Modal Logic

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1 Basic Concepts

1.1 Relational Structures

Definition 1.1. A relational structure $\mathfrak{F}$ is a tuple, where the first component is a non-empty set W called *universe*, and remaining components are relations on W. We assume there is at least one relation on W.

In the following 2 definitions, we assume W be a non-empty set and R a binary relation on W.

Definition 1.2. $R^+ := \bigcap\{R' | R'$ is a transitive binary relation on W and $R \subseteq R'\}$, is called the *transitive closure* of R.

Definition 1.3. $R^* := \bigcap\{R' | R'$ is a reflexive transitive binary relation on W and $R \subseteq R'\}$, is called the *reflexive transitive closure* of R.

Note that transitive closure of a binary relation has nice *finite steps* property, see Exercise 1.1.3.

Definition 1.4. A tree $\mathfrak{T}$ is a structural structure (T, S) where:
(i)T, the set of nodes, contains a unique $r \in T$ (root), such that $\forall t \in TS^*rt$.
(ii)For every $t \neq r$, there is a unique $t' \in T$, such that $St't$
(iii)$\forall t \neg S^+tt$, so S is *acyclic*.

Question 1.1. Why we define tree like that?