Title

Department of Statistics and Applied Probability, University of California Santa Barbara

PSTAT 160B: Stochastic Process

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Abstract

The dynamics of financial markets are highly complex. Mainstream asset pricing models often are inaccurate and fail to account for the role human psychological tendencies play in determining stock price. The empirical data on asset price distributions can be characterized by some stylized facts. In our discussion we will attempt to recreate these stylized facts by incorporating the human tendency of herd behavior. The dynamic complexity of a financial market is due to both its great number of moving elements and the complex structure linking them to each other. In order to simulate this, we will implement the microsimulation models through a variation of the Ehrenfest urn model to thoroughly understand their modeling principles. We will then perform parameter study of α *and* β *to compare the relevant characteristics of the simulation results after tuning the parameters. Following this, we will predict the outcome of each implemented case in order to present a general understanding of the origins of the stylized facts. Mainly, the collective behavior of the interacting elements under the influence of massive external factors with a focus on sensitivity based on behavior of other traders in the market.*

1. Introduction

Empirical observations have detected excess kurtosis in stock returns, suggesting that stock prices in reality are more volatile than in traditional models. We speculate these bursts in volatility come from human tendencies to follow the investment patterns of much of the public, or herd. These tendencies lead us to hypothesize that human psychological tendencies, unrelated to the influx of viable information, drive stock price distributions to be characterized by fatter tails and excess kurtosis. In order to explain some of these phenomena observed in the empirical data, we create a modified Ehrenfest urn to model asset prices in the presence of herd behavior tendencies.

The problems in this field, as discussed above, motivate us to perform a detailed study of the impact of herd-like tendencies on price distributions by the following approaches:

2. Empirical observations of Stock Price returns

Most current theoretical models for showing stock prices to be normally distributed, however, Empirical data of the S&P 500 suggests that distributions of financial returns are fat-tailed with excess kurtosis. Many believe this to be a result of large bursts of volatility due to human tendencies to follow the pack observed in the market.

3. Ehrenfest Urn Model

The Ehrenfest urn model is one of the most instructive and popular models in all of probability, stochastic processing, and physical statistics. The basic Ehrenfest urn consists of 2 urns with *n* balls in the first urn and *N-n* balls in the second urn. At each discrete time step *t*, a natural number from *1* to *N* is chosen at random, and the corresponding ball is forced to change from one urn to the other. In the same fashion, we introduce a discrete random process $X_1, X_2, \ldots X_{t,\ldots}$ where each value represents the number of balls in the first urn at time t.

We generalize the model by allowing the selected ball to change urns with a probability that depends on the number of balls present in the urns.

n = number of bulls

N = total number of people in the economy

b,a = parameters that controls the fluctuation of urn changing

$W_{n,n-1} = (n/N) * (b + N-n) / (a + b + N -1)$

$W_{n,n} = (n/N)*((a + n - 1)/(a + b + N - 1)) + (N-n/N)*((b + N-n - 1)/(a + b + N - 1))$

$W_{n, n+1} = (N-n/N)*((a + n)/(a + b + N - 1))$

4. Alpha and Beta

Within our model, we defined two parameters, $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$, to represent market preference. We denote $\boldsymbol{\alpha}$ to be the parameter controlling the 'bull' preference. This means when increasing $\boldsymbol{\alpha}$ in our Urn model, the market as a whole will increase its preference to act as a 'bull' (i.e demand more of the asset in question). We denote $\boldsymbol{\beta}$ to be the bear preference. This means when we increase $\boldsymbol{\beta}$, the market as a whole will prefer to act as a 'bear' (i.e. supply more of the asset in question). In addition, the structure of our model allows the sum of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ to control the selected individuals sensitivity to shifting towards the side which contains more traders.

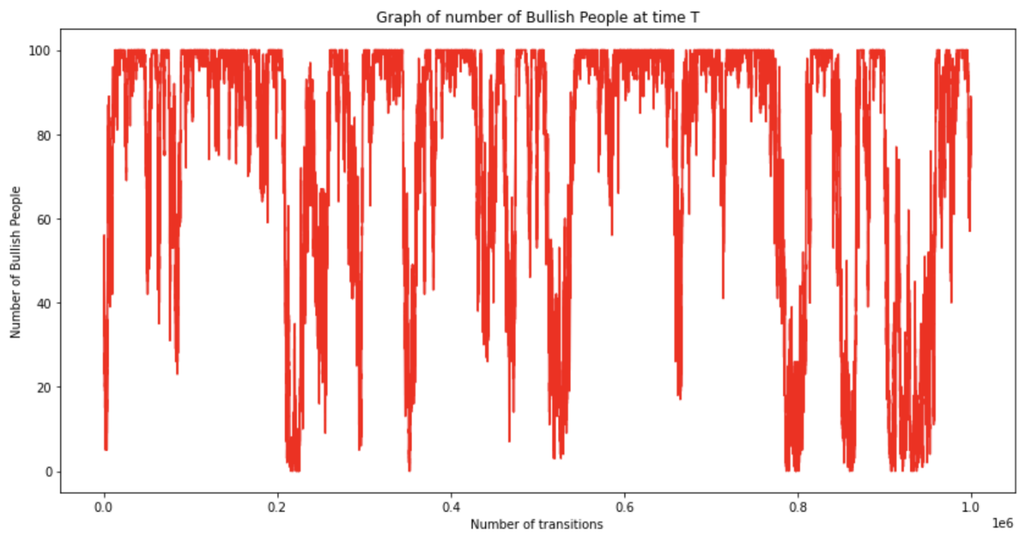
Essentially, if the sum of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ is relatively small in comparison to N, then the selected trader within the market has an increased tendency to move towards the urn with more of his fellow traders. This preference of moving with the herd can be counteracted by making the sum of alpha and beta much larger than N. In this case, the individuals have an increased sensitivity to the alpha and beta parameters, and their corresponding demand preferences.

1. <u>Microstates</u>

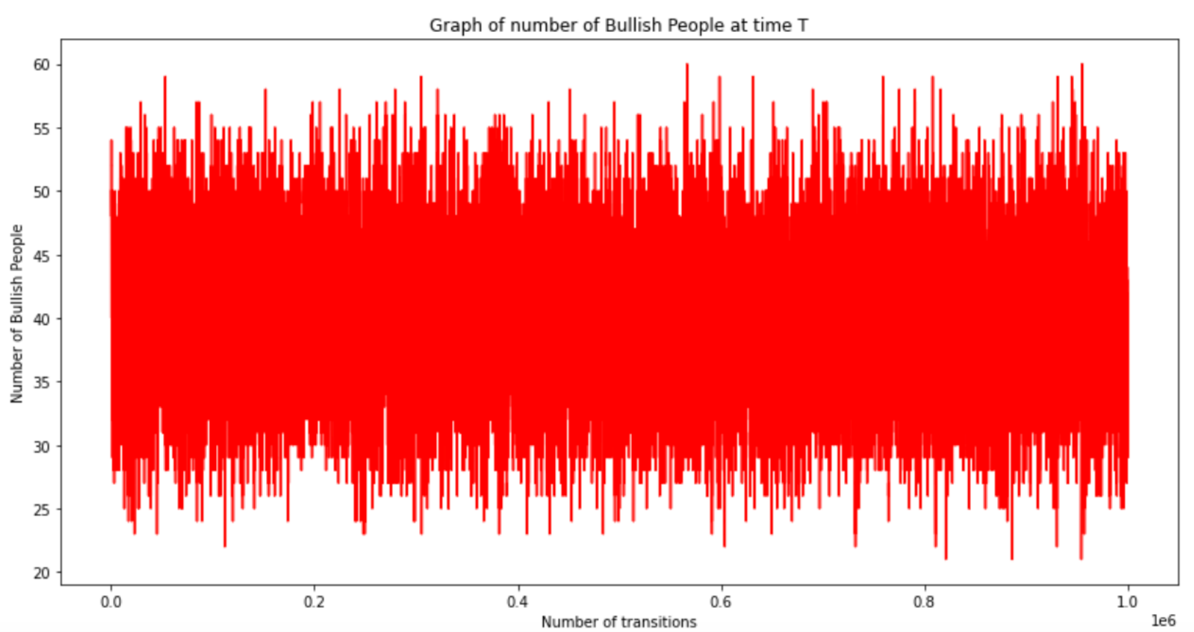
We decided to begin our study with an inside look on the microstates and their long term behavior within the model. We ran simulations allowing us to observe the inside of the urns, i.e the microstates, transitive and long-term behavior with the goal of observing how adjusting $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ really affected the economy. We chose to test three 'types' of markets, which were defined in Geribaldi Penco's, "Ehrenfest Urn Model Generalized: An Exact Approach for Market Participation Models".

These were:

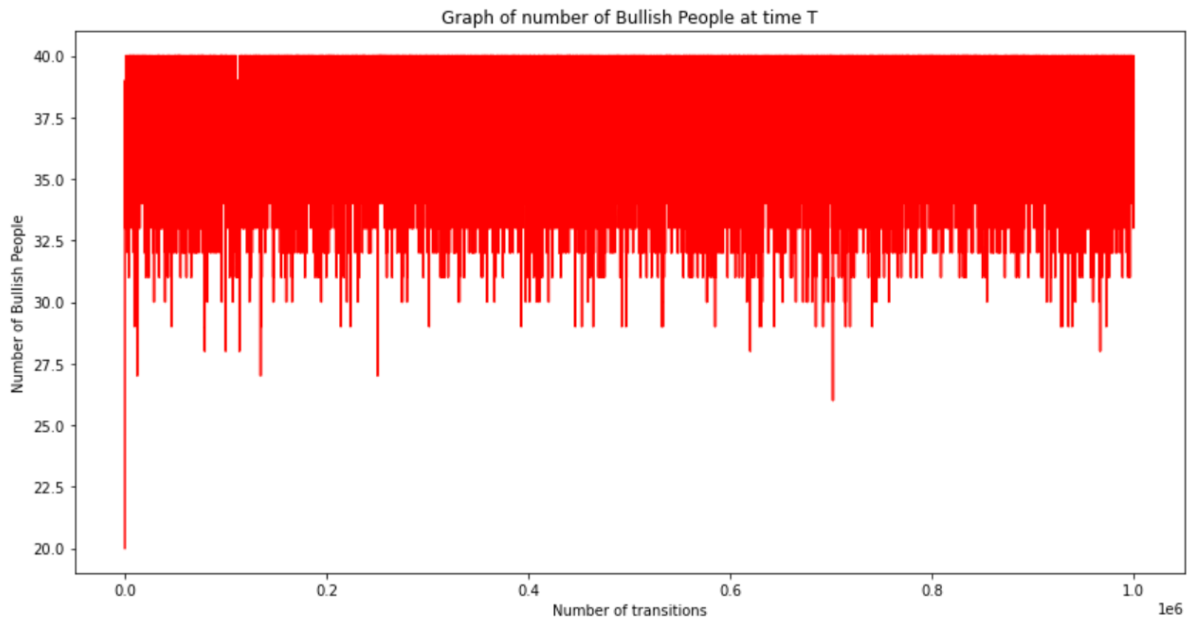
1. $0 < \alpha,\ \beta < 1$: Which was characterized in its steady state by a long time spent around the barriers, with rapid fluctuation across the central region. This is due to the fact that both the alpha and beta parameters were relatively small, leading to increased sensitivity to behave as a herd. With a heightened herd sensitivity, the economy will often get stuck for a while with almost all of the traders as either all bulls or all bears. Occasionally, a few traders shift to the group containing less individuals, and we see a sort of domino effect. The market will suddenly shift to the majority of traders being in the opposite state. With the increased herd sensitivity, the economy will rarely stay evenly split, whichever group has more individuals, will always tend to draw the rest of the traders to it.



2. $\alpha,\beta >> 1$: Which in its steady state consists of a short time spent around the barriers, with long fluctuations across the central region. The economy behaves similarly to the standard ehrenfest urn, with the number of traders tending to be more evenly distributed, as they are less sensitive to the behavior of the herd than in the first model. We can, however, adjust the proportion of traders to favor one state by making either $\boldsymbol{\alpha}$ or $\boldsymbol{\beta}$ larger.



3. $\alpha = \beta = N = 100$: In the steady state of this model we see very short time spent around the barriers, with very long fluctuation across the central region. Setting the parameters in this manner correlates with a very low sensitivity to the behavior of the herd.



6. <u>Macrostates:</u>

Once we have established the dynamics of the microstates we can begin to discuss the overall setup of our experiment. We are going to be using the simulated economy of the Ehrenfest urn model to generate S_t, the history of the price of the asset in discrete time. Specifically, we will increment the price in time t by the excess demand. Excess demand can be found by subtracting the number of bears from the number of bulls at a given time increment t.

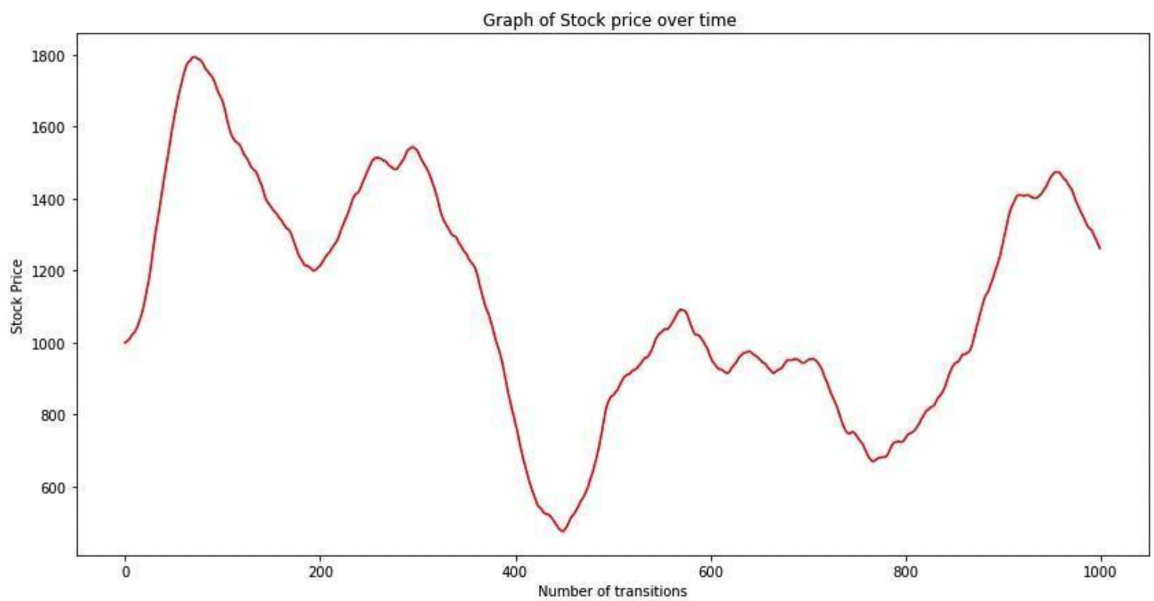
Excess Demand:

Let S_t be the stock price at time t

Let X_t be the number of bulls at time t

\delta p=S_{t+1}-S_t = D(t)=X_t-(N-X_t)=2X_t-N

A sample stock price history would look something like this:



As you can see, the stock price is modeled quite nicely. We note a few key characteristics in the modeling of the stock price as a function of our urn economy:

1) As the number of traders N increases, the stock price has the potential to be more volatile.
2) With low α and β parameters, the stock price tends to have extended periods of tremendous volatility. This is due to the fact that the urn economy tends to spend a lot of time as either all bears ($n = 0$) or all bulls ($n = N$).
3) With high α and low β, the stock price tends to increase on average, and vice versa.

7. **Data collection:**

In order to examine and compare the distributions of returns and stock prices over time with a variety of microstates, we decided to collect six pieces of information from 10,000 simulations we ran for each of the three cases. First, we created an economy of bullish and bearish people, with a total of 100 people. Next, we determined the stationary distribution of the bull and bear economy. We use the beta-binomial distribution defined by Geribaldi Penco, as this is the stationary distribution of our model. Using the beta-binomial distribution, we are able to get the starting number of bullish people; with the remaining people, out of 100, being designated as bears. From the random number of bulls generated, we used the transition probabilities defined by Penco(2000) to simulate the 9000 path time steps to simulate the path of bears/bulls. We then simulated the 9000 time steps 10000 times. Each simulation starts in a different beta binomial distribution due to the fact that this uses the parameters α and β, which varies with each simulation. Next, we created a function which calculates the excess demand. After each step of the 10000 paths, we use this excess demand to calculate the change in price of the stock due to the change in excess demand. Finally, we created a function which converted these stock price paths into returns. We then extracted the returns at steps 3000, 6000, and 9000 and plotted them in histograms to help us understand the distribution of returns over time.

Note:

Let S_t be the stock price at time t

Let X_t be the number of bulls at time t

\delta p=S_{t+1}-S_t = D(t)=X_t-(N-X_t)=2X_t-N

new excess demand = delta p=S_{t+1}-S_t = D(t)=X_t-(N-X_t)=2X_t-N

New prices = previous prices + new excess demand

%return = (new price - previous price)/previous price

Once we collected the data, we created distributions for each of the measurements and compared them to a normal distribution with the mean and standard deviation corresponding to the respective collected dataset. This way we could compare the empirical results to the normal distribution and look for discrepancies between them.

8. Predictions:

Case 1 ($\alpha > 0$ and $\beta < 1$) Prediction:

The first case we analyzed with the goal of mirroring extreme herd sensitivity, with a preference to be bullish. We set our parameters to $\boldsymbol{\alpha} = 0.8, \boldsymbol{\beta} = 0.2$ with our initial bull and bear distribution being randomly drawn from the stationary distribution. We will make a few predictions about what we expect our return and price distributions to look like after running the simulations.

Our first prediction is regarding the mean of the return distributions. We believe the mean of the distributions will be roughly equal to the alpha parameter. This is because the herd sensitivity aspect of the economy works the same in both directions, the only difference leading to one urn being preferred to the other is in the preference parameters. Thus, whichever of the two preference parameters is larger will draw the mean of the return distribution to that side of 0. The reason we hypothesize the mean to be equal to the alpha parameter specifically in this case, is that the alpha parameter is actually equal to the fraction of the sum of the parameters which represents the dominant preference. Thus I hypothesize that on average, roughly 80% of the market will be bullish leading to returns centered around 0.8.

Our second prediction is with regard to the tails of the distribution. We hypothesize that the distribution will innately have heavy tails and as time progresses will develop heavier and heavier tails. The cause of these extremely heavy tails is the 'domino effect' discussed earlier in this paper. As the majority of traders slip into one state, the state begins to draw in more and more traders with the increasing probability. Eventually, we will have a very small fraction or simulations which will remain in an all bull or all bear state for extended periods of time, thus generating extremely high or low price values which will create large swings in returns. We believe that overall the distributions will have a sharp peak around the mean for the simulations which acted in a more 'predictable' manner, surrounded by a very long drawn out tails, representing any of the simulations where the 'domino effect' of the herd played a major role. We believe this attribute of the first case to be the most interesting aspect of our study with regard to explaining real world phenomena.

The third prediction which we make about our first case we analyze is that it will be left skewed. The reason for this left skew is again, the underlying herd sensitivity. When we have alpha larger than beta, the mean of the distribution will obviously be pushed in the positive direction. However, when the herd sensitivity comes into play, we occasionally will have a group of traders who flip to the bearish stance despite the overall market preference to act bullish. Once this happens we will have a period of time where the herd mentality dominates and traders swing to become bears. We then have periods of a bearish majority, driving returns down and skewing the return distribution to the left.

Case 2 (α , β >> 1) Prediction:

The second case was used to contextualize the influence our $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ parameters played in return distributions. We initialized the model parameters to $\boldsymbol{\alpha} = 4000$, $\boldsymbol{\beta} = 6000$, and used a random generator to choose our starting bear/bull values from the analytically calculated stationary distribution.

Our first prediction regarding the second case is that the distribution of returns will be much more gaussian than in the first case. We believe this to be true because the large preference parameters will, for the most part, dominate the underlying herd tendencies of the economy. Specifically, we expect the tails of the second case to be much more condensed than the first. This will lead to a much more gradually sloping mean peak. Our economy will behave much more similarly to a normal ehrenfest urn than in the first case, with a slight preference to one side being the main exception to the standard ehrenfest model. While we expect the distribution of returns to be somewhat similar to a gaussian distribution we expect there to be a right skew to the data. This is for the same reason as the skew in the first case, this time it is in the opposite direction because the economy is bear preferring for these parameters. Specifically, although the majority of the time we will have a bear-heavy market, occasionally a group of traders will be pulled to the bullish urn and the 'domino effect' will occur. It is important to note however, that the skew should be much less apparent in this case than in the first. We believe this to be true due to suppressed relative herd sensitivity.

Case 3 (α = β = N = 100): prediction:

The third case is created with the goal of trying to make an economy where herd behavior and position preference were obsolete. This will allow us to use this state as a basis to contextualize the effects of the herd behavior and preference parameter alone. We initialize the parameters to be $\alpha = \beta = 100$ and again randomly select our initial bull and bear values from the stationary distribution.

Our hypothesis is that the distribution of returns will be almost perfectly normal, with very small standard deviation and a mean of 0. We predict this to be the case because the economy actually prefers to be in a sort of 'neutral state'. We see that our parametric selections drive the bulls to want to be bears and the bears to want to be bulls. With such large parameters driving away from the extremes of all bears or all bulls, we will see tight groupings around a 50-50 split in the data and the price to hover around the starting value the majority of the time.

9. Results:

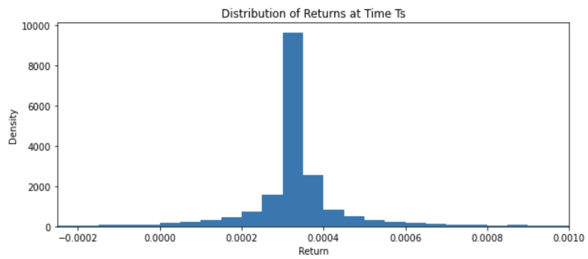
For each simulated case written below, we analyzed our results at three time increments namely Ts = 3000, Tm = 6000, and Th = 9000.

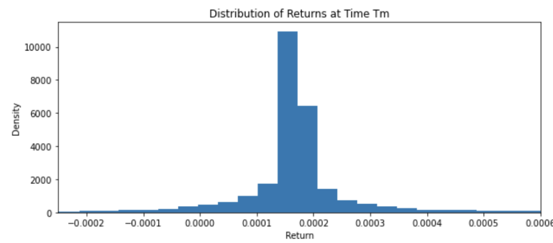
Case 1 (α > 0 and β < 1):

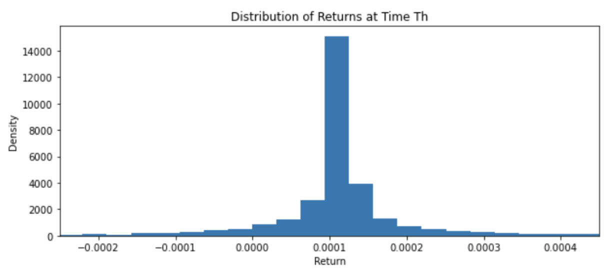
In the first case we analyzed, we wanted to simulate extreme herd sensitivity with a preference to be bullish. Setting the parameters $\alpha = 0.8$, $\beta = 0.2$ with with our initial bull and bear distribution being randomly drawn from the stationary distribution.

Our initial prediction stated that we expect the tails of the distribution to exhibit heavier tails as time progresses. Judging from the tails of the 3 graphs below, our predictions match our observed results as the tails of the distributions become heavier as time increases due to the 'domino effect' discussed earlier in this paper. As our prediction states, there is an extremely sharp peak around the mean indicating our distribution has high kurtosis. This high kurtosis can be attributed to the fact that as the majority of traders tend to slip into one state, that state attracts other traders resulting in large and heavy swings of returns.

In addition, our third prediction states we expect the graph to be left skewed. As our histogram depicts, the distributions at each time step are left skewed which can be attributed to herd sensitivity. More specifically, as one group of bearish traders go against the bullish trends of the market, there exists a period of time where the herd mentality dominates and an increasing number of traders swing from a bullish to bearish stance causing a period of bearish majority. This drives returns down and skews the distribution to the left and contributes to an increase in kurtosis as conveyed in the figures below.



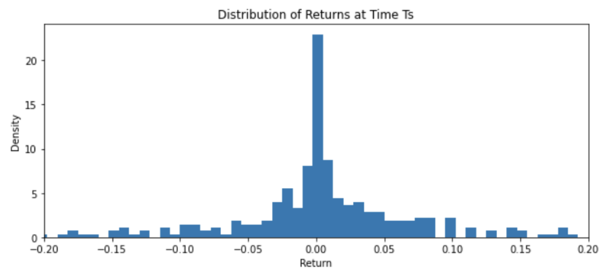


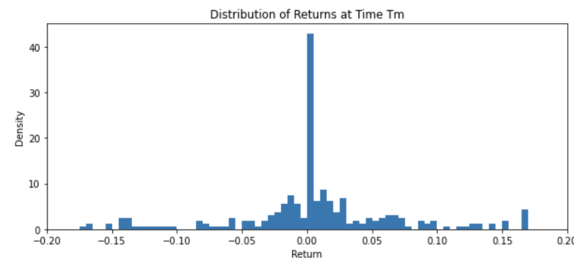


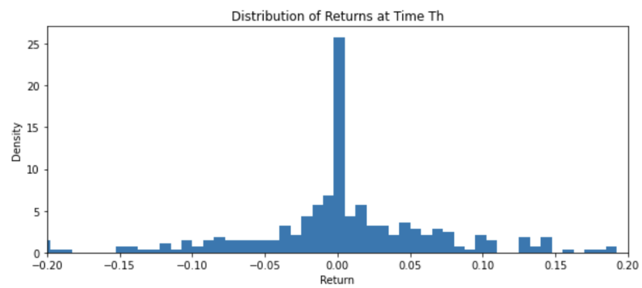
Case 2 (α , β >> 1):

Our goal with this simulation is to contextualize the role our α and β parameters play in determining return distributions. We initialize the model with parameters to α = 4000, β = 6000 and used a random generator to choose our starting bear/bull values from the analytically calculated stationary distribution. Our first prediction states that the resulting distributions will be Gaussian with lower kurtosis compared to the first case due to the expectation of the large parameters to dominate the underlying herd tendencies of the economy.

Our results indicate that the first case produces more of a Gaussian distribution than the second case as shown by the figures below. We observe that the distribution at the first time step Ts is more of a Gaussian distribution compared to our first case, yet the distribution at time Tm is less of a Guassian distribution. The second distribution displays a very high peak as well as non-uniformity centered around the mean signifying an excess level of kurtosis. As our prediction states, the large parameters dominate the underlying herd tendencies of the market with a slightly right skewed distribution around the mean. This is as expected since our β corresponds to bullish tendencies and it is larger than α. However, we notice a higher level of kurtosis in this case than in the previous. In addition, it can be seen in the figure (right) that there are numerous local peaks throughout all time increments. These local peaks and cliffs are attributed to the 'domino effect' mentioned above where a group of bearish traders will be pulled towards bearish tendencies. This is a phenomenon known as suppressed relative herd sensitivity.



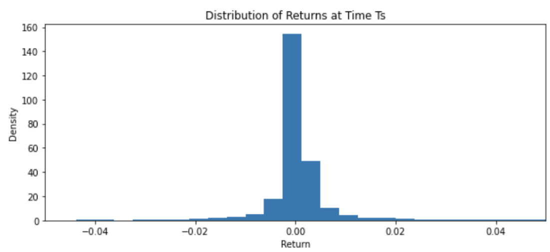


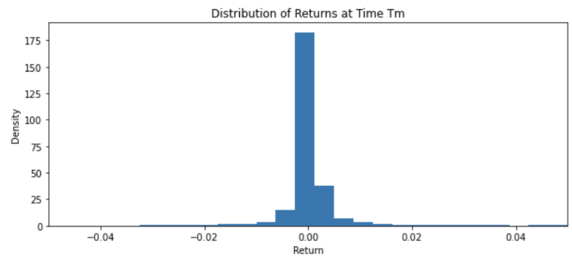


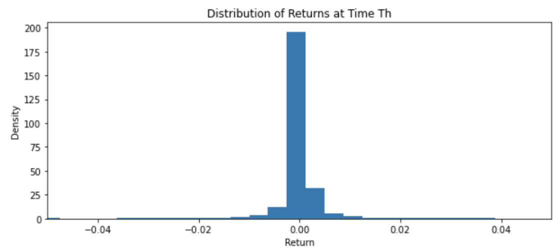
Case 3 ($\alpha = \beta =$ N = 100):

The third case is created with the goal of trying to make an economy where herd behavior and position preference were obsolete, allowing us to use this state as a basis to contextualize the effects of the herd behavior and preference parameter alone. We initialize the parameters to be $\alpha = \beta = 100$ and again randomly select our initial bull and bear values from the stationary distribution.

We predicted the distribution of returns will be completely normal with minimal standard deviation and 0 mean. As depicted in the 3 figures below, we have strayed from our initial hypothesis and notice a non-gaussian distribution at each time increment. Similar to the first case, the distributions display the 'domino effect' of traders slipping in an out of one trading tendency. The only difference is whether traders begin with bearish or bullish investment tendencies. As they are randomly sampled, the initial investment tendency is also randomly decided. Similar to the first case, the distributions across time are left skewed, indicating herd sensitivity. This contributes to traders' tendency to flip from bearish to bullish and vice versa. Lastly, each distribution displays excess kurtosis with high peaks and heavy tails with the tails becoming heavier throughout time.







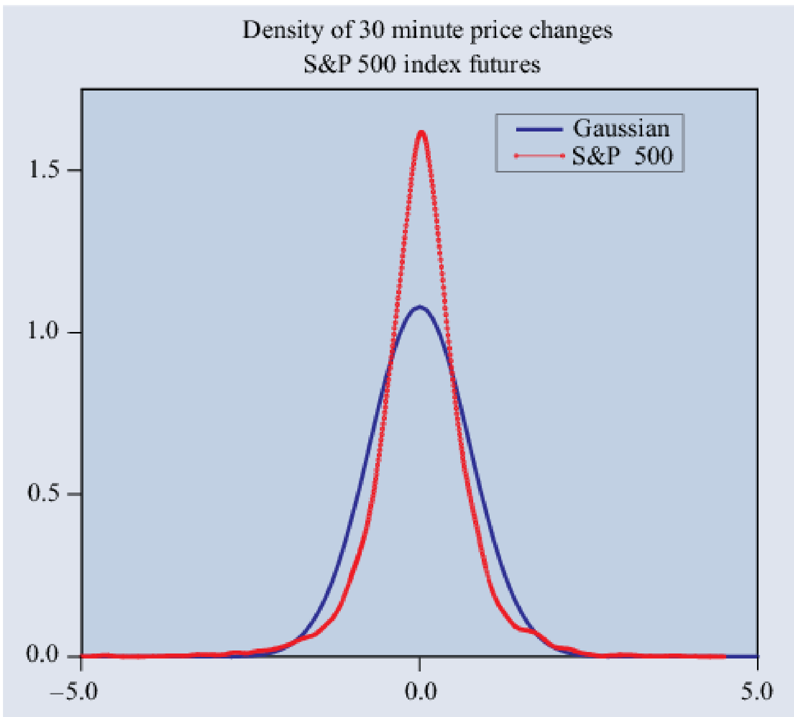
10. Discussion:

As early as the 1960s, Mandelbrot pointed out the insufficiency of the normal distribution for modeling the marginal distribution of asset returns and their heavy-tailed character. Since then, the character of the distribution of price variations has been observed in various markets. One can quantify the distribution through examining the kurtosis of the distribution. According to R Cont, the distribution of returns contains a few stylized facts which are present within our data:

1. Heavy Tails: the distribution follows a sharp peak with a steep fall off.
2. Aggregated Time Scale: as Ts, Tm, and Th increase, the distribution looks more and more like a Gaussian distribution.

3. Local peaks: tt any given time scale, a high degree of variability can be seen. This is quantified by the presence of irregular bursts in time series of a wide variety of volatility estimators.
4. Volume/Volatility Correlation: trading volume is directly correlated with volatility.

According to the paper, we see that the 30 minute density's of the S&P 500 follow a sharp peaked Gaussian distribution with high kurtosis and heavy tails as shown below.



10. Conclusion:

We successfully implemented and simulated the Ehrenfest model described by @cont_herd_2000. Our analysis of the model parameters \alpha and \beta supports their original definitions in all sets of parameters tested. Additionally, we generalize their original interpretations past the original three cases they present.

When trying to contextualize the influence \alpha and \beta played in resulting distributions, we found that mirroring extreme herd sensitivity was possible. Setting \alpha to 0.8 and \beta to 0.2 indicates a preference to be bullish led to a distribution of sharp peaks, heavy tails, large kurtosis, as well as a left skew. Setting traders to have bullish tendencies led to a distribution similar to that of the S&P 500 distribution as time persists. Herd sensitivity is noticed least when traders are equally split to have bullish and bearish investment tendencies as kurtosis is observably lowest when such parameters are set. Lastly, when \beta is larger than \alpha , we discover a sharp peak with a large drop off and interestingly, local peaks where only a select few traders exhibit herd sensitivity. This high kurtosis can be attributed to the fact that as the majority of traders tend to slip into one state and remain within that state over time. However, it can be seen that when the sample of investors are equally split to be bullish and bearish it is less likely for the market to remain in one state for an extended period of time.

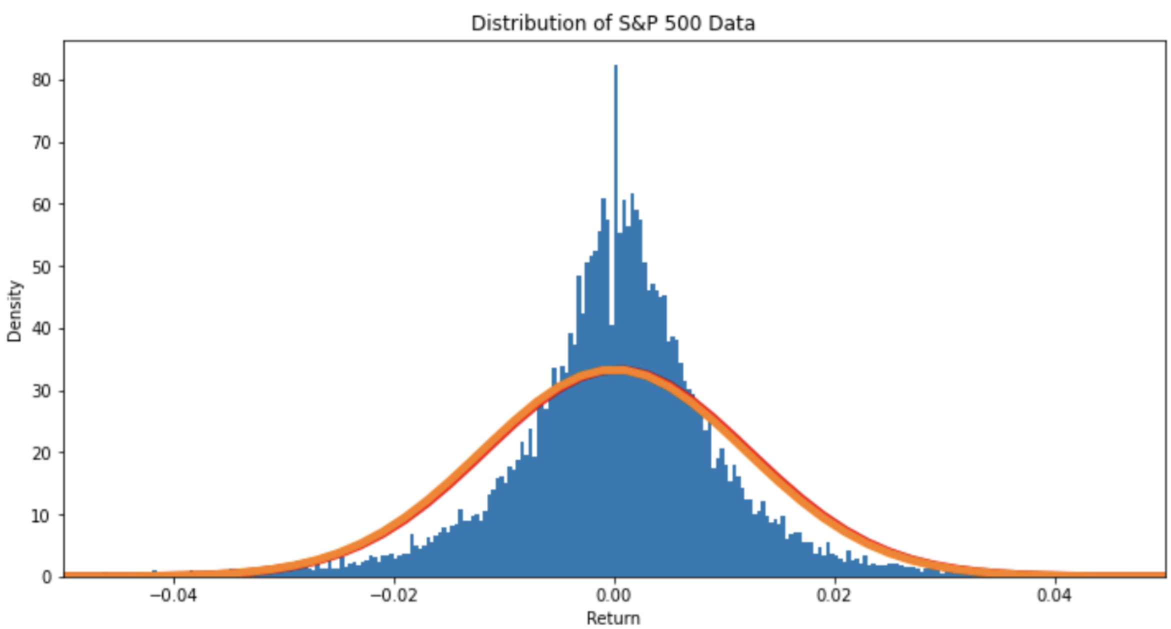
hen discuss the parameters which matched the S&P 500 data and what that means as far as how the SP 500 acts in regard to:

- bull/bear preference
- heard preference/ no preference
- change over time?
-

A,b small:(case 1)

- COMPARE THE HERD SENSITIVE TO THE NON HERD SENSITIVE
    - IT IS REALLY ONE GIANT SKEW WHICH WOULD LEAD TO A FAT TAILS AS AN 'EVENT' COULD LEAD TO PEOPLE ACTING LIKE A HERD!!!!
- LOOK AT THE SHIFTED RETURNS
    - THESE 'EVENTS' HAPPENING OVER AND OVER AGAIN WOULD LEAD TO FAT TAILS
- The sharp peak and 'heavy tails' corresponds to the data being really spread out like what we have been seeing

s&p 500 histogram:

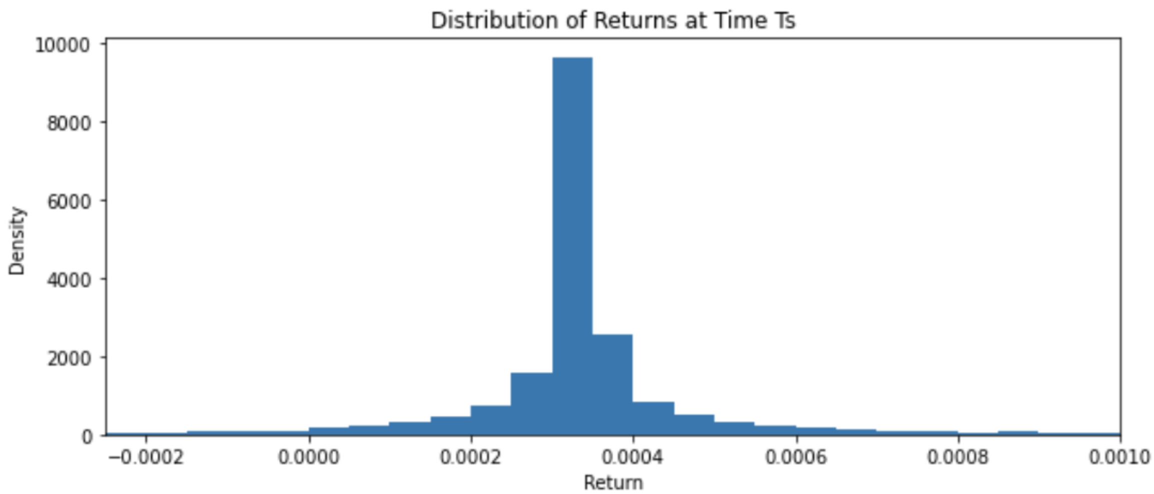


11. <u>Model Histograms:</u>

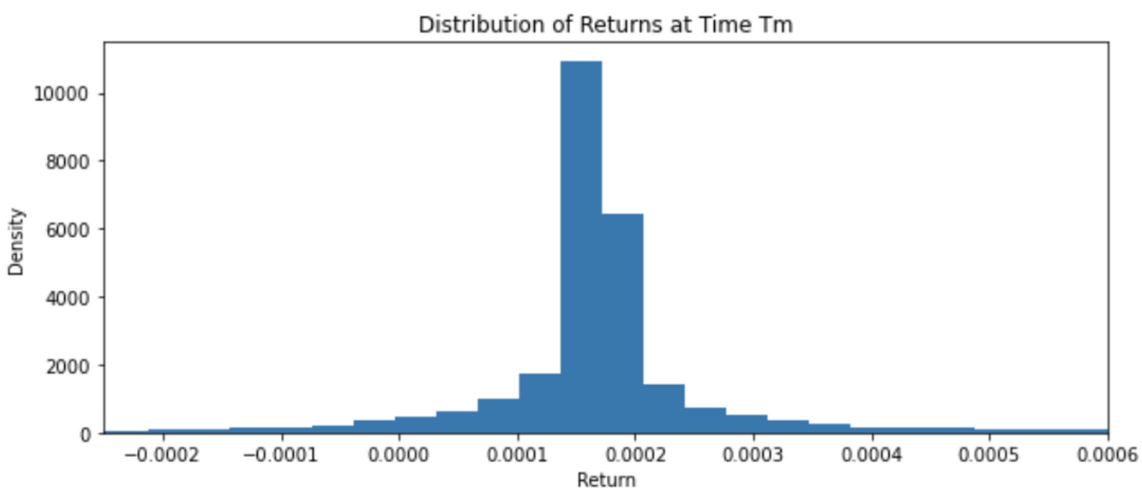
Discussion:

Case 1 (a = 0.8, b = 0.2):

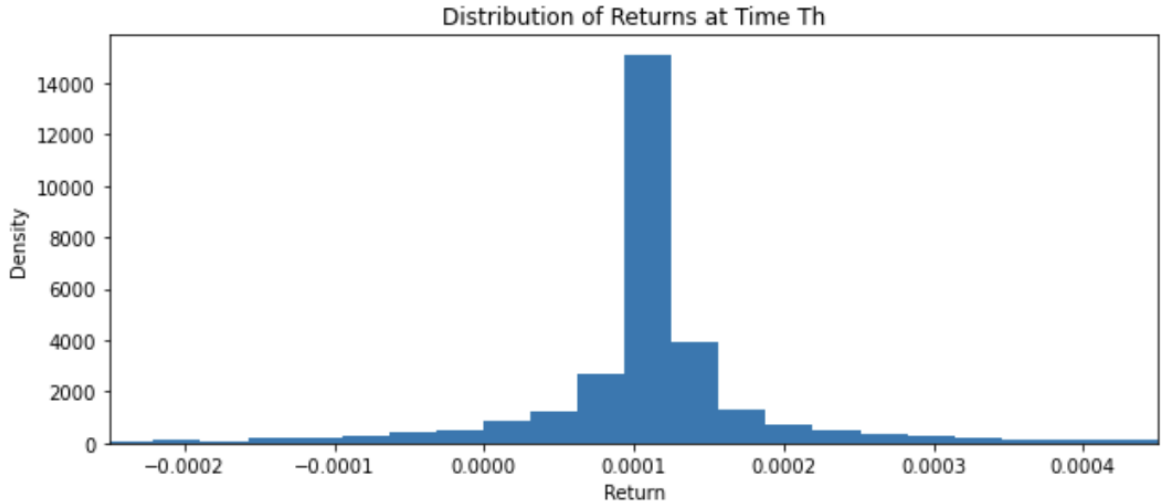
R_Ts:


Distribution of Returns at Time Ts
Density
Return
10000
8000
6000
4000
2000
0
−0.0002
0.0000
0.0002
0.0004
0.0006
0.0008
0.0010

R_Tm:

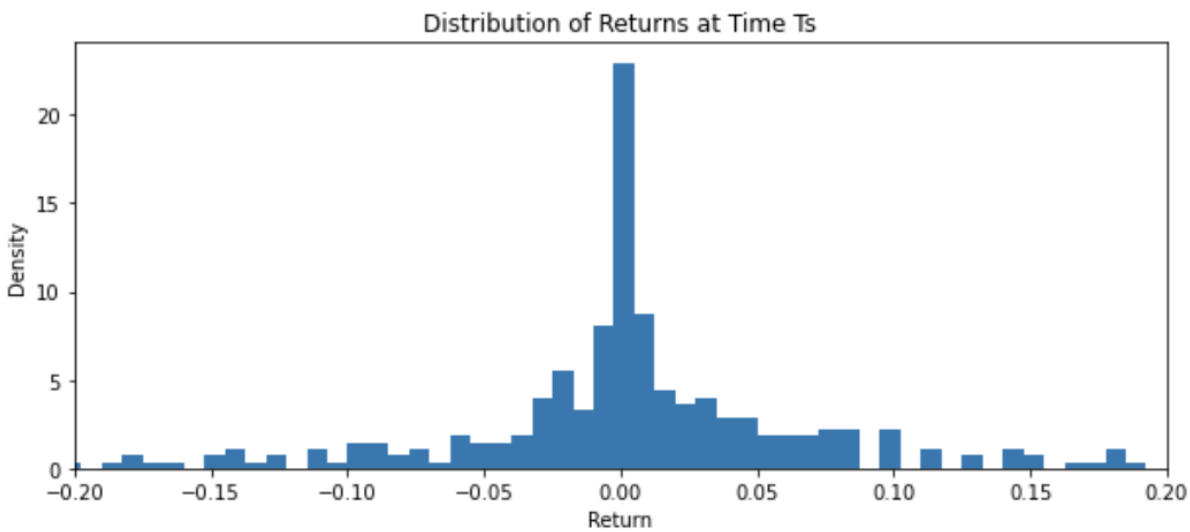

Distribution of Returns at Time Tm
Density
Return
10000
8000
6000
4000
2000
0
−0.0002
−0.0001
0.0000
0.0001
0.0002
0.0003
0.0004
0.0005
0.0006

R_Th:

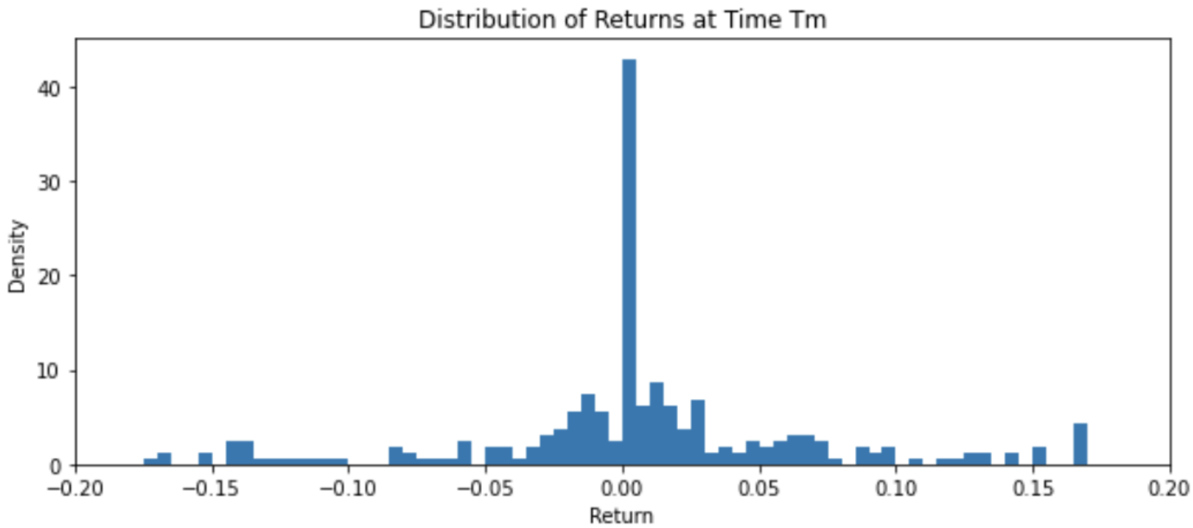


Case 2 (a = 4000, b = 6000):

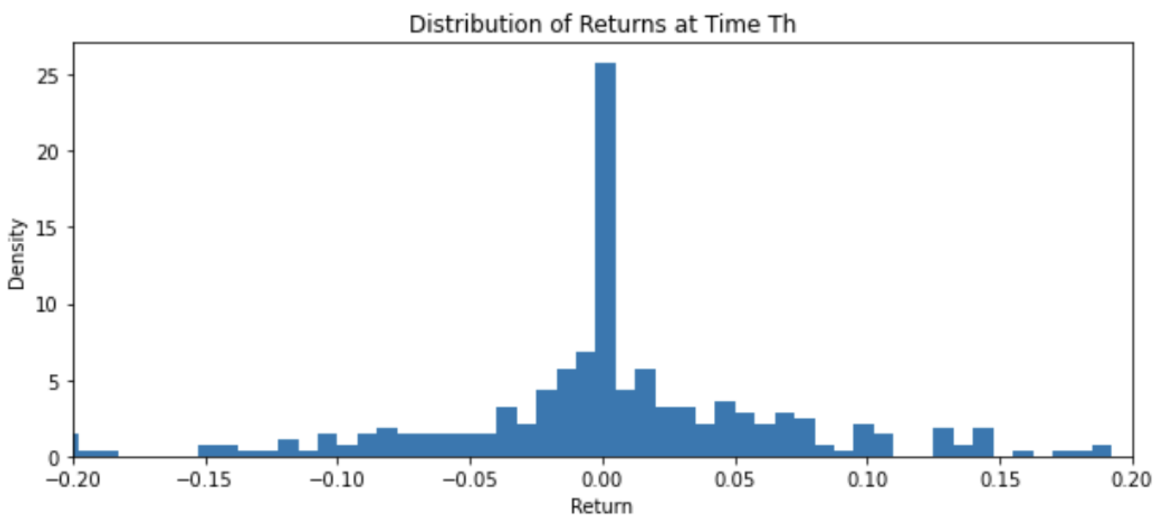
R_Ts:



R_Tm:

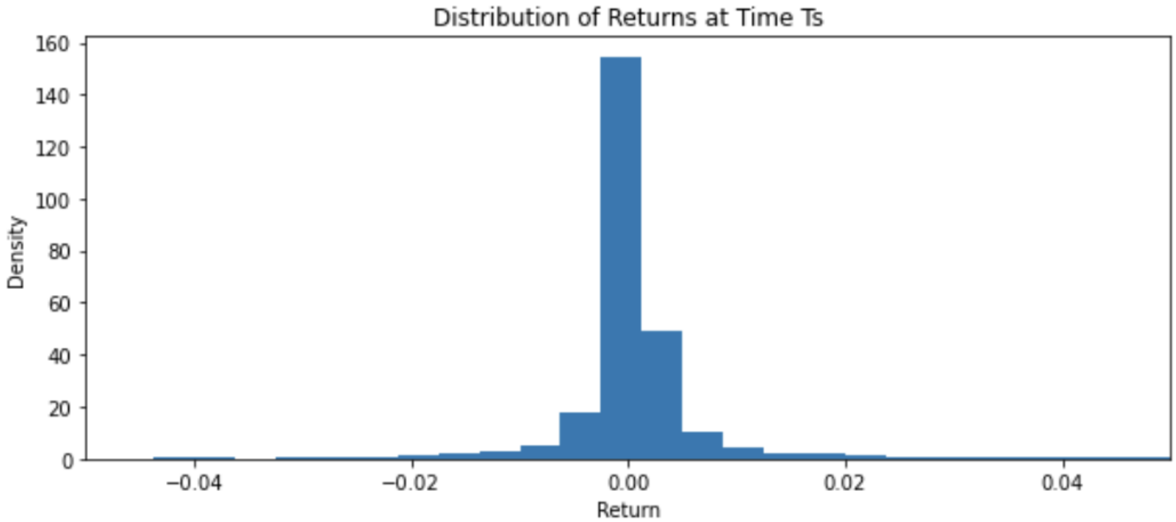


R_Th:

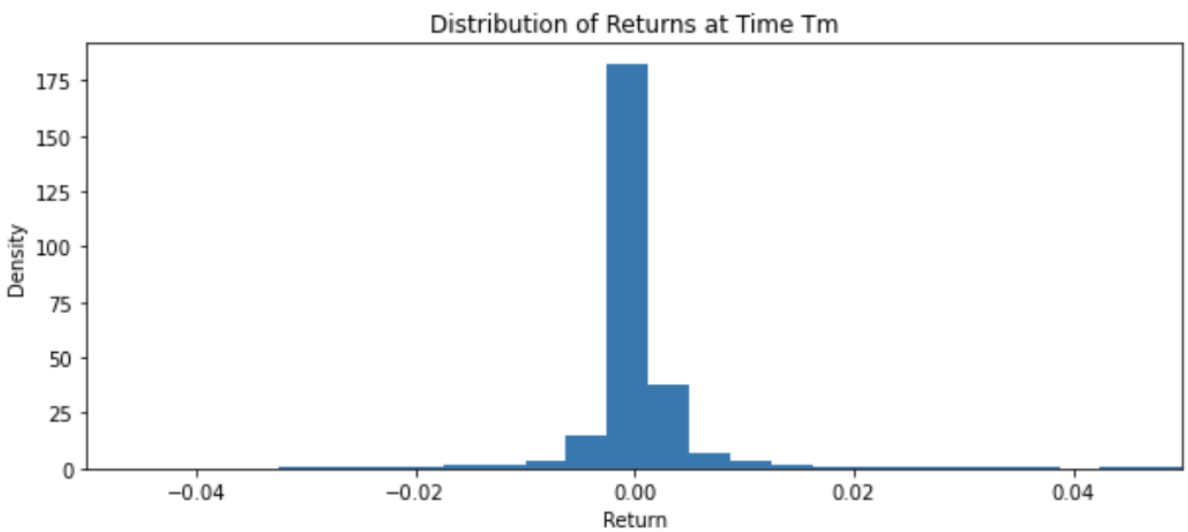


Case 3 (a = 100, b = 100):

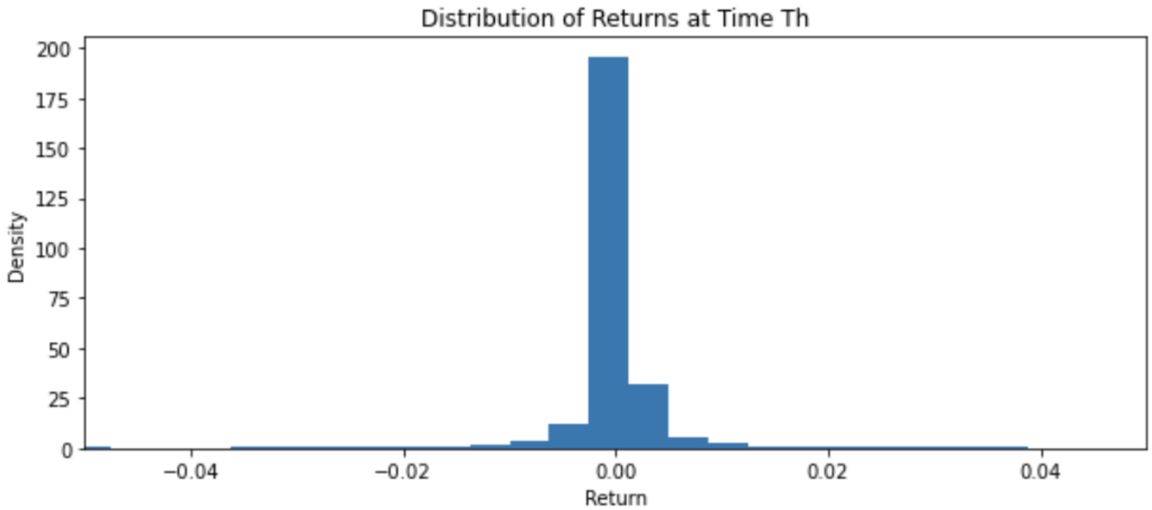
R_Ts:


Distribution of Returns at Time Ts
Density
160
140
120
100
80
60
40
20
0
−0.04
−0.02
0.00
0.02
0.04
Return

R_Tm:


Distribution of Returns at Time Tm
Density
175
150
125
100
75
50
25
0
−0.04
−0.02
0.00
0.02
0.04
Return

R_Th:



12. Conclusion:

References

Cont, R., & Bouchaud, J.-P. (2000). Herd behavior and aggregate fluctuations in financial markets. *Macroeconomic Dynamics*, *4*(2), 170–196. https://doi.org/10.1017/s1365100500015029

Cont, R. (2001). Empirical properties of asset returns: Stylized facts and statistical issues. *Quantitative Finance*, *1*(2), 223–236. https://doi.org/10.1080/713665670

Garibaldi, U., Penco, M. A., & Viarengo, P. (2000). An exact physical approach to market participation models. *Lecture Notes in Economics and Mathematical Systems*, 249–270. https://doi.org/10.1007/978-3-642-55651-7_6

CODE FOR MAKING RETURNS INTO HISTOGRAMS:

```
////
Ts = 3000
Tm = 6000
Th = 9000
```

```
M = 10000

Return_case_1 = pickle.load( open( "sim_1_return.p", "rb" ))

# CASE 1
R_Ts1 = []
R_Tm1 = []
R_Th1 = []
for i in range(0,M):
    R_Ts1.append(Return_case_1[i][Ts-1])
    R_Tm1.append(Return_case_1[i][Tm-1])
    R_Th1.append(Return_case_1[i][Th-1])

 ////

#Graph 1
fig, ax = plt.subplots(figsize=(10, 4))
plt.hist(R_Ts1, density = True, bins = 30000)
plt.xlim(xmin=-0.00025, xmax = 0.001)
plt.title("Distribution of Returns at Time Ts")
plt.xlabel('Return')
plt.ylabel('Density')
plt.show()
'''R_Ts'''

 ////

#Graph 2
fig, ax = plt.subplots(figsize=(10, 4))
plt.hist(R_Tm1, density = True, bins = 10000)
plt.xlim(xmin=-0.00025, xmax = 0.0006)
plt.title("Distribution of Returns at Time Tm")
plt.xlabel('Return')
plt.ylabel('Density')
plt.show()
'''R_Tm'''

 ////

#Graph 3
fig, ax = plt.subplots(figsize=(10, 4))
plt.hist(R_Th1, density = True, bins = 70000)
plt.xlim(xmin=-0.00025, xmax = 0.00045)
plt.title("Distribution of Returns at Time Th")
plt.xlabel('Return')
plt.ylabel('Density')
```

```
plt.show()
'''R_Th'''

 ////

Return_case_2 = pickle.load( open( "sim_2_return.p", "rb" ))

#CASE 2
R_Ts2 = []
R_Tm2 = []
R_Th2 = []
for i in range(0,M):
    R_Ts2.append(Return_case_2[i][Ts-1])
    R_Tm2.append(Return_case_2[i][Tm-1])
    R_Th2.append(Return_case_2[i][Th-1])

 ////
```

```
#Graph 4
fig, ax = plt.subplots(figsize=(10, 4))
plt.hist(R_Ts2, density = True, bins = 500)
plt.xlim(xmin=-0.04, xmax = 0.04)
plt.title("Distribution of Returns at Time Ts")
plt.xlabel('Return')
plt.ylabel('Density')
plt.show()
'''R_Tm'''

 ////

#Graph 5
fig, ax = plt.subplots(figsize=(10, 4))
plt.hist(R_Tm2, density = True, bins = 400)
plt.xlim(xmin=-0.04, xmax = 0.04)
plt.title("Distribution of Returns at Time Tm")
plt.xlabel('Return')
plt.ylabel('Density')
plt.show()
'''R_Tm'''

 ////
```

```
#Graph 6
fig, ax = plt.subplots(figsize=(10, 4))
plt.hist(R_Th2, density = True, bins = 500)
plt.xlim(xmin=-0.05, xmax = 0.05)
plt.title("Distribution of Returns at Time Th")
plt.xlabel('Return')
plt.ylabel('Density')
plt.show()
'''R_Tm'''

 ////

Return_case_3 = pickle.load( open( "sim_3_return.p", "rb" ))

#CASE 3
R_Ts3 = []
R_Tm3 = []
R_Th3 = []
for i in range(0,M):
    R_Ts3.append(Return_case_3[i][Ts-1])
    R_Tm3.append(Return_case_3[i][Tm-1])
    R_Th3.append(Return_case_3[i][Th-1])

 ////

#Graph 7
fig, ax = plt.subplots(figsize=(10, 4))
plt.hist(R_Ts3, density = True, bins = 800)
plt.xlim(xmin=-0.05, xmax = 0.05)
plt.title("Distribution of Returns at Time Ts")
plt.xlabel('Return')
plt.ylabel('Density')
plt.show()
'''R_Tm'''

 ////
```

```
#Graph 8
fig, ax = plt.subplots(figsize=(10, 4))
plt.hist(R_Tm3, density = True, bins = 800)
plt.xlim(xmin=-0.05, xmax = 0.05)
plt.title("Distribution of Returns at Time Tm")
plt.xlabel('Return')
plt.ylabel('Density')
plt.show()
'''R_Tm'''

////
```

```
#Graph 9
fig, ax = plt.subplots(figsize=(10, 4))
plt.hist(R_Th3, density = True, bins = 1000)
plt.xlim(xmin=-0.02, xmax = 0.02)
plt.title("Distribution of Returns at Time Th")
plt.xlabel('Return')
plt.ylabel('Density')
plt.show()
'''R_Tm'''
```

KURTOSIS CALCULATION: TO DO FOR EACH RESULT PLOT

```
from scipy.stats import kurtosis
x =[55, 78, 65, 98, 97, 60, 67, 65, 83, 65] # change list to be R_Ts1, R_Tm1….R_Th3
print(kurtosis(x, fisher=False))
```