1 Propositions

Definition : Proposition

A **proposition** is a declarative sentence (a sentence that declares a fact) is either true or false, but not both. A **compound proposition** is made from existing propositions with *logical operators*.

We use letters to denote **propositional variables** (or **statement variables**), that is, variables that represent proposition.

1.1 Propositional Logic

thu tu thuc hien: phu dinh, hoi, tuyen roi keo theo

Propositions

- **Negation.** The negation of p, denoted by $\neg p$ (or $\overline{p}$), is the statement "not p".

  The truth value of $\neg p$ is the opposite of the truth value of p.

- **Conjuction.** The conjuction $p \wedge q$ is true when both p and q are true and false otherwise.

  Note that the word *"but"* is used instead of *"and"*.

- **Disjunction.** The disjunction $p \vee q$ is the proposition *"p or q"*, false when both p and q are false and true otherwise. Can be called **"inclusive or"**.

- **Exclusive or.** The *exclusive or* of p and q, denoted by $p \oplus q$ is *"or, but not both"*.

Conditional Statements

- **Implication.** The conditional statement $p \to q$ is "if p, then q". It's false when p is true and q is false, and true otherwise. p is called the *hypothesis* (or antecedent or premise) and q is the *conclusion* (or consequence).

"if p, then q"	"p implies q"
"if p, q"	"p only if q"
"p is sufficient for q"	"a sufficient condition for q is p"
"q if p"	"q whenever p"
"q when p"	"q is necessary for p"
"a necessary condition for p is q"	"q follows from p"
"q unless $\neg p$"	

  Especially *"p only if q"*, it's not true when p and q has different truth values.

  And *"q unless* $\neg p$*"*.

- **Bi-Implication.** The *biconditional statement* $p \leftrightarrow q$ is the proposition *"p if and only if q"*.

  Also called *bi-implications*, T when p and q have the same truth value, or $(p \to q) \wedge (q \to p)$

`CONVERSE, CONTRAPOSITIVE, AND INVERSE` Some new statements with a conditional statements $p \to q$.

- ☐ The **converse**: $q \to p$.
- ☐ The **contrapositive**: $\neg q \to \neg p$
- ☐ The **inverse**: $\neg p \to \neg q$

When 2 compound propositions always have the same truth value, we call them **equivalent** (like a conditional statement and its contrapositive).

Precedence and Logical Operators

TABLE 8 Precedence of Logical Operators.	
Operator	*Precedence*
$\neg$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

Logic and Bit Operations

Boolean variables, 1 represents T, 0 represents F.

2 Applications of Propositional Logic

3 Propositional Equivalences

Introduction

Definition : Tautology, Contradiction, Contigency

A compound proposition that is always true whatever the truth values of the propositional variables are, is called a *tautology*. Vice versa, **contradiction**. Neither, **contigency**.

EXAMPLE 1. $p \vee \neg p$ is tautology, $p \wedge \neg p$ is contradiction.

Logical Equivalences

Notation: $p \equiv q$: p and q are *logically equivalence* if $p \leftrightarrow q$ is tautology.

Definition : De Morgan's Laws

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Propositional Satisfiability

A compound proposition is **satisfiable** if there's a way of assigning truth values to its variables that makes it true. We call it a **solution**.

If not, it's **unsatisfiable**. In other words, its negation is tautology.

EXAMPLE. $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$ is true when p, q, and r have the same truth value. Hence, it's *satisfiable*.

Applications of Satisfiability

Solving Satisfiability Problems