How to determine a normal vector to the magnetopause boundary using MVAB and MVAE from spacecraft data

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1 Introduction

This - at present stage rather informal - document is to be regarded as a short handbook in how to determine the normal to the magnetopause (MP) boundary, using observational data. It also deals with how to determine the DeHoffmann-Teller (HT) velocity (DeHoffmann & Teller, 1950) and how to perform the Walén test for magnetic reconnection. A majority of the material presented here concerning normal determination is from the work by Sonnerup & Scheible (2000), while the corresponding material regarding the HT velocity and Walén test is from Khrabov & Sonnerup (2000). These chapters are - as observed by the observational minded person when looking in the reference list - from the book Analysis Methods for Multi-Spacecraft Data by Pashmann & Daly (2000) where much more details are to be found than presented in this document.

In this document techniques using data from a single spacecraft is considered. Techniques using multi spacecraft data (spacecraft timing) are described in chapter 10, p.256 in Pashmann & Daly (2000).

2 Variance analysis

Generally, variance analysis can be applied on just about anything which varies across (in this case) the MP boundary. When determining the MP normal, minimum variance on the magnetic field (MVAB) and maximum variance on the electric field (MVAE) are the (as it seems) most commonly used methods. One alternative is also to use minimum variance on the mass flux (MVA$\rho\mathbf{v}$). MVAB is based on the theory that the normal component of the magnetic field (considering an ideal 1D geometry) is non-varying through the MP boundary. In real life (so to speak - in real space perhaps is a better phrase), there are always perturbations present causing 2D and 3D effects. Hence the normal direction is considered to be in the direction the variance of the magnetic field has its minimum. Corresponding for the electric field, but here the maximum variance is sought according to the theory saying that the electric field component normal to the layer undergoes a large change as it often does at the magnetopause where the tangential flow and/or the tangential magnetic field components usually change direction and/or magnitude by substantial amounts across the layer (Sonnerup & Scheible, 2000).

2.1 MVAB

Here's how to determine the normal using MVAB, see Sonnerup & Scheible (2000) for more theoretical background.

- Construct the variance matrix

$$M_{ij} = \langle B_i B_j \rangle - \langle B_i \rangle \langle B_j \rangle \tag{1}$$

  where $i, j = 1, 2, 3$ denote cartesian components along the x, y, z system, and $\langle ... \rangle$ the mean value. This means that e.g.

$$\langle B_x \rangle = \frac{1}{m} \sum_{i=1}^{m} B_x^{(i)} \tag{2}$$

  where m is the number of data points for the actual MP crossing sequence.

- Find the eigenvalues and the corresponding eigenvectors of the variance matrix 1.
- The eigenvector $\mathbf{x}_3$ corresponding to the smallest eigenvalue, λ_3, is used as the estimator for the vector normal to the current sheet and the eigenvalue itself represents the variance of the magnetic field component along the estimated normal.

2.1.1 Example

Consider the data set on p.218 in Pashmann & Daly (2000). To construct the variance matrix, use e.g. Matlab. There are 16 data points for the actual MP passage. Here we will use column 2-4 - i.e. the respective magnetic field components.

We will construct a 3x3 matrix according to (1). The index notation is the same as Matlab uses for rows and columns (e.g. M_{13} =row 1, column 3 of matrix M). So for this benchmark example we first make a matrix B containing the magnetic field such that column 1 contain B_x, column 2 B_y, and column 3 B_z - where $B_x - B_z$ are the values from the data set on p.218. Hence the line $M_{1,1}$ =mean($B(:,1) * .B(:,1)$)-mean($B(:,1)$)*mean($B(:,1)$) will construct element (1,1) in our variance matrix. To get the full matrix we loop over $i, j = 1, 2, 3$.

Having M we use [V,D]=eig(M) in Matlab to get our eigenvalues/vectors. These should be the same as the ones presented in connection to Figure 8.2 on p.190 in Pashmann & Daly (2000). If not, the Matlab code is wrong.

2.2 MVAE

When the minimum variance of the magnetic field on the actual MP crossings fails, the normal to the MP surface can be determined from maximum variance of the electric field (MVAE) (Sonnerup & Scheible, 2000). The method in this section differs slightly from the one presented in Pashmann

& Daly (2000) (starting on p.209). The main difference is that here the MP velocity is calculated from data (see (6) and (7)) using the plasma velocity and the Alfvén velocity, rather than iterating over the normal velocity u_n as described in the book. The method used here is preferable when we have reconnection present and the MP can be described as a rotational discontinuity.

The MVAE method is based on Faraday's law which requires that the two tangential components of the electric field with respect to a one-dimensional layer in steady-state remains constant through the layer and on its both sides (Sonnerup & Scheible, 2000). That is

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = 0 \tag{3}$$

results in

$$\hat{\mathbf{n}} \times \Delta \mathbf{E} = 0, \tag{4}$$

where $\hat{\mathbf{n}}$ is the normal vector to the current layer. This means that the eigenvector of the variance matrix for the electric field which corresponds to the largest eigenvalue and thus represent the maximum variance direction, is an approximation for the layer normal. Since reality often differs from the assumption of a 1D steady-state discontinuity, there are effects which will affect the result from MVAE. The MP crossings treated in the present analysis and by Phan *et al.* (2004) illustrates very well the oscillating motion of the MP, rather than the ideal assumption of a constant motion. The relative motion between the spacecraft and the MP boundary is also one significant error source. It is therefore important to transform the electric field into a system which moves with the MP. Sonnerup *et al.* (1987) have made a method to minimize the effect of the relative motion between the spacecraft and the MP. When performing the MVAE analysis we consider the following procedure:

- In accordance to the discussion above covering the behaviour of the electric field during the passage of the MP surface, we begin by finding the variance of $\mathbf{E}$. The variance matrix of the convective electric field ($\mathbf{E}^{(m)} = \mathbf{v}^{(m)} \times \mathbf{B}^{(m)}$) is calculated from (Sonnerup & Scheible, 2000)

$$M_{ij}^{E} = < E_i E_j > - < E_i >< E_j >, \tag{5}$$

  where (m) denotes data points, and $< \ldots >$ the average of an enclosed quantity over a set of spacecraft measurements. $< \ldots >$ denotes the average of the enclosed set of data points. The maximum-variance eigenvector $\mathbf{i}_1$ then becomes our initial predictor for the MP normal $\hat{\mathbf{n}}$.

- Having obtained the initial estimate of the MP boundary normal we proceed by calculating the average plasma velocity and the Alfvén velocity in the normal direction, i.e.

$$v_n = < \mathbf{v}^{(m)} > \cdot \mathbf{i}_1 \tag{6}$$

$$u_n = \pm \left\langle \frac{\mathbf{B}^{(m)}}{\sqrt{\mu_0 \rho^{(m)}}} \right\rangle \cdot \mathbf{i}_1. \tag{7}$$

- v_n is the average velocity in the spacecraft frame, while u_n is the velocity of the rotational discontinuity. The MP velocity relative to the spacecraft can then be approximated as

$$\mathbf{V}_{\mathrm{MP}} = (v_n - u_n)\mathbf{i}_1. \tag{8}$$

By subtracting the electric field due to the relative motion of the MP we obtain the electric field in the MP

$$\mathbf{E}^{(m)\prime} = \mathbf{E}^{(m)} - \mathbf{V}_{\mathrm{MP}} \times \mathbf{B}^{(m)}, \tag{9}$$

where $\mathbf{E}$ is the convective electric field.

- The variance analysis is now performed on the corrected electric field $\mathbf{E}'$, resulting in a new estimate for the MP boundary normal, $\mathbf{i}_2$. The new vector $\mathbf{i}_2$ is then applied to (6) and (7), and the iteration is then performed until $\mathbf{i}$ no longer changes. Usually it is sufficient with a few (< 10) iterations for the iteration to converge. For the present ten MP crossings studied, the method of MVAE fails for one MP crossing to produce a value on V_{MP} which is not in conflict with the actual MP motion.

3 HT frame of reference and the Walén test

When analyzing reconnection events, the Walén test typically is used to determine if the MP is a rotational discontinuity. For this the HT frame and velocity also is needed.

In order to obtain the HT frame of reference we seek to find a frame where the mean square of the electric field, $D(\mathbf{V}) =< |(\mathbf{v}^{(m)} - \mathbf{V}) \times \mathbf{B}^{(m)}|^2 >$, is as small as possible (Khrabov & Sonnerup, 2000). The quality of the HT frame is represented by the correlation coefficient C_{HT} between the convective electric field $\mathbf{E}_{\mathrm{c}}^{(m)} = -\mathbf{V}^{(m)} \times \mathbf{B}^{(m)}$ and the HT electric field $-\mathbf{V}_{\mathrm{HT}}^{(m)} \times \mathbf{B}^{(m)}$, together with the the ratio $D(\mathbf{V}_{\mathrm{HT}})/D(0)$. For a HT frame of good quality $C_{\mathrm{HT}} \approx 1$ and $D/D(0) << 1$ (Khrabov & Sonnerup, 2000).

Ideally the Walén relation $\mathbf{v}' = \mathbf{v}^{(m)} - \mathbf{V}_{\mathrm{HT}} = \mathbf{B}^{(m)}(\mu_0\rho^{(m)})^{-1/2}$ holds. For any real phenomena studied however, we always have some non-ideal effects leading to the condition where the plasma velocity $\mathbf{v}'$ is close to the Alfvén velocity $\mathbf{v}' = \mathbf{v}^{(m)} - \mathbf{V}_{\mathrm{HT}} = \pm C_{\mathrm{W}}\mathbf{B}^{(m)}(\mu_0\rho^{(m)})^{-1/2}$, where C_{W} is the Walén correlation coefficient, and often being in the range 0.8-1.0 (Khrabov & Sonnerup, 2000).

References

DeHoffmann, F., & Teller, E. 1950. Magnetohydrodynamic shocks. *Phys. Rev. Lett.*, **80**(692).

Fairfield, D. H. 1971. Average and unusual locations of the Earth's magnetopause and bowshock. *J. Geophys. Res.*, **76**(28), 6700–6716.

Khrabov, A. V., & Sonnerup, B. U. Ö. 2000. DeHoffmann-Teller analysis. *In:* Pashmann, Götz, & Daly, Patrick W. (eds), *Analysis Methods for Multi-Spacecraft Data*. ISSI Scientific Report.

PASHMANN, G., & DALY, P. W. (eds). 2000. *Analysis methods for multi-spacecraft data.* ISSI scientific report (www.issi.unibe.ch).

PHAN, T. D., DUNLOP, M. W., PASCHMANN, G., KLECKER, B., BOSQUED, J. M., RÈME, H. R., BALOGH, A., TWITTY, C., MOZER, F. S., CARLSON, C. W., MOUIKIS, C., & KISTLER, L. M. 2004. Cluster observations of continuous reconnection at the magnetopause under steady interplanetary magnetic field conditions. *Ann. Geophys.*, **22**, 2355–2367.

SONNERUP, B. U. Ö., & SCHEIBLE, M. 2000. Minimum and Maximum Variance Analysis. *In:* PASHMANN, GÖTZ, & DALY, PATRICK W. (eds), *Analysis Methods for Multi-Spacecraft Data.* ISSI Scientific Report.

SONNERUP, B. U. Ö., PAPAMASTORAKIS, I., PASCHMANN, G., & LÜHR, H. 1987. Magnetopause properties from AMPTE/IRM observations of the convection electric field: method development. *J. Geophys. Res.*, **92**(A11), 12,137–12,159.