Triple Integral

EXAMPLE 5. Find the volume of the tetrahedron T bounded by the planes $x+2y+z=2, x=2y, x=0, z=0$.

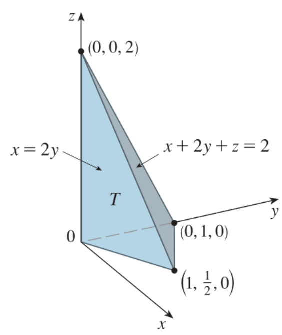

FIGURE 14

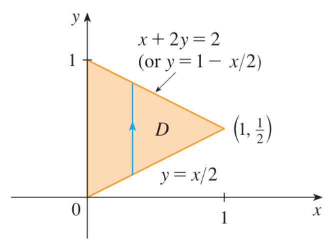

FIGURE 15

$$V(T)=\iiint_T dV=\int_0^1\int_{x/2}^{1-x/2}\int_0^{2-x-2y} dz\,dy\,dx$$
$$=\int_0^1\int_{x/2}^{1-x/2}(2-x-2y)\,dy\,dx=\frac{1}{3}$$

Or we can use the formula

$$V=hS=\left(\frac{1}{3}\right)(2.\frac{1}{2})=\frac{1}{3}$$

1 Triple Integrals in Cylindrical Coordinates

Evaluating Triple Integrals with Cylindrical Coordinates

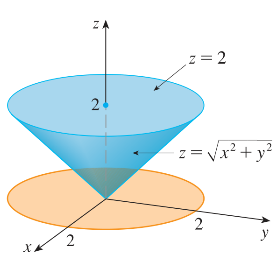

FIGURE 9

EXAMPLE 4. Evaluate $\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2}(x^2+y^2)\,dz\,dy\,dx$.

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{2}(x^2+y^2)\,dz\,dy\,dx=\iiint_E(x^2+y^2)\,dV$$
$$=\int_0^{2\pi}\int_0^2\int_r^2 r^2\,r\,dz\,dr\,d\theta$$
$$=\int_0^{2\pi}d\theta\int_0^2 r^3(2-r)dr$$
$$=2\pi\left[\frac{1}{2}r^4-\frac{1}{5}r^5\right]_0^2=\frac{16}{5}\pi$$

2 Triple Integrals in Spherical Coordinates

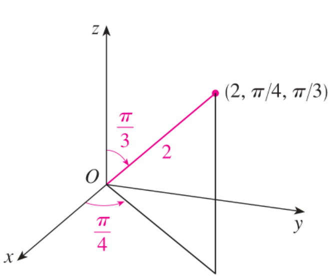

FIGURE 6

EXAMPLE 1. Find the rectangular coordinates of $(2,\pi/4.\pi/3)$.

$$x=\rho\sin\phi\cos\theta=2\sin\frac{\pi}{3}\cos\frac{\pi}{4}=2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)=\sqrt{\frac{3}{2}}$$

$$y=\rho\sin\phi\sin\theta=2\sin\frac{\pi}{3}\sin\frac{\pi}{4}=2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)=\sqrt{\frac{3}{2}}$$

$$z=\rho\cos\phi=2\cos\frac{\pi}{3}=2(\tfrac{1}{2})=1$$

EXAMPLE 2. Find spherical coordinates of $(0, 2\sqrt{3}, -2)$.

$$\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{0 + 12 + 4} = 4$$

So we have

$$\begin{aligned} \cos\phi &= \frac{z}{p} = -\frac{2}{4} = -\frac{1}{2} \qquad \frac{2\pi}{3} \\ \cos\theta &= \frac{x}{\rho\sin\phi} = 0 \qquad \theta = \frac{\pi}{2} \end{aligned}$$

Note. $\theta \neq 3\pi 2$ since $y > 0$.