

1 Propositions

Definition : Proposition

A **proposition** is a declarative sentence (a sentence that declares a fact) is either true or false, but not both. A **compound proposition** is made from existing propositions with *logical operators*.

We use letters to denote **propositional variables** (or **statement variables**), that is, variables that represent proposition.

1.1 Propositional Logic

thu tu thuc hien: phu dinh, hoi, tuyen roi keo theo

Propositions

- **Negation.** The negation of p , denoted by $\neg p$ (or \bar{p}), is the statement "not p ".
The truth value of $\neg p$ is the opposite of the truth value of p .
- **Conjunction.** The conjunction $p \wedge q$ is true when both p and q are true and false otherwise.
Note that the word "*but*" is used instead of "*and*".
- **Disjunction.** The disjunction $p \vee q$ is the proposition "*p or q*", false when both p and q are false and true otherwise. Can be called "**inclusive or**".
- **Exclusive or.** The *exclusive or* of p and q , denoted by $p \oplus q$ is "*or, but not both*".

Conditional Statements

- **Implication.** The conditional statement $p \rightarrow q$ is "if p , then q ". It's false when p is true and q is false, and true otherwise. p is called the *hypothesis* (or antecedent or premise) and q is the *conclusion* (or consequence).

./implication.png

Especially "*p only if q*", it's not true when p and q has different truth values.
And "*q unless $\neg p$* ".

■ **Bi-Implication.** The *biconditional statement* $p \leftrightarrow q$ is the proposition "*p if and only if q*".
 Also called *bi-implications*, T when p and q have the same truth value, or $(p \rightarrow q) \wedge (q \rightarrow p)$

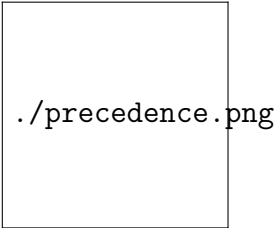
CONVERSE, CONTRAPOSITIVE, AND INVERSE Some new statements with a conditional statements $p \rightarrow q$.

- The **converse**: $q \rightarrow p$.
- The **contrapositive**: $\neg q \rightarrow \neg p$
- The **inverse**: $\neg p \rightarrow \neg q$

When 2 compound propositions always have the same truth value, we call them **equivalent** (like a conditional statement and its contrapositive).

Precedence and Logical Operators

Logic and Bit Operations



Boolean variables, 1 represents T, 0 represents F.

2 Applications of Propositional Logic

3 Propositional Equivalences

Introduction

Definition : **Tautology, Contradiction, Contingency**

A compound proposition that is always true whatever the truth values of the propositional variables are, is called a *tautology*. Vice versa, **contradiction**. Neither, **contingency**.

EXAMPLE 1. $p \vee \neg p$ is tautology, $p \wedge \neg p$ is contradiction.

Logical Equivalences

Notation: $p \equiv q$: p and q are *logically equivalence* if $p \leftrightarrow q$ is tautology.

Definition : **De Morgan’s Laws**

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Propositional Satisfiability

A compound proposition is **satisfiable** if there’s a way of assigning truth values to its variables that makes it true. We call it a **solution**.

If not, it’s **unsatisfiable**. In other words, its negation is tautology.

EXAMPLE. $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$ is true when p , q , and r have the same truth value. Hence, it’s *satisfiable*.

Applications of Satisfiability

Solving Satisfiability Problems