# EC916: Topics in Global Finance

Fatih Kansoy

Lecture - 1 -

*University of Warwick

# Today: Outline

# Course Organisation

- **Synchronous-Live Lectures:**
  - Weeks 16-24 Wednesdays 10:00-11:00 AM via MS Team (1 Hour)
- **Course website**:
  - https://warwick.ac.uk/ec916
- **Course Materials - Moodle**:
  - https://moodle.warwick.ac.uk/course/view.php?id=52928
- **Announcements & News**:
  - Check your emails, MS Teams, and Moodle page regularly

# Course Assessment

- The module is examined in a 2-hour written exam in May (80%) and one 1-hour end-of-term test (20%).
- The EC916 Topics in Global Finance end-of-term test will be held on Week 24, Thu 13:00 - 14:00 March 18 - 13:00 - 14:00, and includes all topics covered in lectures from week 16 to 22.
- Previous years' end-of-term tests are posted on the course website.
- All topics covered up to week 24 will be included in the final exam in May.

# Seminars

- Weeks 17, 19, 21, and 23 (check your Tabula).

**Problem sets:**

- Problem sets will be posted on Moodle prior to the seminar.
- You should work on the problem sets before the seminar.
- Only some problems and questions will be covered during seminars.
- Solutions will be posted online the week after the seminars.

# Pre-Requisite Readings

- A challenging technical course. It covers advanced topics in global finance (including macroeconomics and finance).
- It employs a variety of theoretical and econometric tools (math and stat tools: static and dynamic optimisation, panel data, IV, etc.).
- Need to be very confortable with math and statistics or the term 1 core modules.
- There is a **list of pre-requisite readings**, including mathematics and econometric techniques.
- Students should be familiar with all the pre-requisites, as these are basic tools for the understanding of the course and will not be cover in lectures or seminars.

→ Read "Information for Students.pdf" in Moodle. There is a **list of pre-requisite readings**, including mathematics and econometric techniques and problem sets.

# Readings

- There is no textbook for this course. The required readings consist of a collection of book chapters, journal articles, and working papers. The reading list also includes policy-oriented reports and opinion pieces.
- Required readings are marked with an asterisk in the syllabus *.
- These readings may be subject to change; I will post the required readings before each class on the course website.

# Aim and Approach

- **Aim:** The aim of the module is to equip students with analytical tools and knowledge to study and understand key issues in international finance.

- **Approach:** The course is built around analytical models. Although real-world examples will appear throughout, the course will rely heavily on theoretical analysis. We will use theoretical models to understand current events in the global economy and to analyse policy responses.

# Topics

- Globalisation, Global Finance, Global Imbalances ..................... $\approx$ Lecture 1
- Intertemporal Theory of Current Account ..................... $\approx$ Lecture 2
- Dynamics of Small Open Economies and CA Sustainability ..................... $\approx$ Lecture 3
- International Financial Integration: Models, Evidence, Debates ..................... $\approx$ Lecture 4
- Risk Sharing, Integration, and the Transmission of Shocks ..................... $\approx$ Lecture 5
- Banking, Banking Crises ..................... $\approx$ Lecture 6
- Equilibrium in Financial Markets and Capital Controls ..................... $\approx$ Lecture 7
- Sovereign Debt Crises ..................... $\approx$ Lecture 8
- Balance of Payment Crisis ..................... $\approx$ Lecture 9

# Instructor: Deva Velivela

- Dr. Deva Velivela
- PhD in Economics - The University of Glasgow
- Office: S1.128
- Office Hour: Wednesdays 11:00 - 13:00
- Appointment: Click here to book
- Email: `Deva.Velivela@warwick.ac.uk`

# Instructor: Fatih Kansoy

- PhD in Economics - School of Economic | University of Nottingham
- Worked Nottingham Business School, Joined Warwick 2019
- MSc in Economics - University of Warwick
- Office: S0.86 No online office hours
- Office Hour: Thursday 08:00 - 10:00
- Appointment: `https://calendly.com/kansoy`
- Email: `f.kansoy@warwick.ac.uk`
- Website: `www.kansoy.me`
- **kansoy**

module hashtag is #EC916

Generally academic

# This Lecture

1. Global, Globalisation, Global Finance
2. Global Imnbalances
3. Intertemporal Approach of the Current Account

Readings: -For 1 and 2:

- Trade and Globalization by Esteban Ortiz-Ospina and Diana Beltekian in **Our World in Data**
- Chapters 1 in S. Schmitt-Grohe, Uribe, and Woodford (2019), **International Macroeconomics**

Readings: -For 3:

- Sections 1.1 and 1.2 in M. Obstfeld and K. Rogoff (1996). **Foundations of International Macroeconomics.**
- ******* Chapters 2, and 3 in S. Schmitt-Grohe, Uribe, and Woodford (2019), **International Macroeconomics**..
- Chapters 9 in Julio Garin, Robert Lester, Eric Sims **Intermediate Macroeconomics**

# Global vs International

- Why the course name is Global Finance but not International Finance?
- According to the Merriam-Webster Dictionary:

  **international** “involving two or more countries; occurring between countries,”

  while it defines

  `global` as “involving the entire world”.
- The word **global** is increasingly being used in the context of anything that applies to the whole world instead of the word international.
- Thus, we have global studies, global warming, global economy, global security, and **Global Finance**.

# Globalization

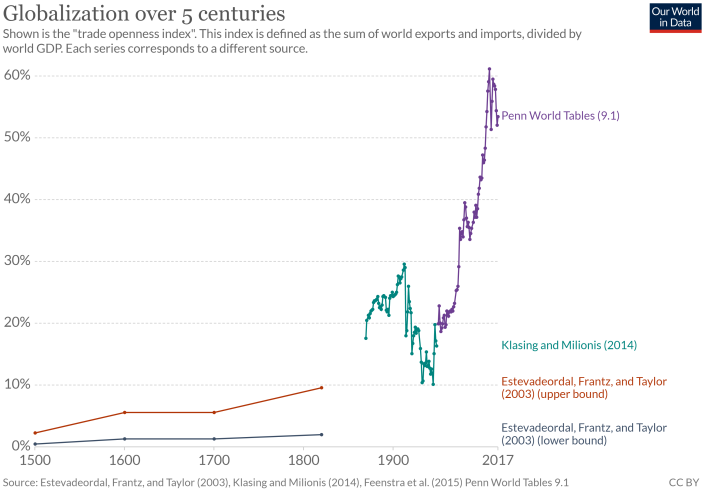
- Then what is globalisation?
- According to PIIE, globalisation is the word used to describe the growing interdependence of the world's economies, cultures, and populations, brought about by cross-border trade in goods and services, technology, and flows of investment, people, and information.


Let's watch a video:
`https://www.piie.com/microsites/globalization/what-is-globalization`

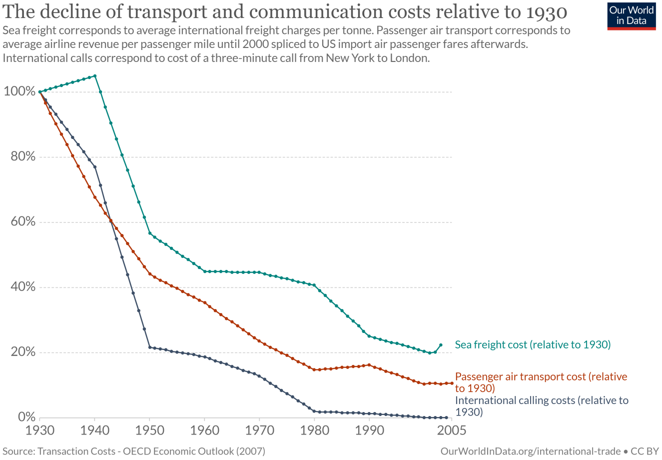
# Globalisation

- Over the last five centuries there are two main globalisation waves.
- Until 1800 there was a long period characterised by persistently low international trade.
- This then changed over the course of the $19^{th}$ century, when technological advances triggered a period of marked growth in world trade – the so-called '**first wave of globalisation**'.
- The first wave of globalisation end with the First World War.

# The First Wave



# The Second Wave



# Globalisation

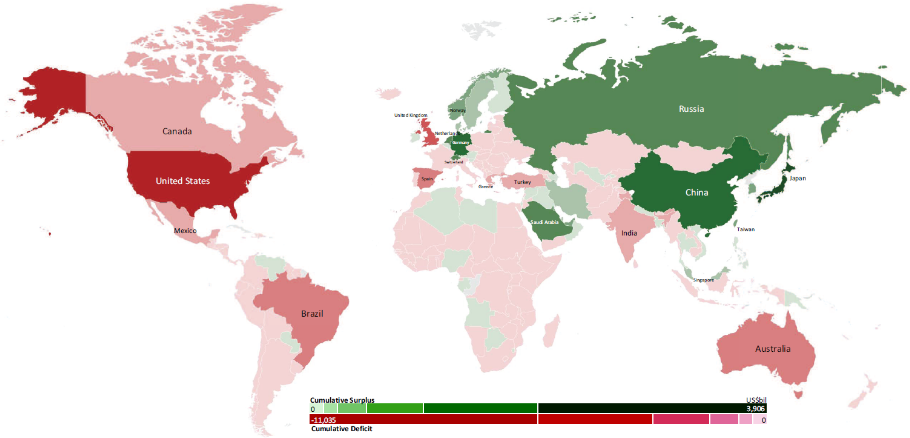
- The world-wide expansion of trade after the Second World War was largely possible because of reductions in transaction costs
- The reductions in transaction costs had an impact, not only on the volumes of trade, but also on the types of exchanges that were possible and profitable.
- The increasing the level of globalisation after the Second World War is known as **the second wave of globalisation**.
- For more visit `https://ourworldindata.org/trade-and-globalization`

# Global Imbalances

- In the past three decades, the world has witnessed the emergence of large external debt positions in some countries and large external asset positions in others.
- The United States became the largest external debtor in the world in the late 1980s and has maintained this position ever since.
- At the same time, China, Japan, and Germany hold large asset positions against the rest of the world.
- This phenomenon has come to be known as global imbalances.

# Global Imbalances

Cumulative Current Account Balances Around the World: 1980-2017



# Global Imbalances

- The map (from SUW-2019 Book ) presents the accumulated current account balances from 1980 to 2017 for 212 countries.
- The map shows that the country with the biggest accumulated current account deficit (brightest red) is the United States.
- The countries that have been financing these deficits (deepest green) are Japan, China, Germany, and oil and gas exporting countries (members of OPEC, Russia, and Norway).
- The United States appears in dark red and China in dark green, reflecting the fact that the former is the world's largest external debtor and the latter one of the world's largest creditors.

# The Trade Balance and the Current Account

- We can now answer the question posed at the beginning: **What determines the trade balance (TB) and the current account (CA)?** The trade balance is the difference between output and consumption,

$$TB_1 = Q_1 - C_1 \qquad // \qquad TB_2 = Q_2 - C_2$$

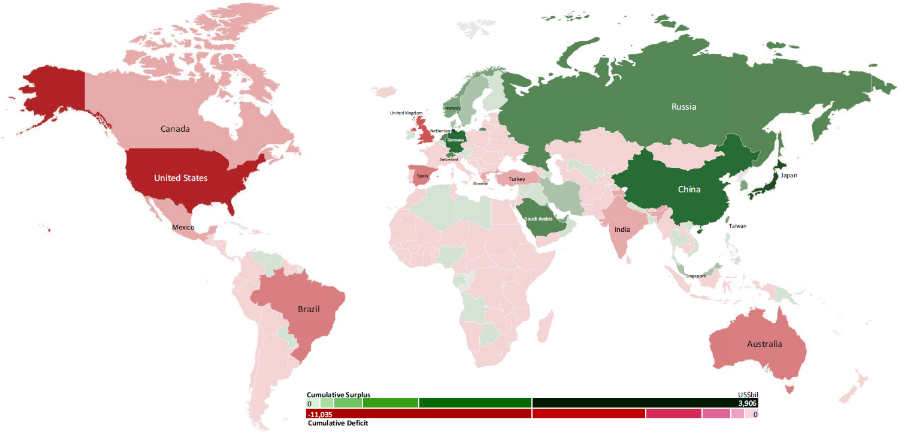
  The current account equals the trade balance plus net investment income (NII)

$$CA_1 = TB_1 + r_0 B_0 \qquad // \qquad CA_2 = TB_2 + r^* B_1$$

- We have shown that the **endogenous variables**, $C_1$ and $C_2$ and $r_1$, depend on **the exogenous variables** $r_0$, $B_0$, $Q_1$, $Q_2$, and $r^*$.
- Therefore, we have that preferences, endowments, the world interest rate, and the initial net international investment position of a country are the determinants of the trade balance and the current account.

# Can a Country Run a Perpetual TB Deficit?

Cumulative Current Account Balances Around the World: 1980-2017



# Can a Country Run a Perpetual TB Deficit?

# Can a Country Run a Perpetual TB Deficit?

- **It depends!** on whether the country is a net debtor or a net creditor.

- If it is a net debtor, that is, if its net international investment position is negative, then the answer is no. For in this case, the country will have to run a trade balance surplus at some point to service its debt.

- If the country is a net creditor of the rest of the world, that is, if its net international investment position is positive, then it can run a perpetual trade deficit and finance it with the interest generated by its net investments abroad.

# Can a Country Run a Perpetual TB Deficit?

Consider an economy that lasts for two periods. It starts period 1 with a net foreign asset position of $B_0^*$. Let $r$ denote the interest rate. Then, the country's net investment income in period 1 is given by $rB_0^*$. Let the trade balance be denoted $TB_1$. Then, the country's net international investment position at the end of period 1 is

$$B_1^* = (1+r)B_0^* + TB_1 \tag{1}$$

A similar expression holds in period 2

$$B_2^* = (1+r)B_1^* + TB_1 \tag{2}$$

At the end of period 2, the country cannot hold assets or debts, because no one will be alive in period 3 to collect (the world ends in period 2). This means that

$$B_2^* = 0 \tag{3}$$

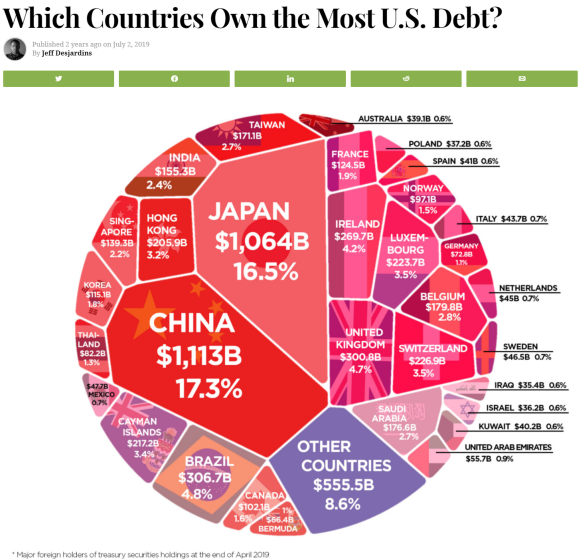
# Can a Country Run a Perpetual TB Deficit?

Combining (1), (2), and (3) yields

$$(1+r)B_0^* = -TB_1 - \frac{TB_2}{(1+r)} \tag{4}$$

- which states that the net foreign asset position (including interest) equals the present discounted value of its future trade deficits.
- It is clear from this expression that if the country is a net debtor, $B_0^* < 0$ then it must run a trade balance surplus at some point.
- However, if the country is a net creditor of the rest of the world, $B_0^* > 0$, then it can afford running trade deficits in both periods.
- This result holds not just for two-period economies, but for economies lasting any number of periods, including an infinite number of periods.
- Since the United States is a net debtor, the present analysis implies that it will have to revert its trade balance deficits at some point in the future.

# Can a Country Run a Perpetual CA Deficit?



# Can a Country Run a Perpetual CA Deficit?

# Can a Country Run a Perpetual CA Deficit?

**The answer to this question is, again, yes,** provided the country's initial net foreign asset position is positive. To see this, recall that, in the absence of valuation changes, the change in the net international investment position is the current account

$$CA_1 = B_1^* - B_0^*$$

Similarly, in period 2 we have

$$CA_2 = B_2^* - B_1^*$$

Combining these two expressions to eliminate $B_1^*$ and recalling that $B_2^* = 0$, we obtain

$$B_0^* = -CA_1 - CA_2$$

which implies that the country can run current account deficits in both periods only if the initial net asset position is positive. This result holds for economies lasting any finite number of periods.

# Current Account

In the absence of valuation changes the current account measures the change in the net international investment position of a country, that is,

$$CA_1 = B_t^* - B_{t-1}^*$$

where $CA_t$ denotes the country's current account in period $t$ and $B_t^*$ the country's net international investment position at the end of period $t$.

- If the current account is in deficit, $CA_t < 0$, then the net international investment position falls,

  $$B_t^* - B_{t-1}^* < 0$$

  .

- Similarly, if the current account displays a surplus, $CA_t > 0$, then the net international investment position improves.

$$B_t^* - B_{t-1}^* > 0$$

# Savings, Investment, and the Current Account

In any period, say period 1, savings, investment, and the current account are linked by the identity

$$CA_1 = S_1 - I_1$$

- This expression is intuitive. Savings in excess of what is needed to finance domestic investment must be allocated to purchases of foreign assets. But the change in the net foreign asset position is precisely the current account.
- To derive the above identity more formally, recall that a country's aggregate supply of goods and services in any given period is the sum of gross domestic product, denoted Q1, and imports, denoted $IM_1$.
- The aggregate demand for goods and services is the sum of private consumption, $C_1$, government consumption, $G_1$, investment, $I_1$, and exports, $X_1$:

$$Q_1 + IM_1 = C_1 + G_1 + I_1 + X_1$$

# Savings, Investment, and the Current Account

$$Q_1 + IM_1 = C_1 + G_1 + I_1 + X_1$$

Now add net investment income, $rB_0^*$, to both sides of the previous expression and recall that the trade balance is the difference between imports and exports, or $TB_1 = X_1 - IM_1$, to get

$$Q_1 + rB_0^* = C_1 + G_1 + I_1 + TB_1 + rB_0^*$$

The sum of GDP and net investment income is known as **National Income**, denoted $Y_1$.
Also, recall that the sum of net investment income and the trade balance is the current account,

$$CA_1 = rB_0^* + TB_1$$

# Domestic Absorption, National Income, and the CA

Thus, we can write

$$y_1 = C_1 + G_1 + I_1 + CA_1 \tag{5}$$

Finally, the difference between national income and private and public consumption is national savings, or

$$S_1 = Y_1 - C_1 - G_1$$

Combining this expression with the one above, we get the expression we were looking for

$$CA_1 = S_1 - I_1$$

# Domestic Absorption, National Income, and the CA

Domestic absorption is defined as the sum of private consumption, government consumption, and investment. Letting A1 denote domestic absorption, we have

$$A_1 = C_1 + G_1 + I_1$$

combining this expression with (5), we can express the current ac- count as

$$CA_1 = Y_1 - A_1$$

, which states that the current account is the gap between national income and the domestic absorption of goods and services.

# Changes in the NIIP and the CA

In the absent of valuation changes, current account surpluses increase a country's net foreign asset position and current account deficits decrease it.
The change in the country's net foreign asset position is

$$B_1^* - B_0^*$$

Thus we have that

$$CA_1 = B_1^* - B_0^*$$

# Changes in the NIIP and the CA

- A country that is a net external debtor cannot run a perpetual trade balance deficit.
- A country that is a net external debtor cannot run a perpetual deficit in the current account. This result applies to economies that last for any finite number of periods.
- We derived four alternative expressions for the current account:

$$CA_t = B_t^* - B_{t-1}^*$$

$$CA_t = rB_{t-1}^* + TB_t$$

$$CA_t = S_t - I_t$$

$$CA_t = Y_t - A_t$$

- You should always keep in mind that all four of the above expressions represent accounting identities that must be satisfied at all times in any economy.

# An Intertemporal Theory of the Current Account

# Introduction

- We have seen that the US is the world's largest external debtor and China is the one of the world's largest creditors.
- Why do some countries borrow and others lend? Why do some countries run trade balance deficits and others trade balance surpluses?
- This chapter addresses these and other related questions by building a model of an open economy to study the determinants of the trade balance and the current account.
- At the heart of this theory is the optimal intertemporal allocation of expenditure.
- Countries/Households will borrow or lend **to smooth consumption over time** in the face of uneven output streams.
- This resource allocation/exchanges across time is called **intertemporal trade**

# Small Open Endowment Economy

- We say that an economy is open when it trades in goods and financial assets with the rest of the world.
- We say that an economy is small when world prices and interest rates are independent of domestic economic conditions.
- The economic size of a country may not be related to its geographic size. Most countries in the world are small open economies.
- For example, Australia and Canada are geographically large, but economically small.
- On the other hand, Japan, Germany, the United Kingdom, and France are geographically small, but economically large.

# The Model Economy

- Consider a two-period small economy in which people live for two periods, $1$ and $2$,
- Households receive endowments $Q_1$ and $Q_2$ in periods, $1$ and $2$, respectively.
- Suppose that goods are perishable in the sense that they cannot be stored from one period to the next.
- However, they can reallocate resources between periods 1 and 2 via the international financial market.
- Specifically, assume that in period 1 each household is endowed with $B_0^*$ units of a bond.
- Initial asset holdings $B_0^*$ inherited from the past, paying the interest rate $r_0$ in period 1 and so generate interest income in the amount of $r_0 B_0^*$ in period 1.
- In period $1$, the household's income is therefore given by the sum of interest income, $r_0 B_0^*$, and the endowment of goods, $Q_1$, that is, period-1 income is equal to $r_0 B_0^* + Q_1$

# The Model Economy

- The household can allocate its income to two alternative uses, purchases of consumption goods, which we denote by $C_1$, and purchases (or sales) of bonds, $B_1^* - B_0^*$, where $B_1^*$ denotes bond holdings at the end of period 1.
- Thus, in period 1 the household faces the following budget constraint:

$$C_1 + B_1^* - B_0^* = r_0 B_0^* + Q_1 \tag{6}$$

- Similarly, in period 2 the household faces a budget constraint stating that consumption expenditure plus bond purchases must equal income,

$$C_2 + B_2^* - B_1^* = r_1 B_1^* + Q_2 \tag{7}$$

where $C_2$ denotes consumption in period 2, $r_1$ denotes the interest rate on bonds held between periods 1 and 2, and $B_2^*$ denotes bond holdings at the end of period 2.

# The Model Economy

- By the **no-Ponzi-game** constraint households are not allowed to leave any debt at the end of period 2, that is, $B_2^*$ must be greater than or equal to zero.
- Also, because the world is assumed to last for only 2 periods, agents will choose not to hold any positive amount of assets at the end of period 2. So $B_2^*$ must be less than or equal to zero. (**Rationality**)
- Thus, asset holdings at the end of period 2 must be exactly equal to

$$B_2^* = 0 \tag{8}$$

- This is referred to this terminal restriction as **the transversality condition.**

# the Intertemporal Budget Constraint

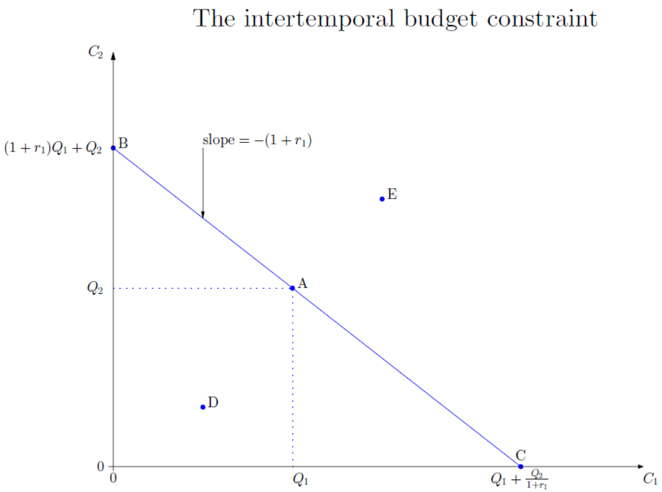
- Combining the period budget constraints (6), (7), and with the transversality condition (8) to eliminate $B_1^*$ and $B_2^*$ gives rise to **the intertemporal budget constraint of the household**:

$$C_1 + \frac{C_2}{1+r_1} = (1+r_0)B_0^* + Q_1 + \frac{Q_2}{1+r_1} \tag{9}$$

- The intertemporal budget constraint states that the present discounted value of consumption (the left-hand side) must be equal to the initial stock of wealth plus the present discounted value of the endowment stream (the right-hand side).

- The household chooses consumption in periods 1 and 2, $C_1$ and $C_2$, taking as given all other variables appearing in the intertemporal budget constraint (9), namely, $r_0$, $r_1$ , $B_0^*$ , $Q_1$, and $Q_2$.

# the Intertemporal Budget Constraint



# the Intertemporal Budget Constraint

- Figure above displays the pairs ($C_1$ and $C_2$) that satisfy the household's intertemporal budget constraint (9).
- For simplicity, we assume that the household's initial asset position is zero, that is, we assume that $B_0^* = 0$
- Then, clearly, the consumption path $C_1 = Q_1$ and $C_2 = Q_2$ (point A in the figure) satisfies the intertemporal budget constraint (9). In words, it is feasible for the household to consume its endowment in each period.
- For example, if the household chooses to allocate all of its lifetime income to consumption in period 2, it can do so by saving all of its period-1 endowment.
- In period 2, its income is the period-1 savings including interest, $(1 + r_1)Q_1$, plus its period-2 endowment, $Q_2$.
- Then, $C_2$ equals $(1 + r_1)Q_1 + Q_2$ and $C_1$ is, of course, nil.
- This consumption path is located at the intersection of the intertemporal budget constraint with the vertical axis (point B in the figure).

# Properties of the Intertemporal Budget Constraint

- At the opposite extreme (point C in the figure), if the household chooses to allocate its entire lifetime income to consumption in period 1, it has to borrow $Q_2/(1+r_1)$ units of goods in period 1.
- Then, $C_1$ equals the period-1 endowment plus the loan,. $Q_1 + Q_2/(1+r_1)$
- In period 2, the household must pay the principal of the loan, $Q_2/(1+r_1)$, plus interest, $(r_1)Q_2/(1+r_1)$, or a total of $Q_2$, to cancel its debt.
- As a result, period-2 consumption is zero, $C_2 = 0$, since all of the period-2 endowment must be used to retire the debt. This consumption path corresponds to the intersection of the intertemporal budget constraint with the horizontal axis (point C in the figure).

# Properties of the Intertemporal Budget Constraint

- The intertemporal budget constraint dictates that if the household wants to increase consumption in one period, it must reduce consumption in the other period.
- Specifically, for each additional unit of consumption in period 1, the household has to give up $(1 + r_1)$ units of consumption in period 2.
- It's downward sloping. Its slope is $-(1 + r_1)$, because if you sacrifice one unit of consumption today and put it in the bank for one period, you get $(1 + r_1)$ units next period.
- The set of feasible consumption paths ($C_1$, $C_2$ ) are those inside or at the borders of the triangle formed by the vertical axis, the horizontal axis, and the intertemporal budget constraint. Points A, B, C, and D are all feasible consumption paths.
- Points outside that triangle, such as point E, are infeasible. They violate the transversality condition because households leave unpaid debts at the end of period 2 ($B_2 < 0$).

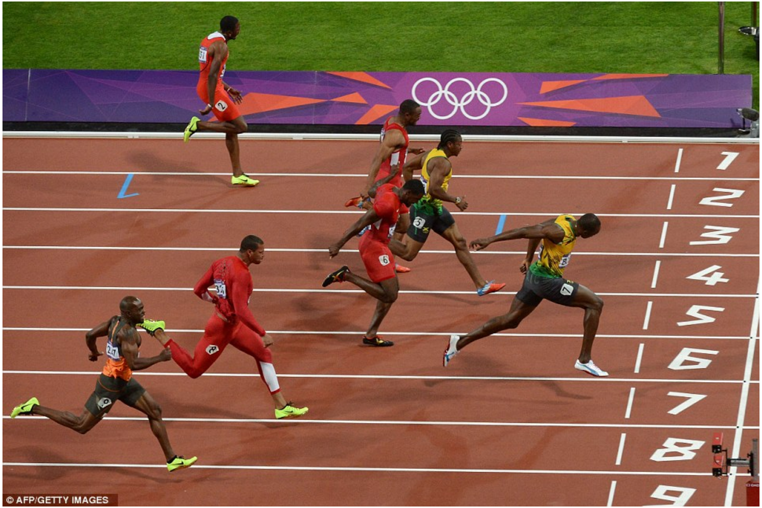
# Life Time Utility Function - What is it?

- We assume that the household's happiness increases with the consumption of goods in periods 1 and 2. Preferences for consumption in periods 1 and 2 are described by the lifetime utility function, which is assumed to be of the form

$$U(C_1) + \beta U(C_2) \tag{10}$$

- This function is known as **the lifetime utility function**. It indicates the level of satisfaction (or felicity) derived by the household from different consumption paths ($C_1$ and $C_2$).
- A utility function is a mathematical function that ranks bundles of consumption goods by assigning a number to each where larger numbers indicate preferred bundles
- Utility functions are ordinal rather than cardinal. A number system is ordinal if we only care about the ranking of the numbers not interested the magnitude of the numbers (cardinal).Thus, utility functions only rank bundles – they only indicate which one is better, not how much better it is than another bundle.

# Life Time Utility Function



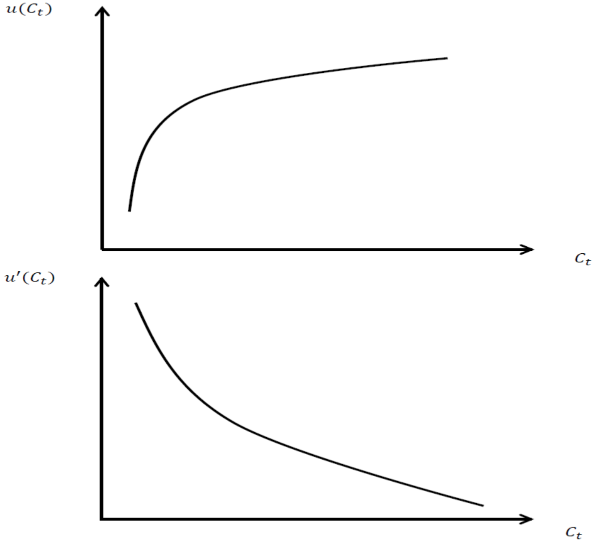
# Life Time Utility Function - Example

- To illustrate, Usain Bolt came 1st and Yohan Blake came 2nd in the London 2012 Olympic final of the 100m sprint.
- The numbers 1 and 2 are ordinal: they tell us that Bolt beat Yohan Blake, ($U_{\text{Bolt}} > U_{\text{Yohan}}$) but do not tell us that he was 1% faster or 10% faster.
- Ordinal means that utility functions only rank bundles – they only indicate which one is better, **not how much better** it is than another bundle.
- The actual finishing times were 9.63 seconds for Bolt and 9.75 for Yohan Blake. These numbers are cardinal: the ranking tells us who won, and the magnitudes tell us about the margin of the win.

# Life Time Utility Function - Properties

- The function $U(\cdot)$ is known as the period utility function and is assumed to be increasing and concave.
- First $U'(\cdot) > 0$ We refer to $U'(\cdot)$ as the marginal utility of consumption.
- Assuming that this is positive just means that **more is better** – more consumption yields more utility.
- Second, we assume that $U''(\cdot) < 0$. This says that there is **diminishing marginal utility**.
- As consumption gets higher, the marginal utility from more consumption gets smaller.
- Figure below plots a hypothetical utility function with these properties in the upper panel, and the marginal utility as a function of $C_t$ in the lower panel.

# Utility Function - Utility and Marginal Utility



# Life Time Utility Function - Examples

$$U(C_t) = \theta C_t, \qquad \theta > 0 \tag{11}$$

The utility function in (11) is a linear utility function. It features a positive marginal utility but the second derivative is zero, so this utility function does not exhibit diminishing marginal utility.

# Life Time Utility Function - Examples

$$U(C_t) = C_t - \frac{\theta}{2} C_t^2, \qquad \theta > 0 \tag{12}$$

The second utility function (12) is called a quadratic utility function. It features diminishing marginal utility, but it does not always feature positive marginal utility – there exists a satiation point about which utility is decreasing in consumption. In particular, if $C_t > 1/\theta$, then marginal utility is negative.

# Life Time Utility Function - Examples

$$U(C_t) = \ln C_t \tag{13}$$

The third utility function, (13) is the log utility function. This utility function is particularly attractive because it is easy to take the derivative and it satisfies both properties laid out above.

# Life Time Utility Function - Examples

$$U(C_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{1}{1-\sigma}, \qquad \sigma > 0 \tag{14}$$

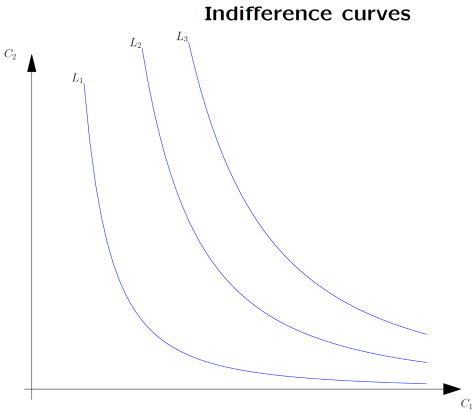
The final utility function is sometimes called the isoelastic utility function. It can be written either of the two ways shown in (14). Because utility is ordinal, it does not matter whether the $\frac{-1}{1-\sigma}$ is included or not. If $\sigma = 1$, then this utility function is equivalent to the log utility function.

# Utility Function - Utility Discount Factor

$$U(C_1) + \beta U(C_2)$$

- The parameter $\beta$ is known as **the subjective discount factor**. The subjective discount factor is a measure of impatience.
- We assume that it is positive but less than one; $0 < \beta < 1$
- Assuming that it is less than one means that the household puts less weight on period 2 utility than period 1 utility. This means that we assume that the household is impatient – it would prefer utility in the present compared to the future.
- The bigger $\beta$ is, the more patient the household is. The smaller is $\beta$, the more impatient the consumer will be. In the extreme case in which $\beta = 0$ consumers care only about present consumption.
- So sometimes we will say that $\beta$ is **the utility discount factor**, while $\frac{1}{1+r_t}$ is **the goods discount factor**.

# the Intertemporal Budget Constraint




Across indifference curves, the lifetime utility increases as one moves northeast in the figure, that is, $L_1 < L_2 < L_3$.

# Indifference Curves

- Figure displays the household's indifference curves. All consumption paths ($C_1$ and $C_2$) on a given indifference curve provide the same level of utility.
- This means that indifference curves are downward sloping. An increase in period-1 consumption requires a decrease in period-2 consumption if the household is to remain indifferent. Indifference curves do not cross one another.
- The negative of the slope of the indifference curve is known as **the intertemporal marginal rate of substitution** of $C_2$ for $C_1$
- If you are consuming very little in period 1 and a lot in period 2, you are willing to give up a lot of period-2 consumption for an additional unit of period-1 consumption. This property of preferences is known as diminishing marginal rate of substitution of $C_2$ for $C_1$

# Indifference Curves

- To obtain the slope of an indifference curve, proceed as follows. Fix the level of lifetime utility at some constant, say L.
- Then the indifference curve associated with a level of lifetime utility L is given by all the paths ($C_2$ and $C_1$) satisfying

$$U(C_1) + \beta U(C_2) = L$$

- Now differentiate this expression with respect to $C_1$ and $C_2$ to obtain

$$U'(C_1)dC_1 + \beta U'(C_2)dC_2 = L$$

  where a prime denotes the derivative of a function with respect to its argument.
- The objects $U'(C_1)$ and $U'(C_2)$ are known as **the marginal utility of consumption** in periods 1 and 2, respectively.

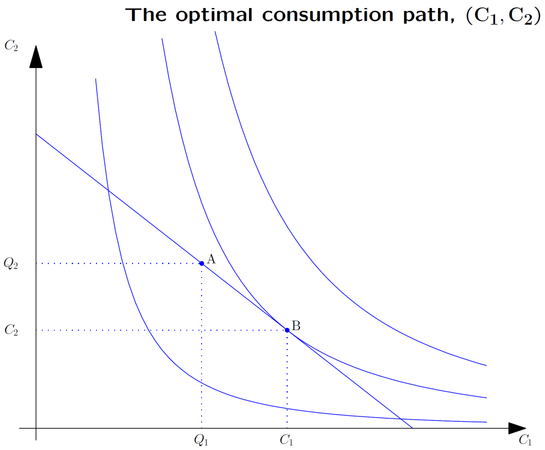
# Concavity

- Rearranging, we obtain that the slope of the indifference curve, $\frac{dC_2}{dC_1}$, is given by

$$\textbf{slope of the indifference curve} = -\frac{U'(C_1)}{\beta U'(C_2)}$$

- The convexity of the indifference curves requires that this slope becomes smaller in absolute value as $C_1$ increases.
- Now, when $C_1$ increases, $C_2$ decreases, because we are moving along a given indifference curve.
- The only way the indifference curve can become flatter as $C_1$ increases and $C_2$ decreases is if $U'(C_1)$ is decreasing in $C_1$, that is, if $U''(C_1)$ is negative, or $U(C_1)$ is concave.
- We have therefore arrived at the result that the convexity of the indifference curve requires that the period utility function $U(\cdot)$ be concave.

# The Optimal Consumption Path



# The Optimal Consumption Path

- To maximize utility, the household chooses a consumption path ($C_1$ and $C_2$) that is on the intertemporal budget constraint and on an indifference curve that provides the highest level of lifetime utility, that is, an indifference curve that is as far northeast as possible.

- At the feasible consumption path that maximizes the household's utility, the indifference curve is tangent to the budget constraint (point **B** in Figure above).

# Deriving the Optimal Consumption Path

- Formally, the household problem is

$$\underset{C_1, C_2}{Max}\Big(U(C_1) + \beta U(C_2)\Big)$$

subject to

$$C_1 + \frac{C_2}{1+r_1} = (1+r_0)B_0^* + Q_1 + \frac{Q_2}{1+r_1}$$

- The household takes as given all objects on the right-hand side of the intertemporal budget constraint. Therefore, to save notation, let's call the right-hand side $\bar{Y}$ :

$$\bar{Y} = (1+r_0)B_0^* + Q_1 + \frac{Q_2}{1+r_1}$$

where $\bar{Y}$ represents the household's lifetime wealth, which is composed of its initial asset holdings and the present discounted value of the stream of income $(Q_1, Q_2)$

# Deriving the Optimal Consumption Path

- We can then rewrite the intertemporal budget constraint as

$$C_2 = (1 + r_1)(\bar{Y} - C_1) \tag{15}$$

- Use this expression to eliminate $C_2$ from the lifetime utility function (10) and then the household maximization problem becomes

$$\underset{C_1}{Max}\Big(U(C_1) + \beta U((1 + r_1)(\bar{Y} - C_1)\Big)$$

To maximise this expression, take the derivative with respect to $C_1$, equate it to zero, and rearrange:

$$U'(C_1) = (1 + r_1)\beta U'(C_2) \tag{16}$$

This optimality condition is known as **the consumption Euler equation.**

# Euler Equation

$$U'(C_1) = (1 + r_1)\beta U'(C_2)$$

- The LHS of the Euler equation represents the utility cost of reducing period-1 consumption by one unit. The RHS represents the utility gain of sacrificing one unit of period-1 consumption.
- If the LHS is greater than the RHS, then the household can increase its lifetime utility by saving less (and hence consuming more) in period 1.
- Conversely, if the LHS is less than the RHS, then the household will be better off saving more (and consuming less) in period 1.
- At the optimal allocation, the left- and right-hand sides of the Euler equation must be equal to each other, so that at the margin the household is indifferent between consuming an extra unit in period 1 and consuming $1 + r_1$ extra units in period 2.
- In other words, the household should pick $C_1$ and $C_2$ so that the marginal utility of period 1 consumption, $u'(C_1)$, equals the marginal utility of period 2 consumption, $\beta U'(C_2)$, multiplied by the gross real interest rate (i.e. one plus the real interest rate).

# Euler Equation

- Rearrange terms ( divide the left- and right- hand sides of the Euler equation by $-\beta U'(C_2)$ to obtain) to write the Euler equation as

$$-\frac{U'(C_1)}{\beta U'(C_2)} = -(1+r_1)$$

- The left-hand side of this expression, $-\frac{U'(C_1)}{\beta U'(C_2)}$ , is the negative of **the marginal rate of intertemporal substitution** of $C_2$ for $C_1$ at the consumption path $(C_1, C_2)$, which, as shown above is the slope of the indifference curve.
- The right-hand side of the expression, $-(1+r_1)$, is the negative of the slope of the budget constraint.
- In particular, getting an additional unit of period 1 consumption requires giving up $1+r_1$ units of period 2 consumption.

# Euler Equation - Example -1

Suppose that the utility function is the natural log like (13). Then the Euler equation can be written:

$$\frac{1}{C_t} = \beta(1 + r_t)\frac{1}{C_{t+1}} \tag{17}$$

This can be re-arranged:

$$\frac{C_{t+1}}{C_t} = \beta(1 + r_t) \tag{18}$$

- The left hand side of (18) is **the gross growth rate of consumption** between $t$ and $t + 1$. Hence, the Euler equation identifies the expected growth rate of consumption as a function of the degree of impatience, $\beta$, and the real interest rate, $r_t$.
- It does not identify the levels of $C_t$ and $C_{t+1}$ - (18) could hold when $C_t$ and $C_{t+1}$ are both big or both small.

# Euler Equation - Example -1

$$\frac{C_{t+1}}{C_t} = \beta(1 + r_t)$$

- Other factors held constant, the bigger is $\beta$, the higher will be expected consumption growth. Likewise, the bigger is $r_t$, the higher will be expected consumption growth.
- $\beta < 1$ means the household is impatient, which incentives consumption in the present at the expense of the future (i.e. makes $C_{t+1}/C_t$ less than one, other things being equal).
- $r_t > 0$ has the opposite effect – it incentives deferring consumption to the future, which makes $C_{t+1}/C_t$ greater than one.
- If $\beta(1 + r_t) = 1$, these two effects offset, and the household will desire $C_{t+1} = C_t$ .

# Euler Equation - Example -2

Suppose that the utility function is the isoelastic like (14). Then the Euler equation can be written:

$$C_t^{-\sigma} = \beta(1+r_t)C_{t+1}^{-\sigma} \tag{19}$$

Take logs of (19) using the **approximation** that $\ln(1+r_t) = r_t$ :

$$-\sigma \ln C_t = \ln \beta + r_t - \sigma \ln C_{t+1} \tag{20}$$

This can be re-arranged to yield:

$$\ln C_{t+1} - \ln C_t = \frac{1}{\sigma} \ln \beta + \frac{1}{\sigma} r_t \tag{21}$$

**-** Since $\ln C_{t+1} - \ln C_t$ is approximately the expected growth rate of consumption between $t$ and $t+1$, this says that consumption growth is positively related to the real interest rate.

**-** The coefficient governing the strength of this relationship is $1/\sigma$.

**-** The bigger is $\sigma$ (loosely, the more concave is the utility function) the less sensitive consumption growth will be to changes in $r_t$, and vice-versa.

# Small Endowment Economy - Autarky

- 2-period endowment economy - output is given and equal to $Q_1$ and $Q_2$.
- Optimisation problem of the representative household:

$$\max U(C_1, C_2) = U(C_1) + \beta U(C_2)$$

$$\text{st} \quad \underbrace{C_1 + \frac{C_2}{1+r}}_{\text{the present discounted value of consumption}} = \underbrace{Q_1 + \frac{Q_2}{1+r}}_{\text{the present discounted value of the endowment stream}}$$

- Inter-temporal **Euler equation** is the same as (16):

$$U'(C_1) = (1+r)\beta U'(C_2)$$

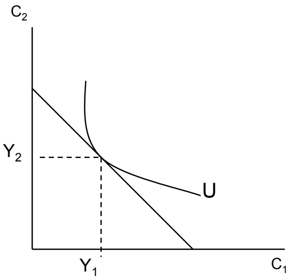
- In the autarky equilibrium $C_1 = Q_1$ and $C_2 = Q_2$.
- The equilibrium autarky interest rate ($r^A$) is given by

$$U'(C_1) = (1+r^A)\beta U'(C_2)$$

This is the **autarky price of future consumption in terms of present consumption.**

# Small Endowment Economy - Autarky

- We can graph the budget constraint as $C_2 = Q_1(1+r) + Q_2 - C_1(1+r)$.




- **In autarky** : $C_1 = Q_1$ and $C_2 = Q_2$.
- **In an open economy**: $C_1 \neq Q_1$ and $C_2 \neq Q_2$.
  - For example, if $\beta = 1/(1+r)$ and $C_1 = C_2 = \bar{C}$ (smooth consumption), but $Q_1 < Q_2$.
  - Then, the country borrows $\bar{C} - Q_1$ on $t = 1$, repays $(1+r)(\bar{C} - Q_1)$ on $t = 2$. Consumes $C_2 = Q_2 - (1+r)(C_1 - Q_1)$.
  - In $t = 1$, the country borrows $\rightarrow$ Current account is deficit: $CA_1 < 0$.

# Small Endowment Economy under Financial Openness

## Open Economy

- The current account equals the increase in net foreign assets

$$CA_t = B_{t+1} - B_t = Q_t + rB_t - C_t$$

- CA in period 1:

$$CA_1 = B_1 - B_0 = Q_1 + rB_0 - C_1 \qquad (\text{and} \quad B_0 = 0)$$

- CA in period 2:

$$CA_2 = Q_2 + rB_1 - C_2 = -(Q_1 - C_1) = -B_1 = -CA_1 \qquad (\text{and} \quad B_2 = 0)$$

- A CA deficit today must be offset by a CA surplus in the future.

# Small Endowment Economy under Financial Openness

- Financial openness allows **inter-temporal trade** and can increase welfare, as it allows consumption to differ from income.

- Example:
  - If the home price of future consumption is relatively low (i.e. $\frac{1}{1+r^A} < \frac{1}{1+r} \Leftrightarrow r^A > r$) because the country wants to consume more today, future consumption is cheap at home.

  - In this case, the home country "import" present consumption and "export" future consumption; i.e. the country runs a CA deficit in period 1 and a surplus in period 2. Put it differently, the country borrows internationally in period 1.

**Open Economy Equilibrium**

- Note that CA deficits/surpluses are not necessarily bad.
- The country's welfare increases by running a deficit in period 1.

# Small Endowment Economy

**CRRA preferences:**

- $U = \frac{C^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$, $\sigma$ is the elasticity of inter-temporal substitution.
- Euler equation

$$C_2 = C_1(1+r)^\sigma \beta^\sigma \tag{22}$$

- Consumption in $t=1$ becomes

$$C_1 = \frac{1}{1+(1+r)^{\sigma-1}\beta^\sigma}\left(Y_1 + \frac{Y_2}{1+r}\right) \tag{23}$$

- The equilibrium price of future consumption in terms of present consumption is

$$\beta\left(\frac{Y_1}{Y_2}\right)^{\frac{1}{\sigma}} = \frac{1}{1+r^A}$$

# Small Endowment Economy

**The shape of the saving schedule**

- Replace $C_2$ into Euler Equation and implicitly differentiate wrt $r$

$$u'(C_1) = \beta(1+r)u'[(1+r)(Y_1 - C_1) + Y_2]$$

$$\frac{dC_1}{dr} = \frac{\beta u'(C_2) + \beta(1+r)u''(C_2)(Y_1 - C_1)}{u''(C_1) + \beta(1+r)^2 u''(C_2)}$$

- With CRRA utility function

$$\frac{dC_1}{dr} = \frac{(Y_1 - C_1) - \sigma C_2/(1+r)}{1 + r + (C_2/C_1)}$$

- A rise in $r$ has an ambiguous effect in $C_1$:

  a. **Substitution effect:** $\sigma C_2/(1+r)$, reduces $C_1$ due to the increase in its relative price.
  b. **Terms-of-trade effect:** $Y_1 - C_1$ on welfare
     . if $C_1 > Y_1$, the $\uparrow$ in the int. rate makes the country poorer, $\downarrow C_1$.
     . if $Y_1 > C_1$, the $\uparrow$ in the int. rate makes the country richer, $\uparrow C_1$.

  * The sign of $\frac{dC_1}{dr}$ depends on the size of a. and b.

# Small Endowment Economy

**The shape of the saving schedule**

- Replace $C_2$ into Euler Equation and implicitly differentiate wrt $r$

$$u'(C_1) = \beta(1+r)u'[(1+r)(Y_1 - C_1) + Y_2]$$

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- With CRRA utility function

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- A rise in $r$ has an ambiguous effect in $C_1$:
  - $a.$ Substitution effect: $\sigma C_2/(1+r)$, reduces $C_1$ due to the increase in its relative price.
  - $b.$ Terms-of-trade effect: $Y_1 - C_1$ on welfare
    - if $C_1 > Y_1$, the $\uparrow$ in the int. rate makes the country poorer, $\downarrow C_1$.
    - if $Y_1 > C_1$, the $\uparrow$ in the int. rate makes the country richer, $\uparrow C_1$.
  - ∗ The sign of $\frac{dC_1}{dr}$ depends on the size of a. and b.

# Small Endowment Economy

## Substitution, Income and Wealth Effects

- Recall optimal consumption in $t = 1$ is (23)

$$C_1 = \frac{1}{1 + (1+r)^{\sigma-1}\beta^{\sigma}}\left(Y_1 + \frac{Y_2}{1+r}\right)$$

- 3 ways in which an interest-rate change can affect consumption:
  - i) Substitution effect: an $\uparrow$ in $r$ makes savings more attractive, $\downarrow C_1$.
  - ii) Income effect: an $\uparrow$ in $r$ allows higher consumption, given the present value of lifetime resources.
  - $\rightarrow$ Whether $i)$ or $ii)$ dominates depends on $(1+r)^{\sigma-1}$.
    - If $\sigma > 1$, substitution effects dominates ($\downarrow C_1$).
    - If $\sigma < 1$, income effect dominates ($\uparrow C_1$).
    - If $\sigma = 1$, they cancel out (log ut. function).
  - iii) Wealth effect: change in lifetime income: $Y_1 + \frac{Y_2}{1+r}$ ($\downarrow C_1$).
- Income and wealth effect are identified as the terms-of-trade effects (ToT).
- The impact of $\Delta r$ on $C_1$ depends on which effects dominates.

# Small Endowment Economy- Extension 1

**Temporary vs. permanent output changes**

- Assume $\beta(1+r) = 1$, then the consumption path should be constant.

- If $Y_1 = Y_2$ then no current account imbalance.

$\rightarrow$ Temporary: If $Y_1$ increases, the price of future consumption becomes relatively more expensive ($r^A < r$). The country runs a CA surplus in 1 and a deficit in 2.

$\rightarrow$ Permanently: If both $Y_1$ and $Y_2$ increase by the same amount, the inter-temporal price does not change. CA is not affected.

$\rightarrow$ **Permanent changes in output don't affect the CA, temporary changes do**

($\uparrow Y \rightarrow$ CA surplus, and $\downarrow Y \rightarrow$ CA deficit ).

# Small Endowment Economy- Extension 2

**Government**

$$C_1 + \frac{C_2}{1+r} = Y_1 - G_1 + \frac{Y_2 - G_2}{1+r}$$

$$CA_t = B_{t+1} - B_t = Y_t + rB_t - C_t - G_t$$

$$\beta \frac{u'(C_2)}{u'(C_1)} = \frac{1}{1+r}$$

- Agents have only $Y_1 - G_1$ and $Y_2 - G_2$ available for consumption
- A temporary increase in $G_1$ lowers output available for consumption in 1 and creates a CA deficit (if CA initially in equilibrium)
- A permanent increase in G has no effect in CA.

# The Interest Rate Parity Condition

- We assume that there is **free capital mobility**, which means that households can borrow and lend freely in the international financial market.

- Let $r^*$ be the world interest rate. Then, free capital mobility guarantees that the domestic interest rate be equal to the world interest rate. That is,

$$r_1 = r^*$$

  We will refer to this condition as **the interest rate parity condition**.

- Any difference between $r_1$ and $r^*$ would give rise to **an arbitrage opportunity** (we will talk more later) that would allow someone to make infinite profits.

- For instance, if $r_1 > r^*$, then one could make infinite amounts of profits by borrowing in the international market and lending in the domestic market.

- Similarly, if $r_1 < r^*$, unbounded profits could be obtained by borrowing domestically and lending abroad. These arbitrage opportunities disappear when $r_1 = r^*$.

# Equilibrium in the Small Open Economy

- As households are identical (assumed) by studying the behaviour of an individual household, we are also learning about the behaviour of the country as a whole.
- This also implies that we can interpret $B_t$, for $t = 0, 1, 2$ , as the country's net foreign asset position or NIIP at the end of period $t$.
- An equilibrium then is a consumption path $(C_1, C_2)$ and an interest rate $r_1$ that satisfy the country's **intertemporal resource constraint**, the consumption Euler equation, and the interest rate parity condition, that is,

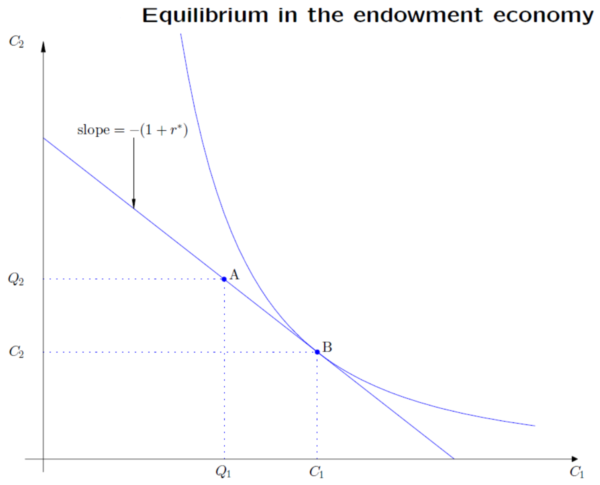
$$C_1 + \frac{C_2}{1 + r_1} = (1 + r_0)B_0^* + Q_1 + \frac{Q_2}{1 + r_1} \tag{24}$$

$$U'(C_1) = (1 + r_1)\beta U'(C_2) \tag{25}$$

$$r_1 = r^* \tag{26}$$

given the exogenous variables $r_0$, $B_0$, $Q_1$, $Q_2$, and $r^*$.

# Graphical Representation of Equilibrium



# Graphical Representation of Equilibrium

- This graph represents the three equilibrium conditions (24) (25), and (26) shown on above that determine $C_1$, $C_2$, and $r_1$.
- The equilibrium is at point B.
- Point **B** is on the intertemporal resource constraint, as required by equilibrium condition (24).
- The indifference curve that crosses point **B** is tangent to the intertemporal budget constraint, whose slope is $-(1 + r_1)$. This means that equilibrium condition (25) holds.
- And the slope of the intertemporal resource constraint is $-(1 + r^*)$, which means that $r^* = r_1$, as required by equilibrium condition (26).

# Adjustment to an anticipated increase in output