# Mobile Robotics - 2021
# Take Home EndSem

Time: 48 Hours Total Marks: 45

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## General Instructions:

- The deadline for this evaluation is on 28/11/21 at 10AM on Moodle. **Late days are not applicable for this evaluation.**
- The submissions are on an individual basis; you are free to reuse code from assignments.
- Do not plagiarize or copy material from any source.
- Keep all answers to the point, verbosity will only make evaluation more difficult.
- Ensure that the questions are in the correct order for the final submission.
- **Submission format:**
    - ZIP folder named <ROLL_NUMBER.zip> containing the following:
    - PDF containing answers to theory questions named as '<ROLL_NUMBER>_theory.pdf'
    - Jupyter Notebook containing solutions to all coding questions named as '<ROLL_NUMBER>_code.ipynb`
    - PDF exported version of the above notebook named as `<ROLL_NUMBER>_code.pdf'
    - The </> are placeholders, do not include it in the file name.

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# Theory Questions

1. **Questions on Fundamental Matrix [5 points]**
   a. Derive Fe = 0, where e is the epipole of the 1st camera seen in the second image **[2 points]**
   b. If the fundamental matrix between images I1 and I2 is F, what is the fundamental matrix between images I2 and I1? Why are they different? **[2 points]**
   c. What is the difference between a fundamental matrix and an essential matrix? How are they related? **[1 point]**

2. **Questions on relationships between image planes [13 points]**
   a. Derive the homography relation for pure rotation that relates a pixel location $x_{i1}$ in frame I1 to the pixel $x_{j2}$ in frame I2. Does such a homography relation hold when there is a camera translation involved? Explain mathematically your answer **[2 + 2 = 4 points]**
   b. What are multiple ways you can relate a pixel $x_{i1}$ in frame I1 to the pixel $x_{j2}$ in frame I2? Be sure to state your assumptions and any additional information used. Write down the equations for those relations **[4 points]**
   c. What are the two homographies involved in Stereo Rectification? What does each such homography accomplish? Why would you call them as homographies? Explain with figures and equations **[1.5 + 1 + 0.5 + 2 = 5 points]**

3. **Questions on Camera Calibration [4 points]**
   a. Write down, in detail, the proof of the DLT algorithm elucidating upon each step. Be sure to highlight the steps in the algorithm that would fail in their purpose if all the correspondences taken lie on a plane. Why is the eigenvector corresponding to the least eigenvalue taken? **[2 + 1 + 1 = 4 points]**

4. **Questions on SLAM/SfM:** For the following questions, clearly write down the variables being solved for and the shape/size of all vectors/jacobians. **[14 points]**
   a. Given 2D points observed in *m* observations and the relative pose between any two observations (in the 2D plane - SE(2)), write down the optimization formulation for SAM (Smoothing and Mapping). **[4 points]**
   b. You are given a series of RGB images across a trajectory (no additional information) and have to estimate the relative pose between images and map the observed environment. In a systematic and concise way, write down the steps you would take to perform such a monocular SLAM with their mathematical equations. Describe how you obtain your initial estimates and optimise for the trajectory. **[10 points]**

## Coding Questions

1. **Optimization**: You are given the function: $exp(a * x) * sin(x) + b$. Implement Levenberg Marquadt using numpy and solve for the parameters of the above function. Optimize for the following parameters: **a=2, b = 1.** Do this for 50 observations that lie between 1 and 20. Plot the loss values over time and data fit curves. Ensure that your initial estimates are not very close to the final parameters. Write down the jacobian formula in the notebook. **[3 points]**

2. **Linear least square:** You are given a bin file from the Kitti raw sequence. Estimate the ground plane from the given bin file. After estimating the ground plane, visualize this in open3d by drawing 200-300 points on the ground with a different color on top of the plot obtained from the LiDAR scan. Use RANSAC to estimate the ground plane. Will this work without RANSAC? Why or Why not? Write down the equation of the ground plane obtained and also mention the parameters used for doing RANSAC. **[6 points]**