

Basic Electrical Engg. (BEE611001)

Module 1 : DC Circuit Analysis

Ohm's law

Kirchhoff's law



Source Transformation

Current and Voltage Division Rule

Thevenin's Theorem

Superposition Theorem

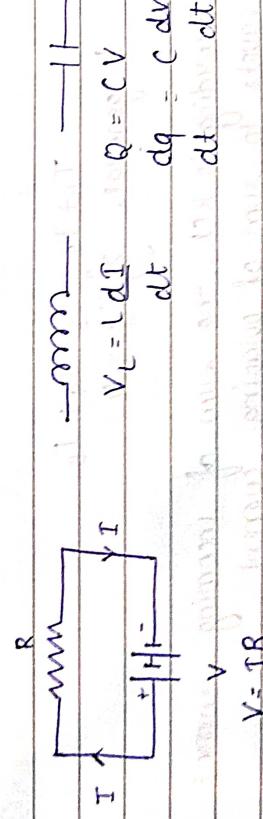
Maximum Power Transfer Theorem

Few Elements -

—~~~~— Resistor / Resistance

—~~~~— Inductor / Inductance

—||— Capacitor / Capacitance



Induction
• When a conductor moves in a magnetic field, an emf is induced in it.
• Faraday's Law of Electromagnetic Induction
• Lenz's Law

Electric Circuits
• A circuit is a closed loop consisting of a source of emf, resistors, inductors, capacitors, and switches.

•

Ohm's law :

$$I \propto V$$

$$I = \frac{V}{R}$$

Here, V = Potential difference across the resistor R

R = Resistance of Resistor

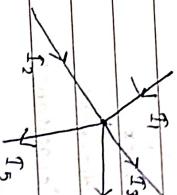
I = Current

$$I \propto V$$

According to Ohm's law, current through resistor R is directly proportional to potential difference across the end of resistor provided that physical conditions (like Temperature, pressure etc.) remain the same.

- Kirchhoff's Current Law:

Node or Junction



$$I_1 + I_2 = I_3 + I_4 + I_5$$

$$\text{In general, } \sum_{j=1}^n I_j = 0$$

According to KCL, the sum of incoming current is equals to sum of outgoing current.

In other words, we can say sum of all the currents about a node point is zero.

* Also known as Nodal Analysis / Node analysis

- Kirchhoff's Voltage law:

$$\sum_{j=1}^n V_j = 0$$

In general,

According to KVL, the sum of all the voltages / voltage drops in a closed circuit is zero.

* Also known as Mesh Analysis.

- Sign Convention for KVL / Mesh Analysis

Fall in potential → -V Rise in potential → +V

+V Rise in potential +IR Fall in potential

* We mark (+) where current is entering.

Question :-  find Equivalent

firstly, solving 3Ω and 6Ω which are connected in parallel & 1Ω & 6Ω are in series.

$\frac{4\Omega}{\text{in}}$

Note, 2Ω and 2Ω are in series.

$\frac{2\Omega}{\text{in}} \parallel \frac{2\Omega}{\text{in}} = 1\Omega$

$\frac{8\Omega}{\text{in}} \parallel \frac{2\Omega}{\text{in}} = 2\Omega$



Now, 4Ω and 6Ω are connected in parallel.



At last, 4Ω , 2.4Ω and 8Ω are connected in series.

So, $R_{\text{eq}} = 4\Omega + 2.4\Omega + 8\Omega$

$$R_{\text{eq}} = 14.4\Omega$$

Voltage Divider

$\frac{V_1}{V} = \frac{R_1}{R_1 + R_2}$ Hence, R_1 & R_2 are connected in series.

So, consequent will be same.

$$\frac{V_1}{V} = \frac{R_1}{R_1 + R_2}$$

By Ohm's law,

$$V_1 = IR_1 \quad \dots \text{(i)}$$

$$V_2 = IR_2 \quad \dots \text{(ii)}$$

$$V = IR_{\text{eq}}$$

i.e.

$$V = I(R_1 + R_2)$$

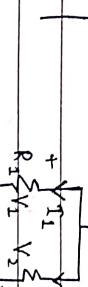
Putting value of I in eq " (i)

$$V_1 = \frac{V}{R_1 + R_2}$$

$$V_2 = \frac{V}{R_1 + R_2}$$

Voltage appearing across one of the series resistances is the total voltage times the ratio of its resistance to the total resistance.

Current Γ divider



$$\Gamma_1 = V_2$$

$$V_1 = \Gamma_1 R_1, \quad V_2 = \Gamma_2 R_2$$

$$\Gamma_1 R_1 = \Gamma_2 R_2$$

$$\Gamma_1 =$$

Using KCL,

$$\Gamma = \Gamma_1 + \Gamma_2$$

$$\Gamma = \frac{\Gamma_2 R_2}{R_1 + R_2} + \Gamma_2$$

$$\boxed{\Gamma_2 = \frac{\Gamma (R_1)}{R_1 + R_2}}$$

$$\boxed{\Gamma_1 = \frac{\Gamma (R_2)}{R_1 + R_2}}$$

Current through one of the through parallel resistors is the total current times the ratio of other resistance to the sum of total resistance.

Ques

$$I = 5 \text{ A}$$

Using Ohm's law

$$V = IR_{\text{eq}}$$

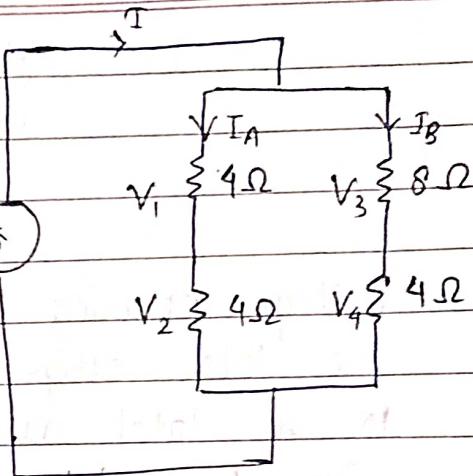
$$R_{\text{eq}} = 8 \times 12 = 96$$

$$R_{\text{eq}} = 8 + 12 = 20$$

$$R_{\text{eq}} = 4.8 \Omega$$

$$V = 5 \times 4.8$$

$$V = 24 \text{ V}$$



$$V_1 = V \left(\frac{R_1}{R_1 + R_2} \right) = 24 \left(\frac{4}{12} \right) = 8 \text{ V}$$

$$V_2 = V - V_1 = 24 - 8 = 16 \text{ V}$$

$$V_3 = 24 \left(\frac{8}{12} \right) = 16 \text{ V}$$

$$V_4 = V - V_3 = 24 - 16 = 8 \text{ V}$$

$$I_A = \frac{V}{R_{\text{eq}}}$$

$$I_A = \frac{24}{8}$$

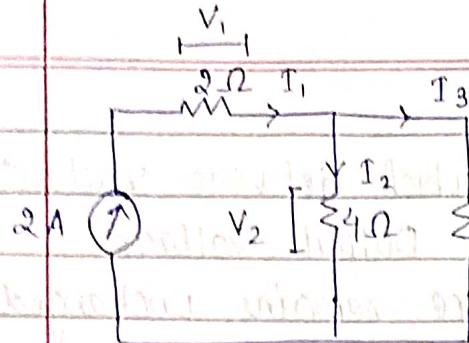
$$I_A = 3 \text{ A}$$

$$I = I_A + I_B$$

$$I_B = I - I_A$$

$$I_B = (5 - 3) \text{ A}$$

$$I_B = 2 \text{ A}$$



I_1 is $2A$ AS, it is in series.
And in Series current remain constant.

$$\text{V}_1 = I_1 R_1$$

$$V_1 = 2 \times 2$$

$$V_1 = 4V$$

4Ω and 6Ω are in parallel

$$\text{So, } R_{\text{eq}} = \frac{4 \times 6}{4+6} = 2.4 \Omega$$

$$V = I R_{\text{eq}}$$

$$V = 2 \times 2.4$$

$$V = 4.8 V$$

Here, Voltage remain constant, as it is connected in parallel.

$$\text{So, } V_2 = V_3 = 4.8 V$$

$$V_2 = I_2 R_2$$

$$\frac{4.8}{4} = I_2$$

$$I_2 = 1.2 A$$

$$V_3 = I_3 R_3$$

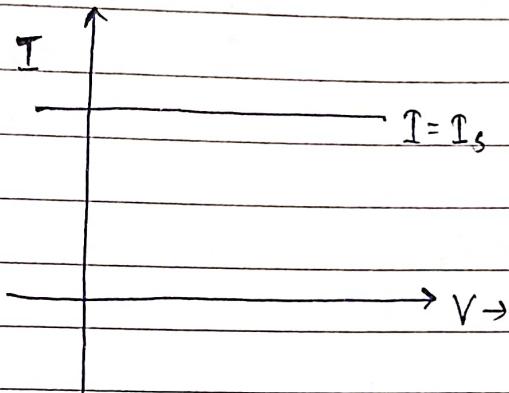
$$\frac{4.8}{6} = I_3$$

$$I_3 = 0.8 A$$

Ideal Current Source :-

It is defined as a source which delivers constant current and is independent of Output Voltage.

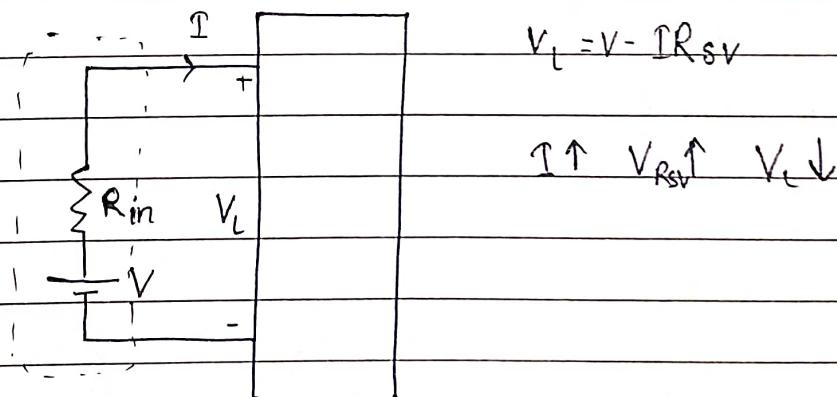
Output current of such source remains unchanged for load resistance varying from 0 to ∞ .

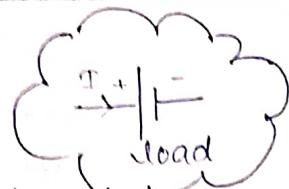


Practical Voltage Source :-

Practical Voltage Source is made as an ideal voltage source in series with resistance.

This resistance is named as Internal Resistance.





Polarity is independent of the source.

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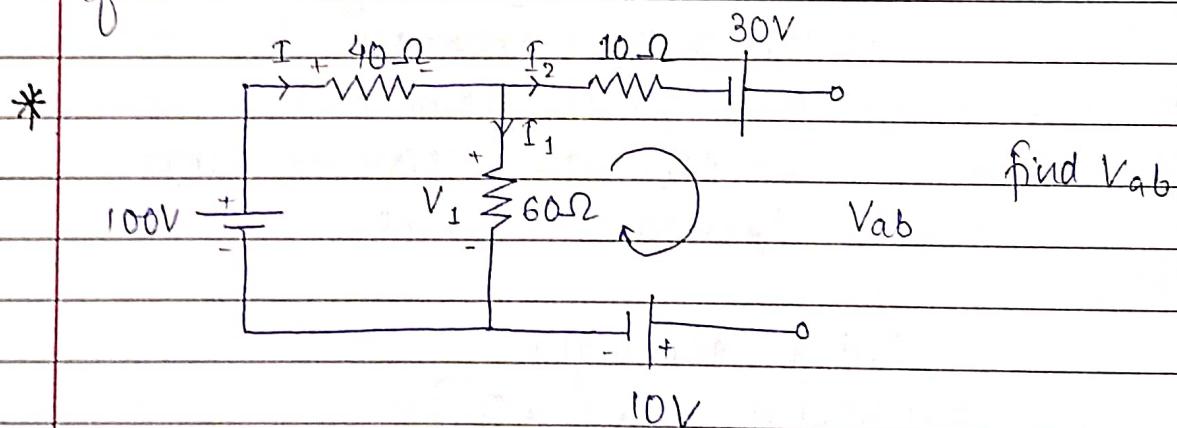
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Mark the polarity

- In a resistance, the polarity depends upon the assumed reference direction of current.
- for any resistance, the end to which current enters is marked as positive.
- The polarity of voltage across the battery does not depend upon the assumed direction of current.

Rule 1: While travelling, if you meet a voltage rise, write voltage with positive sign; write voltage with negative sign if you meet a voltage drop.

Rule 2: While travelling, write the voltage with positive sign if positive is encountered first; write voltage with negative sign, if negative is encountered first.



$$100 - 40I - 60I_1 = 0 \quad (I_1 = I, \text{ As } I_2 \text{ is } 0)$$

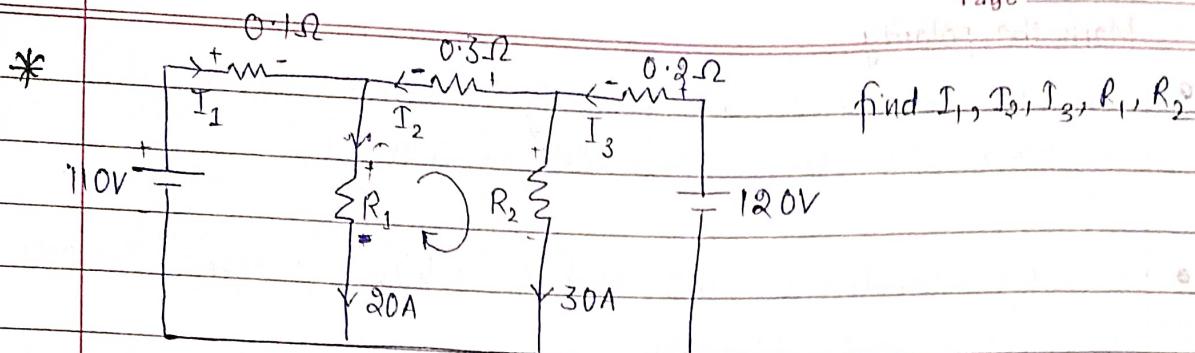
$$100 - 40I - 60I = 0$$

$$100 - 100I = 0$$

$$I = 1 \text{ A}$$

$$60 + 30 - V_{ab} - 10 = 0$$

$$V_{ab} = 80 \text{ V}$$



Applying KVL

$$110 - 0.1I_1 - 20R_1 = 0 \quad \text{(vi)}$$

$$0.3I_2 + R_2 \times 30 + 20R_1 = 0 \quad \text{(vii)}$$

Applying KCL

$$I_1 + I_2 = 20$$

$$I_1 + I_2 + 0I_3 = 20 \quad \text{(i)}$$

$$0I_1 - I_2 + I_3 = 30 \quad \text{(ii)}$$

$$-2I_2 + 2I_3 = 60 \quad \text{(v)}$$

Applying KVL in whole circuit

$$110 - 0.1I_1 + 0.3I_2 + 0.2I_3 - 120 = 0$$

$$-0.1I_1 + 0.3I_2 + 0.2I_3 = 10$$

$$-I_1 + 3I_2 + 2I_3 = 100$$

$$I_1 - 3I_2 - 2I_3 = -100 \quad \text{(viii)}$$

$$I_1 = -100 + 3I_2 + 2I_3 \quad \text{(i)}$$

Putting value in (i)

$$-100 + 3I_2 + 2I_3 + I_2 = 20$$

$$4I_2 + 2I_3 = 120 \quad \text{(iv)}$$

Solving eq (iv) & (v)

$$4I_2 + 2I_3 = 120$$

$$-2I_2 + 2I_3 = 60$$

+ - -

$$6I_2 = 60$$

$$I_2 = 10 \text{ A}$$

Putting value in (i)

$$10 + I_1 = 20$$

$$I_1 = 10 \text{ A}$$

Putting value of I_2 in (ii)

$$-10 + I_2 = 30$$

$$I_2 = 40 \text{ A}$$

Putting value of I_3 in (vi)

$$110 - 0.1 \times 10 - 20R_1 = 0$$

$$109 = 20R_1$$

$$R_1 = 5.45 \Omega$$

Putting value of R_1 and I_2 in (vii)

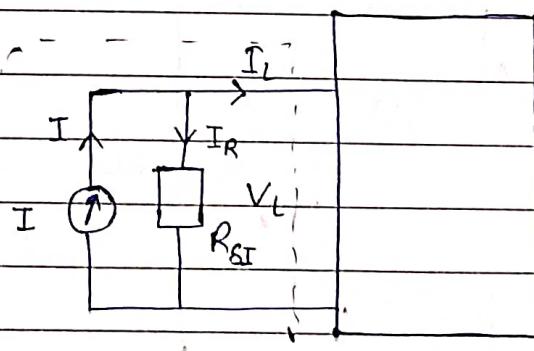
$$0.3 \times 10 - R_2 \times 30 + 20 \times 5.45 = 0$$

$$112 = 30R_2$$

$$R_2 = 3.733 \Omega$$

Practical Current Source :-

Practical Current Source is modelled as an ideal current source in parallel with a resistance. This resistance is called as Internal Resistance.



Using KCL,

$$I = I_L + I_R$$

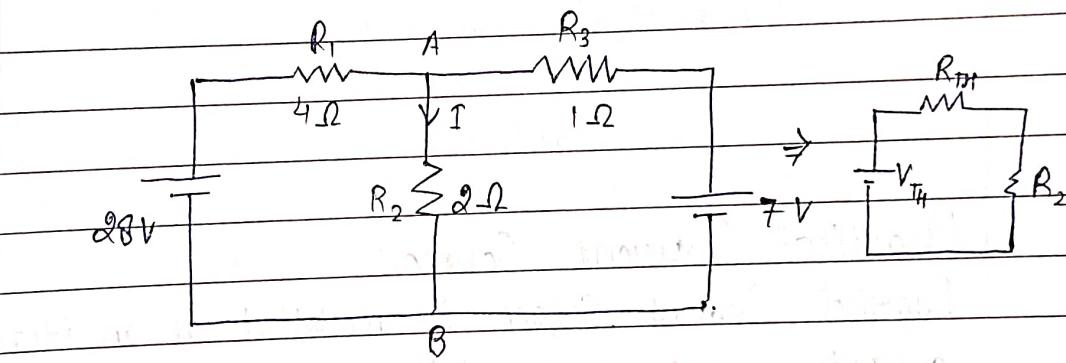
$$I_L = I - I_R$$

$$I_L = \frac{I - V_L}{R_{SI}}$$

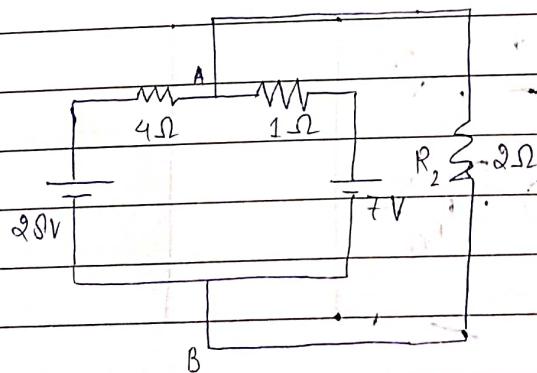
where, $\downarrow I_L = I - \frac{V_L}{R_{SI}}$

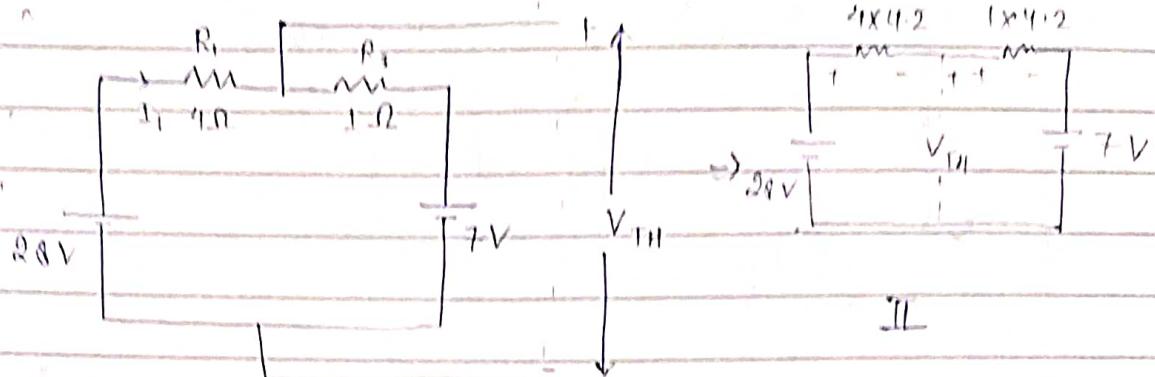
Thevenin's Theorem -

It states that it is possible to simplify any linear circuit containing independent and dependent voltage or current source no matter how complex, to an equivalent circuit with just a single voltage source and a series resistance between any two points in the circuit.



\Downarrow Invert the polarity of the 7V source.





$$28 - 4I_1 = 11.2 - 7 = 0$$

$$2I_1 = 8.8$$

$$I_1 = 4.4 \text{ A}$$

Applying KVL in II loop

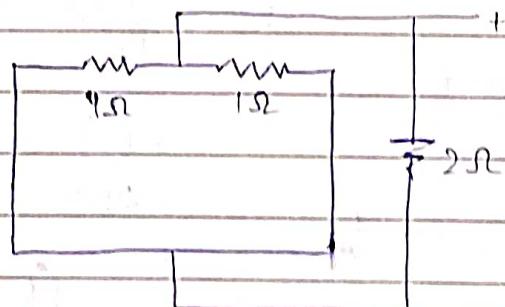
$$28 - 4 \times 4.4 - V_{Th} = 0$$

$$28 - 16.8 = V_{Th}$$

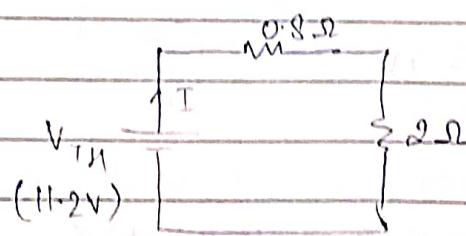
$$V_{Th} = 11.2 \text{ V}$$

for calculating R_{Th} ,

We will short circuit the independent voltage source and open the independent current sources.



$$R_{Th} = \frac{4 \times 1}{4 + 1} = \frac{4}{5} = 0.8 \Omega$$



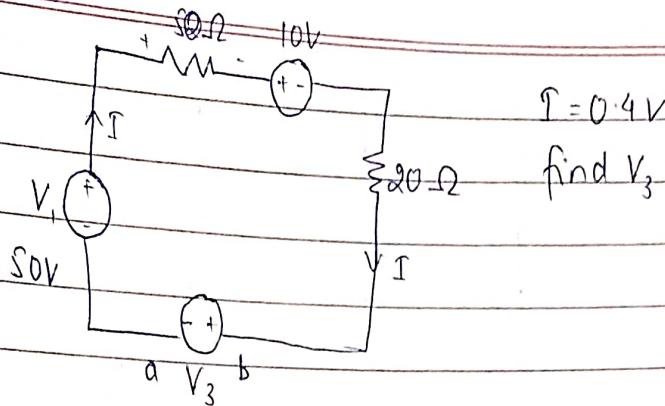
Applying KVL,

$$11.2 - (0.8I) = 0$$

$$11.2 = 0.8I$$

$$I = 14 \text{ A}$$

eg.



Applying KV1

$$50 - (50 \times 0.4) - 10 - V_3 = 0$$

$$40 - 20 - V_3 = 0$$

$$V_3 = 12V$$

$$V_{ab} = -12V$$

$$V_{ba} = 12V$$

Ques An oven takes 16A at 220V. It is desired to reduce the current to 12A.

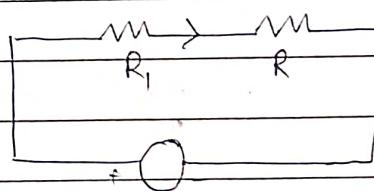
find (i) Resistance which must be connected in Series.

(ii) Voltage across the resistor.

$$I = 12A$$

$$V = IR_1$$

$$R = \frac{220}{16} = 13.75\Omega$$



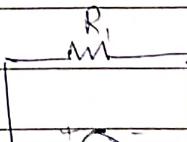
$$I(R_1 + R_2) = V$$

$$12 \left(\frac{220}{16} + R \right) = 220$$

$$\frac{220}{16} + R = 18.33$$

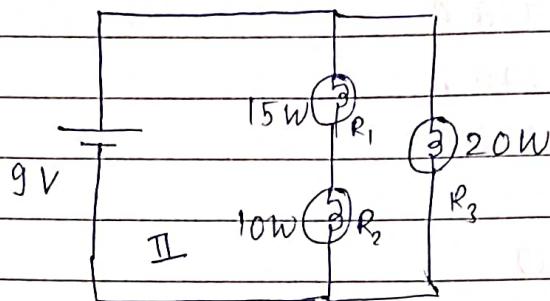
$$R = 18.33 - 13.75$$

$$R = 4.58\Omega$$



$$V = IR = 4.58 \times 12 = 54.96V$$

Ques.



Three bulbs are connected to a ~~9V~~ 9V battery as shown in the figure. Calculate

- Total current supplied by the battery
- Current through each bulb.
- The Resistance of each bulb!

$$P = V^2$$

$$V = IR$$

$$R_3$$

$$9 = I \times 4.05$$

$$20 = 8I$$

$$I_3 = 900$$

$$R_3$$

$$405$$

$$R_3 = 81$$

$$I_3 = 2.22A$$

$$R_3 = 4.05\Omega$$

Applying in Π loop

$$V = IR$$

$$9 = I_1 \left[\frac{P'}{T^2} + \frac{P''}{T_1^2} \right]$$

$$P = T^2 R$$

$R = P / T^2$ [In series T is same]

$$9 = 15 + 10$$

$$I_1$$

$$I_1 = I_2$$

$$I_1 = 25$$

$$9$$

$$I_1 = 2.778A$$

- Total current $I_1 = 2.778 + 2.22 = 5A$

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(ii) Through 15W , $I = 2.778\text{A}$

10W , $I = 2.778\text{A}$

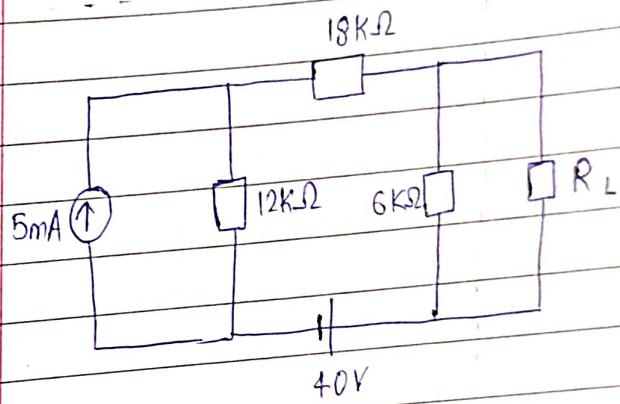
20W , $I'' = 2.222\text{A}$

(iii) $R_3 = 4.05\Omega$

$$R_1 = \frac{P}{I^2} = \frac{15}{(2.778)^2} = 1.944\text{f}\Omega$$

$$R_2 = \frac{P}{I^2} = \frac{10}{(2.778)^2} = 1.296\text{f}\Omega$$

Ques.

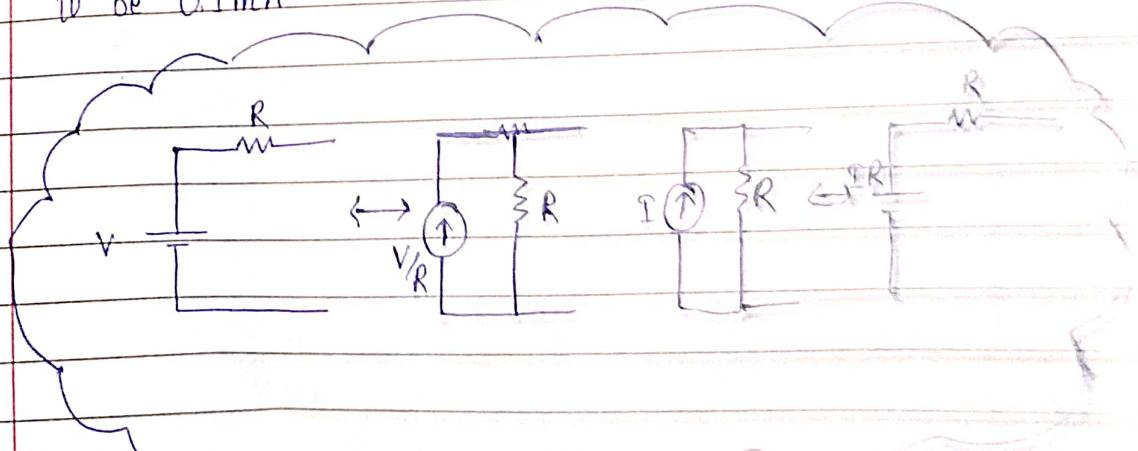


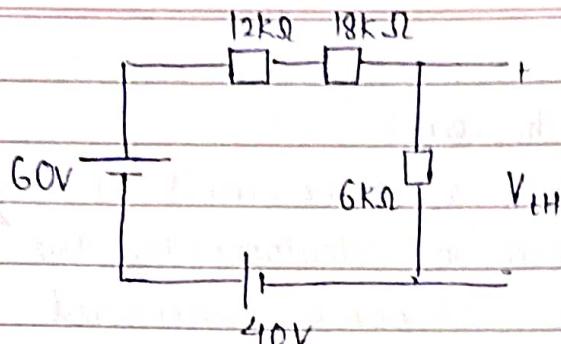
→ Replace the circuit in dotted box by Thevenin's Equivalent Circuit.

→ Find V_{ab} for $R_t = 3\text{k}\Omega$

→ find the value of R_t for which it receives maximum power from the circuit.

→ find the value of R_t that makes current in $6\text{k}\Omega$ to be 0.1mA





Applying KVL

$$60 - 36I + 40 = 0$$

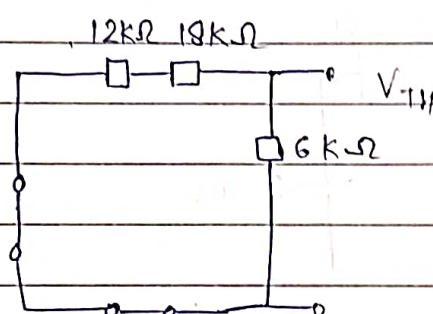
$$20 = 36I$$

$$I = \frac{20}{36} = 5 \text{ mA}$$

$$T = 0.555 \text{ mA}$$

$$V_{TH} = 6 \times 0.555 \text{ mA} = 3.330 \text{ mV} =$$

$$V_{TH} = 6 \times \frac{5}{9} = 3.333 \text{ V}$$

 R_{TH} —

12kΩ & 18kΩ are in series

So, 30 kΩ

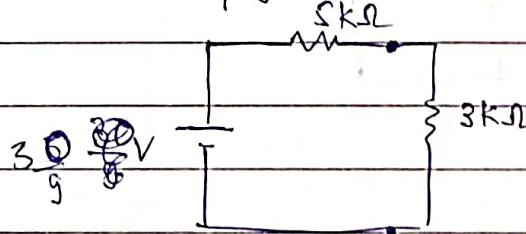
30kΩ & 6kΩ are in parallel.

$$\text{So, } R_{TH} = \frac{30 \times 6}{36} = 5 \text{ k}\Omega$$

36
6

(ii)

Applying KVL,



$$V = IR$$

$$V_{ab} = \frac{30 \times 3 \times 10^3}{9 (5+3) \times 10^3}$$

$$V_{ab} = \frac{90}{9 \times 8} = 1.25 \text{ V}$$

(iii)

for maximum power, $R_L = R_{TH}$ So, $R_L = 5 \text{ k}\Omega$

current source \rightarrow open circuit
voltage source \rightarrow short circuit

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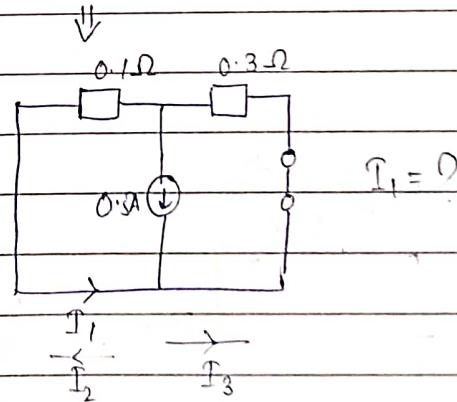
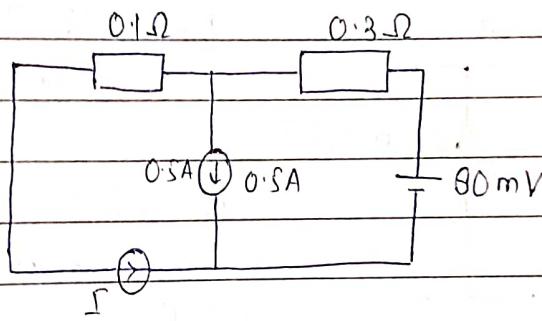
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Principle of Superposition -

(OR Superposition Theorem)

It states that the response in a linear circuit at any point due to multiple sources can be calculated by the summing the effects of each source considered separately, all other sources being made inoperative.

By making a source inoperative means that the voltage source is replaced by a short circuit and current source is being replaced by an open circuit.

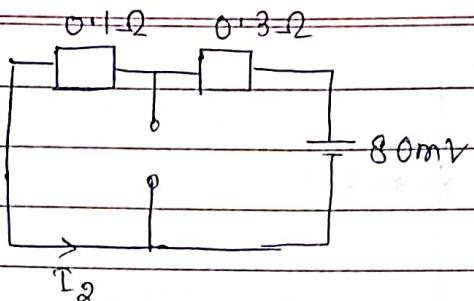


Using Current divider

$$I_2 = \frac{0.5 \times 0.3}{0.1 + 0.3} = \frac{0.15}{0.4} = 0.375\text{A}$$

$$I_1 = -I_2$$

$$I_1 = -0.375\text{A}$$



$$I_2 = \frac{80 \times 10^{-3}}{0.1 + 0.3}$$

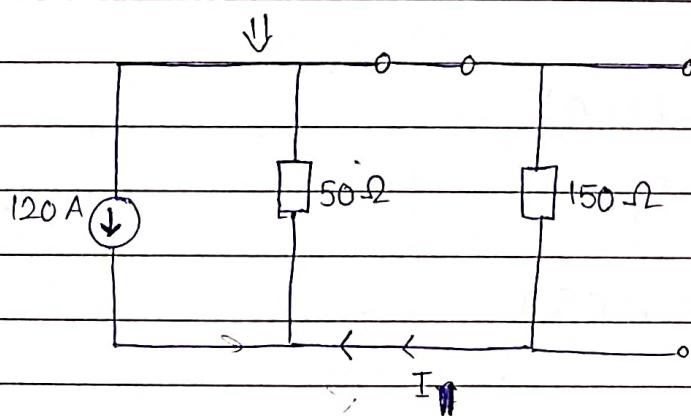
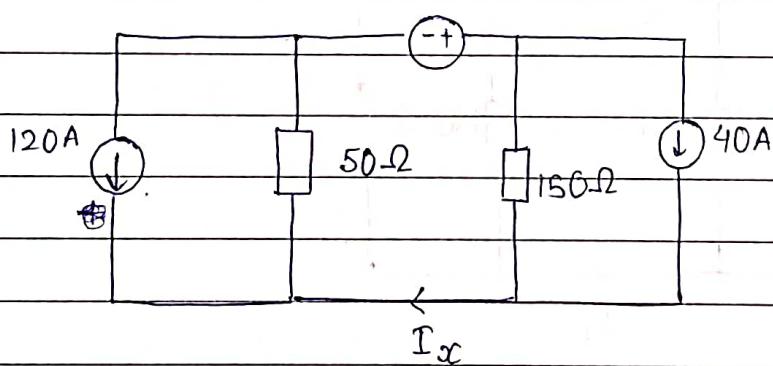
$$I_2 = 0.2 \text{ A}$$

$$I_1 = -0.375 \text{ A}$$

$$I_2 = 0.2 \text{ A}$$

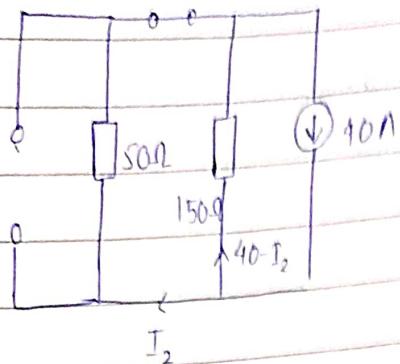
$$I = I_1 + I_2 = -0.375 + 0.2 = -0.175 \text{ A}$$

Ques.



$$I_1 = \frac{120 \times 150}{50 + 150} = 30 \text{ A}$$

$$I_1 = -30 \text{ A} \quad [\text{As dir. is opp.}]$$

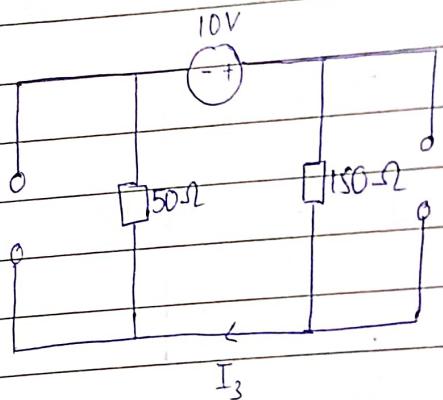


$$\text{Req} \quad 50I_2 = 150(40 - I_2)$$

$$50I_2 = 6000 - 150I_2$$

$$200I_2 = 6000$$

$$I_2 = 30A$$



$$V = IR$$

$$10 = I_3(200)$$

$$\frac{10}{200} = I_3$$

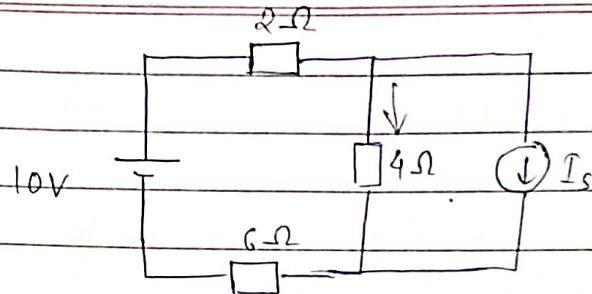
$$I_3 = 0.05A$$

$$I_N = I_1 + I_2 + I_3$$

$$I_N = (-30 + 30 + 0.05)A$$

$$I_N = 0.05A$$

Ques

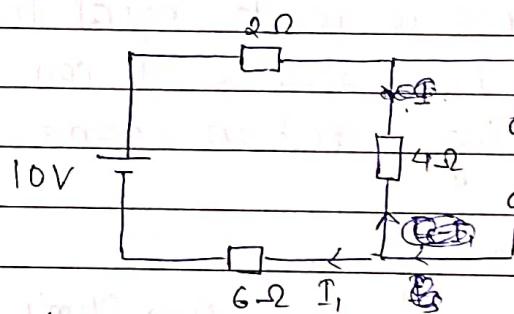


find current I_s to reduce the voltage across 4Ω to 0.8

$$10 - (2+6) I_s = 0$$

$$10 = 8 I_s$$

$$I_s = 1.25 \text{ A}$$

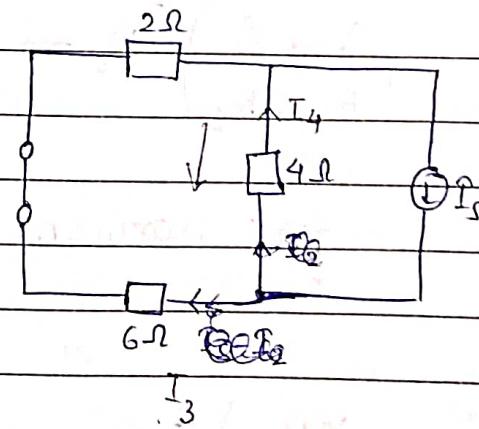


$$V = TR$$

$$10 = I_1 [2+4+6]$$

$$I_1 = 1.0 \text{ A}$$

$$12$$



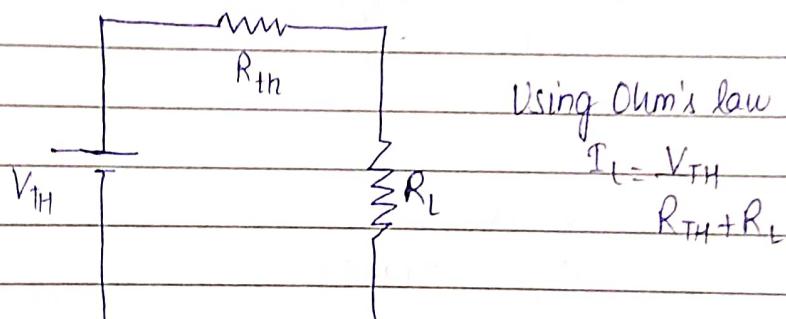
$$I_2 = -I_4 = -I_s \times \frac{8}{12}$$

IMP:

Maximum Power Transfer Theorem -

This theorem states that maximum power is absorbed from a network when the load resistance is equal to the output resistance of the network as seen from the terminals of the load.

When the load resistance is made equal to the output resistance of the circuit, it can be said that impedance matching has been done.



Using Ohm's law

$$I_L = \frac{V_{TH}}{R_{TH} + R_L}$$

$$P_L = I_L^2 R_L$$

$$P_L = \left(\frac{V_{TH}}{R_{TH} + R_L} \right)^2 R_L \quad \text{--- (1)}$$

$$\frac{dP_L}{dR_L} = 0 \quad \text{for maximum power}$$

$$\frac{d}{dR_L} \left(\frac{V_{TH}^2}{(R_L + R_{TH})^2} \times R_L \right) = 0$$

$$-\frac{2V_{TH}^2}{(R_L + R_{TH})^3} \times R_L + \frac{V_{TH}^2}{(R_L + R_{TH})^2} = 0$$

$$\frac{2V_{TH}^2 R_L}{(R_L + R_{TH})^3} = \frac{V_{TH}^2}{(R_L + R_{TH})^2}$$

$$\frac{2R_L}{R_L + R_{TH}} = 1$$

$$2R_L = R_L + R_{TH}$$

$$R_L = R_{TH}$$

Putting value of R_L in ①

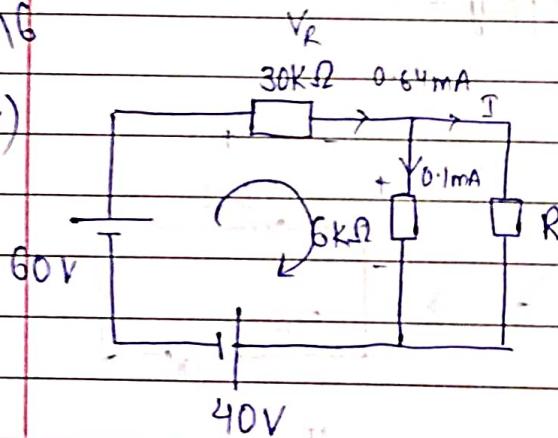
$$P_L = \left(\frac{V_{TH}}{R_{TH} + R_{TH}}\right)^2 R_{TH}$$

$$P_L = \frac{V_{TH}^2}{4R_{TH}}$$

Available Power

Pg 16

(iv)



Using KVL

$$-40 + 60 - V_R - 0.6 = 0$$

$$V_R = 19.4V$$

$$V_R = IR$$

$$19.4 = I \times 30k\Omega$$

$$I = 0.64mA$$

Using KCL,

$$0.64 \text{ mA} = I + 0.1 \text{ mA}$$

$$I = 0.54 \text{ mA}$$

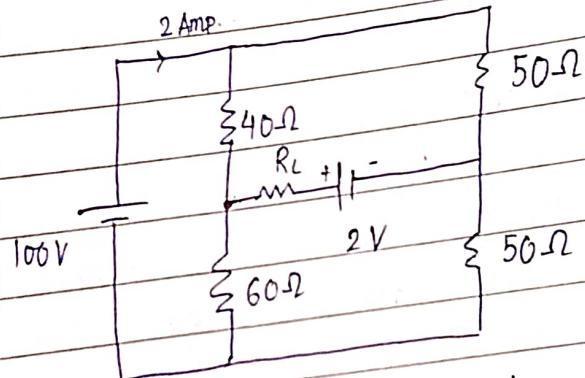
$$V = IR$$

$$0.6 = R$$

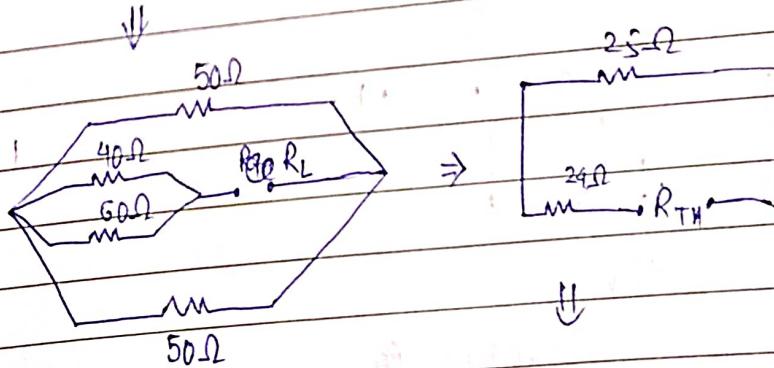
$$0.54$$

$$R = 1.1 \text{ k}\Omega$$

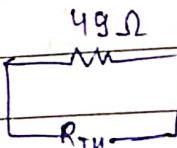
Ques

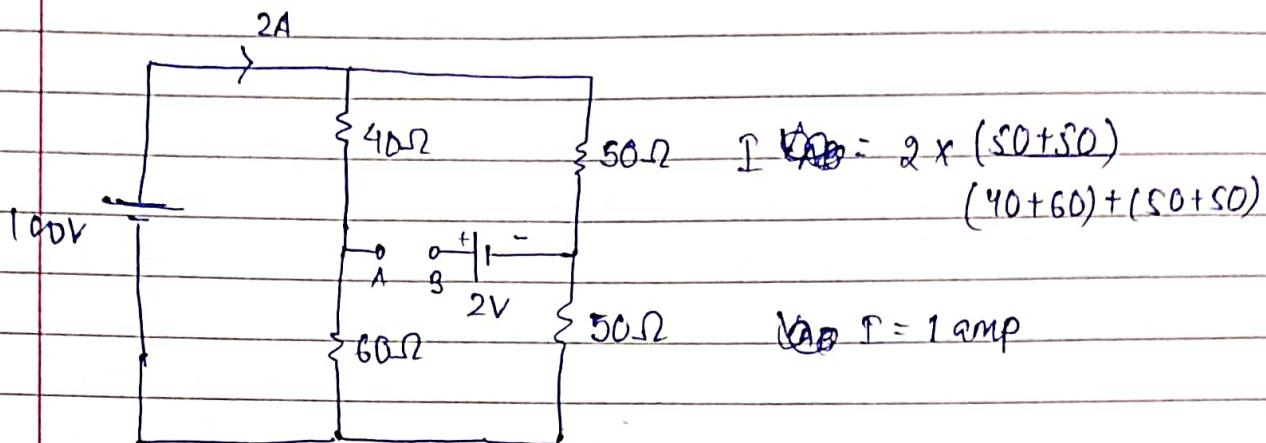


Maximum power supply - ?



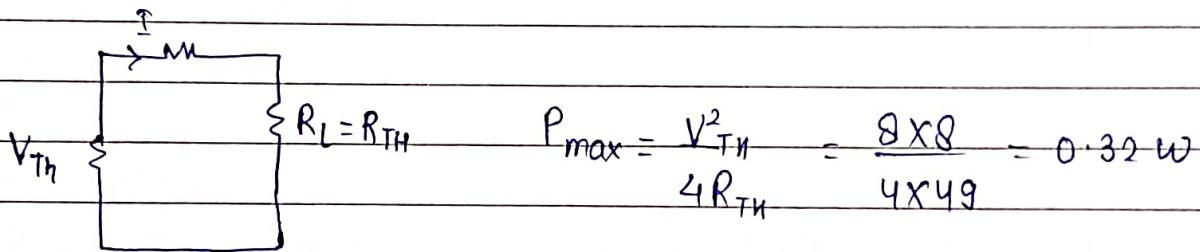
$$R_{TH} = 49 \Omega$$





$$-40I - V_{ab} - 2 + 50 \times 1 = 0$$

$$V_{ab} = 0 \text{ V}$$



Alternating Current

Theory of Cycle -

One complete set of positive and negative value of the function (which goes on repeating) is called as Cycle.

Peak Value -

It is the maximum value either in the positive or the negative of the quantity.

Time Period -

It is the duration of time required for the quantity to complete one cycle.

Frequency -

It is the number of oscillations/cycle that occur in a second.

Angular Frequency -

It is equal to the number of radian covered in a second. Denoted by ω .

$$\omega = 2\pi f = 2\pi$$

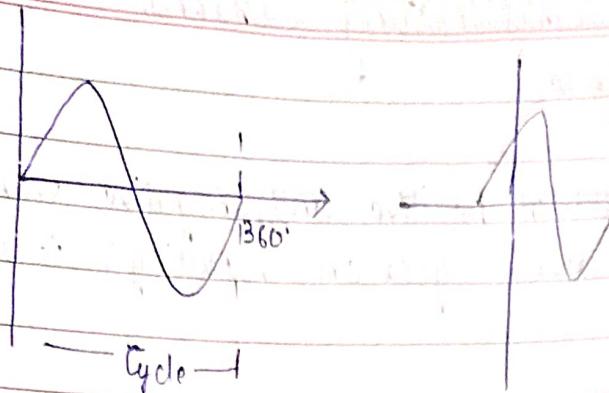
Its SI unit is rad sec⁻¹.

Phase -

It is the fraction of the time period or cycle that has elapsed since it last passed from the chosen zero position or origin.

$$\text{phase} = \frac{t}{T}$$

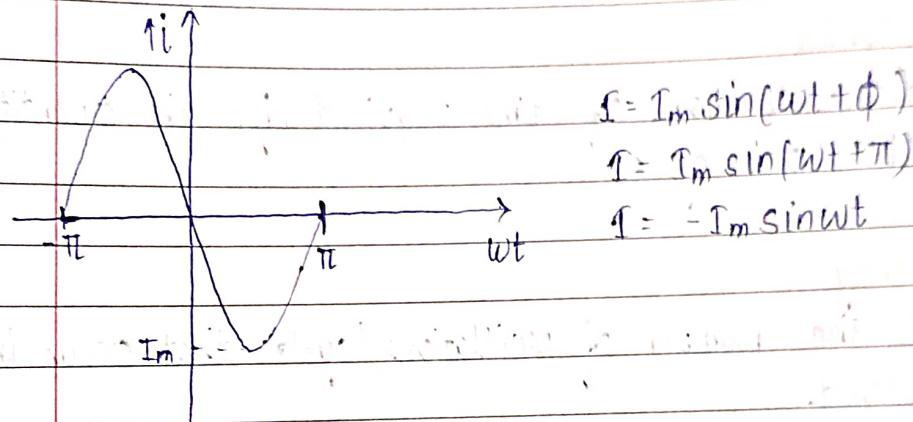
where t is Time Period.



$$I_m \sin(wt + \theta)$$

left side $\rightarrow +\theta$

right side $\rightarrow -\theta$



→ If ϕ is positive number, the waveform is shifted towards left and if ϕ is a negative number, the waveform is shifted towards right.

→ If ϕ is positive, it is angle of lead.

→ If ϕ is negative, it is angle of lag.

Note:-

• For a full cycle of sinusoidal waveform it has same area in the positive and negative loops. Hence, the algebraic sum of two areas is zero.

So, we can say that average value of sine wave over one full cycle is 0.

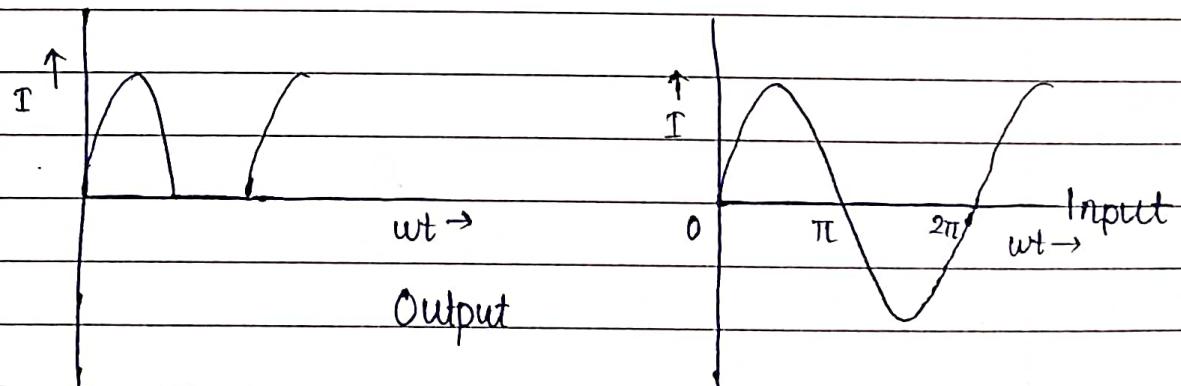
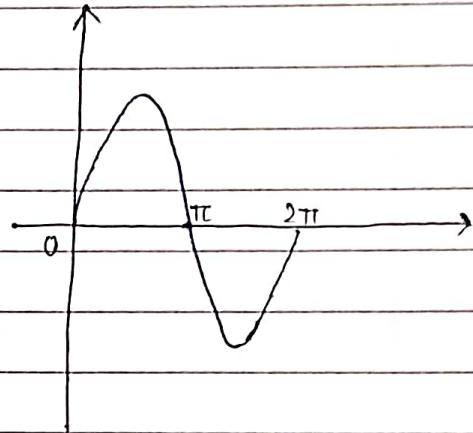
Therefore, for a sinusoidal waveform we define average value of over half cycle only.

- For any symmetrical periodic waveform the average value is calculated over only half cycle.

for ~~full~~ wave -

$$\begin{aligned}
 I_{avg} &= \frac{1}{\pi} \int_0^{\pi} I_m \sin wt d(wt) \\
 &= \frac{I_m}{\pi} \int_0^{\pi} \sin wt d(wt) \\
 &= \frac{I_m}{\pi} \left[-\cos wt d(wt) \right]_0^{\pi} \\
 &= \frac{I_m}{\pi} \left[-\cos \pi + \cos 0 \right]
 \end{aligned}$$

$I_{avg} = \frac{2I_m}{\pi}$



$$I_{avg} = \frac{1}{2\pi} \int_0^{2\pi} I d(wt) = \frac{1}{2\pi} \int_0^{2\pi} I_m \sin wt d(wt) = \frac{I_m}{2\pi} \left[-\cos wt \right]_0^{2\pi}$$

$$I_{avg} = \frac{I_m}{2\pi} \left[-\cos 2\pi + \cos 0 \right] = 0$$

Complex plane -

$$\begin{aligned} Z &= a + jb \quad \text{(1) Rectangular form} \\ \theta &= \tan^{-1}\left(\frac{b}{a}\right) \end{aligned}$$

$$Z = R + j\omega L$$

Here, Z is impedance

$$Z = R - j$$

ωC

Putting value of a & b in (1)

$$Z = r \cos \theta + j r \sin \theta \quad \text{(trigonometric form)}$$

$$Z = r e^{j\theta} \quad \text{(exponential form)}$$

$$Z = r \angle \theta \quad \text{(Polar form)}$$

Operations -

$$Z_1 Z_2 = |Z_1| |Z_2| e^{j(\theta_1 + \theta_2)}$$

$$\frac{Z_1}{Z_2} = \frac{|Z_1| e^{j\theta_1}}{|Z_2| e^{j\theta_2}}$$

Ques. A sinusoidal current of $10 \cos 60^\circ A$ is added to another sinusoidal current of $20 \cos 0^\circ A$. Find the resultant current.

$$Z = r \cos \theta + j r \sin \theta$$

$$Z = 10 \cos 0^\circ + j 10 \sin 0^\circ$$

$$Z_2 = \frac{20 \cos 0^\circ + j 20 \sin 0^\circ}{2}$$

$$Z_2 = 10 \cos 0^\circ + j 10 \sin 0^\circ$$

$$\begin{aligned} Z_1 + Z_2 &= 20 \cos 0^\circ + j 10 \sin 0^\circ \\ &= 10(2 + j\sqrt{3}) \end{aligned}$$

$$Q = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{V_2}{V_1}\right) = 40.8^\circ$$

$$\begin{aligned} R &= \sqrt{b^2 + a^2} = \sqrt{900 + 3600} = \sqrt{4500} = 26.45\Omega \\ Z &= R + jX = 26.45 + j10 \\ Z_{eq} &= 26.45 \angle 40.8^\circ \end{aligned}$$

Ques Determine the average and rms value of the resultant current in a wire carrying simultaneously a DC current of 10A and a sinusoidal current of peak value of 10A find the average and rms value.

$$I_{DC} = 10A$$

$$I_{AC} = 10\sin\omega t$$

$$\text{Resultant } I = 10 + 10\sin\omega t$$

$$\begin{aligned} I_{avg} &= \int_{-\pi}^{\pi} (10 + 10\sin\omega t) dt = \frac{1}{2\pi} \int_{-\pi}^{\pi} (10 + 10\sin\omega t)^2 d(\omega t) \\ &= 10 + 0 \end{aligned}$$

$$I_{avg} = \frac{1}{2\pi} \int_{-\pi}^{\pi} (10 + 10\sin\omega t)^2 d(\omega t)$$

$$I_{avg} = 10$$

$$I_{rms} = \sqrt{\frac{1}{2\pi} \int_{-\pi}^{\pi} (10 + 10\sin\omega t)^2 d(\omega t)}$$

$$I_{rms} = \sqrt{\frac{100}{2\pi} \int_{-\pi}^{\pi} (1 + \sin\omega t + \sin^2\omega t) d(\omega t)}$$

$$I_{rms} = \sqrt{\frac{200}{2\pi} \int_{-\pi}^{\pi} (1 + \cos\omega t + \frac{1 - \cos2\omega t}{2}) d(\omega t)}$$

$$I_{rms} = \sqrt{\frac{200}{2\pi} \int_{-\pi}^{\pi} (2 + \cos\omega t + \sin\omega t) d(\omega t)}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{n} \int_0^T V^2 dt} = \sqrt{1 + 2(1 - \cos \omega t)}$$

Q. 87

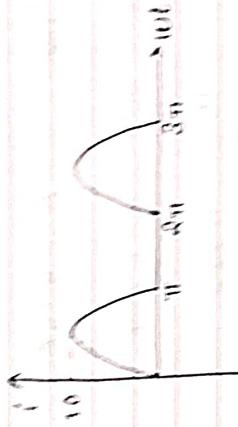
$$V_{\text{rms}} = \sqrt{\frac{1}{n} \int_0^T V^2 dt}$$

$$V_{\text{rms}} = \sqrt{100}$$

resultant
a DC current
line of

total AV_{rms}

find power consumed by load
when above current
is flowing through it



$$\text{Power} = \int_0^T V^2 dt = \int_0^T \left[100 + 200 \sin^2 \omega t \right] dt = \frac{1}{2} \int_0^T [100 + 100 \cos 2\omega t] dt$$

$$\text{Power} = \frac{1}{2} \int_0^T [100 + 100 \cos^2 \omega t] dt = \frac{1}{2} \int_0^T [100 + 50 + 50 \cos 2\omega t] dt = \frac{1}{2} \int_0^T [150 + 50 \cos 2\omega t] dt$$

$$\text{Power} = \frac{1}{2} \left[150t + 25 \sin 2\omega t \right]_0^T = \frac{1}{2} \left[150T + 25 \sin 2\omega T \right] = \frac{1}{2} \left[150T + 25 \sin 2\pi \right] = \frac{1}{2} \left[150T \right] = 75T$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T V^2 dt} = \sqrt{\frac{1}{T} \int_0^T [150t + 25 \sin 2\omega t] dt} = \sqrt{\frac{1}{T} \left[150t^2 + 25 \left(\frac{-1}{2} \cos 2\omega t \right) \right]} = \sqrt{\frac{1}{T} \left[150T^2 + 25 \left(\frac{1}{2} \right) \right]} = \sqrt{\frac{1}{T} \left[150T^2 + 12.5 \right]} = \sqrt{150T^2 + 12.5} = \sqrt{150T^2} = \sqrt{150} T$$

$$V_{\text{rms}} = \sqrt{\frac{1}{2n} \int_0^{2\pi} V^2 dt} = \sqrt{\frac{1}{2n} \left[150t^2 + 25 \left(\frac{1}{2} \cos 2\omega t \right) \right]} = \sqrt{\frac{1}{2n} \left[150t^2 + 12.5 \right]} = \sqrt{\frac{1}{2n} \left[150 \left(\frac{2\pi}{\omega} \right)^2 + 12.5 \right]} = \sqrt{\frac{1}{2n} \left[150 \cdot \frac{4\pi^2}{\omega^2} + 12.5 \right]} = \sqrt{\frac{1}{2n} \left[150 \cdot 4 \cdot 3.14^2 + 12.5 \right]} = \sqrt{\frac{1}{2n} \left[150 \cdot 4 \cdot 9.86 + 12.5 \right]} = \sqrt{\frac{1}{2n} \left[5910 + 12.5 \right]} = \sqrt{\frac{1}{2n} \left[5922.5 \right]} = \sqrt{\frac{5922.5}{2n}} = \sqrt{2961.25/n}$$

$$P = V_{\text{rms}} R = \sqrt{2961.25/n} \cdot 250 \Omega$$



Find I_{rms} , I_{av} , Φ

$$I_{\text{av}} = \frac{10 \times 2 + 0}{3 - 0} = \frac{20}{3} \text{ A}$$

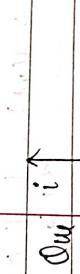
$$I_{\text{rms}} = \sqrt{\int_0^3 i^2 dt} = \sqrt{\int_0^1 10^2 dt + \int_1^3 0^2 dt}$$

$$I_{\text{rms}} = \sqrt{\frac{1}{3} \times 10^2 [t]_0^2} = \sqrt{\frac{100}{3} [2-0]}$$

$$I_{\text{rms}} = \sqrt{\frac{200}{3}} \text{ A} = 8.16 \text{ A}$$

$$\text{form factor } \frac{I_{\text{rms}}}{I_{\text{av}}} = \frac{8.16 \times 3}{20} \approx 1.224$$

$$\text{peak factor } \frac{I_m}{I_{\text{rms}}} = \frac{10}{8.16} \approx 1.225$$



Equation of AB

$$I_{\text{av}} = \frac{1}{2} \times 5 \times 10 = \frac{50}{10} = 5 \text{ A}$$

$$I_{\text{rms}} = \sqrt{\frac{1}{5} \int_0^5 i^2 dt} = \sqrt{\frac{1}{5} \int_0^5 4t^2 dt} = \sqrt{\frac{1}{5} \times \frac{4}{3} t^3} = \sqrt{\frac{16}{15} t^3} = \sqrt{\frac{16}{15} \times 12.5} = \sqrt{\frac{160}{15}} = 5.12 \text{ A}$$

$$I_{\text{rms}} = \sqrt{\frac{4}{5} \left[\frac{t^3}{3} \right]_0^5} = \sqrt{\frac{4}{5} \times \frac{12.5}{3}} = \sqrt{\frac{100}{3}} = 5.74 \text{ A}$$

$$\text{form factor } \frac{I_{\text{rms}}}{I_{\text{av}}} = \frac{5.74}{5} = 1.148$$

$$\text{peak factor } \frac{I_m}{I_{\text{rms}}} = \frac{10}{5.74} = 1.732$$

$$\text{Peak factor } \frac{I_m}{I_{\text{av}}} = \frac{10}{5} = 2$$