

# EC201 Macroeconomics

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Class - 5 -

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# This Week: Outline

1. The Journey of Humatity

2. REVIEW QUESTIONS

2.1 QUESTIONS 1

2.2 QUESTIONS 2

3. CLASS QUESTIONS

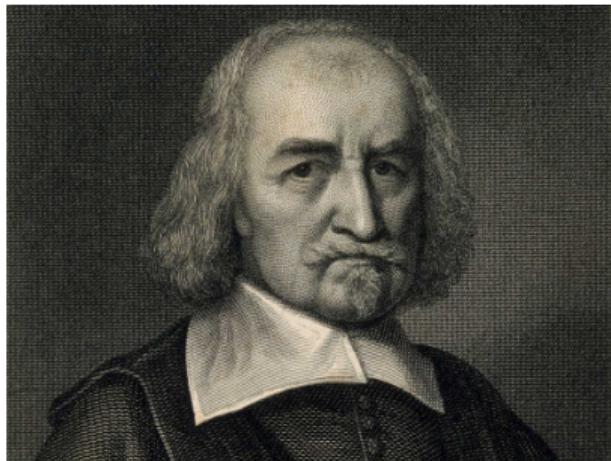
3.1 QUESTION 1

3.2 QUESTION 2

4. Self Study Questions

# Thomas Hobbes: nasty, brutish, and short

- ▶ For most of history, standards of living were extremely low, not much different from that in the poorest countries of the world today.
- ▶ Evidence suggests, for example, that wages in ancient Greece and Rome were approximately equal to wages in Britain in the fifteenth century or France in the seventeenth.
- ▶ In the words of seventeenth-century English philosopher Thomas Hobbes, life was Thomas Hobbes: “**solitary, poor, nasty, brutish, and short**”



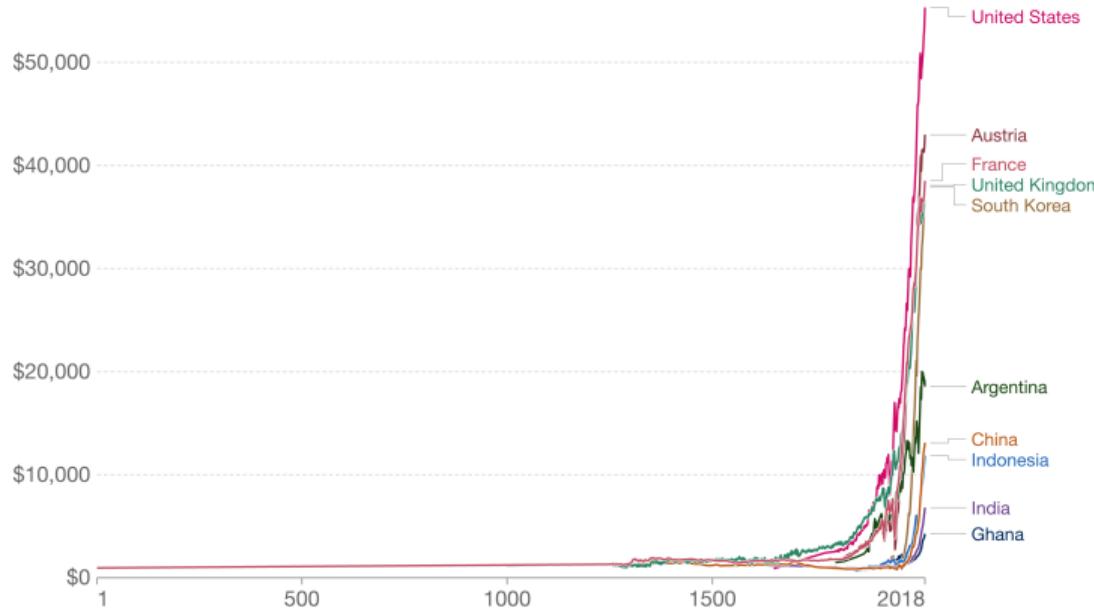
THOMAS HOBBES: 1588 - 1679

# A long, long time ago in a galaxy far, far away

## GDP per capita, 1 to 2018

This data is adjusted for differences in the cost of living between countries, and for inflation. It is measured in constant 2011 international-\$.

Our World  
in Data



Source: Maddison Project Database 2020 (Bolt and van Zanden, 2020)

OurWorldInData.org/economic-growth • CC BY

# Things that didn't exist 20 years ago:

- ▶ iPhone
- ▶ Facebook
- ▶ YouTube
- ▶ Instagram
- ▶ Twitter
- ▶ TikTok
- ▶ Android
- ▶ Bitcoin
- ▶ Tesla
- ▶ iPad
- ▶ Gmail
- ▶ WhatsApp
- ▶ Spotify
- ▶ Airbnb
- ▶ Uber

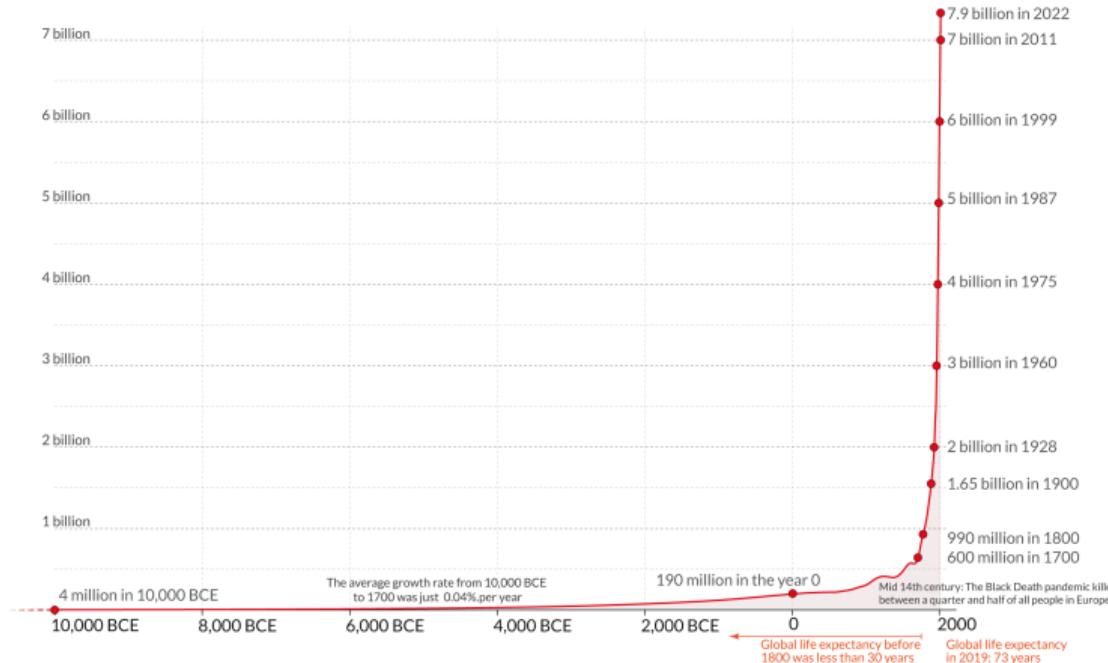
Our lives and habits are completely different than our parents

# Not just getting richer but getting crowded too: World Population



## The size of the world population over the last 12,000 years

Demographers expect rapid population growth to end by the end of the 21st century. The UN demographers expect a population of about 11 billion in 2100.



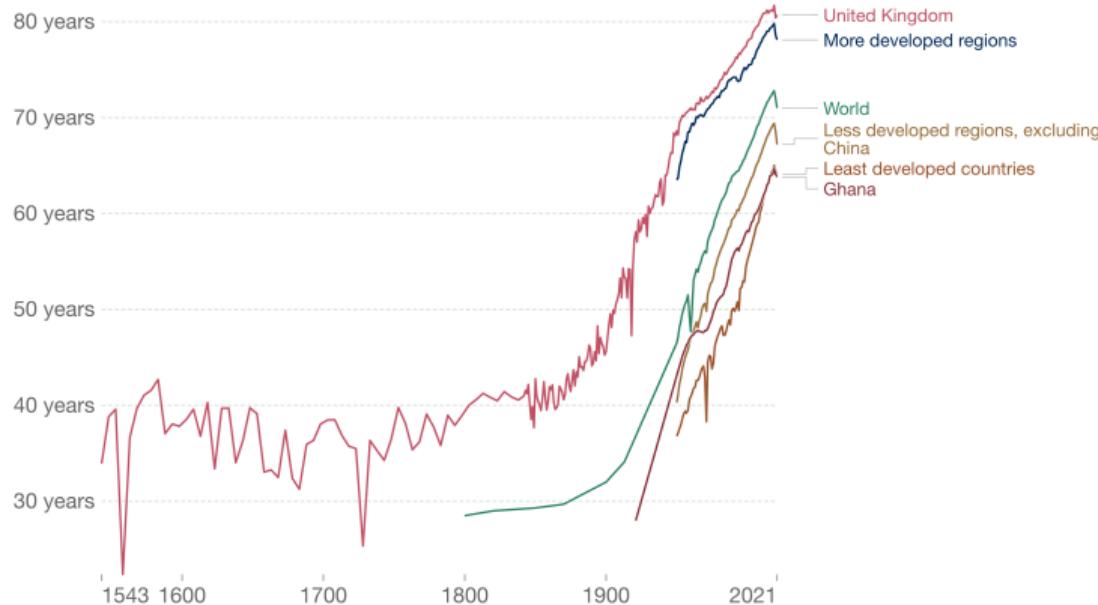
Based on estimates by the History Database of the Global Environment (HYDE) and the United Nations. On [OurWorldInData.org](http://OurWorldInData.org) you can download the annual data.  
This is a visualization from [OurWorldInData.org](http://OurWorldInData.org).

Licensed under CC-BY-SA by the author Max Roser.

# Life expectancy: not as in before

Life expectancy, 1543 to 2021

Our World  
in Data



Source: UN WPP (2022); Zijdemar et al. (2015); Riley (2005)

Note: Shown is the 'period life expectancy'. This is the average number of years a newborn would live if age-specific mortality rates in the current year were to stay the same throughout its life.

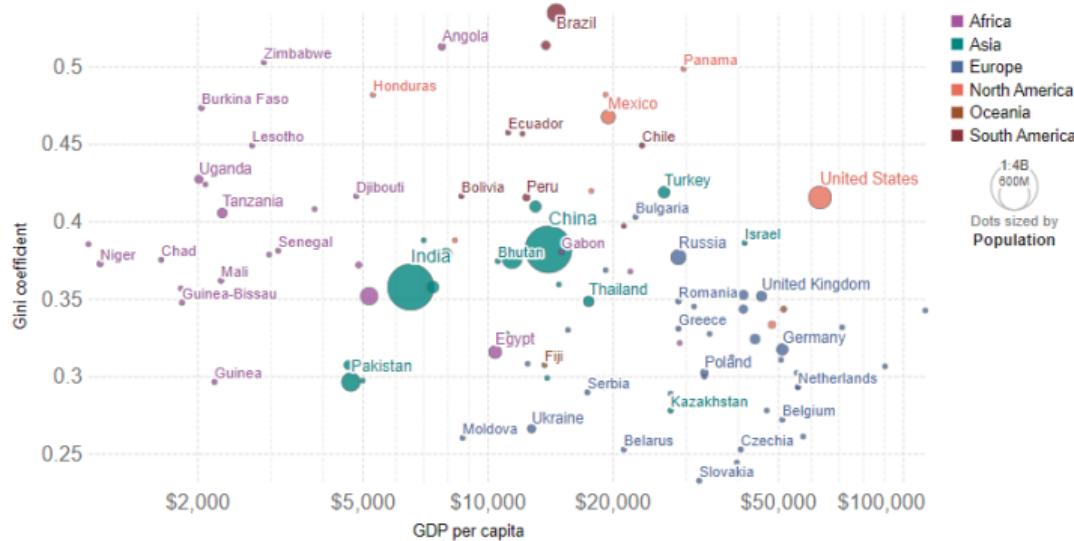
OurWorldInData.org/life-expectancy • CC BY

# Inequality

## GDP per capita vs. income inequality, 2019

GDP is adjusted for inflation and for differences in the cost of living between countries. The Gini coefficient is a measure of the inequality of the income distribution in a population. Higher values indicate a higher level of inequality.

Our World  
in Data



Source: World Bank PIP; Feenstra et al. (2015), Penn World Table 10.0

Note: This data is measured in international-\$ at 2017 prices. Depending on the country and year, it relates to either disposable income or consumption per capita.

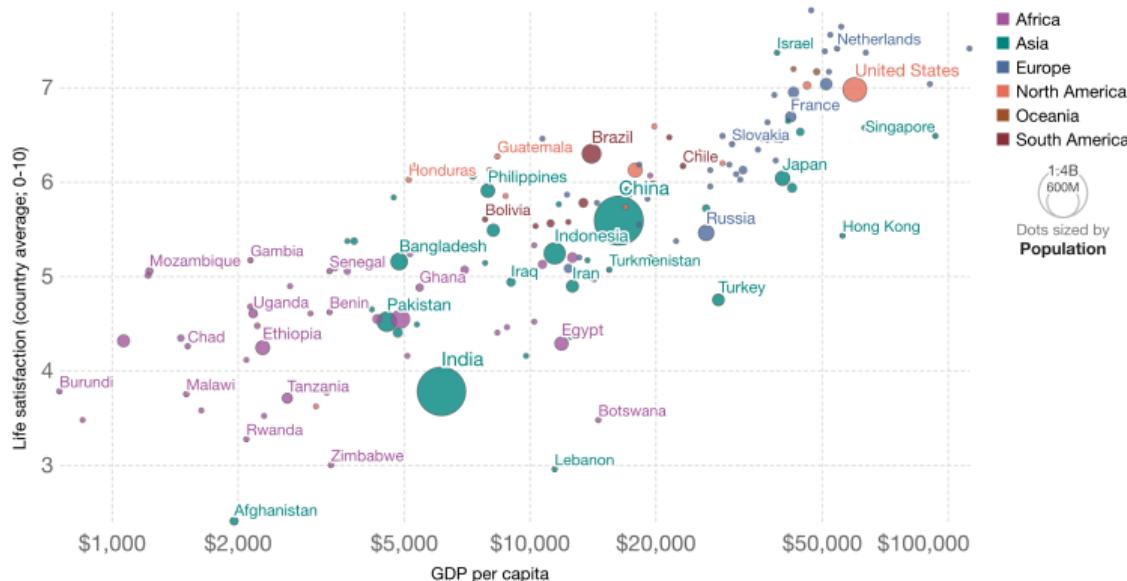
[OurWorldInData.org/income-inequality](https://ourworldindata.org/income-inequality) • CC BY

# Richer and Happier

## Self-reported life satisfaction vs GDP per capita, 2020

Our World  
in Data

The vertical axis shows the national average of the self-reported life satisfaction on a scale ranging from 0-10, where 10 is the highest possible life satisfaction. The horizontal axis shows GDP per capita adjusted for inflation and cross-country price differences.



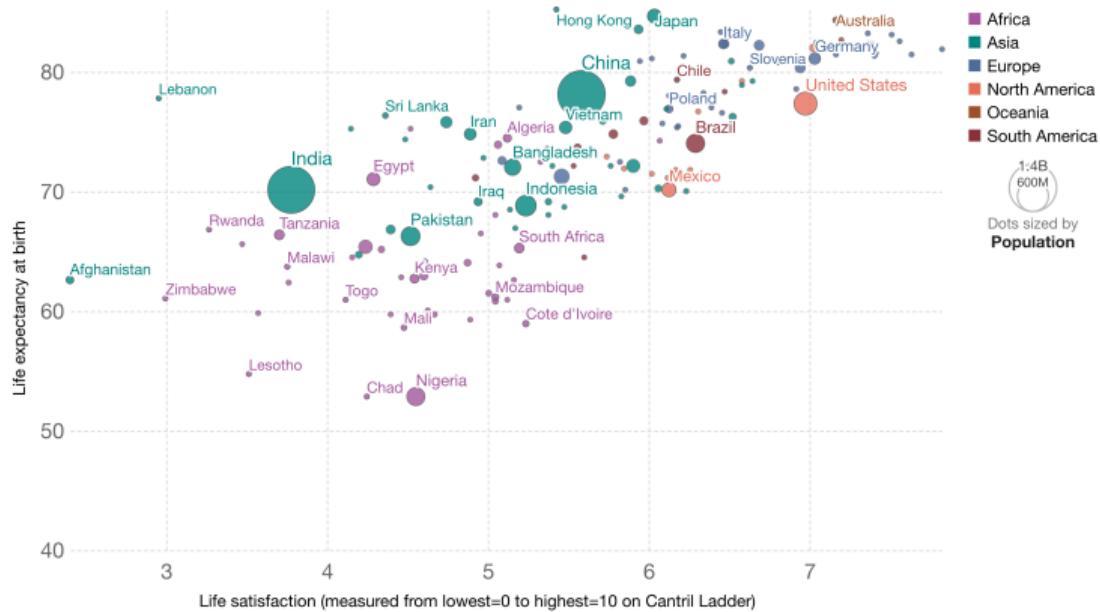
Source: World Happiness Report (2022); Data compiled from multiple sources by World Bank  
[OurWorldInData.org/happiness-and-life-satisfaction/](http://OurWorldInData.org/happiness-and-life-satisfaction/) • CC BY

# Be happy and live long

## Life satisfaction vs. life expectancy, 2020

Our World  
in Data

The vertical axis shows life expectancy at birth. The horizontal axis shows self-reported life satisfaction in the Cantril Ladder (0-10 point scale with higher values representing higher life satisfaction).



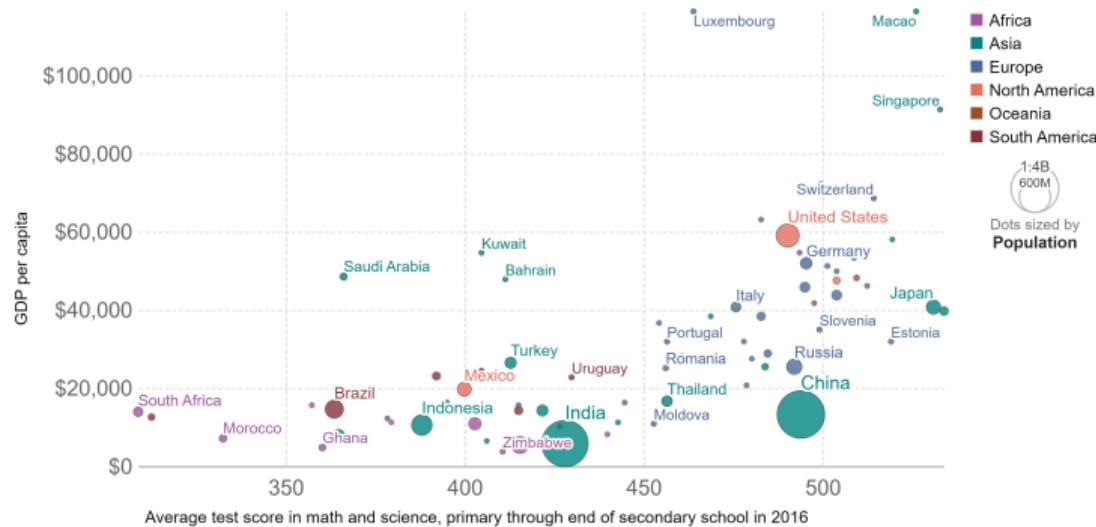
Source: United Nations – Population Division (2022); World Happiness Report (2022) OurWorldInData.org/happiness-and-life-satisfaction • CC BY

# High GDP High Math and Science

Our World  
in Data

## Average test score in mathematics and science vs. GDP per capita, 2016

Shown on the x-axis are Hanushek and Woessmann's combination of scores from international student achievement tests. The scores are standardized to the PISA test scale, so that the OECD countries have a mean of 500 and a standard deviation of 100. The test scores are here plotted against GDP per capita in 2016, but the test scores are not from a particular year, and instead are the average of all standardized math and science test scores each country participated in.



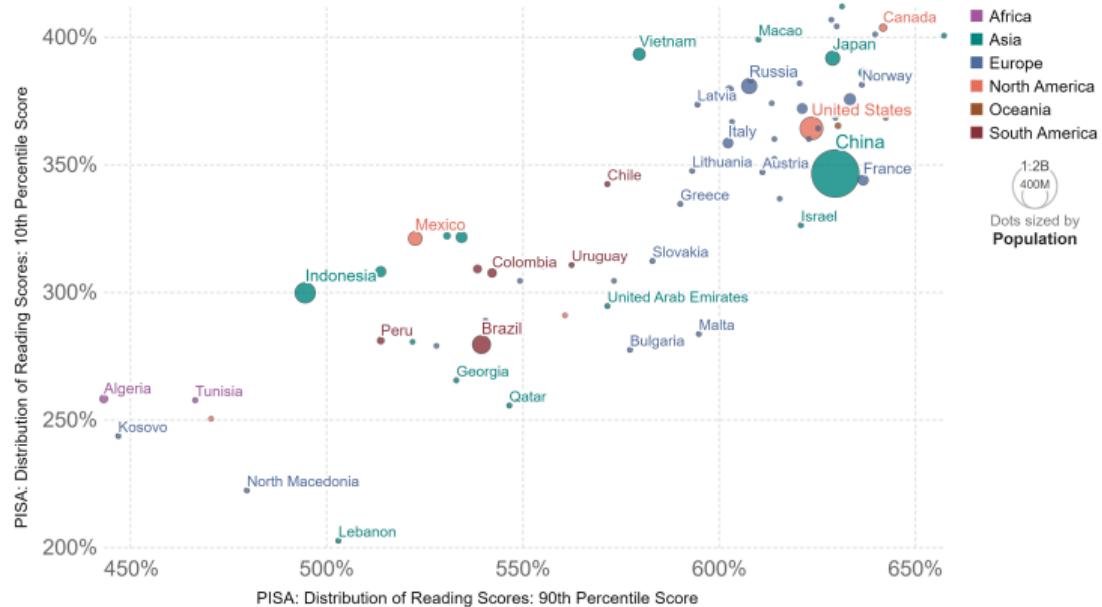
Source: Data compiled from multiple sources by World Bank, Hanushek and Woessmann (2012)  
OurWorldInData.org/quality-of-education • CC BY

# PISA and GDP

Our World  
in Data

## PISA score of the best and worst student by country, 2015

The visualization shows the correlation between the PISA scores in the reading dimension at the 90th and the 10th percentile.



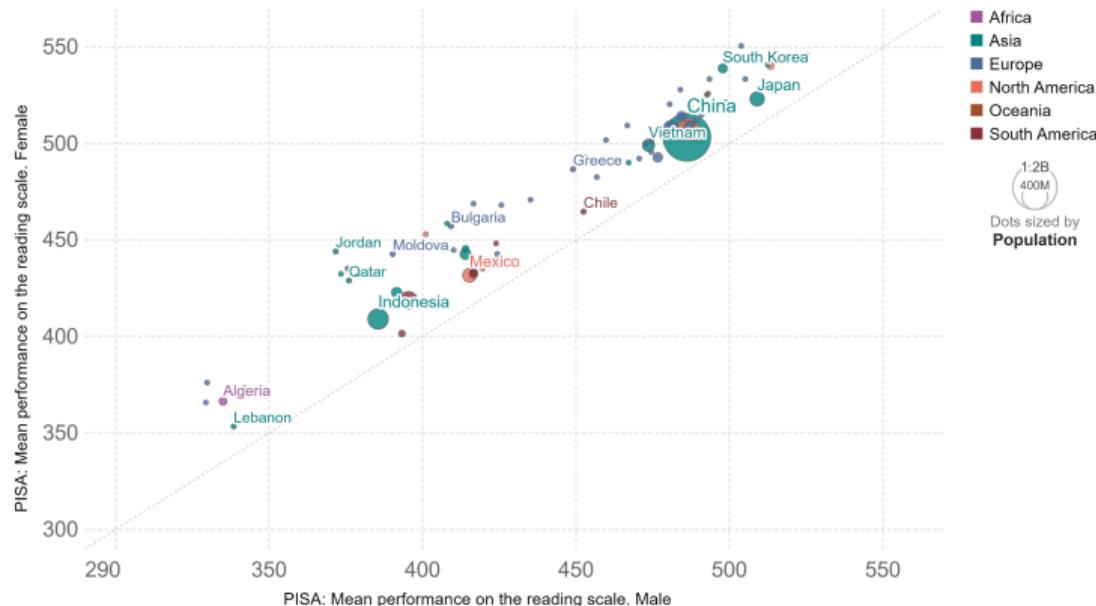
Source: OECD Programme for International Student Assessment (PISA)

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# Everywhere: Girls > Boys

## PISA: Mean performance on the reading scale, by sex, 2015

Average score of 15-year-old students on the PISA reading scale. The metric for the overall reading scale is based on a mean for participating OECD countries set at 500, with a standard deviation of 100.



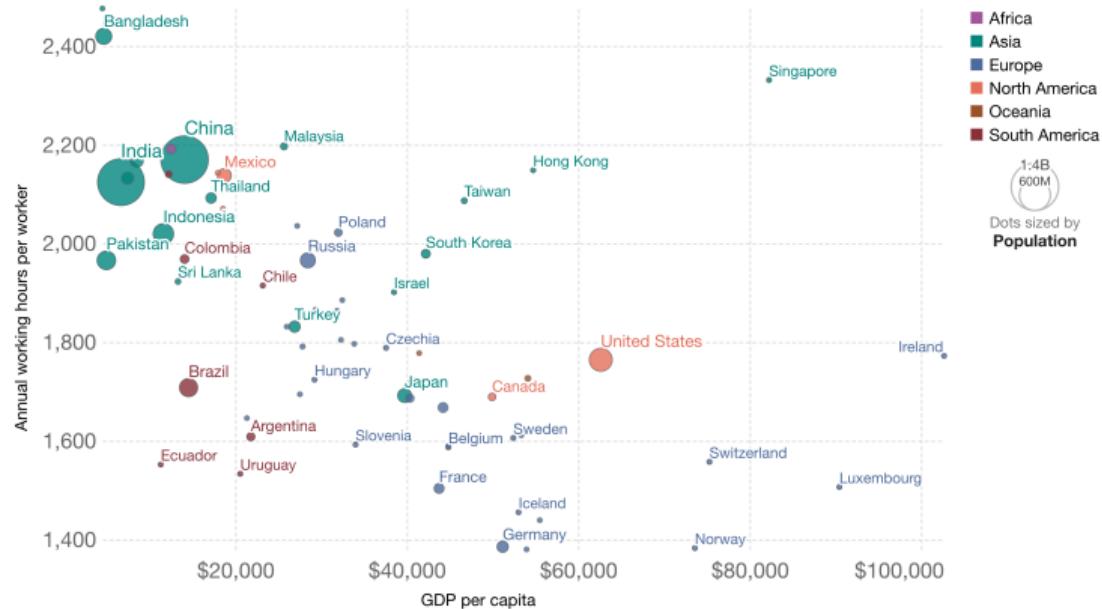
Source: OECD Programme for International Student Assessment (PISA)

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## **Get richer and work less**

### Annual working hours vs. GDP per capita

Working hours are the annual average per worker. GDP per capita is adjusted for differences in the cost of living between countries, and for inflation.



Source: Feenstra et al. (2015); Penn World Table 10.0

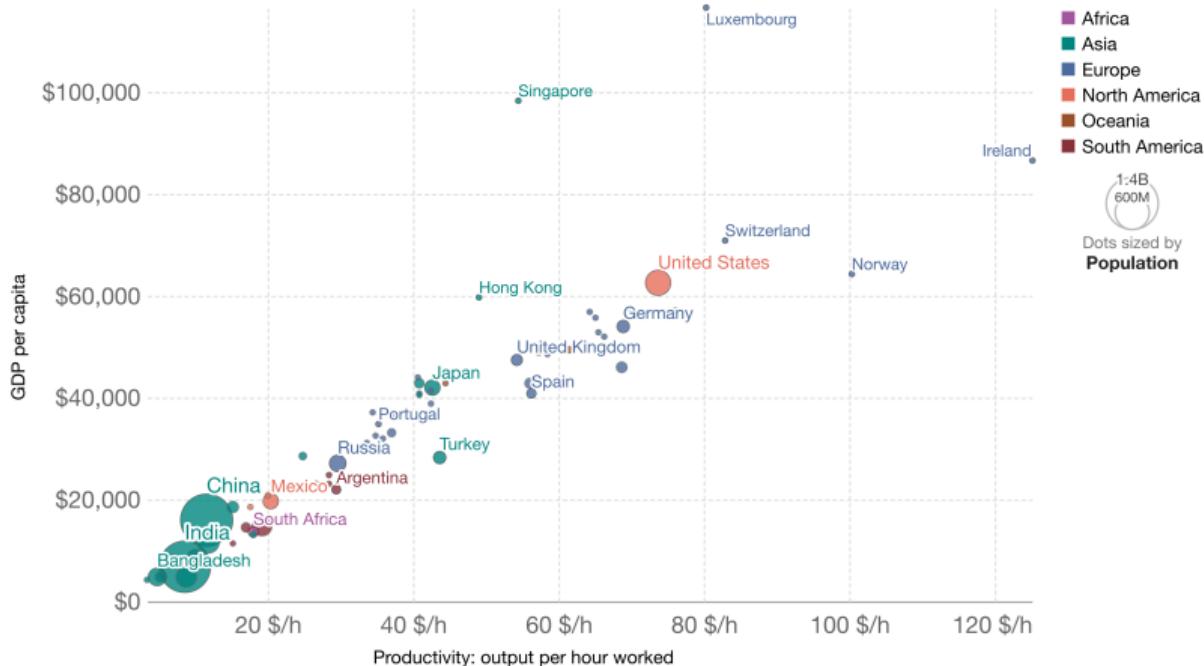
OurWorldInData.org/working-hours • CC BY

# Work but effectively please!

## GDP per capita vs. labor productivity, 2019

Gross domestic product (GDP) per capita measured in international-\$, versus labor productivity measured as GDP per hour worked.

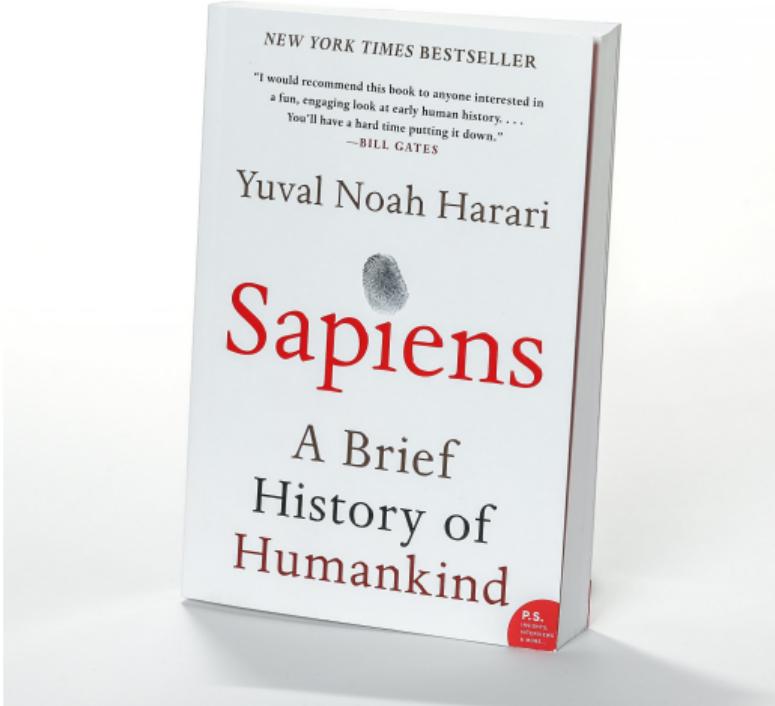
Our World  
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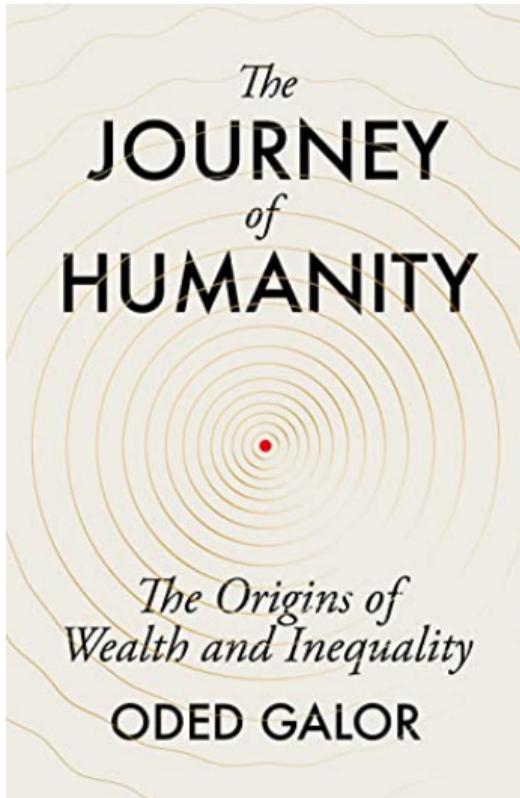
Source: Data compiled from multiple sources by World Bank, Feenstra et al. (2015), Penn World Table 10.0

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## Book Advices



YUVAL NOAH HARARI A Historian.



ODED GALOR An Economist.

## Review Question 1

- ▶ List at least three reasons why higher GDP per capita does not necessarily imply higher **welfare**.
- ▶ Does this mean we should not care that much about cross-country income differences?
- ▶ Explain why or why not.

## Question 1

- ▶ What does the GDP include?

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$$Y = C + I + G + NX$$

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# Question 1

- ▶ What does the GDP include?

$$Y = C + I + G + NX$$

- ▶ What does the GDP exclude?
- ▶ Many...

- \* Life Expectancy
- \* Leisure Time / Working Time
- \* Inequality
- \* Pollution / Weather
- \* Education / Health ...

## Reason for Welfare $\neq$ GDP

- ▶ Higher GDP might be brought about by a higher labour supply and less leisure time
- ▶ Higher levels of production might cause higher levels of pollution, which might decrease utility
- ▶ Higher GDP might raise individual aspirations, so that a given amount of consumption might generate less utility
- ▶ Human welfare depends on many factors which are not directly captured by GDP, such as health, education, life expectancy, weather..

## But...

- ▶ Still, we care greatly about cross-country income differences since GDP appears to be strongly correlated with alternative measures incorporating some of the factors listed above (while being at the same time easier to measure and compare across countries compared to such measures).
- ▶ For example, according to Jones and Klenow (2016) GDP per person is an informative indicator of welfare across a broad range of countries: the two measures have a correlation of **0.98**.
- ▶ Moreover, a country's productive capacity is likely to be a key factor in generating improvements in several welfare-relevant domains, such as health and educational opportunities

TABLE 7—WELFARE ACROSS COUNTRIES IN 2007: MACRO DATA

Country	Welfare $\lambda$	Per capita income	log ratio	Decomposition			
				LifeExp	C/Y	Leisure	C ineq.
United States	100.0	100.0	0.000	0.000 77.8	0.000 0.845	0.000 836	0.000 0.658
Sweden	91.2	79.4	0.139	0.181 80.9	-0.186 0.701	0.010 807	0.135 0.404
France	91.1	70.3	0.259	0.176 80.8	-0.085 0.776	0.063 629	0.106 0.471
Japan	82.6	71.3	0.147	0.265 82.5	-0.154 0.724	-0.028 912	0.063 0.554
Norway	81.0	112.8	-0.331	0.148 80.4	-0.598 0.464	0.019 780	0.100 0.483
Germany	77.4	74.4	0.039	0.098 79.5	-0.195 0.695	0.047 687	0.089 0.506
Ireland	69.6	96.4	-0.325	0.069 79.0	-0.454 0.536	-0.022 896	0.082 0.519
Hong Kong	59.0	83.4	-0.345	0.239 82.4	-0.433 0.548	-0.151 1,194	-0.000 0.658
Singapore	56.7	117.1	-0.726	0.139 80.4	-0.685 0.426	-0.180 1,251	-0.000 0.658
South Korea	45.3	58.3	-0.252	0.078 79.3	-0.290 0.632	-0.116 1,120	0.076 0.531
Argentina	21.8	26.2	-0.181	-0.121 75.1	-0.108 0.759	0.048 684	-0.000 0.658
Chile	19.7	30.9	-0.451	0.029 78.5	-0.254 0.655	-0.026 908	-0.199 0.912
Thailand	10.9	18.1	-0.507	-0.158 73.5	-0.207 0.687	-0.043 951	-0.099 0.794
South Africa	4.5	17.4	-1.351	-0.931 51.0	-0.053 0.801	0.061 636	-0.427 1.135
Botswana	4.3	25.1	-1.767	-0.852 52.1	-0.574 0.476	-0.008 859	-0.333 1.048
Vietnam	4.0	5.9	-0.378	-0.082 74.2	-0.269 0.645	-0.020 893	-0.006 0.668
Zimbabwe	3.1	8.3	-0.972	-0.983 0.155	-0.050 0.155	-0.094 -0.050	

# Recommendation...



The book ends by making the case that GDP was a good measure for the twentieth century but is increasingly inappropriate for a twenty-first-century economy driven by innovation, services, and intangible goods.

# Question?

## Review Questions 2

Consider an economy where the production function is

$$Y(K, L) = 10K^{1/2}L^{1/2}$$

Assume that the supply of capital and labor are fixed at  $\bar{K} = 100$  and  $\bar{L} = 36$ .

► Compute the equilibrium levels of

- \* output,  $Y$
- \* the wage rate,  $w$
- \* the rental rate of capital,  $r$

## Review Questions 2

By substituting  $\bar{K} = 100$  and  $\bar{L} = 36$  in the production function, we get

$$Y = 10 \times 10 \times 6 = 600$$

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$$\omega = MPL = 5 \frac{\bar{K}^{\frac{1}{2}}}{\bar{L}^{\frac{1}{2}}} = \frac{25}{3}$$

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By substituting  $\overline{K} = 100$  and  $\overline{L} = 36$  in the production function, we get

$$Y = 10 \times 10 \times 6 = 600$$

Factor prices are given by

$$\omega = MPL = 5 \frac{\overline{K}^{\frac{1}{2}}}{\overline{L}^{\frac{1}{2}}} = \frac{25}{3}$$

$$r = MPK = 5 \frac{\overline{L}^{\frac{1}{2}}}{\overline{K}^{\frac{1}{2}}} = 3$$

# Question?

# Class Questions

## Class Question 1

Consider the model of production discussed in the lecture. Assume that the production function is

$$Y(K, L) = K^\alpha L^{1-\alpha}$$

and that the supplies of capital and labour are fixed and exogenous.

- (a) Write down an expression for the equilibrium wage and rental rate of capital. Represent graphically the equilibrium in the market for labour and capital.

## Question 1a

$$\omega^* = (1 - \alpha) \left( \frac{\bar{K}}{\bar{L}} \right)^\alpha$$

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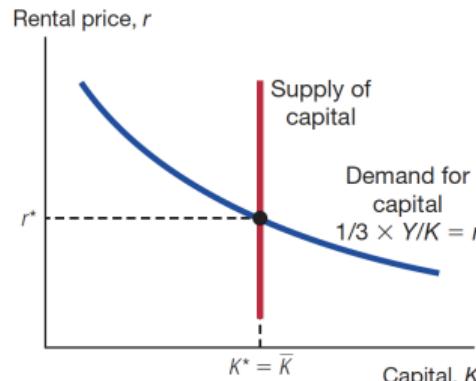
$$r^* = \alpha \left( \frac{\bar{L}}{\bar{K}} \right)^{1-\alpha}$$

# Question 1a

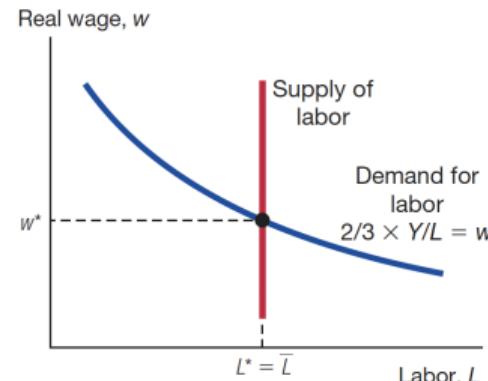
$$\omega^* = (1 - \alpha) \left( \frac{\bar{K}}{\bar{L}} \right)^\alpha$$

$$r^* = \alpha \left( \frac{\bar{L}}{\bar{K}} \right)^{1-\alpha}$$

## Supply and Demand in the Capital and Labor Markets



(a) The capital market



(b) The labor market

## Question 1b

Assume now that the government introduces an employment subsidy  $s \in (0, 1)$ , so that firms pay only a fraction  $s$  of the employees' wages. How are the equilibrium levels of labour, capital, the wage rate and the rental rate of capital affected? Show both analytically and graphically.

## Question 1b

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$$\omega^1 = \left( \frac{1-\alpha}{1-s} \right) \left( \frac{\bar{K}}{\bar{L}} \right)^\alpha$$

## Question 1b

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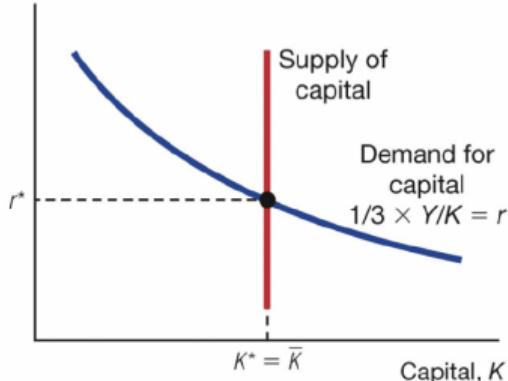
$$r^2 = \alpha \left( \frac{\bar{L}}{\bar{K}} \right)^{1-\alpha}$$

The equilibrium levels of capital and labour do not change. The wage increases, while the rental rate of capital is unaffected. **Graph:** the labour demand curve shifts up in the labour market graph; nothing happens in the market for capital.

# Question 1b

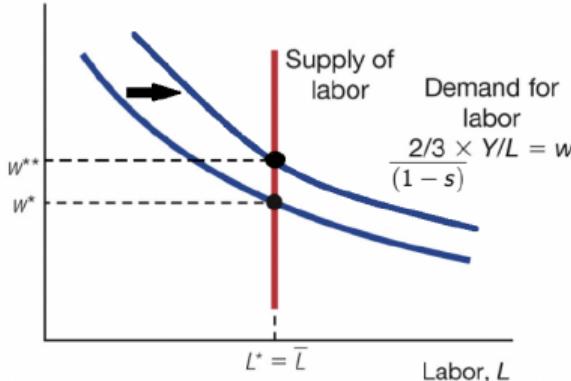
## Supply and Demand in the Capital and Labor Markets

Rental price,  $r$



(a) The capital market

Real wage,  $w$



(b) The labor market

$$\omega^{**} = \left( \frac{1-\alpha}{1-s} \right) \left( \frac{\bar{K}}{\bar{L}} \right)^\alpha$$

$$r^* = \alpha \left( \frac{\bar{L}}{\bar{K}} \right)^{1-\alpha}$$

$$\omega^{**} > \omega^*.$$

## Question 1c

Assume now instead that the government introduces a tax on revenues  $\tau \in (0, 1)$  (payroll and capital costs are not deductible). How are the equilibrium levels of labour, capital, the wage rate and the rental rate of capital affected compared to part (a)? Show both analytically and graphically.

## Question 1c

Assume now instead that the government introduces a tax on revenues  $\tau \in (0, 1)$  (payroll and capital costs are not deductible). How are the equilibrium levels of labour, capital, the wage rate and the rental rate of capital affected compared to part (a)? Show both analytically and graphically.

$$\omega^2 = (1 - \tau)(1 - \alpha) \left( \frac{\bar{K}}{\bar{L}} \right)^\alpha$$

## Question 1c

Assume now instead that the government introduces a tax on revenues  $\tau \in (0, 1)$  (payroll and capital costs are not deductible). How are the equilibrium levels of labour, capital, the wage rate and the rental rate of capital affected compared to part (a)? Show both analytically and graphically.

$$\omega^2 = (1 - \tau)(1 - \alpha) \left( \frac{\bar{K}}{\bar{L}} \right)^\alpha$$

$$r^2 = (1 - \tau)\alpha \left( \frac{\bar{L}}{\bar{K}} \right)^{1-\alpha}$$

The equilibrium levels of capital and labour do not change. Both the wage and the rental rate of capital decrease.

Graph: the labour demand curve and the capital demand curve both shift down.

## Question 1.d

Finally, assume that the government implements a lump-sum transfer to firms  $T > 0$ . How are the equilibrium levels of labour, capital, the wage rate and the rental rate of capital affected compared to part (a)? Show both analytically and graphically.

## Question 1.d

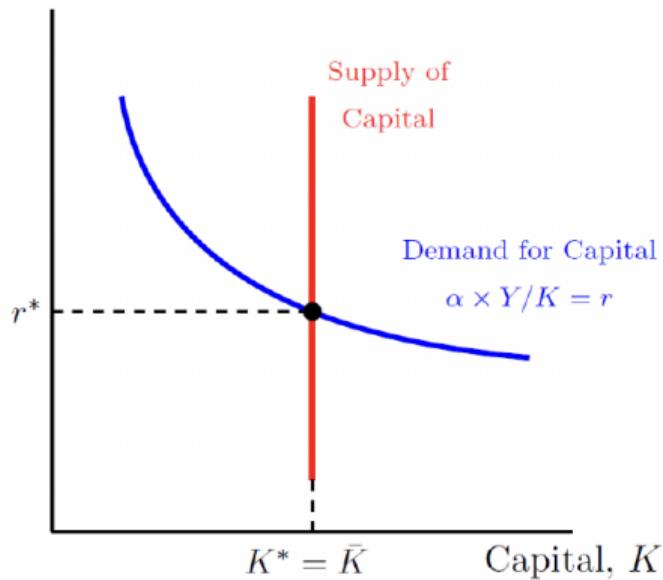
Finally, assume that the government implements a lump-sum transfer to firms  $T > 0$ . How are the equilibrium levels of labour, capital, the wage rate and the rental rate of capital affected compared to part (a)? Show both analytically and graphically.

All equilibrium quantities are identically to part (a). No changes in the graphs.

## Question 1b

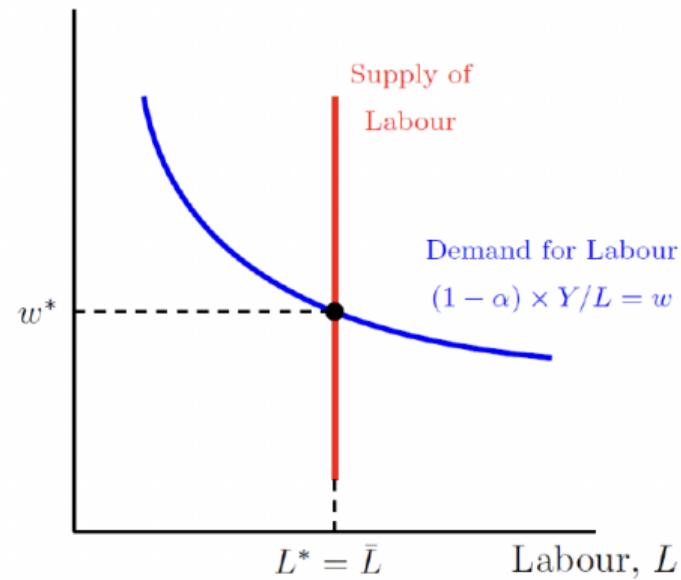
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Rental price,  $r$



The capital market

Real wage,  $w$



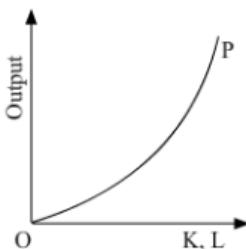
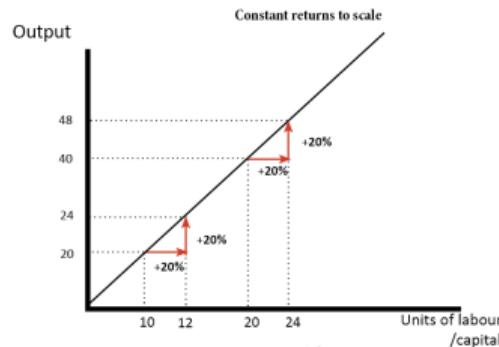
The labour market

# Question?

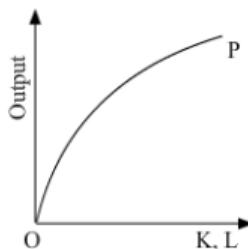
# Question 2

# CONSTANT RETURN TO SCALE

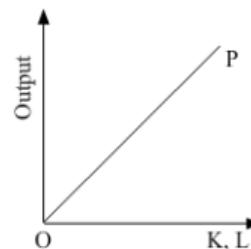
Graphically;



(a) Increasing Returns  
to Scale



(b) Decreasing Returns  
to Scale



(c) Constant Returns  
to Scale

## Question 2

Do the following production functions display increasing, constant or decreasing returns to scale in K and L? Assume that A is some positive fixed number.

(a)  $Y = K^{\frac{1}{2}} L^{\frac{1}{2}}$

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**Constant.**  $(\gamma K)^{\frac{1}{2}}(\gamma L)^{\frac{1}{2}} = \gamma Y$  for any  $\gamma > 0$

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(b)  $Y = K^{\frac{2}{3}}L^{\frac{2}{3}}$

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(b)  $Y = K^{\frac{2}{3}}L^{\frac{2}{3}}$

**Increasing.**  $(\gamma K)^{\frac{2}{3}}(\gamma L)^{\frac{2}{3}} = \gamma^{\frac{4}{3}}Y > \gamma Y$  for any  $\gamma > 1$

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**Constant.**  $(\gamma K)^{\frac{1}{2}}(\gamma L)^{\frac{1}{2}} = \gamma Y$  for any  $\gamma > 0$

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**Increasing.**  $(\gamma K)^{\frac{2}{3}}(\gamma L)^{\frac{2}{3}} = \gamma^{\frac{4}{3}}Y > \gamma Y$  for any  $\gamma > 1$

(c)  $Y = K^{\frac{1}{3}}L^{\frac{1}{2}}$

**Decreasing.**  $(\gamma K)^{\frac{1}{3}}(\gamma L)^{\frac{1}{2}} = \gamma^{\frac{5}{6}}Y < \gamma Y$  for any  $\gamma > 1$

## Question 2

Do the following production functions display increasing, constant or decreasing returns to scale in K and L? Assume that A is some positive fixed number.

(a)  $Y = K^{\frac{1}{2}}L^{\frac{1}{2}}$

**Constant.**  $(\gamma K)^{\frac{1}{2}}(\gamma L)^{\frac{1}{2}} = \gamma Y$  for any  $\gamma > 0$

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- ▶ Consider the production function

$$Y_c = \overline{A}_c K_c^\alpha (h_c L_c)^{1-\alpha}$$

where  $h_c$  is *human capital* per worker, i.e. the productivity of a given worker in country  $c$ . The value of  $h_c$  might be different across countries due to different levels of educational attainment, health, experience, and so on.

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- (b) Derive an expression for the equilibrium wage in country  $c$ . What factors might explain why wages are different across countries?

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$$w_c = (1 - \alpha) \overline{A}_c k_c^\alpha (h_c)^{1-\alpha}$$

Wages might differ across countries because of differences in productivity, physical capital per worker and human capital per worker.

# Self Study Questions

(c) How would you calibrate  $\alpha$ ?

$\alpha = 1/3$  to match the capital share.

## Self Study Questions

- (d) In India, physical capital per worker and GDP per worker are respectively 8% and 10% of the corresponding quantity in the US. Some economists argue that the income gap between the two countries might be entirely explained by differences in human capital per worker, while  $A_{US} = A_{INDIA}$ . How much should  $h_{INDIA}/h_{US}$  be for this to be true?

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$$\frac{y_{INDIA}}{y_{US}} = \frac{\bar{A}_{INDIA}}{\bar{A}_{US}} \left( \frac{k_{INDIA}}{k_{US}} \right)^\alpha \left( \frac{h_{INDIA}}{h_{US}} \right)^{1-\alpha}$$

$$0.1 = \frac{\bar{A}_{INDIA}}{\bar{A}_{US}} (0.08)^{1/3} \left( \frac{h_{INDIA}}{h_{US}} \right)^{2/3}$$

If  $\frac{\bar{A}_{INDIA}}{\bar{A}_{US}} = 1$ , then  $h_{INDIA}/h_{US} = 0.11$ . Human capital per worker should be about 10 times higher in the US.