Department of Electronic & Telecommunications Engineering

University of Moratuwa

EN4384 – Wireless and Mobile Communications

Simulation Assignment

Name	**Index No**
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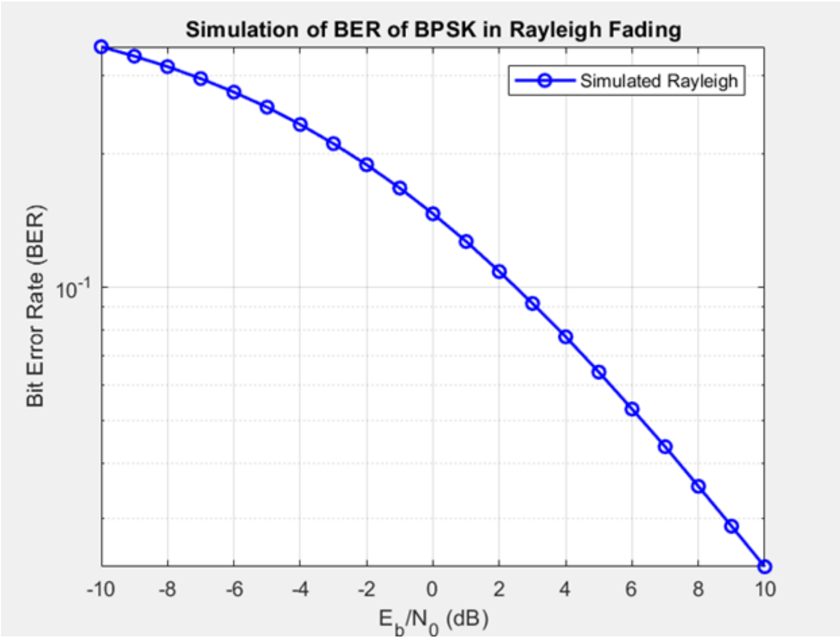
Task 1: Average bit error rate (BER) evaluation in fading channels

(a) Simulated average BER of binary phase shift keying (BPSK) in a Rayleigh fading channel



(b) The average probability of error over a fading channel is given by:

$$P_{e,\text{avg}} = \int_0^\infty P_e f_\beta(\beta)\, d\beta$$

Where:

- $P_{e,\text{avg}}$: Average probability of error
- P_e: Error probability for a given channel realization
- γ: Signal-to-noise ratio (SNR)
- $f_\beta(\beta)$: PDF of the output SNR

For a BPSK in Rayleigh fading channel without diversity:

$$P_e = Q\left(\sqrt{2\gamma\beta}\right), \quad f_\beta(\beta) = \frac{1}{\bar{\gamma}} e^{-\frac{\beta}{\bar{\gamma}}}$$

where:

- $Q(x)$: Q-function (tail probability of a standard normal distribution), defined as:
$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2}\, dt$$
- $\bar{\gamma}$: Average Signal-to-noise ratio (SNR)

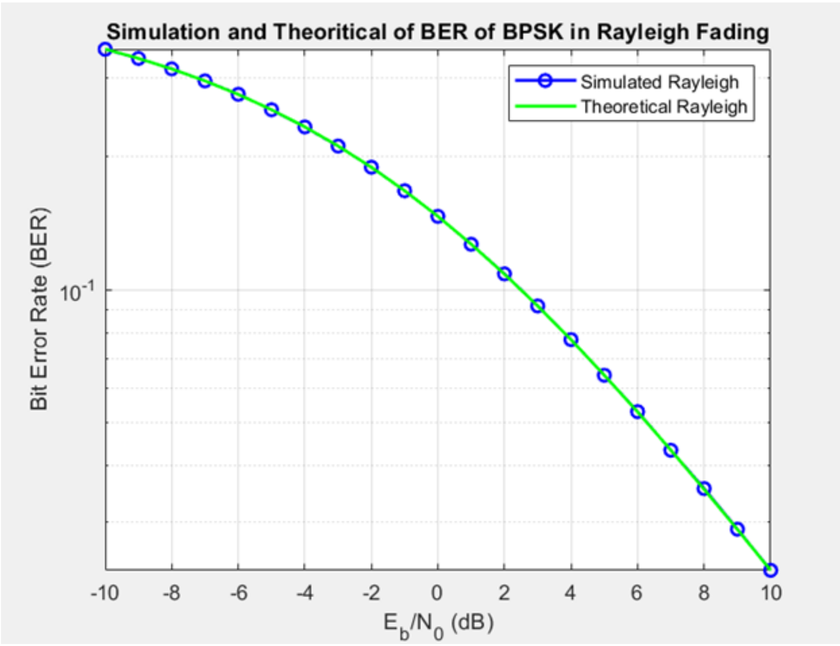
Assuming $\bar{\gamma} = 1$, the expression simplifies to:

$$P_{e,\text{avg}} = \int_0^\infty Q\left(\sqrt{2\gamma\beta}\right) e^{-\beta}\, d\beta$$

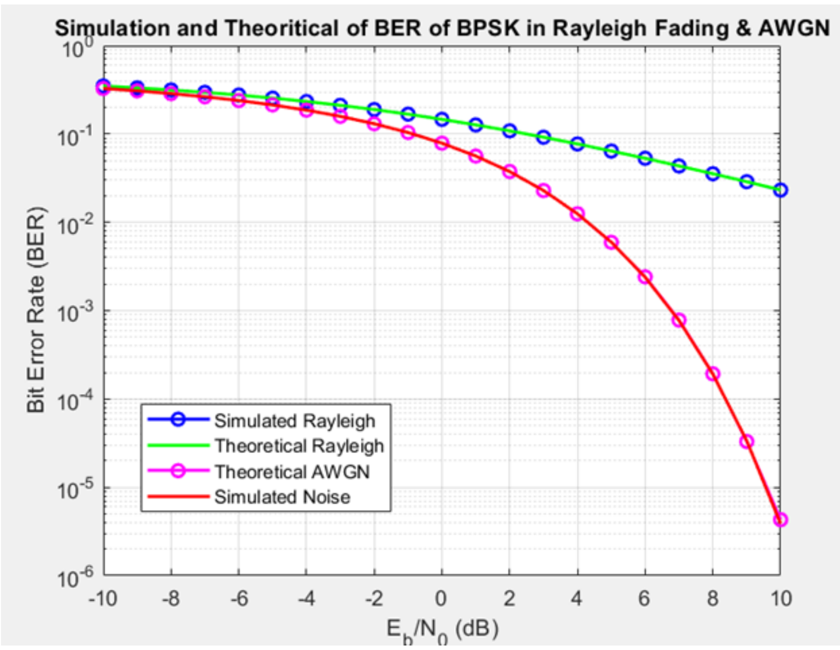
By solving the integral using integration by parts, the result is:

$$P_{e,\text{avg}} = \frac{1}{2}\left[1 - \sqrt{\frac{\gamma}{\gamma+1}}\right]$$

(c) Simulated and Theoritical of BER of BPSK in Rayleigh Fading.



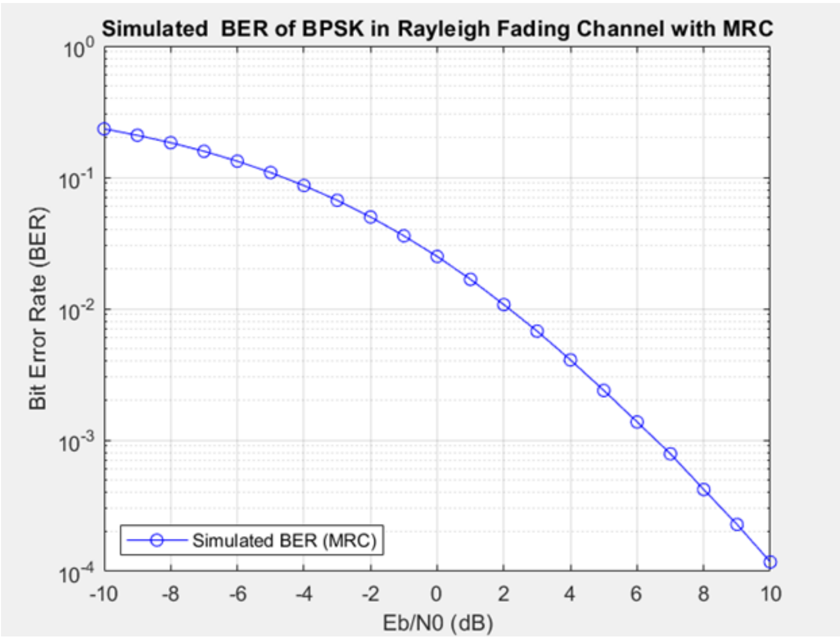
(d) Comparison of BER of BPSK in Rayleigh Fading and AWGN



According to above results the AWGN curve shows a steeper slope and a lower BER at a given SNR, while the Rayleigh fading curve has a slower slope and a higher BER for the same SNR.

Task 2: Comparing diversity combining schemes

(a) Simulated average BER of binary phase shift keying (BPSK) in a Rayleigh fading channel with MRC (Maximal Ratio Combining).



(b) The average probability of error for Maximal Ratio Combining (MRC) under Rayleigh fading is given by:

$$\bar{P}_b = \int_0^\infty Q\left(\sqrt{2\gamma}\right) p_{\gamma_\Sigma}(\gamma)\, d\gamma$$

where:

- $Q(x)$: The Q-function, which represents the tail probability of the standard normal distribution.
$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2}\, dt$$
- $p_{\gamma_\Sigma}(\gamma)$: The PDF of the combined SNR γ_Σ with M diversity branches:
$$p_{\gamma_\Sigma}(\gamma) = \frac{\gamma^{M-1} e^{-\gamma/\bar{\gamma}}}{\bar{\gamma}^M (M-1)!}, \quad \gamma \geq 0$$
- $\bar{\gamma}$: The average SNR per branch.

By Substituting:

$$\bar{P}_b = \int_0^\infty Q\left(\sqrt{2\gamma}\right) \frac{\gamma^{M-1} e^{-\gamma/\bar{\gamma}}}{\bar{\gamma}^M (M-1)!}\, d\gamma$$

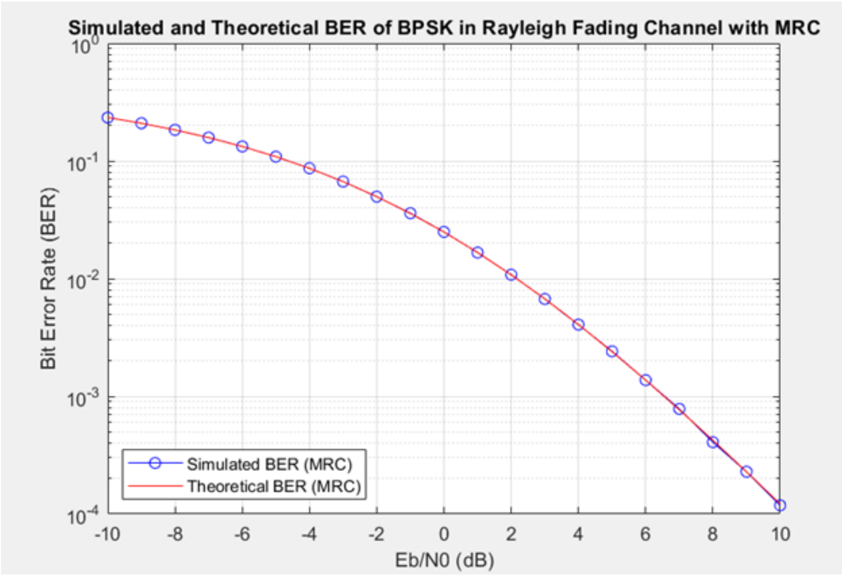
So by solving above, for BPSK modulation in i.i.d. Rayleigh fading channels, the average error probability is derived as:

$$\bar{P}_b = \left(\frac{1-\Gamma}{2}\right)^M \sum_{m=0}^{M-1} \binom{M-1+m}{m} \left(\frac{1+\Gamma}{2}\right)^m$$

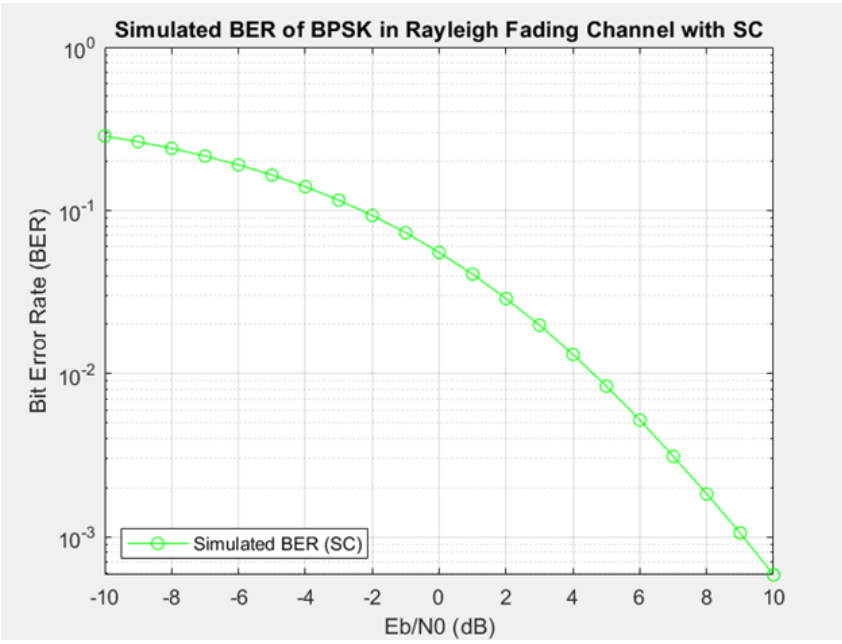
where:

- $\Gamma = \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}$ is a normalized parameter representing the impact of the average SNR $\bar{\gamma}$.

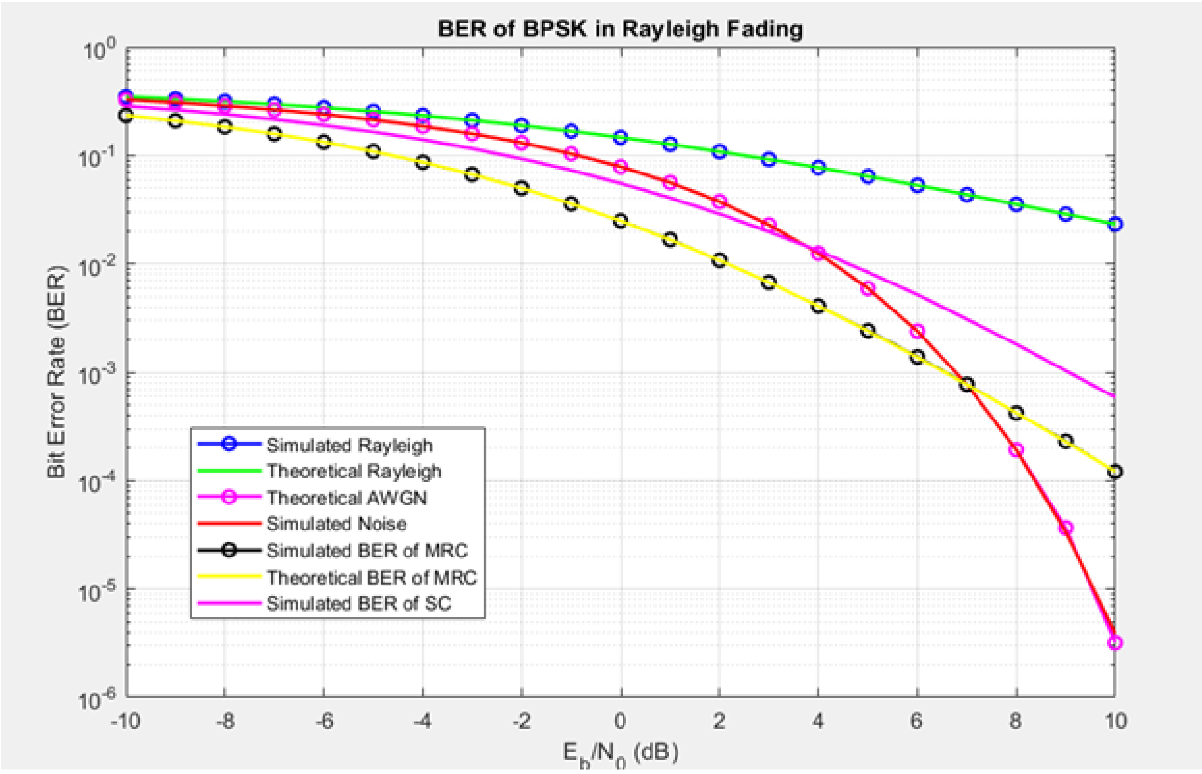
Simulated and Theoritical of BER of BPSK in Rayleigh Fading with MRC.



(c) Simulated average BER of binary phase shift keying (BPSK) in a Rayleigh fading channel with SC (Selection Combining).



(d) Comparison of BER for BPSK in Rayleigh Fading: MRC, SC, and AWGN.



(e) Following are the theoretical aspects of the diversity and array gains for different antenna configurations at the receiver.

- N: Number of antennas at the receiver
- **Diversity Gain**:
  - MRC: N
  - SC: N
- **Array Gain**:
  - MRC: N
  - SC: $\sum_{i=1}^{N} \frac{1}{i}$

Since here $N = 3$, we have:

Gain	MRC	SC
Diversity gain	3	3
Array gain	3	$\frac{11}{6}$

Diversity gain determines the slope of the plot of BER vs. SNR, while Array gain causes a leftward shift in the graph. Based on the above results and the plot, the Diversity Gain is the same for MRC and SC, but the Array Gain is higher for MRC, indicating better overall performance.

Task 3 : SVD based decoupling of MIMO channel

```
Transmit Power: 1 W
Equal Power Allocation Rate: 3.3534 bps/Hz
Channel Inversion Allocation Rate: 0.92875 bps/Hz
Best Eigenmode Allocation Rate: 3.3716 bps/Hz
Waterfilling Allocation Rate: 4.2083 bps/Hz
----------------------------------------------------
Transmit Power: 2 W
Equal Power Allocation Rate: 5.1559 bps/Hz
Channel Inversion Allocation Rate: 1.671 bps/Hz
Best Eigenmode Allocation Rate: 4.315 bps/Hz
Waterfilling Allocation Rate: 5.9909 bps/Hz
----------------------------------------------------
Transmit Power: 5 W
Equal Power Allocation Rate: 8.1913 bps/Hz
Channel Inversion Allocation Rate: 3.2975 bps/Hz
Best Eigenmode Allocation Rate: 5.5967 bps/Hz
Waterfilling Allocation Rate: 8.9174 bps/Hz
----------------------------------------------------
Transmit Power: 10 W
Equal Power Allocation Rate: 11.0475 bps/Hz
Channel Inversion Allocation Rate: 5.2194 bps/Hz
Best Eigenmode Allocation Rate: 6.6039 bps/Hz
Waterfilling Allocation Rate: 11.6269 bps/Hz
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