3.

$$\int_1^2\int_0^2 (y+2xe^y)\,dx\,dy$$
$$=\int_1^2 \left[yx+x^2e^y\right]_0^2\,dy=\int_1^2 2y+4e^y\,dy$$
$$=\left[y^2+4e^y\right]_1^2=3+4(e^2-e)$$

4.

$$\int_0^1\int_0^1 ye^{xy}\,dx\,dy$$
$$=\int_0^1 \left[e^{xy}\right]_0^1\,dy=\int_0^1 e^y-1\,dy$$
$$=\left[e^y-y\right]_0^1=e-2$$

5.

$$\int_0^1\int_0^x \cos\left(x^2\right)dy\,dx$$
$$=\int_0^1 x\cos\left(x^2\right)dx=\frac{1}{2}\int_0^1\cos\left(x^2\right)dx^2$$
$$=\frac{1}{2}\left[\sin\left(x^2\right)\right]_0^1=\frac{1}{2}(\sin 1-\sin 0)$$

6.

$$\int_0^1\int_x^{e^x} 3xy^2\,dy\,dx$$
$$=\int_0^1\left[xy^3\right]_0^{e^x}\,dx=\int_0^1 xe^{3x}\,dx$$
$$=\frac{1}{3}\int_0^1 x\left(e^{3x}\right)'\,dx$$
$$=\frac{1}{3}\left[\left(xe^{3x}\right)\Big|_0^1-\int_0^1 e^{3x}\,dx\right]$$
$$=\frac{1}{3}\left[e^3-\frac{1}{3}e^3\right]=\frac{2}{9}e^3$$

7.

$$\int_0^\pi\int_0^1\int_0^{\sqrt{1-y^2}} y\sin x\,dz\,dy\,dx$$
$$=\int_0^\pi \sin x\,dx\int_0^1\int_0^{\sqrt{1-y^2}} y\,dz\,dy$$

Let $z=r\cos\theta,\ y=r\sin\theta$. Then

$$D=\{(r,\theta)\mid 0\le\theta\le 2\pi, 0\le r\le 1\}$$

$$\begin{aligned}\int_0^1\int_0^{\sqrt{1-y^2}} y\,dz\,dy&=\int_0^{2\pi}\int_0^1 r\sin\theta\, r\,dr\,d\theta\\&=\int_0^{2\pi}\sin\theta\,d\theta\int_0^1 r^2\,dr\\&=2\left[-\cos\theta\right]_0^\pi\left[\frac{r^3}{3}\right]_0^1=\frac{2}{3}\end{aligned}$$

Hence $\int\limits_0^\pi\int_0^1\int_0^{\sqrt{1-y^2}} y\sin x\,dz\,dy\,dx=1\left(\frac{2}{3}\right)=\frac{2}{3}$.

8.

$$\int_0^1\int_0^y\int_x^1 6xyz\,dz\,dx\,dy$$
$$=\int_0^1\int_0^x\int_y^1 6xyz\,dz\,dy\,dx$$
$$=\int_0^1\int_0^x 3xy^3-3xy\,dy\,dx$$
$$=\int_0^1\left[\frac{3}{4}xy^4-\frac{3}{2}xy^2\right]_0^x\,dx$$
$$=\int_0^1\frac{3}{4}x^5-\frac{3}{2}x^3\,dx=\left[\frac{1}{8}x^6-\frac{3}{8}x^4\right]_0^1$$
$$=-\frac{1}{4}$$