EC201 Macroeconomics

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Class - 5 -

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This Week: Outline

Review Questions

1. In the RBC model, business cycles are driven by exogenous technology shocks. Which of the following could be the real-world counterpart of a negative technology shock?
   (a) A reduction in labour supply
   (b) Bad weather
   (c) A wave of panic among consumers and investors
   (d) All of the above
   (e) None of the above

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   The Solow Residual is defined as $A_t = \frac{Y_t}{K_t^{\alpha} L_t^{1-\alpha}}$. The data needed to compute it are real GDP, total hours worked and the capital stock at time t; moreover, we need to calibrate α (which is usually set to 1/3 to match the capital share of income). There are many reasons why A_t might fail to capture the level of technology; for example, inputs might be mismeasured, or the assumed Cobb-Douglas production function might be a poor approximation of the real world production process.

Problem – 1 –

[**Derivation of the RBC Model**] This question guides you through the derivation of the RBC model. We assume the following functional forms:

$$Y_t = A_t K_t^{\alpha} h_t^{1-\alpha}$$
$$u(c_t) + v(1-h_t) = \log c_t + \theta \log(1-h_t)$$

(a) Set up and solve the representative firm's profit maximization problem.

(b) Set up and solve the representative household's utility maximization problem, assuming perfect foresight.

(c) Combine the equations derived above and re-write them as a system of 3 dynamic equations in K_t, h_t, c_t.

(d) Suppose $\delta = 1$, i.e. full depreciation of capital. Under this special assumption, it turns out that the solution for consumption takes the following simple form (you do not need to show this):

$$c_t = (1-\alpha\beta) Y_t$$

Substitute this result into the equation summarizing the optimal labour supply choice. How does h_t respond to a positive technology shock in this model? Is this consistent with what we see in the data?

(e) Suppose now that $\theta = 0$, i.e. the household does not care about leisure. How does h_t respond to a positive technology shock in this case? Is this consistent with what we see in the data?

a Set up and solve the representative firm's profit maximisation problem??

Problem 1.a

Firm's maximisation problem:

$$\max_{K_t, h_t} A_t K_t^{\alpha} h_t^{1-\alpha} - w_t h_t - r_t K_t \tag{1}$$

Problem 1.a

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$$\max_{K_t, h_t} A_t K_t^{\alpha} h_t^{1-\alpha} - w_t h_t - r_t K_t \tag{1}$$

First order conditions:

$$(1-\alpha) A_t \left(\frac{K_t}{h_t}\right)^{\alpha} = w_t \tag{2}$$

$$\alpha A_t \left(\frac{h_t}{K_t}\right)^{1-\alpha} = r_t \tag{3}$$

b

b Set up and solve the representative household's utility maximization problem, assuming perfect foresight.

Problem 1.b

$$\max_{\{c_t, h_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[log c_t + \theta log(1 - h_t) \right]$$

Subject to

$$K_{t+1} = (1 - \delta)K_t + r_t K_t + w_t h_t - c_t, \quad \forall t$$

Problem 1.b

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Subject to

$$K_{t+1} = (1-\delta)K_t + r_t K_t + w_t h_t - c_t, \quad \forall t$$

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Big[log c_t + \theta log(1 - h_t) \Big] - \lambda_t \Big[K_{t+1} - (1-\delta)K_t - r_t K_t - w_t h_t + c_t \Big] \tag{4}$$

Problem 1.b

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left[log c_t + \theta log(1 - h_t) \right] - \lambda_t \Big[K_{t+1} - (1 - \delta) K_t - r_t K_t - w_t h_t + c_t \Big]$$

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$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \Longrightarrow \beta^t \frac{1}{c_t} = \lambda_t$$

Problem 1.b

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$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \implies \beta^t \frac{1}{c_t} = \lambda_t$$
$$\frac{\partial \mathcal{L}}{\partial h_t} = 0 \implies \beta^t \frac{\theta}{1 - h_t} = \lambda_t w_t$$

Problem 1.b

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Problem 1.b

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Problem 1.b

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Big[log c_t + \theta log(1 - h_t) \Big] - \lambda_t \Big[K_{t+1} - (1-\delta)K_t - r_t K_t - w_t h_t + c_t \Big]$$

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Problem 1.b

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Big[log c_t + \theta log(1-h_t)\Big] - \lambda_t \Big[K_{t+1} - (1-\delta)K_t - r_t K_t - w_t h_t + c_t\Big]$$

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \Longrightarrow \beta^t \frac{1}{c_t} = \lambda_t$$

$$\frac{\partial \mathcal{L}}{\partial h_t} = 0 \Longrightarrow \beta^t \frac{\theta}{1-h_t} = \lambda_t w_t$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = 0 \Longrightarrow \lambda_t = \lambda_{t+1}(1 + r_{t+1} - \delta)$$

Problem 1.b

Remember

$$\implies \beta^t \frac{1}{c_t} = \lambda_t$$
$$\implies \beta^{t+1} \frac{1}{c_{t+1}} = \lambda_{t+1}$$

Substitute out the Lagrangian multipliers to get

Problem 1.b

Remember

$$\implies \beta^t \frac{1}{c_t} = \lambda_t$$
$$\implies \beta^{t+1} \frac{1}{c_{t+1}} = \lambda_{t+1}$$

Substitute out the Lagrangian multipliers to get

$$\frac{\theta}{1-h_t} = \frac{w_t}{c_t}$$
$$\frac{1}{c_t} = \beta(1 + r_{t+1} - \delta)\frac{1}{c_t + 1}$$

Together with the budget constraint, these equations (one for every t) implicitly define the solution to the household problem.

c Combine the equations derived above and re-write them as a system of 3 dynamic equations in K_t, h_t, c_t.

Problem 1.c

Remember

$$(1-\alpha)A_t\left(\frac{K_t}{h_t}\right)^{\alpha} = w_t \tag{5}$$

$$\alpha A_t\left(\frac{h_t}{K_t}\right)^{1-\alpha} = r_t \tag{6}$$

$$A_t K_t^{\alpha} h_t^{1-\alpha} = r_t K_t + w_t h_t \tag{7}$$

What did we have?

$$\begin{aligned}
\frac{\theta}{1-h_t} &= \frac{w_t}{c_t} \\
\frac{1}{c_t} &= \beta(1+r_{t+1}-\delta)\frac{1}{c_t+1} \\
K_{t+1} &= (1-\delta)K_t + r_t K_t + w_t h_t - c_t
\end{aligned}$$

Problem 1.c

Substituting factor prices into the households' FOCs,

$$\frac{\theta}{1-h_t} = \frac{(1-\alpha)A_t\left(\frac{K_t}{h_t}\right)^{\alpha}}{c_t} \tag{8}$$

$$\frac{1}{c_t} = \beta\left(1+\alpha A_{t+1}\left(\frac{h_{t+1}}{K_{t+1}}\right)^{1-\alpha}-\delta\right)\frac{1}{c_{t+1}} \tag{9}$$

$$K_{t+1} = (1-\delta)K_t + A_t K_t^{\alpha} h_t^{1-\alpha} - c_t \tag{10}$$

7– Labour Supply

8– Intertemporal Consumption-Saving Decision

9– Capital Accumulation Equation

d Suppose $\delta = 1$ i.e. full depreciation of capital. Under this special assumption, it turns out that the solution for consumption takes the following simple form (you do not need to show this):

$$c_t = (1 - \alpha\beta)Y_t$$

Substitute this result into the equation summarising the optimal labour supply choice. How does h_t respond to a positive technology shock in this model? Is this consistent with what we see in the data?

Problem 1.d

$$\frac{\theta}{1-h_t} = \frac{(1-\alpha)A_t\left(\frac{K_t}{h_t}\right)^{\alpha}}{(1-\alpha\beta)Y_t}$$

$$\frac{(1-\alpha)\frac{Y_t}{h_t}}{(1-\alpha\beta)Y_t} = \frac{1-\alpha}{(1-\alpha\beta)h_t}$$

This implies that h_t is constant and independent from A_t. In this model, a positive technology shock has no impact on h_t. This is not consistent with the data, given then empirically total hours worked are strongly pro-cyclical.

e Suppose now that $\theta = 0$ i.e. the household does not care about leisure. How does h_t respond to a positive technology shock in this case? Is this consistent with what we see in the data?

Problem 1.e

If there is no disutility of labour, the household will choose to work as much as possible, $h_t = 1$. As above, h_t is constant and independent from A_t, contrary to what we see in the data.