Steady State AC Analysis
Tutorial sheet No. 2

1) $f = 50\,Hz$, $I_{rms} = 20A$

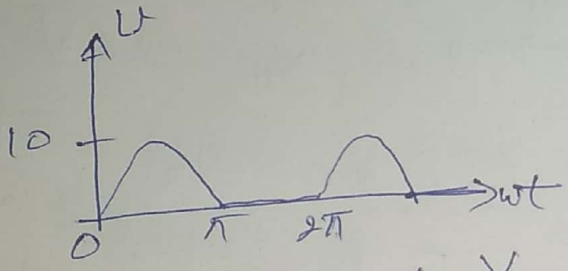
$I_m = \sqrt{2}\, I_{rms} = 20\sqrt{2}$

$\omega = 2\pi f = 2\pi \times 50 = 100\pi$

$i = I_m \sin \omega t$

$= 20\sqrt{2} \sin(100\pi t)$ Ans

2)



$T = 2\pi$, $V_m = 10\,V$

$V_{av} = \frac{1}{T}\int_0^T v\, d\omega t$

$= \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin\omega t\, d\omega t + \int_{\pi}^{2\pi} 0\, d\omega t\right]$

$= \frac{1}{2\pi} \times V_m \left[\int_0^{\pi} \sin\omega t\, d\omega t\right] = \frac{V_m}{2\pi}\left(-\cos\omega t\right)_0^{\pi}$

$= \frac{V_m}{2\pi}\left[-\cos\pi + \cos 0\right]$

$= \frac{V_m}{2\pi}\left[1 + 1\right] = \frac{V_m}{\pi} = \frac{10}{\pi} = 3.18\,V$ Ans

$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2\, d\omega t} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} (V_m \sin\omega t)^2\, d\omega t + 0}$

$= \left[\frac{V_m^2}{2\pi}\int_0^{\pi} \sin^2\omega t\, d\omega t\right]^{1/2} = \left(\frac{V_m^2}{2\pi}\int_0^{\pi} \frac{1}{2} - \frac{\cos 2\omega t}{2}\, d\omega t\right)^{1/2}$

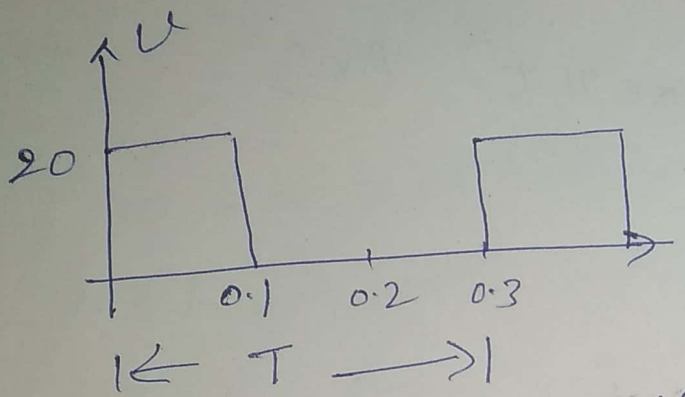
$= \left[\frac{V_m^2}{2\pi}\left[\frac{\omega t}{2} - \frac{\sin 2\omega t}{4}\right]_0^{\pi}\right]^{1/2} = \left[\frac{V_m^2}{2\pi}\left[\frac{\pi}{2} + 0 - \frac{0}{4} + \frac{0}{4}\right]\right]^{1/2}$

$$V_{rms} = \frac{V_m}{2} = \frac{10}{2} = 5\,V \quad \text{Ans}$$

$$k_f = \frac{V_{rms}}{V_{av}} = \frac{5}{3.18} = 1.57 \quad \text{Ans.}$$

$$k_p = \frac{V_m}{V_{rms}} = \frac{10}{5} = 2 \quad \text{Ans}$$

3)



$T = 0.3\,sec$

$V_m = 20\,V$

$v = 20 \quad 0 \le t < 0.1$

$= 0 \quad 0.1 \le t < 0.3$

$$V_{av} = \frac{1}{T}\int_0^T v\,dt = \frac{1}{0.3}\int_0^{0.3} v\,dt = \frac{1}{0.3}\left[\int_0^{0.1} 20\,dt + 0\right]$$

$$= \frac{1}{0.3} \times 20\left[\int_0^{0.1} dt\right] = \frac{20 \times 0.1}{0.3} = \frac{20}{3}$$

$$\boxed{V_{av} = 6.67\,V} \quad \text{Ans}$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2\,dt} = \sqrt{\frac{1}{0.3}\left(\int_0^{0.1} 20^2\,dt\right)}$$

$$= \sqrt{\frac{400}{0.3} \times 0.1}$$

$$= 11.5\,V \quad \text{Ans}$$

4) (A) $v = \sqrt{2}(50)\sin(377t - 35^\circ)$

$V_{rms} = \frac{V_m}{\sqrt{2}}$; $V_m = \sqrt{2} \times 50$

$= 50V$

Phase $= -35^\circ$

Compare with $v = V_m \sin(\omega t + \theta)$

$\theta = -35^\circ$

(B) $v = \sqrt{2} \times (90.4)\sin(754t + 48^\circ)$

$V_m = \sqrt{2} \times 90.4$

$V_{rms} = \frac{V_m}{\sqrt{2}} = 90.4V$

$\theta = 48^\circ$

(C) $v = 83.6\cos(400t - 15^\circ)$

$v = 83.6\sin(400t - 15 + 90)$; $\sin(90+\theta) = \cos\theta$

$v = 83.6\sin(400t + 75^\circ)$

$V_m = 83.6$, $\theta = 75^\circ$

$V_{rms} = \frac{83.6}{\sqrt{2}} = 59.1V$

(D) $i = 3.46\cos(815t + 30^\circ) = 3.46\sin(815t + 30 + 90)$

$= 3.46\sin(815t + 120^\circ)$

$I_m = 3.46A$, $\theta = 120^\circ$

$I_{rms} = \frac{3.46}{\sqrt{2}}$, $\theta = 120^\circ$

$= 2.45A$, $\theta = 120^\circ$

5 (a) $6 + j9 = 10.8\angle 56.3^\circ$ → use scientific calculator to convert

(b) $-21.4 + j33.3 = 39.6\angle 122.7^\circ$

6 (a) $6.21 + j3.24 + 4.134 - j9.47$

$= 6.21 + 4.13 + j(3.24 - 9.47)$

$= 10.34 - 6.23$

(b) $7.34 - j1.29 - (5.62 + j0.92)$ (4)

$= 7.34 + (-5.62) - j1.29 - j0.92$

$= 1.72 - j6.23$

(c) $-24 + j12 - (-36 - j16) - (17 - j24)$

$= -24 + 36 - 17 + j(12 + 16 + 24)$

$= -5 + j52$

(7) (a) $(4 + j2)(3 + j4) = 4.472 \angle 26.56 \times 5 \angle 53.13$

$= 22.36 \angle 79.69^\circ = 4 + j21.998$

* use scientefic calculator

(b) $(6 + j2)(3 - j5)(2 - j3)$

$= (6.3245 \angle 18.4349^\circ)(5.83 \angle -59.036)(3.6 \angle -56.31$

$= 132.644 \angle -96.9111$

$= -15.96 - j131.68$