



MASTER THESIS

Radiation study on the Superconducting Solenoid Magnet for $\mu^- \rightarrow e^-$ Conversion Experiment

和訳：ミューオン電子転換探索実験による放射線環境下における超伝導ソレノイド磁石に関する研究

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Chapter 1

Introduction

Although the Higgs boson has been found in 2012 by CMS and ATLAS collaboration ⁽²⁾, there are no clues of new physics in last running with 3.5 TeV per beam. the Standard Model still remains many unsolved problems such as neutrino oscillation, the lack of dark matter candidate, fine-tuning and matter anti-matter asymmetry. On the other hand, it is known that the quarks and neutrinos are mixing as the Cabibbo-Kobayashi-Maskawa (CKM) matrix and Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, respectively ⁽³⁾. However, as the same family with neutrino, charged lepton flavor violation (cFLV) is still not found. As the prediction of lower branching ratio from the physics beyond the Standard model (BSM), new physics is possible to be probed through the cLFV.

Coherent muon to electron transition (COMET) experiment is planning to seek the cLFV channel of $\mu^- N \rightarrow e^- N$ from 2017. To achieve the better sensitivity, the most intense muon beamline in J-PARC is under designing and construction.

1.1 Charged lepton flavor violation

In Standard Model, the neutrino oscillation is constricted and massless, in addition, the lepton number is totally conserved when they decay, like

$$\begin{array}{ccccccc} \mu^- & \rightarrow & e^- & + & \nu_\mu & + & \bar{\nu}_e \\ L_\mu : & 1 & 0 & 1 & 0 & & \\ L_e : & 0 & 1 & 0 & -1 & & \end{array}$$

where the electron lepton number L_e and muon lepton number L_μ on left side is conserved on right side. This decay is the ordinary muon decay and also called Michel decay. However this is not the only decay for muon, all kinds of muon decay is listed in table 1.1 ⁽⁴⁾.

Unlike to the michel decay, decay mode of charged lepton like $\mu^- \rightarrow e^- \gamma$ has very small probability of decay, apparently, its electron and muon lepton number are not conserved on two sides. The branching ratio predicted by standard model is less than 10^{-54} , which the details are described in Appendix. Otherwise, the physics beyond the standard model such as SUSY-Seasaw, SUSY-GUT etc. predicts the branching ratio higher than 10^{-15} , which is possible to be obversed. The reaction like this is called charged lepton flavor

violation.

Decay modes	Fraction (Γ_i/Γ)	Confidence level	Reference
$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$	$\approx 100\%$	(5)	
$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu \gamma$	$1.4_{\pm 0.4}\%$		
$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu e^+ e^-$	$3.4_{\pm 0.4} \times 10^{-5}$		
Flavor violating modes			
$\mu^- \rightarrow e^- \nu_e \bar{\nu}_\mu$	$< 1.2\%$	90 %	(6)
$\mu^- \rightarrow e^- \gamma$	$< 2.4 \times 10^{-12}$	90 %	(7)
$\mu^- \rightarrow e^- e^+ e^-$	$< 1.0 \times 10^{-12}$	90 %	(8)
$\mu^- \rightarrow e^- \gamma \gamma$	$< 7.2 \times 10^{-11}$	90 %	

Table 1.1: Decay modes and branching ratio of muon.

1.2 Searches for cLFV with muon

$\mu \rightarrow e\gamma$, $\mu \rightarrow eee$ and $\mu \rightarrow e$ conversion are three main decay modes for cFLV seeking, and each mode has its different view in $\mu \rightarrow e$ transition. In these modes, $\mu \rightarrow e\gamma$ and $\mu \rightarrow eee$ belong to a dipole-type like interaction, however, $\mu \rightarrow e$ conversion is a four-fermion like interaction. The effective Lagrangian which includes both dipole-type and four-fermion operators is given by ⁽⁹⁾.

$$\mathcal{L} = \frac{1}{1 + \kappa} \frac{m_\mu}{\Lambda^2} \bar{\mu}_R \sigma^{\mu\nu} e_L F_{\mu\nu} + \frac{\kappa}{1 + \kappa} \frac{1}{\Lambda^2} (\bar{\mu}_L \gamma^\mu e_L) (\bar{q}_L \gamma_\mu q_L) \quad (1.1)$$

where κ is a parameter which governs the relative size of dipole-type and four-fermion operators. Parameter Λ represents the effective mass (energy) scale of new physics. The first and second term of this equation are meant to the dipole-like ($\kappa \ll 1$) and four-fermion interaction ($\kappa \gg 1$), which correspond to $\mu \rightarrow e\gamma$ and $\mu \rightarrow e$ conversion, respectively.

1.2.1 $\mu \rightarrow e\gamma$

$\mu \rightarrow e\gamma$ is a 2-body back-to-back decay with monoenergetic e and γ . The way to search this decay is to reconstruct the time and momentum of these back-to-back monoenergetic e and γ to 105.7 MeV. It is difficult to distinguish the signals from Michel decay and $\mu \rightarrow e\gamma$. Thus, its experimental sensitivity is totally constricted by the detector. The earliest $\mu \rightarrow e\gamma$ search dates back to 1947 with branching ratio of 0.1 (90% C.L.) ⁽¹⁰⁾, and the most recent experiment for seeking this mode is the MEG experiment.

MEG experiment searches the $\mu^+ \rightarrow e^+ \gamma$ in Paul Scherrer Institute (PSI) with $3 \times 10^7 \mu^+ / sec$. The data is taken from 2009 to 2010, and MEG experiment is now under upgrading to the phase-II experiment. As the result, MEG collaboration presents a new upper limit on the branching ratio of this decay of 5.7×10^{-13} (90% C.L.) with 3.6×10^{14} stopped muons on target ⁽¹¹⁾.

Unfortunately, with the branching ratio of 5.7×10^{-13} , there is no signal of interest found finally. The MEG-II experiment will start data-taking from 2016 to 2019 with 180 days per year and aim to reach the branching ratio of 6×10^{-14} ⁽¹²⁾.

1.2.2 $\mu \rightarrow eee$

The first experiment for searching $\mu \rightarrow eee$ decay is taken in 1958 at Nevis cyclotron with the branching ratio of 3.0×10^{-5} ⁽¹³⁾. Recent best branching ratio is 10^{-12} which is measured by SINDRUM-I experiment. A new experiment called Mu3e purposes to start from 2016 to reach a branching ratio of $< 10^{-16}$ with the requirement of muon stop rate of about $2 \times 10^9 \mu^+/\text{sec}$. Mu3e has two phases, which phase-I and phase-II will taken in $\pi E5$ beamline and HiMB beamline of PSI respectively.

Unlike to $\mu \rightarrow e\gamma$ decay, $\mu^+ \rightarrow e^+e^+e^-$ is a 3-body decay. The reconstruction of signal of positron and electron must be from the same vertex. Thus, the challenges of Mu3e are the vertex, timing and momentum resolution⁽¹⁴⁾. Its experimental sensitivity also limited by detector.

1.2.3 $\mu \rightarrow e$ conversion

$\mu \rightarrow e$ conversion is a different process from the other two, because the processes of $\mu \rightarrow e\gamma$ and $\mu \rightarrow eee$ are detector-limited owing to accidental background, whilst, the $\mu \rightarrow e$ conversion has no accident background but the background from beam, which depends on the beam qualities. The past experiments for searching $\mu^- \rightarrow e^-$ conversion is listed in table 1.2.

Target	Upper limit	Place/Collaboration	Year
Cu	$< 1.6 \times 10^{-8}$	SREL	1972
³² S	$< 7 \times 10^{-11}$	SIN	1982
Ti	$< 1.6 \times 10^{-11}$	TRIUMF	1985
Ti	$< 4.6 \times 10^{-12}$	TRIUMF	1988
Pb	$< 4.9 \times 10^{-10}$	TRIUMF	1988
Ti	$< 4.3 \times 10^{-12}$	PSI/SINDRUM II	1993
Pb	$< 4.6 \times 10^{-11}$	PSI/SINDRUM II	1996
Ti	$< 6.1 \times 10^{-13}$	PSI/SINDRUM II	1998
Au	$< 7 \times 10^{-13}$	PSI/SINDRUM II	2006

Table 1.2: History of $\mu^- \rightarrow e^-$ conversion experiments.

Signal and background

When the $\mu^- N \rightarrow e^- N$ conversion is occurred, the event signal of electron is mono-energetic with energy

$$E_{\mu e} = m_\mu - B_\mu - E_{rec} \quad (1.2)$$

where m_μ and B_μ are mass of muon and bind energy of the 1s monic atom respectively. E_{rec} is the nuclear recoil energy, which is given by $E_{rec} \approx (m_\mu - B_\mu)^2 / 2m_N$. It is very small and can be neglected. The signal depends on the nuclei, for instance, it is 105.0 MeV for Al stopping target. This mono-energetic signal is beyond the background, and one peak behind electron background can be observed in $\mu^- N \rightarrow e^- N$ conversion. That peak depends on the atom number of stopping target, the coherent $\mu \rightarrow e$ conversion peak for SINDRUM-II experiment (Ti target) is 95 MeV/c.

As for the background, the Michel decay in orbit (DIO) emits a electron and dominates the intrinsic background of $\mu \rightarrow e$ conversion. Without the direct background like DIO, there are many background

such as beam related background listed in table 1.3 will cause the indistinguishable signal in CDC during the reconstruction of signal.

Intrinsic physics backgrounds	
Muon decay in orbit	$\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e$
Radiative muon capture (external)	$\mu^- + A \rightarrow \nu_\mu + A' + \gamma$
Radiative muon capture (internal)	$\mu^- + A \rightarrow \nu_\mu + A' + e^+ + e^-$
Neutron emission	$\mu^- + A \rightarrow \nu_\mu + A' + n$
Charged particle emission	$\mu^- + A \rightarrow \nu_\mu + A' + p$ (or d or α)
Beam related backgrounds	
Radiative pion capture (external)	$\pi^- + A \rightarrow \gamma + A', \gamma \rightarrow e^- + e^+$
Radiative pion capture (internal)	$\pi^- + A \rightarrow e^+ + e^- + A'$
Beam electrons	e^- scattering
Muon decay in flight	μ^- decays in flight to produce e^-
Pion decay in flight	π^- decays in flight to produce e^-
Neutron induced backgrounds	neutrons hit material to produce e^-
Anti-proton induced backgrounds	anti-proton hit material to produce e^-
Other background	
Cosmic-ray induced background	
Room neutron induced background	
False tracking	

Table 1.3: Potential backgrounds for searching $\mu^- N \rightarrow e^- N$ conversion.

SINDRUM-II experiment

The most precision branching ratio is measured by SINDRUM-II experiment with gold target at PSI. As the spectrum shown in figure 1.1, the steeply falling distribution expected from muon DIO, using 53 MeV/c negative muon, there are two events found at high momentum region, but it just outside the region of interest. These two events might have been induced by cosmic rays or radiative pion capture or etc. The upper limit of this branching ratio ($\mu^- + Au \rightarrow e^- + Au$) is set to 7×10^{-13} .

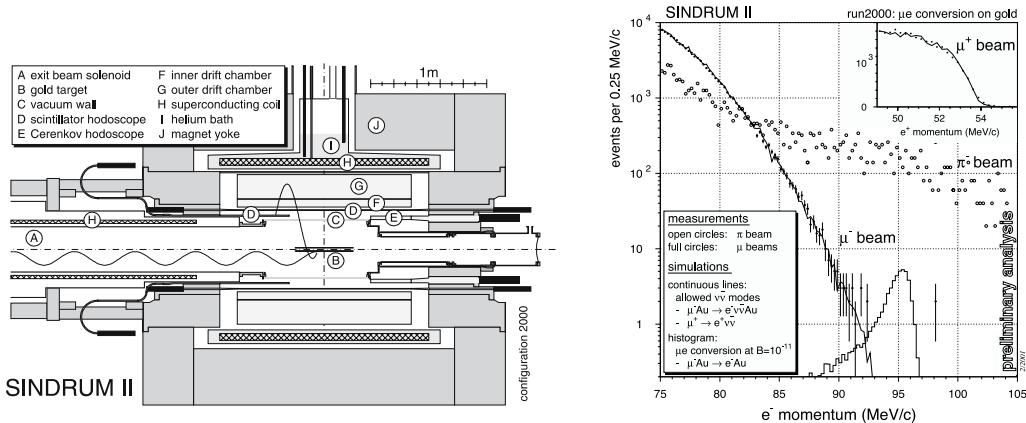


Figure 1.1: SINDRUM-II experiment observes a $\mu^- + Ti \rightarrow e^- + Ti$ conversion with branching ratio of 7×10^{-13} . Two signal is found in high energy region but it is out of the range of $\mu^- \rightarrow e^-$ signal.

COMET and Mu2e experiment are aiming to achieve the branching ratio less than 10^{-17} , which is 4 orders less than the previous experimental result. It is able to be validated by using high intense, pulsed

muon beam.

Extinction

Because the DC beam is used in PSI, and it is hard to distinguish the signal from capture or beam. To solve the issue of background, the pulsed proton should be employed.

In the case of COMET experiment, since the muon in aluminium have lifetimes of about 864 nsec, a pulsed proton beam with interval time of $1.17 \mu\text{sec}$ is employed to eliminate the prompt background. As shown in figure 1.2, the prompt background like π^- decay is occurred after 100 nsec beyond the proton beam. Due to the long lifetime of muonic atoms, the time window is opened from 700 nsec to distinguish the background events and signal events.

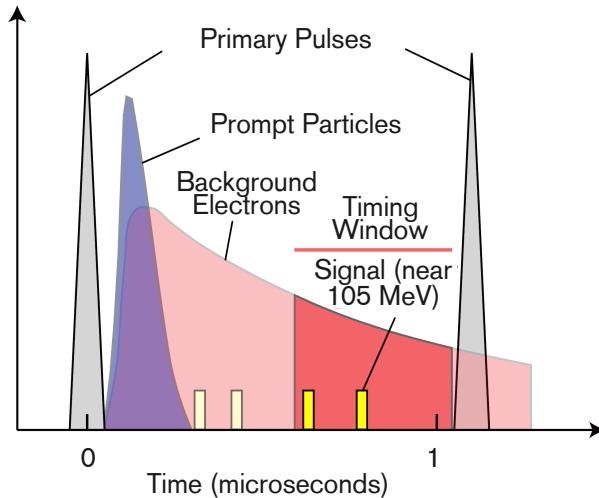


Figure 1.2: The time concept of proton beam. The time window will open from 700 nsec to take data.

The proton beam energy requires a pulsed beam with low production of antiproton and extinction, which will produce the backgrounds. Therefore, the best choice is 8 GeV for COMET experiment, and the its limit intensity is $4.4 \times 10^{13} \text{ pps}$. Antiproton can produce the signal like electron and tracked by CDC when it hits the material. The proton extinction factor is defined as

$$R_{Ext} = \frac{N_{leak}}{N_{pulse}} \quad (1.3)$$

where N_{leak} is the proton leaked from the pulse, it becomes the background if it comes into the time window, and N_{pulse} is the proton number in the pulse.

1.3 COMET experiment

The COMET experiment is designed to search the following $\mu \rightarrow e$ conversion and be carried out at J-PARC.

$$\mu^- + N(A, Z) \rightarrow e^- + N(A, Z) \quad (1.4)$$

It has two steps for searching cLFV, which are phase-I and phase-II. phase-I experiment is to measure the background and search for cLFV because the $\mu \rightarrow e$ conversion totally depends on the beam qualities. phase-II experiment is to search the $\mu \rightarrow e$ conversion with the sensitivity of 3×10^{-17} . As shown in figure 1.3, 8 GeV proton hits the target to produce pions, then these pions are captured by magnetic field and transported to the stopping target to generate the signal of $\mu \rightarrow e$ conversion. The background measurement is taken at the end of 90 degree bending.

To achieve the experimental sensitivity of 3×10^{-17} which corresponds to the branching ratio, a total 2×10^{18} muons stopped in stopping target is needed. Thus, the high intense muon source with about $10^{11} \mu^-/\text{sec}$ is required for COMET experiment. As for COMET experiment, one COMET dedicated beamline is under construction in J-PARC.

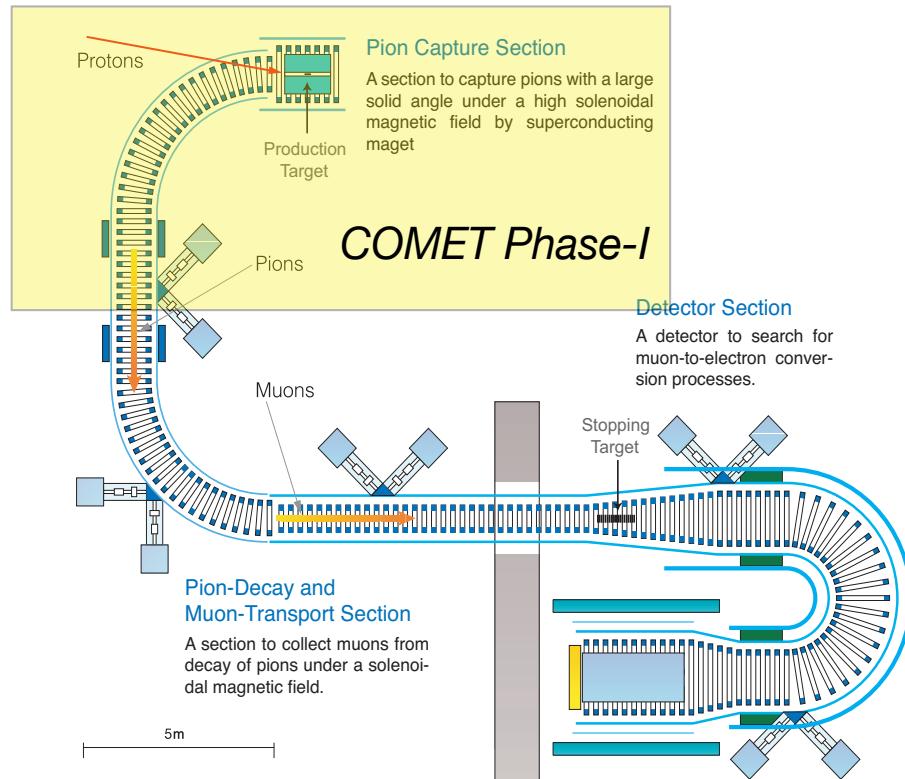


Figure 1.3: A schematic layout of COMET. 8 GeV proton from B-line hits the target and produce pion first, these pions are captured by pion capture solenoid, and decays to muons in muon transport solenoid. Then, the muon with low momentum is captured by stopping target, meanwhile, an electron with 105 MeV/c from $\mu \rightarrow e$ conversion will be transported to E-cal to take a measurement by detector solenoid.

1.3.1 Proton beamline

To obtain the more precision branching ratio of $\mu \rightarrow e$ conversion, more stopped muon is required. Stopped muon can be gained from two ways, increasing the operating time and increasing the muon beam intensity. Since the most intense muon beamline over the world is located in PSI, and it only provides the muon with 4×10^8 , a new high intense muon beamline is necessary for future particle physics. Table 1.4 lists the current and future muon beamline, apparently, COMET muon beamline will be the most intense muon beamline over the world in future.

Lab/Beamline	Energy and power	Present μ rate [Hz]	Future μ rate [Hz]
PSI:			
LEMS	590 MeV, 1.3 MW, DC	4×10^8	
π E5	590 MeV, 1.3 MW, DC	1.6×10^8	
HiMB	590 MeV, 1 MW, DC		$4 \times 10^{10} (\mu^+)$
J-PARC:			
MUSE D-line	3 GeV, 1 MW, Pulsed	3×10^7	
MUSE U-line	3 GeV, 1 MW, Pulsed		$4 \times 10^8 (\mu^+)$
COMET	8 GeV, 56 kW, Pulsed		$> 10^{11} (\mu^-)$
FNAL:			
Mu2e	8 GeV, 25 kW, Pulsed		$5 \times 10^{10} (\mu^-)$
TRIUMF:			
M20	500 MeV, 75 kW, DC	2×10^6	
KEK:			
Dai Omega	500 MeV, 2.5 kW, Pulsed	4×10^5	
RAL-ISIS:			
RIKEN-RAL	800 MeV, 160 kW, Pulsed	1.5×10^6	
RCNP:			
MUSIC	400 MeV, 400 W, Pulsed	$10^8 (\mu^+)$	
DUBNA:			
Phasatron Ch: I-III	660 MeV, 1.65 kW, Pulsed	3×10^4	

Table 1.4: Current running muon beamline around the world. Current most intense muon beamline is in PSI, COMET muon beamline will be 3 orders higher than PSI muon beamline.

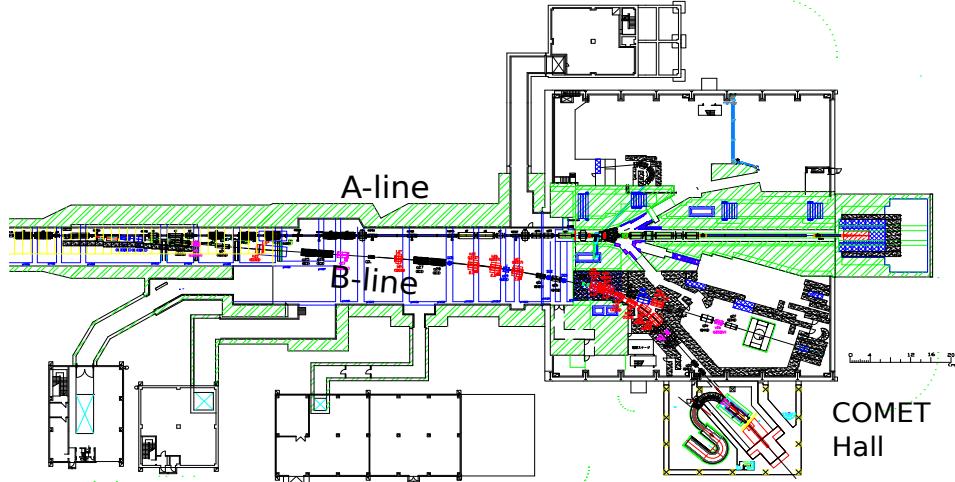


Figure 1.4: Dedicated proton beamline for COMET experiment. A new proton beamline, B-line is under construction. COMET Hall is located beside Hadron Hall.

To accommodate the COMET experiment, a new beamline (B-line) is under construction. As shown in figure 1.4, A-line is the existing beamline from the main ring into the Nuclear and Particle Physics (NP) Hall. B-line is a dedicated proton for COMET experiment.

Both phase-I and phase-II require a pulsed, 8 GeV proton beam slow-extracted from the J-PARC main ring into COMET experimental hall. The proton beam intensity and production target for phase-I and phase-II is different, which is shown as follows. In table 1.5, the requirement of COMET is compared with Mu2e as well.

COMET	Target	Beam intensity [pps]	Power [kW]	stopped muon [μ^-/sec]	Sensitivity
phase-I	graphite	2.5×10^{12}	3	1.30×10^9	3.1×10^{-15}
phase-II	tungsten	4.4×10^{13}	56	$> 10^{11}$	$< 3 \times 10^{-17}$

Table 1.5: Different requirement of proton beam in phase-I and phase-II.

1.3.2 Superconducting solenoids

Because the background depends on the muon beam quality, to eliminate the background, the muons must be captured and transported to stopping target efficiently, which can be carried out by using superconducting solenoid magnet system. The COMET superconducting magnets must be designed with requirements

1. high magnetic field (5 Tesla) around the production target to capture the pions.
2. because the trapped charged particle such as pion becomes background signal, smooth magnetic field is needed to eliminate the trapping of charged particles from pion capture solenoid to muon transport solenoid.
3. due to the pion decay, the muon transport solenoid must has enough length.
4. appropriate magnetic field to select the low momentum muons and transport them to stopping target.
5. low magnetic field for detector to track the signal electron.

Requirement of magnetic field

Muons are transported to the stopping target through the muon transport solenoid, which consists of curved superconducting magnets. The muon transport solenoid must be designed to select the muons with low momentum and eliminate muons with high momentum ($p_\mu > 75 \text{ MeV}/c$), in addition, it also must be long enough for the pion decay. Due to the beam dispersion in muon transport solenoid, the selection of charged particle depends on their momentum. The magnitude of drift is given by

$$D = \frac{1}{qB} \left(\frac{s}{R} \right) \frac{p_L^2 + p_T^2/2}{p_L} = \frac{1}{qB} \left(\frac{s}{R} \right) \frac{p}{2} \left(\cos\theta + \frac{1}{\cos\theta} \right) \quad (1.5)$$

where q is the electric charge of particle, B is the magnetic field, s and R are the path length and the radius of curvature of solenoid respectively. s/R is the bending angle, p_L and p_T [GeV/c] are longitudinal and transverse momentum. To keep the center of the helical trajectories of muons in the bending plane, a compensating dipole field parallel to the drift direction is applied, which is given by

$$B_{comp} = \frac{1}{qR} \frac{p_0}{2} \left(\cos\theta_0 + \frac{1}{\cos\theta_0} \right) \quad (1.6)$$

Superconducting magnet system

Superconducting magnet system plays a key role in pion capture, muon transport and signal tracking. In the case of COMET phase-I experiment, the 8 GeV proton with high intensity is injected into superconducting magnet and hits the target to produce pion, then these backward pions are captured by pion capture solenoid, which provides 5 Tesla magnetic field at peak.

Muons with low momentum from the decay of captured pions are transported to stopping target by the muon transport solenoid with 3 Tesla. Because the high momentum muon and the other particles will cause the background on detector, the muon transport solenoid is designed with 90 degree curve to throw out these particles. After the muon captured by stopping target, the emitted electron from stopping target will be bent by detector solenoid which provides 1 Tesla magnetic field and tracked in cylindrical drift chamber (CDC).

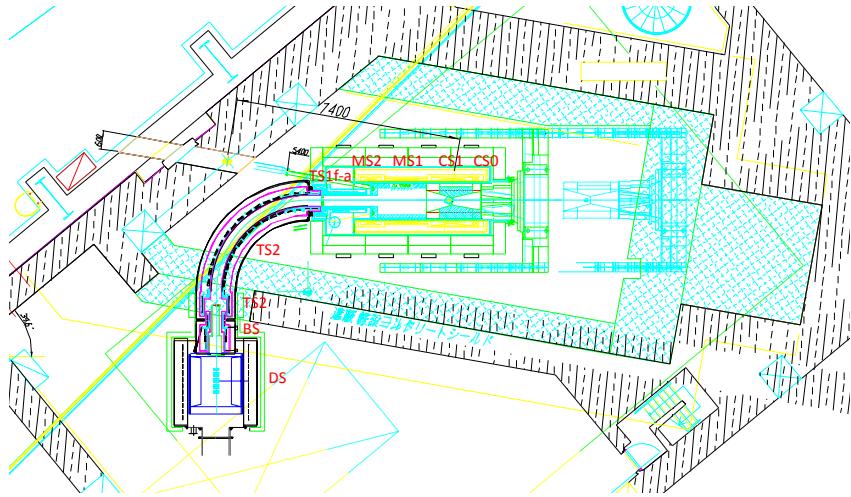


Figure 1.5: Schematic layout of superconducting magnet system for COMET phase-I experiment.

The schematic layout of COMET superconducting solenoid is given in figure 1.5. Pion capture solenoid consists of CS0, CS1, MS1, MS2 and TS1a-TS1f superconducting coils. Meanwhile, muon transport solenoid includes TS2, TS3 and BS coils, and detector solenoid has 12 DS coils.

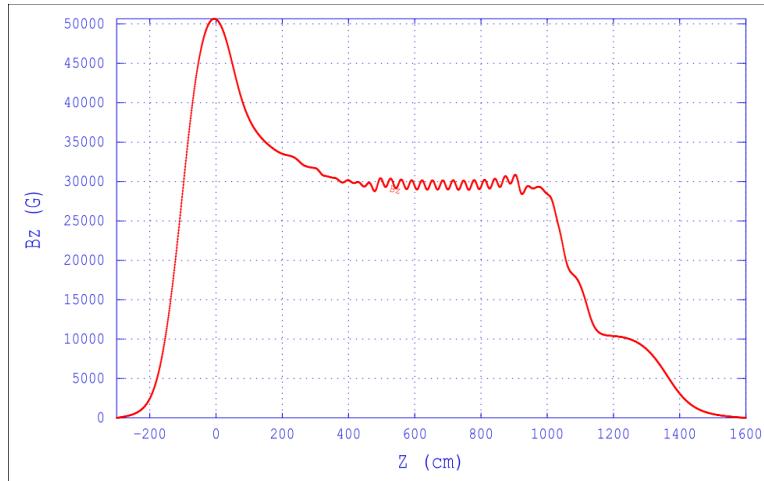


Figure 1.6: Magnetic field of COMET superconducting magnets along the axis.

1.4 Objective

The branching ratio of $\mu^- \rightarrow e^-$ conversion is predicted to around 10^{-15} by the physics beyond the Standard model such as SUSY-GUT, SUSY-Seasaw. To obtain that experimental sensitivity for searching $\mu^- \rightarrow e^-$ conversion less than 10^{-17} , a 8 GeV, 4.4×10^{13} pps proton beam is employed to produce high intense muon beam of $10^{11} \mu^-/\text{sec}$ with low momentum.

As the significant part of COMET muon beamline, the pion capture solenoid in superconducting magnet system surrounds a production target with high magnetic field of 5 Tesla and small radius of 672 mm. Its main issue is the radiation, which cause the over-heat and quench of superconducting magnet due to the thermal conductivity degradation of cooling path and stabilizer. Thus, the radiation effects on COMET superconducting solenoid are studied in this thesis.

In chapter 2, the current design details of superconducting magnets for COMET experiment, pion capture solenoid, and muon transport solenoid, will be explained.

In chapter 3, to investigate how much radiation will be produced in COMET experiment, Monte Carlo code is modified to carry out the calculation with COMET condition. Using the real geometry of COMET experiment, the prompt radiation for phase-II and the residual radiation after phase-I experiment is estimated. In addition, one radiation shield is designed for protecting the superconducting coils of pion capture solenoid.

In chapter 4, the radiation resistance of the material and device, GFRPs, quench protection diode, insulation tape and pure aluminium, which are used in COMET superconducting magnet system has been tested.

In chapter 5, considering the thermal conductivity will degrade after irradiation, the most important CS1 coils where close to the production target is under risk of quench and over-heat. Thus, the cooling and quench simulation is estimated.

Chapter 2

Superconducting solenoid system for COMET experiment

COMET phase-I experiment is aiming to measure the background and search the $\mu \rightarrow e$ conversion with the sensitivity of 10^{-14} . Its superconducting magnet system consists of pion capture solenoid, muon transport solenoid and detector solenoid. In phase-II experiment, nowadays' 90 degree bent transport solenoid will be extended to 180 degree curved solenoid, and another spectrometer solenoid will be added.

2.1 Pion capture solenoid

Pion capture solenoid shown in figure 2.1 is most critical magnet for capturing the pions from production target with a large solid angle. Considering the valleys of magnetic field causes the trapping of charged particle, the magnetic field distribution must be smooth. The details of the magnetic design of pion capture solenoid is shown in table 2.1.

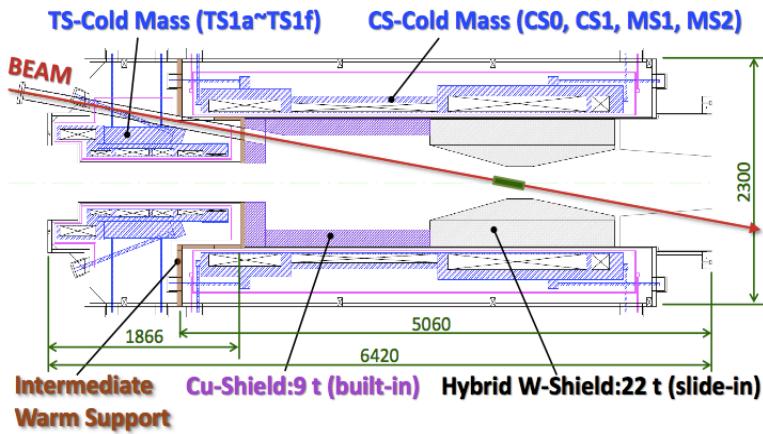


Figure 2.1: Schematic layout of pion capture solenoid.

All of the superconducting coils in pion capture solenoid is connected together with the same current provided by one power supply. Furthermore, conduction cooling is employed in whole COMET superconducting magnet system to minimize the tritium production under high radiation environment by reaction of ${}^3\text{He}(n,p){}^3\text{H}$. Also the conduction cooling eliminates a helium vessel, which requires significant addi-

tional structure to the superconducting coil assembly resulting move heat load by the irradiation. From all aspect, the conduction cooling is the best choice for COMET superconducting magnet system. aluminium-stabilized conductors is used in all the coils for reducing the heat load and preventing the over heat of coils after quench.

Coils	B_z [T]	Inner radius [mm]	Outer radius [mm]	Length [mm]	Current [A]	Current density [A/mm ²]
CS0	4.0	662	806	175	2700	33.75
CS1	5.0	662	806	1350	2700	33.75
MS1	4.0	662	742	1425	2700	33.75
MS2	3.0	662	774	700	2700	33.75
TS1a	3.0	250	266	200	2700	33.75
TS1b	3.0	250	298	240	2581	32.26
TS1c	3.0	250	314	200	2700	33.75
TS1d	3.0	250	314	200	2619	32.74
TS1e	3.0	250	298	200	2538	31.73
TS1f	3.0	400	496	350	2916	36.45

Table 2.1: Details of coils for pion capture solenoid.

2.1.1 Coils structure

For stabilization and quench protection of the coils, superconducting wires are all merged with 5N aluminium (0.1% Ni) due to its low density. The aluminium stabilized conductor is chosen because aluminium is lighter than copper, which is employed as superconducting stabilizer popularly, such that it can be more transparent to the irradiation. Here, 0.1% nickel mixed is to enhance the mechanical property of conductor.

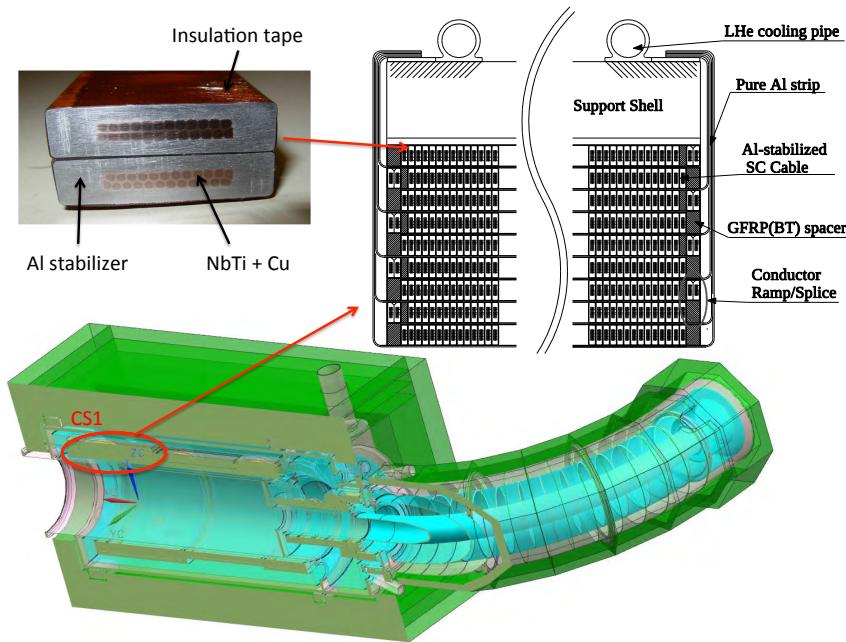


Figure 2.2: Coil and conductor structure of CS1.

The aluminium-stabilized conductor with the size of 15.0×4.7 mm² is wound with two layers of insulation tape, which is made of polyimide film covered by boron free glass with BT and epoxy resin.

Here, The reason why BT and epoxy resin is used because it has good mechanical property under the high radiation. In addition, the insulated conductor is impregnated by the BT and epoxy resin mixed with silica filler during the coils winding.

The coil structure of CS1 and detail of design parameters are shown in figure 2.2 and table 2.2 respectively. There are 9 layers superconducting coils totally, and each layer is cooled by inserting 1 mm thick pure aluminium strip with RRR of 2000. Each aluminium strip is installed between two conductor layers, and connected to the liquid helium cooling pipe. Support shell with 5 cm thick is located on the top of superconducting coils to suppress the deformation of coils.

Item	Value
Conductor	aluminium stabilized SC cable Al:Cu:NbTi = 7.3:0.9:1
Cable dimensions	$15.0 \times 4.7 \text{ mm}^2$ (without insulation) $15.3 \times 5.0 \text{ mm}^2$ (with insulation)
Cable insulation	Polyimide film/Boron-free glass cloth /BT-epoxy prepreg
Magnet length	$\sim 6 \text{ m}$
Number of coils	10
Operation current	2700 A
Max. field on conductor	5.5 T ($T_{cs} = 6.5 \text{ K}$)
Stored energy	47 MJ
Coil layer	9 (CS0+CS1), 5 (MS1), 7 (MS2), 1~6 (TSa~TS1f)
Quench protection	NA

Table 2.2: Details of design parameters for capture solenoid magnet.

2.1.2 Production target

Production target is to make amounts of pions in the capture solenoid. Phase-I is going to use the IG-43 graphite target rod with length of 60 cm, radius of 2 cm and density of 1.82 g/cm^3 . 3.2 kW proton beam will deposit a heat load of approximately 100 W in the graphite target. The temperature of target grows up to 190°C at peak when 100 W energy is deposited in target. A target support system accurately locates the target within the solenoid inner shield.

To achieve more pion production in COMET phase-II experiment, the target will be changed to pure tungsten rod with length of 16 cm and radius of 0.3 cm after the implementation of phase-I. In addition, due to 56 kW proton beam power, phase-II target must be cooled on low temperature. The proton beam also has to be fitted to a gaussian beam with 0.2 cm radius.

2.1.3 Radiation shield

A radiation shield is installed inside the CS and MS coils to protect superconducting coils from radiation. This radiation shield has to be designed for both phase-I and phase-II experiment due to the high residual radiation. The neutron came from production target directly is about $3 \times 10^{12} \text{ n/cm}^2/\text{sec}$ in COMET phase-II experiment, which corresponds to 10^{24} n/m^2 . Both NbTi superconducting cable and stabilizers will degrade

if they are irradiated by neutrons with over 10^{22} n/m². Therefore, due to a small radius of capture solenoid, a powerful radiation shield is needed to stop or decrease the radiation in short distance. The most ideal radiation shield is made of pure tungsten with density of 19.25 g/cm³.

However, to reduce the costs, a new radiation shield is necessary, and there are 3 candidates for the shielding design, which are the copper, tungsten and stainless steel. Tungsten is most ideal material for shielding but it has bad thermal conductivity, thus, the water cooling should be installed into tungsten shield. Copper has good thermal conductivity, but the eddy current will generate inside it during the hysteresis, so it needs to be cut in somewhere to prevent the eddy current. Stainless steel has good shielding ability but higher residual radiation. The details of shielding design will show in next chapter.

2.1.4 Quench protection system

Quench is caused by a hot spot in superconducting magnet. When a hot spot quenches, temperature will rise suddenly, and propagate to all magnet. Superconducting magnets has a risk of burn-out at temperature of quench.

Because the current becomes the joule heat after quench, decreasing the current quickly can suppress the temperature rise after quench. There are two ways to speed up the current decay. As shown in figure 2.3, one is to use the dump resistor in superconducting magnet circuit. Once the quench is detected, the power supply will be turned off and current will flow towards to the dump resistor. One quench protection diode is a switch for the quench protection circuit. It is connected with superconducting wire and cooled to 4.2 K during operation. The time constant is written as

$$\tau = \frac{L}{R_{dump}} \quad (2.1)$$

where L is the inductance of magnets and R_{dump} is the resistance of dump resistor respectively.

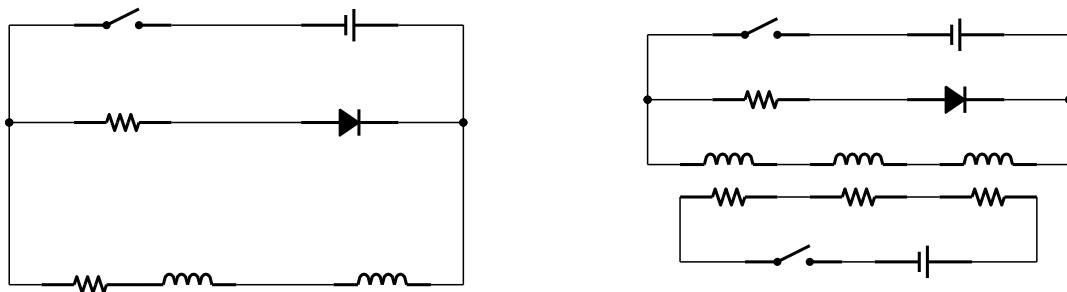


Figure 2.3: Schematics of quench protection circuit.

Higher resistance of dump resistor makes current decay quicker. In the case of the pion capture solenoid, it is covered by the aluminium stabilizer. Once quench is occurred, the current will flow into the aluminium stabilizer and copper matrix. The current must decay more quickly due to the self resistance of stabilizer and matrix. The time constant is written as

$$\tau = \frac{L}{R_{dump} + R_{self}} \quad (2.2)$$

Another way is to make the superconducting magnet quench to increase the self resistance by external heater. It needs external power supply to heat the heater.

A tentative design of the quench protection system for pion capture solenoid is shown in figure 2.4. All superconducting magnets in pion capture solenoid are connected together. The power supply for capture solenoid is 2700 A, and 4 reserve trim power supplies and dump resistors are to reduce the current for TS1b, TS1d, TS1e and TS1f. A dump resistor for capture solenoid is 0.185Ω , supposed the power supply is 500 V. Once quench is occurred in one of the superconducting magnet of pion capture solenoid, the power supply will be turned off, and the magnet stored energy is extracted by the dump resistors. As for the current design, the quench back heater will not be employed in pion capture solenoid due to the high resistivity of aluminium stabilizer after quench.

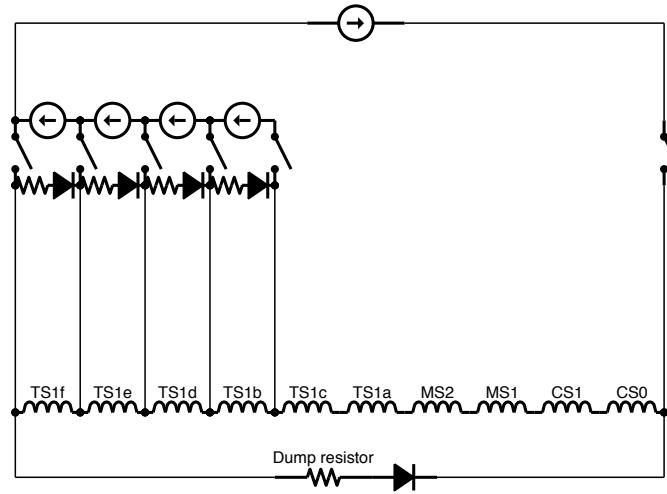


Figure 2.4: Schematics of quench protection system for pion capture solenoid.

2.2 Muon transport solenoid

The muon transport solenoid consists of 16 coils without stabilizer and a dipole coil with 0.056 Tesla compensating field is attached on the outer surface of each solenoid, which is shown in figure 2.5.

Coil	B_z [T]	B_y [T]	Length [mm]	Inner radius [mm]	Outer radius [mm]	Current [A]	Current density [A/mm ²]
TS2a	3	NA	255	234	249	210	72
TS2-1	3	0.06	205	234	264	210	95
TS2-2~15	3	0.06	205	234	272	210	95
TS2-16	3	0.06	205	234	254	210	94
TS3	3	NA	600	400	437	190	85.5

Table 2.3: Coils parameters for muon transport solenoid.

All of the coils of muon transport solenoid are connected to each other. Details of each coil and design parameters are listed in table 2.3 and 2.4.

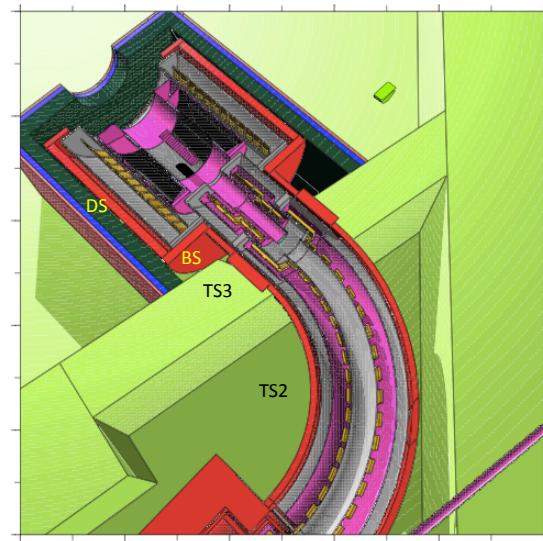


Figure 2.5: Schematic layout of muon transport solenoid and detector solenoid.

Item	Value
Conductor	NbTi/Cu wire Cu:NbTi = 6:1
Cable dimensions (solenoid)	ϕ 1.5 mm (without insulation) ϕ 1.56 mm (with insulation)
Cable dimensions (dipole)	ϕ 1.2 mm (without insulation) ϕ 1.3 mm (with insulation)
Cable insulation	Polyimide-imide enamel (AIW), PVF (TS2-15, TS3)
Magnet length	\sim 6 m
Curvature radius	3 m
Number of solenoid coils	18
Number of dipole coils	16 pairs
Operation current	210 A (solenoid) 165 A (dipole)
Field on axis	\sim 3 T (solenoid) \sim 0.056 T (dipole)
Stored energy	5.6 MJ
Total inductance	254 H
Refrigeration	conduction from forced flow 2-phase LHe piping (7-10 g/sec)
Quench protection	semi-active quench back heater

Table 2.4: Design parameters of transport solenoid.

2.2.1 Quench protection

The coils in transport solenoid is not stabilized by aluminium and with high inductance. To decrease the current quickly, the self resistivity must be high enough. Thus, one semi-active quench back heater is employed in transport solenoid, which is shown in figure 2.6.

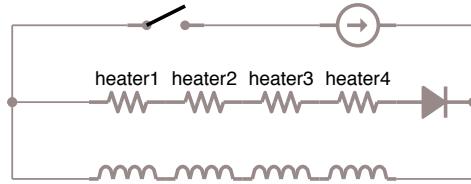


Figure 2.6: Semi-active quench protection circuit for muon transport solenoid.

Unlike to the pion capture solenoid, quench back heaters are installed in one quench protection circuit with superconducting coils. Once a coil of muon transport solenoid quenches, the switch will be turned off, then current will flow to the heater. Every heater will be activated by its own current and heat every coil in muon transport solenoid.

Diode is a switch to prevent the current flowing to the heaters during the operation and turn on the heater when quenched. In the case of muon transport solenoid, it operates in the cold mass (4.2 K). The radiation may cause the turn-on voltage reduced. Considering the radiation may cause the degradation of forward voltage and temperature rising, the radiation test is described in next chapter.

Chapter 3

Radiation estimation

Since high intensity proton beam with 8 GeV will be used in COMET experiment, the radiation is an important issue for superconducting magnet design. In this chapter, the estimations of prompt and residual radiation are described. Moreover, one radiation shield is optimized for protecting the pion capture solenoid.

3.1 Hadronic models

Monte Carlo code is usually employed in high energy physics to simulate the physical process. Recently, it is also playing increasingly role in the other studies, like heavy ion therapy⁽¹⁵⁾, nuclear fusion reactor et al. Several hadron models in some Monte Carlo codes which is used in the paper is introduced as follows.

3.1.1 PHITS

Particle and Heavy Ion Transport code System (PHITS) is developed by Japan Atomic Energy Agency (JAEA)⁽¹⁶⁾. The PHITS code is written in FORTRAN, and it can deal with the transport of almost all particles over a wide energy range.

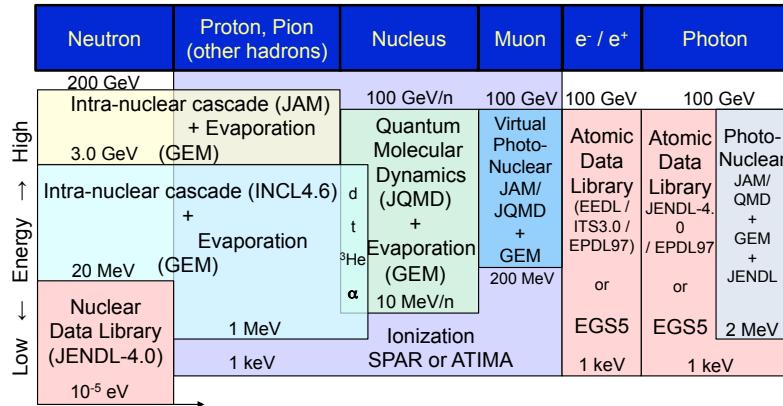


Figure 3.1: The physical model used in PHITS code. Default hadronic model is JAM_INCL4.6_JENDL, moreover, JAM_BERT_JENDL and JAM_INC-ELF_JENDL are able to be selected in PHITS code.

PHITS uses its own models for nuclear collisions and for hadron reactions. The details of physical model in PHITS are shown in figure 3.1. As for 8 GeV hadron collision, the Japanese nuclear library, JENDL-4.0,

is employed in the low energy region (≤ 20 MeV). Between 20 MeV and 3 GeV, there are three kinds of different model in hadron reaction, which are the INCL4.6⁽¹⁷⁾, Bertini and INC-ELF⁽¹⁸⁾. The JAM model is used in the range higher than 3 GeV.

3.1.2 FLUKA

FLUKA is a fully integrated particle physics Monte Carlo simulation package developed by CERN⁽¹⁹⁾. Two hadronic models are included in FLUKA, the Generalized IntraNuclear Cascade (GINC) model in high energy region (≥ 5 GeV) which is treated in the Glauber-Gribov formalism and PEANUT model in the low energy region⁽²⁰⁾.

3.1.3 GEANT4

GEANT4 is the most powerful toolkit for simulation of high energy physics experiment⁽²¹⁾. Like PHITS, GEANT4 is using the G4 neutron data library (G4NDL) in the energy less than 20 MeV. Between 20 MeV and 5 GeV, three models, Binary, Bertini and INCL++, can be selected in G4. G4 also provides the FTF String model from the range of 5 GeV to 20 TeV and QG String model to 100 TeV.

3.2 Modification of PHITS code

PHITS code has no powerful function like GEANT4, which cannot bend the charged particle with field map, no muon physics induced besides the Michel decay and etc. Here, the modification of PHITS code will be introduced to achieve the realistic simulation of COMET experiment. The main optimization of PHITS code is listed as follows.

- Bridged PHITS and the accelerator simulation code TURTLE.
- Enable to read the external field map.
- The charged particle can be bended in user defined field.
- The output file from PHITS can be read into GEANT4 code.
- Enable to use the ROOT toolkit in PHITS.

3.2.1 Proton beam

The proton beam source is simulated by the accelerator simulation code, TURTLE. As the TURTLE simulation shown in figure 3.2, one bunch of the proton beam where locates in front of production target is gaussian distribution with size of $2\text{ cm} \times 0.8\text{ cm}$ and momentum of $8.9\text{ MeV}/c$. Its momentum vector is point to the target.

PHITS can read the external file as a source, but due to the different coordinate used in PHITS and TURTLE. One script has been written to convert the coordinate and pretend the PHITS dump file to be read. The proton beam used in radiation estimation is from TURTLE data for phase-I and gaussian beam for phase-II, respectively.

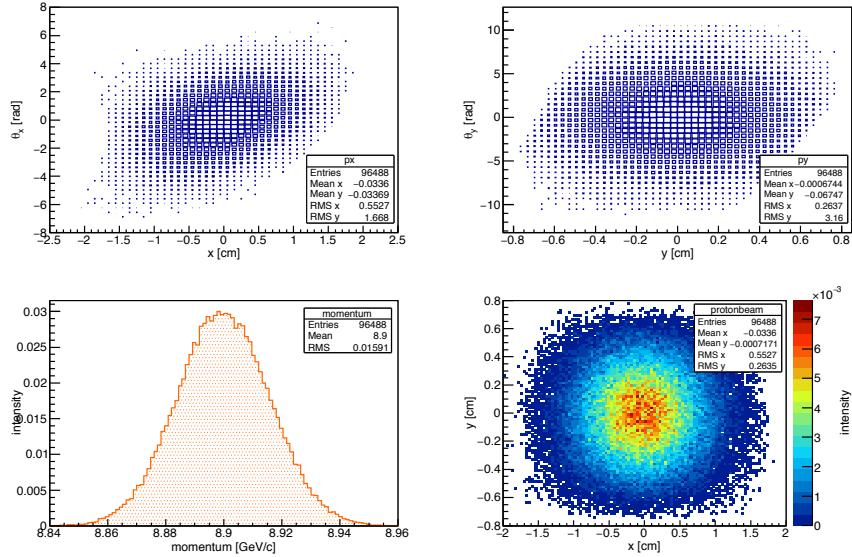


Figure 3.2: Proton beam distribution simulated by TURTLE code. Top two are the momentum distribution of x and y axis, respectively. Bottom two are the momentum distribution and spatial distribution. Here, intensity means the normalized intensity.

3.2.2 Magnetic field

As for current PHITS, it still cannot bend the charged particles in field map. However, the radiation estimation is affected by the magnetic field sensitively. To solve this issue, one magnetic field interpolation subroutine is included into PHITS code. The details of the magnetic field interpolation is described in appendix. The charged particles are bended with a circular path in the magnetic field which is written as

$$R = \frac{p_{\perp}}{eB} = \frac{m\gamma\beta c}{eB} = p_{\perp} \cdot \frac{3.336}{B} \quad (3.1)$$

where p_{\perp} is the transverse momentum with unit of GeV/c and B is the magnetic field with unit of Tesla. To confirm the magnetic field interpolation, the uniform field map with 5 Tesla has been tested and its tracking is shown in 3.3. Following the equation 3.1, the bending radius of incident negative pion with 0.113 GeV/c momentum and 10 degree in 5 Tesla is 1.2 cm. The radius from the simulation is totally agree with the calculation.

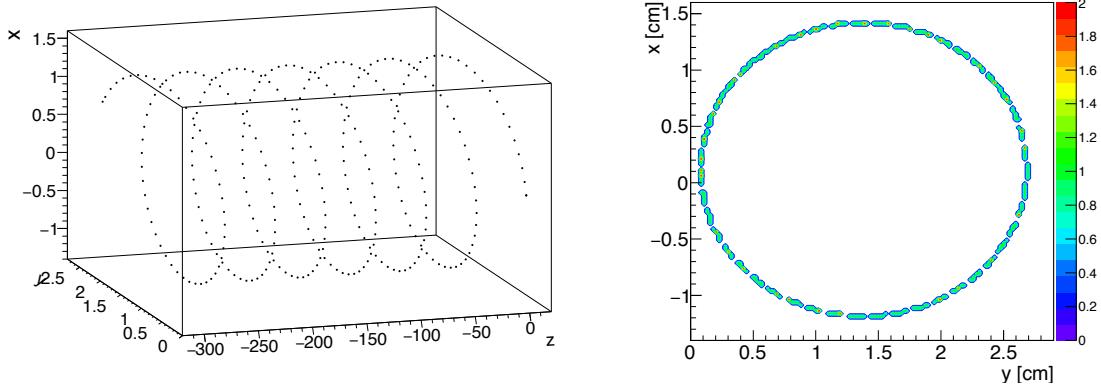


Figure 3.3: Tracking of 40 MeV (113 MeV/c) negative pion with uniform 5 Tesla field. Pion is shoted at (0,0,0) with 10 degree.

The field map included into COMET simulation is calculated by Finite Element Method (FEM) with iron yoke, which is shown in figure 3.4. There are still some missing part of field map, like the field around the iron yoke and proton beam transport pipe. It affects the proton beam transportation definitely, but this map is enough for the radiation study.

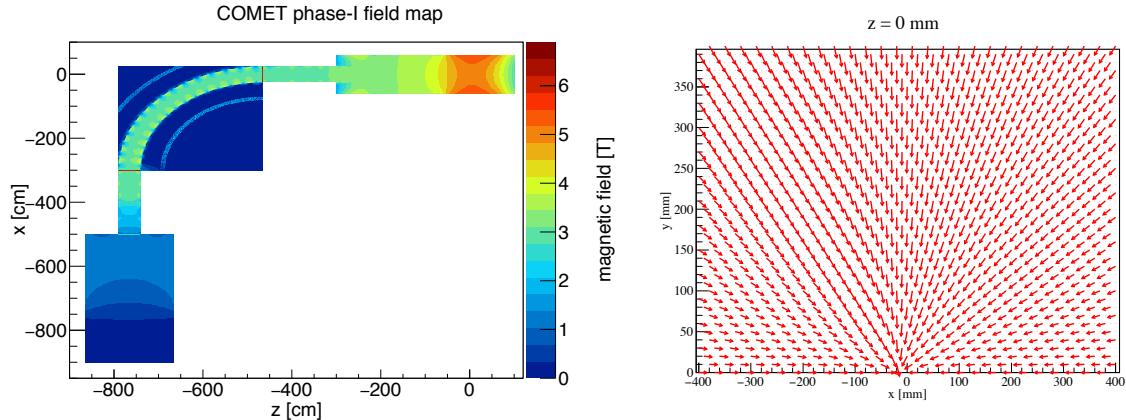


Figure 3.4: Magnetic field map of phase-I experiment used in PHITS simulation. It is calculated by FEM with iron yoke. Magnetic field vector at $z = 0$ cm is shown on the right.

3.3 Prompt radiation

3.3.1 Physical model comparison

Since PHITS does not include a lot of physics model involved muon such as muon decay on the orbits, specific X-ray of muon capture, rate decay of muon and etc. but GEANT4 did, the comparison of particle yield between GEANT4 and PHITS is necessary to make sure whether the missing physics model can be neglected in radiation estimation or not.

The comparison is taken by using a very simple geometry, which is only one graphite target with 60 cm length and 2 cm radius. Incident proton beam is 8 GeV with gaussian distribution. Magnetic field is included in the region around target with 30 cm radius and 140 cm length. As the result shown in figure 3.6 and table 3.1, the integral yield of pion and muon from target predicted by PHITS is higher than GENAT4's

with factor 12.9%. For the neutron, the production yield has 17% difference between GEANT4 and PHITS.

	μ^- [n/proton]	π^- [n/proton]	μ^+ [n/proton]	π^+ [n/proton]	neutron [n/proton]
GEANT4	0.033	0.613	0.053	0.786	2.891
PHITS	0.034	0.723	0.044	0.876	3.464
σ	-3%	-15%	21%	-10%	-17%

Table 3.1: Comparison of PHITS with GEANT4. σ represents (G4-PHITS)/PHITS.

The backward production of muon and pion at high energy region is difficult to predict because of the lack of experimental data. Especially for the COMET experiment which employs the backward muon, prediction of backward muon and pion is quite different from the different physics model. Thus, as the comparison of backward pion and muon at 1 m far from the target, not only the momentum spectrum but also the momentum direction are similar. Because the pion only comes from the high energy cascade model, the different must be between JAM and QGSP.

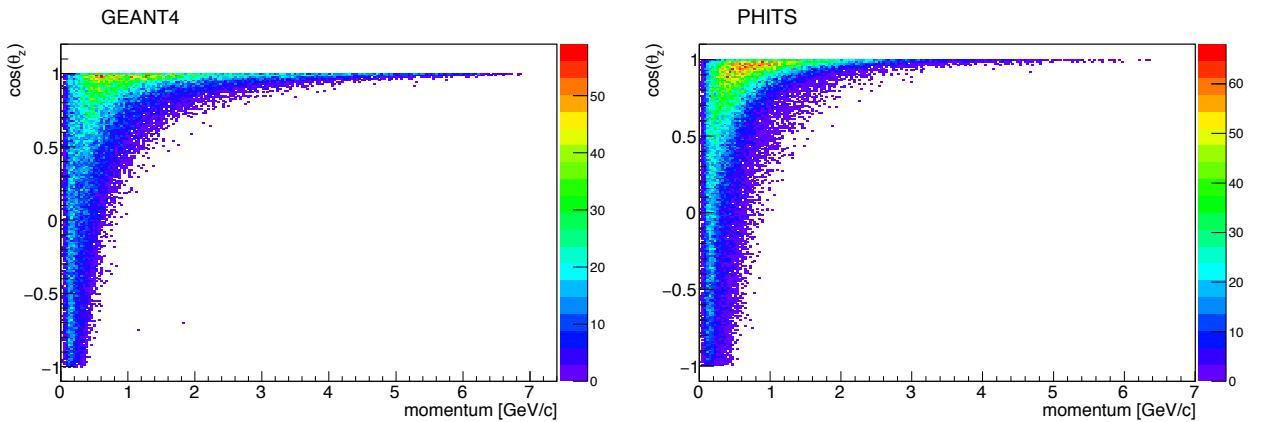


Figure 3.5: The comparison of momentum distribution of QGSP_BERT-ENDF with JAM_BERT_JENDL. Both momentum spectrum and distribution are very similar for these two models.

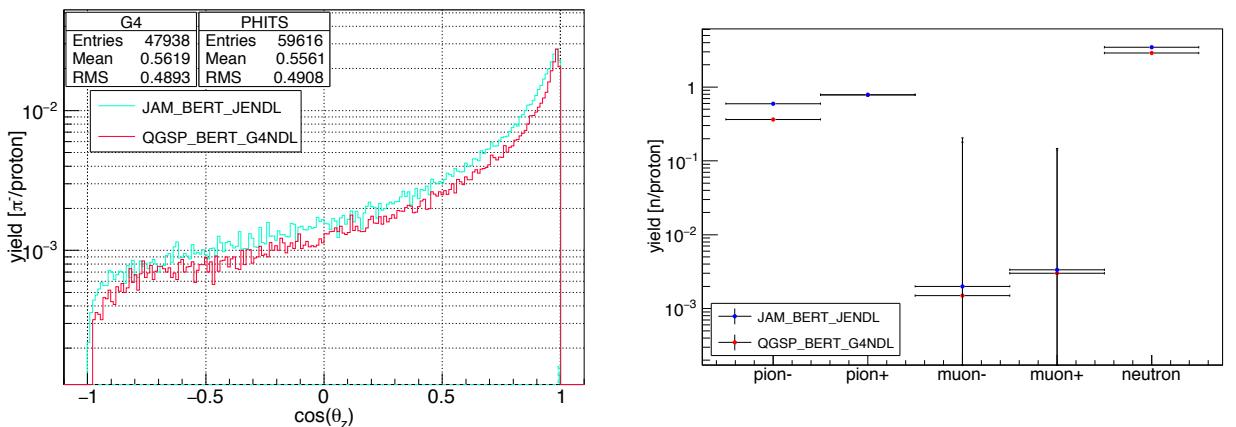


Figure 3.6: Compared the backward particle yield and integral yield with GEANT4 and PHITS. Left one is the backward pion and muon yield, red and blue line represent the yield calculated by QGSP_BERT_G4NDL and JAM_BERT_JENDL respectively. Right one shows the integral yield of pion and muon from target, red and blue point are the result of QGSP_BERT_G4NDL and JAM_BERT_JENDL.

3.3.2 Geometry

The geometry plays an important role in radiation simulation, especially for the high energy physics. Every detail of COMET phase-I geometry is established in PHITS simulation in terms of the design drawing from KEK.

As shown in figure 3.7, the COMET experimental hall is separated by concrete wall with 1 m thickness at end of 90 degree bending and 2 m at beam dump. The concrete wall between experimental hall and control room and accelerator hall is with 3 m thickness to prevent the radiation leaking. Beam dump consists of iron yoke with $1.5 \times 1.5 \times 1$ m and $2.5 \times 2.5 \times 1$ m holes which is same to the Mu2e beam absorber design and concrete⁽²²⁾. Ceiling board is made of iron yoke with thickness of 0.6 m, gas (air) with thickness of 0.7 m and concrete with 3.7 m thickness. Shielding supporter structure is made of stainless steel (SUS-304) with two holes for proton beam exit and target changing. Radiation shield is employed the composite shield which will be described later. In the muon beamline, two 1 mm titanium vacuum windows are set up in the end of pion capture solenoid and muon transport solenoid respectively. Cosmic shield is consist of 20 cm concrete with 50% iron, 10 cm polyethylene and 10 cm lead.

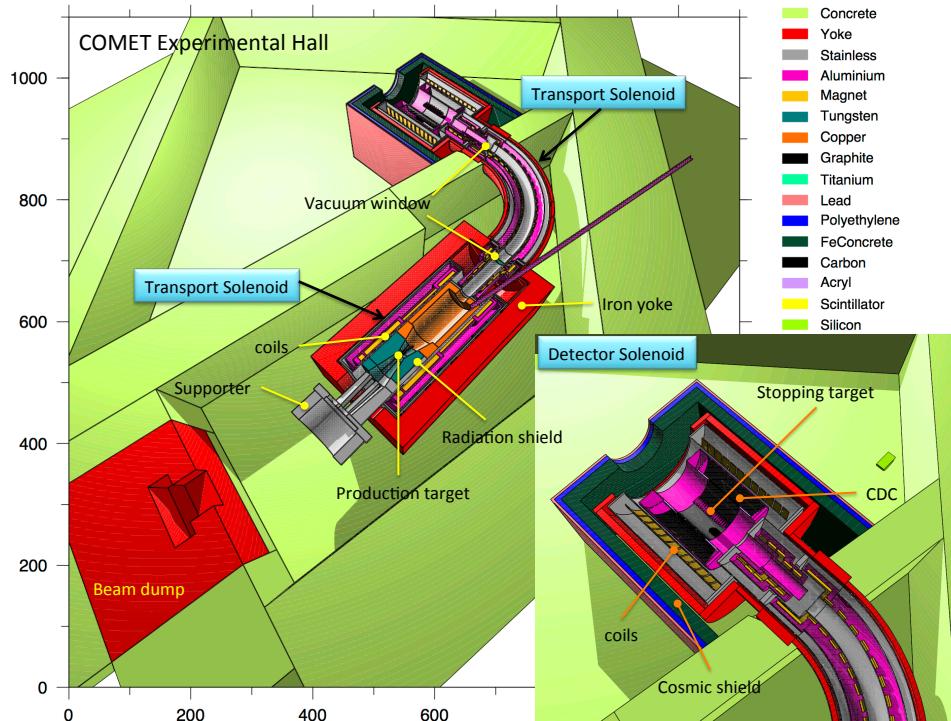


Figure 3.7: COMET phase-I experiment geometry in PHITS simulation.

3.3.3 Radiation estimation

Since only backward pion is employed to create muon in high magnetic field, the main radiation is neutron and photon because they are not affected with magnetic field. As implemented the geometry and magnetic field, the neutron and photon distribution is calculated and shown in figure 3.9. Because the geometry is only for phase-I experiment, the production target is replaced from graphite to pure tungsten target, and the proton beam intensity is changed to 4.4×10^{13} pps to estimate the radiation for phase-II experiment. The neutron energy and photon is cut from 10^{-12} MeV and 1 keV respectively. Electron is not included because

the electron is calculated by Kerma factor, which is not reliable.

Furthermore, to prevent neutrons leak to the detector room, the concrete wall with 2 m thick is replaced by iron yoke. Even if the iron yoke is employed as a wall here, it still has 5×10^6 n/cm²/sec neutrons at least which leak from experimental room to detector room which is 5 times higher than phase-I experiment. The neutron along the muon beamline is with low energy and it is easy to be capture or scattered in the detector solenoid. While, as for the photon, it comes from the neutron capture, muon capture, bremsstrahlung et al. Thus, the photon in the detector solenoid is about 10^9 n/cm²/sec at peak, which is 100 times higher than phase-I experiment at same place.

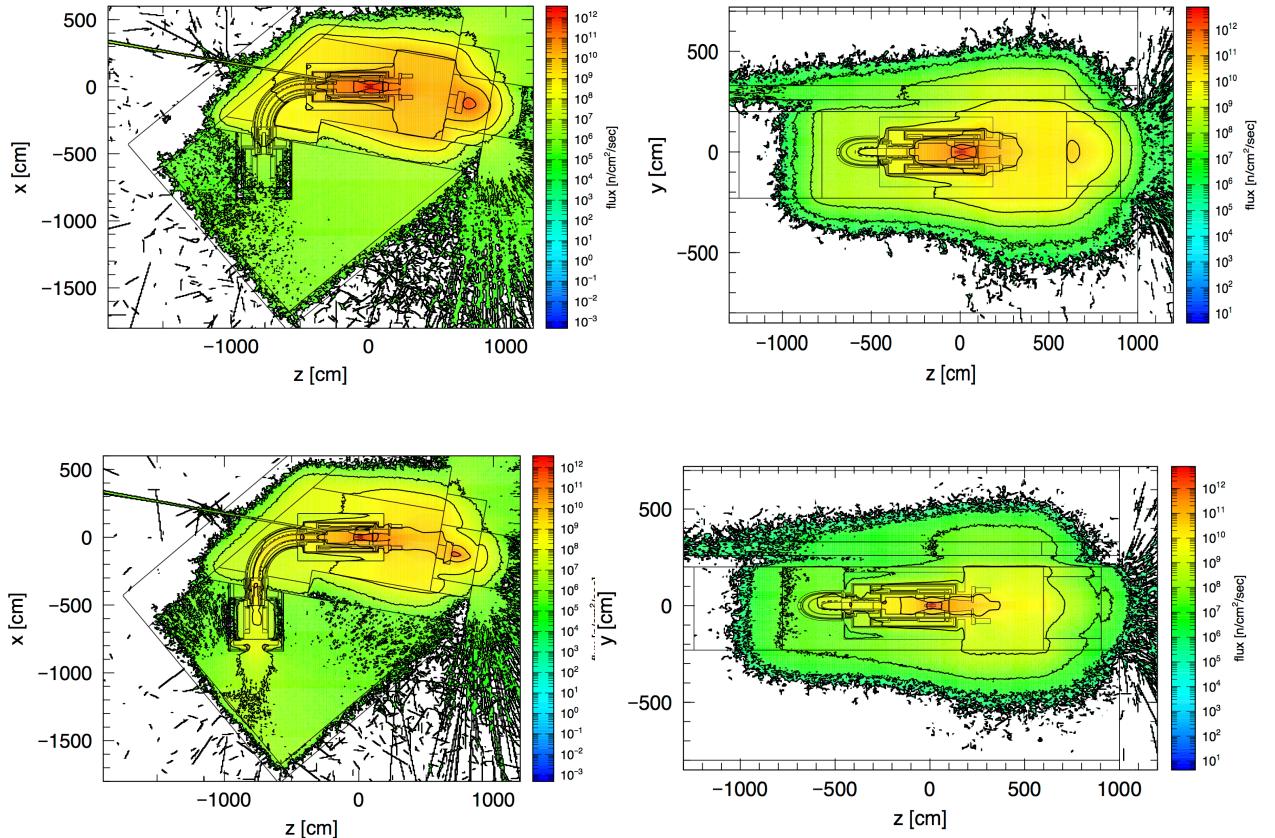


Figure 3.9: Neutron and photon distribution for COMET phase-II experiment. Top two show the neutron distribution from x-z and y-z view. Bottom two are the distribution of photon from x-z and y-z view.

Figure 3.10 shows the energy spectrum of muon, photon and neutron for phase-II experiment at the end of 90 degree bending. Compared with the other particle, neutron and photon is most dominated particle along the beamline. Almost neutron is in the range of low energy, and photon is with energy from 0.01 MeV to 100 MeV.

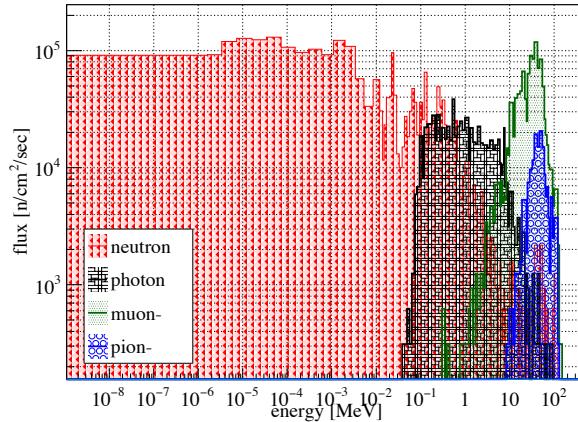


Figure 3.10: Energy spectrum of muon, photon and neutron from both experimental hall and beamline at the end of 90 degree bending along the axis. Most of neutrons are at the low energy region, and photon are all higher than 10 keV.

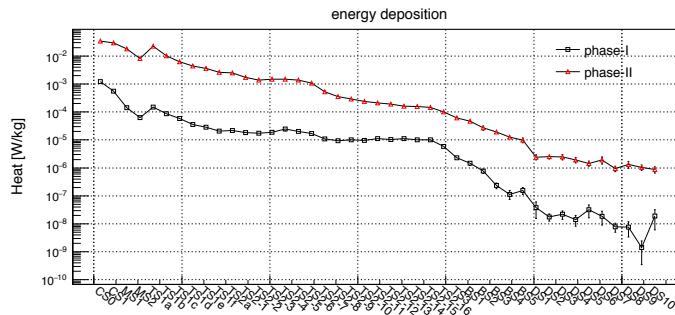


Figure 3.11: The energy deposition of each magnets in phase-I and phase-II experiment. Black and red point represent the energy deposition of phase-I and phase-II experiment respectively. The energy deposition of phase-II is one order higher than phase-I's, and its peak is located in CS0 coil.

The radiation remains their energy into superconducting coils, which will cause the hot spot inside the magnets. Once the hot spot is generated in the superconducting magnet, these magnets are at risk from quench. The heat load of each magnet for phase-I and phase-II experiment are shown in figure 3.11. Compared with phase-I experiment, heat load of phase-II experiment is 10 times higher than phase-I experiment at least. The peak of heat load is at CS0 coil which 0.03 W/kg corresponding to 0.73 MGy for 280-day operation.

3.4 Radiation shielding optimization

The heat and radiation shield (HRS) is located around the production target to protect the superconducting from the radiation exposure. This radiation shield consists of two parts, one is for protecting MS magnets, another one is for protecting CS magnets. It is unable to take long time to change the radiation shield after phase-I experiment due to the residual radiation. Thus, radiation shield must be designed for both phase-I and phase-II experiment.

Pure tungsten is most ideal material as the radiation shield due to its high density. However, it has several disadvantages such as high costs, heavy mass and complex fabrication. Therefore, some parts of radiation

shield can be replaced by the other materials.

3.4.1 Radiation shield for MS

Inside the superconducting magnets, radiation shield must be made of material without magnetism. Thus, there are two candidates for MS radiation shield, copper and stainless steel. The heat load of each CS and MS magnet when the copper and stainless steel is used as radiation is estimated by PHITS code in table 3.2. It shows that copper shield has better shielding ability than stainless steel shield.

	CS0 [W]	CS1 [W]	MS1 [W]	MS2 [W]	CS + MS [W]
SUS304	8.03	51.82	27.80	10.49	98.13
Copper	7.56	50.14	22.99	8.73	89.42

Table 3.2: Heat load of stainless steel and copper shield.

Considering the residual radiation of radiation shield, the activated isotopes and activity after 1-month operation and 10-month cooling are investigated and shown in table 3.3. Apparently, copper shield's activity is quite lower than stainless steel shield's. Furthermore, the dose equivalent of copper and stainless steel shield are $1.51 \mu\text{Sv}/\text{h}$ and $7.02 \mu\text{Sv}/\text{h}$ respectively.

Nuclei	Activity [MBq]	Half life [year]	Nuclei	Activity [MBq]	Half life [year]
^{60}Co	192	5.27	^{55}Fe	4950	2.73
^{57}Co	111	0.75	^{54}Mn	1500	0.86
^{63}Ni	80.9	100	^{49}V	878	0.90
^{58}Co	56.9	0.19	^{57}Co	585	0.75
^{54}Mn	54.7	0.86	^{58}Co	387	0.19
^3H	30.9	12.3	^{51}Cr	92.6	0.08
^{55}Fe	29.3	2.73	^{59}Fe	29.4	0.12
^{49}V	16.7	0.90	^{63}Ni	26.5	100
^{56}Co	5.52	0.21	^3H	26.0	12.3
^{65}Zn	5.13	0.67	^{60}Co	21.7	5.27

Table 3.3: Activated isotopes of copper (left) and stainless steel (right).

From the aspect of shielding ability and residual radiation, copper is the better material for MS radiation shield. Noting that the induced current will be generated by the changing of magnetic field, the copper should be cut in somewhere.

3.4.2 Radiation shield for CS

Proton is injected into pion capture solenoid with 10 degree, thus, a hot spot is created on the one side of CS1 and CS0 coils, which is the one of the reason why quench occurs. Thus, the radiation shield for CS requires

- Limit the maximum heat load in coils.
- Limit the maximum radiation induced damage in coils.
- Reduce the mass of radiation shield.

- Pion yield should not be reduced significantly.

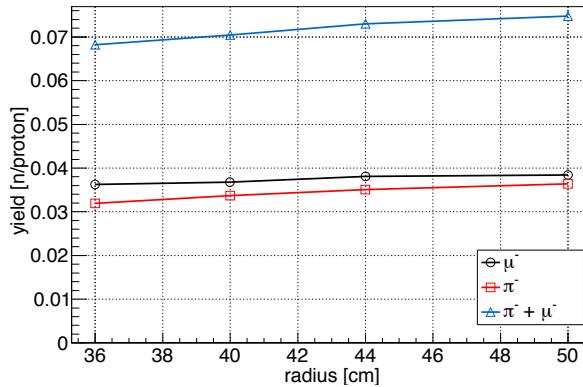


Figure 3.12: A relation of muon and pion yield with shielding backward radius. Current shielding backward radius is 36 cm. Black line: μ^- , red line: π^- , blue line: $\mu^- + \pi^-$.

As for the muon yield, it does not depend on the material of radiation but the backward radius of shield. Figure 3.12 shows that the muon and pion yield increases about 9% after the radius is changed from 36 cm to 50 cm.

To reduce the costs and mass of radiation shield, the low radiation region which corresponds to the side far to incident proton enables to be fabricated by another kind of material. As for the material, the ideal material is pure tungsten but only the tungsten alloy is cheaper and easy to fabricate. Thus, the details of tungsten alloy (AN-1800) are listed in table 3.4.

Composition [wt.%]	Ni: 3.0 ± 0.25 ; Cu: 2.0 ± 0.25 ; W: 95.0 ± 0.5
Density [g/cm ³]	18.0 ± 0.2
Hardness	$HV 320 \pm 50$
Tensile strength	620 MPa
Offset yield strength	500 MPa
Elastic modulus	310 GPa
Thermal conductivity	104 W/m/K
Thermal expansion coefficient	$5.5 \times 10^{-6} \text{ K}^{-1}$

Table 3.4: Material property of tungsten alloy (AN-1800).

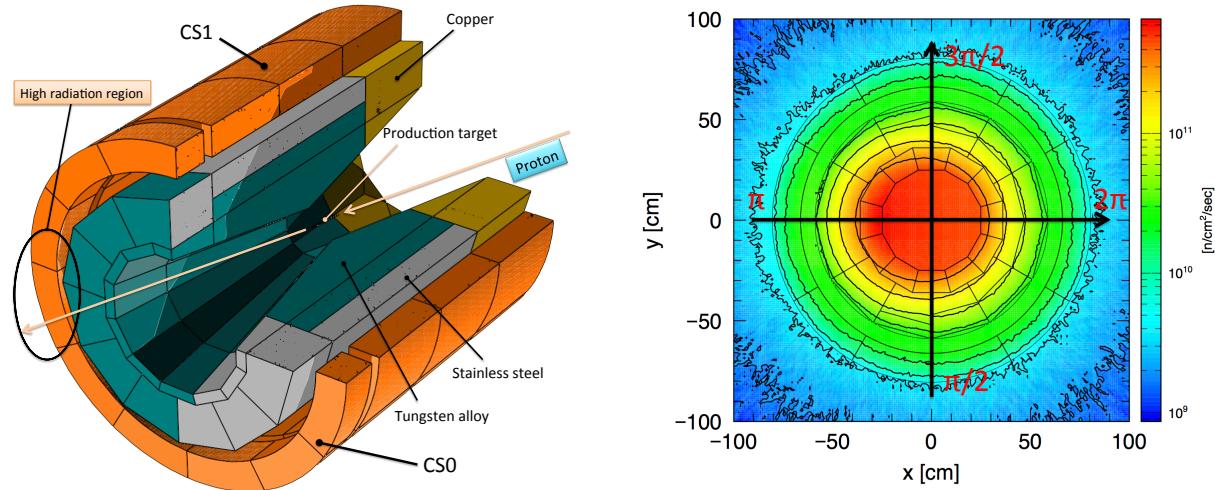


Figure 3.13: The concept of COMET radiation shield. It consists of tungsten alloy, stainless steel and copper. Tungsten alloy and stainless steel are employed on the high and low radiation side respectively. On the downstream of target, copper is used. Right figure shows the distribution of neutron from x-y view. Azimuthal angle is also defined in here. The peak of neutron flux is located at 180 degree of CS coils.

As shown in figure 3.13, the radiation shield is designed which consists of tungsten alloy, copper and stainless steel. The DPA, neutron fluence and heat load of several shielding design concepts along azimuthal direction is compared in figure 3.14 and 3.15. Details of several shielding designs are listed as follows.

- Ideal pure tungsten shield: made of pure tungsten with a cylindrical shape.
- Ideal tungsten alloy shield: made of tungsten alloy with a cylindrical shape.
- Ideal copper shield: made of copper with a cylindrical shape.
- Realistic tungsten alloy shield: made of tungsten alloy with a polygonal shape.
- Realistic composites shield: made of tungsten alloy, copper and stainless steel with a polygonal shape.

After these comparisons, we come out the conclusion

- The maximum DPA of CS1 coils increases over 4 times when the pure tungsten is replaced by copper.
- Compared with tungsten alloy, pure tungsten has stronger shielding ability.
- Shielding ability of cylindrical shape is stronger than polygonal shape.
- The peak of DPA, heat load and neutron fluence is same between composites shield and tungsten alloy shield.

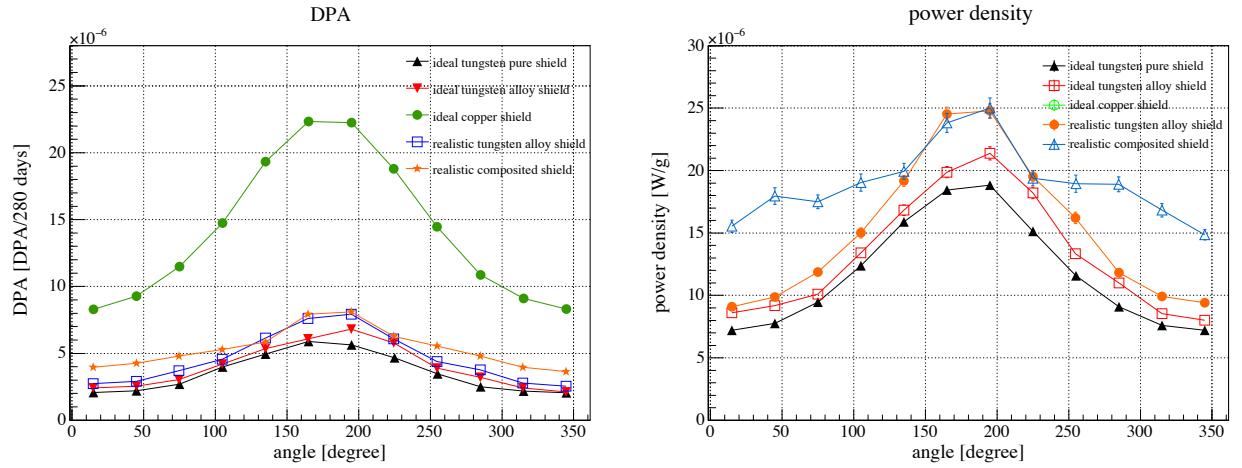


Figure 3.14: Compared the azimuthal DPA and energy deposition of middle part of CS1 coil in different version of shielding design. The long CS1 coil is cut to 3 parts along the z axis and 12 parts along the azimuthal axis.

The details of shielding design is listed in table 3.5.

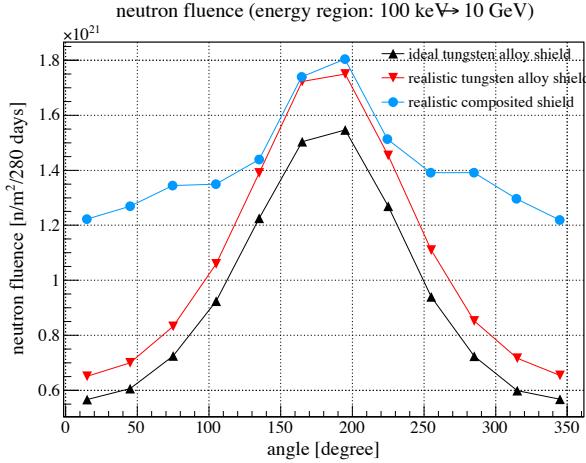


Figure 3.15: High energy neutron ($E \geq 0.1 \text{ MeV}$) along the azimuthal direction of CS1 coils.

CS1	Composited shield	Pure tungsten shield
peak neutron fluence [$n/m^2/280 \text{ days}$]	2.35×10^{21}	2.26×10^{21}
peak power density [W/g]	2.50×10^{-5}	2.40×10^{-5}
power [W]	57.5	36.6
peak dose [MGy/280 days]	0.79	0.76
peak DPA [DPA/280 days]	1.05×10^{-5}	0.98×10^{-5}
tungsten mass [t]	15.6	28
total mass [t]	21.5	28

Table 3.5: Details of the shielding design. CS1 is cut to 3 parts along the z axis, 12 parts along the azimuthal direction.

3.5 Residual radiation estimation

To achieve the muon with higher intensity, the graphite target will be replaced by pure tungsten target after phase-I experiment finished. Thus, the residual radiation plays an important role in target changing

and maintenance of the superconducting magnets. The residual radiation has been investigated by using FLUKA code and PHITS code.

3.5.1 Residual radiation

The residual radiation of each part of solenoid is estimated by PHITS and DCHAIN-SP code⁽²³⁾. DCHAIN-SP code is a dedicated software for the residual radiation developed by JAEA. Using the result of nucleus yield from PHITS code, the photon flux and its intensity is able to be calculated by DCHAIN-SP. Then, the distribution of residual radiation is estimated by

$$E(\epsilon) = f(\epsilon) \cdot \phi(\epsilon) \quad (3.2)$$

where $E(\epsilon)$ and $\phi(\epsilon)$ are the effective dose and the gamma ray flux calculated in DCHAIN-SP. The fluence-to-effective dose conversion coefficients $f(\epsilon)$ is shown in reference⁽²⁴⁾ with unit of Sv/cm^2 .

The operation time schedule used in calculation are 1-year cooling after 1-month running with intensity of 2.5×10^{12} pps, which are all base on the phase-I experiment. Residual radiation of vacuum vessel, magnet, radiation shield and iron yoke are given by figure 3.16. To reduce the residual radiation as much as possible, considering copper is hard to weld with stainless steel, some parts of vacuum vessel is able to be changed to iron. As listed in table 3.6, residual radiation of stainless steel is 10 times higher than iron's. Thus, the stainless steel is replaced by iron for vacuum vessel outside the superconducting magnet due to magnetism of iron.

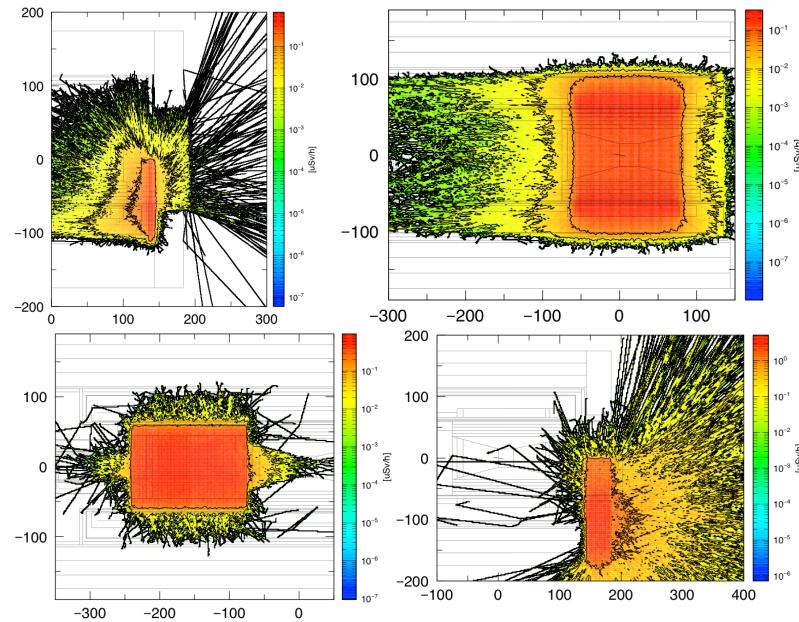


Figure 3.16: Residual radiation of vacuum vessel, magnet, radiation shield and iron yoke with 1 month operation and 10 month cooling.

	Stainless steel	Iron
Radiation dose [$\mu\text{Sv}/\text{h}$]	7.00	0.65

Table 3.6: Residual radiation of vacuum vessel between CS0 and iron yoke.

The residual radiation of superconducting magnet is calculated as one conductor which consists of aluminium, copper and NbTi with density of 4.0 g/cm³. The activity and radiation dose given by figure 3.17 shows that the maximum radiation dose is 0.33 μ Sv/h after 1 month operation and 10 month cooling for phase-I experiment and the peak of activation is at CS0 and CS1. Isotopes like ⁹³Nb, ²²Na, ³H, ⁶⁰Co et al. with long half life are generated in CS1 and CS0 coils.

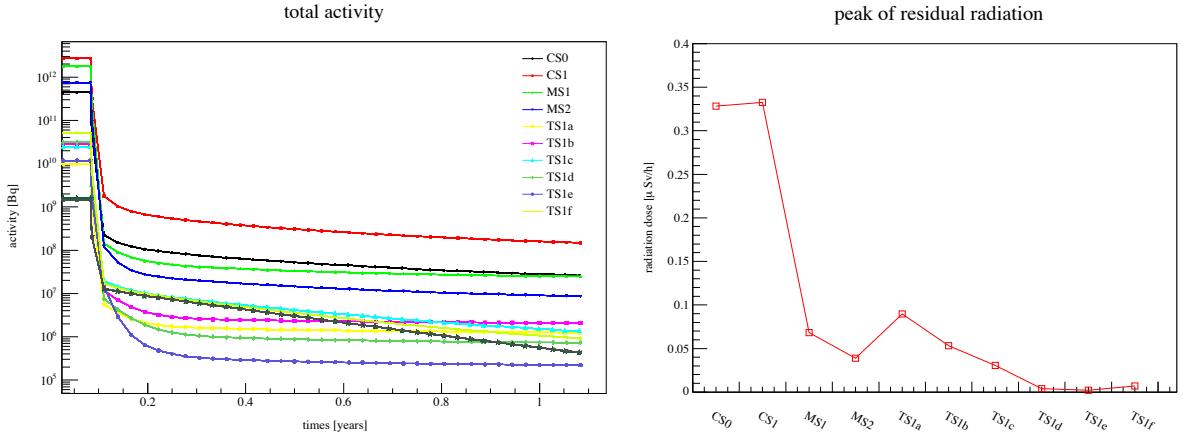


Figure 3.17: Activity and peak of dose equivalent of each magnets with 1-month operation and 10-month cooling.

Because PHITS and DCHAIN-SP only can calculate the residual radiation from a defined region, it may cause the average of the radiation if the geometry is cut to many tiny meshes. Figure 3.18 (left) shows the total dose equivalent predicted by PHITS and DCHAIN-SP. It is added from many parts of small regions. The maximum dose equivalent is predicted to 0.75 mSv/h for 1-month operation and 10-month cooling at production target. To ensure the dose equivalent, we also compare this result with FLUKA code which can simulate the residual radiation dose without region cutting. M. Brugger et al. present the residual radiation calculated from FLUKA code has good agreement experimental data⁽²⁵⁾.

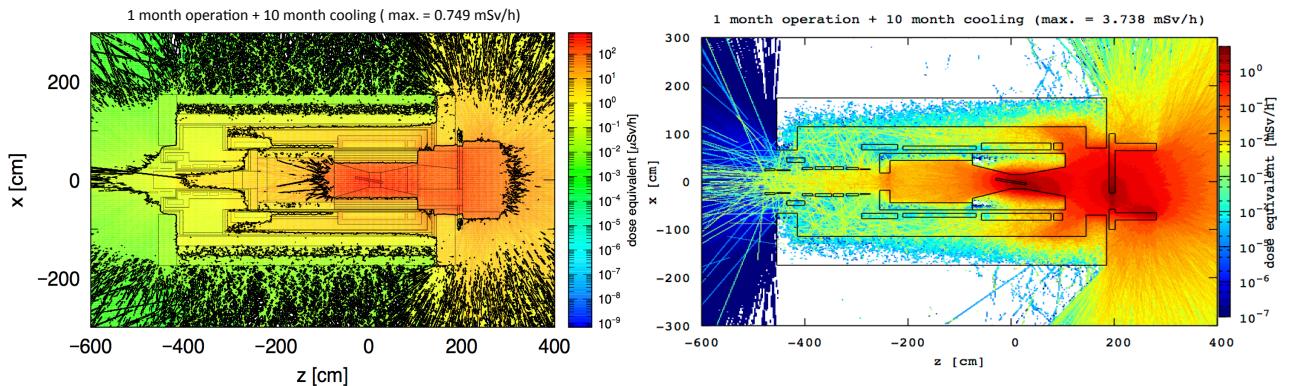


Figure 3.18: Compared the residual radiation distribution of pion capture solenoid between PHITS and FLUKA. Left and right represent the PHITS and FLUKA result respectively.

As the result shown in figure 3.18 (right), the maximum dose equivalent is predicted to 3.74 mSv/h with same time schedule, which is 5 times higher than PHITS result. In figure 3.7, the dose equivalent at different position along the axis is compared. Besides the place where peak is, the dose equivalent is quite similar between the FLUKA and PHITS. Difference may be owing to the mesh cutting of PHITS calculation around

target.

Code	$z = 0$ [mSv/h]	$z = 300$ [mSv/h]	$z = -300$ [mSv/h]	$z = -500$ [mSv/h]
FLUKA	3.74	0.04	0.009	0.002
PHITS	0.75	0.03	0.01	0.002

Table 3.7: Compared the residual radiation along the z axis. $z=0$ is at the production target, $z=-300$ is 3 m far from the target on upstream of target, $z=300$ is 3 m far from the target on downstream of target, and $z=-500$ is 5 m far from the target.

Because of the target changing after phase-I experiment, this residual radiation is still too high for co-workers. Table 3.8 gives the maximum residual radiation after cooling for 10 months, 1.75 year and 2 years.

Time [year]	Maximum residual radiation [mSv/h]
0.83	3.738
1.75	1.125
2.00	1.013

Table 3.8: Maximum residual radiation after cooling for 10 months, 1.75 year and 2 years.

From this estimation, the residual radiation will be reduced about 3 times after cooling for 2 years compared with 10 month cooling. Considering the beam dump is not included into this simulation, working for 1 hour is possible for co-workers.

Chapter 4

Irradiation test for superconducting magnet material

Since the superconducting solenoid of COMET experiment is exposed under the high radiation environment, the superconducting solenoid has a risk of quench due to the radiation damage in the some part of solenoid. Thus, the study of radiation damage on the material used in COMET superconducting magnet system is quite significant. In this chapter, the irradiation test on GFRPs, Quench protection diode and pure aluminium will be introduced.

4.1 Glass Fiber Reinforced Plastic

Glass Fiber Reinforced Plastics (GFRP) are widely used in many engineering applications because of their superior properties such as higher specific strength, specific modulus, anti-corrosion and thermal insulation⁽²⁶⁾. In the case of COMET experiment, GFRPs are employed as the insulation spacer for superconducting magnets which suffer high magnetic field and radiation. Due to the worse radiation resistance for the Glass-Epoxy (G10), 3 kinds of GFRPs what impregnate the Bismaleimide-Triazine (BT), Bismaleimide (BMI) and Cyanate Ester (CE) in the S-2 glass fiber have been developed to enhance the radiation resistance and mechanical properties. Here Boron is excluded from the S-2 glass in order to reduce the influence from neutron.

4.1.1 Experiment

Each sample of GFRPs (G10, CE, BT and BMI) is prepared as Japanese Industrial Standards (JIS), which is with 100 mm long, 10 mm wide and 0.5 mm thick. The irradiation test of GFRPs is taken in the Takasaki Advanced Radiation Research Institute by using 1 MeV uniform electron beam and Research Reactor Institute, Kyoto University (KUR) by using 34 MeV accelerator electron beam. After irradiation, the tensile strength of irradiated samples is measured as well. Figure 4.1 shows how GFRP samples break after irradiation.

For the 1 MeV electron irradiation test, radiation dose is measured by cellulose triacetate (CTA) tape⁽²⁷⁾

and it can be estimated from the equation as follows.

$$\text{dose} = \frac{D_e - D_0}{K} \cdot \frac{0.125}{t} \cdot f \quad (4.1)$$

where the unit of dose is kGy. D_e and D_0 is the light absorption before and after irradiation. K is the constant for irradiation which is defined to 0.0063 for electron and 0.0081 for photon. t and f are the thickness of CTA tape and correction term, respectively. Average radiation dose is about 0.8 kGy/sec·mA in terms of this equation. Each sample of G10, CE, BT and BMI is irradiation with 50, 100 and 200 MGy totally. As the results shown in figure 4.2 (left), BT and BMI are definitely stronger than G10 and CE without irradiation. The tensile strength of G10 and CE drops to about 150 MPa after 200 MGy irradiation. Similarly, BMI also has a tendency to degrade. While the BT, the tensile strength has no big change after 200 MGy irradiation.

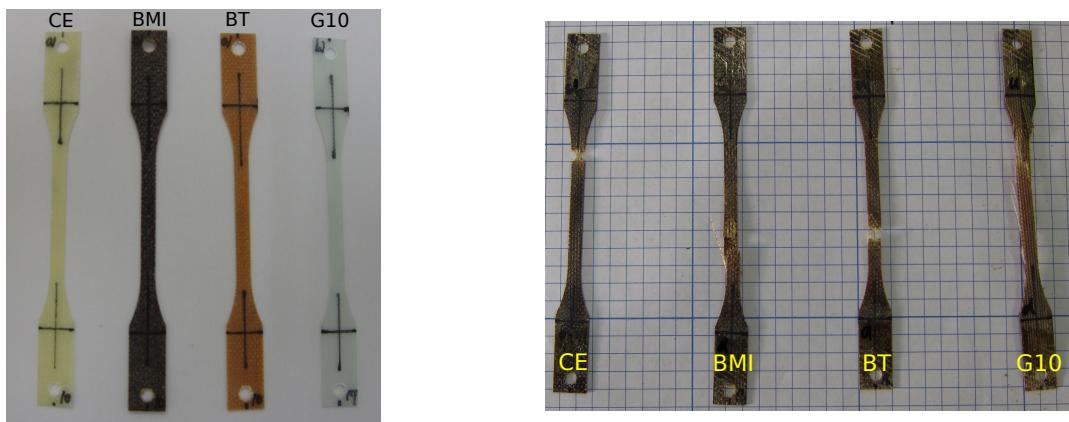


Figure 4.1: GFRP samples before the irradiation are shown on the left. The samples are tensioned until break after irradiation (right).

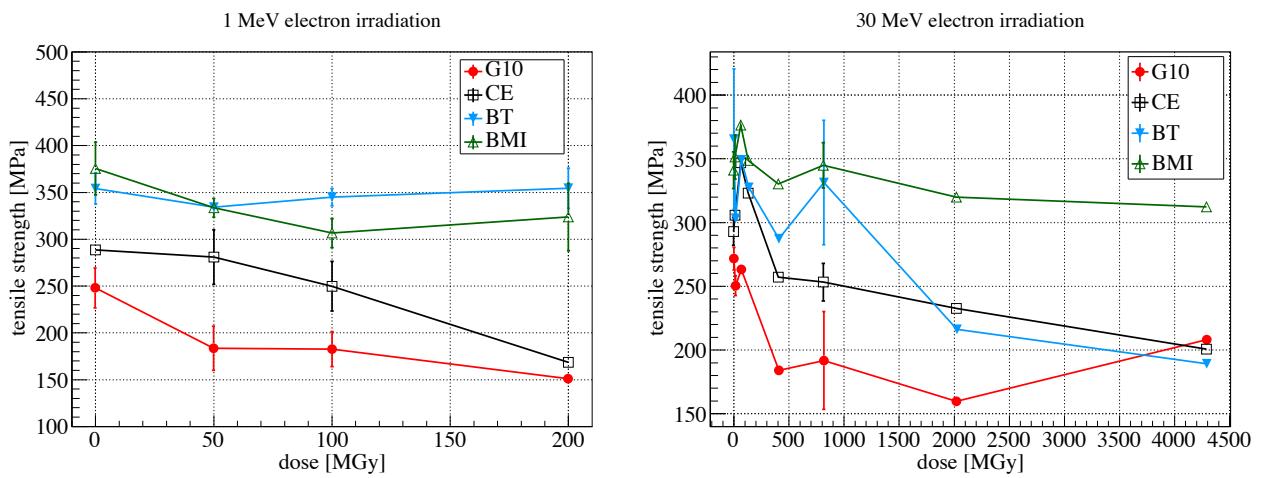


Figure 4.2: The tensile strength of irradiated samples (G10, CE, BMI and BT) when it breaks. Left and Right figure represent 1 MeV electron irradiation test in JAEA, Takasaki and 30 MeV electron irradiation test in LINAC, KURRI.

To investigate the reaction of high energy electron on GFRPs, irradiation with 34 MeV electron has been tested by using accelerator. Because of the electron with high energy, not only the electron but also the photon from Bremsstrahlung and neutron from the photonuclear reaction interact on GFRPs. LINAC

accelerator at KUR provides a gaussian beam with the width of 30 mm. GFRP sample is cooled by water during the irradiation (30°C). Unlike to the 1 MeV irradiation, CTA tape is unable to be used to measure the radiation dose because its energy is out of the range of the measurement ability of CTA tape. Thus, in this case, radiation dose is estimated by PHITS code, which is 1.25 kGy/sec·mA. Each sample is irradiated with 9, 30, 45, 90, 270, 540, 1340 and 2840 MGy totally. In figure 4.2, all samples start to degrade from 90 MGy and they drop to around 200 MPa besides BMI. The flexural strength test and outgassing test for GFRPs are described in reference ⁽²⁸⁾. Considering the mechanical property, outgassing and radiation resistance, BT will be employed as insulation spacer in COMET superconducting magnets.

4.2 Insulation tape

The conductor of pion capture solenoid is wound by two layers of insulation tape so called pre-preg tape as electrical insulation. This insulation tape shown in figure 4.3 is made of polyimide film covered by boron free glass cloth with BT and epoxy resin to enhance the mechanical property and radiation resistance under the high radiation environment.

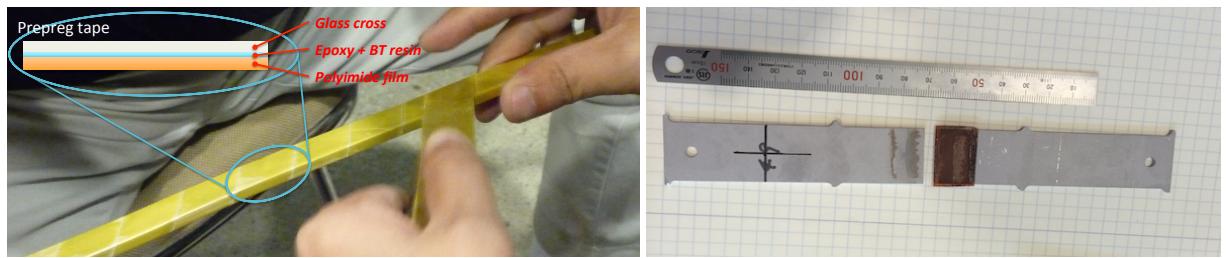


Figure 4.3: Insulation tape is made of glass cross and polyimide film impregnated by BT and epoxy resin (left). The two pieces of insulation tapes are sandwiched with aluminium strip to take tensile test (right).

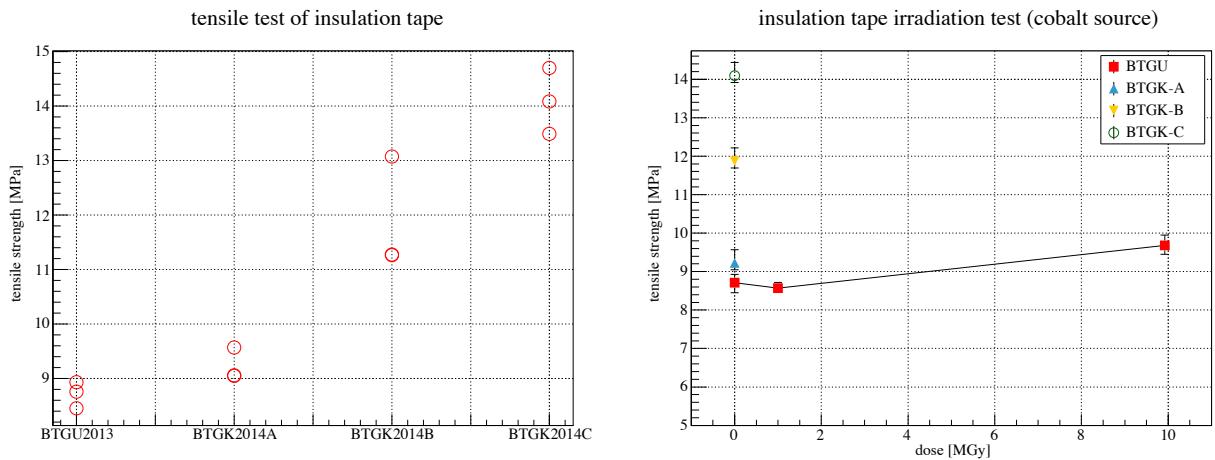


Figure 4.4: 3 samples of each BTGU, BTGK-A, BTGK-B and BTGK-C are tensioned without irradiation (left). BTGU shows no degradation after irradiated with 10 MGy at limit by cobalt γ ray source.

Four samples, BTGU, BTGK-A, BTGK-B and BTGK-C, made of same material but different rate of BT and epoxy resin are prepared for tensile test. Here, mixing the BT and epoxy resin with different rate can adjust the viscosity of insulation tape. Two samples are struck and sandwiched with aluminium strip

(Al-GU-UG-Al) as JIS standards for tensile test are cured by 170°C and 8 hours. Figure 4.3 shows how the pre-preg tape breaks after tensile test.

The result of tensile test for the sample without irradiation is given in figure 4.4, which shows the BTGK-C is the strongest one in these sample. However, the glass layer and resin layer is not struck before the thermal process, which is hard to employ it as the insulation tape in superconducting coil. Considering the mechanical properties may reduce after irradiation, the irradiation of BTGU samples is taken in Takasaki Advanced Radiation Research Institute with cobalt radiation source. As the result shown in figure 4.4 (right), the tensile strength of BTGU sample does not reduce after irradiation, however, surprisingly, the tensile strength increases about 10% after 10 MGy irradiation.

4.3 Quench protection diode

Quench protection diode is a switch of the dump resistor. When magnets quench, the power supply will be cut off, then this diode will be turned on and current will flow toward the dump resistor. The operating voltage will increase after irradiation⁽²⁹⁾, which causes the over heat of quench protection diode. Moreover different manufacturer fabricates diode with different electrical property. Here, a dedicated quench protection diode for COMET superconducting magnet has been irradiated by high energy neutron. Its electrical characteristics under low temperature and radiation is investigated.

4.3.1 Experiment

The irradiation test for quench protection diode is taken with COMET detector group together at the Tendem accelerator, Kyushu university. The fast neutron is produced from carbon target with incident 9 MeV deuteron, which creates neutron by deuteron-deuteron reaction. 5 kinds of samples are set up in front of the production target with the distance of 3 cm as the sequence of MPPC, APD, Artix-7, ROESTI and diode, which is shown in figure 4.5.

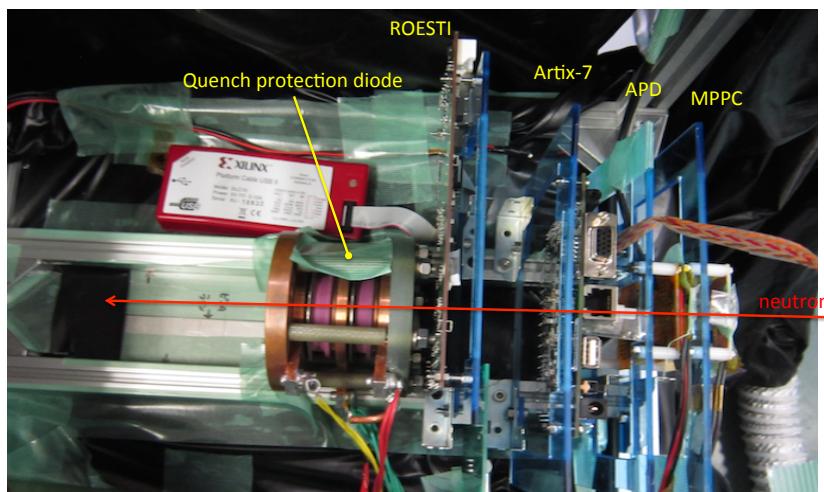


Figure 4.5: Sample setup for neutron irradiation test. Diode is set up in front of MPPC, APD, Artix-7 and ROESTI where is 30 cm far from the neutron production target.

Because diode is with the radius of 3 cm and length of 10 cm, which is possible to stop the neutron, it is

located at last of samples where 30 cm far from the production target. Its turn-on voltage is measured during the irradiation with 5 A power supply, and neutron flux is measured by activation analysis with aluminium, nickel and gold wire (0.07% Fe). Temperature of diode during the irradiation is recorded by thermometer.

4.3.2 Neutron measurement

Activation analysis is a common and easy way of neutron measurement, but with the bad energy resolution. aluminium, nickel and gold wire (0.07% Fe) are employed and stuck on the surface of each sample. The photon from neutron reaction on each activation sample which is list in table 4.1 is measured by HPGe detector.

Element	Reaction	Threshold energy	Half life	Emitted radiation energy [MeV]
aluminium	$^{27}\text{Al}(\text{n}, \text{p})^{27}\text{Mg}$	1.9 MeV	9.46 min	β^- (1.75), γ (0.84, 1.013)
aluminium	$^{27}\text{Al}(\text{n}, \alpha)^{24}\text{Na}$	3.27 MeV	15 h	β^- (1.389), γ (1.369, 2.754)
Gold	$^{197}\text{Au}(\text{n}, \gamma)^{198}\text{Au}$	(-)	2.7 d	β^- (0.962), γ (0.412)
Gold	$^{197}\text{Au}(\text{n}, 2\text{n})^{196}\text{Au}$	7.36 MeV	6.18 d	β^- , γ (0.356)
Nickel	$^{58}\text{Ni}(\text{n}, 2\text{n})^{57}\text{Ni}$	12.09 MeV	36 h	β^+ , γ (0.511, 1.37)

Table 4.1: Neutron activation reactions.

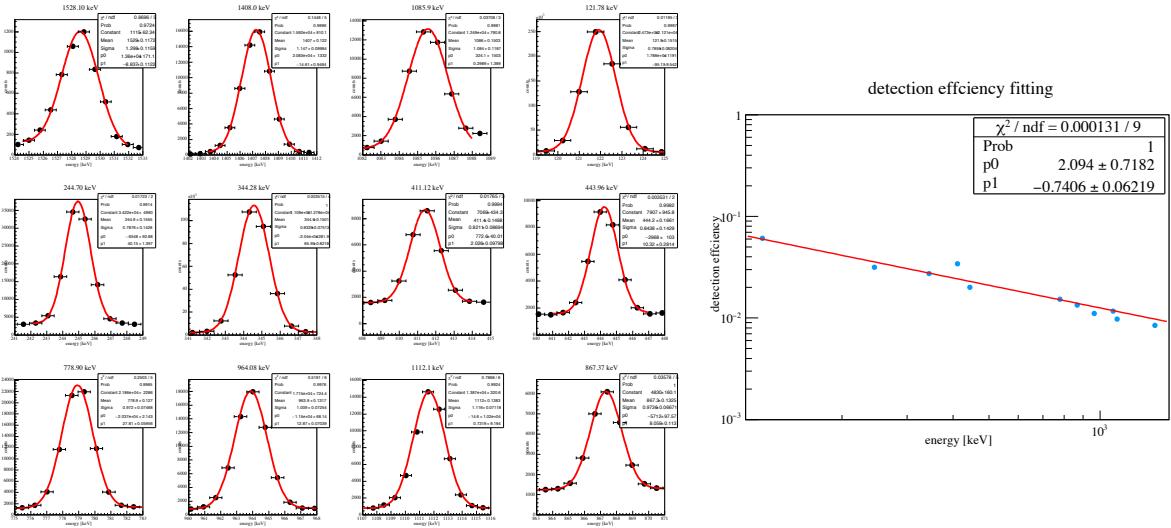


Figure 4.6: The detection efficiency is calibrated by using ^{152}Eu radiation source. Several peak of ^{152}Eu is fitted as convolution on left, and the detection efficiency of HPGe is fitted on right.

^{152}Eu with 2.79×10^7 Bq is employed as the calibration source to estimate the detection efficiency of HPGe detector. As shown in figure 4.6, each peak of radiation source ^{152}Eu is fitted as gaussian and linear function. The detection efficiency is fitted as

$$\epsilon = a \cdot E^b \quad (4.2)$$

where the counts for the calculation of detection efficiency ϵ is within 1.78σ .

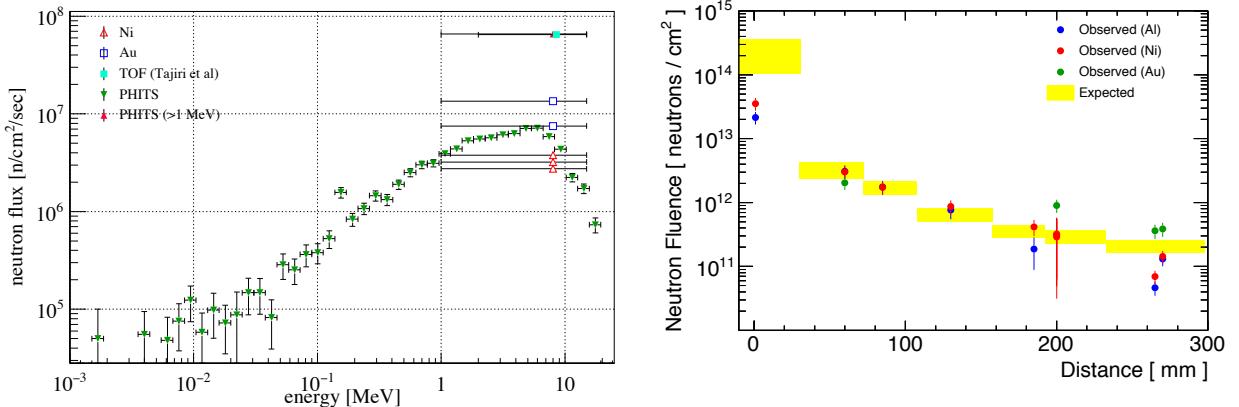
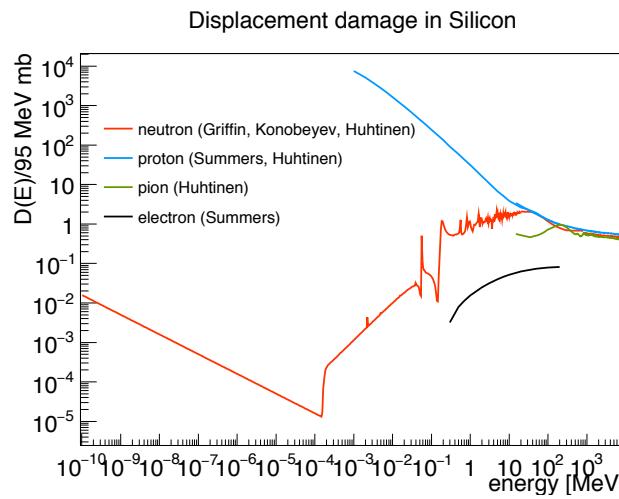
a	b
2.094 ± 0.7182	-0.7406 ± 0.06219

Table 4.2: Fitting parameters for detection efficiency.

Neutron flux measured by each activation sample is given by ⁽³⁰⁾

$$\phi = \frac{C \cdot A \cdot \lambda}{\sigma \cdot N_A \cdot \gamma \cdot m \cdot I \cdot \epsilon \cdot (1 - e^{-\lambda t_0}) \cdot (e^{-\lambda t_1} - e^{-\lambda t_2})} \quad (4.3)$$

where C is the number of counts under the peak of each activation sample. σ and γ are the cross section of a reaction which is picked up from JENDL data library and probability that a photon is emitted per decay of the isotope respectively. N_A is the Avogadro constant, and I is a weight fraction of isotope with atomic mass A . m is the mass of activation sample. ϵ and $t_2 - t_1$ are the detection efficiency and counting time.

**Figure 4.7:** Neutron flux is measured by Au wire, Al and Ni foils. The comparison of experimental data with simulation (left). The neutron flux on the place where samples are set up is shown on right.**Figure 4.8:** Factor of displacement cross section in silicon. The displacement damage cross section for 1 MeV neutrons is set as a normalizing value: $D_n(1\text{MeV}) = 95 \text{ MeVmb}$.

As shown in figure 4.7, neutron flux on diode measured by gold wire is higher nickel's, which is $1.35 \times 10^7 \text{ n/cm}^2/\text{sec}$. From the prediction of PHITS code and previous measurement with liquid scintil-

lator, the neutron flux is 6×10^7 n/cm²/sec which is about 5 times higher than the measurement of gold wire. It is possible that a part of neutron is stopped at flange or the other irradiation sample and causes the different between measurement and prediction. Thus, in this case, we trust the measurement and the total neutron fluence irradiated until the end of experiment is estimated for 10¹² n/cm².

4.3.3 Electrical property of diode

To investigate the electrical properties of diode, the turn-on voltage is measured during the irradiation, then the voltage on the operating current is predicted by fitting the V-I curve of diode. The relation of turn-on voltage (forward voltage at 100 mA) and neutron fluence is shown in figure 4.9. It reduces with neutron fluence and slope of V-I curve becomes bigger after irradiation. Temperature from the beginning of the experiment until the end of the experiment is controlled around 25.6°C.

Figure 4.10 shows the fitting for each V-I curve as the function 4.4, and fitting parameters are listed in table 4.3.

$$I = I_0(e^{-V/V_T} - 1) \quad (4.4)$$

where I and V are the diode current and voltage across the diode. V_T is thermal voltage and I_0 is the saturation current, which are assumed the fitting parameters here.

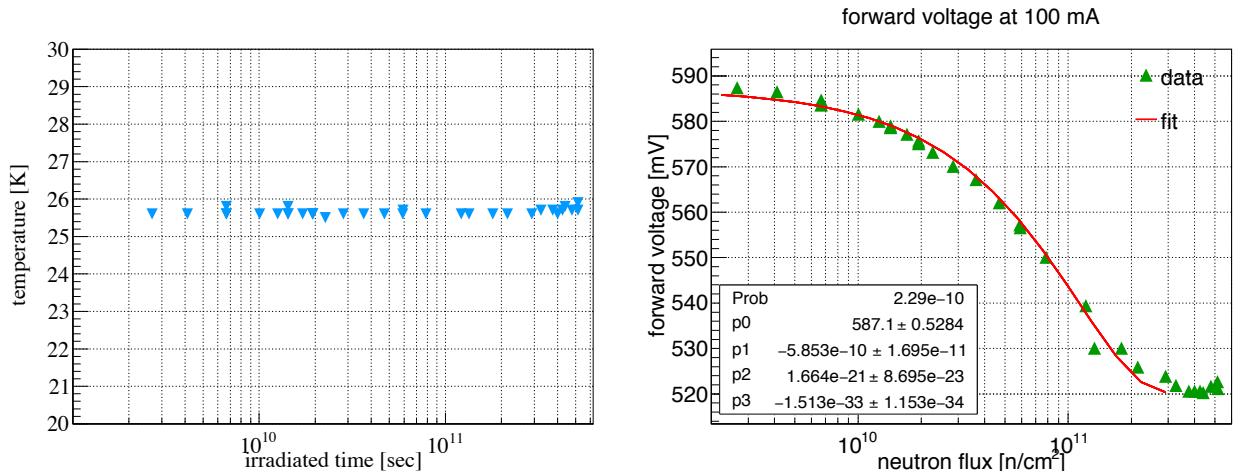


Figure 4.9: The temperature during the irradiation (left). The turn-on voltage decreases with the neutron fluence (right).

irradiated time [sec]	I_0	V_T	irradiated time [sec]	I_0	V_T
0	2.04883×10^{-7}	23.0918	500	1.97681×10^{-7}	23.034
1064	2.73547×10^{-7}	22.6213	1505	3.75863×10^{-7}	22.2084
4274	1.35549×10^{-6}	20.5447	9884	5.35415×10^{-6}	18.7640
14008	9.77339×10^{-6}	17.9582	68468	4.15128×10^{-5}	15.7070

Table 4.3: Fitting parameters for V-I curve.

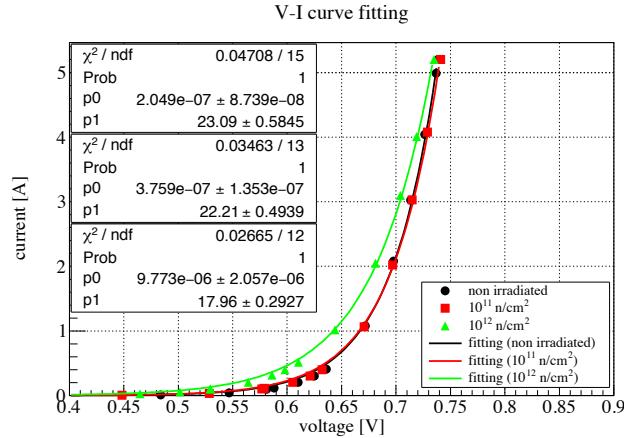


Figure 4.10: Measured turn-on voltage for quench protection diode. It is fitted as diode V-I function. Black and green point represent the original V-I curve and V-I curve after 10^{12} n/m^2 irradiation.

4.3.4 Electrical property at cryogenic temperature

Since Quench protection diode is connected to the magnets with superconducting wire, the diode must be cooled to the temperature same as superconducting wire, 4.2 K. Therefore, the electrical property at cryogenic temperature needs to be investigated as well.

For the measurement at cryogenic temperature, Quench protection diode is cooled directly by liquid nitrogen at 77 K which shown in figure 4.11. Unlike to the result of irradiation, not only the turn-on voltage but also the V-I curve increases in 77 K. In figure 4.12, the V-I curve at 300 K and 7 K can be fitted by

Temperature [K]	I_0	V_T	Temperature [K]	I_0	V_T
300	8.088×10^{-7}	21.05	77	1.969×10^{-12}	25.95

Table 4.4: Fitting parameters for forward voltage of diode at 300 K and 77 K.

Using this fitting parameters, the forward voltage at 300 K and 77 K can be predicted to 0.92 V and 1.25 V respectively. Compared with the case of room temperature, 0.33 V is increased during the cooling.



Figure 4.11: Measurement of the forward voltage of diode at cryogenic temperature.

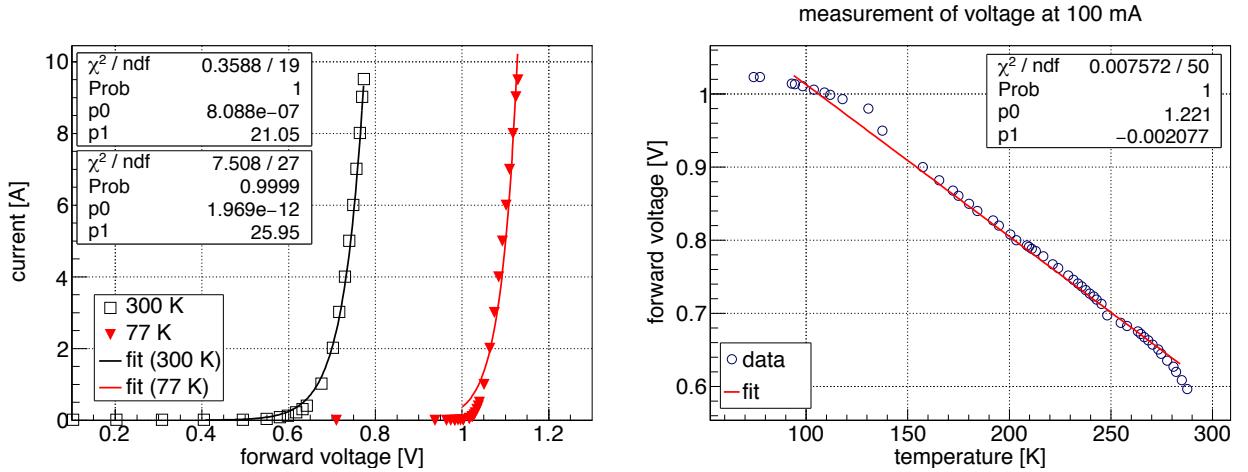


Figure 4.12: Forward V-I curve measured at room temperature and nitrogen temperature without irradiation (left). The measurement of turn-on voltage at 100 mA from room temperature to 77 K (right).

4.3.5 Forward voltage at 210 A

Since the operating current for transport solenoid is 210 A, a relation between fitted forward voltage at 210 A and neutron fluence is shown in figure 4.13. The forward voltage increases following the neutron fluence which is assumed as the exponential function.

$$V = \exp(-0.09373 + 8.832 \times 10^{-14} \cdot \phi) \quad (4.5)$$

In the case of COMET phase-II experiment, the neutron at the place of quench protection diode is about $4.02 \times 10^5 \text{ n/cm}^2/\text{sec}$ with the energy lower than 1 MeV according to PHITS simulation. As for the 280-day operation, the total neutron fluence is estimated to about $9.73 \times 10^{12} \text{ n/cm}^2$ for quench protection diode. Because the displacement cross section of silicon at low energy region is lower than the case of high energy region, irradiation for 10^{12} n/cm^2 at the range from 1 MeV reaches to the effects of $9.73 \times 10^{12} \text{ n/cm}^2$ irradiation. After 10^{12} n/cm^2 irradiation, the forward voltage at operating current is increased 0.08 V.

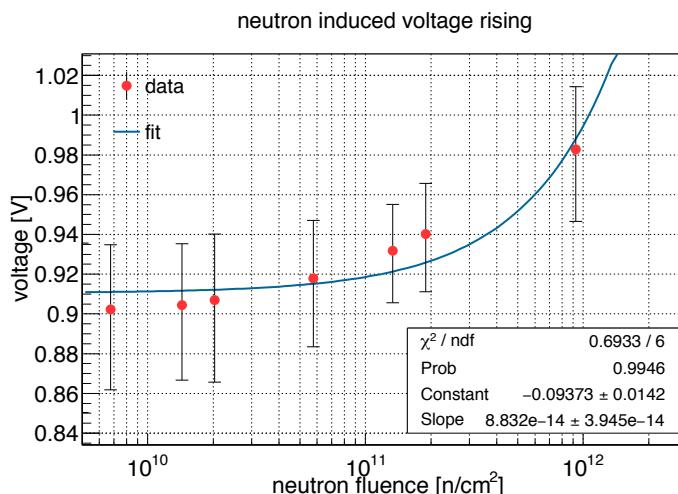


Figure 4.13: Relative increase of the forward voltage versus neutron fluence at room temperature.

Turn-on voltage at 100 mA is proportional to the temperature linearly as

$$V = 1.221 - 0.002077 \cdot T \quad (4.6)$$

Assuming the forward voltage at 210 A is also linear to the temperature, the forward voltage is predicted to 1.33 V at 4.2 K and 210 A without irradiation.

However, the forward voltage is estimated to 0.98 V at 210 A and room temperature after irradiation of 10^{12} n/cm² neutrons. Supposed its forward voltage is 1.5 V at 4.2 K and 210 A with irradiation of 10^{12} n/cm² neutrons, its heat generation is the product of forward voltage and current, which is 315 W. The temperature of quench protection diode is calculated by one dimensional finite element method. Diode consists of silicon with length of 8 cm and radius of 3 cm. Its thermal conductivity shown in figure 4.14 is taken and fitted from reference ⁽³¹⁾⁽³²⁾. Two sides of diode is cooled by liquid helium at 4.2 K, and heat is generated inside the diode. Current decays with the resistance of heater ($R = 2.38 \Omega$, $L = 254 H$). As a result, the maximum temperature will be 11 K during the cooling. Considering the worst case, the temperature of diode will increase up to 70 K in 10 sec without cooling.

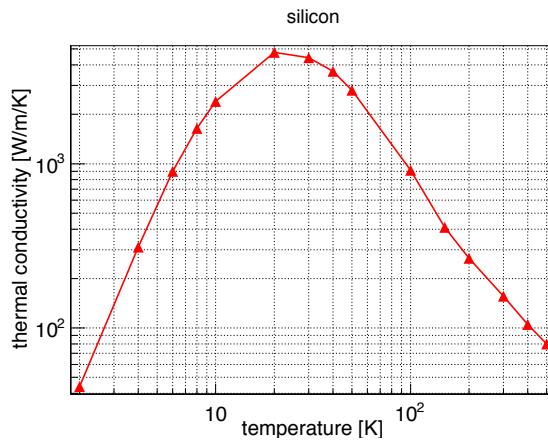


Figure 4.14: Experimental data of the thermal conductivity for silicon.

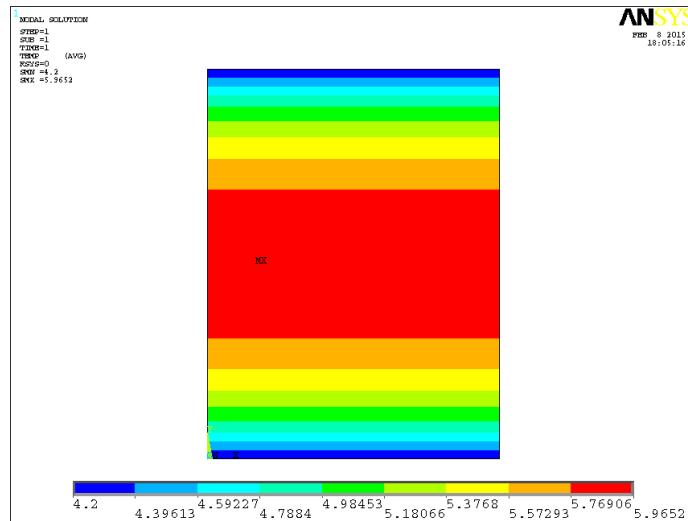


Figure 4.15: The temperature distribution of quench protection diode during the operation calculated by ANSYS.

To confirm the temperature, the temperature of quench protection diode is also estimated by ANSYS. The thermal conductivity is approached linearly between the two data points. Diode is cooled by liquid helium at 4.2 K on the top and bottom of surface. Heat generation is set to 315 W homogeneously without current decay, which is the worst circumstance for diode. As a result, its maximum temperature is about 6.0 K.

4.4 Radiation test for conductor

The degradation of electrical resistivity occurs when the stabilizer irradiated by radiation due to the production of Frenkel pairs. In the cryogenic temperature, since the displaced atoms are difficult to return to the defect, a depletion region is possible to be generated under the long time irradiation. Radiation damage effects of conductor are necessary to investigate because it may cause the overheat of superconducting coils due to the RRR decreasing and quench.

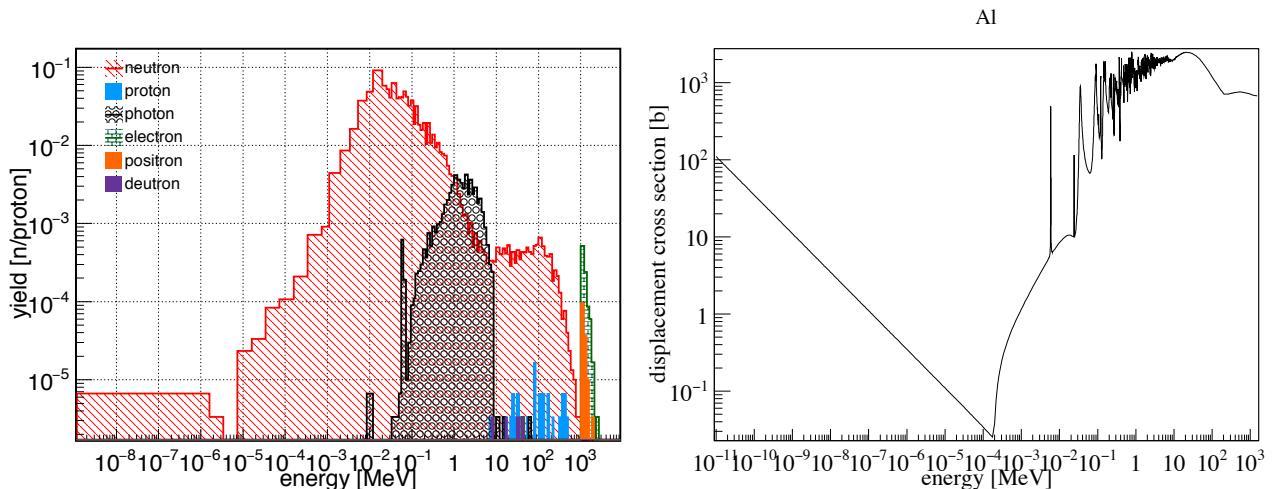


Figure 4.16: all kinds of particles which hit the CS1 coil (left). The displacement cross section of aluminium for neutron (right).

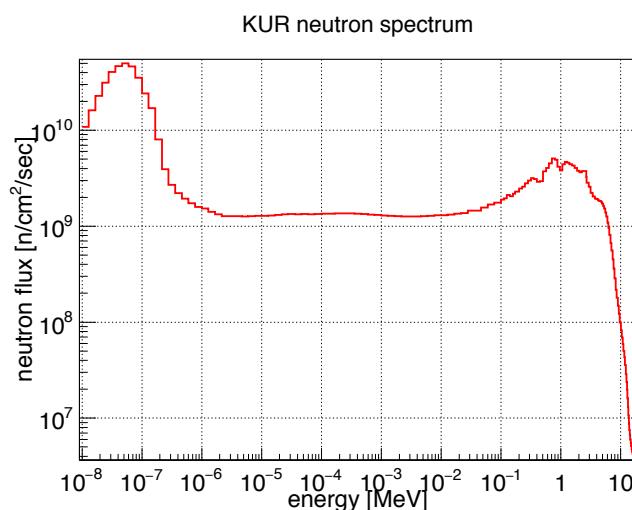


Figure 4.17: Neutron spectrum at core of KUR reactor.

To ensure what kinds of particle hit the superconducting coils, the simulation is taken by PHITS and its yield is shown in figure 4.16. Compared with the other particles, the neutron is most biggest issue for superconducting coils. Furthermore, the energy of neutron in the range from 0.1 MeV dominates the radiation damage, which is about 100 b. Thus, irradiation test of pure aluminium and copper for conductor is studied in Kyoto University Research Reactor Institute (KURRI) with reactor neutron. The pure aluminium and copper samples are set up in the one of beamline from reactor where can irradiate with cryogenic temperature. Neutron fluence is 1.4×10^{11} n/m²/sec for fast neutron in terms of the KUR study⁽³³⁾. Figure 4.17 shows the energy spectrum at center of reactor, which is higher than the place where we take irradiation test with factor 30.34.

As the result, we obverse that the resistivity increases about 0.03 nΩ·m for aluminium and 0.01 nΩ·m for copper after after 10^{20} n/m² neutron irradiation. It is also confirmed that the resistivity recover by the thermal cycle to the room temperature by 100% for Al and about 90% for Cu. Thus, due to the neutron induced resistivity, the resistivity of stabilizer at cryogenic temperature should be modified as

$$\rho(T) = \rho_0(T) + \Delta\rho_n(\Phi) + \Delta\rho_{mag}(B) \quad (4.7)$$

where $\rho_0(T)$ is the resistivity at cryogenic temperature without magnetic field and irradiation. $\Delta\rho(\Phi)$ and $\Delta\rho(B)$ are the neutron induced resistivity which depends on the neutron fluence and magnetoresistivity, respectively. Thus, the RRR during the irradiation is given by

$$RRR = \frac{\rho_{RT}}{\rho_0 + f \cdot \phi \cdot t} \quad (4.8)$$

where the factor f is shown in table 4.5.

Element	Factor [nΩ·m ³]
aluminium	3×10^{-22}
Copper	1×10^{-22}

Table 4.5: Factor for the neutron induced resistivity.

Since the aluminium with RRR of 2000 and 400 will be employed as strip and stabilizer respectively, the relation between RRR and neutron fluence is shown in figure 4.18. The RRR of aluminium strip and stabilizer for 280-day operation will become 40 or lower for capture solenoid according to the maximum neutron fluence of 2×10^{21} n/m² which is predicted by PHITS code (JAM_INCL4.6_JENDL4 model). Details of RRR degradation for main superconducting magnets of pion capture solenoid is listed in table 4.6 and figure 4.18 (right).

Name	neutron flux [n/cm ² /sec]	DPA [DPA/280 days]	RRR (Al strip)	RRR (stabilizer)
CS0	1.03×10^{10}	3.44×10^{-5}	39	36
CS1	5.12×10^9	2.97×10^{-5}	77	67
MS1	4.11×10^9	1.82×10^{-5}	95	80
MS2	1.20×10^9	8.11×10^{-6}	293	185

Table 4.6: Details of RRR degradation for pion capture solenoid after 280-day operation.

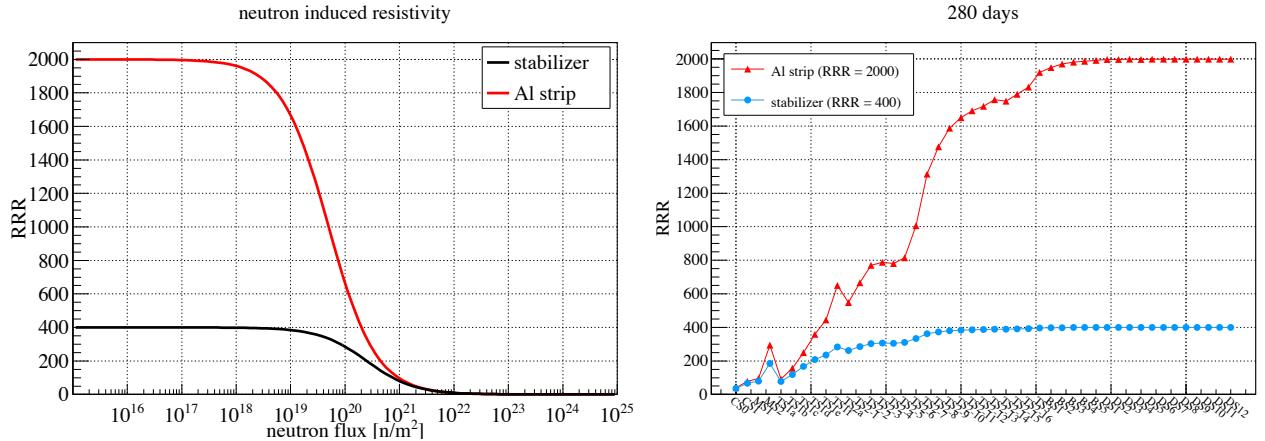


Figure 4.18: Predicted the RRR-neutron curve from the neutron irradiation test in KUR.

4.4.1 Discussion

The radiation induced damage of material can be estimated by one parameter called Displacement Per Atom (DPA) which is given by

$$DPA = \sigma_{dpa} \cdot \phi_i \quad (4.9)$$

where σ_{dpa} and ϕ_i are the displacement cross section and particle flux respectively. The displacement cross section depends on the material, and is a predicting value. Through the DPA, the radiation damage in different energy region is able to be compared. Using the neutron flux and displacement cross section of aluminium in figure 4.16 and the neutron spectrum of KUR, the DPA for KUR irradiation test can be estimated as follows.

DPA model	DPA calculation [DPA]
BCA + MD	1.0×10^{-5}
ABBN	2.6×10^{-5}
NRT (Mu2e)	2.5×10^{-5}
NRT (COMET)	3.0×10^{-5}

Table 4.7: DPA estimation for the neutron irradiation test of pure aluminium. NRT (COMET) is estimated PHITS code, and the others are predicted by Mu2e collaboration with same experiment.

Here, BCA + MD, ABBN, and NRT (Mu2e) are the estimation with different DPA models from Mu2e collaboration. Our estimation is about 3.0×10^{-5} DPA. J.A. Horak et al. measure the neutron induced resistivity of metals, as a result, the induced resistivity for aluminium is $382.3 \text{ n}\Omega\cdot\text{cm}$ per 5.6×10^{-4} Frenkel pair ⁽³⁴⁾. In addition, reference ⁽¹⁾ shows many results of Frenkel pair resistivity of aluminium in table 4.8. It shows that J.A. Horak's data is the maximum value compared with the other references. The relation between DPA and RRR by using these prediction from experimental data is shown in 4.19.

For the COMET phase-II experiment, the DPA peak of CS1 coil is about 1.1×10^{-5} DPA for 280-day operation from the prediction of PHITS code with NRT model. The aluminium strip will not degrade lower than 100 for 280-day operation from this estimation. However, the design has to consider the worst situation for superconducting coils, the uncertainty of the prediction from neutron DPA and neutron fluence needs to be reconsidered.

ρ_{FP} [$\mu\Omega \cdot m$]	Type	Reference
3.9±0.6	Exp D	(35)
4.2±0.8	Exp D	(36)
3.2±0.6	Exp D	(37)
3.4	Exp T(p)	(38)
1.32	Exp T(p)	(39)
1.35	Exp T(p)	(40)
4.0±0.6	Evl E	(41)
4.2±0.5	Evl E	(42)
4.3	Evl S	(43)
6.8	Adp	(34)
2.0±1.0	KEK	

Table 4.8: The Frenkel pair resistivity ρ_{FP} of aluminium. Methods of the data derivation: Exp D is X-ray diffraction method, Exp T is the threshold energy determination for electron irradiation of single crystals at low temperature. Exp T(p) is for the electron irradiation of polycrystals, Evl S is the estimation made with the help of the systematics. Adp is the data adopted by the authors of cited work ⁽¹⁾. KEK is the neutron irradiation test in KUR.

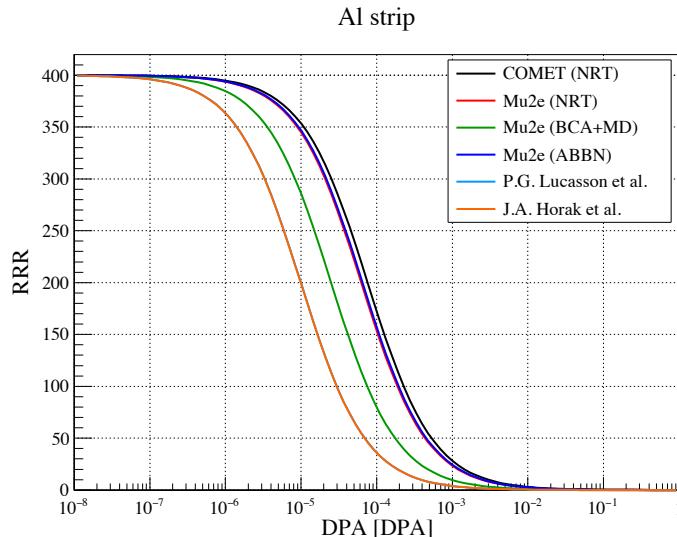


Figure 4.19: A relation between DPA and RRR. Orange line is the maximum radiation induced resistivity, and the others are the prediction of KUR irradiation test with different displacement cross section model.

Chapter 5

Thermal stability and quench protection of superconducting solenoids

Radiation will not only generate the heat inside the coils but also cause the damage of superconducting material, which is possible to lead the degradation of cooling strip. Due to the conduction cooling, the consequence of aluminium strip degradation is over-heat and quench, which are the major issue for COMET superconducting magnets. Here, the thermal property of the most dangerous coil, CS1 coil, is analysed.

5.1 Quench estimation

Quench plays a significant role in superconducting magnet performance. Since over 2000 A current is flowing inside the superconducting wire, the resistance of NbTi will increase suddenly when superconducting state turns to the normal state. If this current cannot decay quickly, the quenching temperature will increase highly and cause the unrecoverable burn-out of superconducting wire.

In Sep. 2008, the accident caused by quench occurred in LHC superconducting magnet. As the consequence, the bus bar is burned out in the interconnect. It costs half year to fix the superconducting magnet in LHC accident. As for the COMET experiment, it is not possible to do the maintenance during the experiment due to the residual radiation of the solenoid. Thus, the quench estimation is necessary to know the temperature after magnet quenched.

One parameter called MIITs is estimated for the worst situation of quench. To calculate the MIITs, the specific heat and thermal conductivity from cryogenic temperature to room temperature is required. The specific heat and electrical resistivity of copper, aluminium and NbTi are fitted with experimental data correctly.

5.1.1 Specific heat

There are two models for lattice specific heat, the Debye model and Einstein model⁽⁴⁴⁾. In Debye model, the phonon energy is written as

$$U = \int d\omega D(\omega) \left(\frac{\hbar\omega}{e^{\hbar\omega/\tau} - 1} \right) \quad (5.1)$$

where $D(\omega)$ is the density of state, ω is the wave number. The density of state is described as $V\omega^2/2\pi^2v^3$, than the specific heat can be solved as

$$C_V = \frac{\partial U}{\partial T} = 9Nk_B\left(\frac{T}{\theta}\right)^3 \int_0^{x_D} dx \frac{x^4 e^x}{(e^x - 1)^2} \quad (5.2)$$

where θ is the Debye temperature and equals to $\frac{\hbar v}{k_B} \cdot (\frac{6\pi^2 N}{V})^{1/3}$. x_D comes from the cut off wave number and equals to θ/T . In the cryogenic temperature which is less than the Debye temperature ($T \ll \theta$), The specific heat can be approach as

$$C_V \cong \frac{12\pi^4}{5} Nk_B\left(\frac{T}{\theta}\right)^3 \quad (5.3)$$

Debye model describes the contribution of lattice on specific heat. However, the contribution of electrons is treated approximately as the free-electron model. The electronic contribution to specific is

$$C_V = \gamma T = \frac{1}{2}\pi^2 Nk_B \frac{T}{T_F} \quad (5.4)$$

where $\gamma = \pi^2 D(\epsilon_F) k_B^2 / 3$ is called the Sommerfeld constant and T_F is the Fermi temperature, respectively. Combining the lattice and electronic contribution, the specific heat can be written as

$$C_V \cong \gamma T + \beta T^3 \quad (5.5)$$

As for the aluminium, the Debye temperature is 428 K and the Sommerfeld constant is 1.35 mJ/mol/K². However, it does not agree with the experimental data. Therefore, the specific heat of aluminium is fitted as polynomial function with different range of temperature. The details is shown in table 5.1 and the fitting result is given in figure 5.1. The experimental data is given in reference ⁽⁴⁵⁾. The fitting function does not agree with the experimental in the range from 0 K to 2 K. Because the initial temperature is 4.5 K, it will not affect the calculation here.

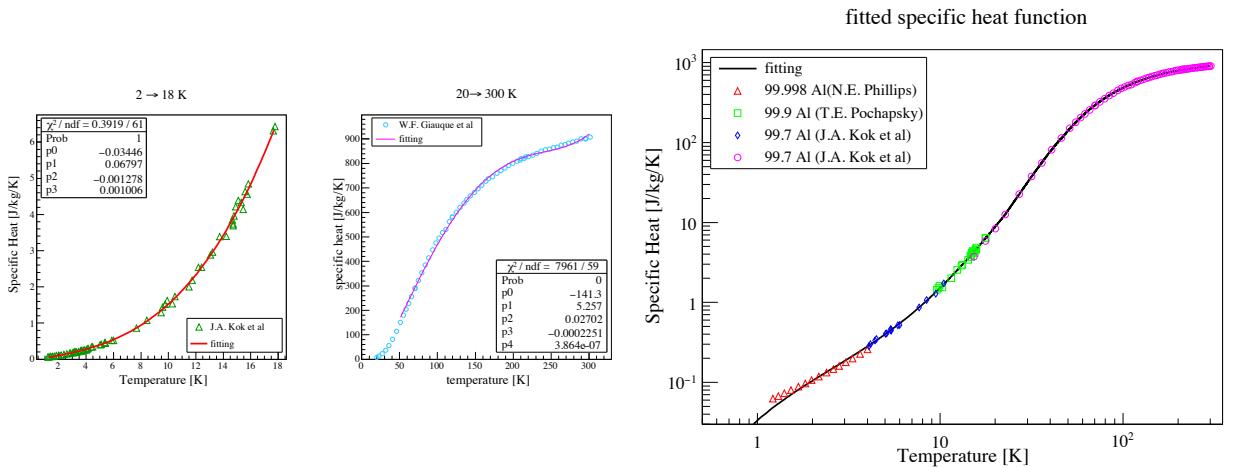


Figure 5.1: The specific heat of aluminium is fitted from the range from 4.5 K to 22.67 K, 22.67 K to 46 K and 46 K until 350 K respectively.

$$C = p_0 + p_1 \cdot T + p_2 \cdot T^2 + p_3 \cdot T^3 \quad (5.6)$$

T_{lower}	T_{upper}	p_0	p_1	p_2	p_3
0	22.67	-0.03446	0.06797	-0.001278	0.001006
22.67	46	7.88×10^{13}	6.93201	-0.07139	46.4363
46	350	6.273517	-0.5469	0.000925	-156.932

Table 5.1: The details for fitting the aluminium specific heat.

The Copper specific heat is fitted as polynomial function with the experimental data in reference ⁽⁴⁵⁾. Here p_0 and $p_2 \cdot T^2$ are the correction of specific heat. $p_1 \cdot T$ and $p_3 \cdot T^3$ are the lattice specific heat and electronic specific heat, respectively. The fitting parameters are listed in table 5.2 and the result is shown in figure 5.2.

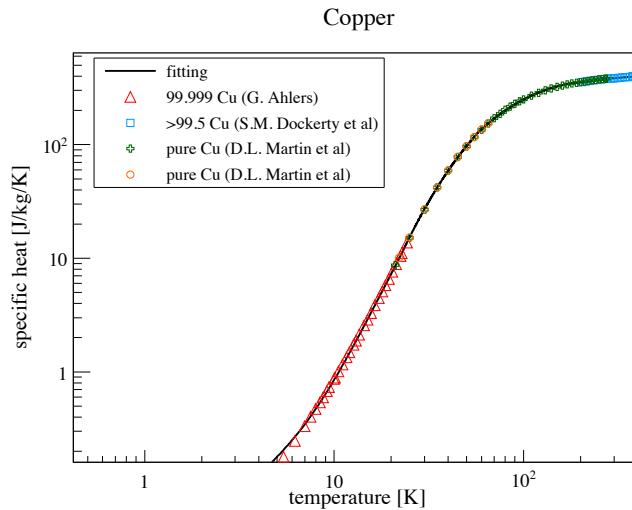


Figure 5.2: Fitted copper specific heat curve.

T_{lower}	T_{upper}	p_0	p_1	p_2	p_3
0	23	-0.104251	0.0832507	-0.0118811	0.00131157
23	55	25.2218	-3.41019	0.144053	-0.000939968
55	250	-158.094	6.40481	-0.0273429	4.08432×10^{-6}

Table 5.2: Fitting parameters for copper specific heat.

T_{lower}	T_{upper}	p_0	p_1	p_2	p_3	p_4
0	$T_c(B)$	0	$64 \times B$	0	49.1	0
$T_c(B)$	26.358	0	928	0	16.24	0
28.358	50.99	41383	-7846.1	553.71	11.9838	-0.2177
50.99	165.8	-1.53×10^6	83022	-716.3	2.976	-0.00482
165.8	496.54	1.24×10^6	13706	-51.66	0.09296	-6.29×10^{-5}
496.54	1000	2.45×10^6	955.5	-0.257	0	0

Table 5.3: The fit parameters for NbTi specific heat.

The NbTi specific heat is same to the CUDI parameters which is given in ROXIE's database ⁽⁴⁶⁾. The fit

parameters are reported in table 5.3.

$$C = \rho_{NbTi} \cdot (p_0 + p_1 \cdot T + p_2 \cdot T^2 + p_3 \cdot T^3 + p_4 \cdot T^4) \quad (5.7)$$

5.1.2 Electrical resistivity

Because of collisions of free electrons with impurities, lattice and phonons, it generates the electrical resistivity. The electrical resistivity is obtained from the free electron gas model.

$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau} \quad (5.8)$$

In the room temperature, the collisions of electrons with phonon is dominated the electrical resistivity. However, only the collisions of electrons with impurities and defects occurs in the cryogenic temperature. Therefore, the total resistivity can be written as

$$\rho = \rho_L + \rho_i \quad (5.9)$$

where ρ_L is the resistivity from the phonon collisions, ρ_i is the resistivity from the scattering of electron waves with impurities. ρ_L will become 0 when the temperature is close to 0 K. There are two ways to fit the electrical resistivity.

- Fit the electrical resistivity directly.
- Fit the thermal conductivity first, then convert it with Wiedemann-Franz law.

In reference ⁽⁴⁷⁾, it gives the function to fit the copper electrical resistivity. Its function contains temperature and RRR dependence like

$$\rho(T, RRR) = \rho_0 + \rho_i + \rho_{i0} \quad (5.10)$$

ρ_0 and ρ_i are same to the two terms in equation 5.9, which indicate the resistivity in cryogenic temperature and room temperature, respectively. ρ_{i0} is the correction term of ρ_0 and ρ_i . Hence ρ_0 depends on the material purity and structure, while ρ_i depends on the temperature. Each term are given by

$$\rho_0 = \frac{\rho_{RT}}{RRR} \quad (5.11)$$

$$\rho_i = \frac{p_1 \cdot T^{p_2}}{1 + p_1 \cdot p_3 \cdot T^{(p_2-p_4)} \cdot \exp\left\{-\left(\frac{p_5}{T}\right)^{p_6}\right\}} \quad (5.12)$$

$$\rho_{i0} = p_7 \cdot \frac{\rho_i \cdot \rho_0}{\rho_i + \rho_0} \quad (5.13)$$

As for the copper, the electrical resistivity at room temperature is $1.553 \times 10^{-8} [\Omega \cdot m]$, and the same fitting parameters is employed in MIITs simulation which is listed in table 5.4.

p_1	p_2	p_3	p_4	p_5	p_6	p_7
1.171×10^{-17}	4.49	3.841×10^{10}	1.14	50	6.428	0.4531

Table 5.4: NIST fitting parameters for copper electrical resistivity.

On the other hand, the electrical resistivity is able to be achieved from the thermal conductivity. Reference⁽⁴⁸⁾ mentions the predicting of thermal conductivity which based on a set of semi-empirical equations presented by Hust et al.⁽⁴⁹⁾. The thermal conductivity is written as

$$K = \frac{1}{W_0 + W_i + W_{i0}} \quad (5.14)$$

where W_0 and W_i represent the electron-defect and electron-phonon interactions respectively, which is similar to the NIST equation. W_{i0} is necessary to produce acceptable fits. Each term is given as follows.

$$W_0 = \frac{\beta}{T} \quad (5.15)$$

$$W_i = \frac{p_1 \cdot T^{p_2}}{1 + p_1 p_3 T^{(p_2+p_4)} \exp\left\{-\left(\frac{p_5}{T}\right)^{p_6}\right\}} + W_c \quad (5.16)$$

$$W_{i0} = p_7 \cdot \frac{W_i W_0}{W_i + W_0} \quad (5.17)$$

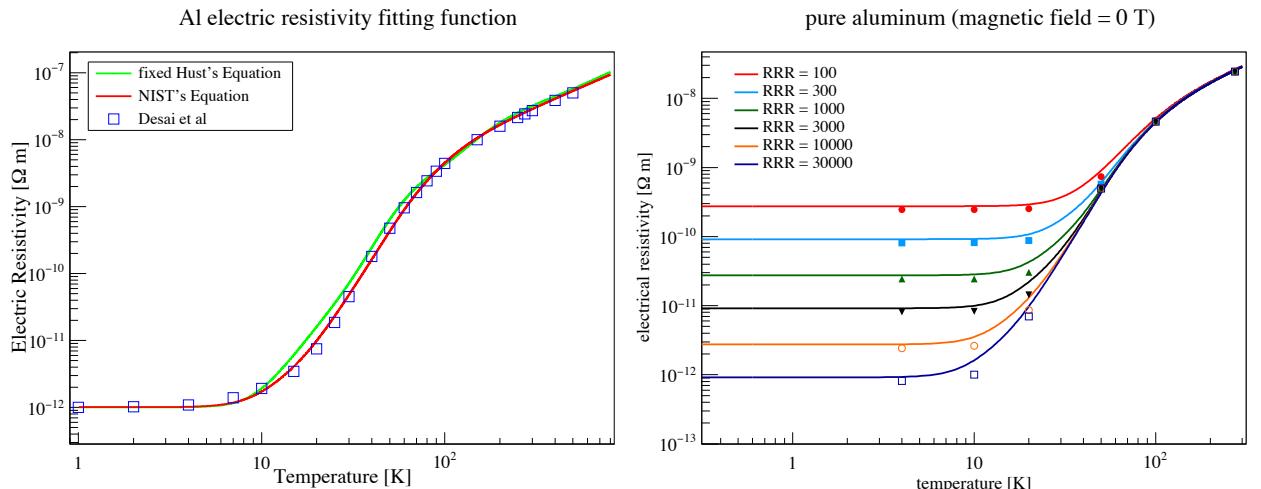
The parameter β is the function of the residual resistivity ρ_0 , which can be represented as Wiedemann-Franz law.

$$\beta = \frac{\rho_0}{L_0} = \frac{\rho_{RT}}{L_0} \cdot \frac{1}{RRR} \quad (5.18)$$

The W_c is correction parameter for only the pure aluminium which is compared with experimental data.

$$W_c = -0.0005 \cdot \ln\left(\frac{T}{330}\right) \cdot \exp\left\{-\left(\frac{\ln(T/380)}{0.6}\right)^2\right\} - 0.0013 \cdot \ln\left(\frac{T}{110}\right) \cdot \exp\left\{-\left(\frac{\ln(T/94)}{0.5}\right)^2\right\} \quad (5.19)$$

As for the pure aluminium, the fitting parameter for NIST and Hust equation is listed in table 5.5 and 5.6 respectively.



(a) Fixed the Hust's equation and NIST's equation of aluminium electric resistivity.

(b) Comparison of the aluminium electric resistivity with different RRR.

Figure 5.3: Fitted aluminium resistivity.

p_1	p_2	p_3	p_4	p_5	p_6	p_7
1.671×10^{-17}	4.36	2.841×10^{10}	1.18	64	4.428	1.2031

Table 5.5: NIST fitting parameter for pure aluminium.

p_1	p_2	p_3	p_4	p_5	p_6	p_7
4.716×10^{-8}	2.446	623.6	-0.16	130.9	2.5	0.8168

Table 5.6: Hust fitting parameter for pure aluminium.

Figure 5.3a shows the comparison of the Hust and NIST equation with experimental data by using these fitting parameters for pure aluminium ($\text{RRR} = 10000$)⁽⁵⁰⁾. The fixed Hust equation is higher than the experimental data in the range from 10 K to 100 K. Hence the Hust equation still needs to be fixed in future.

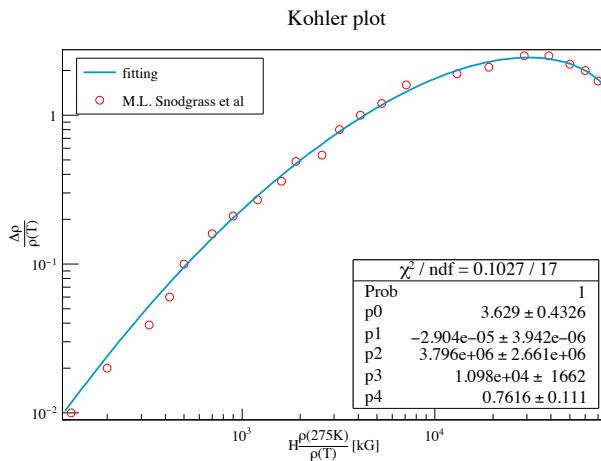
In figure 5.3a, only the aluminium with 10000 of RRR is compared to the fitting function. While, the pure aluminium with different RRR must be agreed to the fitting function and it is shown in figure 5.3b. (The experimental data is obtained from Prof. Nakamoto.) The modified NIST equation is employed in here. The fitting function is higher than experimental data about factor 0.2, and its difference is only shown in low temperature. As for the high temperature, it has good agreement with data.

Magnetoresistivity

Due to the external magnetic field, the mean free path will be shortened and the collisions between electrons and phonon is increased, which leads to the electrical resistivity increasing. The Kohler's plot is the most common expression of the magnetoresistivity. The Corruccini⁽⁵¹⁾ gives the equation to calculate the magnetoresistivity in reference^{(52) (53)}. The electrical resistivity in the high field can be written as

$$\rho(B, T) = \rho(0, T) + \rho(0, T) \cdot \frac{h^2 \cdot (p_0 + p_1 \cdot h)}{p_2 + p_3 \cdot h + p_4 \cdot h^2} \quad (5.20)$$

$$h = B \cdot \frac{\rho_{ref}}{\rho(0, T)} \cdot 10 \quad (5.21)$$


Figure 5.4: Fitted the Kohler plot with experimental data for pure aluminium.

where ρ_{ref} is $2.75 \times 10^{-8} [\Omega \cdot m]$ and $\rho(0, T)$ is the electrical resistivity without magnetic field. As for the aluminium, the fitting parameters are listed in table 5.7 and the fitted Kohler plot is shown in figure 5.4.

p_0	p_1	p_2	p_3	p_4
3.62857	-2.90419×10^{-5}	3.79649×10^6	10975.9	0.761609

Table 5.7: Fitting parameters for pure aluminium magnetoresistivity.

As for the copper, the same fitting equation for copper magnetoresistivity in ROXIE is employed here. It is fitted as the polynomial function which is given as follows.

$$\rho(B, T) = \rho(0, T) + \rho(0, T) \cdot 10^h$$

$$h = p_0 + p_1 \cdot \log_{10}x + p_2 \cdot (\log_{10}x)^2 + p_3 \cdot (\log_{10}x)^3 + p_4 \cdot (\log_{10}x)^4$$

$$x = \frac{1.553 \times 10^{-8} \times B}{\rho(0, T)}$$

p_0	p_1	p_2	p_3	p_4
-2.662	0.3168	0.6229	-0.1839	0.01827

Table 5.8: Fitting parameters for copper magnetoresistivity.

5.1.3 Thermal conductivity

Thermal conductivity can be obtained from the electric resistivity by using Wiedemann-Franz law, which is written as

$$k = \frac{L_0 \cdot T}{\rho} \quad (5.22)$$

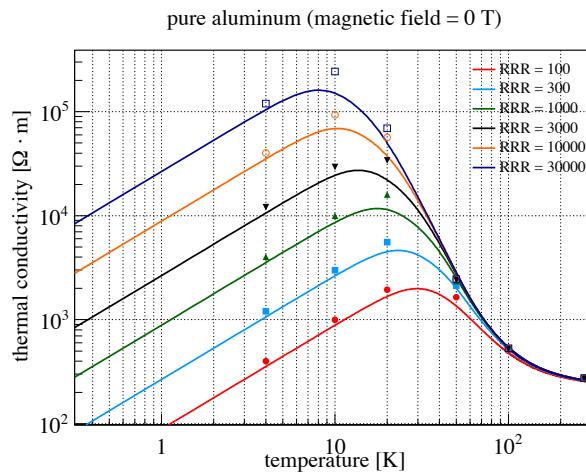


Figure 5.5: Thermal conductivity of aluminium with different RRR.

where ρ is the resistivity, and Lorentz constant L_0 is defined as

$$L_0 = \frac{\pi^2}{3} \left(\frac{k_B}{e} \right)^2 = 2.44 \times 10^{-8} W \cdot \Omega / K^2 \quad (5.23)$$

The thermal conductivity used in thermal estimation is shown in figure 5.5.

5.1.4 MIITs

Because over 2000 A current will generate the joule heat inside the coils, the wire has a risk of burn-out after a quench. Usually, there is external sump resistor to dissipate the stored energy in magnets. Once the quench has been detected, the power supply will be switched off and the current decays due to the dump resistor. The current decay can be presented as

$$I(t) = I_0 \cdot \exp\left(-\frac{R}{L}t\right) \quad (5.24)$$

where L is the inductance of magnet. The speed of current decay depends on the inductance and resistance and the current can totally decay in the time of $\tau = L/R$. The quenching temperature can be restrained if the current decays quickly.

Due to the joule heat $I^2(t)R(T)$, a relation between the time dependence of the current after quench and the maximum temperature can be established. The temperature change during the a time interval dt can be written as

$$\frac{C(T)}{R(T)}dT = I^2(t)dt \quad (5.25)$$

where specific heat $C(T)$ and resistance $R(T)$ depend on the temperature, and the current decay shown in equation 5.24 $I(t)$ depends on the time. Separating and integrating the time term and the temperature term, one relation can be obtained

$$f(T_{max}) = \int_0^\infty I^2(t) \cdot dt = \int_{T_0}^{T_{max}} \frac{C(T)}{R(T)} \cdot dT \quad (5.26)$$

The unit of the time integral over the square of the current is usually defined as $10^6 \cdot A^2 \cdot sec$, which sometimes is called as "MIITs". As for COMET superconducting magnet, the conductor consists of NbTi superconducting filament, copper matrix and aluminium stabilizer. Thus, the specific heat $C(T)$ and resistance $R(T)$ need to average like

$$C(T) = \gamma_{Cu} \cdot C_{Cu}(T) \cdot A_{Cu} + \gamma_{NbTi} \cdot C_{NbTi}(T) \cdot A_{NbTi} + \gamma_{Al} \cdot C_{Al}(T) \cdot A_{Al} \quad (5.27)$$

$$R(T) = \left(\frac{A_{Cu}}{\rho_{Cu}(T)} + \frac{A_{Al}}{\rho_{Al}(T)} \right)^{-1} \quad (5.28)$$

where A and γ are the cross section and density respectively. Noting that the unit of $C(T)$ must be $[J/K/m^3]$ and the unit of each material's specific heat is $[J/K/kg]$. The reason why the resistance of NbTi is not included because the resistance of NbTi becomes so high that the current only flows into aluminium stabilizer and copper matrix when quench happens. The parameters used in MIITs calculation is listed as follows.

I ₀	R	L	τ	T ₀	Al:Cu:NbTi
2700 A	0.185 Ω	12.69 H	68.65 sec	4.5 K	7.3:1:1

Table 5.9: The parameters for MIITs estimation of CS1.

The resistance in table 5.9 is the resistance of dump resistor, which is calculated by assuming the allowable voltage of 500 V. The MIITs curve with 5.5 Tesla for different RRR is shown in figure 5.6. MIITs is calculated from the integral time term of current decay in equation 5.26 using the parameters in table 5.9, which is $250 \text{ MA}^2\text{sec}$ and corresponds to 260 K (RRR=100). RRR is assumed to about 100 after 90-day irradiation in COMET experiment in the worst case.

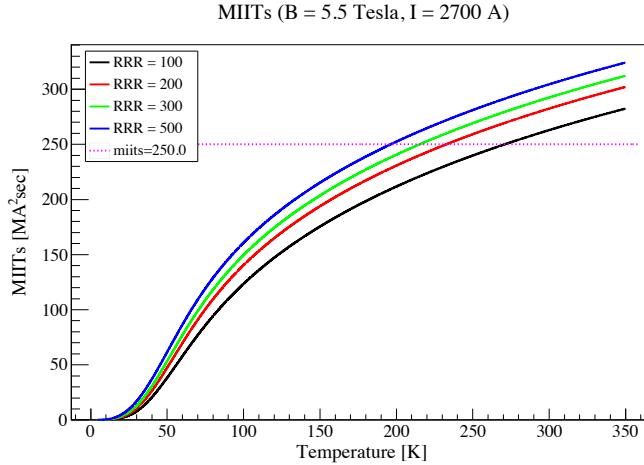


Figure 5.6: The MIITs calculation of CS1 coils with 5.5 Tesla magnetic field and 2700 A current.

Figure 5.7a shows the prediction of maximum temperature from MIITs in different magnetic field. Even the magnetic field becomes 0 Tesla, the maximum temperature is still close to 200 K if RRR drops to 100. Due to the conduction cooling, 200 K is quite high for the superconducting magnets. Following the temperature increasing, it is possible to lead to the consequence of unstuck insulation tape. To avoid the overheat after quench, the voltage should be increased. Figure 5.7b gives the relation between voltage and MIITs predicted maximum temperature. To reduce the maximum temperature after quench, the MIITs should be restrained under $200 \text{ MA}^2\text{sec}$ and the voltage must be higher than 600 V.

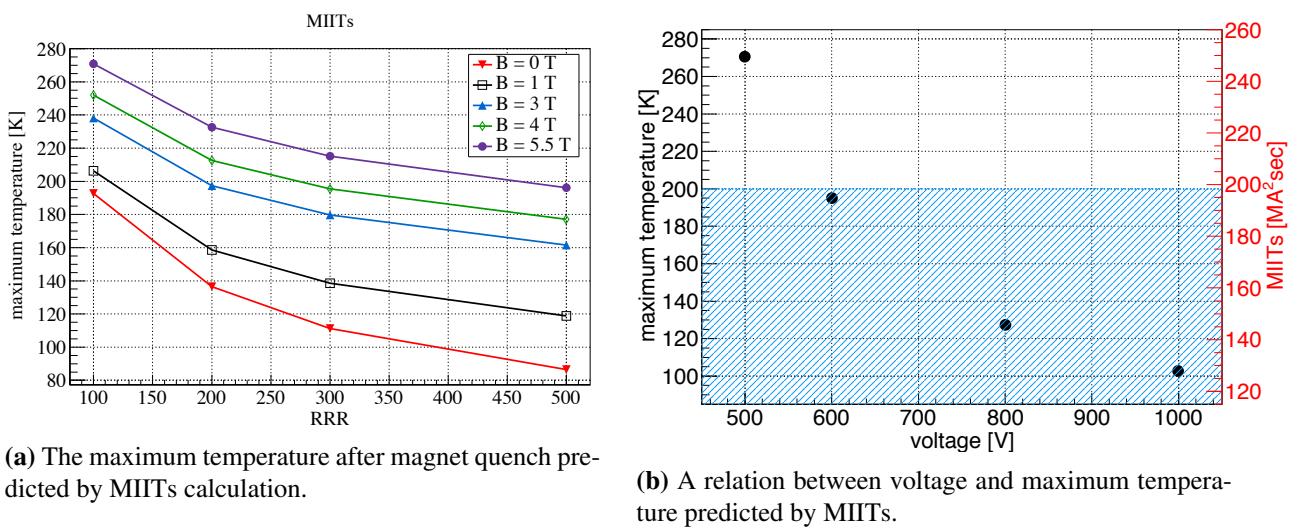


Figure 5.7: Improvement of power supply according to the results of MIITs estimation.

However, MIITs can only estimate the maximum temperature of the worst situation. As the case of

COMET superconducting magnets, there are aluminium stabilizer is enclosed around the coils. The current decay will be more quickly for the real case due to the aluminium stabilizer self resistance. Furthermore, the magnetic field decay is not considered in MIITs estimation as well. Thus, the quench simulation is necessary for estimation of the temperature after quench.

5.2 Cooling estimation

The thermal conductivity of aluminium stabilizer and copper matrix will degrade owing to the radiation, which leads to the overheating of the superconducting coil during the operation. To analyse the thermal stability of superconducting magnet, a thermal simulation code for COMET superconducting magnet has been developed. This code bases on the three dimensional equation⁽⁵⁴⁾, which is written as

$$\gamma C \frac{dT}{dt} = k_x \frac{\partial^2 T}{\partial x^2} + k_y \frac{\partial^2 T}{\partial y^2} + k_z \frac{\partial^2 T}{\partial z^2} + W \quad (5.29)$$

where γ , C, k and W are material's density, specific heat, thermal conductivity and heat generation, respectively. The specific heat only affects the speed of temperature change, and maximum temperature change during the cooling is affected by the term of thermal conductivity and heat generation. Considering the insulation tape has the worse thermal conductivity, the thermal conductivity of conductor is able to be approached like

$$\frac{\sum_{i=1}^{\infty} l_i}{k} = \sum_{i=1}^{\infty} \frac{l_i}{k_i} \quad (5.30)$$

The temperature margin for COMET superconducting magnets can be solved by using the result of energy deposition and neutron flux from PHITS code.

5.2.1 Geometry

The geometry which used in thermal analysis is shown in figure 5.8 and 2.2. CS1 coils have 9 layers and 270 turns totally, and aluminium strips with RRR of 2000 are inserted into each layer.

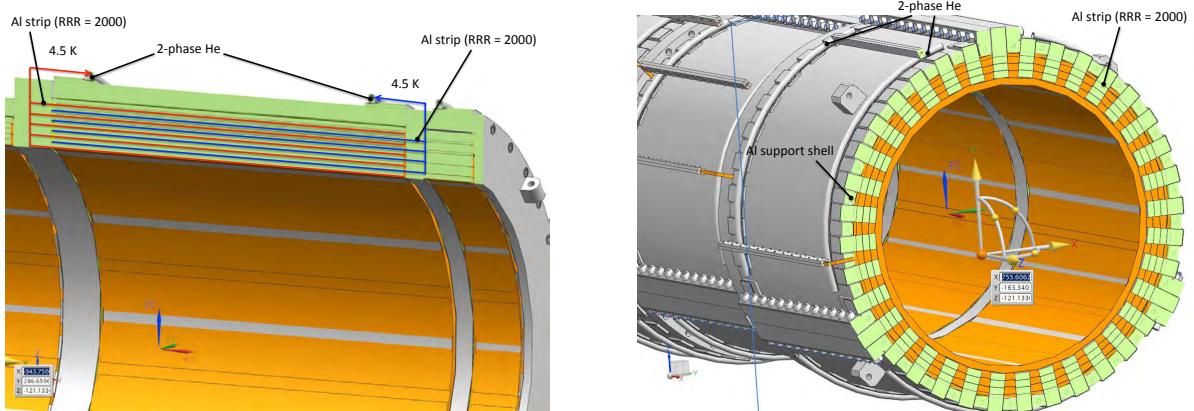


Figure 5.8: Details of the geometry used in magnet thermal simulation.

Here it is approached to 270 conductors along the axis 19 layers (conductor + strip) along the radius. Each aluminium strip is inserted from single side due to the GFRP spacer in the edge of coil. Aluminium

strips with long 25 cm are connected to the 2 phase LHe cooling pipe with 4.5 K. On the azimuthal direction, the thermal conductivity of aluminium is defined with factor 0.7 because there is resin between the aluminium strips. On the top of coils, one aluminium supporter with 5 cm suppresses the coils from hoop stress. The magnetic field is set to 5.5 Tesla in whole coil.

5.2.2 Heat generation

During the operation, heat generation inside the superconducting magnet is easy to cause the quench of magnet. Several main heat generations are listed in table 5.10. Thermal radiation is the absorption or emitting of the electromagnetic wave energy. AC loss is occurred by the alternating magnetic field. Because the prompt radiation contributes 172.5 W in heat load, the others can be neglected in simulation.

	Heat load [W]		Heat load [W]
Thermal radiation	4.16	Supporter	4.41
Current lead	2.00	Residual gas	0.20
AC loss	0.50	Radiation	172.48

Table 5.10: Heat generation for the capture solenoid.

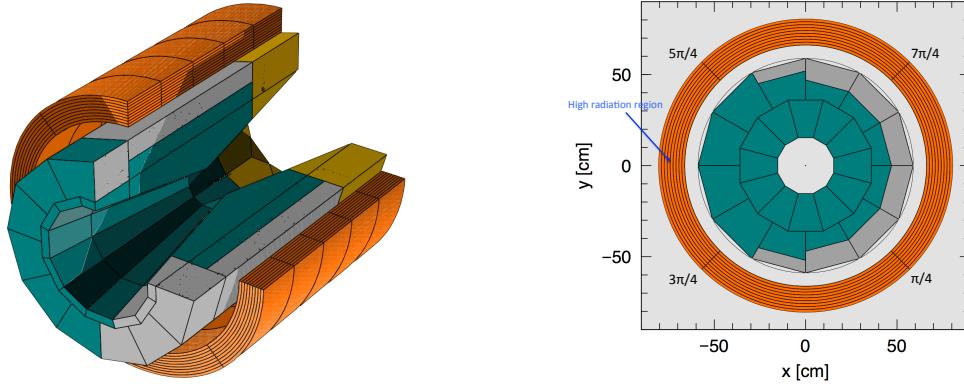


Figure 5.9: Mesh cutting of CS1 coils.

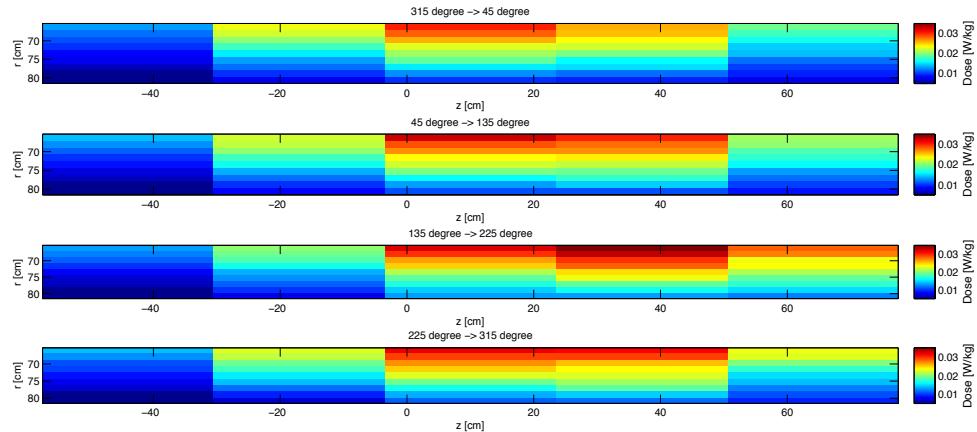


Figure 5.10: Calculated the energy deposition distribution with realistic radiation shield geometry.

The heat load (energy deposition) is calculated by PHITS code with realistic shielding geometry in figure 5.9. Superconducting coils consist of NbTi, Al and Cu with 3.80 g/cm^3 are cut for 5 parts along the z direction, 4 parts along the azimuthal direction and 9 parts along the radius direction in calculation. As the result shown in figure 5.10, the peak is located on the place 40 cm far from the production target along the z direction, and from $3\pi/4$ to $5\pi/4$ along the azimuthal direction. The energy deposition of inner layer is about 3 times higher than the outer layer's.

5.2.3 Maximum temperature of cooling

The parameters used in thermal analysis for CS1 coil is listed in table 5.11. Due to asymmetric distribution of radiation in CS1 coil, heat generation map and neutron induced RRR map calculated by Monte Carlo code are utilized. The relation between neutron fluence ($E_n > 0.1 \text{ MeV}$) and RRR degradation is employed in this simulation because it predicts the worst situation. Thermal conductivity is converted by Wiedemann-Franz law from the electrical resistivity. The specific heat only affects the speed of heat transfer and thermal conductivity has no remarkable change in range from cryogenic temperature to about 20 K for both copper and aluminium, therefore the specific heat and thermal conductivity are set to constant which does not change with temperature here. However, the thermal conductivity of insulation tape is still unknown, it is supposed to be 0.01 W/m/K which is worse than the Kapton tape. Its thermal conductivity although depends on the temperature, setting it to one constant is the worst circumstances for CS1 coil.

Al strip thickness	1mm	Conductor cross section	$15 \text{ mm} \times 4.73 \text{ mm}$
Insulation tape thickness	$0.075 \text{ mm} \times 2$	Insulation thickness	0.8 mm
Al:Cu:NbTi	7.3:1:1	Supporter shell thickness	50 mm
End strip to helium	25 cm	Al density	2700 kg/m^3
Conductor density	4000 kg/m^3	Insulation tape density	1420 kg/m^3
Tape thermal conductivity	0.01 W/m/K	Al specific heat	0.225 J/kg/K
Conductor specific heat	0.16 J/kg/K	Tape specific heat	1090 J/kg/K

Table 5.11: Parameter setting in thermal estimation of CS1 coils.

Due to the finite difference method used in thermal simulation, the result will grow up to infinite if the geometry's mesh and time mesh is not proper in calculation. Here, as for the issue of meshing, the proper mesh for thermal analysis in this time is founded and listed in table 5.12.

Axis	Meshing	Length
z	45	135 cm
ϕ	4	503 cm
r	19	14.4 cm
time	0.0001 sec	

Table 5.12: Meshing set in thermal analysis of CS1 coils.

Superconducting coils have a risk of quench when its temperature closes to the current sharing temperature. Thus, the peak of temperature must be suppressed under the current sharing temperature for securing

the coils from quench. According to K. H. Mess's book, the current sharing temperature is given by

$$T_{cs} = T_0 + (T_c - T_0) \cdot \left(1 - \frac{I}{I_c(T_0)}\right) \quad (5.31)$$

where T_0 is the initial temperature, I is the operating current, and $I_c(T_0)$ is the critical current in initial temperature. The current sharing for COMET CS1 coils is 6.5 K (Dr. Sasaki's work).

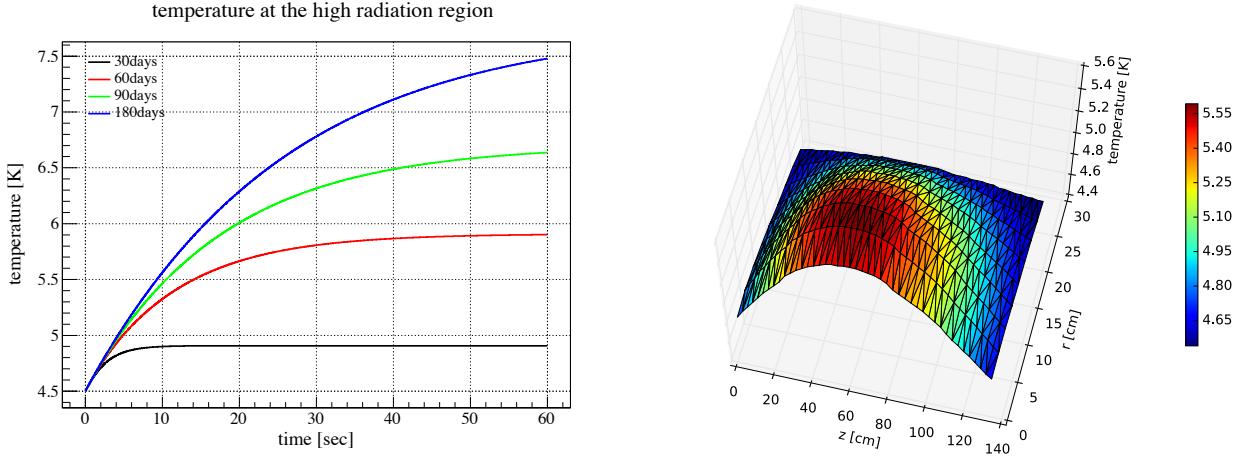


Figure 5.11: Calculated the temperature rising during the operation.

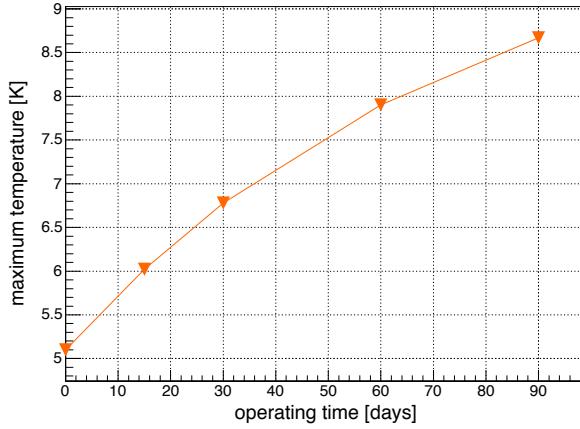


Figure 5.12: Maximum temperature for the original coil structure.

Figure 5.11 gives the temperature distribution of 9 layers of conductor and the temperature change with time. As for figure shown on the left, following the operation time increases, it needs more time to become stable. The temperature peak is in the middle of CS1 and innermost layer. Maximum temperature given in figure 5.12 and 5.13 is the peak temperature in CS1 coils. As for the original design, which shown in figure 5.12, the peak temperature will reach to the current sharing temperature after 25-day operation. Considering the simulation uncertainty, only 10-day continuous operation is allowed for phase-II experiment. Since the superconducting magnet needs almost 1 month to be cooled down to 4.2 K, the plan of 10-day operation wastes a lot of beam time for experiment. Therefore, the design must be optimized as follows.

- Insert the aluminium strips from two sides, and 3/4 part along the azimuthal direction is cooled from

both side.

- Shorten the length of aluminium strip which connects to the cooling pipe from 25 cm to 5 cm.
- Enlarge the thickness of inner aluminium strip from 1 mm to 3 mm.

Because the coils will be wound to the next layer in the edge of solenoid, only 3/4 part of coils is possible to be cooled from both side. It may be hard to wind the coils if the thickness of all aluminium strips are increased. However, as the figure 5.10 already shown, the peak of energy deposition is in the inner layer of all coils. Thus, increasing the thickness of inner strip is not only possible to reduce the temperature peak but also easy to wind. After these modifications have been employed, in figure 5.14, the maximum temperature for 60 day continuous operation is 6.2 K (orange line). Operating for more than 30 days is feasible for phase-II experiment.

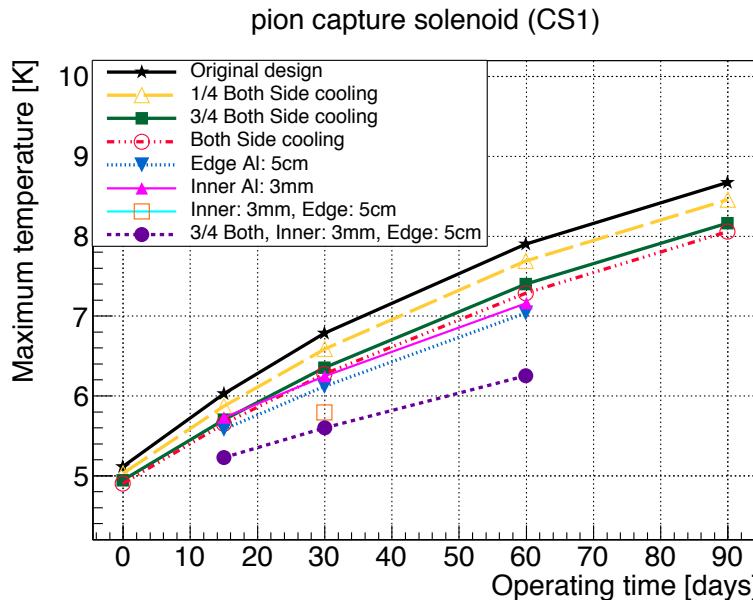


Figure 5.13: The maximum temperature against the operation time.

In previous parameter setting, the only unknown parameter is the thermal conductivity of the insulation tape. The thermal conductivity of ordinary Kapton tape at 4.2 K is around 0.01 W/m/K and it increases with temperature ⁽⁵⁵⁾. As for the insulation tape, it develops for COMET experiment to enhance the radiation resistance, thus there is no reference shown its thermal conductivity. Figure 5.14 gives the relation between the thermal conductivity of insulation tape and the peak temperature for 30 day operation. The maximum temperature will rise close to the current sharing temperature for 30 day operation if the thermal conductivity is worse than 0.003 W/m/K. Therefore, the thermal conductivity of insulation tape is necessary to take a measurement in future.

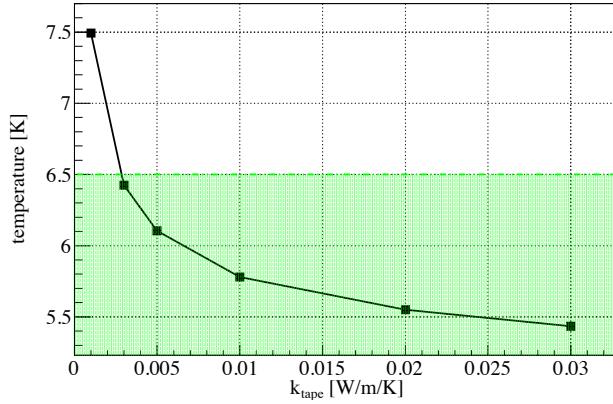


Figure 5.14: the thermal conductivity dependence of insulation tape.

5.2.4 Discussion

As mentioned in previous chapter, different result of neutron production and energy deposition is obtained in terms of different physics model. Nowadays' heat calculation bases on the model of JAM_INCL_JENDL. The figure 5.15 on left shows the difference between JAM_INCL_JENDL and QGSP_BERT_ENDF model. The difference is almost with factor 10 at TS1d and with factor 1.5 at CS1. Although the GEANT4 predicts higher heat load, the maximum temperature will be 5.35 K with 0.03 W/kg heat generation and aluminium RRR of 50.

We suppose there is factor of ± 0.3 in different physics model. The figure 5.15 shows the temperature changes with the factor of energy deposition for 90 day operation. The maximum temperature grows up with factor. Supposed that the energy deposition predicted by the other physics model is higher than JAM_INCL_JENDL by factor 1.5, cooling for 30 days is not allowed for phase-II experiment.

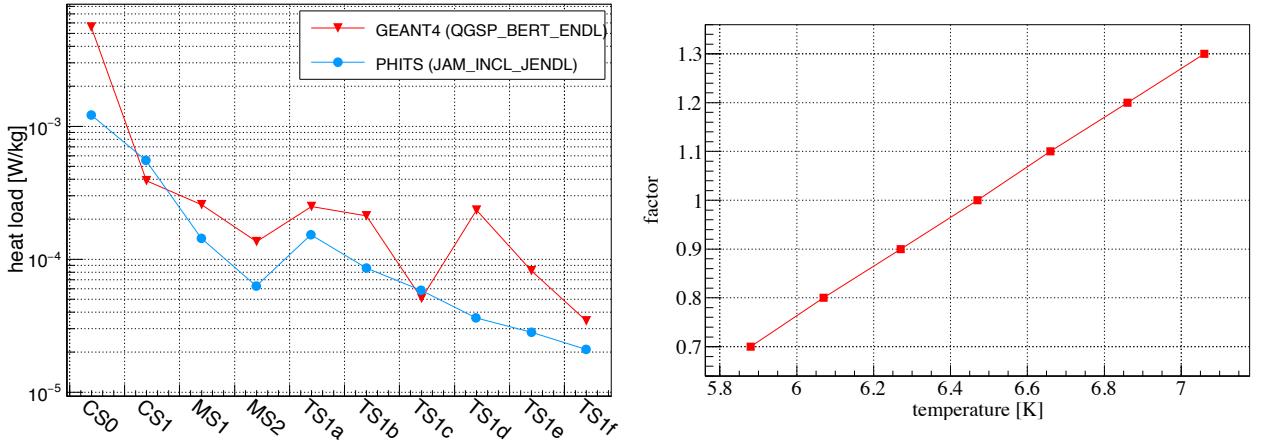


Figure 5.15: The temperature peak increases linearly with the energy deposition factor.

Improvement of CS1 coil

Due to cooling issue of the CS1 coils, the new scenario should be considered if the CS1 cannot be cooled for a long time. There are two ways to optimize the CS1 coils to enlarge the operating time.

- Using the original design but enlarged the thickness of aluminium strip.
- Separating the CS1 coils to two parts.

As shown in figure 5.16, the maximum temperature calculated with 90 day operation is able to be reduced by increasing the thickness of aluminium strip. If the thickness of aluminium strip can be increased from 1 mm to 2 mm, the peak temperature of 90 day operation is possible to be reduced under current sharing temperature. However, increasing the thickness of aluminium strip may make the coil winding become difficult. It still needs to discuss in future.

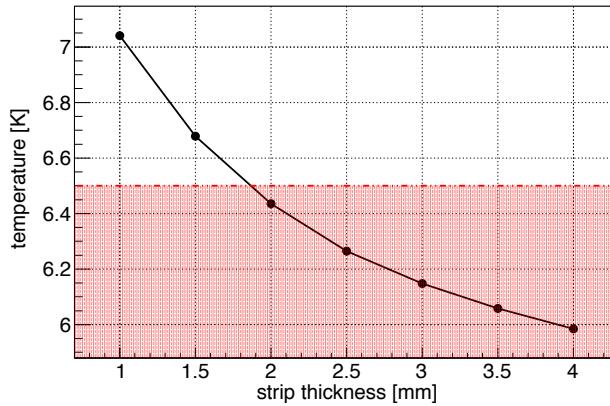


Figure 5.16: The CS1 coils can be optimized by increasing the thickness of aluminium strip.

The second scenario is to separate the CS1 coils to two parts. Since the energy deposition peak in the middle of the CS1 coils, separating the CS1 coils and inserting the aluminium strip in the middle of the CS1 coils, which is shown in figure 5.17, can help CS1 to be cooled down very well.

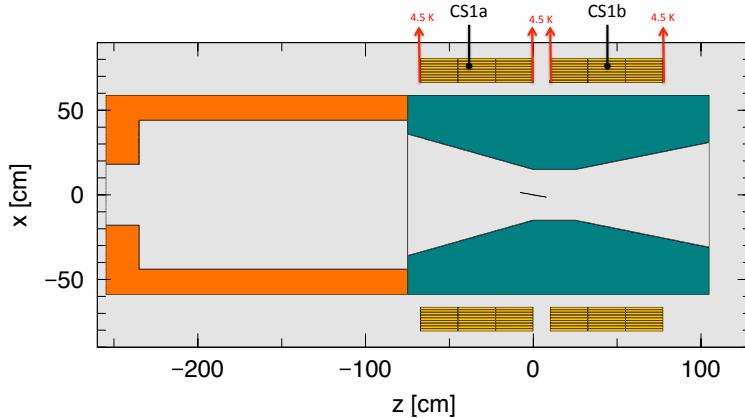


Figure 5.17: The scenario of CS1 separating.

To validate this scenario, the cooling, quench, magnetic field, muon yield and mechanical issue must be estimated.

Heat load and neutron flux

Assuming current CS1 coil is cut to two equal parts, CS1-a and CS1-b, the heat load and neutron flux may be changed inside CS1. The neutron flux for 90-day operation and heat load are calculated by PHTIS

with the geometry shown in figure 5.17. Its results are shown in figure 5.19 and 5.18. The peak of neutron flux (> 1 MeV) is 7.5×10^{20} n/m² for 90-day operation, obviously, the peak of neutron flux grows up to 8.4×10^{20} n/m² after optimization. As for the heat load, its peak decreases from 0.033 W/kg to 0.031 W/kg before and after optimization. After the CS1 coil cutting, the peak of both heat load and neutron flux are located on the edge of CS1-a coil, which is easy to cool down efficiently.

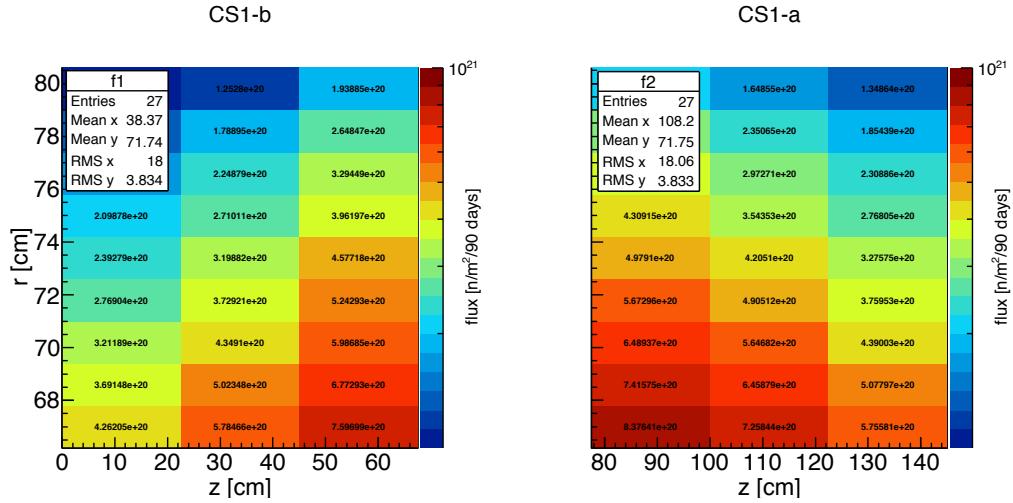


Figure 5.18: Neutron flux for 90-day operation.

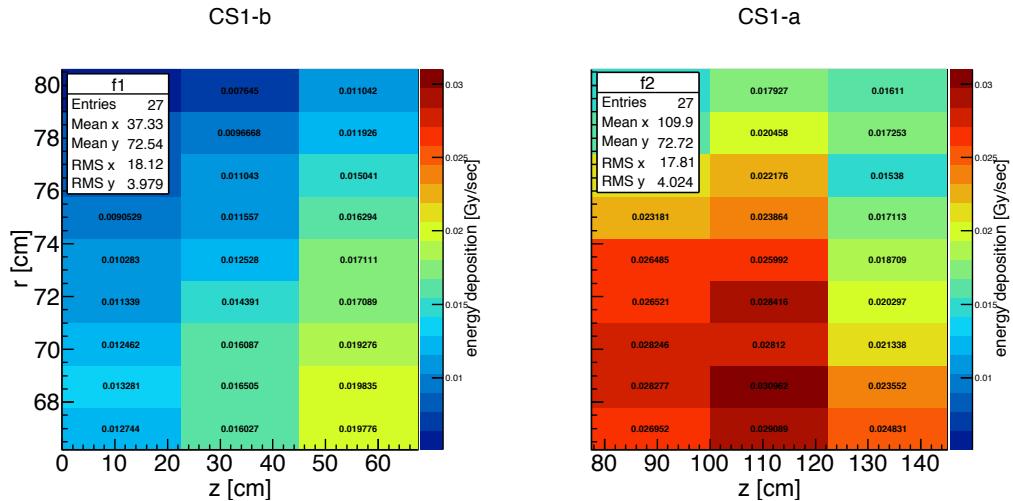


Figure 5.19: Heat load of CS1-a and CS1-b coils.

Thermal stability and quench

The temperature during is estimated the previous program, and the heat generation and RRR distribution uses the distribution in figure 5.19 and 5.18. Considering the peak of heat load and neutron flux is located in CS1-a coil, here only the temperature of CS1-a is estimated. Figure 5.20 (left) shows the temperature distribution of CS1-a coil. Its maximum temperature is 5.6 K for 90-day operation, which is totally lower than the current sharing temperature. Operation for over 90 days is available if the CS1 coil is separated to two equal parts.

After the coil separate to two parts, its aluminium self resistance is reduced in terms of the decreasing of stabilizer. Because the aluminium self resistance helps the current decay after quench, its temperature of CS1-a after quench may be higher than the case before separating. The maximum temperature after quench is shown in figure 5.20. Its current decays to about zero in 80 sec, and the temperature at quench spot is 120 K after quench.

From the aspect of thermal stability, separating the CS1 to two equal parts is available, but the magnetic field will decrease after separating and the muon yield will be affected. Also the mechanical property is important issue for the coil separating, the design still needs to be discussed.

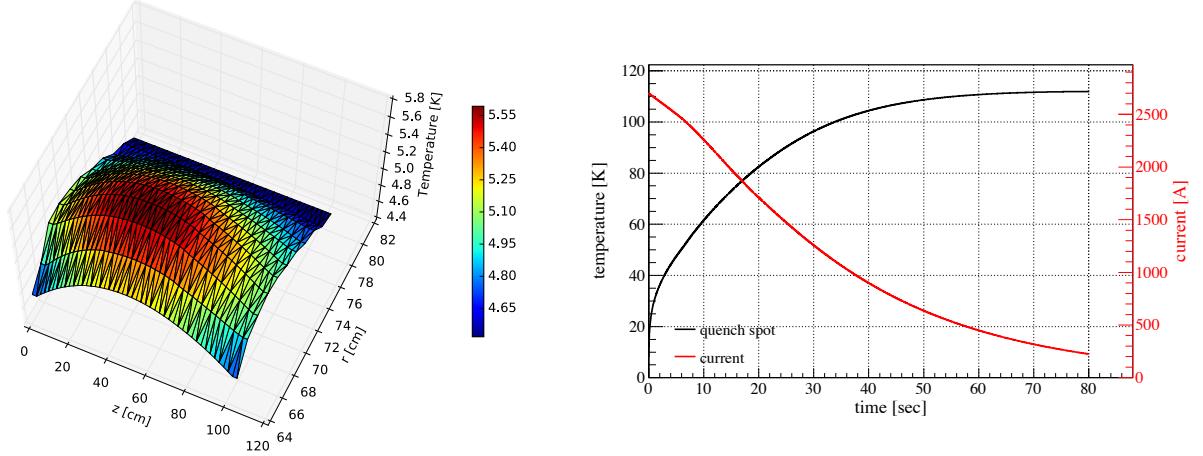


Figure 5.20: Left: temperature distribution of CS1-a coil during operation. Right: The temperature at quench spot and current decay of CS1-a coil.

Chapter 6

Conclusion

To search for the $\mu^- \rightarrow e^-$ conversion with more precision experimental sensitivity, COMET experiment requires the high intense muon. 8 GeV proton beam with 4.4×10^{13} pps is going to be employed to achieve the high intense muon beam, Thus as the major issue for designing a superconducting magnet system to generate muons, the radiation on COMET superconducting magnet system has been studied in this thesis.

Firstly, radiation on both COMET phase I and phase II are estimated with realistic geometry by using Monte Carlo code.

- COMET superconducting magnets suffer from 1.03×10^{10} n/cm²/sec neutron irradiation (> 0.1 MeV) for phase-II experiment, which corresponds to 2.49×10^{21} n/m² neutron fluence totally.
- About 3.74 mSv/h residual radiation is still remained after 1-month operation and 10-month cooling for phase-I experiment.
- A new radiation shield is designed for both phase-I and phase-II experiment to protect CS and MS superconducting coils from the radiation.

Secondly, considering the radiation induced damage on material used in COMET superconducting magnet system, several irradiation tests for GFRPs, quench protection diode, insulation tape and 5N aluminium is carried out.

- The tensile strength of BT is about 375 MPa without irradiation and drops to about 320 MPa after 200 MGy irradiation, which is stronger than G10 and CE
- The forward voltage of quench protection diode increases about 0.08 V after total 9.7×10^{11} n/m² neutron irradiation.
- With fitting the cryogenic property of quench protection diode, we estimate its forward voltage will grow up from 0.9 V to about 1.5 V at 210 A and 4.2 K after 10^{12} n/m² neutron irradiation, and its temperature rises from 4.2 K to 11 K.
- Among the several samples of insulation tape, BTGK-C has good mechanical property which has tensile strength of 14 MPa.
- The enhanced tensile strength of insulation tape has been observed after 10 MGy irradiation by cobalt source.
- According to the neutron irradiation test of 5N aluminium, the RRR of stabilizer drops lower than 100 after 3-month operation.

Thirdly, due to the degradation of the stabilizer and aluminium strip which may lead to the issue of cooling and quench, the temperature during the cooling is estimated by solving the three dimensional heat equation. As for the most dangerous coil in COMET superconducting magnet system, CS1, which is longest one and close to the production target, its temperature margin is estimated to 6.8 K for 30-day operation which exceeds to the current sharing temperature, 6.5 K. Therefore, to design a superconducting magnet what can operate under the high radiation environment for long time, original design of CS1 solenoid is optimized as follows.

- Shorten the length of aluminium strip from edge of coils to cooling pipe.
- Enlarge the length of inner aluminium strip.
- Insert the aluminium strip into each layer from two sides.

The temperature can be controlled around 5.6 K for 30-day operation with these modification. As for the quench issue, the maximum temperature of pion capture solenoid is estimated by the worst circumstances. The quench will lead the temperature to 250 K with 500 V power supply if the RRR of aluminium drops to 100. The voltage of power supply should be increase to 600 V to secure the temperature after quench.

Therefore, as for nowadays' design for the superconducting magnets of CS1 coils, 30 day operation is fully possible. However, if the experiment requires for more operation time, the optimization of design should focus on how to cool the CS1 coils efficiently.

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Appendices

A Michel decay

Michel decay is the most dominated decay mode of muon. It can be calculated by Fermi theory, and the muon decay rate can be written as

$$\Gamma(\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu) = \frac{1}{\tau_\mu} = \frac{G_F^{(e)} G_F^{(\mu)} m_\mu^5}{192\pi^3} \quad (1)$$

where the weak couplings to the electron and muon are $G_F^{(e)}$ and $G_F^{(\mu)}$, respectively. The weak coupling is measured by experiment and can be concluded that $G_F^{(e)} = G_F^{(\mu)} = G_F^{(\tau)} = 1.16638 \times 10^{-5} GeV^{-2}$. The muon differential deacay rate is given by⁽⁵⁶⁾

$$\begin{aligned} \frac{d^2\Gamma(\mu^- \rightarrow e^- \nu \bar{\nu})}{dx \cdot d(\cos\theta_e)} = & \frac{m_\mu}{4\pi^3} W_{e\mu}^4 G_F^2 \sqrt{x^2 - x_0^2} \\ & \times [F_{IS}(x) - P_\mu \cos\theta_e F_{AS}(x)] \\ & \times [1 + \vec{P}_e(x, \theta_e) \cdot \hat{\zeta}] \end{aligned}$$

where $W_{e\mu} = (m_\mu^2 + m_e^2)/2m_\mu$, $x = E_e/W_{e\mu}$ and $x_0 = m_e/W_{e\mu}$, respectively. θ_e is the angle between the muon polarization \vec{P}_μ and the electron momentum e^- . $\hat{\zeta}$ is the directional vector of the measurement of electron spin polarization. The function $F_{IS}(x)$ and $F_{AS}(x)$ are the isotropic and anisotropic parts of the electron spectrum, which can be written as

$$F_{IS}(x) = x(1-x) + \frac{2}{9}\rho(4x^2 - 3x - x_0^2) + \eta_0(1-x) \quad (2)$$

$$F_{AS}(x) = \frac{1}{3}\epsilon\sqrt{x^2 - x_0^2}\{1 - x + \frac{2}{3}\delta[4x - 3 + (\sqrt{1 - x_0^2} - 1)]\} \quad (3)$$

where $\rho, \eta, \epsilon, \delta$ are called Michel parameters. In Standard model, they are $\rho = \frac{3}{4}$, $\eta = 0$, $\eta = 1$ and $\delta = \frac{3}{4}$, while, the experimental values are

$$\rho = 0.7518 \pm 0.0026, \eta = -0.007 \pm 0.013, \delta = 0.7486 \pm 0.0038, \epsilon = 1.0027 \pm 0.0084 \quad (4)$$

which the SM value is very close to the experimental value.

B cLFV in Standard Model

The most popular explanation of charged lepton flavor violation in Standard model is one W boson is emitted through the neutrino oscillation⁽⁵⁷⁾ of $\nu_\mu \rightarrow \nu_e$. Feynman diagram of $\mu^- \rightarrow e^- \gamma$ is shown in figure 1.

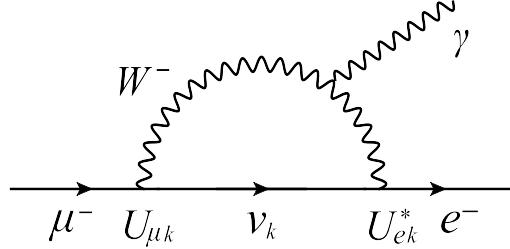


Figure 1: Feynman diagram of $\mu^- \rightarrow e^- \gamma$ predicted by massive neutrino. cLFV occurs through a neutrino oscillation, and γ is emitted from a W^-

The treatment of neutrino oscillation for three flavors is developed by 3×3 PMNS matrix with mixing angle, which is written as

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad (5)$$

To solve the neutrino oscillation, we start from the time evolution of the wave function, which is determined by the time evolution of the mass eigenstates and can be written as

$$|\psi(\mathbf{x}, t)\rangle = U_{\mu 1}^* |\nu_1\rangle e^{-i\phi_1} + U_{\mu 2}^* |\nu_2\rangle e^{-i\phi_2} + U_{\mu 3}^* |\nu_3\rangle e^{-i\phi_3} \quad (6)$$

using the PMNS matrix, the probability of $\nu_\mu \rightarrow \nu_e$ can be described as

$$P(\nu_\mu \rightarrow \nu_e) = |\langle \nu_e | \psi(\mathbf{x}, t) \rangle|^2 = -4 \sum_{k=1,2,3} U_{ek} U_{\mu k}^* \sin^2(\Delta_{ji}) \quad (7)$$

where $\Delta_{ji} = (\phi_j - \phi_i)/2 = (m_j^2 - m_i^2) \cdot L / 4E_\nu$. L is the neutrino oscillation occurring distance and $(m_j^2 - m_i^2)$ is the difference between the square of neutrino mass. Thus, the decay rate of $\nu_\mu \rightarrow \nu_e$ is fixed by involving the neutrino oscillation and given by

$$\begin{aligned} \Gamma(\mu^- \rightarrow e^- + \gamma) &= \frac{G_F^2 m_\mu^5}{192\pi^3} \frac{\alpha}{2\pi} P(\nu_\mu \rightarrow \nu_e) \\ &\simeq \frac{G_F^2 m_\mu^5}{192\pi^3} \frac{\alpha}{2\pi} |U_{e3} U_{\mu 3}^* \frac{\Delta m^2}{m_W^2}|^2 \end{aligned} \quad (8)$$

where α, m_W is the fine structure constant and the mass of W boson. The ratio of the partial width $\Gamma(\mu^- \rightarrow e^- \gamma)$ to the Michel decay rate gives the branching ratio

$$Br(\mu^- \rightarrow e^- \gamma) = \frac{\Gamma(\mu^- \rightarrow e^- + \gamma)}{\Gamma(\mu^- \rightarrow e^- \bar{v}_e v_\mu)} = \frac{\alpha}{2\pi} |U_{e3} U_{\mu 3}^* \frac{\Delta m^2}{m_W^2}|^2 < 10^{-54} \quad (9)$$

C cLFV in supersymmetry

Supersymmetry (SUSY) is the most possible candidate in the physics beyond the Standard model, which can explain the hierarchy problem and Higgs' mass perfectly. It is a proposed extension of SM by including the superpartner for each particle. For instance, each fermion has its superpartner, boson, and vice versa. In the prediction of SUSY model which shown in figure 2, the cLFV is occurred by the slepton mixing.

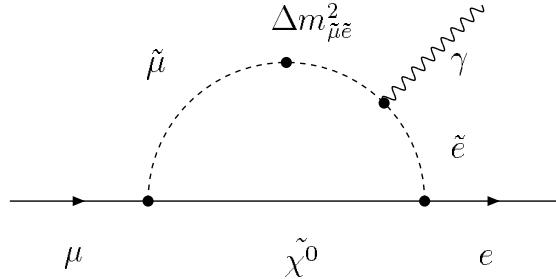


Figure 2: Feynman diagram of $\mu^- \rightarrow e^- \gamma$ predicted by slepton mixing. it occurs through its superpartner $\tilde{\mu}$ and \tilde{e} mixing.

The branching ratio of $\mu \rightarrow e\gamma$ is given similarly to the SM by⁽⁵⁸⁾

$$Br(\mu \rightarrow e\gamma) = \frac{\alpha}{2\pi} \left| \sum_k \tilde{U}_{ek} \tilde{U}_{\mu k}^* \frac{\Delta m_k^2}{m_S^2} \right|^2 \left(\frac{m_W}{m_S} \right)^4 \quad (10)$$

where m_S is the typical scale of scalar masses, \tilde{U} is the mixing matrix between sleptons and leptons, and Δm_k^2 is the square of mass difference between the k th and 1st sleptons.

From figure 2, the mixing $\Delta m_{\tilde{e}\tilde{\mu}}^2$ between a smuon $\tilde{\mu}$ and a selectron \tilde{e} plays a key role in cLFV. This slepton mixing parameter, $\Delta m_{\tilde{e}\tilde{\mu}}^2$ is given by the slepton mass matrix $m_{\tilde{l}}^2$

$$(m_{\tilde{l}}^2)_{ij} = (m_l m_l^\dagger)_{ij} + (\tilde{m}_l^2)_{ij} = \begin{pmatrix} m_{\tilde{e}\tilde{e}}^2 & \Delta m_{\tilde{e}\tilde{\mu}}^2 & \Delta m_{\tilde{e}\tilde{\tau}}^2 \\ \Delta m_{\tilde{\mu}\tilde{e}}^2 & m_{\tilde{\mu}\tilde{\mu}}^2 & \Delta m_{\tilde{\mu}\tilde{\tau}}^2 \\ \Delta m_{\tilde{\tau}\tilde{e}}^2 & \Delta m_{\tilde{\tau}\tilde{\mu}}^2 & m_{\tilde{\tau}\tilde{\tau}}^2 \end{pmatrix} \quad (11)$$

where $(m_l m_l^\dagger)_{ij}$ is the lepton mass matrix and $(\tilde{m}_l^2)_{ij}$ is the term of SUSY breaking, respectively. As the scenario of SUSY, SUSY-GUT and SUSY-Seasaw contribute to the off-diagonal terms.

SUSY-GUT

Supersymmetric grand unified theories (SUSY-GUT) is the extension of grand unified theories with supersymmetry. Three gauge coupling constants of electromagnetic, weak and strong interaction are unified to SU(5) coupling constant at scale of 10^6 GeV, which is also called GUT scale. Due to the generation of the off-diagonal elements in slepton mass matrix, the cLFV is occurred with big branching ratio. $\Delta m_{\tilde{\mu}\tilde{e}}^2$ is given by

$$\Delta m_{\tilde{\mu}\tilde{e}}^2 \propto \frac{3m_0^2 + A_0^2}{8\pi^2} h_t^2 V_{td}^* V_{ts} \ln \frac{M_{GUT}}{M_{R_3}} \quad (12)$$

where m_0 and A_0 are the universal scalar mass and the universal trilinear coupling respectively. V_{td} and V_{ts} are the elements in the CKM matrix. M_{R_3} is the mass of the right-hand neutrino. M_{GUT} is the GUT scale ($\approx 2 \times 10^{18}$ GeV). and h_t is the Yukawa coupling constant of top quark. Figure 3 shows the prediction of the branching ratio of $\mu \rightarrow e$ conversion with Ti target in SUSY-GUT model⁽⁵⁶⁾, which is higher than the prediction of SM with the order of 1040. Here, $m_{\tilde{e}_R}$ is the mass of right-hand electron, and μ is the parameter defined by Higgs and Higgsino.

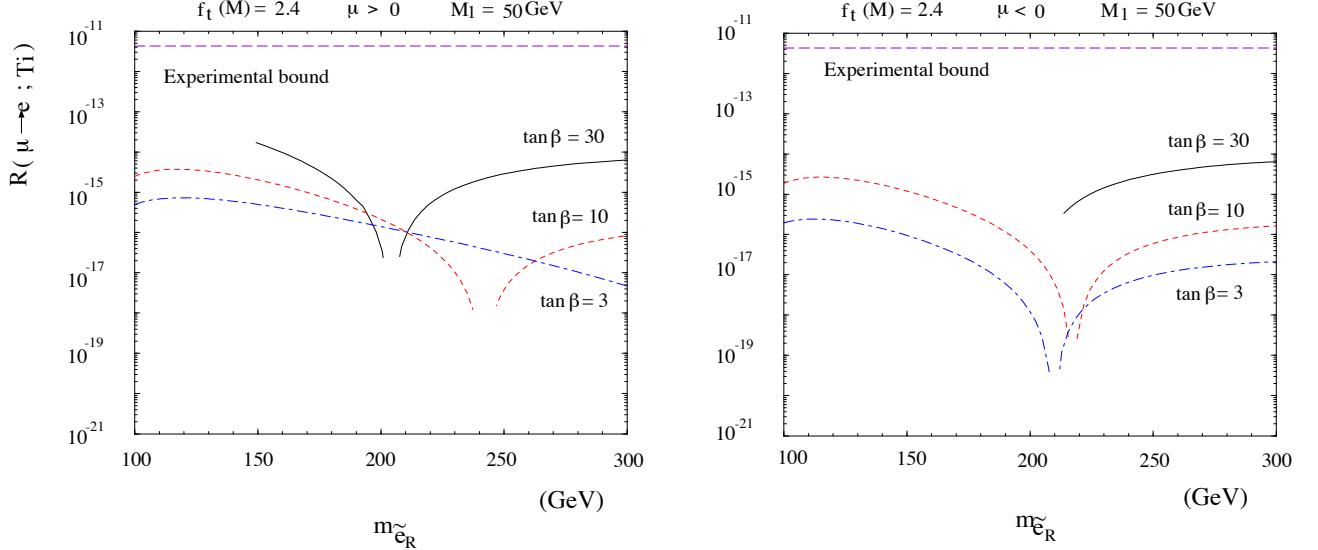


Figure 3: Prediction of branching ratio of $\mu \rightarrow e$ conversion in SUSY-GUT SU(5). Mass of right-hand \tilde{e} can be observed indirectly if the $\mu \rightarrow e$ conversion experiment gets more precision sensitivity.

SUSY-Seasaw

SUSY-Seasaw model is another scenario for the prediction of cLFV by including the right-hand neutrino. From the right-hand neutrino mass term, the slepton mixing can be obtained as well. The slepton mixing between muon and electron depends on the solar neutrino oscillation and atmospheric neutrino oscillation⁽⁵⁹⁾. Thus, $\Delta m_{\mu\tilde{e}}^2$ can be written as

$$\Delta m_{\mu\tilde{e}}^2 \propto \frac{3m_0^2 + A_0^2}{8\pi} h_\tau^2 U_{\tau e}^* U_{\tau\mu} \ln \frac{M_{GUT}}{M_{R_3}} \quad (13)$$

where $U_{\tau e}^*$ and $U_{\tau\mu}$ are the elements of PMNS matrix. h_τ is the Yukawa coupling constant of τ . The prediction of branching ratio of $\mu \rightarrow e\gamma$ by SUSY-Seasaw is given in figure 4. Similar to the prediction of SUSY-GUT model, it also predicts the higher branching ratio for cLFV.

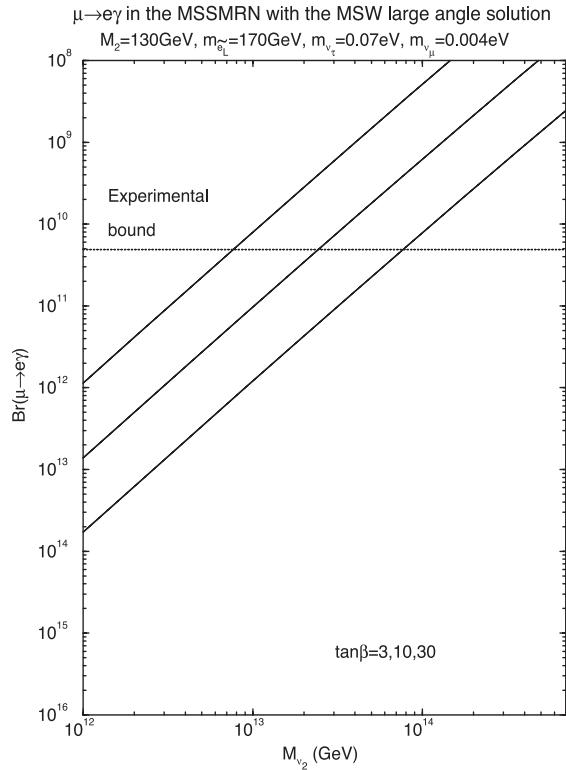


Figure 4: Prediction of branching ratio of $\mu \rightarrow e\gamma$ in SUSY-Seasaw model.

D Impact of cLFV

Because of the higher branching ratio predicted by BSM, in addition, the branching ratio depends on the experimental sensitivity, the discovery of cLFV plays an important role in seeking for the BSM. In W. Altmannshofer's paper, the sensitivities of several flavor observables to a variety of BSM models are listed in figure 5.

"DNA" of flavour physics effects for the most interesting observables in a selection of SUSY and non-SUSY models
★★★ signals large effects, ★☆ visible but small effects and ★ implies that the given model does not predict sizable effects in that observable.

	AC	RVV2	AKM	δLL	FBMSSM	LHT	RS
$D^0 - \bar{D}^0$	★★★	★	★	★	★	★★★	?
ϵ_K	★	★★★	★★★	★	★	★☆	★★★
$S_{\psi\phi}$	★★★	★★★	★★★	★	★	★★★	★★★
$S_{\phi K_S}$	★★★	★☆	★	★★★	★★★	★	?
$A_{CP}(B \rightarrow X_S \gamma)$	★	★	★	★★★	★★★	★	?
$A_{7,8}(B \rightarrow K^* \mu^+ \mu^-)$	★	★	★	★★★	★★★	★☆	?
$A_9(B \rightarrow K^* \mu^+ \mu^-)$	★	?					
$B \rightarrow K^{(*)} \nu \bar{\nu}$	★						
$B_s \rightarrow \mu^+ \mu^-$	★★★	★	★				
$K^+ \rightarrow \pi^+ \nu \bar{\nu}$	★	★★★	★★★				
$K_L \rightarrow \pi^0 \nu \bar{\nu}$	★	★★★	★★★				
$\mu \rightarrow e \gamma$	★★★						
$\tau \rightarrow \mu \gamma$	★★★	★★★	★	★★★	★★★	★★★	★★★
$\mu + N \rightarrow e + N$	★★★						
d_n	★★★	★★★	★★★	★☆	★★★	★	★★★
d_e	★★★	★★★	★☆	★	★★★	★	★★★
$(g-2)_\mu$	★★★	★★★	★☆	★★★	★★★	★	?

Figure 5: The size of the observable flavor effect for a variety of SUSY and non-SUSY models.

where AC, RVV2, AKM, δLL , FBMSSM, LHT and RS are Abelian U(1) flavor symmetry model, non-Abelian model, Antush-King-Malinsky SU(3) flavor symmetry model, flavor models with pure CKM-like left handed currents, flavor-blind MSSM, little Higgs with T-parity and Randall-Sundrum model⁽⁶⁰⁾. In all these decay modes, $\mu \rightarrow e$ conversion and $\mu \rightarrow e\gamma$ contributes a large effect on every theory, which means the BSM is easy to be found in these two modes with high probability.

E Magnetic field interpolation

Suppose there is field map, it only gives us the coordinate (x, y, z), magnetic field vector (B_x, B_y, B_z) and magnetic field (B), so we have many points with magnetic field. If the particle goes through this point, the magnetic field is already there. However, the magnetic field has to be interpolated when the particle goes through the point we haven't. The simple way to interpolate the field is to weight the field in a box.

There is box which vertices are the known coordinate of field map. Each vertices can be written as

$$\begin{aligned}
\vec{r}_1 &= (x_{lt}, y_{lt}, z_{lt}) \\
\vec{r}_2 &= (x_{gt}, y_{lt}, z_{lt}) \\
\vec{r}_3 &= (x_{gt}, y_{gt}, z_{lt}) \\
\vec{r}_4 &= (x_{gt}, y_{gt}, z_{gt}) \\
\vec{r}_5 &= (x_{lt}, y_{gt}, z_{lt}) \\
\vec{r}_6 &= (x_{lt}, y_{gt}, z_{gt}) \\
\vec{r}_7 &= (x_{lt}, y_{lt}, z_{gt}) \\
\vec{r}_8 &= (x_{gt}, y_{lt}, z_{gt})
\end{aligned} \tag{14}$$

where the x_{gt} is the greater value and x_{lt} is the less value along the x direction. Thus, the weight of x, y and z axis in this box can be written as

$$t = \frac{x_0 - x_{lt}}{x_{gt} - x_{lt}}, \quad u = \frac{y_0 - y_{lt}}{y_{gt} - y_{lt}}, \quad v = \frac{z_0 - z_{lt}}{z_{gt} - z_{lt}} \tag{15}$$

where (x_0, y_0, z_0) is the position of the particle and (t, u, v) is the weight on this position. (x_0, y_0, z_0) must be inside this box. Therefore, the magnetic field of this position is

$$\begin{aligned}
B_i(\vec{r}_0) = & (1-t)(1-u)(1-v)B_i(\vec{r}_1) \\
& + (1-u)(1-v)tB_i(\vec{r}_2) \\
& + tu(1-v)B_i(\vec{r}_3) \\
& + tuvB_i(\vec{r}_4) \\
& + (1-t)u(1-v)B_i(\vec{r}_5) \\
& + (1-t)uvB_i(\vec{r}_6) \\
& + (1-t)(1-u)vB_i(\vec{r}_7) \\
& + t(1-u)vB_i(\vec{r}_8)
\end{aligned} \tag{16}$$

$(x_{lt} \leq x_0 \leq x_{gt}, y_{lt} \leq y_0 \leq y_{gt}, z_{lt} \leq z_0 \leq z_{gt})$