

Assignment1

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1 11.16.4.6 QUESTION

Three letters are dictated to three persons and an envelope is addressed to each of them, the letters are inserted into the envelopes at random so that each envelope contains exactly one letter. Find the probability that at least one letter is in its proper envelope.

The total number of ways to insert the letters into the envelopes is $3! = 6$, as there are three letters and three envelopes, and each letter can be inserted into any of the three envelopes.

To count the number of ways that none of the letters are in their proper envelopes, we can use the principle of derangements. The number of derangements of a set of n elements is the number of permutations of the elements such that no element appears in its original position. The number of derangements of a set of n elements is denoted by $!n$ and can be calculated recursively as follows: $!n = (n-1)(!(n-1) + !(n-2))$, with $!1 = 0$ and $!2 = 1$. Therefore, the number of ways to insert the three letters into the envelopes such that none of the letters are in their proper envelopes is $!3 = 2$.

The probability that at least one letter is in its proper envelope is therefore $1 - (\text{number of ways none are in proper envelope} / \text{total number of ways}) = 1 - 2/6 = 2/3$.

Therefore, the probability that at least one letter is in its proper envelope is $2/3$.