1. $\int_C (xy+x+y)\,dx+(xy+x-y)\,dy,\ \ C = x^2+y^2 = R^2$
Sử dụng công thức Green:

$$\begin{cases} \dfrac{\partial P}{\partial y} = x+1 \\ \dfrac{\partial Q}{\partial x} = y+1 \end{cases}$$

$\rightarrow \dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y} = y - x$

$$I = \iint\limits_{x^2+y^2\le R^2} (y-x)\,dx\,dy$$
$$= \iint\limits_{x^2+y^2\le R^2} y\,dx\,dy - \iint\limits_{x^2+y^2\le r^2} x\,dx\,dy$$

$\rightarrow I = 0$
D đối xứng qua Ox, $F(x,y)$ lẻ đối với y thì $I = 0$;
hoặc khi đối xứng qua Oy - lẻ với x.

2. $I = \int_C e^x\left(1+\dfrac{y^2}{2}\right)dx - (y-\sin y)\,dy$, C là đường gấp khúc nối $O(0,0), A(1,1), B(0,2)$
Ta có: $\dfrac{\partial Q}{x} - \dfrac{\partial P}{\partial y} = -e^x y$

$$I = e^x(1+\frac{y^2}{2}) - e^x y = \iint\limits_C -e^x y\,dx\,dy$$
$$= -\int_0^1 \int_x^{2-x} e^x y\,dy = \frac{1}{2}\int_0^1 e^x(4x-4)\,dx$$
$$= 4-2e$$

3. $I = \int_C (xy+e^x\sin x+x+y)\,dx - (xy-e^{-y}+x-\sin y)\,dy,\ C: x^2+y^2 = 2x$
$\dfrac{\partial Q}{\partial x} - \dfrac{\partial P}{\partial y} = -y-x-2$, mà $\iint\limits_C y\,dx\,dy = 0$
$I = -\iint\limits_C (y+x+2)\,dx\,dy = -\iint\limits_D (x+2)\,dx\,dy$

Đặt $x = r\cos\phi,\ y = r\sin\phi \Rightarrow -\dfrac{\pi}{2} \le \phi \le \dfrac{\pi}{2},\ 0 \le r \le 2\cos\phi$

$$I = -\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\phi \int_0^{2\cos\phi} (r\cos\phi+2)r\,dr = -3\pi$$