Predicates and Quantifiers

Last Thursday of Feb

1 Introduction

Predicates

Statements involving variables, such as $x > 3$, $x = y + 3$, $x + y = z$. They are neither true nor false when the values are not specified.

PRECONDITIONS AND POSTCONDITIONS Predicates are used to establish the correctness of computer programs, if it always produces the desired output. The statements describing valid input are **preconditions** and the conditions that the output should satisfy are **postconditions**.

Quantifiers

Quantification expresses the extent to which a predicate is true over a *range* of elements. The area of logic that deals with predicates and quantifiers is the **predicate calculus**.

Definition : Universal Quantification

The *universal quantification* of $P(x)$ is $\forall x P(x)$, *(domain must always be specified).*

$$\forall x P(x) = P(x_1) \wedge P(x_2) \wedge \cdots \wedge P(x_n)$$

False when there's a **counterexamplen**.
$P(x)$: the value of the **propositional function** P at x

Bien 1 luong tu thanh menh de : luong tu hoa. 1. luong tu hoa pho quat (voi moi) 2. luong tu hoa ton tai (ton tai x)

Viec chi ro khong ian tac dong la rat quan trong. gia chi chan ly cua 1 cau duoc luong tu hoa phu thuoc vao viec ko gian co phan tu nao.

VD $x^2 \geq x$ and real numbers VS natural numbers.

nhu vay de cminh $\forall x P(x)$ F, tim 1 gia tri x la $P(x) = F$

Definition : Existential Quantification

The *existential quantification* of $P(x)$ is $\exists x P(x)$.

$$\exists x P(x) = P(x_1) \vee P(x_2) \vee \cdots \vee P(x_n)$$

It's helpful to think in terms of looping and searching when determining the truth value of a quantification.

THE UNIQUENESS QUANTIFIER. $\exists_1$ or $\exists_!$ states "There exists a unique x such that $P(x) = T$".

Quantifiers with Restricted Domains

luong tu hoac gan value thi x bi rang buoc. x bi rang buoc boi chu "ton tai"

1 chuw cai bieu thi giong nhau nhung ko chong cheo

Logical Equivalences Involving Quantifiers

Statements

Negating Quantified Expressions

$$\neg(\forall P(x)) \equiv \exists(\neg P(x))$$

Translating from English into Logical Expressions

EXAMPLE 1. Express the Statement *"Every student in this class has studied calculus"*.

$$\forall x(S(x) \to C(x))$$

Using Quantifiers in System Specification

Examples from Lewis Carol

hoc bai 5 thi chung minh la hop thuc

Logic Programming

PROLOG

2 Nested Quantifiers

Introduction

Understanding Statements Involving Nested Quantifiers

EXAMPLE 1.

$$\forall x \forall y((x > 0) \wedge (y < 0) \to (xy < 0))$$

EXAMPLE 2.

$$\forall x(C(x) \vee \exists y(C(y) \wedge F(x, y)))$$

OF QUANTIFICATION AS LOOPS

The Order of Quantifiers

3 Rules of Inference

hop thuc la gi: khi cac gia dinh thi dung thi ket luan cung dung

$$(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \to q$$