Assignment1

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1 11.16.4.6 Question

Q: Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

solution:

We define the sample space of this experiment as S, the set of all possible ways that the letters can be inserted into the envelopes. Since there are $3! = 6$ possible permutations of the three letters, each permutation is equally likely to occur, and we have $|S| = 6$.

Let A_1, A_2, and A_3 be the events that Person 1, Person 2, and Person 3, respectively, receive the correct letter. We want to find the probability of the event that at least one letter is in its proper envelope, which can be expressed as:

$$P(A_1 + A_2 + A_3)$$

By the principle of inclusion-exclusion, we have:

$$P(A_1+A_2+A_3) = P(A_1)+P(A_2)+P(A_3)-P(A_1.A_2)-P(A_1.A_3)-P(A_2.A_3)+P(A_1.A_2.A_3)$$

The first three terms on the right-hand side represent the probabilities that exactly one person receives the correct letter. We have:

$$P(A_1) = \frac{1}{3}$$

$$P(A_2) = \frac{1}{3}$$

$$P(A_3) = \frac{1}{3}$$

The next three terms on the right-hand side represent the probabilities that exactly two persons receive the correct letters. We have:

$$P(A_1.A_2) = \frac{1}{6}$$

$$P(A_1.A_3) = \frac{1}{6}$$

$$P(A_2.A_3) = \frac{1}{6}$$

The last term on the right-hand side represents the probability that all three persons receive the correct letters, which is $1/6$. Substituting these values into the inclusion-exclusion formula, we get:

$$P(A_1 + A_2 + A_3) = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} - \frac{1}{6} - \frac{1}{6} - \frac{1}{6} + \frac{1}{6}$$

Simplifying, we get:

$$P(A_1 + A_2 + A_3) = \frac{2}{3}$$

Therefore, the probability that at least one letter is in its proper envelope is $\frac{2}{3}$.