dt$$

$$= -(-(t-2)) + 1$$

$$= t-1$$



$0 < t-2 < 1$

$2 < t < 3$

$$m(t) = \int_{t-2}^{1} d\tau$$

$$= 1 - (t-2)$$

$$= 1 - t + 2$$

$$= (t+3)$$

$$m(t) = \begin{cases} -\frac{1}{2}(t+4)^2 & -4 \le t \le -3 \\ 11/2 + 5t + t^2 & -3 \le t \le -2 \\ -\frac{1}{2}(t+1)^2 & -2 \le t \le -1 \\ 0 & -1 \le t \le 1 \\ t-1 & 1 \le t \le 2 \\ -t+3 & 2 \le t \le 3 \\ 0 & \text{everywhere else} \end{cases}$$