Creating Customer Segments by RFM score Using Clustering

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Steps

- 1. Import Libraries
- 2. Import Data
- 3. Preprocess Data
- 4. Explore Data
- 5. Implement Algorithms

Import Libraries

In general, import all libraries before importing data. However, for learning purpose, import libraries in each step as needed. This is will give a better understanding of the libraries and their functions.

```
In [1]: # Import Libraries necessary for this project
#For all data processing functions
#import panda as pd

#For all numerical processing
#import numpy as np

#from IPython.display import display # Allows the use of display() for DataFra
mes
#import matplotlib.pyplot as plt

# Import supplementary visualizations code visuals.py
#import visuals as vs
```

Import Data

In general, import all needed libraries before importing data. If this is a learning execise, import libraries in each step as needed. This is will give a better understanding of the libraries.

In [2]: #import necessary libraries
import pandas as pd
from IPython.display import display

Load the dataset into pandas dataframe
raw_data = pd.read_excel("Online_Retail.xlsx")
print ("Dataset has {} rows(samples) with {} columns(features) each.".format(*
raw_data.shape))

display the top 5 rows of the dataset
raw_data.head(5)

Dataset has 541909 rows(samples) with 8 columns(features) each.

Out[2]:

	InvoiceNo	StockCode	Description	Quantity	InvoiceDate	UnitPrice	CustomerID	Co
0	536365	85123A	WHITE HANGING HEART T- LIGHT HOLDER	6	2010-12-01 08:26:00	2.55	17850.0	Unit Kinç
1	536365	71053	WHITE METAL LANTERN	6	2010-12-01 08:26:00	3.39	17850.0	Uni Kin
2	536365	84406B	CREAM CUPID HEARTS COAT HANGER	8	2010-12-01 08:26:00	2.75	17850.0	Unit Kinç
3	536365	84029G	KNITTED UNION FLAG HOT WATER BOTTLE	6	2010-12-01 08:26:00	3.39	17850.0	Unit Kinç
4	536365	84029E	RED WOOLLY HOTTIE WHITE HEART.	6	2010-12-01 08:26:00	3.39	17850.0	Unit Kinç

In [3]: #summary of dataset's distribution
 raw_data.describe()

Out[3]:

	Quantity	UnitPrice	CustomerID
count	541909.000000	541909.000000	406829.000000
mean	9.552250	4.611114	15287.690570
std	218.081158	96.759853	1713.600303
min	-80995.000000	-11062.060000	12346.000000
25%	1.000000	1.250000	13953.000000
50%	3.000000	2.080000	15152.000000
75%	10.000000	4.130000	16791.000000
max	80995.000000	38970.000000	18287.000000

Out[4]:

	InvoiceNo	StockCode	Description	Quantity	InvoiceDate	UnitPrice	CustomerID	
141	C536379	D	Discount	-1	2010-12-01 09:41:00	27.50	14527.0	l
154	C536383	35004C	SET OF 3 COLOURED FLYING DUCKS	-1	2010-12-01 09:49:00	4.65	15311.0	l F
235	C536391	22556	PLASTERS IN TIN CIRCUS PARADE	-12	2010-12-01 10:24:00	1.65	17548.0	l F
236	C536391	21984	PACK OF 12 PINK PAISLEY TISSUES	-24	2010-12-01 10:24:00	0.29	17548.0	l F
237	C536391	21983	PACK OF 12 BLUE PAISLEY TISSUES	-24	2010-12-01 10:24:00	0.29	17548.0	l F

In [5]: from datetime import datetime #calculate recency for each transaction currentDT = pd.to_datetime(datetime.now().date()) raw data['Recency']= (pd.to datetime(datetime.now().date())-pd.to datetime(raw data.InvoiceDate)).dt.days # store the returned items in a dataset returned invoices=raw data[raw data.Quantity<0] returned_invoices['Quantity']=returned_invoices['Quantity']*-1 # remove transactions that has negative or 0 quantity or with unit price raw_data=raw_data[(raw_data.Quantity>0) & (raw_data.UnitPrice>0)] # get the list of items that are purchased and then returned keys = ['CustomerID', 'StockCode','Quantity'] i1 = raw_data.set_index(keys).index i2 = returned invoices.set index(keys).index raw_data_filtered=raw_data[~i1.isin(i2)]

C:\Users\kanja\Anaconda3\envs\py36\lib\site-packages\ipykernel_launcher.py:8:
SettingWithCopyWarning:

A value is trying to be set on a copy of a slice from a DataFrame. Try using .loc[row_indexer,col_indexer] = value instead

See the caveats in the documentation: http://pandas.pydata.org/pandas-docs/stable/indexing.html#indexing-view-versus-copy

```
In [6]: import numpy as np

#Calculate revenue(monetization) generated by each customer
def total_price(raw_data_filtered):
    x = (raw_data_filtered.Quantity * raw_data_filtered.UnitPrice)
    return np.sum(x)

data=pd.DataFrame()
data['monetization']=raw_data_filtered.groupby('CustomerID').apply(total_price)

#get the recency of the account
data['recency']=raw_data.groupby('CustomerID').agg({'Recency': np.min})

#get the total number of transactions
data['frequency']=raw_data.groupby('CustomerID').Recency.nunique()

#save customer_id which is an index in the data frame
CustomerID=data.index
# show summary of the data distribution
data.describe()
```

Out[6]:

	monetization	recency	frequency
count	4324.000000	4324.000000	4324.000000
mean	1924.122274	2493.968085	3.873265
std	8228.468418	100.039894	5.956480
min	2.900000	2402.000000	1.000000
25%	305.475000	2419.000000	1.000000
50%	663.630000	2452.000000	2.000000
75%	1624.420000	2543.000000	4.000000
max	278528.420000	2775.000000	132.000000

Preprocess Data

```
In [7]: from sklearn import preprocessing

print('Is there null value in the data frame? {}.' .format('Yes' if (data.isnu ll().values.any()) else 'No' ))
   if (data.isnull().values.any()):
        print(data.isnull().sum())

#normalize all the columns(features) so that all the values in the column lie between 0 and 1
   #this way each features will get equal preference regardless of their actual r ange
   n_data=pd.DataFrame()
   n_data = pd.DataFrame(preprocessing.normalize(data),columns=data.columns)
   n_data.head(5)
```

Is there null value in the data frame? No.

Out[7]:

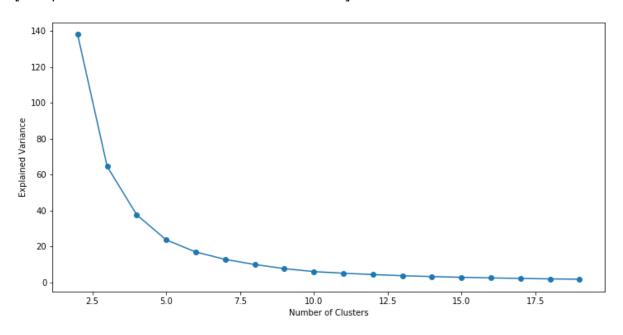
	monetization	recency	frequency
0	0.873333	0.487121	0.001418
1	0.587270	0.809390	0.001307
2	0.587636	0.809126	0.000334
3	0.122377	0.992484	0.000366
4	0.499385	0.866376	0.002488

Explore Data

In [8]: #Elbow method & silhouette score import matplotlib.pyplot as plt from sklearn.metrics import silhouette score from sklearn.cluster import KMeans # Pretty display for notebooks %matplotlib inline cluster range = range(2, 20) cluster_errors = [] for num clusters in cluster range: clusters = KMeans(num_clusters) clusters.fit(n_data) preds = clusters.predict(n data) cluster errors.append(clusters.inertia) score = silhouette_score(data, preds, metric='euclidean') print ("For K-means n_clusters = {}. The average silhouette_score is : {}" .format(num_clusters, score)) clusters df = pd.DataFrame({ "num clusters":cluster range, "cluster errors": cluster_errors }) plt.figure(figsize=(12,6)) plt.ylabel('Explained Variance') plt.xlabel('Number of Clusters') plt.plot(clusters df.num clusters, clusters df.cluster errors, marker = "o")

For K-means n clusters = 2. The average silhouette score is: 0.5898505014145 For K-means n clusters = 3. The average silhouette score is: 0.4828100242967 8055 For K-means n clusters = 4. The average silhouette score is: 0.4654911613248 501 For K-means n clusters = 5. The average silhouette score is: 0.4531849302091 061 For K-means n clusters = 6. The average silhouette score is: 0.4327111002623 7675 For K-means n clusters = 7. The average silhouette score is: 0.4240064158466 For K-means n_clusters = 8. The average silhouette_score is : 0.4201046290238 078 For K-means n clusters = 9. The average silhouette score is: 0.3846998789804 1297 For K-means n clusters = 10. The average silhouette score is : 0.388102440660 8065 For K-means n_clusters = 11. The average silhouette_score is : 0.358432650358 64397 For K-means n clusters = 12. The average silhouette score is: 0.359343803481 83653 For K-means n clusters = 13. The average silhouette score is: 0.359339297782 39027 For K-means n_clusters = 14. The average silhouette_score is : 0.348634587252 3635 For K-means n clusters = 15. The average silhouette score is : 0.330701837747 95064 For K-means n clusters = 16. The average silhouette score is: 0.310689520053 For K-means n_clusters = 17. The average silhouette_score is : 0.313485826037 69914 For K-means n clusters = 18. The average silhouette score is : 0.302180640784 12055 For K-means n_clusters = 19. The average silhouette_score is : 0.286872762192 7523

Out[8]: [<matplotlib.lines.Line2D at 0x1b06a378518>]



Implement Algorithms

```
In [9]:
        #Select a few observations to sample from the dataset
        indices = [4000, 3000, 2, 1400, 1111]
        # Create a DataFrame of the chosen samples
        samples = pd.DataFrame(n_data.loc[indices], columns = n_data.keys()).reset_ind
        ex(drop = True)
        samples_z = pd.DataFrame(n_data.loc[indices], columns = n_data.keys()).reset_i
        ndex(drop = True)
        print ("Chosen samples of dataset:")
        #calculate the z score to understand the location of the feauture for each cus
        tomer in density plot
        for col in n_data.columns:
            col zscore = col + ' zscore'
            samples_z[col_zscore] = (samples_z[col] - data[col].mean())/data[col].std
        ()
        samples_z
```

Chosen samples of dataset:

Out[9]:

	monetization	recency	frequency	monetization_zscore	recency_zscore	frequency_z
0	0.020554	0.999789	0.000399	-0.233835	-24.919741	-0.650194
1	0.072459	0.997371	0.000384	-0.233828	-24.919766	-0.650196
2	0.587636	0.809126	0.000334	-0.233766	-24.921647	-0.650205
3	0.982372	0.186938	0.000620	-0.233718	-24.927867	-0.650157
4	0.674233	0.738514	0.002441	-0.233755	-24.922353	-0.649851

```
In [10]: #n_data.drop(['CustomerID'], axis = 1, inplace = True)
#n_data.drop(['cluster'], axis = 1, inplace = True)
```

In [11]: #Find cluster centers and size using Kmeans # Loop through clusters from sklearn.metrics import silhouette score from sklearn.cluster import KMeans range n clusters = range(5,6) for n_clusters in range_n_clusters: #Apply your clustering algorithm of choice to the reduced data clusterer = KMeans(n_clusters=n_clusters).fit(n_data) #Predict the cluster for each data point preds = clusterer.predict(n_data) #Predict the cluster for each transformed sample data point sample preds = clusterer.predict(samples) #cluster centers centers = clusterer.cluster centers score=silhouette_score(n_data, preds, metric='euclidean') #centroids of each cluster centers df = pd.DataFrame(centers) centers_df.columns=data.columns cluster_size=np.bincount(preds) centers_df.insert(loc=0, column='cluster_size', value=cluster_size) centers df int64=centers df#.astype('int64') print ("For n_clusters = {}".format(n_clusters)) print("Silhouette Coefficient: %0.3f" % score) display(centers_df_int64)

> For n_clusters = 5 Silhouette Coefficient: 0.591

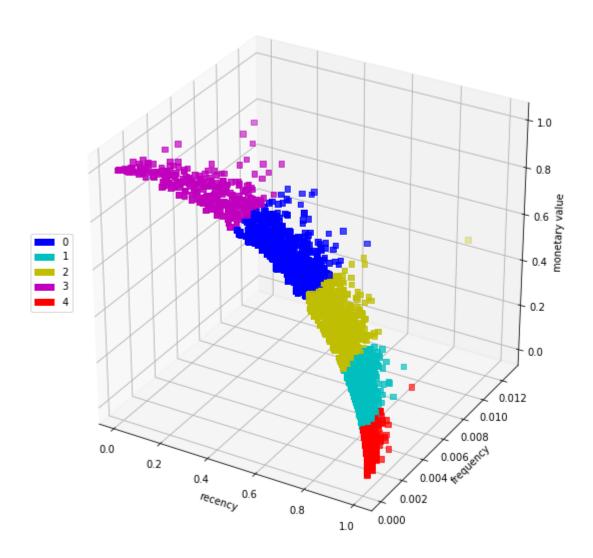
	cluster_size	monetization	recency	frequency
0	486	0.776287	0.622749	0.001787
1	1058	0.302308	0.950841	0.001054
2	659	0.543670	0.835189	0.001539
3	298	0.947510	0.291997	0.001593
4	1823	0.104288	0.993267	0.000548

	monetization	recency	frequency	customer_id	cluster
0	0.873333	0.487121	0.001418	12347.0	0
1	0.587270	0.809390	0.001307	12348.0	2
2	0.587636	0.809126	0.000334	12349.0	2
3	0.122377	0.992484	0.000366	12350.0	4
4	0.499385	0.866376	0.002488	12352.0	2

Out[12]:

	monetization_x	recency_x	frequency_x	customer_id	cluster_x	monetization_y	rece
0	4310.00	2404	7	12347.0	0	0.873333	0.48
1	1797.24	2477	4	12348.0	2	0.587270	0.80
2	1757.55	2420	1	12349.0	2	0.587636	0.80
3	334.40	2712	1	12350.0	4	0.122377	0.99
4	1405.28	2438	7	12352.0	2	0.499385	0.86

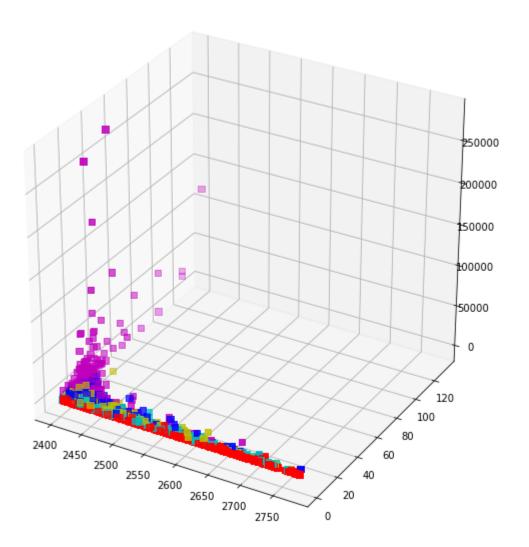
In [13]: import matplotlib.pyplot as plt from mpl toolkits.mplot3d import Axes3D import pandas as pd import numpy as np from matplotlib.colors import LinearSegmentedColormap fig = plt.figure(figsize=(10,10)) ax = fig.add_subplot(111, xlabel='recency',ylabel='frequency',zlabel='monetary value',projection='3d') x = np.array(n_data['recency']) y = np.array(n_data['frequency']) z = np.array(n_data['monetization']) colors=['b', 'c', 'y', 'm', 'r'] custom_cmap = LinearSegmentedColormap.from_list("my_colormap", colors) clusters=[0,1,2,3,4] # marker="s" for square, s=30 is for size of the square ax.scatter(x,y,z, marker="s", c=n_data["cluster"], s=30, cmap=custom_cmap) import matplotlib.patches as mpatches recs = []for i in range(0,len(colors)): recs.append(mpatches.Rectangle((0,0),1,1,fc=colors[i])) plt.legend(recs,clusters,loc=6) plt.show()



```
In [14]: import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D
import pandas as pd
import numpy as np

fig = plt.figure(figsize=(10,10))
ax = fig.add_subplot(111, projection='3d')
x = np.array(data['recency'])
y = np.array(data['frequency'])
z = np.array(data['monetization'])

ax.scatter(x,y,z, marker="s", c=data["cluster"], s=40, cmap=custom_cmap)
plt.show()
```



```
In [15]: #Find cluster centers and size using GaussianMixture
         # Loop through clusters
         from sklearn.metrics import silhouette score
         from sklearn.mixture import GaussianMixture
         if 'customer_id' in n_data.columns:
             n_data.drop(['customer_id'], axis = 1, inplace = True)
         if 'cluster' in n data.columns:
             n_data.drop(['cluster'], axis = 1, inplace = True)
         range n clusters = range(2,10)
         for n in range_n_clusters:
             #Apply your clustering algorithm of choice to the reduced data
             clusterer = GaussianMixture(n components=n).fit(n data)
             #Predict the cluster for each data point
             preds = clusterer.predict(n_data)
             #Predict the cluster for each transformed sample data point
             sample preds = clusterer.predict(samples)
             #cluster centers
             centers = clusterer.means_
             score = silhouette_score(n_data, preds, metric='mahalanobis')
             #centroids of each cluster
             centers df = pd.DataFrame(centers)
             cluster size=np.bincount(preds)
             centers_df.insert(loc=0, column='cluster_size', value=cluster_size)
             centers_df_int64=centers_df#.astype('int64')
             print ("For GMM with n_clusters = {}. The average silhouette_score is : {}
         ".format(n, score))
             display(centers df int64)
             #Display the predictions
             for i, pred in enumerate(sample preds):
                 print ("Sample point", i, "predicted to be in Cluster", pred)
```

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For GMM with n_clusters = 2. The average silhouette_score is : 0.394642546398 23076

	cluster_size	0	1	2
0	1811	0.638615	0.707206	0.001552
1	2513	0.147678	0.985305	0.000660

Sample point 0 predicted to be in Cluster 1
Sample point 1 predicted to be in Cluster 1
Sample point 2 predicted to be in Cluster 0
Sample point 3 predicted to be in Cluster 0
Sample point 4 predicted to be in Cluster 0
For GMM with n_clusters = 3. The average silhouette_score is : 0.327332213852
85916

	cluster_size	0	1	2
0	1919	0.109815	0.992436	0.000562
1	1094	0.774514	0.581494	0.001703
2	1311	0.357744	0.928162	0.001165

Sample point 0 predicted to be in Cluster 0
Sample point 1 predicted to be in Cluster 0
Sample point 2 predicted to be in Cluster 1
Sample point 3 predicted to be in Cluster 1
Sample point 4 predicted to be in Cluster 1
For GMM with n_clusters = 4. The average silhouette_score is : 0.320551463734
3055

	cluster_size	0	1	2
0	1096	0.298934	0.951748	0.001040
1	638	0.873075	0.448000	0.001689
2	803	0.573427	0.811920	0.001595
3	1787	0.102225	0.993554	0.000544

Sample point 0 predicted to be in Cluster 3
Sample point 1 predicted to be in Cluster 3
Sample point 2 predicted to be in Cluster 2
Sample point 3 predicted to be in Cluster 1
Sample point 4 predicted to be in Cluster 2
For GMM with n_clusters = 5. The average silhouette_score is : 0.331821803528
13645

	cluster_size	0	1	2
0	482	0.769045	0.632002	0.001773
1	1075	0.294663	0.953275	0.001029
2	674	0.535875	0.840108	0.001533
3	316	0.943201	0.303478	0.001621
4	1777	0.101901	0.993600	0.000543

```
Sample point 0 predicted to be in Cluster 4
Sample point 1 predicted to be in Cluster 4
Sample point 2 predicted to be in Cluster 2
Sample point 3 predicted to be in Cluster 3
Sample point 4 predicted to be in Cluster 0
For GMM with n_clusters = 6. The average silhouette_score is : 0.292953809554
6279
```

	cluster_size	0	1	2
0	893	0.249165	0.967329	0.000919
1	307	0.945638	0.297042	0.001607
2	405	0.790824	0.606421	0.001781
3	447	0.610085	0.789586	0.001656
4	1646	0.094661	0.994555	0.000525
5	626	0.424102	0.903598	0.001323

```
Sample point 0 predicted to be in Cluster 4
Sample point 1 predicted to be in Cluster 4
Sample point 2 predicted to be in Cluster 3
Sample point 3 predicted to be in Cluster 1
Sample point 4 predicted to be in Cluster 3
For GMM with n_clusters = 7. The average silhouette_score is : 0.291490406504
9743
```

	cluster_size	0	1	2
0	143	0.978918	0.183445	0.001369
1	905	0.245142	0.968366	0.000907
2	446	0.594085	0.801896	0.001638
3	1616	0.093403	0.994709	0.000523
4	606	0.415565	0.907740	0.001308
5	271	0.891876	0.445302	0.001860
6	337	0.758993	0.647765	0.001724

```
Sample point 0 predicted to be in Cluster 3
Sample point 1 predicted to be in Cluster 3
Sample point 2 predicted to be in Cluster 2
Sample point 3 predicted to be in Cluster 0
Sample point 4 predicted to be in Cluster 2
For GMM with n_clusters = 8. The average silhouette_score is : 0.262648119260
8048
```

	cluster_size	0	1	2
0	579	0.355997	0.933360	0.001162
1	127	0.982233	0.168221	0.001278
2	1457	0.085811	0.995581	0.000506
3	363	0.663026	0.746172	0.001744
4	301	0.805760	0.588769	0.001750
5	865	0.215021	0.975726	0.000834
6	437	0.509083	0.859209	0.001493
7	195	0.915805	0.396843	0.001847

```
Sample point 0 predicted to be in Cluster 2
Sample point 1 predicted to be in Cluster 2
Sample point 2 predicted to be in Cluster 6
Sample point 3 predicted to be in Cluster 1
Sample point 4 predicted to be in Cluster 3
For GMM with n_clusters = 9. The average silhouette_score is : 0.228974117992
4036
```

	cluster_size	0	1	2
0	686	0.268523	0.962535	0.000972
1	256	0.820462	0.568929	0.001757
2	967	0.064880	0.997550	0.000475
3	486	0.404402	0.913471	0.001289
4	185	0.919021	0.389809	0.001856
5	323	0.695316	0.716427	0.001779
6	126	0.982656	0.166173	0.001269
7	402	0.549830	0.833649	0.001543
8	893	0.152435	0.987840	0.000641

```
Sample point 0 predicted to be in Cluster 2
Sample point 1 predicted to be in Cluster 2
Sample point 2 predicted to be in Cluster 7
Sample point 3 predicted to be in Cluster 6
Sample point 4 predicted to be in Cluster 5
```

```
In [16]:
         from sklearn.cluster import DBSCAN
         from sklearn.metrics import silhouette score
         from sklearn import metrics
         if 'customer_id' in n_data.columns:
             n_data.drop(['customer_id'], axis = 1, inplace = True)
         if 'cluster' in n data.columns:
             n_data.drop(['cluster'], axis = 1, inplace = True)
         clusterer_DB = DBSCAN(eps=.01, metric='euclidean', min_samples=1).fit(n_data)
         labels_DB = clusterer_DB.labels_
         #display(labels DB)
         display(len(set(labels_DB)))
         score_DB=silhouette_score(n_data, labels_DB, metric='euclidean')
         print("Silhouette Coefficient: %0.3f" % score DB )
         n_data['cluster']=labels_DB
         n_data['customer_id']=CustomerID
         n_data.head(10)
         n_data[["customer_id", "cluster"]].to_csv('finaldatawithcluster_DB_monthly_fre
         quency', index=False)
         5
```

Silhouette Coefficient: -0.151

```
In [17]:
         from sklearn.cluster import AffinityPropagation
         from sklearn import metrics
         if 'customer id' in n data.columns:
             n_data.drop(['customer_id'], axis = 1, inplace = True)
         if 'cluster' in n_data.columns:
             n_data.drop(['cluster'], axis = 1, inplace = True)
         # Compute Affinity Propagation
         af = AffinityPropagation().fit(n_data)
         cluster_centers_indices = af.cluster_centers_indices_
         labels = af.labels_
         n_clusters_ = len(cluster_centers_indices)
         import matplotlib.pyplot as plt
         from itertools import cycle
         print(n_clusters)
         n_data['cluster']=labels
         n_data['customer_id']=CustomerID
         n_data.head(10)
         n_data[["customer_id", "cluster"]].to_csv('finaldatawithcluster_AP_monthly_fre
         quency', index=False)
```

5

In [18]: | #AgglomerativeClustering if 'customer id' in n data.columns: n data.drop(['customer id'], axis = 1, inplace = True) if 'cluster' in n data.columns: n_data.drop(['cluster'], axis = 1, inplace = True) from sklearn.cluster import AgglomerativeClustering # Affinity = {""euclidean", "l1", "l2", "manhattan", # "cosine"} # Linkage = {"ward"}#, "complete", "average"} Hclustering = AgglomerativeClustering(n_clusters=5, affinity='euclidean', link age='ward') Hclustering.fit predict(n data) #ms = np.column_stack((ground_truth, Hclustering.labels_)) #df = pd.DataFrame(ms, # columns = ['Ground truth','Clusters']) #pd.crosstab(df['Ground truth'], df['Clusters'], # marains=True) labels_HC = Hclustering.labels_ display(labels HC) len(set(labels_HC)) n data['cluster']=labels HC n_data['customer_id']=CustomerID.astype('Int64') display(n data.head(10)) #true_centers_Kmeans_df_int64['cluster'] = pd.Series(true_centers_Kmeans_df_in t64.index, index=true_centers_Kmeans_df_int64.index) n_data[['customer_id','cluster']].to_csv('customerwithcluster_HC_monthly_frequ ency.csv', index=False)

array([0, 2, 2, ..., 3, 2, 2], dtype=int64)

	monetization	recency	frequency	cluster	customer_id
0	0.873333	0.487121	0.001418	0	12347
1	0.587270	0.809390	0.001307	2	12348
2	0.587636	0.809126	0.000334	2	12349
3	0.122377	0.992484	0.000366	3	12350
4	0.499385	0.866376	0.002488	2	12352
5	0.034132	0.999417	0.000384	3	12353
6	0.379191	0.925318	0.000351	1	12354
7	0.172965	0.984928	0.000377	1	12355
8	0.757363	0.652994	0.000808	4	12356
9	0.930942	0.365168	0.000150	0	12357

```
In [19]: #data.drop(['customer id'], axis = 1, inplace = True)
         #data.drop(['cluster'], axis = 1, inplace = True)
         from scipy.spatial.distance import squareform, pdist
         import seaborn as sns
         distance DB=pd.DataFrame(squareform(pdist(n data.iloc[:, 1:])))
         #display(distance_DB)
         distance_DB.describe()
         all_numbers=distance_DB.values.tolist()
         flat_list = [item for sublist in all_numbers for item in sublist]
         flat list
         np.histogram(flat_list)
         n, bins, patches = plt.hist(flat_list)
         print("n: ", n, sum(n))
         print("bins: ", bins)
         for i in range(len(bins)-1):
             print(bins[i+1] -bins[i])
         print("patches: ", patches)
         print(patches[1])
         # sort the data:
         sorted_flat_list = np.sort(flat_list)
         # calculate the proportional values of samples
         p = 1. * np.arange(len(flat list)) / (len(flat list) - 1)
         # plot the sorted data:
         fig = plt.figure()
         ax1 = fig.add_subplot(121)
         ax1.plot(p, sorted_flat_list)
         ax1.set xlabel('$p$')
         ax1.set_ylabel('$x$')
         ax2 = fig.add subplot(122)
         ax2.plot(sorted flat list, p)
         ax2.set xlabel('$x$')
         ax2.set ylabel('$p$')
         sns.distplot(flat_list);
```

> [3537756. 3164962. 2806100. 2435148. 2066820. 1674352. 1304024. 932666. 200692.] 18696976.0 574456. 594.00003449 1188.00006898 1782.00010347 bins: 0. 2376.000 13796 2970.00017245 3564.00020694 4158.00024143 4752.00027592 5346.00031041 5940.00034491] 594.000034491 594.000034491

594.000034491

594.000034491

594.000034491

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594.000034491

594.000034491

patches: <a list of 10 Patch objects>

Rectangle(594,0;594x3.16496e+06)

