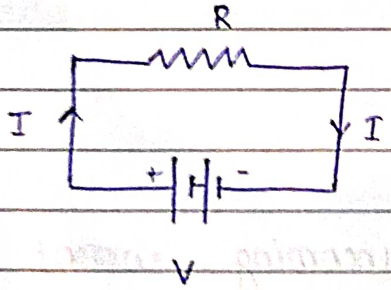
Basic Electrical Engg. (BEEG 1001)

Module 1: DC Circuit Analysis

- Ohm's law
- Kirchhoff's law
  - KCL
  - KVL
- Source Transformation
- Current and Voltage Division Rule
- Theorem's :- Thevenin's Theorem
  - Superposition Theorem
  - Maximum Power Transfer Theorem

Few Elements -

Resistor / Resistance
Inductor / Inductance
Capacitor / Capacitance



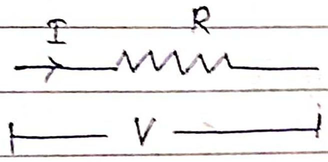
$V = IR$

$V_L = L\frac{dI}{dt}$

$Q = CV$

$\frac{dq}{dt} = C\frac{dv}{dt}$

- Ohm's law:

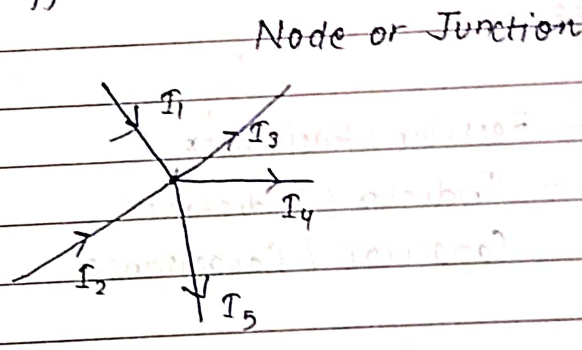


Here, V = Potential difference across the resistor R
R = Resistance of Resistor
I = Current

$I \propto V$

According to Ohm's law, current through resistor R, directly proportional to potential difference across the end of resistor provided that physical conditions (like temperature, pressure, etc.) remain the same.

- Kirchoff's Current Law:



$I_1 + I_2 = I_3 + I_4 + I_5$

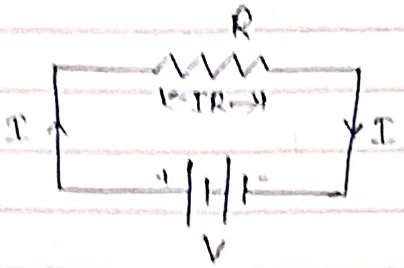
In general, $\sum_{j=1}^{n} I_j = 0$

According to KCL, the sum of incoming current is equals to sum of outgoing current.

In other words, we can say sum of all the currents about a node point is zero.

* Also known as Nodal Analysis / Node Analysis.

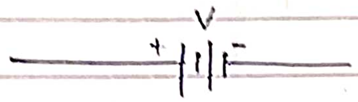
- Kirchhoff's Voltage law :

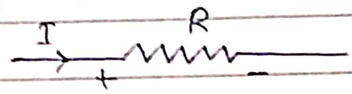


In general,
$$\sum_{j=1}^{n} V_j = 0$$

According to KVL, the sum of all the voltages / voltage drops in a closed circuit is zero.
* Also known as Mesh Analysis.

- Sign Convention for KVL / Mesh Analysis



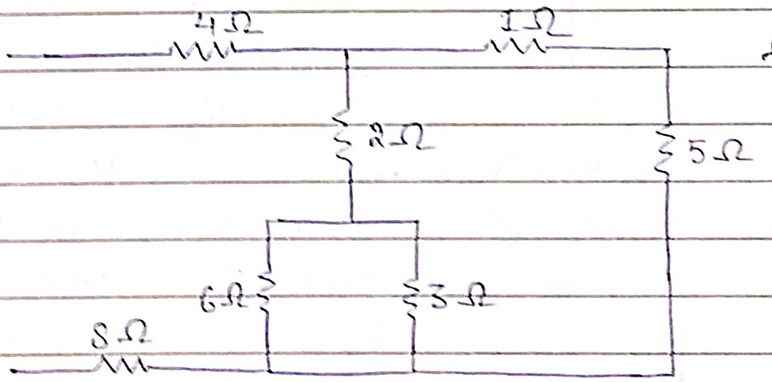


fall in potential $\rightarrow$ $-V$ fall in potential $\rightarrow$ $-IR$

$+V$ $\leftarrow$ rise in potential $+IR$ $\leftarrow$ rise in potential

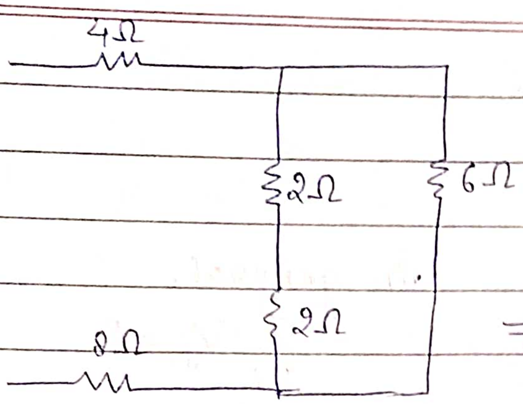
* We mark (+) where current is entering.

Question :-



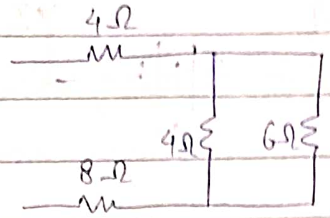
find Requivalent

firstly, solving 3Ω and 6Ω which are connected in parallel & 1Ω & 6Ω are in series.

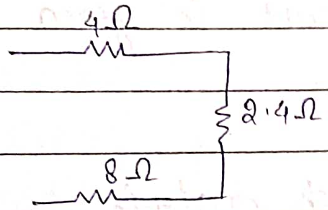


Now, 2Ω and 2Ω are in series.

So,



Now, 4Ω and 6Ω are connected in parallel.

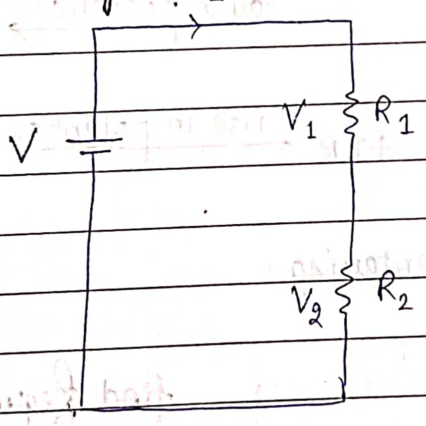


At last, 4Ω, 2.4Ω and 8Ω are connected in series.

So, $R_{eq} = 4\Omega + 8\Omega + 2.4\Omega$

$R_{eq} = 14.4\Omega$

Voltage Divider



Here, R_1 & R_2 are connected in series.

So, current will be same.

$V_1 = IR_1$ --- (i)

By ohm's law,

$V_2 = IR_2$ --- (ii)

$\cancel{V = V_1 + V_2}$

As well as,

$V = IR_{eq}$

i.e.

$V = I(R_1 + R_2)$

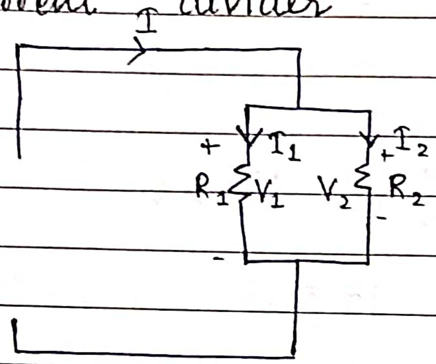
Putting value of I in eq^n (i)

$$V_1 = V\left(\frac{R_1}{R_1 + R_2}\right)$$

$$V_2 = V\left(\frac{R_2}{R_1+R_2}\right)$$

Voltage appearing across one of the series resistances is the total voltage times the ratio of its resistance to the total resistance.

- Current divider



$$V_1 = V_2$$

$$V_1 = I_1 R_1 \quad , \quad V_2 = I_2 R_2$$

$$I_1 R_1 = I_2 R_2$$

$$I_1 = \frac{I_2 R_2}{R_1}$$

Using KCL,

$$I = I_1 + I_2$$

$$I = \frac{I_2 R_2}{R_1} + I_2$$

$$\boxed{I_2 = I\left(\frac{R_1}{R_1+R_2}\right)} \qquad \boxed{I_1 = I\left(\frac{R_2}{R_1+R_2}\right)}$$

Current through one of the through parallel resistors is the total current times is the ratio of other resistance to the sum of total resistance.

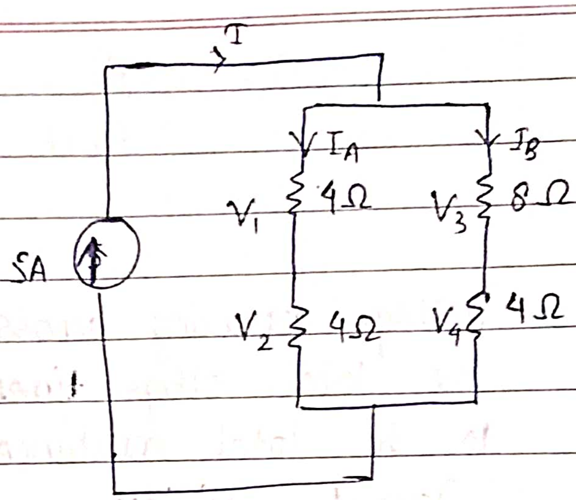
Que $I = 5A$

Using Ohm's law

$V = I R_{eq}$

$R_{eq} = \dfrac{8 \times 12}{8+12} = \dfrac{96}{20}$

$R_{eq} = 4.8\,\Omega$



$V = 5 \times 4.8$

$V = 24.0\,V$

$V_1 = V\left(\dfrac{R_1}{R_1+R_2}\right) = 24\left(\dfrac{4}{8}\right) = 12\ V$

$V_2 = V - V_1 = 24 - 12 = 12\ V$

$V_3 = 24\left(\dfrac{8}{12}\right) = 16\ V$

$V_4 = V - V_3 = 24 - 16 = 8\ V$

$I_A = \dfrac{V}{R'_{eq}}$

$I_A = \dfrac{24}{8}$

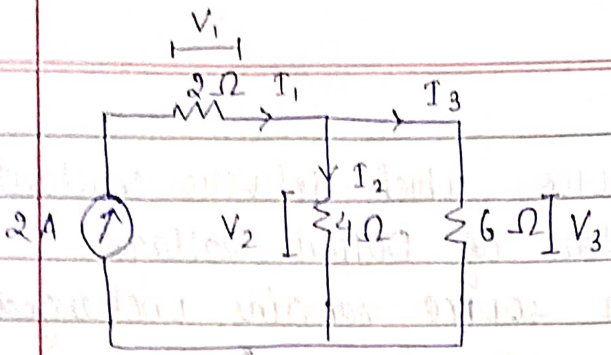
$I_A = 3\ A$

$I = I_A + I_B$

$I_B = I - I_A$

$I_B = (5 - 3)\,A$

$I_B = 2A$



I_1 is 2A AS, it is in series. And in Series current remain constant.

$V_1 = I_1 R_1$
$V_1 = 2 \times 2$
$V_1 = 4V$

4Ω and 6Ω are in parallel
So, $R_{eq} = \frac{4 \times 6}{4+6} = 2.4\Omega$

$V = I R_{eq}$
$V = 2 \times 2.4$
$V = 4.8\ V$

Here, Voltage remain constant, as it is connected in parallel.

So, $V_2 = V_3 = 4.8\ V$

$V_2 = I_2 R_2$
$\frac{4.8}{4} = I_2$

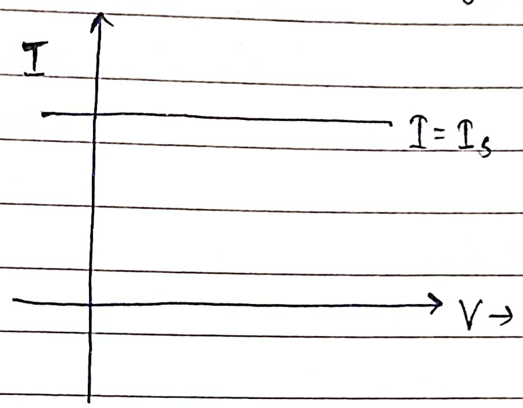
$I_2 = 1.2\ A$

$V_3 = I_3 R_3$
$\frac{4.8}{6} = I_3$

$I_3 = 0.8\ A$

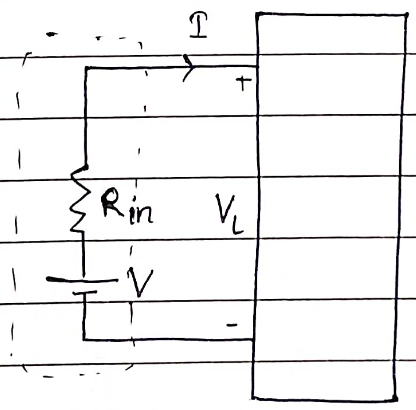
Ideal Current Source :-

It is defined as a source which delivers constant current and is independent of Output Voltage. Output current of such source remains unchanged for load resistance varying from 0 to ∞.



Practical Voltage Source :-

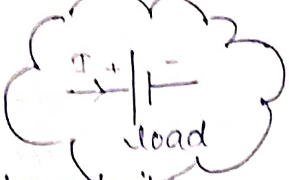
Practical Voltage Source is mo as an ideal voltage source in series with resistance. This resistance is named as Internal Resistance.



$V_L = V - IR_{SV}$

$I\uparrow \quad V_{RSV}\uparrow \quad V_L\downarrow$

Polarity is independent of the ...



Mark the polarity

- In a resistance, the polarity depends upon the assumed reference direction of current.

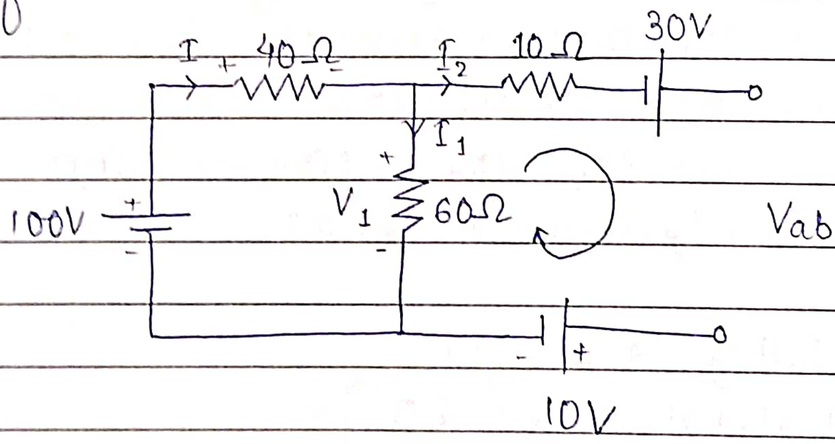
- For any resistance, the end to which current enters is marked as positive.

- The polarity of voltage across the battery does not depend upon the assumed direction of current.

Rule 1: While travelling, if you meet a voltage rise, write voltage with positive sign; write voltage with negative sign if you meet a voltage drop.

Rule 2: While travelling, write the voltage with positive sign if positive is encountered first; write voltage with negative sign, if negative is encountered first.

*



find V_{ab}

$100 - 40I - 60I_1 = 0$ $\quad (I_1 = I,$ As I_2 is $0)$

$100 - 40I - 60I = 0$

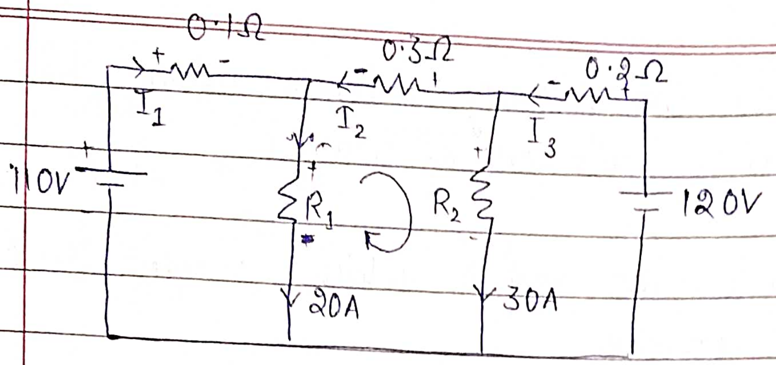
$100 - 100I = 0$

$I = 1\text{ A}$

$60 + 30 - V_{ab} - 10 = 0$

$V_{ab} = 80V$

*



find I_1, I_2, I_3, R_1, R_2

Applying KVL

$110 - 0.1 I_1 - 20 R_1 = 0$ ...(vi)

$0.3 I_2 - R_2 \times 30 + 20 R_1 = 0$ ...(vii)

Applying KCL

$I_1 + I_2 = 20$

$I_3 = I_2 + 30$

$I_1 + I_2 + 0 I_3 = 20$ .... (i)

$0 I_1 - I_2 + I_3 = 30$ .... (ii)

$-2I_2 + 2I_3 = 60$ -- (v)

Applying KVL in whole circuit

$110 - 0.1 I_1 + 0.3 I_2 + 0.2 I_3 - 120 = 0$

$-0.1 I_1 + 0.3 I_2 + 0.2 I_3 = 10$

$-I_1 + 3I_2 + 2I_3 = 100$

$I_1 - 3I_2 - 2I_3 = -100$ ... (iii)

$I_1 = -100 + 3I_2 + 2I_3$ ... (i)

Putting value in (i)

$-100 + 3I_2 + 2I_3 + I_2 = 20$

$4I_2 + 2I_3 = 120$ .. (iv)

Solving eqn (iv) & (v)

$4I_2 + 2I_3 = 120$

$-2I_2 + 2I_3 = 60$

$+ \quad - \quad -$

$6I_2 = 60$

$I_2 = 10A$

Putting value in (i)

$$10 + I_2 = 20$$
$$I_2 = 10A$$

Putting value of I_2 in (ii)

$$-10 + I_3 = 30$$
$$I_3 = 40A$$

Putting value of I_1 in (vi)

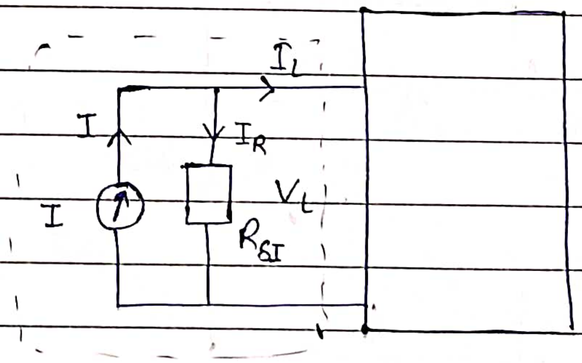
$$110 - 0.1 \times 10 - 20R_1 = 0$$
$$109 = 20R_1$$
$$R_1 = 5.45\,\Omega$$

Putting value of R_1 and I_2 in (vii)

$$0.3 \times 10 - R_2 \times 30 + 20 \times 5.45 = 0$$
$$112 = 30R_2$$
$$R_2 = 3.733\,\Omega$$

Practical Current Source :-

Practical Current Source is modelled as an ideal current source in parallel with a resistance. This resistance is called as Internal Resistance.



Using KCL,

$$I = I_L + I_R$$
$$I_L = I - I_R$$

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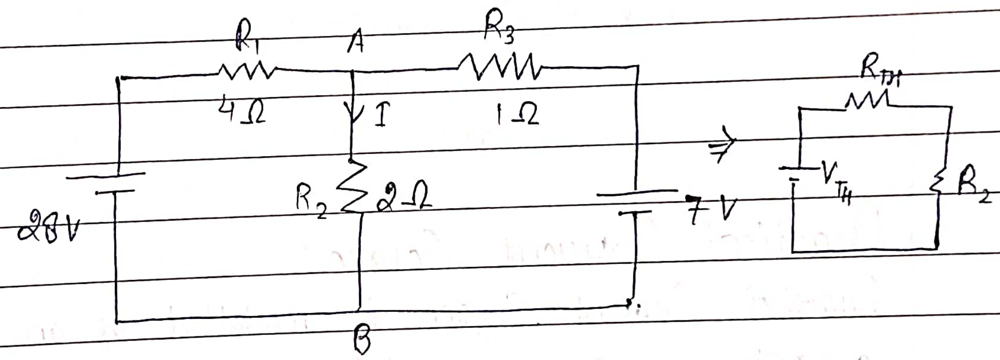
$$\boxed{I_L = I - \frac{V_L}{R_{SI}}}$$

where, $\downarrow I_L = I - \frac{V_L \uparrow}{R_{SI}} \uparrow$

Thevenin's Theorem -

It states that it is possible to simplify any linear circuit containing independent and dependent voltage or current source no matter how complex, to an equivalent circuit with just a single voltage source and a series resistance between any two points in the circuit.



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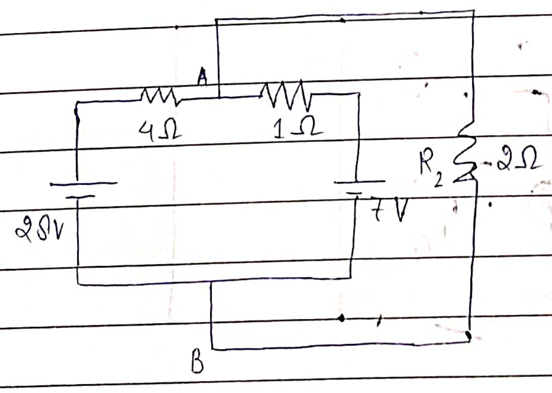
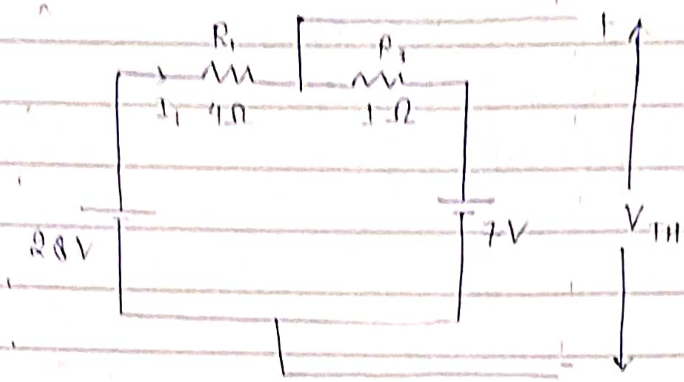
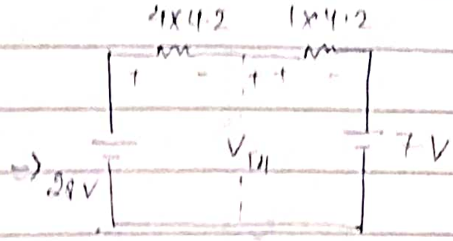






II

$28 - 4I_1 - I_1 - 7 = 0$

$21 = 5I$

$I_1 = 4.2\,A$

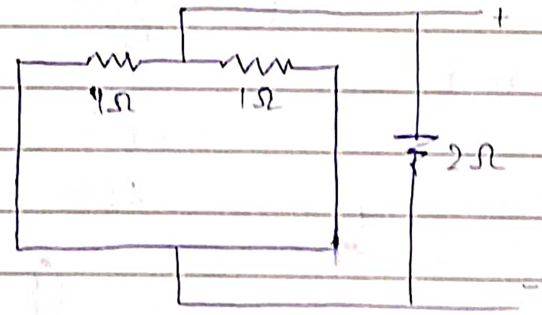
Applying KVL in II loop

$28 - 4 \times 4.2 - V_{TH} = 0$

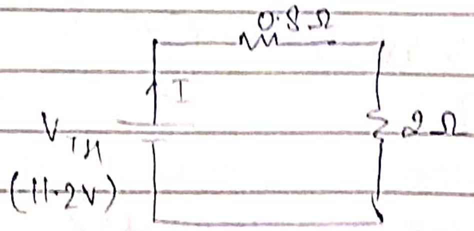
$28 - 16.8 = V_{TH}$

$V_{TH} = 11.2V$

For calculating R_{TH},
We will short circuit the independent voltage source and open the independent current sources.



$$R_{TH} = \frac{4 \times 1}{4+1} = \frac{4}{5} = 0.8\,\Omega$$



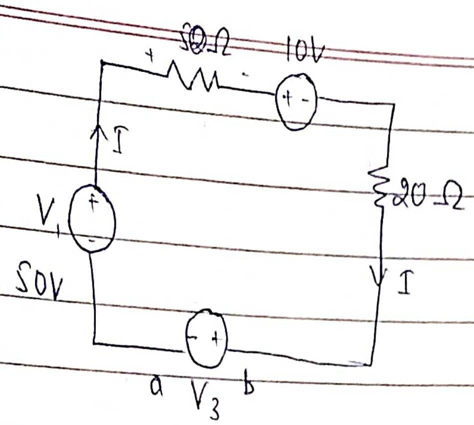
Applying KVL,

$11.2 - (0.8 + 2)I = 0$

$11.2 = 2.8I$

$I = 4A$

eg.



$I = 0.4V$
find V_3

Applying KVL

$50 - (70 \times 0.4) - 10 - V_3 = 0$

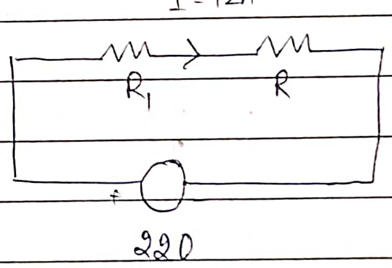
$40 - 28 - V_3 = 0$

$V_3 = 12\ V$

$V_{ab} = -12V$

$V_{ba} = 12V$

Que An oven takes 16 A at 220V. It is desired to reduce the current to 12 A.
find (i) Resistance which must be connected in Series.
(ii) Voltage across the resistor.

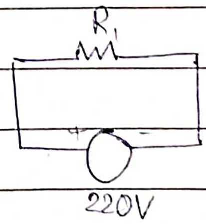


$V = IR_1$

$R_1 = \frac{220}{16} = 13.75\Omega$

$I(R_1 + R_2) = V$

$12\left(\frac{220}{16} + R\right) = 220$



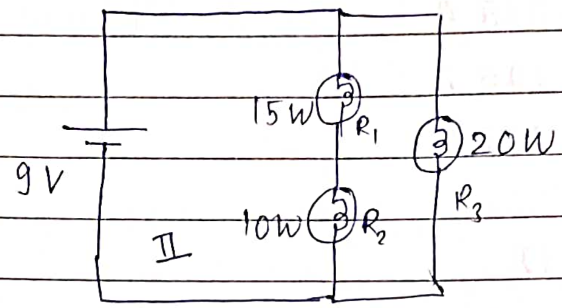
$\frac{220}{16} + R = 18.33$

$R = 18.33 - 13.75$

$R = 4.58\ \Omega$

$V = IR = 4.58 \times 12 = 54.96\ V$

Que.



Three bulbs are connected to a ~~...~~ 9 V battery as shown in the figure Calculate
(i) Total current supplied by the battery
(ii) Current through each bulb.
(iii) The Resistance of each bulb!

$P = \frac{V^2}{R_3}$

$20 = \frac{81}{R_3}$

$R_3 = \frac{81}{20}$

$R_3 = 4.05\,\Omega$

Applying in I loop

$V = IR$

$9 = I_1\left[\frac{P'}{I_1^2} + \frac{P''}{I_1^2}\right]$

$9 = \frac{15+10}{I_1}$

$I_1 = \frac{25}{9}$

$I_1 = 2.778\,A$

$P = I^2 R$

$R = \frac{P}{I^2}$ [In series I is same]

$I_1 = I_2$

$V = IR$

$9 = I \times 4.05$

$I_3 = \frac{900}{405}$

$I_3 = 2.22\,A$

(i) Total current I, $= 2.778 + 2.222 = 5\,A$

Pg 16

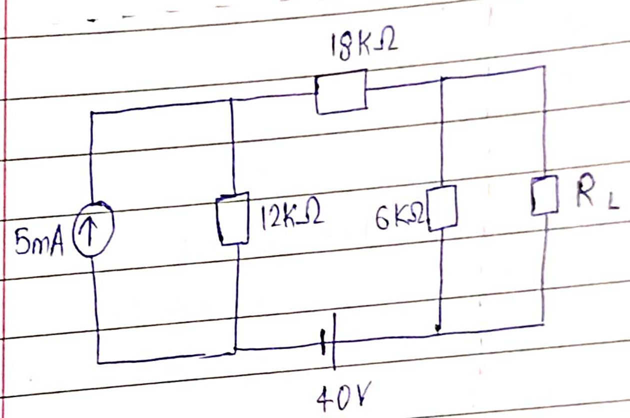
(ii) Through 15 W, $I = 2.778$ A
$10W, I' = 2.778$ A
$20W, I'' = 2.222$ A

(iii) $R_3 = 4.05\Omega$

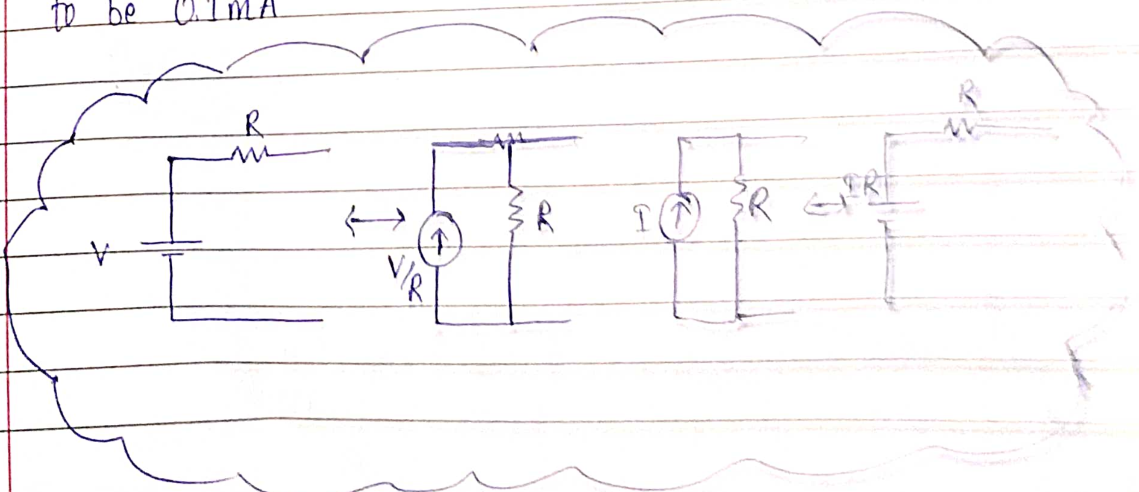
$R_1 = \frac{P}{I_1^2} = \frac{15}{(2.778)^2} = 1.944\Omega$

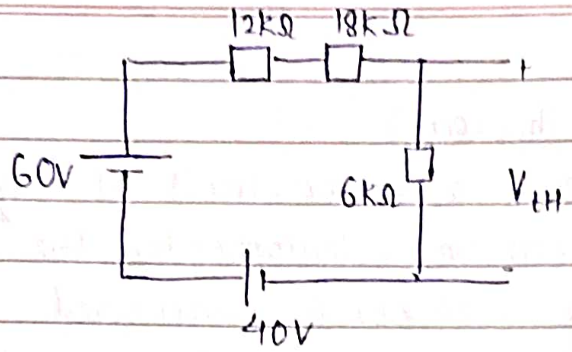
$R_2 = \frac{P}{I_2^2} = \frac{10}{(2.778)^2} = 1.296\Omega$

Que.



→ Replace the circuit in dotted box by Thevnin's Equivalent Circuit.
→ Find V_{ab} for $R_L = 3K\Omega$
→ find the value of R_L for which it receives maximum power from the circuit.
→ find the value of R_L that makes current in 6KΩ to be 0.1mA





Applying KVL

$60 - 36I + 40 = 0$

$20 = 36I$

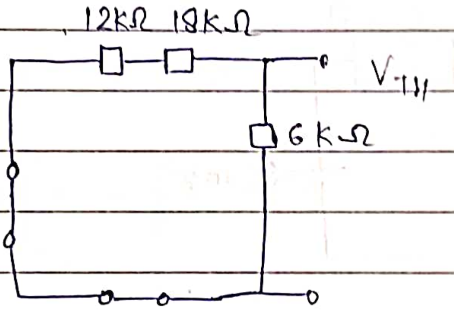
$I = \frac{20}{36} = \frac{5}{9}\ mA$

$I = 0.555 mA$

~~$V_{TH} = 6 \times 0.555 mV = 3.330 mV$~~

$V_{TH} = 6 \times \frac{5}{9} = 3.333\ V$

R_{TH} —



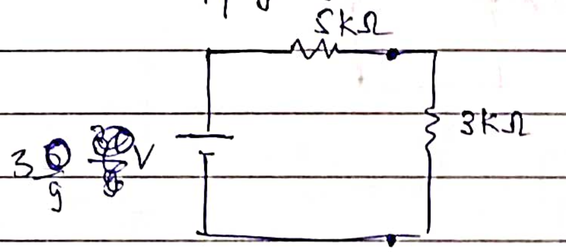
12kΩ & 18kΩ are in series

So, 30 kΩ

30kΩ & 6kΩ are in parallel.

So, $R_{TH} = \frac{30 \times 6}{36} = 5 k\Omega$

(ii) Applying KVL,



$V = IR$

$V_{ab} = \frac{30 \times 3 \times 10^3}{9\ (5+3) \times 10^3}$

$V_{ab} = \frac{90}{9 \times 8} = 1.25\ V$

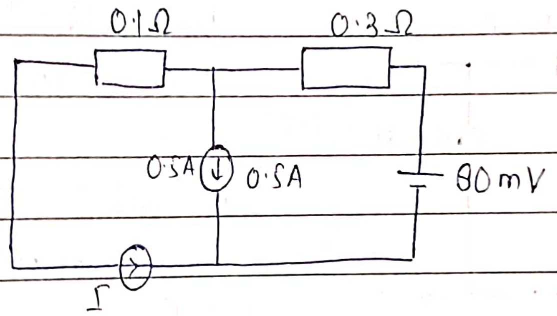
(iii) for maximum power, $R_L = R_{TH}$ So, $R_L = 5k\Omega$

current source → open circuit
Voltage source → short circuit

Principle of Superposition -
(OR Superposition Theorem)

It states that the response in a linear circuit at any point due to multiple sources can be calculated by the summing the effects of each source considered separately, all other sources being made inoperative.

By making a source inoperative means that the voltage source is replaced by a short circuit and current source is being replaced by an open circuit.



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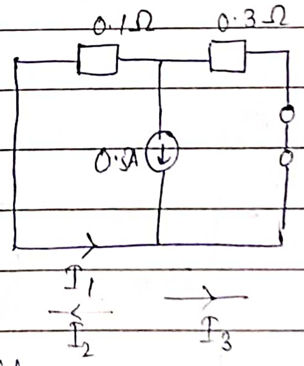

$I_1 = 0$

Using Current divider

$$I_2 = \frac{0.5 \times 0.3}{0.1 + 0.3} = \frac{0.15}{0.40} = 0.375A$$

$I_1 = -I_2$

$I_1 = -0.375A$

FREEMIND

Date

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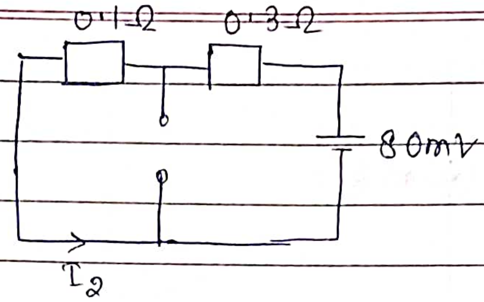


$$I_2 = \frac{80 \times 10^{-3}}{0.1 + 0.2}$$

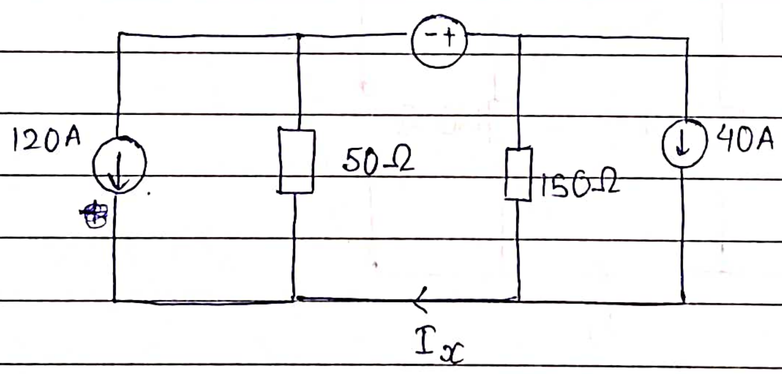
$I_2 = 0.2\ A$

$I_1 = -0.375A$

$I_2 = 0.2\ A$

$I = I_1 + I_2 = -0.375 + 0.2 = -0.175\ A$

Que.



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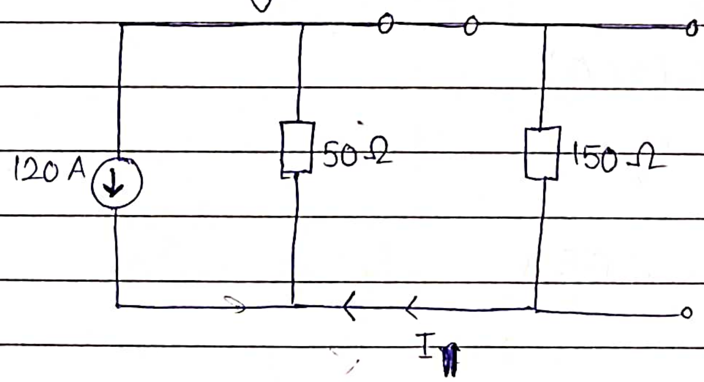
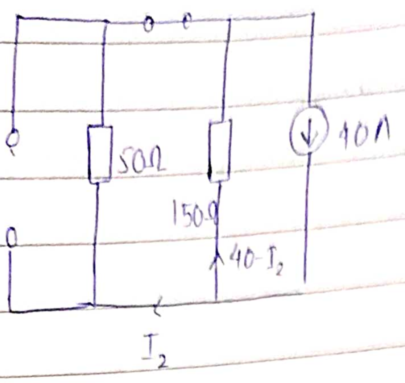
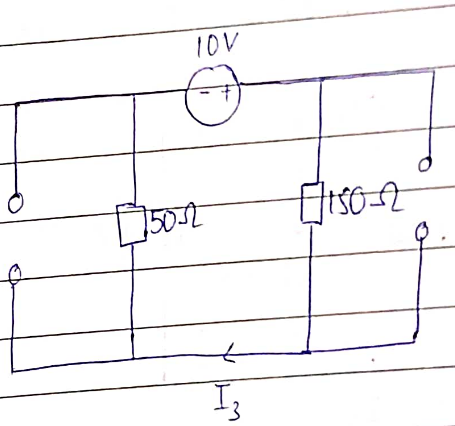


$$I_1 = \frac{120 \times 50}{50 + 150} = 30\ A$$

$I_1 = -30A$ [As dir.ⁿ is opp.]



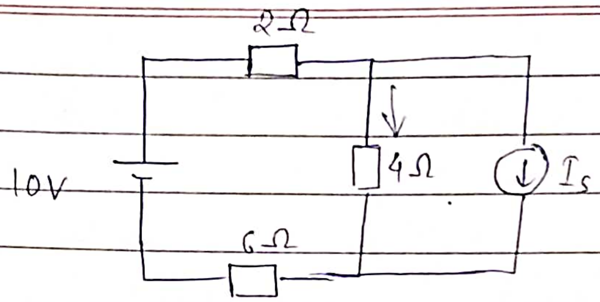
$$50 I_2 = 150(40 - I_2)$$
$$50 I_2 = 6000 - 150 I_2$$
$$200 I_2 = 6000$$
$$I_2 = 30\ A$$



$$V = IR$$
$$10 = I_3(200)$$
$$\frac{10}{200} = I_3$$
$$I_3 = 0.05\ A$$

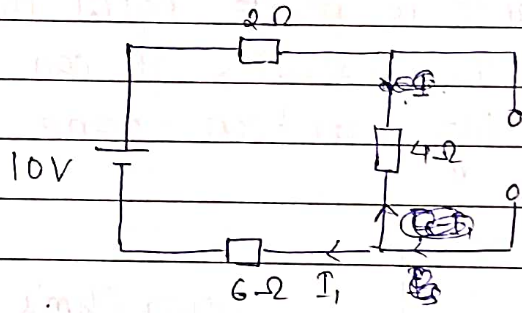
$$I_x = I_1 + I_2 + I_3$$
$$I_x = (-30 + 30 + 0.05)A$$
$$I_x = 0.05A$$

Que

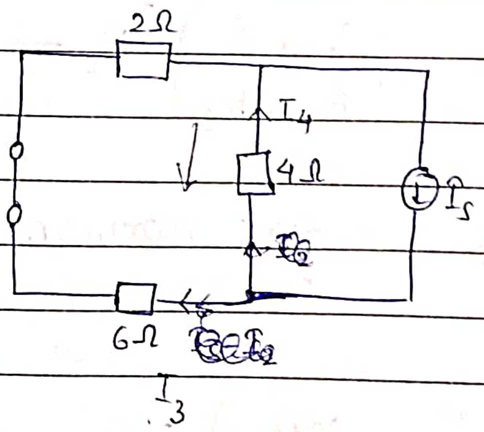


find current I_s to reduce the voltage across 4Ω to 0.8

$$10-(2+6)I_s=0$$
$$10=8I_s$$
$$I_s=1.25\ A$$



$$V=IR$$
$$10=I_1\left[2+4+6\right]$$
$$I_1=\frac{10}{12}\ A$$



$$I_2=-I_4=-\frac{I_s\times 8}{12}$$

$R_L = R_{TH}$

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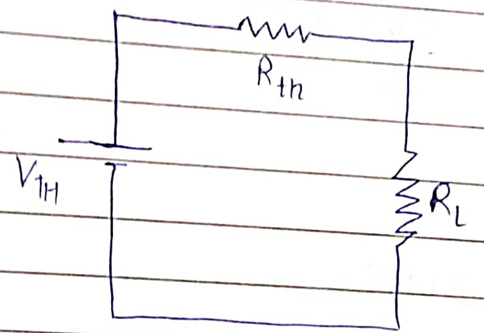
Imp. • **Maximum Power Transfer Theorem** -

→ This theorem states that maximum power is absorbed from a network when the load resistance is equal to the output resistance of the network as seen from the terminals of the load.

→ When the load resistance is made equal to the output resistance of the circuit, it can be said that impedance matching has been done.



Using Ohm's law

$$I_L = \frac{V_{TH}}{R_{TH}+R_L}$$

$$P_L = I_L^2 R_L$$

$$P_L = \left(\frac{V_{TH}}{R_{TH}+R_L}\right)^2 R_L \quad (1)$$

$$\frac{dP_L}{dR_L} = 0 \quad \text{for maximum power}$$

$$\frac{d}{dR_L}\left(\frac{V_{TH}^2}{(R_L+R_{TH})^2} \times R_L\right) = 0$$

$$-\frac{2V_{TH}^2}{(R_L+R_{TH})^3}\times R_L + \frac{V_{TH}^2}{(R_L+R_{TH})^2} = 0$$

$$\frac{2\cancel{V_{TH}^2}R_L}{(R_L+R_{TH})^3} = \frac{\cancel{V_{TH}^2}}{(R_L+R_{TH})^2}$$

$$\frac{2R_L}{R_L+R_{TH}} = 1$$

$$2R_L = R_L + R_{TH}$$

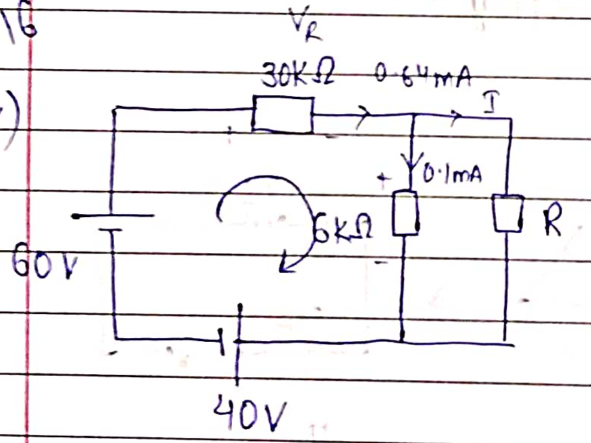
$$R_L = R_{TH}$$

Putting value of R_L in (1)

$$P_L = \left(\frac{V_{th}}{R_{th}+R_{th}}\right)^2 R_{th}$$

$$\boxed{P_L = \frac{V_{th}^2}{4R_{TH}}}$$ Available Power

Pg-16 (iv)



Using KVL

$$-40+60-V_R-0.6=0$$

$$V_R = 19.4V$$

$$V_R = IR$$

$$19.4 = I\times 30\,k\Omega$$

$$I = 0.64mA$$

Using KCL,

$0.64 mA = I + 0.1 mA$

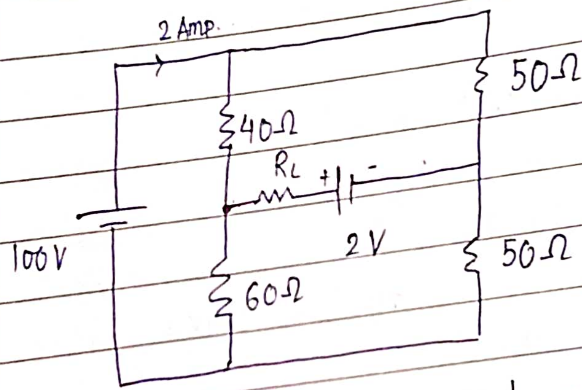
$I = 0.54 mA$

$V = IR$

$\frac{0.6}{0.54} = R$

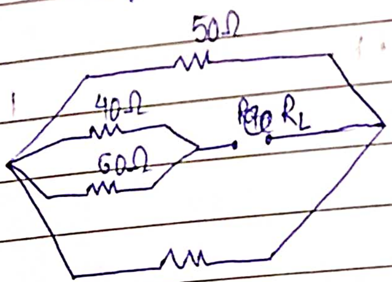
$R = 1.1 k\Omega$

Que



Maximum power supply - ?

⇓



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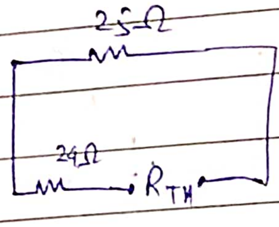

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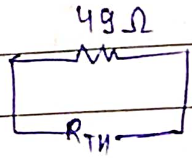
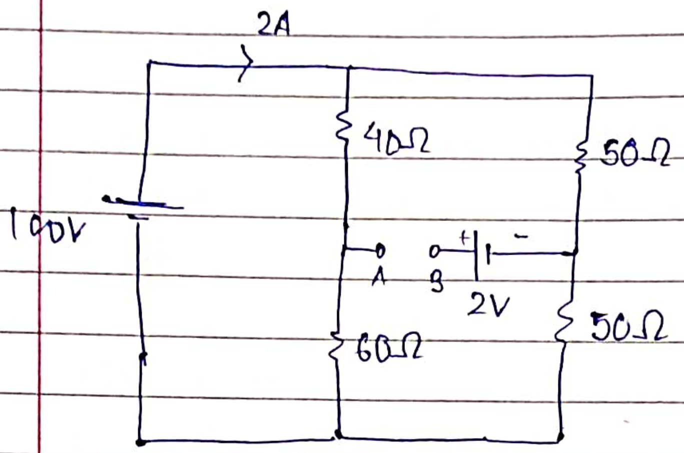


$R_{TH} = 49 \Omega$

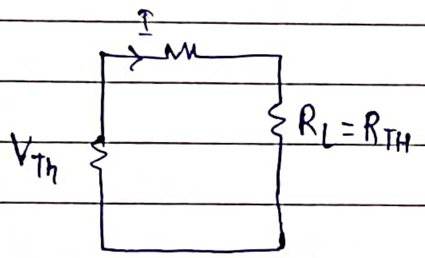


$$I = 2 \times \frac{(50+50)}{(40+60)+(50+50)}$$

$$I = 1\ \text{amp}$$

$$-40I - V_{ab} - 2 + 50 \times 1 = 0$$

$$V_{ab} = 8\,V$$



$$P_{max} = \frac{V_{TH}^2}{4R_{TH}} = \frac{8 \times 8}{4 \times 49} = 0.32\,W$$

Alternating Current

Theory of Cycle -

One complete set of positive and negative value of the function (which goes on repeating) is called as Cycle.

Peak Value -

It is the maximum value either in the positive or the negative of the quantity.

Time Period -

It is the duration of time required for the quantity to complete one cycle.

Frequency -

It is the number of oscillations/cycle that occur in a second.

Angular Frequency -

It is equal to the number of radian covered in a second. Denoted by ω.

$$W = 2\pi f = \frac{2\pi}{T}$$

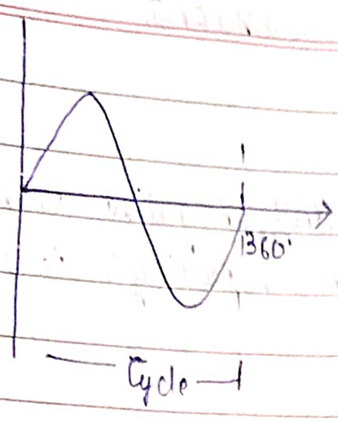
It's SI unit is rad sec^{-1}.

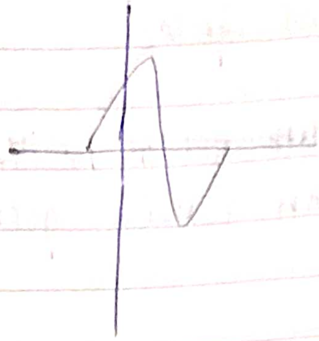
Phase -

It is the fraction of the time period or cycle that has elapsed since it last passed from the chosen zero position or origin.

$$\text{phase} = \frac{t}{T}$$

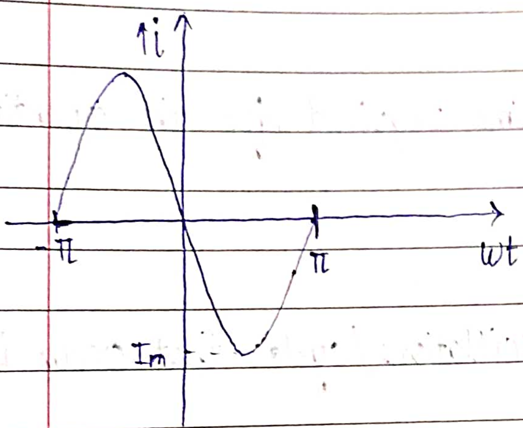
where t is Time Period.





$I_m \sin(\omega t + \theta)$

left side → $+\theta$
right side → $-\theta$



$I = I_m \sin(\omega t + \phi)$
$I = I_m \sin(\omega t + \pi)$
$I = -I_m \sin \omega t$

→ If ϕ is positive number, the waveform is shifted towards left and if ϕ is a negative number, the waveform is shifted towards right.

→ If ϕ is positive, it is angle of lead.

→ If ϕ is negative, it is angle of lag

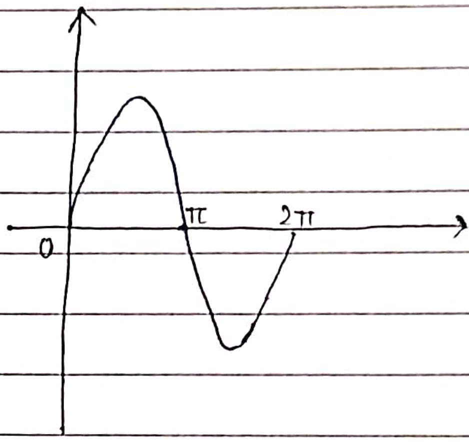
Note:-

- For a full cycle of sinusodial waveform it has same area in the positive and negative loops. Hence, the algebraic sum of two areas is zero.
  So, we can say that average of value of sine wave over one full cycle is 0.

Therefore, for a sinusoidal waveform we define average value ~~of~~ over half cycle only.

- For any symmetrical periodic waveform the average value is calculated over only half cycle.

For ~~half~~ full wave -



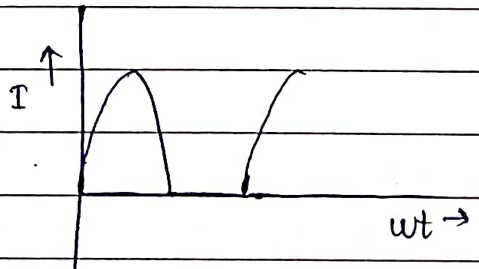
$$I_{avg} = \int_0^{\pi} \frac{I_m \sin \omega t}{\pi - 0} \, d(\omega t)$$

$$= \frac{I_m}{\pi} \int_0^{\pi} \sin \omega t \, d(\omega t)$$

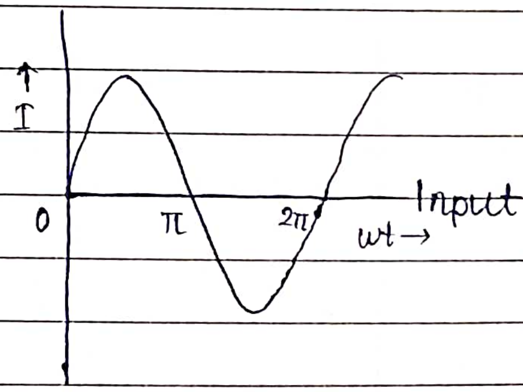
$$= \frac{I_m}{\pi} \left[-\cos \omega t \, d(\omega t) \right]_0^{\pi}$$

$$= \frac{I_m}{\pi} \left[-\cos \pi + \cos 0 \right]$$

$$\boxed{I_{avg} = \frac{2 I_m}{\pi}}$$



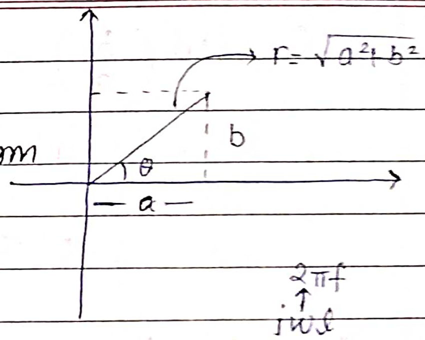
Output



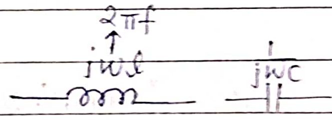
$$I_{avg} = \int_0^{2\pi} \frac{I \, d(\omega t)}{2\pi - 0} = \frac{1}{2\pi} \int_0^{2\pi} I_m \sin \omega t \, d(\omega t) = \frac{I_m}{2\pi} \left[-\cos \omega t \right]_0^{2\pi}$$

$$\cancel{I_{avg} = \frac{I_m}{2\pi} \left[-\cos 2\pi + \cos 0 \right] = 0}$$

Complex plane -



$Z = a + jb$ — (1) Rectangular form
$\theta = \tan^{-1}\left(\frac{b}{a}\right)$



$Z = R + j\omega L$ $a = r\cos\theta$
Here, Z is impedance $b = r\sin\theta$
$Z = R - \frac{j}{\omega C}$

Putting value of a & b in (1)
$Z = r\{\cos\theta + j\sin\theta\}$ ---- (trigonometric form)
$Z = re^{j\theta}$ ---- (exponential form)
$Z = r\angle\theta$ ---- (Polar form)

Operations -

$Z_1 Z_2 = r_1 r_2 \angle \theta_1 + \theta_2$
$\frac{Z_1}{Z_2} = \frac{r_1}{r_2} \angle \theta_1 - \theta_2$

Que. A sinusoidal current of $10\angle 0^\circ$ A is added to another sinusoidal current of $20\angle 60^\circ$ A. find the resultant current.

$Z = r\{\cos\theta + j\sin\theta\}$
$Z = 10\{\cos 0^\circ + j\sin 0^\circ\}$ $Z_2 = 20\{\cos 60^\circ + j\sin 60^\circ\}$
$Z_1 = 10$ $Z_2 = \frac{20 \times 1}{2} + j \times \frac{\sqrt{3} \times 20}{2}$ 10

$Z_2 = 10 + j10\sqrt{3}$

$Z_1 + Z_2 = 20 + j\,\cancel{10}\,10\sqrt{3}$
$= 10(2 + j\sqrt{3}) =$

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$\theta = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right) = 40.89$

$R = \sqrt{b^2+a^2} = \sqrt{400+300} = \sqrt{700} = 26.457$

$Z = Re^{i\theta} \quad R\angle\theta$

$Z = 26.45\angle 40.98$

Que Determine the average and rms value of the resultant current in a wire carrying simultaneously a DC current of 10A and a sinusoidal current of peak value of 10A find the average and rms value.

$I_{DC} = 10+0j$

$I_m = 10$

$I_{AC} = 10\sin\omega t$

Resultant $I = 10 + 10\sin\omega t$

$I_{av} = \frac{\int_0^{2\pi} 10+10\sin\omega t \; d(\omega t)}{2\pi - 0} = \frac{1}{2\pi}\int_0^{\pi} 10+10\sin\omega t \; d(\omega t)$

$I_{av} = \frac{1}{2\pi}$ Wrong

$I_{avg} = 10$

$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi}(10+10\sin\omega t)^2 \, d(\omega t)}$

$I_{rms} = \sqrt{\frac{100}{2\pi}\int_0^{2\pi}\left[1+\sin^2\omega t + 2\sin\omega t\right]_0^{2\pi} d(\omega t)}$

$I_{rms} = \sqrt{\frac{50}{\pi}\int_0^{2\pi} 1 + 2\sin\omega t + \frac{1-\cos 2\omega t}{2} \; d(\omega t)}$

$I_{rms} = \sqrt{\frac{50}{\pi}\left[\omega t - 2\cos\omega t + \frac{\omega t}{2} + \frac{\sin 2\omega t}{2}\right]_0^{2\pi}}$

6.457

e resultant
a DC current
lue of

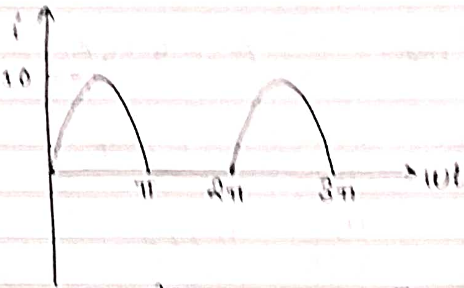
$d(\omega t)$

$$I_{rms} = \sqrt{\frac{50}{\pi}\left[2\pi - 2 + 2\frac{\pi}{2} + 2\cos 0^\circ\right]}$$

$$I_{rms} = \sqrt{\frac{50}{\pi} \times 3\pi}$$

$$I_{rms} = \sqrt{150}$$

Tutorial ~~No~~ –

1\. 

Find Power consumed by 10Ω resistor when above current is flowing through it.

$$I_{av} = \frac{\int_0^{2\pi} I\, d(\omega t)}{2\pi - 0} = \frac{\int_0^{\pi} I_m \sin\omega t\, d(\omega t) + \int_{\pi}^{2\pi} 0\, d(\omega t)}{2\pi - 0} = \frac{1}{2\pi}\int_0^{\pi} 10 \sin\omega t\, d(\omega t)$$

$$I_{av} = \frac{5}{\pi}\int_0^{\pi} \sin\omega t\, d(\omega t) = \frac{5}{\pi}\left[-\cos\omega t\right]_0^{\pi} = -\frac{5}{\pi}\left[\cos\pi - \cos 0\right]$$

$$I_{av} = -\frac{5}{\pi}\left[-1-1\right] = \frac{10}{\pi} = 3.18\,A$$

$$I_{rms} = \sqrt{\frac{\int_0^{2\pi} i^2\, dt}{2\pi - 0}} = \sqrt{\frac{\int_0^{\pi} I_m^2 \sin^2\omega t\, d(\omega t) + \int_{\pi}^{2\pi} 0\, d(\omega t)}{2\pi}}$$

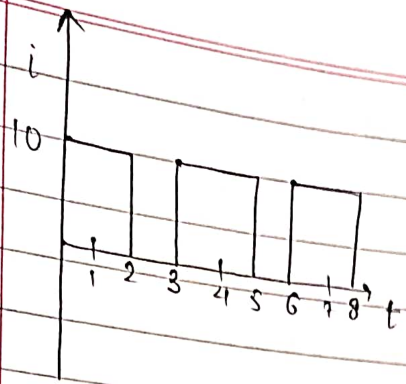
$$I_{rms} = \sqrt{\frac{I_m^2}{2\pi}\int_0^{\pi} \sin^2\omega t\, d(\omega t)} = \frac{I_m}{\sqrt{2\pi}}\sqrt{\left[\frac{1 - \cos 2\omega t}{2}\right]_0^{\pi}}$$

$$I_{rms} = \frac{I_m}{2\sqrt{\pi}}\left[\sqrt{\pi - \cos 2\pi - 0 + \cos 0}\right] = \frac{10}{2\sqrt{\pi}}\left[\sqrt{\pi - 1 + 1}\right]$$

$$I_{rms} = 5$$

$$P = I_{rms}^2 R = 5 \times 5 \times 10 = 250\,W$$

Que.



find I_{rms}, I_{av}??

$$I_{av} = \frac{10\times2+0}{3-0} = \frac{20}{3} A$$

$$I_{rms} = \sqrt{\frac{\int_0^3 i^2 dt}{3-0}} = \sqrt{\frac{1}{3}\int_0^2 i^2 dt + \int_2^3 0\, dt}$$

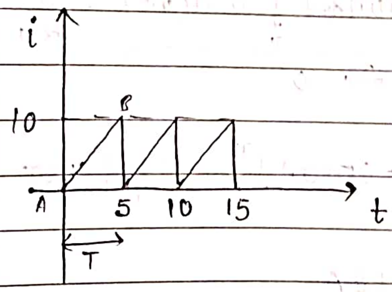
$$I_{rms} = \sqrt{\frac{1}{3}\times10^2[t]_0^2} = \sqrt{\frac{100}{3}[2-0]}$$

$$I_{rms} = \sqrt{\frac{200}{3}} A = 8.16 A$$

$$\text{form factor} = \frac{I_{rms}}{I_{av}} = \frac{8.16\times3}{20} = 1.224$$

$$\text{Peak factor} = \frac{I_m}{I_{rms}} = \frac{10}{8.16} = 1.225$$

Que



Equation of AB

$y = mx + c^0$

$10 = m5$

$m = 2$

$i = 2t$

$$I_{av} = \frac{\frac{1}{2}\times5\times10}{5-0} = \frac{50}{10} = 5A$$

$$I_{rms} = \sqrt{\frac{\int_0^5 i^2 dt}{5-0}} = \sqrt{\frac{1}{5}\int_0^5 i^2 dt} = \sqrt{\frac{1}{5}\int_0^5 4t^2 dt}$$

$$I_{rms} = \sqrt{\frac{4}{5}\left[\frac{t^3}{3}\right]_0^5} = \sqrt{\frac{4}{5}\times\frac{125}{3}} = \sqrt{\frac{100}{3}} = 5.77$$

$$\text{form factor} = \frac{I_{rms}}{I_{av}} = \frac{5.77}{5} = 1.1547$$

$$\text{Peak factor} = \frac{I_m}{I_{rms}} = \frac{10}{5.77} = 1.732$$

volt Ampere reactive

RL, RC

Date Imp.

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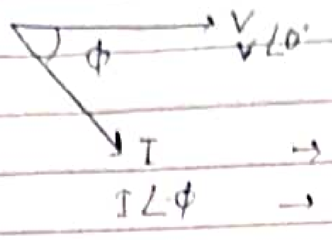
Phasors -

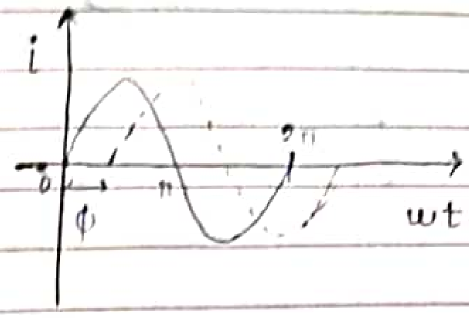


$V = V_m \sin\omega t$

$I = I_m \sin(\omega t - \phi)$

→ V leads I by ϕ.

→ I lags V by ϕ.



→ Power factor - $\cos\phi$

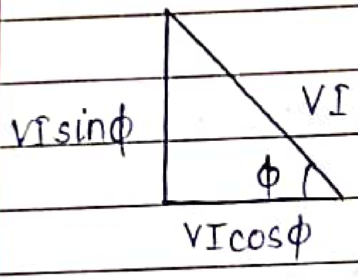
Power → Reactive $= VI\sin\phi$ (VAR)

Apparent $= VI$ (VA)

Real $= VI\cos\phi$ (Watt)

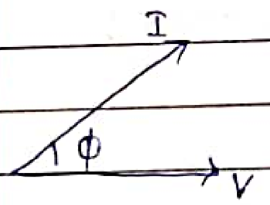
where, V & I are RMS values

Powertriangle -



→ lag power factor → I lags V

→ lead power factor → I leads V



$\cos\phi$ (lead)

Qu $V = 55\sin(\omega t)$ V

$i = 6.1\sin(\omega t - \frac{\pi}{5})$ A

$\phi = \frac{\pi}{5}$

Find Real Power, Reactive power, Apparent power,

Find the instantaneous power at $\omega t = 0.3$

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$\cos\left(\frac{\pi}{5}\right) = 0.809$

$V_{rms} = \frac{V_m}{\sqrt{2}} = 38.89$

$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{6.1}{\sqrt{2}} = 4.31$

Apparent $= V_{rms} I_{rms} = 167.6159$

Real Power $= V_{rms} I_{rms} \cos\phi = 135.60$

Reactive Power $= V_{rms} I_{rms} \sin\phi = 98.522$

$p = VI$

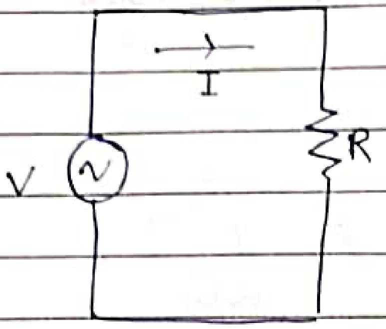
$p = V_m \sin\omega t \; I_m \sin(\omega t - \phi)$

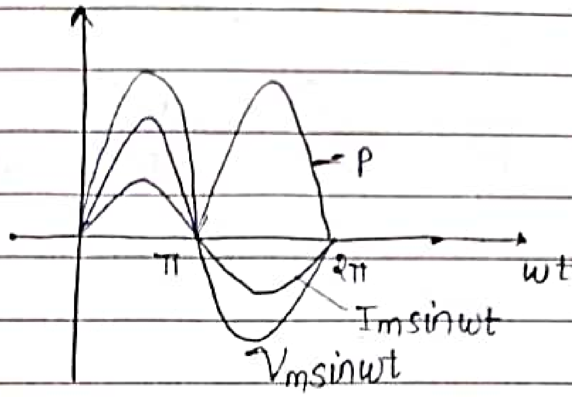
$p = 2 V_m I_m \sin\omega t \sin(\omega t - \phi)/_2$

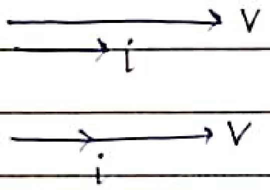
$p = \frac{V_m I_m}{2} 2 \sin\omega t \sin(\omega t - \phi)$

$p = \frac{V_m I_m}{2} \left[\cos(\omega t - \omega t + \phi) - \cos(\omega t + \omega t - \phi) \right]$

Purely Resistive Ckt





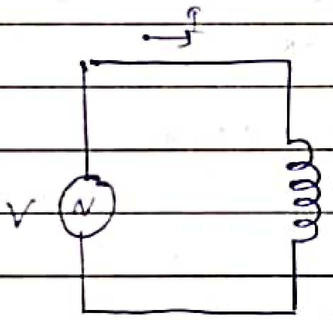


$V = V_m \sin \omega t$

$i = \frac{V}{R} = \frac{V_m \sin \omega t}{R}$

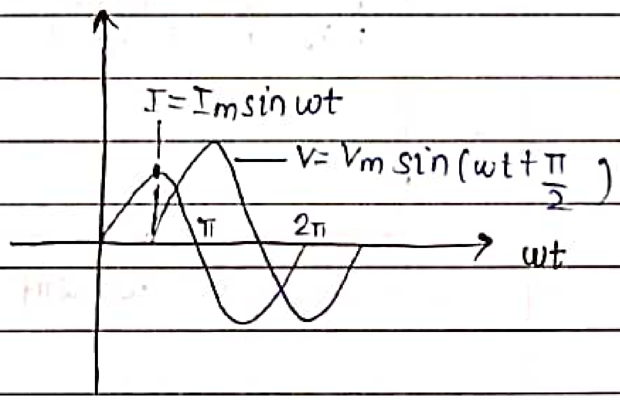
$i = I_m \sin \omega t$

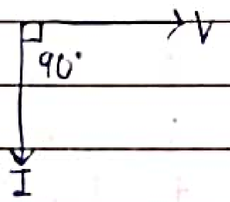
Purely Inductive ckt



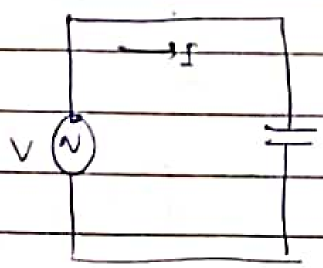
$X_L = \omega L$

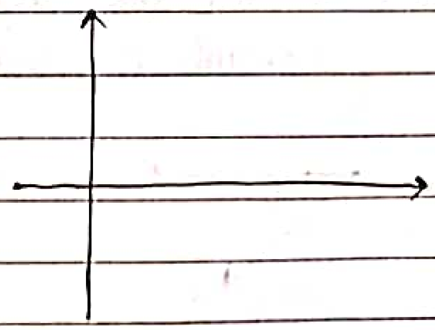
$X_L = 2\pi f L$





Purely Capacitive Ckt

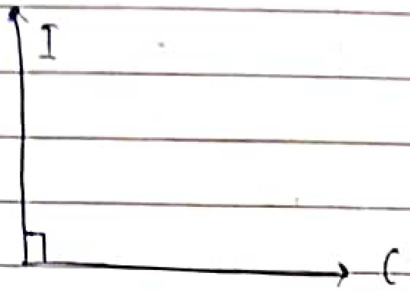




$V = V_m \sin \omega t$

$i = C\frac{dV}{dt} = C\frac{d}{dt}(V_m \sin \omega t) = C V_m \cos \omega t \cdot \omega$

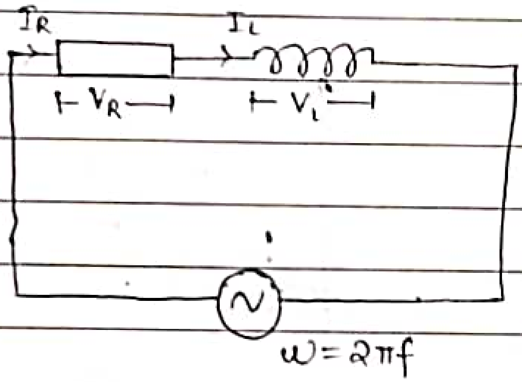
$i = C \omega V_m \sin(\omega t + \frac{\pi}{2})$

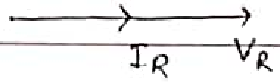


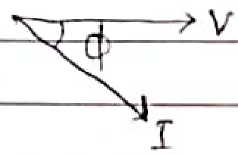
$I \angle 90°$ $\qquad X_C = \frac{1}{\omega C}$

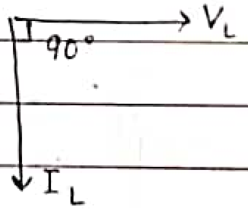
Imp.

RL Circuit -









$Z = R + j\omega L$

where Z is Impedance

R is Resistance

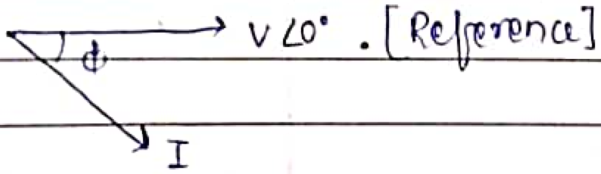
L is Inductance

ωL is Inductive Reactance

$Z = r \angle \phi$

$r = \sqrt{R^2 + (\omega L)^2}$

$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$



$I = \frac{V}{Z}$

$= \frac{V \angle 0°}{r \angle \phi}$

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$$= \frac{V}{Z} \angle -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$V = V_m \sin \omega t = V\angle 0$$

$$I = \frac{V_m}{\sqrt{R^2 + (\omega L)^2}} \cdot \sin\left(\omega t - \tan^{-1}\left(\frac{\omega L}{R}\right)\right)$$

$$P = VI$$

$$P = V_m \sin(\omega t) I_m \sin(\omega t - \phi)$$

$$P = V_m I_m \sin(\omega t) \sin(\omega t - \phi)$$

$$P = \frac{V_m I_m}{2} [\cos\phi - \cos(2\omega t - \phi)]$$

$$P = \frac{V_m I_m}{2} 2 \sin \omega t \sin(\omega t - \phi)$$

$$P = \frac{V_m I_m}{2} [\cos(\omega t - (\omega t - \phi)) - \cos(\omega t + \omega t - \phi)]$$

$$P = \frac{V_m I_m}{2} [\cos(\omega t - \omega t + \phi) - \cos(2\omega t - \phi)]$$

$$P = \frac{V_m I_m}{2} [\cos\phi - \cos(2\omega t - \phi)]$$

$$P = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} [\cos\phi - \cos(2\omega t - \phi)]$$

$$\boxed{P = VI\cos\phi - VI\cos(2\omega t - \phi)}$$ ... instantaneous power

The second term in the above expression represent a sinusoidal wave form of angular frequency, 2ω; Its average value is 0. However, first term $VI\cos\phi$ is constant with time.

Avg. power | Actual power | Real power = $VI\cos\phi$

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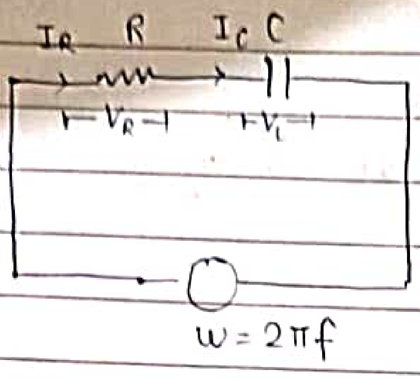
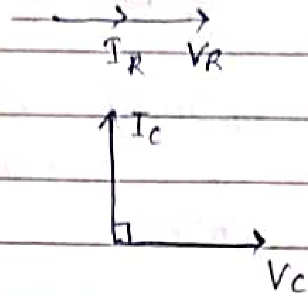
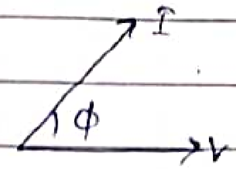
RC Ckt -







[illegible]

$$Z = R + \frac{1}{jwC} \qquad Z = r\angle\phi \qquad \phi = \tan^{-1}\left\{\frac{-1/wC}{R}\right\}$$

$$Z = R - \frac{j}{wC} \qquad r = \sqrt{R^2 + \left(\frac{1}{wC}\right)^2} \qquad \phi = \tan^{-1}\left(\frac{-1}{wCR}\right)$$

$$I = \frac{V\angle 0}{r\angle -\tan^{-1}\left(\frac{1}{wCR}\right)}$$

$$I = \frac{V}{r}\angle \tan^{-1}\left(\frac{1}{wCR}\right)$$

$$V = V_m \sin wt$$
$$I = I_m \sin(wt + \phi)$$

$$P = VI$$
$$P = V_m I_m \sin wt \sin(wt + \phi)$$
$$P = \frac{V_m I_m}{2} \cdot 2 \sin wt \sin(wt + \phi)$$

$$P = \frac{V_m I_m}{2}\left[\cos(wt - wt - \phi) - \cos(wt + wt + \phi)\right]$$

$$P = \frac{V_m I_m}{2}\left[\cancel{\cos(-\phi) - \cos(2wt + \phi)}\right]$$

$$P = \frac{V_m I_m}{\sqrt{2}\,\sqrt{2}}\left[\cancel{\cos\phi - \cos(2wt + \phi)}\right]$$

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$\boxed{P = VI\cos\phi - VI\cos(2\omega t + \phi)}$

A metal filament lamp rated at 750W, 100 V is to be used on a 230V, 50Hz Supply, by connecting a capacitor of suitable value in series. Determine -

Vi (?)

$V\angle 0' \to V\angle\theta$

$I\angle\phi \to I\angle\theta - \phi$

Power factor $= \cos(\theta - \phi)$

115μF (i) the Capacitance required

64.12° (ii) phase angle

0.435 leading (iii) power factor

1725 VA (iv) Apparent power

1553 VA (v) Reactive power