NASA REPORT: GRAVITATIONAL ANALYSIS FOR APOLLO 11 MISSION

I. Introduction

This report presents an analysis of the gravitational environment that the Apollo 11 mission will navigate during its journey to the Moon. Understanding the gravitational interactions between Earth and Moon is crucial for trajectory planning, fuel calculations, and ensuring the safety and success of the mission.

We explored 4 main parts during this investigation: Gravitational Potential of Earth, Gravitational Potential of the Earth and Moon System, Gravitational Force Field of the Earth and Moon System, and finally the possible altitude of our Saturn V rocket at the end of the burn process.

In all, we provide an indepth report on the possible fields - potential and force - that the Saturn V rocket might face by providing descriptive graphs such as heatmaps, contour plots, and stream plots. All graphs were made possible by python libraries of matplotlib.pyplot and numpy for array transformation and data storage. For all potential field and gravitational field exploration, the points of interest range from -1.5 * distance from Earth to Moon to 1.5 * the same distance.

II. The Gravitational Potential of the Earth-Moon System

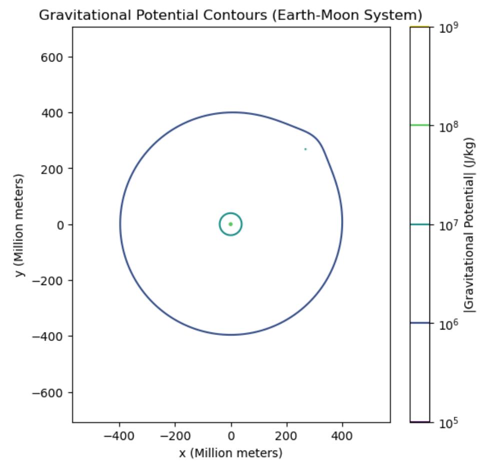
The gravitational potential field represents the potential energy per unit mass at various points in space due to the gravitational influence of massive bodies. In the Earth-Moon system, this potential field is dominated by Earth's gravity near Earth and gradually transitions to lunar influence as distance from Earth increases. It is determined by the following equation where G is the gravitational constant, M is the mass of the Earth/Moon, and r is the distance between the center of Earth/Moon and the point of interest.

$$\Phi(r) = -\frac{GM}{r}$$

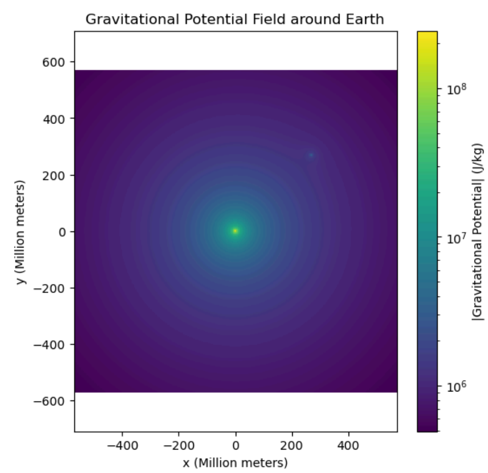
Both potentials of Earth and Moon were calculated separately and summed to provide the final potential at any given point.

Key Observations from Potential Field Analysis:

1. **Potential Contours**: The circular contours shown in the potential field diagram (Image 2) illustrate equipotential surfaces around Earth. These contours indicate regions where a spacecraft would require the same energy to escape the gravitational influence.



2. **Gradient of Potential**: The heat map visualization shows the intensity of the gravitational potential field around Earth. The bright green-yellow center represents the highest potential magnitude, which rapidly decreases with distance following an inverse relationship. One can see two dots with the bigger one representing the Earth and the smaller one (off center) representing the Moon.



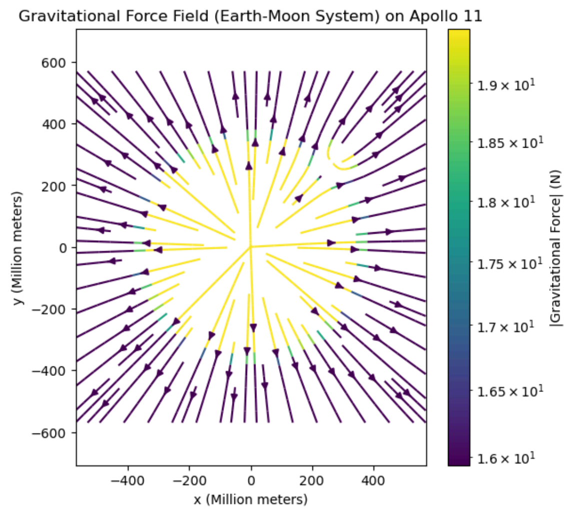
III. The Gravitational Force of the Earth-Moon System

The gravitational force field represents the direction and magnitude of the acceleration a spacecraft will experience at various points in the Earth-Moon system. The following equation is used to calculate the Gravitational Force that could be felt by our rocket at any given point in space. The parameters are: Mass of Earth/Moon, mass of the Apollo Rocket, and the distance between the Earth/Moon and the points of interest.

$$\vec{F}_{21} = -G\frac{M_1 m_2}{|\vec{r}_{21}|^2}\hat{r}_{21}$$

Key Observations from Force Field Analysis:

1. **Force Vector Field**: The following image displays the gravitational force field as vectors, with arrows pointing in the direction of the net gravitational force. Near Earth, these vectors point radially inward toward Earth's center, while a transition region exists where the Moon's influence becomes significant. This plot was created by using the streamplot function of matplotlib where the x and y vectors of the force field are given as function arguments for each point in the x,y plane.



Understanding these force fields is essential for predicting the spacecraft's behavior during all mission phases, from launch to lunar orbit insertion and return trajectory.

IV. Projected Performance of the Saturn V Stage 1

The Saturn V rocket's first stage must generate sufficient thrust to overcome Earth's strong gravitational pull near the surface and achieve the initial velocity needed for the multi-stage journey to the Moon. Therefore, to that end we calculated the burn time and the altitude of the rocket at the end of the burn time.

Performance Calculations:

1. **Burn Time Calculation**: We obtained the total burn time by simply taking wet mass minus dry mass divided by the burn rate.
2. **Altitude Calculation**: We were able to calculate altitude by making a function which returns the change in velocity which was integrated to get the final translation.

$$h = \int_0^T \Delta v(t) dt$$

And the change of velocity at any given time is calculated by the following equation where m0 is the initial mass, m(t) = m0 - m_burn_rate*t (mass at time t), vE is the fuel exhaust velocity and g is the gravitational acceleration.

$$\Delta v(t) = v_e \ln\left(\frac{m_0}{m(t)}\right) - gt$$

3. **Comparison with Test Results**: Our results indicate that the burn time equal to about 127 seconds and the altitude is 64546.8 meters. Recent prototype testing, however, showed the first stage burning for approximately 160 seconds and reaching an altitude of 70 km. Our theoretical calculations under predicted the values.
4. **Projected vs. Actual Performance**: Our idealized calculations likely overestimated performance because we neglected atmospheric drag, which is significant in the lower atmosphere. Additionally, the rocket's mass changes continuously as fuel is consumed, affecting the acceleration profile throughout the burn.

The Saturn V first stage is projected to perform adequately for the Apollo 11 mission requirements, with sufficient margin to account for real-world factors not included in our simplified gravitational model.

V. Discussion and Future Work

Simplifications and Approximations:

1. **Two-Body Approximation**: Our analysis treated the Earth-Moon system as a two-body problem, neglecting the gravitational influences of the Sun and other planets. While this is reasonable for near-Earth and near-Moon calculations, it may introduce small errors for the trans-lunar trajectory.
2. **Static Body Masses**: We calculated gravitational fields in a static reference frame, whereas the actual Earth-Moon system is dynamic, with the Moon orbiting Earth and both bodies orbiting the Sun.
3. **Neglected Factors**: Our Saturn V performance calculations did not account for atmospheric drag, changing vehicle mass, or specific engine performance variations with altitude.

Future Work:

1. **Incorporate Atmospheric Effects**: Future analysis should include atmospheric drag models that vary with altitude to provide more accurate predictions for the initial launch phase.
2. **Dynamic Simulation**: Develop a time-dependent simulation that accounts for the motion of Earth and Moon during the mission duration.
3. **Multi-Body Gravitational Model**: Expand the gravitational model to include Solar influence, which becomes increasingly relevant for longer-duration missions.

Conclusion

This analysis provides a fundamental understanding of the gravitational conditions Apollo 11 will encounter. The Earth-Moon system has a complex but manageable gravitational field, and the mission trajectory has been designed with safety and backup measures in mind.

The Saturn V’s first stage has the necessary power to escape Earth’s gravity during launch. While our simplified gravitational model offers a useful initial estimate, further refinements are needed to include additional factors that could affect the mission’s actual performance.

These findings support Director Kranz’s confidence in the mission plan while also identifying areas where further analysis could improve our understanding of Apollo 11’s gravitational challenges.