Tutorial Sheet (Electromagnetics)

Q.1 The conduction current flowing through a wire with conductivity $\sigma = 6\times10^{7}$ S/m and relative permittivity $\varepsilon_r = 1$ is given by $I_c = 6 \sin \omega t$ (mA). If $\omega = 10^{10}$ radian, find the displacement current. **Ans : $I_D = 8.85\times10^{-12} \cos \omega t$ Amp.**

Q.2 A parallel plate capacitor with plate area of 5 cm^2 and plate separation of 3 mm has a voltage of $50 \sin 10^3 t$ applied to its plates. Calculate the displacement current assuming $\varepsilon = 2\varepsilon_0$. Ans : $I_D = 1.47 \times 10^{-7} \cos 10^3 t$ Amp

Q.3 In a material for which $\sigma = 5$ s/m and $\varepsilon_r = 1$, the electric field intensity $E = 250 \sin 10^{10} t$ V/m. Find the conduction and displacement current densities and the frequency at which they have equal magnitude. Ans:

$J_c = 1.25 \times 10^3 \sin 10^{10} t$ A/m^2, $J_D = 22.13 \cos 10^{10} t$ A/m^2, $f = 8.99 \times 10^{10}$ Hz

Q.4 The relative permittivity of distilled water is 81. Calculate refractive index and velocity of light in it. Ans : $\mu = 9$, $v = 3.33 \times 10^7$ m/s

Q.5 Show that equation of continuity $div\ J + \frac{\partial \rho}{\partial t} = 0$ is contained in Maxwell equation.

Q.6 Assuming that all the energy from a 1000 watt lamp is radiated uniformly, calculate the average values of the intensities of electric and magnetic fields of radiation at a distance of 2m from the lamp? Ans : $E = 86.59$ V/m, $H = 0.23$ A/m

Q.7 If earth receives 2 cal $min^{-1} cm^{-2}$ solar energy what are the amplitudes of electric and magnetic fields of radiation? Ans : $E_0 = 1026.8$ V/m, $H_0 = 2.73$ A/m

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$J = \frac{i_c}{A}$ and $J = \sigma E$

thus $\frac{i_c}{A} = \sigma E \Rightarrow E = \frac{i_c}{\sigma A}$ (I)

now $J_d = \frac{\partial D}{\partial t} = \varepsilon\left(\frac{\partial E}{\partial t}\right) = \varepsilon \frac{\partial}{\partial t}\left(\frac{i_c}{\sigma A}\right)$

$J_d = \frac{\varepsilon}{\sigma A} \frac{\partial i_c}{\partial t}$ (II)

$\varepsilon / \varepsilon_0 = \varepsilon_r \Rightarrow \varepsilon = \varepsilon_0 \varepsilon_r = 1\,\varepsilon_0 \Rightarrow \varepsilon = \varepsilon_0$

thus $J_d = \frac{\varepsilon_0}{\sigma A} \frac{\partial}{\partial t}(6 \sin \omega t \times 10^{-3}$ Ampere/m$^2)$

thus $i_d = J_D \times A = \frac{\varepsilon_0}{\sigma} \frac{\partial}{\partial t}(6 \sin \omega t)\, 10^{-3}$ Ampere

$i_d = 8.85 \times 10^{-12} \times 10^{-3} \times 6 \times \omega \cos \omega t / 6 \times 10^{7}$

$i_d = 8.85 \times 10^{-12} \times 10^{-3} \times 6 \times 10^{10} \cos \omega t / 6 \times 10^{7} = 8.85 \times 10^{-12} \cos \omega t$ Ampere Ans

Relative permiability

$\mu_r = \mu / \mu_0$

$\mu = \mu_0 \mu_r$

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$J_D = \frac{\partial D}{\partial t} = \frac{\partial}{\partial t}(\varepsilon E) = \varepsilon \frac{\partial}{\partial t}\left(\frac{V}{d}\right) = \frac{\varepsilon}{d} \frac{\partial V}{\partial t}$

$i_d = A \times J_d = \frac{A \varepsilon}{d} \frac{\partial V}{\partial t} = \frac{A(2\varepsilon_0)}{d} \frac{\partial}{\partial t}(50 \sin 10^3 t)$

$= \frac{(5 \times 10^{-4}) \times 2 \times (8.85 \times 10^{-12})}{3 \times 10^{-3}}\, 50 \times 10^3 \cos 10^3 t$

$= \frac{88.5 \times 10^{-16+3+3} \times 50}{3} \cos 10^3 t$

$= 1475 \times 10^{-10} \cos 10^3 t$

$= 1.475 \times 10^{-7} \cos 10^3 t$ Ampere Ans

$E = V/d$

E magnitude of the electric field between the plate

V potential difference between plate

d separation of the plates

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(a) conduction current $J_c = \sigma E$

$J_c = 5 \times 250 \sin 10^{10} t = 1250 \sin 10^{10} t = 1.25\times10^{3} \sin 10^{10} t$ A/m2

Ans

(b) $J_D = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t} = \varepsilon_0 \varepsilon_r \frac{\partial E}{\partial t} = \varepsilon_0 \times 1 \times \frac{\partial}{\partial t}(250 \sin 10^{10} t)$

$= 8.85\times10^{-12} \times 250 \times 10^{10} \times \cos 10^{10} t$ A/m2 $\Rightarrow 22.12 \cos(10^{10} t)$ Ampere/m2

Ans

(c) Let the electric field is given as

$E = E_0 e^{i\omega t}$ where ω is the angular frequency and $i = \sqrt{-1}$

$J_d = \varepsilon \frac{\partial E}{\partial t} = \varepsilon i \omega E$

$|J_d| = \varepsilon \omega E$ Thus $\left|\frac{J_c}{J_d}\right| = \frac{\sigma E}{\varepsilon \omega E} = \frac{\sigma}{\varepsilon \omega}$

We know that $J_c / J_d = \sigma / \varepsilon\omega$

according to questions $J_c = J_d$ therefore $\sigma = \omega \varepsilon$

also $\omega = 2\pi f$ thus $f = \frac{\omega}{2\pi} = \frac{\sigma/\varepsilon}{2\times3.14}$

$= \frac{5/8.85\times10^{-12}}{2\times3.14} = \frac{0.565\times10^{12}}{2\times3.14} = \frac{5.65\times10^{11}}{2\times3.14}$

$= 0.899\times10^{11} = 8.99\times10^{10}$ Hz

Ans

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conducting medium or non conducting medium $v = \frac{1}{\sqrt{\mu\varepsilon}}$ ~~for non conducting medium~~

Distilled water is non magnetic medium, the refractive index is given by

$n = \frac{c}{v} = \frac{\sqrt{\mu\varepsilon}}{\sqrt{\mu_0\varepsilon_0}} = \sqrt{\frac{\varepsilon}{\varepsilon_0}}$ for non magnetic medium $\mu = \mu_0$

$n = \frac{c}{v} = \sqrt{\varepsilon_r} = \sqrt{81} = 9$ so the refractive index is 9 Answer

Now $v = \frac{c}{n} = \frac{3\times10^{8}}{9} = 3.33\times10^{7}$ m/sec Answer

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The Divergence of the curl of any vector field (A) is always zero.

i.e $\nabla\cdot(\nabla\times A)=0$ where A is any vector or scalar quantity.

According to maxwell fourth eqn we know that

$$\text{curl } H = J+\frac{\partial D}{\partial t}$$

taking the divergence of both side we get

$$\underbrace{\nabla\cdot(\text{curl } H)}_{0} = \nabla\cdot\left(J+\frac{\partial D}{\partial t}\right)$$

$$= \nabla\cdot J + \frac{\partial}{\partial t}(\nabla\cdot D)$$

since we know that $\nabla\cdot D=\rho$

$$\nabla\cdot J+\frac{\partial\rho}{\partial t}=0 \Rightarrow \boxed{\text{div } J+\frac{\partial\rho}{\partial t}=0}$$

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$S = EH$ watt/m2 $= \frac{1000}{4\pi r^2}$ given that $r = 2$ m

$$S=EH=\frac{1000}{4\pi(2)^2}=\frac{1000}{16\pi}=\frac{1000}{16\times3.14}=19.90 \text{ watt/m2}$$

we know that $E/H = 376.7$ ohm

$$EH\times\frac{E}{H}=E^2=19.90\times376.7=7496.3$$

$$\boxed{E=86.59 \text{ volt/m}} \qquad H=\frac{S}{E}=\frac{19.90}{86.59}=0.23 \text{ Ampere/m}$$

$$\boxed{H=0.23 \text{ Ampere/m}}$$

Ans

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As we know that $S = E \times H$ in magnitude i.e. $S = EH\sin 90^\circ = EH$

$\therefore S = 2 \text{ cal}\cdot\text{min}^{-1}\text{cm}^{-2} = \frac{2 \times 4.2 \times 10^{+4}}{60}$ Joule $\text{sec}^{-1}\text{m}^{-2}$

since 1 cal. = 4.2 Joule

$1 (\text{m})^2 = 10^4 (\text{cm})^2$

$S = 1.4 \times 10^{+3}$ Joule $\text{sec}^{-1}\text{m}^{-2}$ = 1400 Joule $\text{sec}^{-1}\text{m}^{-2}$

$S = 1400$ Watts/m^2

$S = EH = 1400$ watt/m^2

in free space

$$\frac{E}{H} = 376.7 \text{ ohm}$$

thus

$$EH \times \frac{E}{H} = 1400 \times 376.7 \Rightarrow E^2 = 1400 \times 376.7$$

$$E = \sqrt{1400 \times 376.7} = 726.2 \text{ volt/m}$$

Now $H = \frac{S}{E} = \frac{1400}{726.2} = 1.927 \quad \frac{\text{watt/m}^2}{\text{volt/m}} = \frac{\text{watt}}{\text{m}^{\cancel{2}}} \times \frac{\cancel{\text{m}}}{\text{volt}} = \frac{\text{watt}}{\text{m} \times \text{volt}}$

$H = 1.927$ Ampere/m

Amplitude of electric and magnetic field of radiation

$$E_0 = E\sqrt{2} = 726.2 \times \sqrt{2} = 1026.8 \Rightarrow 1026.8 \text{ volt/m}$$

$$H_0 = H\sqrt{2} = 1.927 \sqrt{2} = 2.725 \text{ Ampere/m}$$

Answer

$$\boxed{E = E_{rms} = \frac{E_0}{\sqrt{2}} \Rightarrow E_0 = E\sqrt{2}}$$