3.

$$\int_1^2 \int_0^2 (y + 2xe^y)\, dx\, dy$$
$$= \int_1^2 \left[yx + x^2 e^y\right]_0^2\, dy = \int_1^2 2y + 4e^y\, dy$$
$$= \left[y^2 + 4e^y\right]_1^2 = 3 + 4(e^2 - e)$$

4.

$$\int_0^1 \int_0^1 ye^{xy}\, dx\, dy$$
$$= \int_0^1 \left[e^{xy}\right]_0^1\, dy = \int_0^1 e^y - 1\, dy$$
$$= \left[e^y - y\right]_0^1 = e - 2$$

5.

$$\int_0^1 \int_0^x \cos(x^2)\, dy\, dx$$
$$= \int_0^1 x\cos(x^2)\, dx = \frac{1}{2}\int_0^1 \cos(x^2)\, dx^2$$
$$= \frac{1}{2}\left[\sin(x^2)\right]_0^1 = \frac{1}{2}\sin 1$$

6.

$$\int_0^1 \int_x^{e^x} 3xy^2\, dy\, dx$$
$$= \int_0^1 \left[xy^3\right]_x^{e^x}\, dx = \int_0^1 xe^{3x} - x^4\, dx$$
$$= \frac{1}{3}\int_0^1 x\left(e^{3x}\right)'\, dx - \frac{1}{5}x^5\Big|_0^1$$
$$= \frac{1}{3}\left[\left(xe^{3x}\right)\Big|_0^1 - \int_0^1 e^{3x}\, dx\right] - \frac{1}{5}$$
$$= \frac{1}{3}\left[e^3 - \frac{1}{3}e^3 + \frac{1}{3}\right] - \frac{1}{5} = \frac{2e^3+1}{9} - \frac{1}{5}$$

7.

$$\int_0^\pi \int_0^1 \int_0^{\sqrt{1-y^2}} y\sin x\, dz\, dy\, dx$$
$$= \int_0^\pi \sin x\, dx \int_0^1 \int_0^{\sqrt{1-y^2}} y\, dz\, dy$$

Let $z = r\cos\theta,\ y = r\sin\theta$. Then

$$D = \{(r,\theta) \mid 0 \le \theta \le 2\pi, 0 \le r \le 1\}$$

$$\begin{aligned}\int_0^1 \int_0^{\sqrt{1-y^2}} y\, dz\, dy &= \int_0^{2\pi} \int_0^1 r\sin\theta\, r\, dr\, d\theta \\ &= \int_0^{2\pi} \sin\theta\, d\theta \int_0^1 r^2\, dr \\ &= 2\left[-\cos\theta\right]_0^\pi \left[\frac{r^3}{3}\right]_0^1 = \frac{2}{3}\end{aligned}$$

Hence $\int\limits_0^\pi \int_0^1 \int_0^{\sqrt{1-y^2}} y\sin x\, dz\, dy\, dx = 1\left(\frac{2}{3}\right) = \frac{2}{3}$.

8.

$$\int_0^1 \int_0^y \int_x^1 6xyz\ dz\, dx\, dy$$
$$= \int_0^1 \int_0^x \int_y^1 6xyz\ dz\, dy\, dx$$
$$= \int_0^1 \int_0^x (3xy - 3xy^3)\, dy\, dx$$
$$= \int_0^1 \left[\frac{3}{2}xy^2 - \frac{3}{4}xy^4\right]_0^x\, dx$$
$$= \int_0^1 \left(\frac{3}{2}x^3 - \frac{3}{4}x^5\right)\, dx = \left[\frac{3}{8}x^4 - \frac{1}{8}x^6\right]_0^1$$
$$= \frac{1}{4}$$