

Assignment 2

Introduction

In lab 6, We implement the Voronoi Parallel Linear Enumeration algorithm in 2D with Sutherland-Hodgman polygon clipping algorithm and KDTree.

In lab 7, We add weight parametres to our Voronoi diagram and use L-BFGS to solve the semi-discrete optimal transport.

Context:

Nodesets.h :

in this header file, we define the `Node` for KDtree and some operator for `vector<double>` which we use to store k-dim values. in every Node we store the `left child`, `right child`, `parents` nodes and `value` of k-dim vector, `axis` records the axis for current node, `idx` to mark every vector(use when solving the k-nn problem) .

Polygon.h:

Here we define the `Point` class for vector in 2-dim, `Line` class and `Polygon` class. the intersection function `intersect`(for line with line) and `internorm` (for points and line) and the side determine function `side` and `side2`

SutherlandHodgman.h:

this includes the very basic Sutherland-Hodgman algorithm for 2 polygons

```
Polygon SutherlandHodgman(Polygon subjectPolygon, Polygon clipPolygon){
```

`SH2` for clipping with points with weight.

```
vector<Polygon> SHDiagram(vector<Point> sites,vector<double> ws)
```

`SHDiagram` to generate voronoi diagram

KDtree.h:

`cmp` is used to compare vector at certain dimension

`PwithD` is the structure to store vectors with distance(to certain point, don't have to calculate every time)

`knear` is the class for the k-nn problem. It's a queue ordered by distance to some point, we also provide `.add()`, `.pop()` and `.popfront()` function for insert or delete elements.

`KDTree` is the class to search vectors. in this class we have

```
Node* find(vector<double> d)
```

to find the nearest (not d itself) node of point d.

```
knear FindKnearest(vector<double> p, int k)
```

to find the k nearest neighbors of point p(p itself not included)

Voronoi2D.h

it is a voronoi diagram generator using kdtree.

we have `vectoP` and `Pmtovec` these two transformer between `Point` and `vector<double>`[3]

The function `clips` is almost the same as in SH, which is used to clip with points

`voronoiCell` is the function to avoid clip with far away points.

`voronoiDiagram` is the function to generate voronoi Diagram with KDtree.

gfunction.h

It is the g-function and g-gradient function using voronoi diagram for L-BFGS.

[optimal.h](#)

It is the optimal function

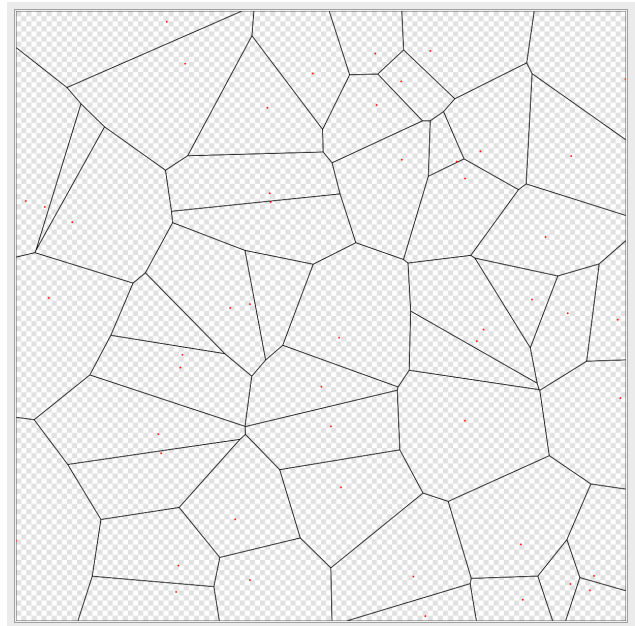
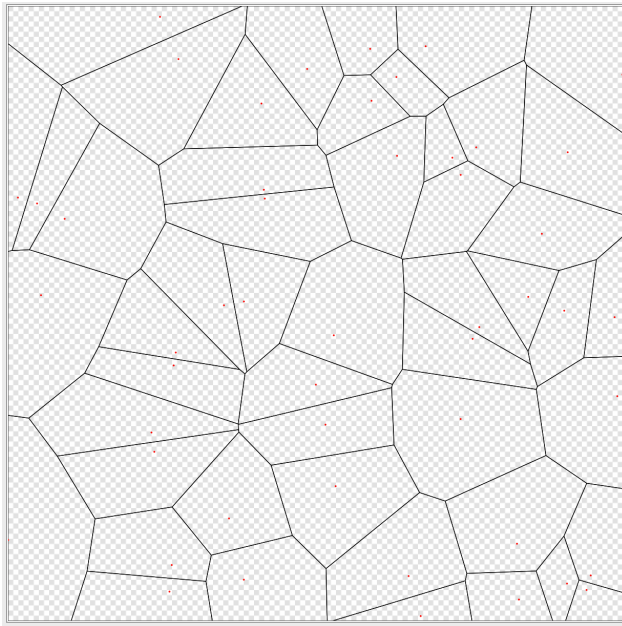
[lloyd.h](#)

It is to generate uniform spread random sites.

[fluid.h](#)

It is the stimulation of water drop.

In lab 6, we generate the voronoi diagram with SH algorithm and KTtree. the following left one is the diagram generated by 50 random sites. (see image/voronoi_KD50.svg)

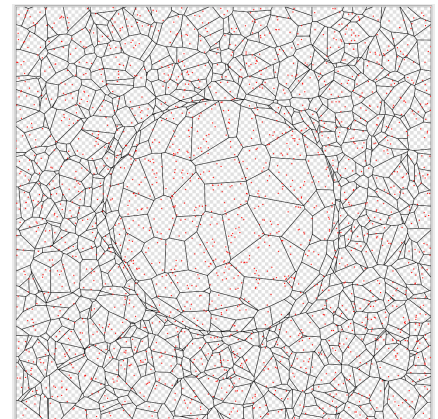
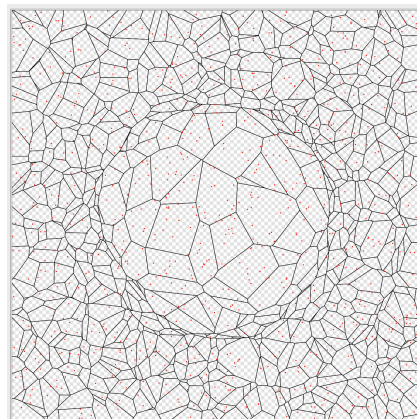
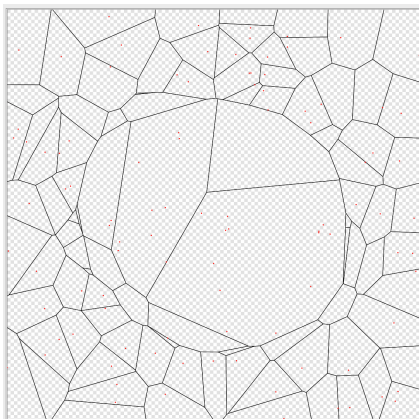


In lab7, we will generate power diagram. The right graph is generated with same sites but with weight of $w_i = i / 25000$. (see image/voronoi_50pow.svg)

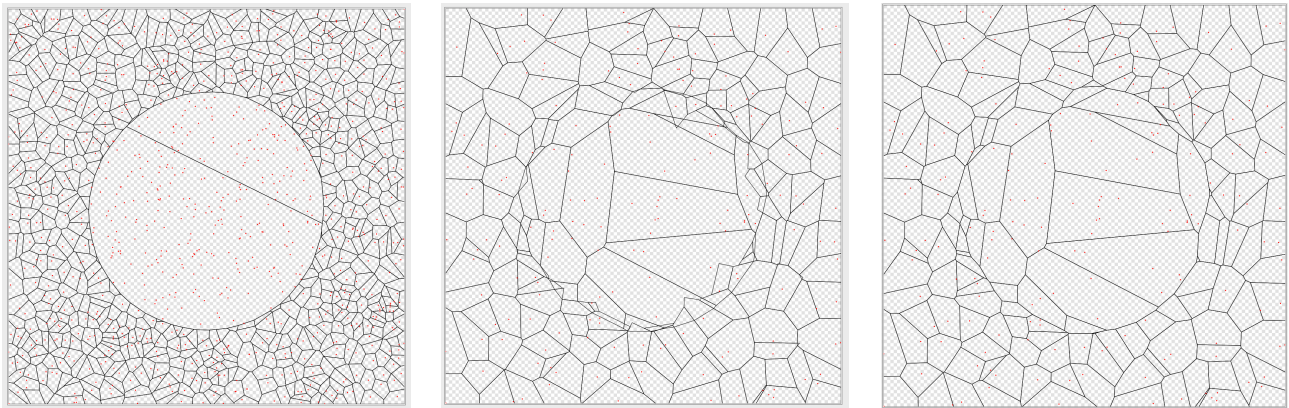
Until now, i tried several sites/random weights using the two method and gets the same diagram. So I suppose both of them are right.

Then we tried to generate optimal transport.

We use $\lambda_i = \exp(-\|y_i - C\|^2 / 0.02)$ the following image is $n=100$, $n=1000$, $n=2000$ from left to right (see image/opt_SH_100, svg, opt_SH_1000.svg, opt_SH_2000.svg)



if we take lambda as lecture notes $\lambda_i = \exp(-\|y_i - C\|/0.02)$ with $n=1000$ we have the following graph:(see image/opt_SH_norm)



If we use KDtree in this part, it will give some quite strange image. However, if I use KDtree in LFGBS and use the output weight with SH and KD to generate graph, I could get a good one with SH but wrong one with KD. (centre for KD and right one for SH, see image/opt_KD200, svg,opt_SH200.svg)

(I have spend lots of time debugging it and still haven't solve it. if you have time, could you please see my code?)

*change the #define SH and #define KD in the top of optimal.h and main.h to set the method.

Lab8 is the stimulation of water.

We first set uniformly spread sites then select a circle as initial water particles.

we set mass=200 for water particles and -1 for air particles(we assume that air only move by force from water)