Date :

WAVE OPTICS

(i) Interference :-
- Coherent Sources
- Principle of superposition
- YDSE
- General Eq :- $y = a \sin(\omega t + \phi)$

  y → Resultant Disp.
  a → Amplitude
  ω → Angular frequency
  φ → Phase diff.

- Division of wavefront
- Fresnel bi Prism Experiment
- Division of Amplitude
  ↳ Newtons Ring Experiment

Coherent Sources

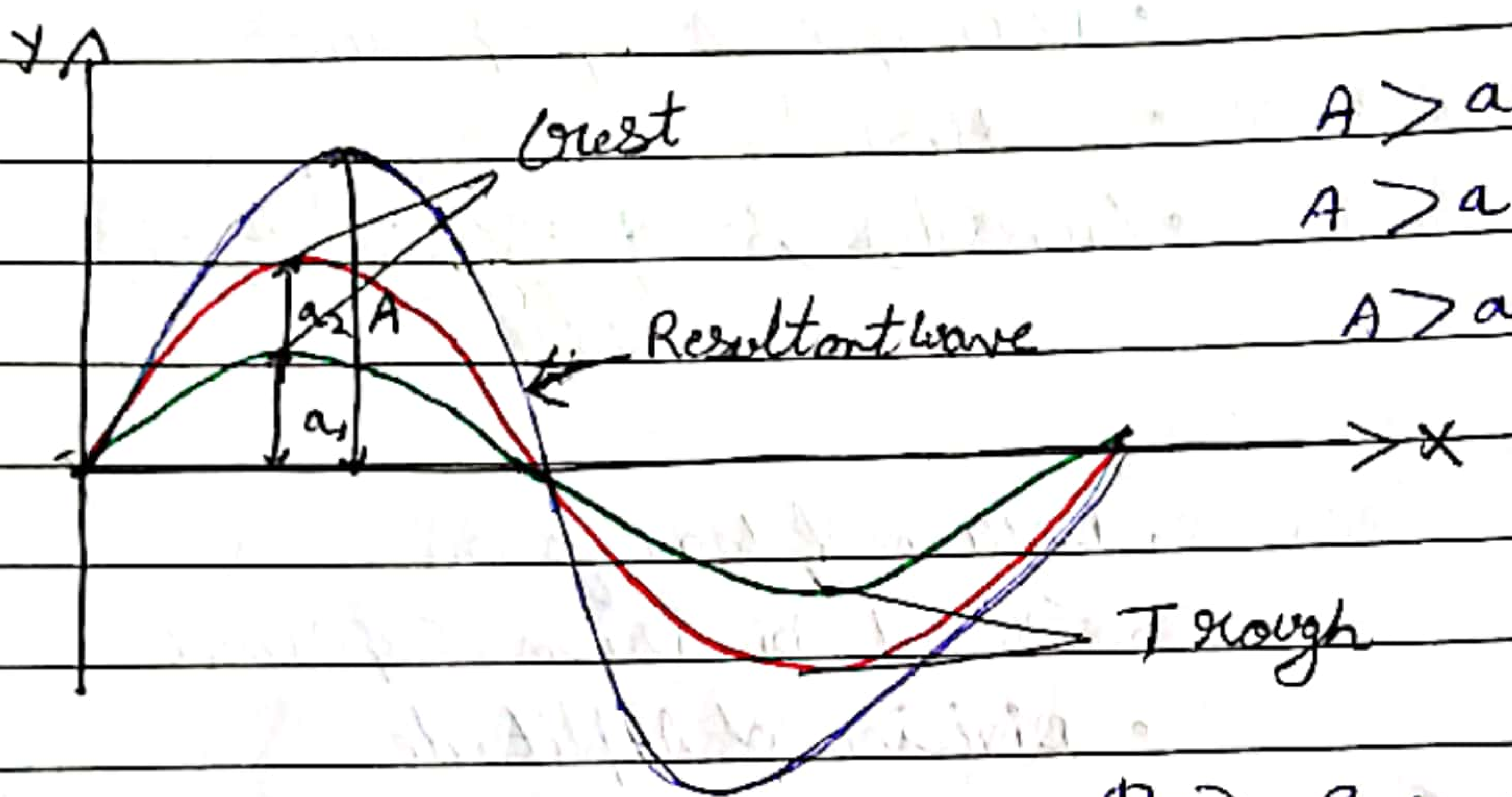
The two sources of light, whose frequencies are same & the Phase difference b/w the waves emitted by which remains constant w.r.t time.

Wavelets :- Each Point on a wavefront act as a centre of new disturbances & emits its own set of Spherical waves called Secondary wavelets.

Interference :- The Phenomenon of redistribution of light energy in a medium on account of superposition of light waves from two coherent source.

⇒ TYPES of Interference

* Constructive Interference :- Where Intensity of resultant wave is maximum.



$A > a_1$

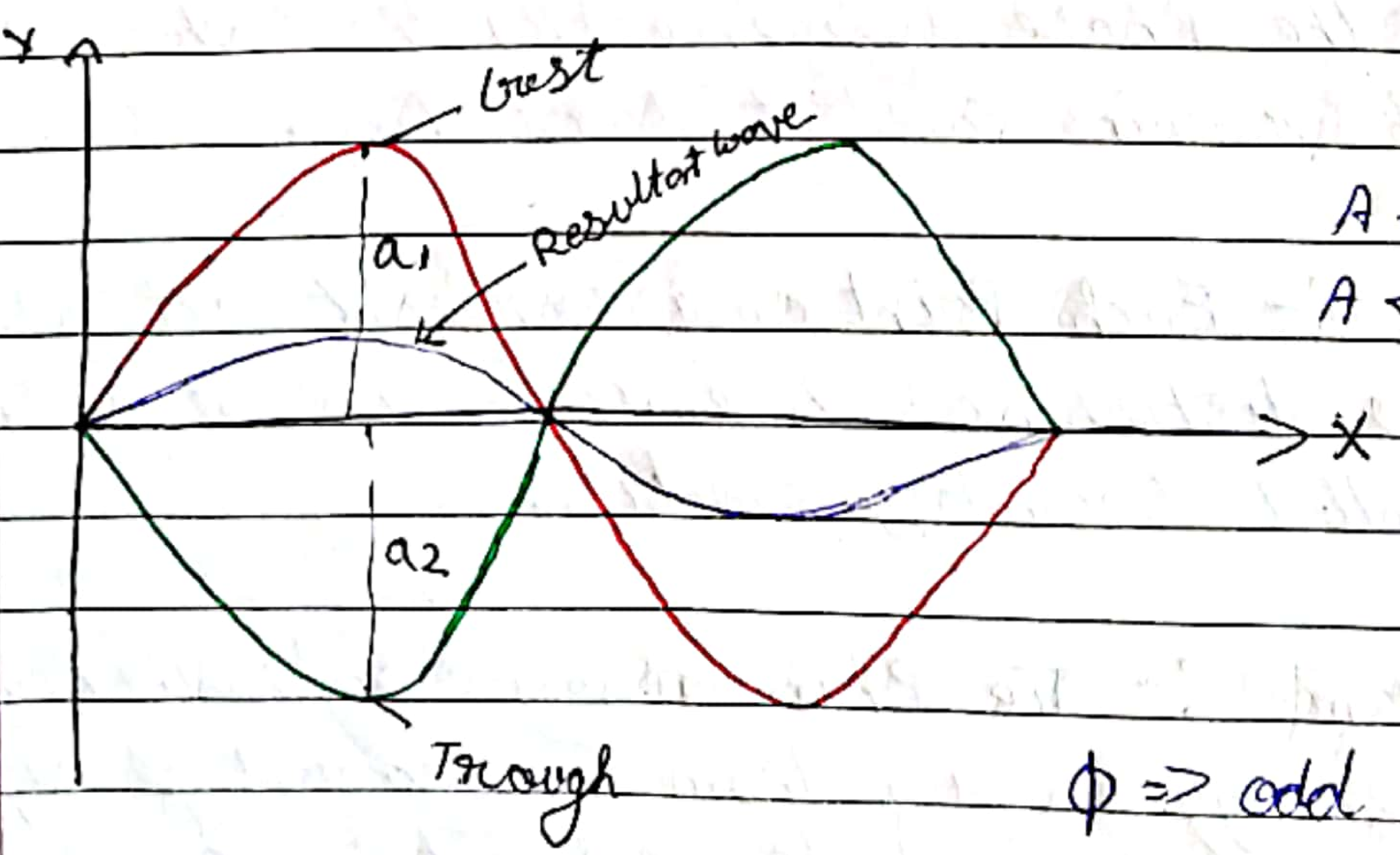
$A > a_2$

$A > a_1 + a_2$

$I > I_1 + I_2$

$\phi \Rightarrow$ Even Multiple of π

* Destructive Interference :- Where Intensity of resultant wave is minimum & waves are in opposite Phase.



$A < a_1$

$A < a_2$

$\phi \Rightarrow$ odd multiple of π

⇒ Principle of superposition

Resultant disp. $(\vec{y}) = \vec{y_1} + \vec{y_2} + \vec{y_3} + \cdots \vec{y_n}$

$$Y_1 = a_1 \sin(\omega t + \phi)$$
$$Y_2 = a_2 \sin(\omega t)$$

$$Y = Y_1 + Y_2$$

⇒ Expression of resultant displacement, Amplitude & Intensity obtain due to superposition of two light waves.

Two light waves having amplitude a_1 & a_2, travelling in same direction, superposing at same time t, ϕ is phase diff. & ω be angular freq.

- Displacement equation of I light wave

$$Y_1 = a_1 \sin(\omega t) \quad -①$$

- Displacement equation of II light wave

$$Y_2 = a_2 \sin(\omega t + \phi) \quad -②$$

Using Principle of super Position

Resultant disp.

$$Y = Y_1 + Y_2$$
$$Y = a_1 \sin \omega t + a_2 \sin(\omega t + \phi)$$

$$Y = a_1 \sin \omega t + a_2 (\sin \omega t \cos \phi + \cos \omega t \sin \phi)$$

$$Y = a_1 \sin \omega t + a_2 \sin \omega t \cos \phi + a_2 \cos \omega t \sin \phi$$

$$Y = (a_1 + a_2 \cos \phi) \sin \omega t + (a_2 \sin \phi) \cos \omega t \quad - (3)$$

Let,

$a_1 + a_2 \cos \phi = A \cos \theta$, $-(4)$ where A & θ are new

$a_2 \sin \phi = A \sin \theta$ $-(5)$ constants

Then,

$$Y = \cancel{A} A \cos \theta \sin \omega t + A \sin \theta \cos \omega t$$

$$\boxed{Y = A(\sin(\omega t + \theta))}$$

Squaring of 4 & 5 & adding.

$$A^2 \cos^2 \theta + A^2 \sin^2 \theta = (a_1 + a_2 \cos \phi)^2 + (a_2 \sin \phi)^2$$

$$A^2 = a_1^2 + a_2^2 \cos^2 \phi + 2 a_1 a_2 \cos \phi + a_2^2 \sin^2 \phi$$

$$A^2 = a_1^2 + a_2^2 + 2 a_1 a_2 \cos \phi$$

(i) $\boxed{A = \sqrt{a_1^2 + a_2^2 + 2 a_1 a_2 \cos \phi}}$ Amplitude of resultant wave

As, $I \propto A^2 \Rightarrow I = K A^2$

$$I = [a_1^2 + a_2^2 + 2 a_1 a_2 \cos \phi] K$$

Also

(ii) $\boxed{I = [I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi]}$

$$\left[\begin{array}{l} I_1 = K a_1^2 \\ I_2 = K a_2^2 \end{array}\right]$$

expression of I in Square Cosine form

$$I = K\left[a_1^2 + a_2^2 + 2a_1a_2\left(2\cos^2\frac{\phi}{2} - 1\right)\right]$$

$$= K\left[a_1^2 + a_2^2 \cancel{+} - 2a_1a_2 + 4a_1a_2\cos^2\frac{\phi}{2}\right]$$

(iii) $$\boxed{I = K\left[(a_1 - a_2)^2 + 4a_1a_2\cos^2\frac{\phi}{2}\right]}$$ If $K = 1$

⇒ Condition for Constructive Interference

- Intensity should be maximum.

For max. I $\cos\phi = 1$

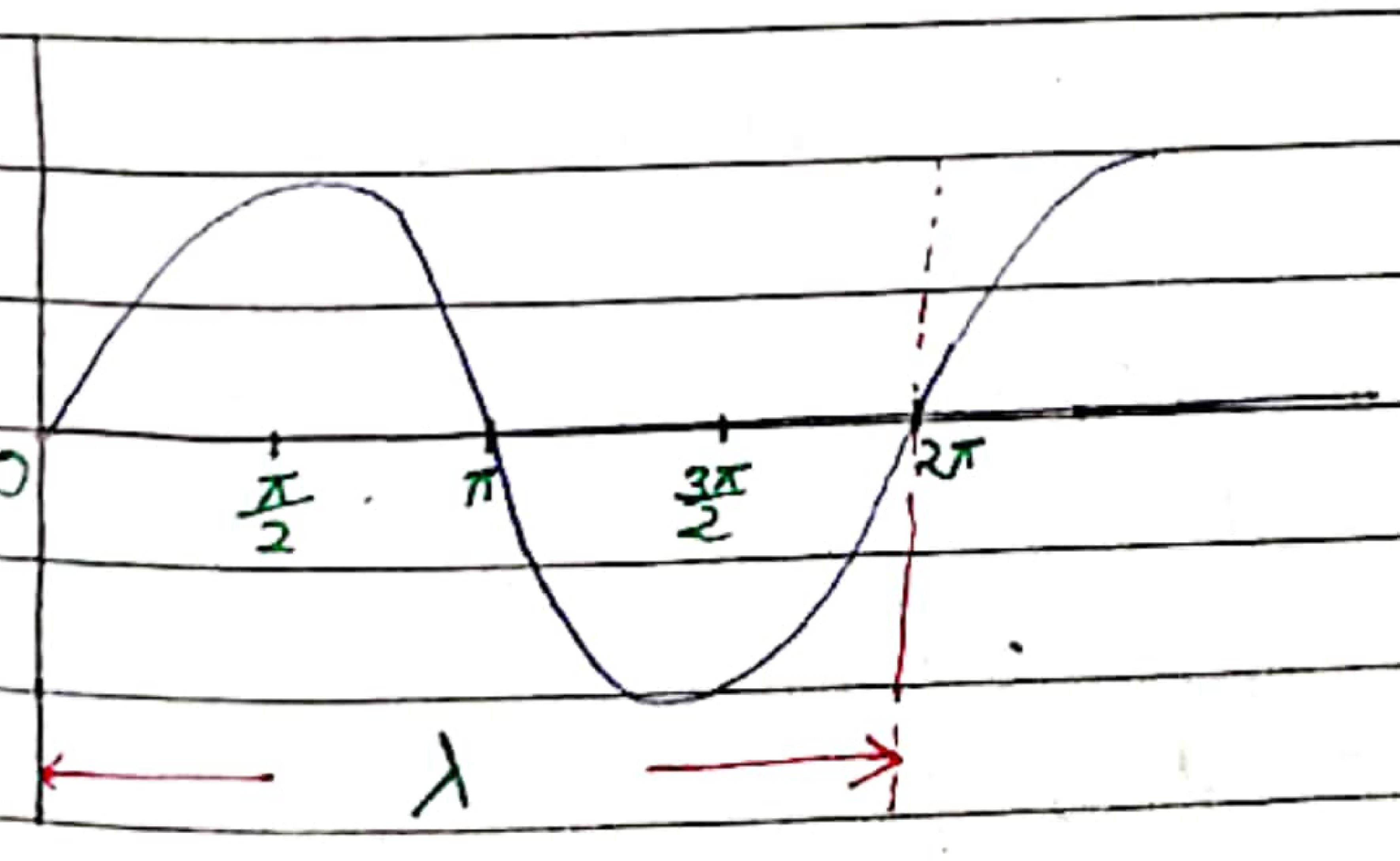
so, for

$\cos\phi = 1$

Phase diff ⇒ $\phi = [0, 2\pi, 4\pi \ldots 2n\pi]$

$\boxed{\phi = 2n\pi}$: where $n = 0, 1, 2 \ldots$

(even multiple of π)



Phase diff. (ϕ)		Path diff. (Δ)
2π	→	λ
1	→	$\lambda/2\pi$
ϕ	→	$\frac{\lambda\phi}{2\pi}$

* $\boxed{\text{Path diff.} (\Delta) = \frac{\lambda}{2\pi}\phi}$

if $\phi = 2n\pi$

$\boxed{\Delta = n\lambda} \Rightarrow 2n\frac{\lambda}{2}$, where $n = 0, 1, 2 \ldots$
(even multiple of $\lambda/2$)

$I_{max} = (a_1 + a_2)^2 = (\sqrt{I_1} + \sqrt{I_2})^2$

⇒ Condition for Destructive Interference

- Intensity should be minimum.

For min I, $\cos\phi = -1$

So for,

$\cos\phi = -1$

Phase diff ⇒ $\phi = \pi, 3\pi, 5\pi \ldots (2n+1)\pi$

$\boxed{\phi = (2n+1)\pi}$: where $n = 0, 1, 2 \ldots$
(odd multiple of π)

$\Delta = \frac{\lambda (2n+1)\pi}{2\pi} = \boxed{(2n+1)\frac{\lambda}{2}}$

where $n = 0, 1, 2 \ldots\ldots$
(odd Multiple of $\lambda/2$)

$I_{min} = a_1^2 + a_2^2 - 2a_1a_2$
$= (a_1 - a_2)^2$

$\boxed{I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2}$

⇒ If Amplitude of super impose wave is equal

i.e when $a_1 = a_2 = a$

$$I = a^2 + a^2 + 2a^2 \cos\phi$$

$$= 2a^2 (1 + \cos\phi)$$

$$= 2a^2 \cdot 2\cos^2 \frac{\phi}{2}$$

$$= \boxed{4a^2 \cos^2 \frac{\phi}{2}} = \boxed{4I \cos^2 \frac{\phi}{2}}$$

* $I_{max} = 4a^2 = 4I$

* $I_{min} = 0$

Average Intensity (I_{av})

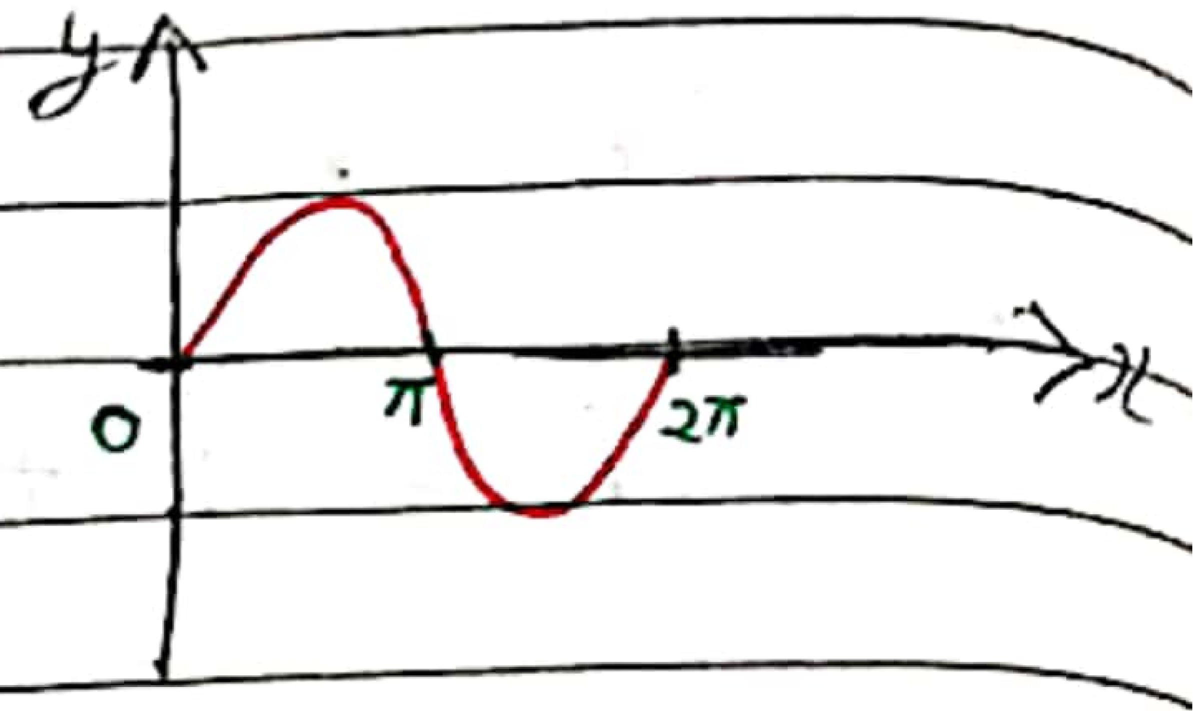
* Average of maximum & minimum values of intensity.

$$\boxed{I_{av} = \frac{I_{max} + I_{min}}{2}}$$

$$= \frac{(a_1 + a_2)^2 + (a_1 - a_2)^2}{2} \Rightarrow \boxed{\begin{matrix} a_1^2 + a_2^2 \\ \text{OR} \\ I_1 + I_2 \end{matrix}}$$

I_{av} = Sum of intensities of two superimposing waves.

* Average of all Points of Intensities



$$I_{av} = \frac{\int_0^{2\pi} I \, d\phi}{\int_0^{2\pi} d\phi}$$

$$= \frac{\int_0^{2\pi} [a_1^2 + a_2^2 + 2a_1a_2 \cos\phi] \, d\phi}{\int_0^{2\pi} d\phi}$$

$$= \frac{\left[a_1^2\phi + a_2^2\phi + 2a_1a_2 \sin\phi\right]_0^{2\pi}}{[\phi]_0^{2\pi}}$$

$$= \frac{a_1^2[\phi]_0^{2\pi} + a_2^2[\phi]_0^{2\pi}}{[\phi]_0^{2\pi}}$$

$$\boxed{I_{av} = a_1^2 + a_2^2}$$

OR

$$\boxed{I_{av} = I_1 + I_2}$$

⇒ Energy Conservation

Intensity before super imposition $= I_1 + I_2$

Intensity after super imposition $(I_{avg}) = I_1 + I_2$

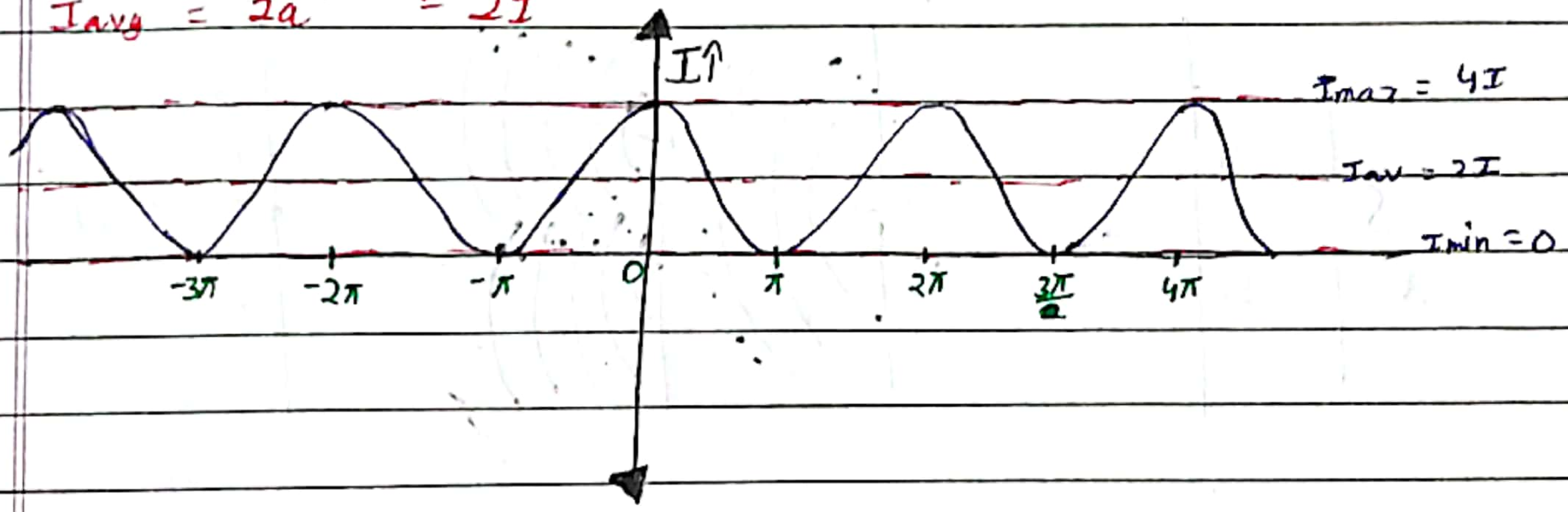
* So, there is no violation of the law of conservation of energy in the Phenomenon of Interference.
* The energy is simply redistributed in constructive & Destructive interference.

Intensity distribution curve

(i) When $a_1 = a_2 = a$, Resultant $I = 2a^2(1+\cos\phi)$

imp. Points

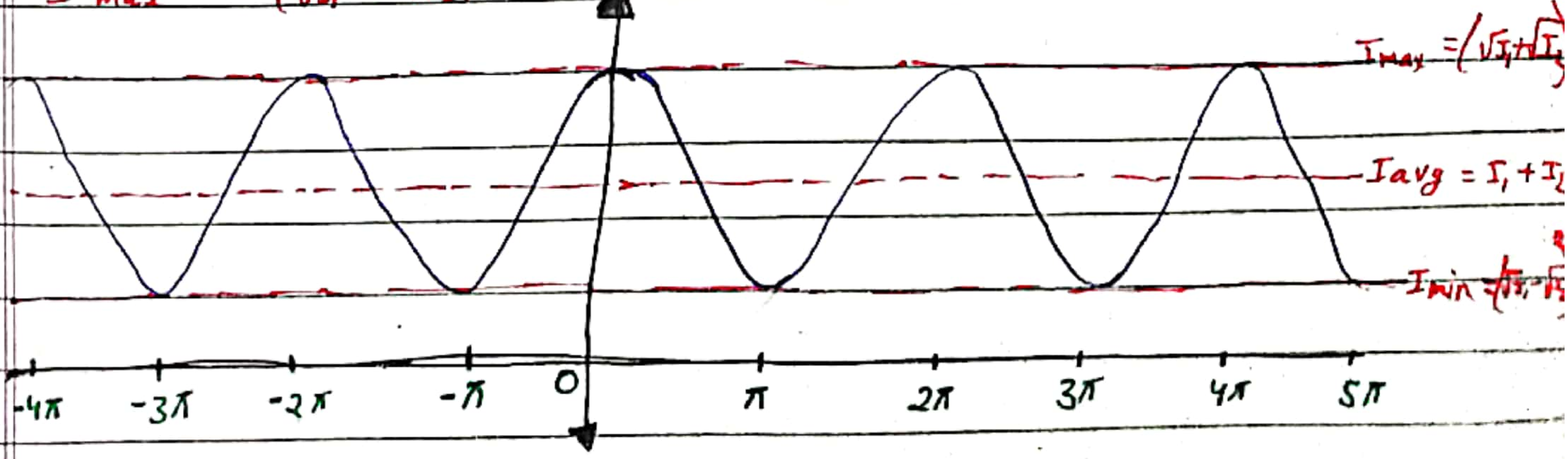
* $I_{max} = 4a^2 = 4I$, for $\cos\phi = 1$, $\phi = 2n\pi$ $n = 0, 1, 2 - - -$
* $I_{min} = 0$, for $\cos\phi = -1$, $\phi = (2n+1)\pi$ $n = 0, 1, 2 - - -$
* $I_{avg} = 2a^2 = 2I$



(ii) Amplitudes a_1, a_2

imp Points

* $I_{avg} = I_1 + I_2$
* $I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$
* $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$



Methods of Obtaining Interference

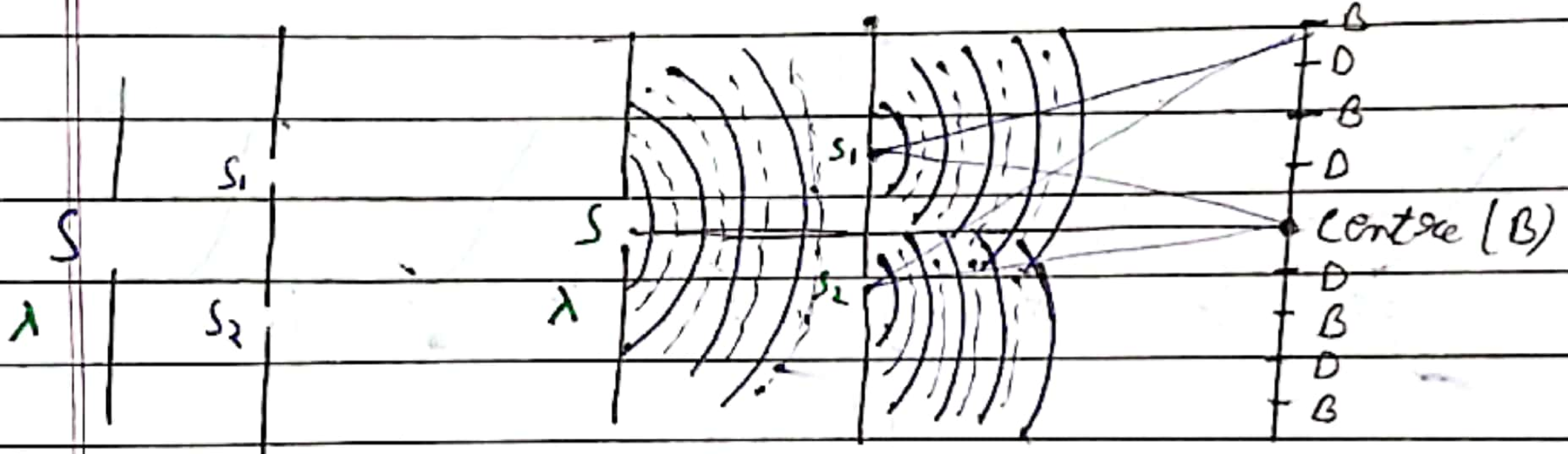
(i) Division of wave front

(ii) Division of Amplitude

⇒ Division of wave front

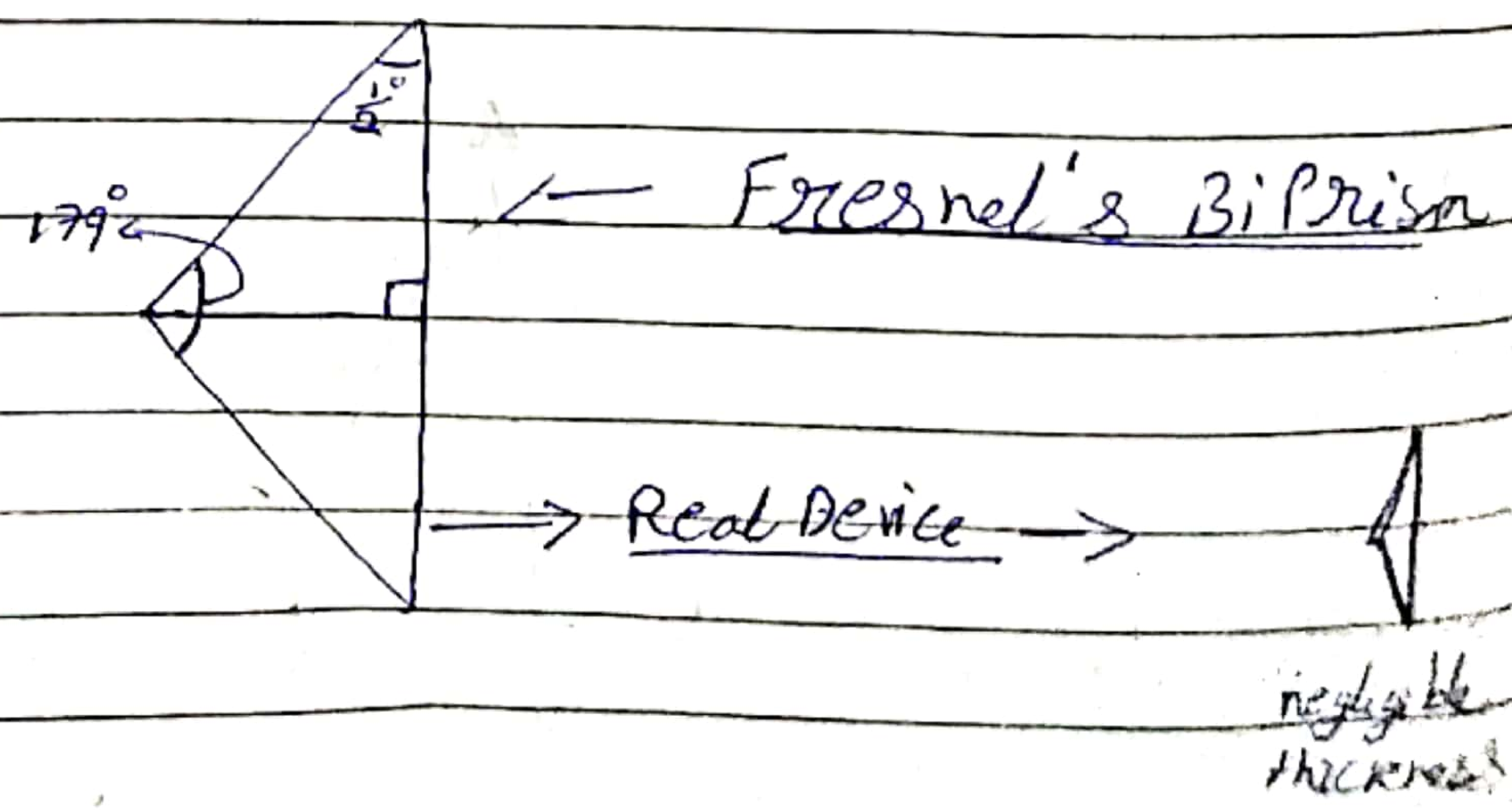
* Wave front - Locus of all Points where waves are in Phase.

Ex :- Young's Double SLIT Experiment.

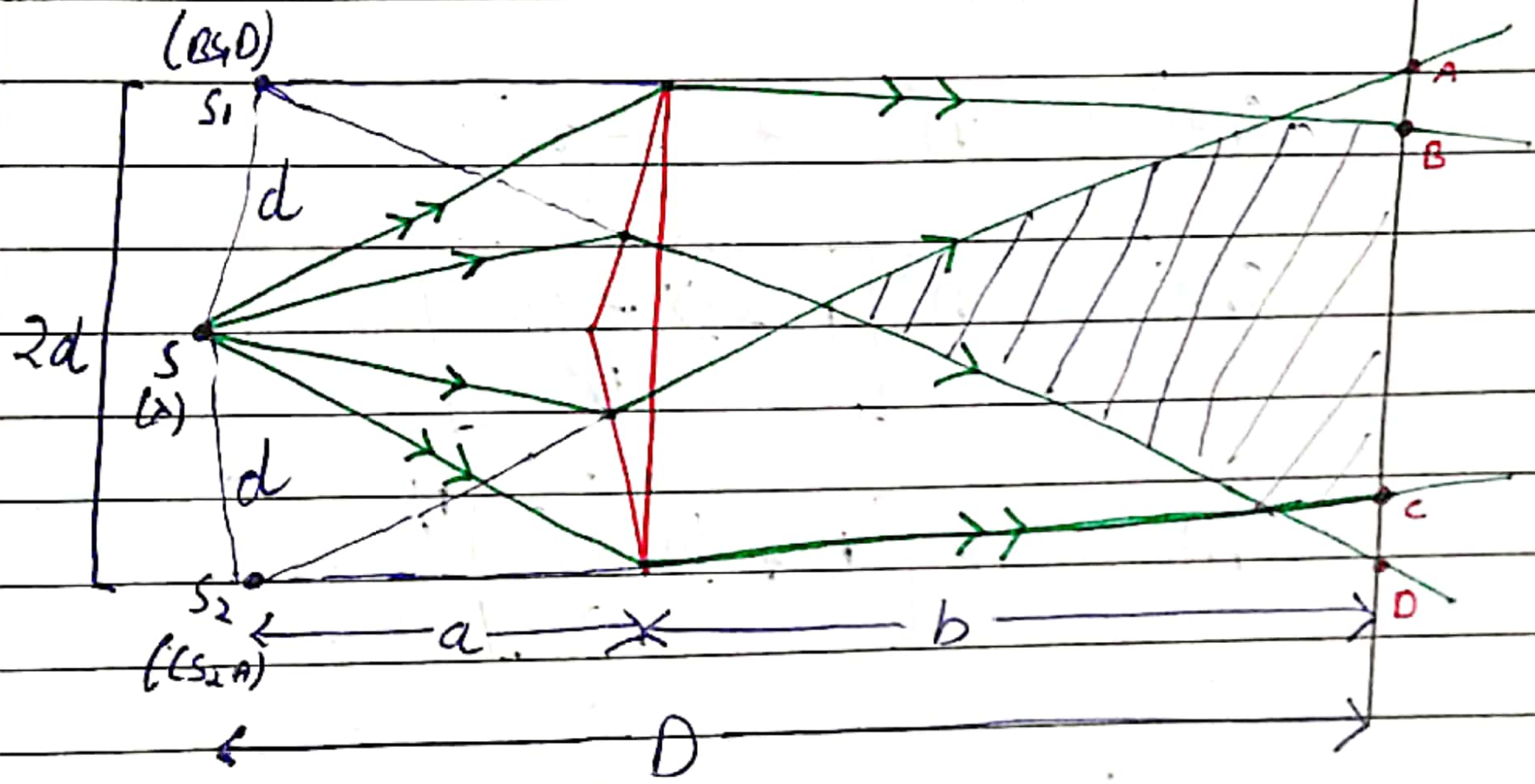


⇒ Ex :- Fresnel's Bi Prism Experiment

Optical Device



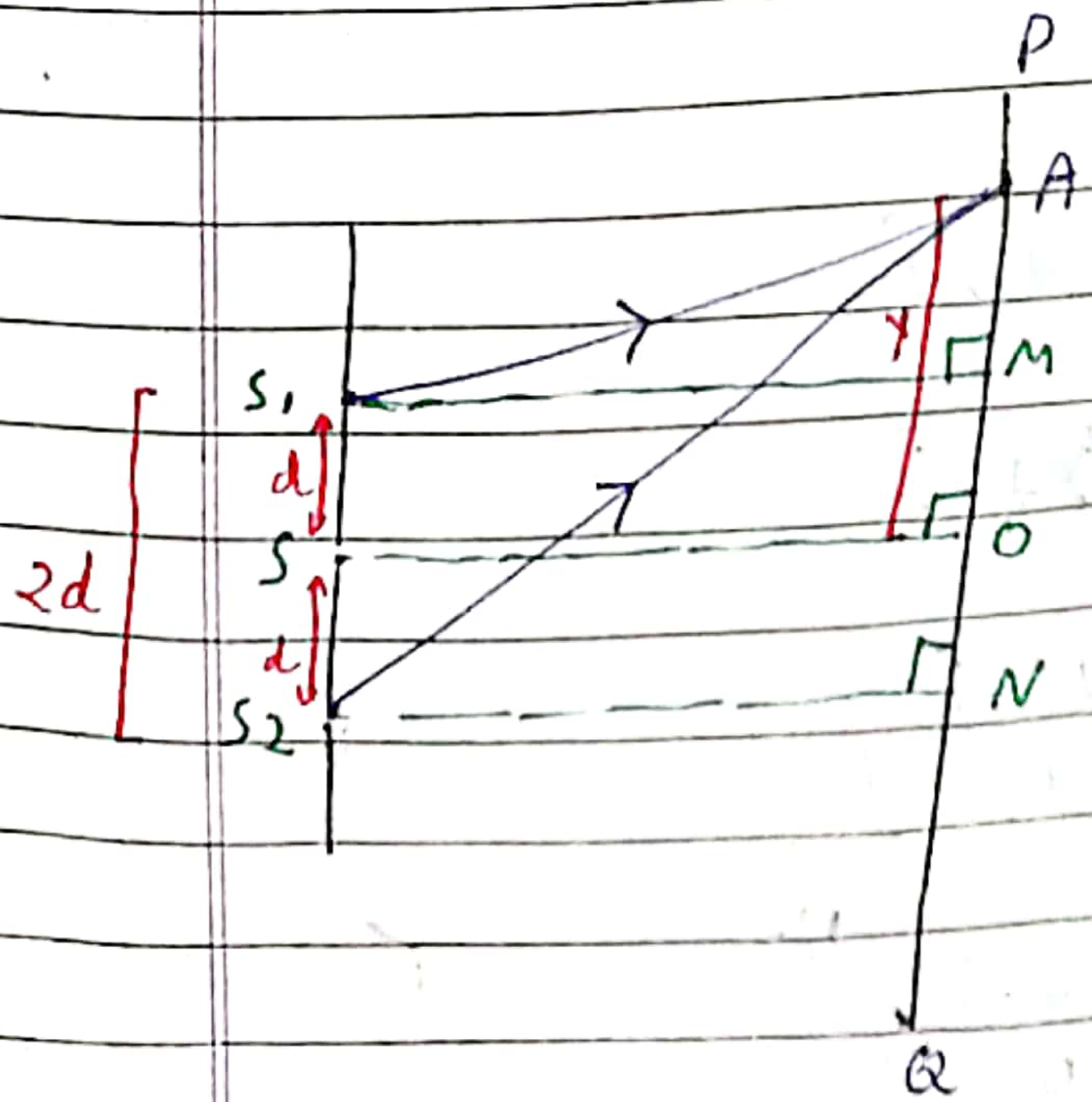
DIAGRAM



$D = a + b$

THEORY

To derive the Position of bright & dark fringes on screen & to determine the fringe width.



Path diff. b/w light rays reaching at Point A from Source A S_1 & S_2

$\Delta(\text{Path diff.}) = S_2A - S_1A$ —(i)

Draw $S_1M \perp PQ$
$SO \perp PQ$
$S_2N \perp PQ$

from

ΔS_1AM

$$(S_1A)^2 = (S_1M)^2 + (AM)^2$$

$$(S_1A)^2 = D^2 + (y-d)^2$$

$$(S_1A)^2 = D^2\left[1 + \frac{(y-d)^2}{D^2}\right]$$

$$S_1A = D\left[1 + \frac{(y-d)^2}{D^2}\right]^{1/2}$$

Also

$y-d$ is very small in comparison to D, so expanding by binomial theorem.

* $(1+x)^n = 1 + nx + (-- \; --) \rightarrow$ neglect

$$S_1A = D\left[1 + \frac{1}{2}\frac{(y-d)^2}{D^2}\right] \quad \text{— (ii)}$$

from

ΔS_2AN

$$(S_2A)^2 = (S_2N)^2 + (AN)^2$$

$$(S_2A)^2 = D^2 + (y+d)^2$$

$$(S_2A)^2 = D^2\left[1 + \frac{(y+d)^2}{D^2}\right]$$

$$S_2A = D\left[1 + \frac{(y+d)^2}{D^2}\right]^{1/2}$$

Again $y+d << D$

so, expanding binomially

$$S_2A = D\left[1 + \frac{1}{2}\frac{(y+d)^2}{D^2}\right] \quad \text{— (3)}$$

Now

Put (2) & (3) in eq (1)

$$\Delta = D\left[1 + \frac{1}{2}\frac{(y+d)^2}{D^2}\right] - D\left[1 + \frac{1}{2}\frac{(y-d)^2}{D^2}\right]$$

$$\Delta = D\left[\frac{1}{2D^2}\left((y+d)^2 - (y-d)^2\right)\right]$$

$$\Delta = \frac{y^2 + d^2 + 2yd - y^2 - d^2 + 2yd}{2D}$$

$$\Delta = \frac{4yd}{2D}$$

$$\boxed{\Delta = \frac{2yd}{D}} \quad \text{— (4)}$$

* For n^{th} bright fringe: $\Delta = n\lambda$ $[n = 0, 1, 2 \ldots]$

$$n\lambda = \frac{2yd}{D}$$

(Distance of n^{th} bright fringe from Centre) $\boxed{Y_n = \frac{nD\lambda}{2d}}$

* For n^{th} dark fringe:

$$\Delta = (2n-1)\frac{\lambda}{2}, \quad (2n-1)\frac{\lambda}{2} = \frac{2yd}{D}$$

$$\boxed{Y_n = \frac{(2n-1)\lambda D}{d}} \quad n = 1, 2 \ldots$$

* Fringe width :- It is the distance b/w two consecutive bright or dark fringe.

For bright fringe : If y_n & y_{n+1} be the distances of n^{th} & $(n+1)^{th}$ fringe from centre of screen then.

$$y_{n+1} = \frac{(n+1)D\lambda}{2d}, \quad y_n = \frac{nD\lambda}{2d}$$

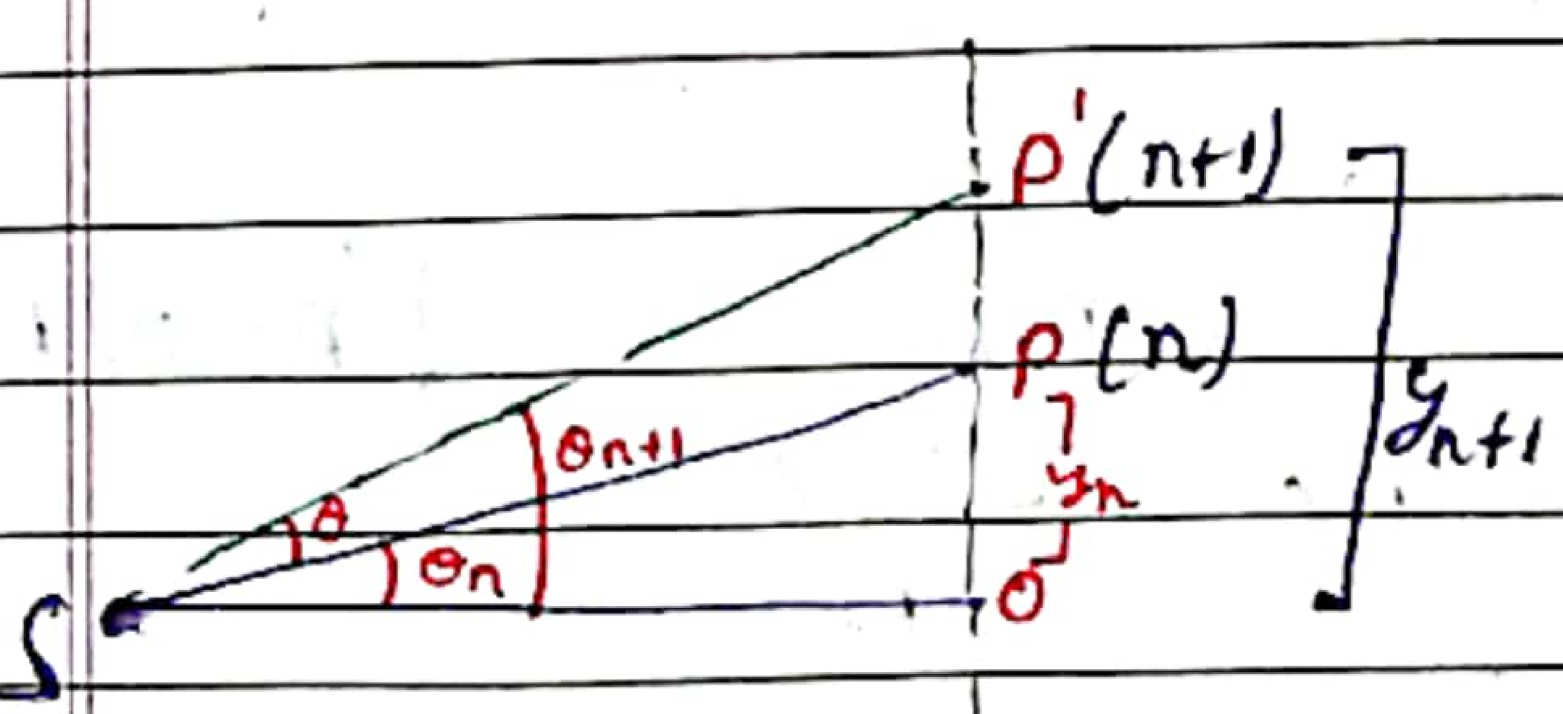
Then

$$\text{fringe width } (\beta) = y_{n+1} - y_n$$
$$= \frac{(n+1)D\lambda}{2d} - \frac{nD\lambda}{2d}$$

$$\boxed{\beta = \frac{D\lambda}{2d}}$$

* Angular fringe width (β_θ): Angular separation b/w two consecutive bright or dark fringes.

$$\beta_\theta = [\beta_{n+1} - \beta_n]_\theta$$

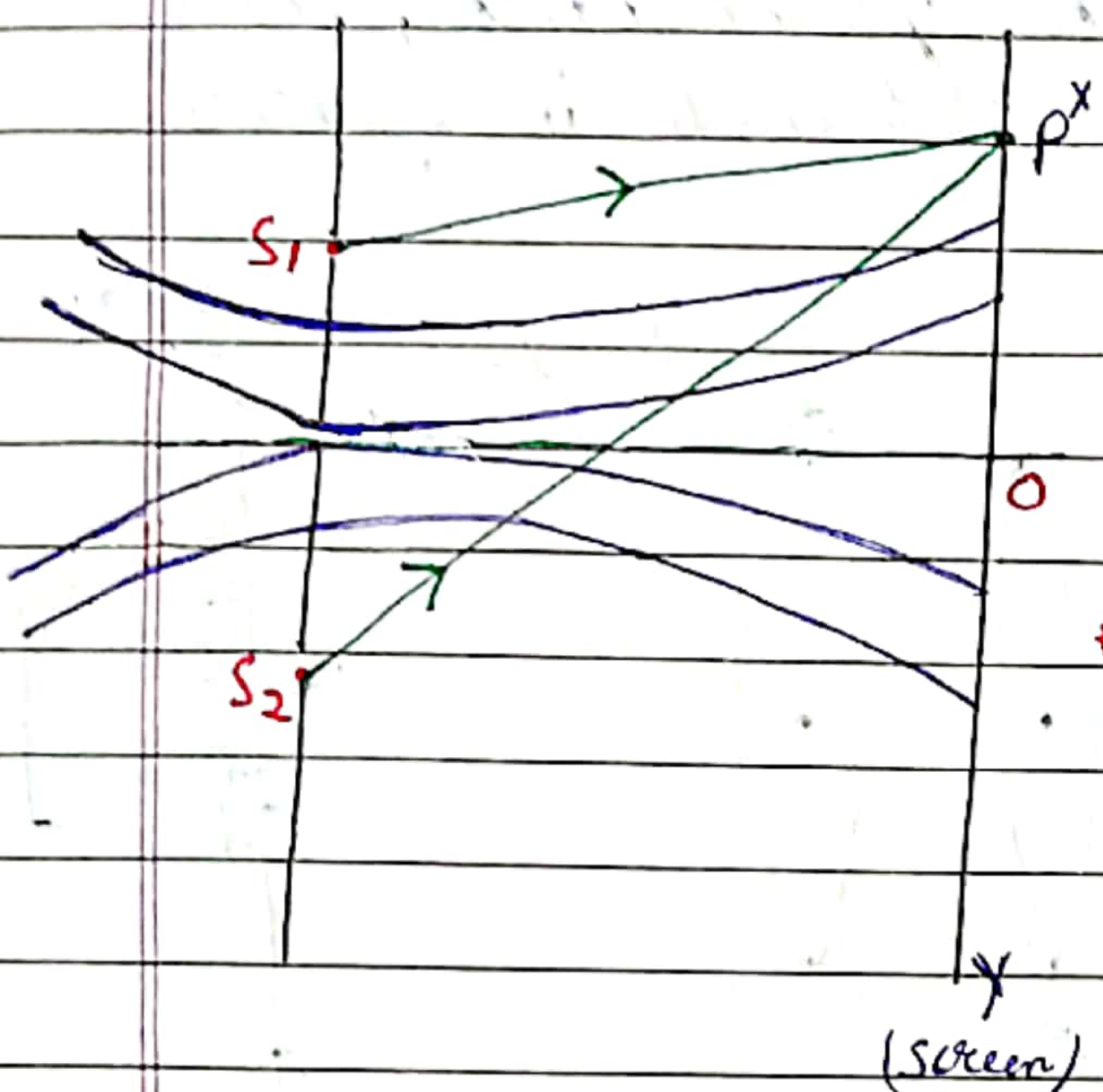


$$\beta_\theta = \frac{y_{n+1}}{D} - \frac{y_n}{D}$$

$$\boxed{\beta_\theta = \frac{\beta}{D} = \frac{\lambda}{2d}}$$

$$\tan\theta_n = \frac{y_n}{D} \simeq \theta_n \quad \text{as } y_n \ll D$$
$$\tan\theta_{n+1} = \frac{y_{n+1}}{D} \simeq \theta_{n+1} \quad \text{as } y_{n+1} \ll D$$

* Shape of Interference fringes in YDSE / Biprism



- Condition of maximum intensity
  $\Delta = S_2P - S_1P = n\lambda \quad n = 0, 1 \ldots$
- Condition of minimum intensity
  $\Delta = S_2P - S_1P = \frac{(2n-1)\lambda}{2} \quad n = 1, 2 \ldots$

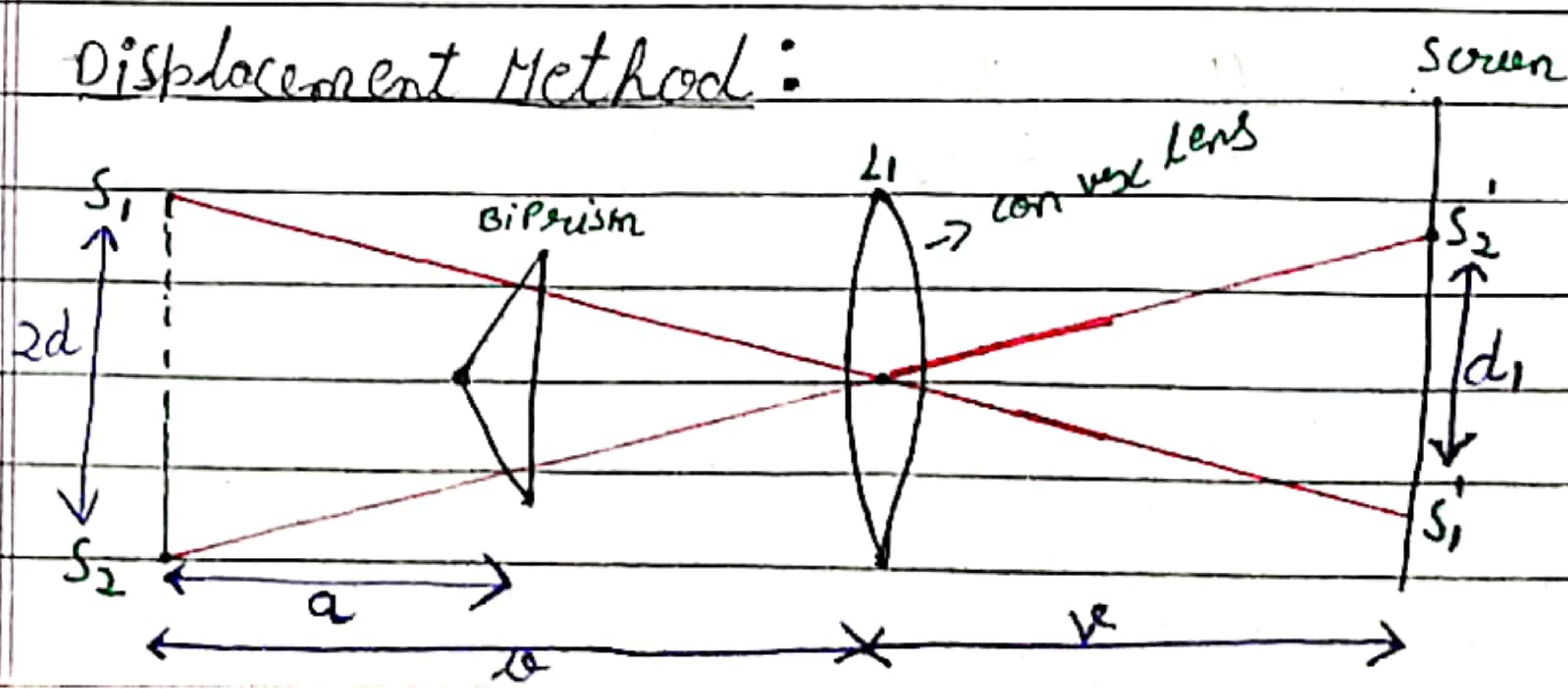
* For a given value of n, locus of Points of maximum / min. intensity is

$$\boxed{S_2P - S_1P = \text{Constant.}}$$

This is equation of Hyperbola with S_1 & S_2 as foci. But here ecentricity is very high $\{10^6\}$. So the Hyperbolic fringes on screen appear to be nearly straight lines. As λ is extremely small, $(S_2P - S_1P)$ is very small $(D >> 2d)$ & the fringe Pattern on screen consist of nearly Parallel, equidistant bright & dark lines.

⇒ Determination of 2d (Distance b/w S_1 & S_2)

(a) Displacement Method:



$u + v = D$

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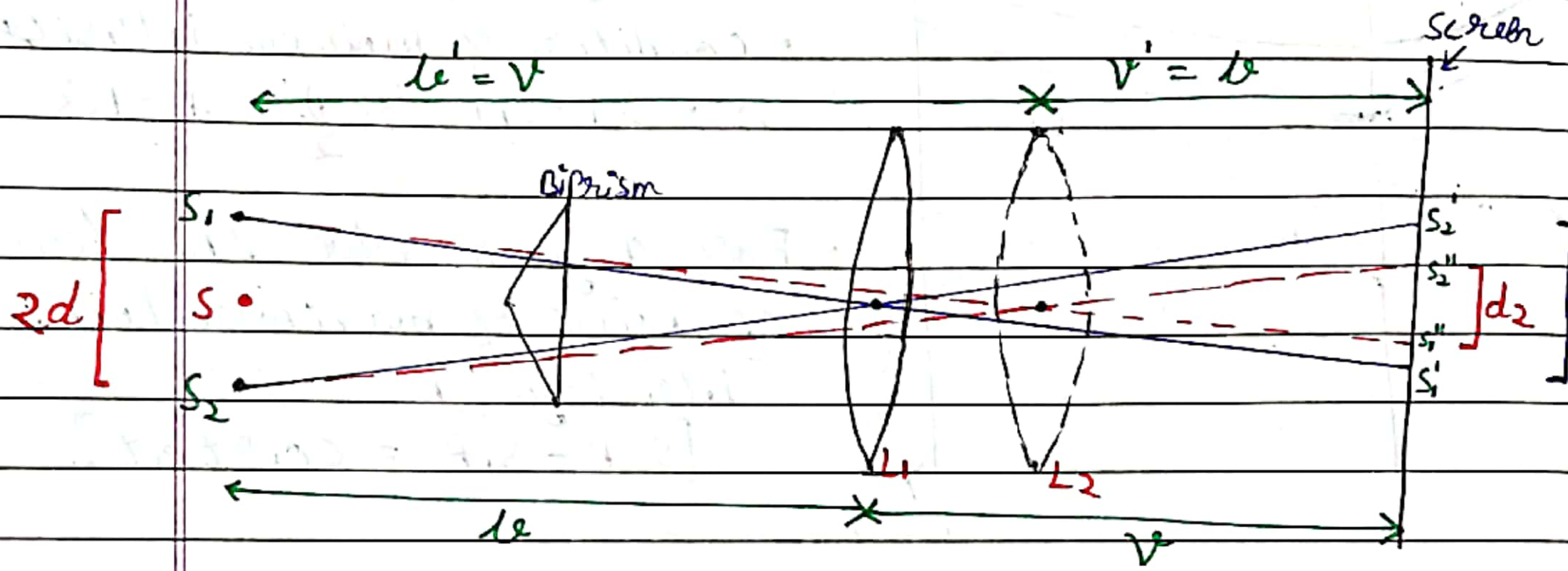
In this method a convex lens of short focal length is placed b/w biprism & screen. By moving lens L_1 along the optical bench, two positions L_1 & L_2 are obtained in eyepiece.



d_1 & d_2 = distance b/w images on screen

For lens L_1

$$\frac{v}{u} = \frac{d_1}{2d} \quad \text{—(i)}$$

For lens L_2

$$\frac{v'}{u'} = \frac{d_2}{2d}$$

$$\Rightarrow \frac{u}{v} = \frac{d_2}{2d} \quad \text{—(ii)}$$

eq (i) × (ii)

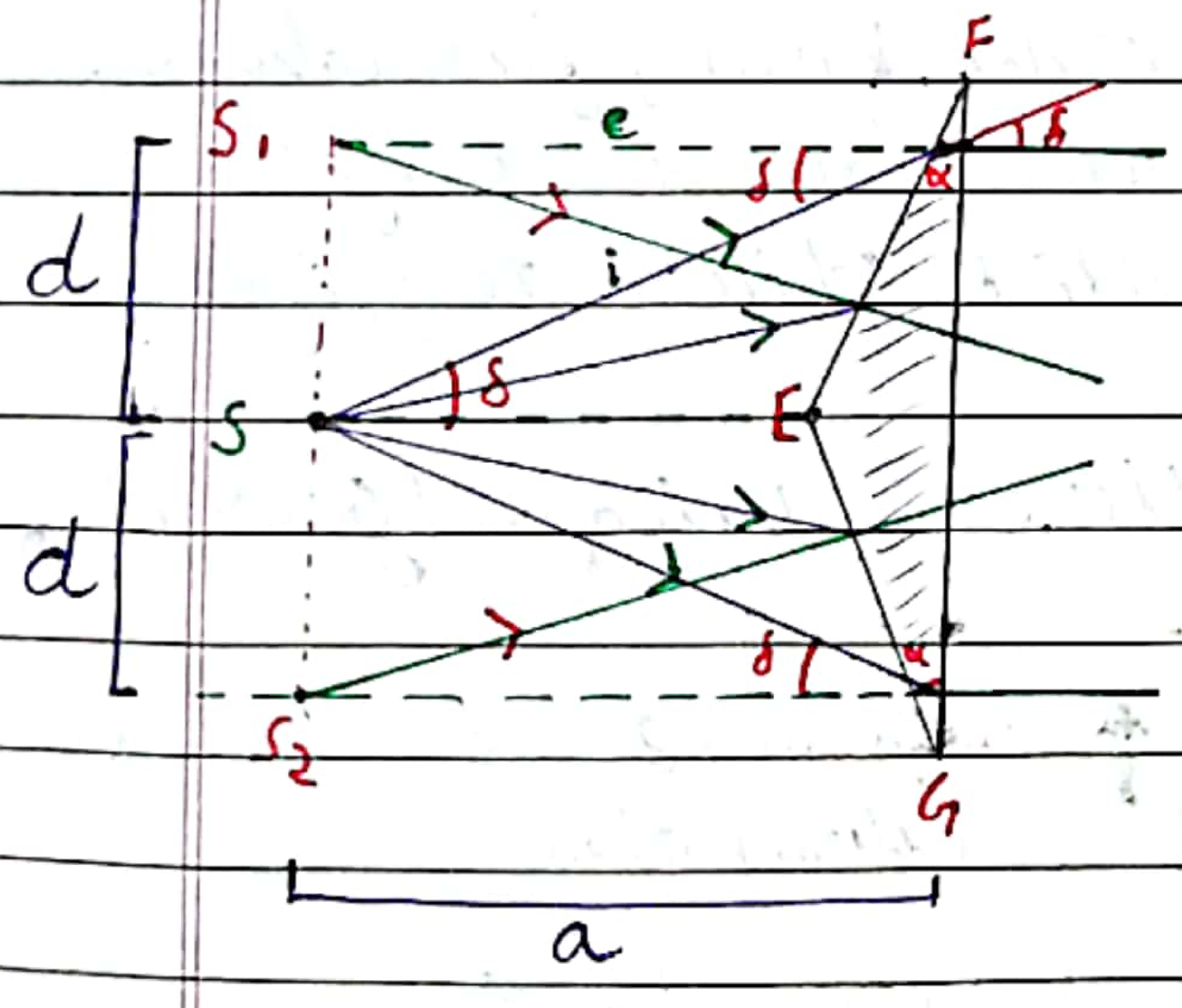
$$\frac{v}{u} \times \frac{u}{v} = \frac{d_1 \times d_2}{(2d)^2}$$

* $\boxed{2d = \sqrt{d_1 d_2}}$

(b) <u>Deviation Method:</u>

<u>For a Prism of very small refracting angle α, deviation produced in a ray is given by :-</u>

$\delta = (\mu - 1)\alpha$: • <u>where μ is the refractive Index of Prism.</u>
• α is Angle of Prism.



<u>Angle of deviation</u>

$$\delta = (\mu - 1)\alpha$$

$\tan\delta = \frac{d}{a}$ (In ΔSS_2G) : a is dist. b/w slit & Bi Prism.

As $d \lll a$; $\delta = \frac{d}{a}$

$$\frac{d}{a} = (\mu - 1)\alpha \Rightarrow d = a(\mu - 1)\alpha$$

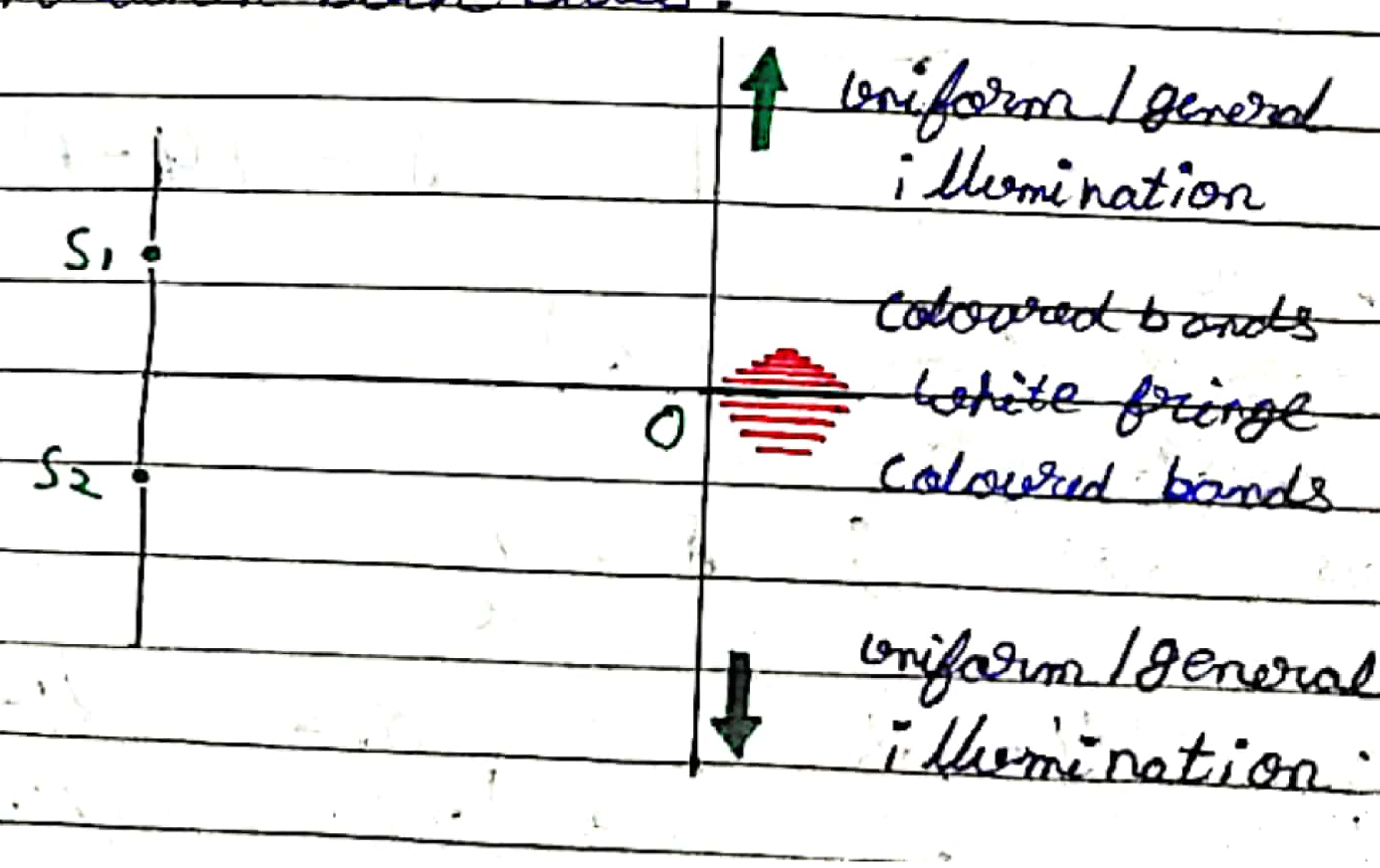
* $\boxed{2d = 2a(\mu - 1)\alpha}$

White Light fringes in BiPrism Experiment

- White Light :- It is a mixture of so many wave lengths varying from 4400 Å (violet) to 7500 Å (Red).

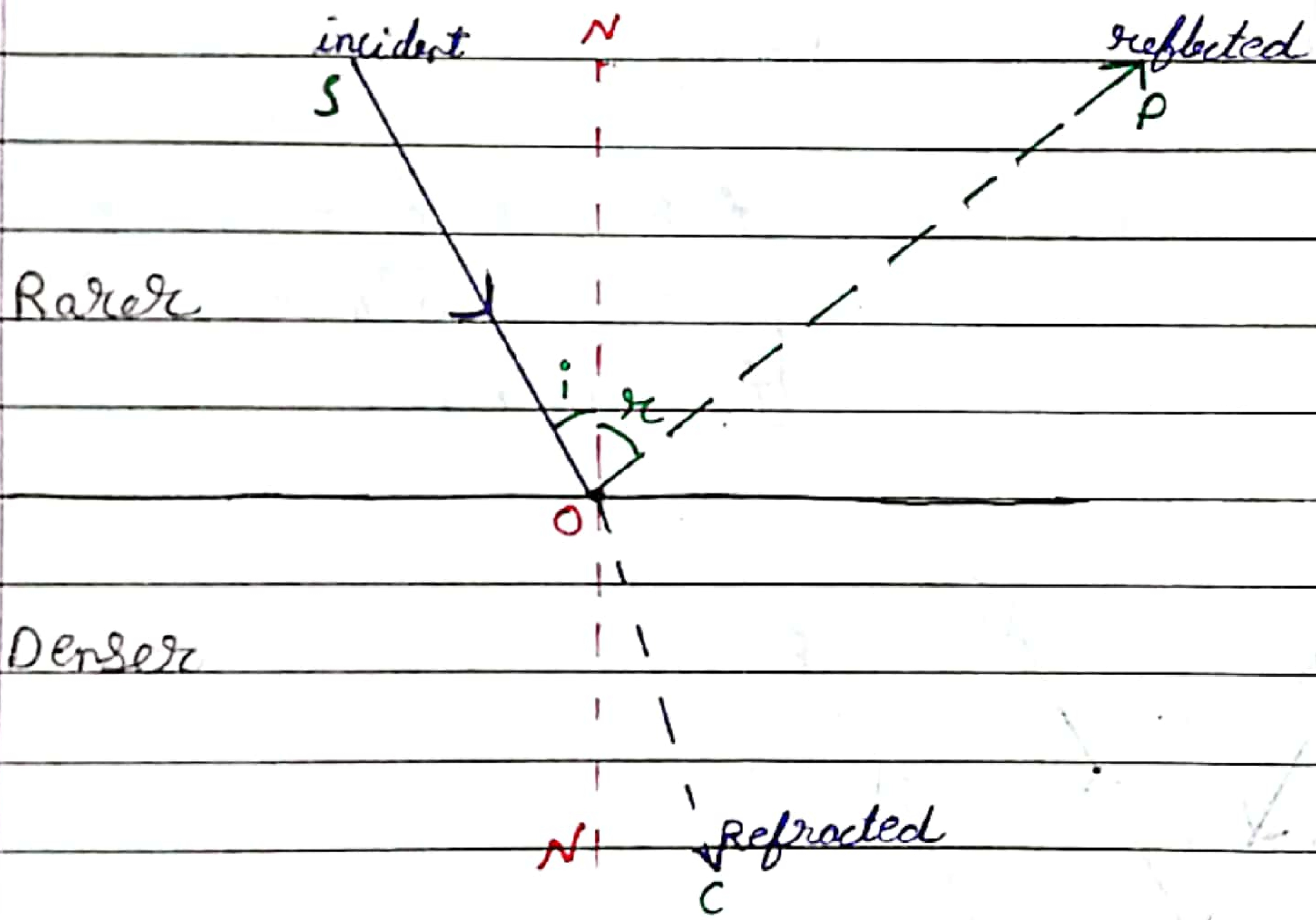
⇒ When Monochromatic source of light is replaced by White light in BiPrism experiment then according to formula $\beta = \frac{\lambda D}{2d}$, each Pair of Monochromatic source of white light Produces its own system of fringes with different fringe widths. The resultant Pattern Consists of :-

(i) Zero order band which is white.
(ii) On both sides of Zero order band, few coloured fringes are observed. In coloured fringes, the inner ends are reddish while outer edges are violet.
(iii) After coloured fringes, there is uniform / General illumination both sides.



Change of Phase on Reflection : STOKE'S TREATMENT

When a light ray get reflected in rarer medium from the surface of denser medium, Additional Path change(Δ) of $\lambda/2$ or Phase change (ϕ) of π takes Place.



$$\Delta = \underbrace{SO + OP}_{\lambda} \pm \lambda/2 \rightarrow \text{Additional Path Change}$$

$$\Delta = \frac{3\lambda}{2}, \frac{\lambda}{2}$$

$$\phi = \pi$$

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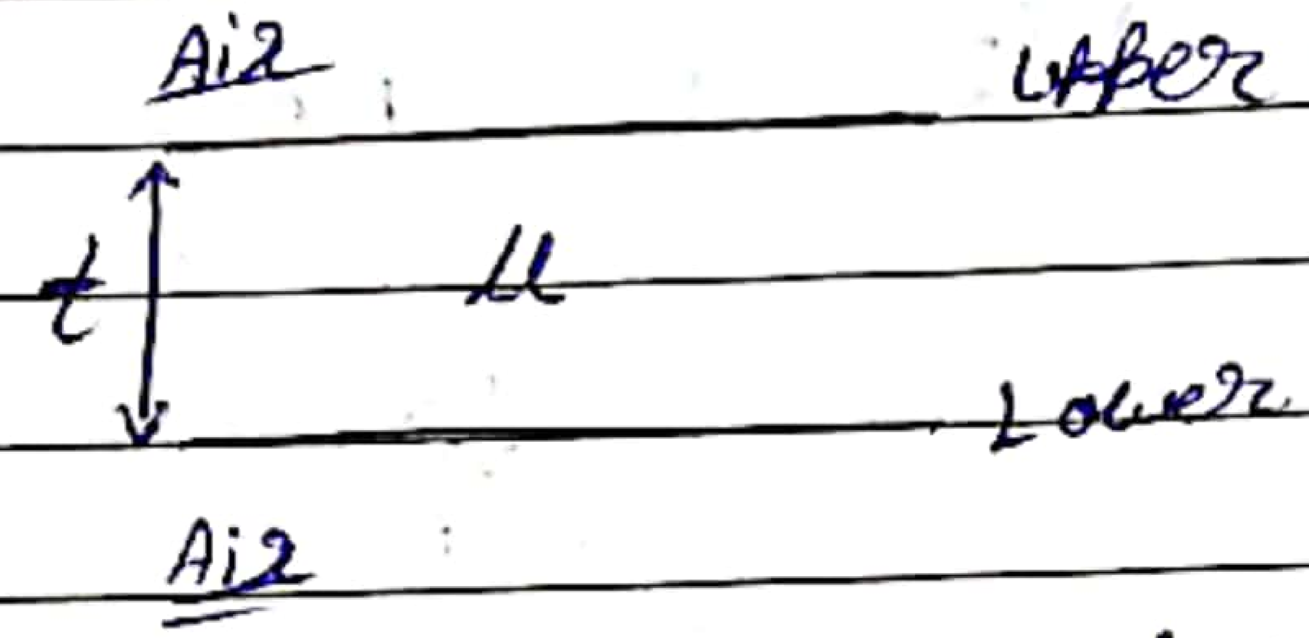
⇒ Division of Amplitude

(i) Interference in Parallel Sided thin film.

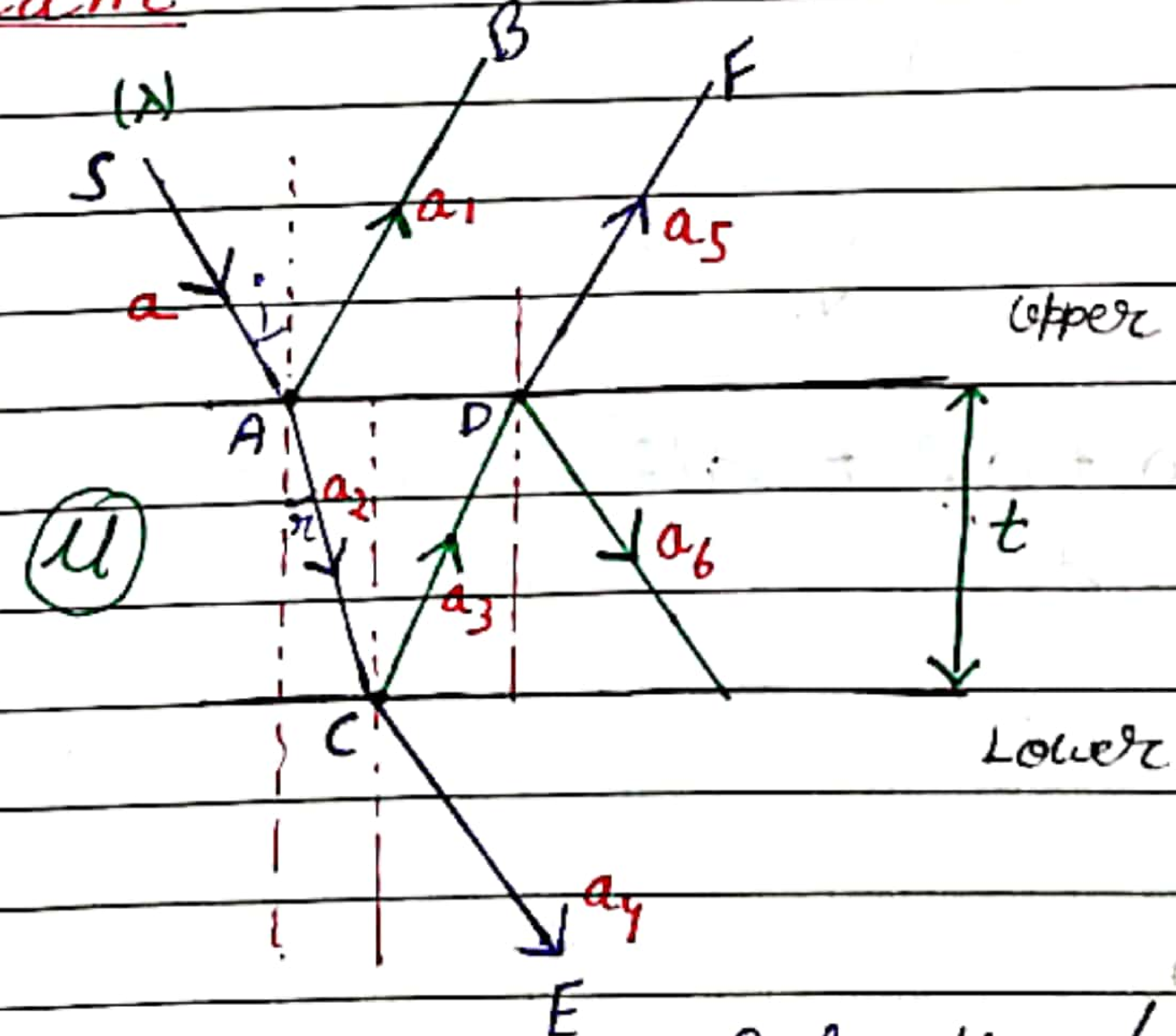


$t \approx 4000\,\mathring{A} - 8000\,\mathring{A}$

$\approx 4\times10^{-7}m - 8\times10^{-7}m$

$\boxed{t \approx 10^{-7}m}$

Diagram



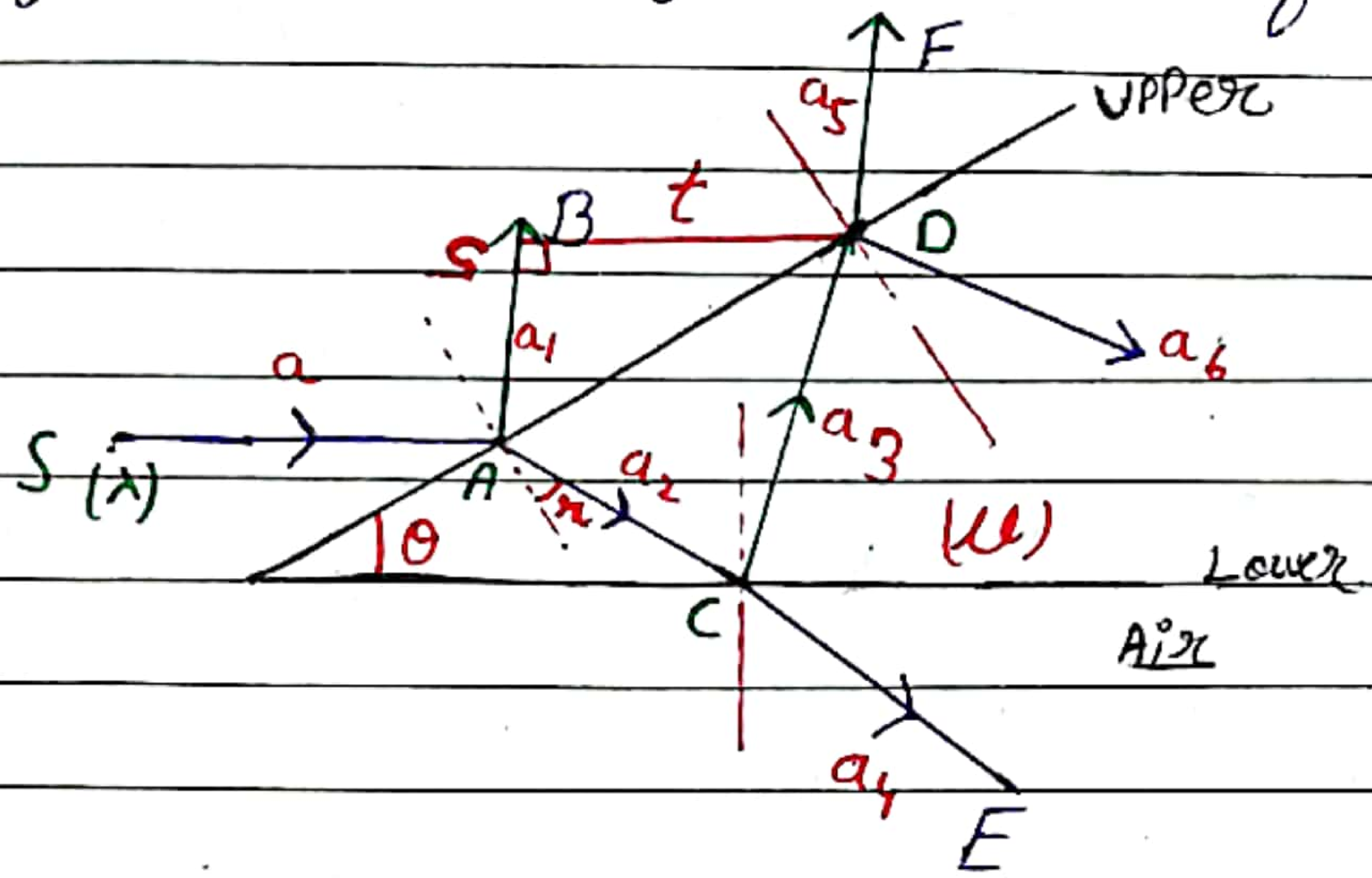
Refraction (Transmitted light)

$\Delta = 2\mu t\cos(r)$

From Stoke's Law

$$\boxed{\Delta = 2\mu t\cos r \pm \lambda/2}$$

(ii) Interference in wedge shaped thin film



: θ is angle of wedge

Partial Reflection & Refraction

Path difference

$$\Delta_{AB} = SA + AG$$
$$\Delta_{DF} = SA + AC + CD$$

$$\Delta = SA + AC + CD - (SA + AG)$$

$$\boxed{\Delta = 2\mu t \cos(r+\theta)}$$

μ: is refractive index of Denser Medium
t: is $\perp^r$ distance (DG)
r: is refracted ray angle with normal

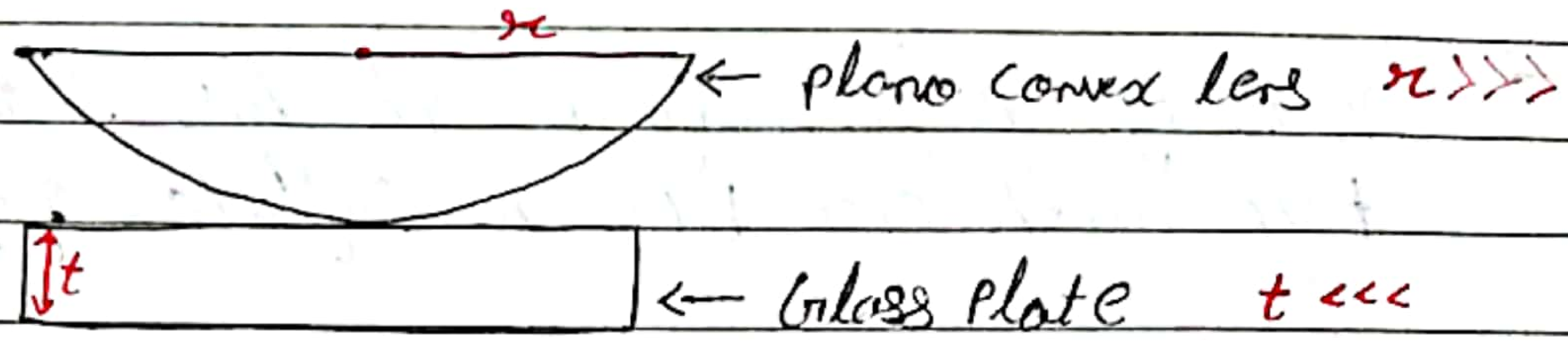
From Stoke's Law

* $$\boxed{\Delta = 2\mu t [\cos(r+\theta)] \pm \lambda/2}$$

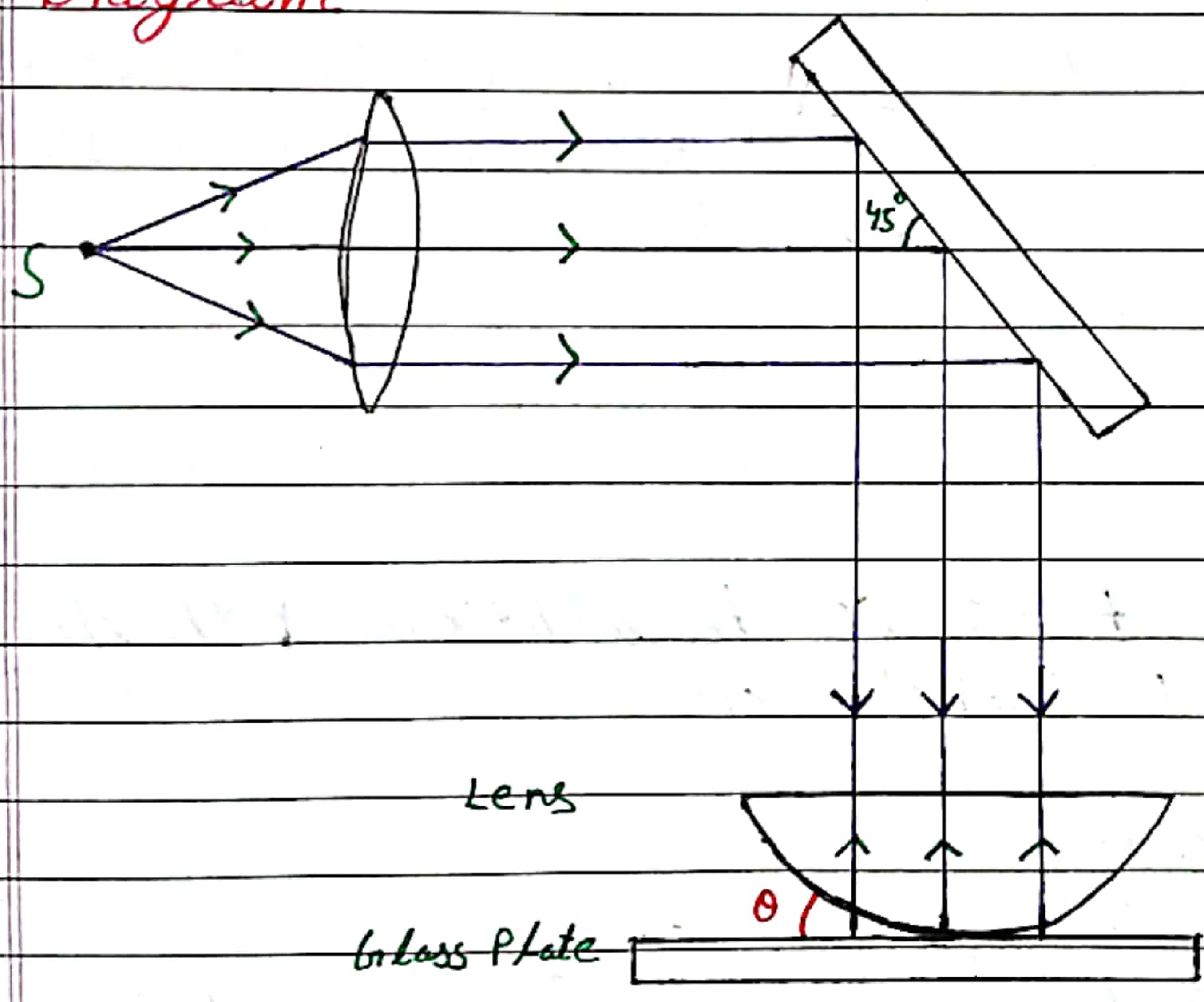
Newton's Ring Experiment

(i) Main equipment

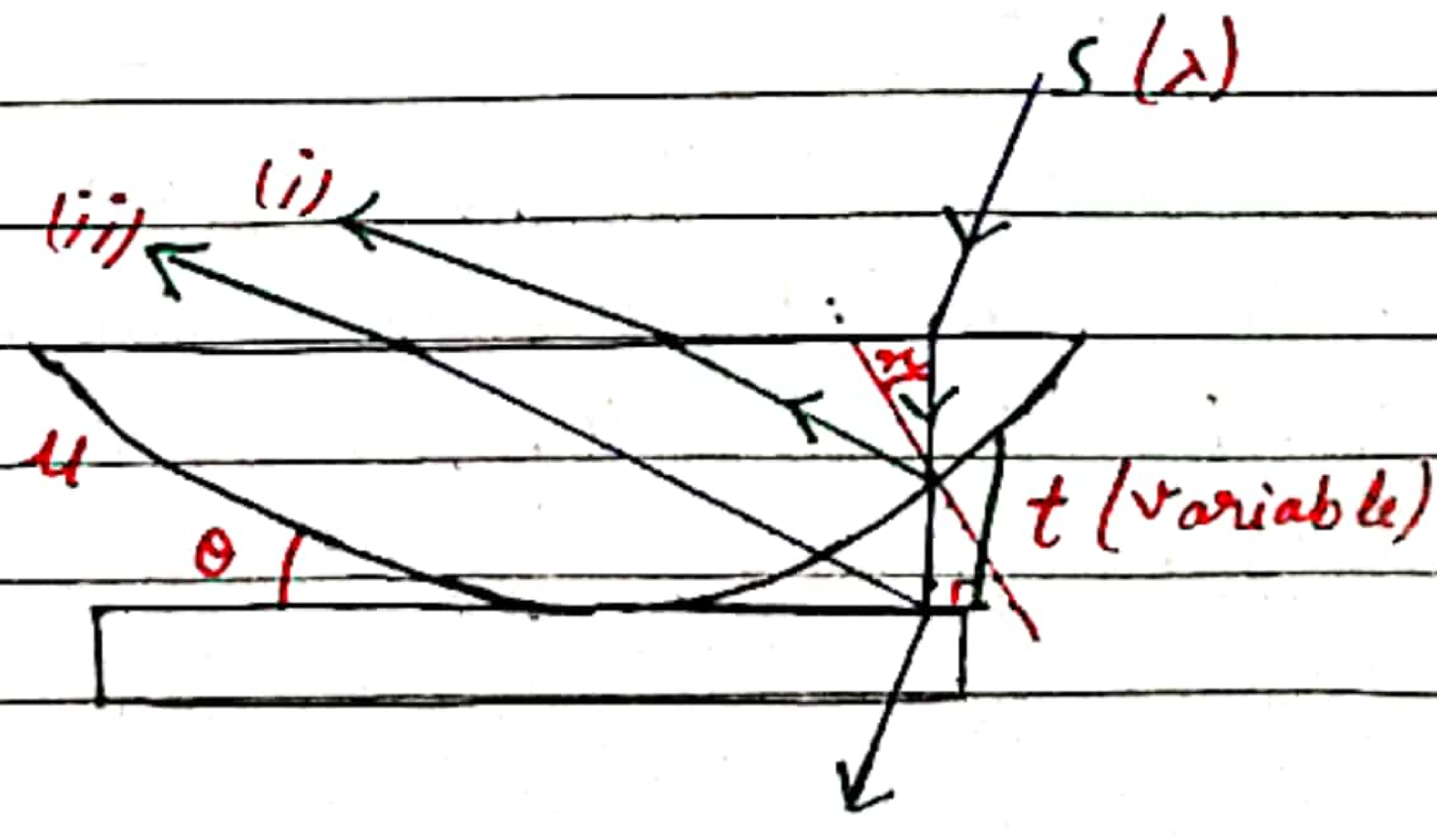
A Plano Convex lens & a glass Plate.



(ii) Diagram



(iii) Phenomenon



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(iv) Theory

In newton ring experiment interference occurs b/w the light rays reflected from lower surface of Plano convex lens & upper surface of Glass Plate.

As wedge is forming b/w the lens & the glass Plate, Path difference b/w the reflected rays is :-

$$\Delta = 2\mu t \cos(r+\theta) \pm \lambda/2 \quad -(i)$$

μ = Refractive index of medium b/w lens & glass Plate
t = variable thickness or $\perp^r$ distance b/w lens & glass Plate.
r = angle of refraction
θ = angle of wedge
λ = wavelength of light

As, Here $\pm \lambda/2$ is the additional Path change due to Stoke's Law.

=> In exp. setup:

Angle of incidence $(i) = 0^\circ$ (Normal incidence)
then,
- Angle of reflection $(r) = 0^\circ$
- As radius of curvature of Plano convex lens is very large So θ is very small.

So,

$$\text{for } \cos(\pi + \theta) = \cos\theta \approx 1$$

Then eq (i) become

$$\Delta = 2\mu t \pm \lambda/2 \quad -(ii)$$

⇒ Condition of Maxima

$$\Delta = n\lambda \qquad n = 0, 1, 2 \ldots$$

$$2\mu t + \lambda/2 = n\lambda$$

$$\boxed{2\mu t = (2n-1)\lambda/2} \qquad n = 1, 2, 3 \ldots$$

⇒ Condition of Minima

$$\Delta = (2n-1)\lambda/2 \qquad n = 1, 2, 3 \ldots$$

$$2\mu t - \lambda/2 = (2n-1)\lambda/2$$

$$\boxed{2\mu t = n\lambda} \qquad n = 1, 2, 3 \ldots$$

* Why center is Dark?

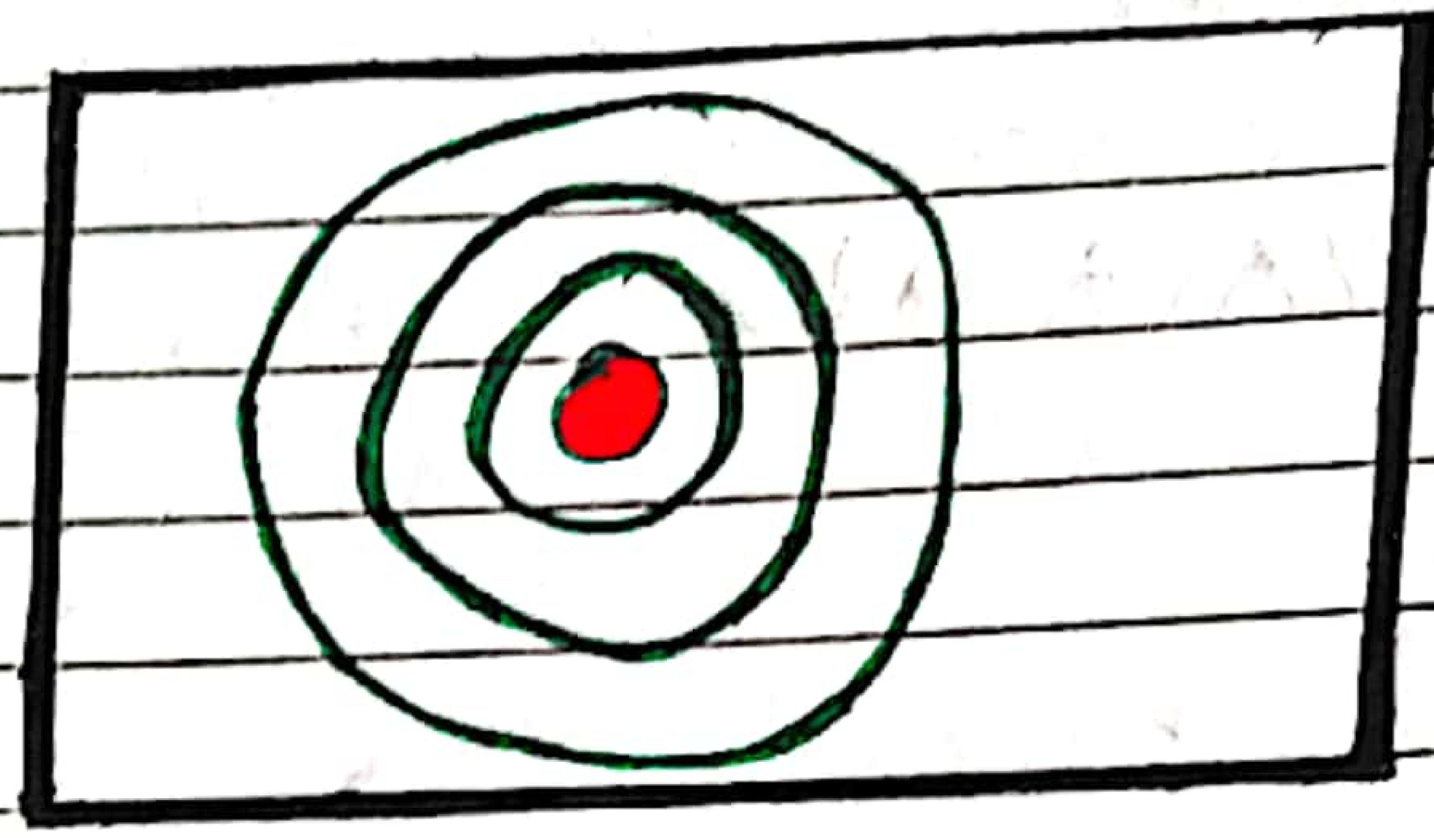
Soln ⇒ At Point of contact of Plano convex lens & glass Plate.

$$t = 0.$$

$$\Delta = 2 \times \mu \times (0) \pm \lambda/2$$

$$\Delta = \pm \lambda/2 \quad (\text{Dark}) \Rightarrow (2n-1)\frac{\lambda}{2} \quad (n=1)$$

(v) Shape of fringes :



The Path difference between the reflected ray & incident ray depends upon the thickness of the air gap between lens & the base. As the lens is symmetric along its axis, the thickness is constant along the circumference of a ring of a given radius. Hence, Newton's rings are circular.

(vi) Calculation

A.c to geometrical Property of circle we have,

$$AC \times CB = OC \times CD$$

$$(r_n)^2 = t(2R-t)$$

$$= 2Rt - t^2$$

Since

$e << R$ $\therefore$ e^2 is negligible

$$(r_n)^2 = 2tR$$

$$\therefore \boxed{t = \frac{(r_n)^2}{2R}}$$

* Maxima

$$2\mu t = (2n-1)\frac{\lambda}{2}$$

$$2\mu\left(\frac{r_n^2}{2R}\right) = (2n-1)\frac{\lambda}{2}$$

$$r_n^2 = (2n-1)\frac{R\lambda}{2\mu}$$

$$\boxed{\begin{gathered} D_n^2 = 2(2n-1)R\lambda \\ D_n \propto \sqrt{2n-1} \end{gathered}}$$: For air $\mu = 1$

* Minima

$$2\mu t = n\lambda$$

$$\cancel{2}\mu\frac{(r_n)^2}{\cancel{2}R} = n\lambda$$

$$\boxed{\begin{gathered} (D_n)^2 = 4nR\lambda \\ D_n \propto \sqrt{n} \end{gathered}}$$: For air $\mu = 1$

⇒ Wave length of Newton Ring

We know that, $(D_n)^2 = 4n\lambda R$ [For Minima]

∴ for $(n+P)^{th}$ ring

$$(D_{n+P})^2 = 4(n+P)\lambda R$$

$$(D_{n+P})^2 - (D_n)^2 = 4P\lambda R$$

$$\boxed{\lambda = \frac{(D_{n+P})^2 - (D_n)^2}{4PR}}$$

Diffraction

The Phenomenon of bending of light round the corners of any obstacle or aperture when the size of the obstacle or aperture is of the order of wavelength of the light used is called diffraction.

Diffraction is mainly of two types:-

i) Fresnel's diffraction
ii) Fraunhofer's diffraction

⇒ Fresnel's diffraction :- In Fresnel's diffraction, the source of light & the screen are kept at finite distance from the diffracting obstacle or aperture.

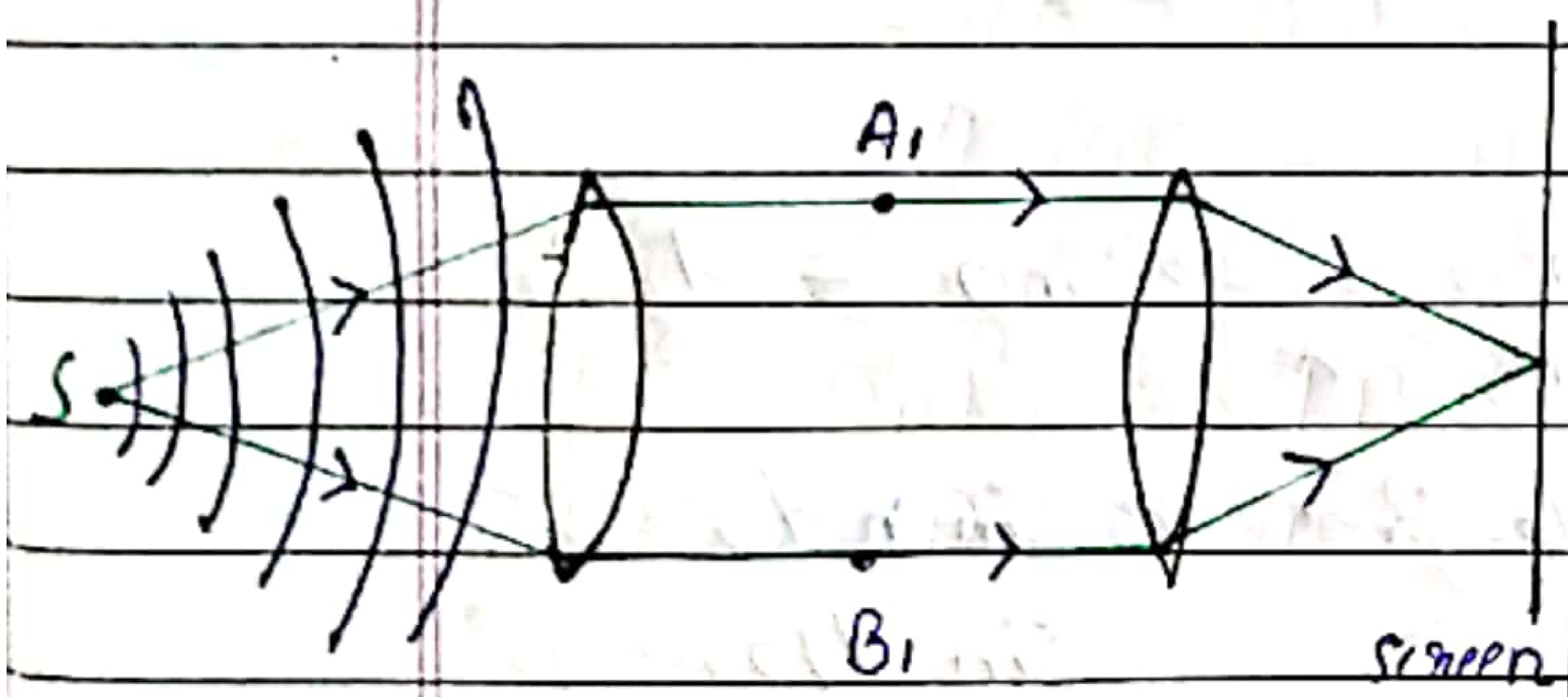
⇒ Fraunhofer's diffraction :- In Fraunhofer's diffraction the source of light & the screen are effectively at infinite distance from the diffracting obstacle or aperture.

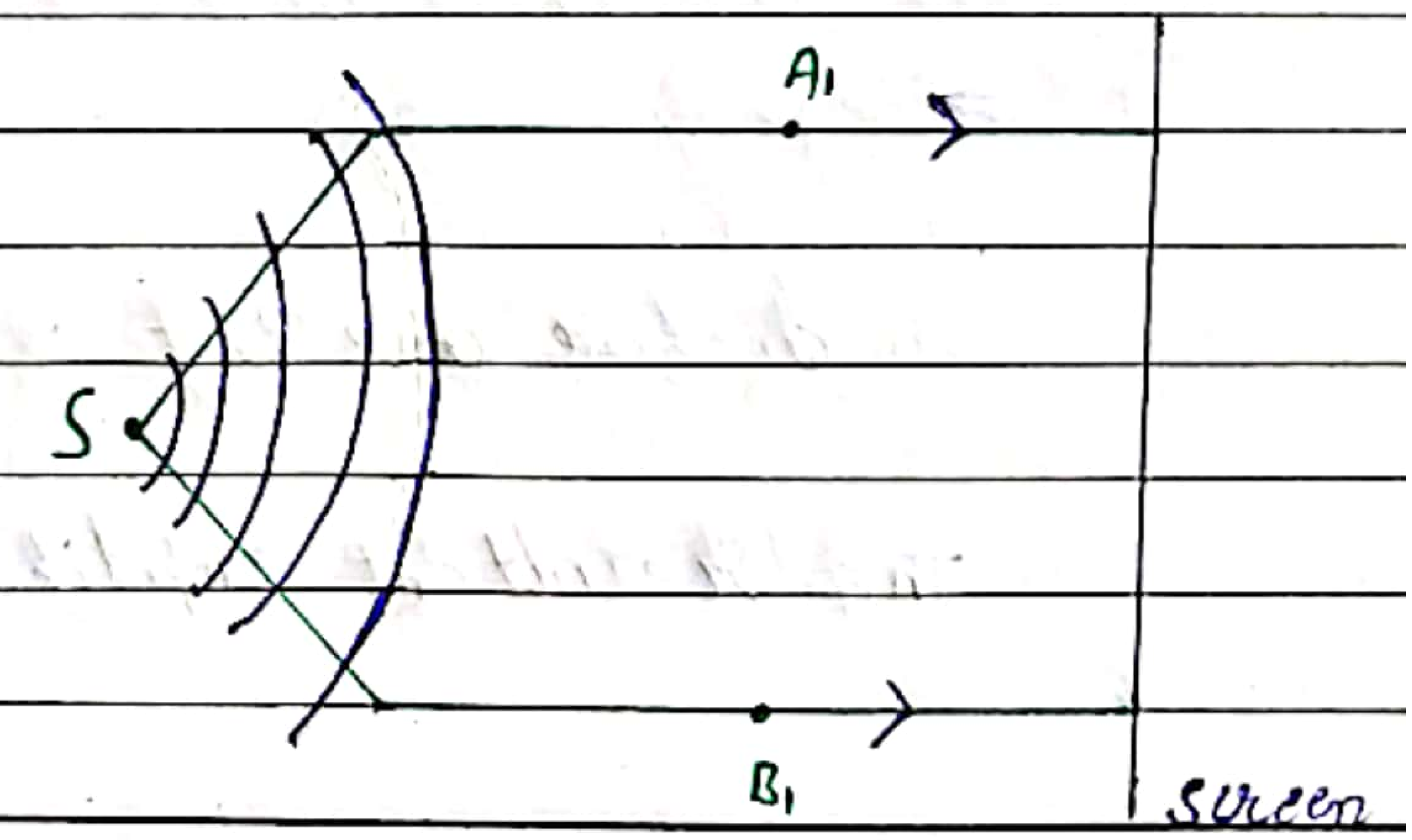
The distribution of bright & dark fringes in diffraction is called diffraction Pattern.

Difference b/w Fresnel & Fraunhofer diffraction

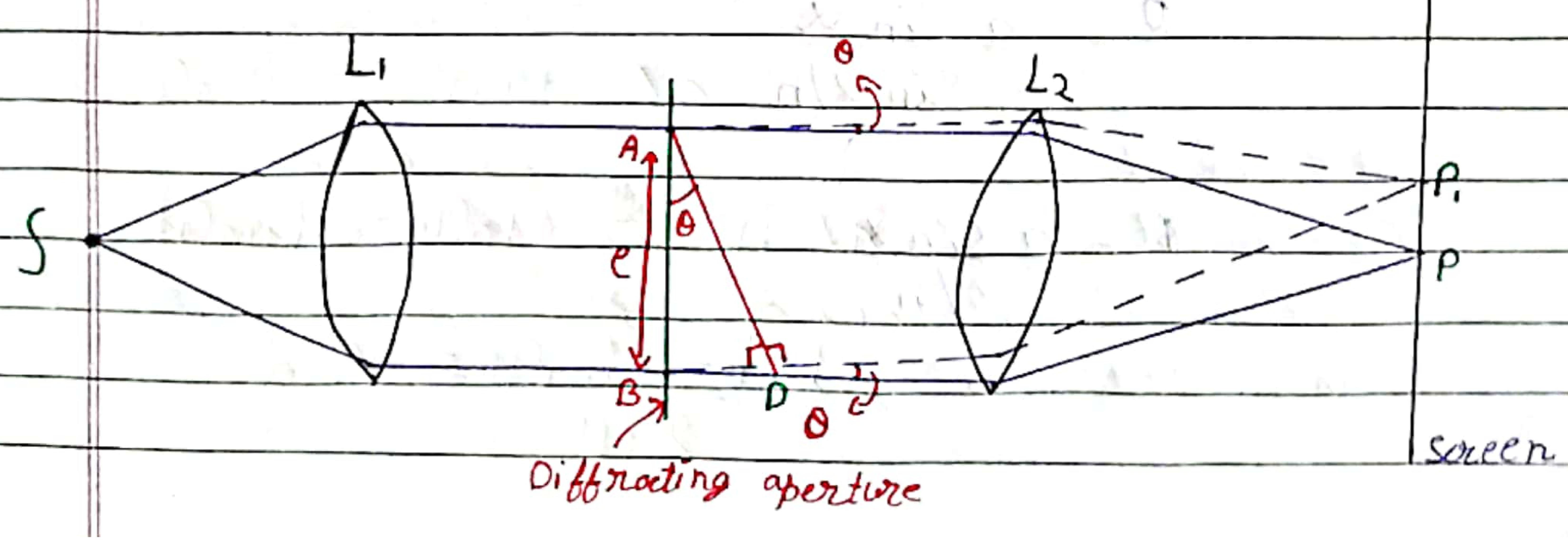
Fraunhofer's	Fresnel's
i) Source of light & Screen are at infinite distance	i) source & screen are at finite distance from the

from diffracting bstacle.	diffracting obstacle.
ii) Two convex lenses are used.	ii) No lens is used.
iii) Incident wavefront is plane wavefront	iii) Incident wavefront is either spherical or cylindrical.
iv) Centre of diffraction pattern is always bright.	iv) Centre ~~to~~ may be dark or bright.
v) Single slit, double slit or N slit (grading) is used for diffraction pattern	v) Fresnel's zone is used to get diffraction zone.





⇒ Fraunhofer's Diffraction due to single slit



⇒ Derivation

AD is $\perp^{r}$

ΔBAD

$$\sin\theta = \frac{BD}{AB} = \frac{BD}{e} \Rightarrow BD = e\sin\theta$$

$$\Delta = e\sin\theta$$

$$\delta = \frac{2\pi}{\lambda}\Delta \text{ (Path difference)}$$

$$= \frac{2\pi}{\lambda} e\sin\theta$$

* Slit is of n Parts

$$\delta \text{ for one Part} = \frac{1}{n}\cdot\frac{2\pi}{\lambda} e\sin\theta = d$$

The resultant amplitude $R \Rightarrow \frac{a\sin nd/2}{\sin d/2}$

$$= \frac{a\sin(\pi e\sin\theta/\lambda)}{\sin(\pi e\sin\theta/n\lambda)}$$

$$R = \frac{a\sin\alpha}{\sin\alpha/n}$$; where $\alpha = \frac{\pi e\sin\theta}{\lambda}$

$$R = \frac{a\sin\alpha}{\alpha/n}$$; $\frac{\alpha}{n}$ is very small

$\sin\theta \approx \theta$

$$R = na\frac{\sin\alpha}{\alpha} = \boxed{A\frac{\sin\alpha}{\alpha}} \quad ; A = na$$

$$\boxed{I = R^2 = \left(A\frac{\sin\alpha}{\alpha}\right)^2} \quad ; \alpha = \frac{\pi}{\lambda} e\sin\theta$$

$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad ; I_0 = A^2$$

* Minima

$$\sin\alpha = 0$$

where $\alpha = \pm m\pi \quad ; m = 1, 2, 3 \ldots$

$$e\frac{\pi}{\lambda}\sin\theta = m\pi$$

$$\boxed{e\sin\theta_n = \pm m\lambda} \quad ; m = 1, 2, 3 \ldots$$

$m \neq 0$

* Maxima

Secondary maxima:-

$$\frac{\sin\alpha}{\alpha} = 1$$

$$\alpha = \pm\frac{(2n+1)}{2}\pi \quad ; n = 1, 2, 3 \ldots$$

$$\boxed{e\sin\theta_n = \pm\frac{(2n+1)\lambda}{2}}$$

* Central maxima

$$\alpha = 0$$

$$\boxed{\sin\theta = 0}$$

$$\theta = 0$$

Diffraction due to n slits (Diffraction grating)
(Transmission grating)

Width of one grating $= e + d$

We know that

$$\delta = \frac{2\pi}{\lambda} \times \Delta \text{ (Path diff.)}$$

then,

$$\delta = \frac{2\pi}{\lambda}(e+d)\sin\theta = 2\beta$$

Let the Phase diff. of any two Consecutive grating is 2β.

Amplitude of single grating $= \frac{A \sin\alpha}{\alpha} = a$

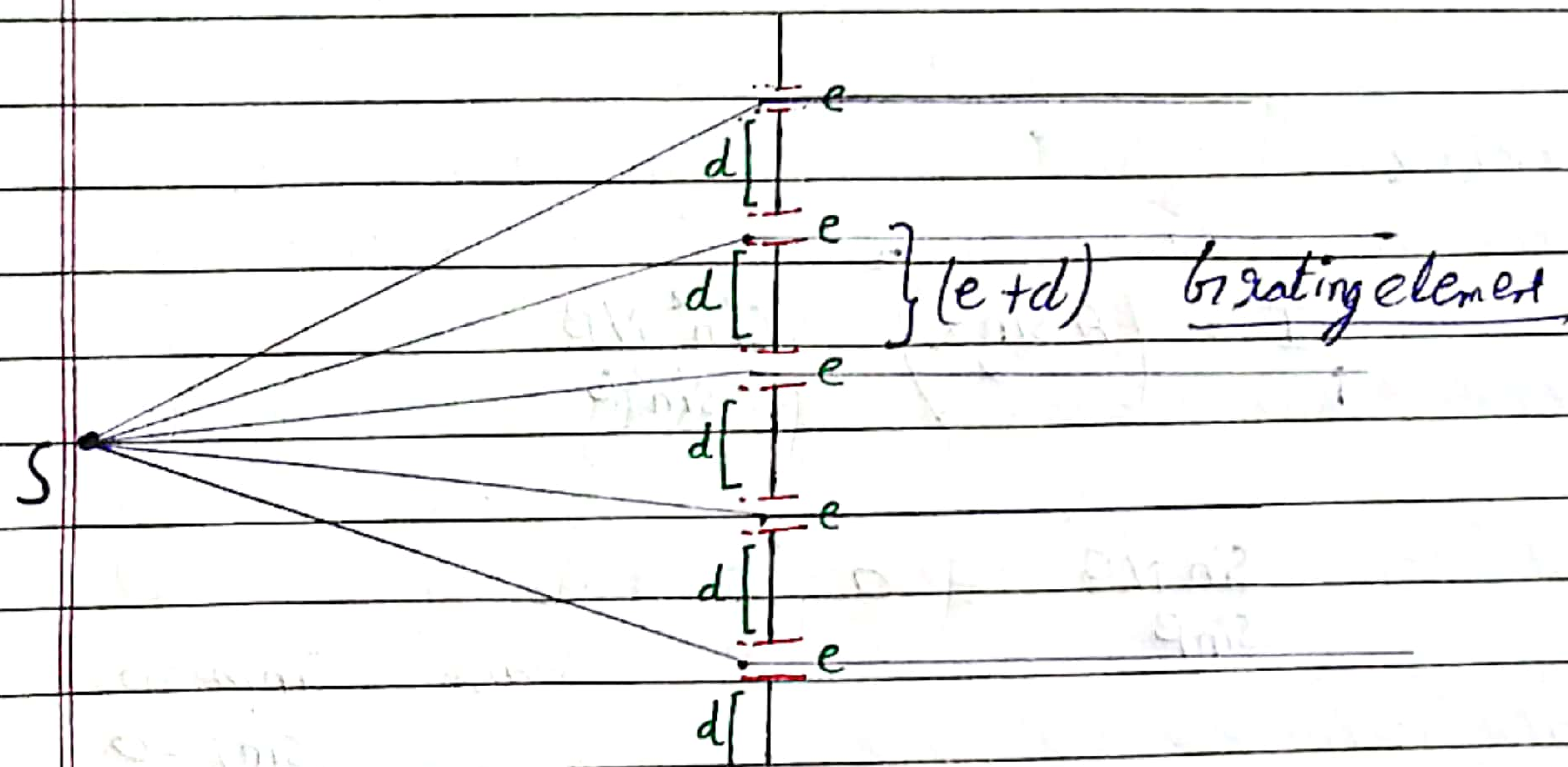
Amplitude due to N grating $= \frac{A\sin\alpha}{\alpha} \cdot \frac{\sin N\beta}{\sin\beta}$

$$\boxed{I = \left(\frac{A\sin\alpha}{\alpha}\right)^2 \cdot \frac{\sin^2 N\beta}{\sin^2\beta}}$$

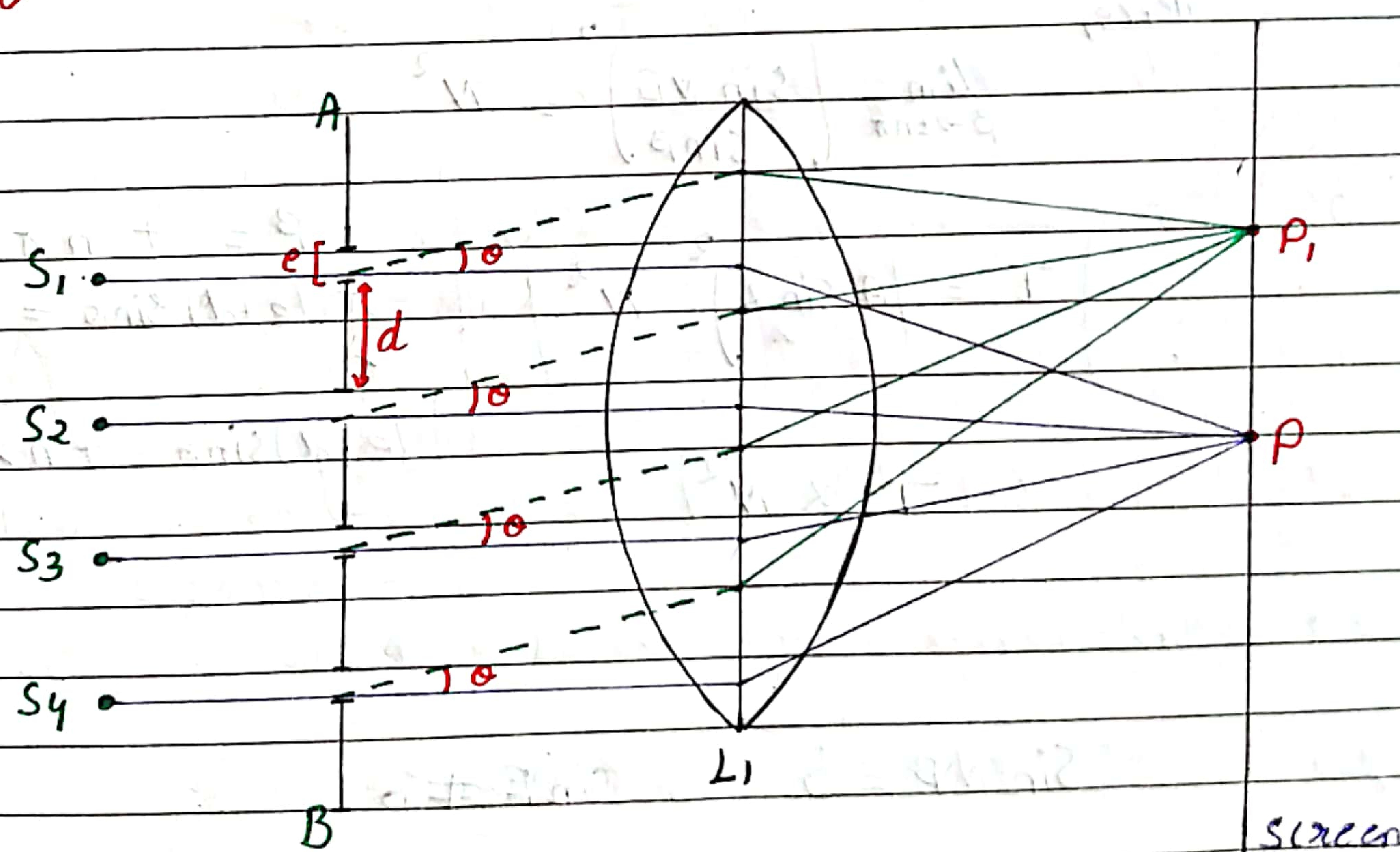
Theory

A Plane transmission grating placed perpendicular to the plane of paper. e be the width of each slit & d the width of each opaque part. Then $(e+d)$ is grating element.

Date :



Diagram



* **Maxima**

$$I = \left(\frac{A\sin\alpha}{\alpha}\right)^2 \frac{\sin^2 N\beta}{\sin^2\beta}$$

$$\frac{\sin N\beta}{\sin\beta} = 0$$ [By using L Hopital's Rule]

Because $\sin N\beta = 0$
$\sin\beta = 0$

$$\lim_{\beta\to n\pi} \frac{N\cos N\beta}{\cos\beta} = \pm N$$

Hence,

$$\lim_{\beta\to\pm n\pi}\left(\frac{\sin N\beta}{\sin\beta}\right)^2 = N^2$$

$$\boxed{I = \left(\frac{A\sin\alpha}{\alpha}\right)^2 N^2}$$

$$\boxed{I \propto N^2}$$

$$\beta = \pm n\pi$$

$$\frac{\pi}{\lambda}(e+d)\sin\theta = \pm n\pi$$

$$\boxed{(e+d)\sin\theta = \pm n\lambda} \quad n = 0, 1, 2 \ldots$$

* **Minima**

$$\sin N\beta = 0 \;\; \& \;\; \sin\beta \neq 0$$

$$\frac{\sin N\beta}{\sin\beta} = 0 \Rightarrow \sin N\beta = 0$$

$$N\beta = \pm m\pi$$

$$\boxed{\beta = \pm\frac{m\pi}{N}}$$

Put β

$$2\beta = \frac{2\pi}{\lambda}(e+d)\sin\theta$$

$$\frac{2m\pi}{N} = \frac{2\pi}{\lambda}(e+d)\sin\theta$$

$$\boxed{(e+d)\sin\theta = \left(\frac{m\lambda}{N}\right)}$$ $[m = 1, 2, 3 \cdots (N-1)]$

* In diffraction