

# EC201 Macroeconomics

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Class - 7 -

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# This Week: Outline

1. REVIEW QUESTIONS PROBLEM 1 & 2

2. CLASS QUESTIONS PROBLEM 1

3. CLASS QUESTIONS PROBLEM 2

1. Derive the IS equation, i.e. the equilibrium relationship between short-run output and the real interest rate. Intuitively, why is this relationship downward sloping?
2. Differently from the RBC model, in the Keynesian model monetary policy has real effects. Discuss intuitively which features of the Keynesian model are responsible for this difference, and illustrate in words how in this model the Central Bank can influence the real interest rate and real output.

**1** Derive the IS equation, i.e. the equilibrium relationship between short-run output and the real interest rate. Intuitively, why is this relationship downward sloping?

The IS equation can be derived by combining the equations for consumption, investment and government purchases,

$$C_t = (\bar{a}_c + a_{c,t})\bar{Y}_t$$

$$G_t = (\bar{a}_g + a_{g,t})\bar{Y}_t$$

$$I_t = [\bar{a}_i + a_{i,t} - \bar{b} (R_t - \bar{R})] \bar{Y}_t$$

with the National Account Identity

$$Y_t = C_t + I_t + G_t$$

Substituting, one gets

$$Y_t = [\bar{a}_c + a_{c,t} + \bar{a}_g + a_{g,t} + \bar{a}_i + a_{i,t} - \bar{b}(R_t - \bar{R})] \bar{Y}_t$$

$$\tilde{Y}_t = \frac{Y_t - \bar{Y}_t}{\bar{Y}_t} = a_t + \bar{a}_c + \bar{a}_g + \bar{a}_i - 1 - \bar{b}(R_t - \bar{R}) = a_t - \bar{b}(R_t - \bar{R})$$

where  $a_t = a_{c,t} + a_{g,t} + a_{i,t}$ .

Intuitively, the relationship is downward sloping because a higher interest rate reduces firms' investment in the final good by increasing the return of investing in the financial market or by increasing the cost of borrowing to finance such investment.

**2** Differently from the RBC model, in the Keynesian model monetary policy has real effects. Discuss intuitively which features of the Keynesian model are responsible for this difference, and illustrate in words how in this model the Central Bank can influence the real interest rate and real output.

- The key feature of the Keynesian model responsible for this difference is sticky inflation.
- The Central Bank can influence directly the nominal interest rate; while in the RBC model changes in the nominal interest rate (or, equivalently, in the money supply) are counteracted by proportional changes in inflation with no effect on the real interest rate, in the Keynesian model inflation reacts only slowly to monetary policy, which therefore influences the real interest rate.
- In turn, changes in the real interest rate affect output by impacting firms' demand for investment (see the solution for question 1 for the details).



# CLASS QUESTIONS PROBLEM 1

**[Monetary Policy Regimes].** Consider the Keynesian AS-AD model. Assume that short-run output satisfies

$$\tilde{Y}_t = a_t - \bar{b} (\mathcal{R}_t - \bar{R})$$

the Central Bank sets the real interest rate according to

$$\mathcal{R}_t - \bar{R} = \bar{m}(\pi_t - \bar{\pi}) + \varepsilon_t^{\mathcal{M}}$$

and firms' setting behaviour implies

$$\pi_t = \pi_{t-1} + \bar{v}\tilde{Y}_t + \bar{o}_t$$

- (a) Derive the AD and AS equations and display them graphically on a diagram with  $\pi_t$  on the y-axis and  $\tilde{Y}_t$  on the x-axis. What are the slopes of the two curves?

Now consider two countries, Doveland ( $D$ ) and Hawkland ( $H$ ), which are identical to each other except for the fact that  $\bar{m}^H > \bar{m}^D$ .

- (b) Compare the AS-AD diagram for the two countries. How do they differ?
- (c) Suppose that both countries are at their long-run equilibrium at time  $t$ . Suddenly, at time  $t + 1$  a political crisis in Eastern Europe leads to an increase in the global price of natural gas, leading to higher production costs in both countries. Compare both graphically and intuitively the contemporaneous (i.e. at time  $t + 1$ ) impact of this shock across the two countries.
- (d) Suppose that at time  $t + 2$  the shock has dissipated, and gas prices are back to the normal level. Compare graphically and intuitively what happens in the two economies.

**a** Derive the AD and AS equations and display them graphically on a diagram with  $\pi_t$  on the y-axis and  $\tilde{Y}_t$  on the x-axis. What are the slopes of the two curves?

## Problem 1.a

To derive the AD equation, combine the IS curve and the monetary policy rule

$$\tilde{Y}_t = a_t - \bar{b}\bar{m}(\pi_t - \bar{\pi}) + \bar{b}\epsilon_t^M \quad (1)$$

which can be written as

$$\pi_t = \frac{a_t}{\bar{b}\bar{m}} - \frac{1}{\bar{b}\bar{m}}\tilde{Y}_t + \bar{\pi} + \frac{1}{\bar{m}}\epsilon_t^M \quad (2)$$

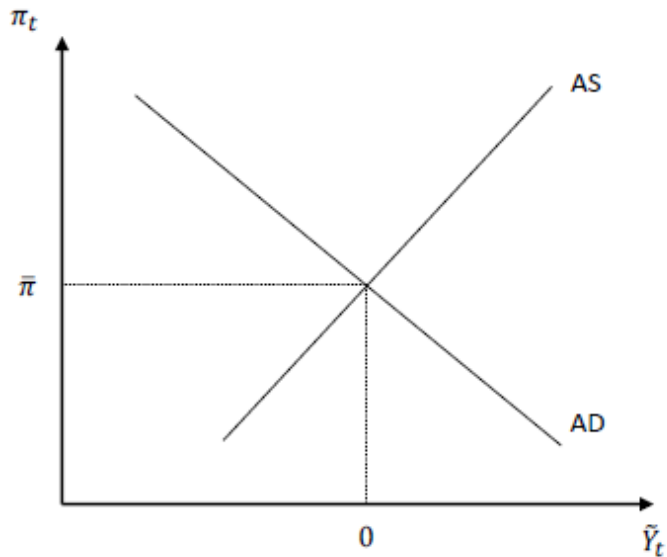
from which we see that the slope of the AD curve is  $-\frac{1}{\bar{b}\bar{m}}$

The AS curve is simply given by the equation characterising firms' setting behaviour,

$$\pi_t = \pi_{t-1} + \bar{v}\tilde{Y}_t + \bar{o} \quad (3)$$

and its slope is  $\bar{v}$ .

## Problem 1.a



**b** Now consider two countries, Doveland (D) and Hawkland (H), which are identical to each other except for the fact that  $\bar{m}^H > \bar{m}^D$

Compare the AS-AD diagram for the two countries. **How do they differ?**

## Problem 1.b

The key difference is that the AD curve is flatter in Hawkland (higher  $\bar{m}$ ).

Figure I: Doveland

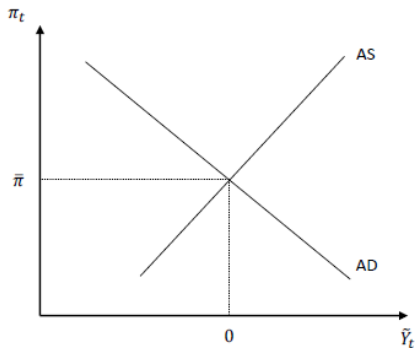
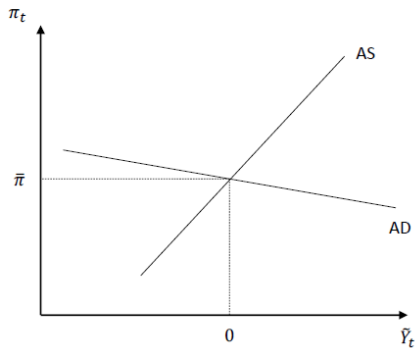


Figure II: Hawkland



# Bank of England



## Bank of England - Hawk/Dove Analysis

InTouch Capital Markets

Last Update: 08 February 2023


Name	Position	Hawk Scale	Vote Direction		
Most Dovish			02 Feb	15 Dec	03 Nov
Tenreyro	Ext Member	<div><div></div></div>	→	→	↑
Dhingra	Ext Member	<div><div></div></div>	→	→	↑
Cunliffe	Dep, Stability	<div><div></div></div>	↑	↑	↑
Broadbent	Dep Gov	<div><div></div></div>	↑	↑	↑
Bailey	Governor	<div><div></div></div>	↑	↑	↑
Pill	Chief Economist	<div><div></div></div>	↑	↑	↑
Ramsden	Dep Gov	<div><div></div></div>	↑	↑	↑
Haskel	Ext Member	<div><div></div></div>	↑	↑	↑
Mann	Ext Member	<div><div></div></div>	↑	↑	↑
Most Hawkish					

### Notes

↑ vote to hike, → vote to keep rate unch, ↓ vote to cut



# US vs EU

<div>  <b>FOMC - Hawk/Dove Analysis</b> </div> <div>InTouch Capital Markets</div>					
Last Update: 08 February 2023					
Name	Position	Hawk Scale	Voter?		
Most Dovish			2023	2024	2025
<b>Brainard</b>	Vice Chair	<div></div>	✓	✓	✓
<b>Cook</b>	Board	<div></div>	✓	✓	✓
<b>Goolsbee</b>	Chicago	<div></div>	✓	✗	✓
George	Kansas City	<div></div>	✗	✗	✓
<b>Harker</b>	Philadelphia	<div></div>	✓	✗	✗
Bostic	Atlanta	<div></div>	✗	✓	✗
<b>Logan</b>	Dallas	<div></div>	✓	✗	✗
Collins	Boston	<div></div>	✗	✗	✓
<b>Williams</b>	New York	<div></div>	✓	✓	✓
<b>Jefferson</b>	Board	<div></div>	✓	✓	✓
<b>Barr</b>	Board	<div></div>	✓	✓	✓
<b>Powell</b>	Chairman	<div></div>	✓	✓	✓
<b>Bowman</b>	Board	<div></div>	✓	✓	✓
Daly	San Francisco	<div></div>	✗	✓	✗
Barkin	Richmond	<div></div>	✗	✓	✗
Mester	Cleveland	<div></div>	✗	✓	✗
<b>Waller</b>	Board	<div></div>	✓	✓	✓
<b>Kashkari</b>	Minneapolis	<div></div>	✓	✗	✗
Bullard	St Louis	<div></div>	✗	✗	✓
Most Hawkish					

<div> <b>ECB - Hawk/Dove Analysis</b> </div> <div>Prepared for ITC Markets by Econostream Media</div>					
Last Update: 08 February 2023					
Name	Position	Hawk Scale	Voter?		
Most Dovish			16 Mar	04 May	15 Jun
Panetta	Exec Board	<div></div>	✓	✓	✓
Stournaras	Greece	<div></div>	✗	✓	✓
Visco	Italy	<div></div>	✓	✓	✓
Lane	Exec Board	<div></div>	✓	✓	✓
Hernández de Cos	Spain	<div></div>	✓	✗	✓
Herodotou	Cyprus	<div></div>	✗	✓	✓
Centeno	Portugal	<div></div>	✓	✓	✗
Makhlouf	Ireland	<div></div>	✓	✓	✓
de Guindos	Vice President	<div></div>	✓	✓	✓
Vujčić	Croatia	<div></div>	✗	✓	✓
Lagarde	President	<div></div>	✓	✓	✓
Scicluna	Malta	<div></div>	✓	✗	✗
Villeroi	France	<div></div>	✓	✓	✗
Rehn	Finland	<div></div>	✓	✓	✓
Reinesch	Luxembourg	<div></div>	✓	✗	✓
Schnabel	Exec Board	<div></div>	✓	✓	✓
Elderson	Exec Board	<div></div>	✓	✓	✓
Vasle	Slovenia	<div></div>	✓	✓	✗
Šimkus	Lithuania	<div></div>	✓	✗	✓
Kazāks	Latvia	<div></div>	✗	✓	✓
Kažimír	Slovakia	<div></div>	✓	✓	✓
Müller	Estonia	<div></div>	✓	✓	✓
Wunsch	Belgium	<div></div>	✓	✓	✓
Nagel	Germany	<div></div>	✓	✓	✓
Holzmann	Austria	<div></div>	✓	✗	✗
Knot	Netherlands	<div></div>	✗	✓	✓
Most Hawkish					

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'Emphasise the pain': Jay Powell keeps hawkish tone even as inflation eases

Fed chair tries to manage investor expectations following shift down to half-point rate rises

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Tt's good to see progress, but let's just understand we have a long way to go to get back to price stability'. Fed chair Jay Powell told reporters on Wednesday © Evelyn Hockstein/Reuters

Cobly Smith in Washington DECEMBER 15 2022

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As Federal Reserve officials prepared to kick off their final policy meeting of 2022 on Tuesday, they received some welcome news: inflation, which has been running at multi-decade highs for more than a year, fell decisively in

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Eurozone inflation

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# Christine Lagarde's hawkishness reflects Europe's stickier inflation problem

ECB president admits eurozone price pressures may rise again in early 2023

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Christine Lagarde dashed hopes the ECB could stop its rate rises soon: 'We are not slowing down. We are in for the long game' © Sanziana Perju/ECB

Martin Arnold in Frankfurt

DECEMBER 15 2022

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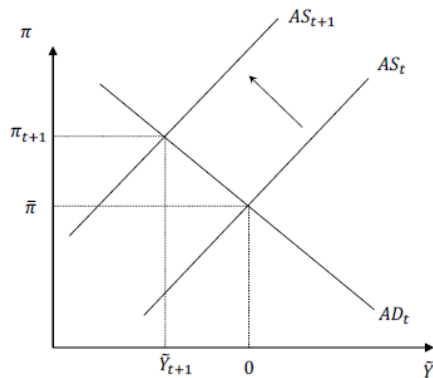
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# Hawks vs Dove

In general, **hawks** are the members who tend to want tighter monetary policy to temper inflation and growth whilst **doves** tend to want looser monetary policy to support growth and inflation.

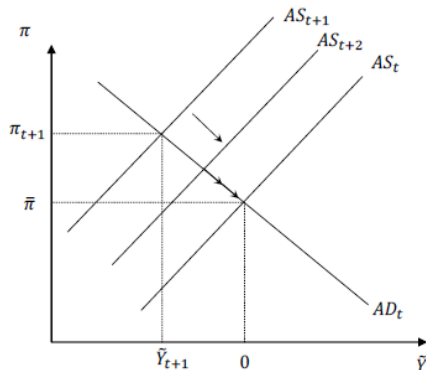
# A Supply-Driven Business Cycle

- ▶ At time  $t$ , the economy is on its long-run equilibrium:  
 $\pi_t = \bar{\pi}$ ,  $\tilde{Y}_t = 0$ ,  $R_t = \bar{R}$
- ▶ Suppose that at time  $t + 1$ , due to political turmoil in the Middle East, oil prices increase
- ▶ This increases production costs and forces firms to increase prices:  $\pi_{t+1} > 0$
- ▶ The shock is temporary and lasts only one period:  
 $\pi_{t+2} = \pi_t = 0$
- ▶ At time  $t + 1$ , the AS curve shifts upwards
  - ▶  $\pi_{t+1} > \bar{\pi}$ , due to the higher production costs
  - ▶  $R_{t+1} > \bar{R}$ , the CB reacts to higher inflation through a contractionary policy (moderating the increase in inflation, but creating a recession)



# A Supply-Driven Business Cycle

- ▶ At time  $t + 2$ , the shock has disappeared and the AS curve moves back towards the original position
- ▶ However, expected inflation ( $=\pi_{t+1}$ ) is now higher than before ( $=\bar{\pi}$ )  $\rightarrow AS_{t+2}$  is still above  $AS_t$
- ▶ The transition continues gradually until the AS curve is back to the original position
- ▶ During the transition, as inflation decreases the Central Bank relaxes monetary policy (lowering  $R_t$  and raising  $\tilde{Y}_t$ )
- Supply shocks create **stagflation**: contemporaneous stagnation of economic activity and inflation
- Overall, the model tells us that if the business cycle is driven by supply shocks, inflation should be countercyclical (higher inflation during recessions)



c Suppose that both countries are at their long-run equilibrium at time  $t$ . Suddenly, at time  $t + 1$  a political crisis in Eastern Europe leads to an increase in the global price of natural gas, leading to higher production costs in both countries.

Compare both graphically and intuitively the contemporaneous (i.e. at time  $t + 1$ ) impact of this shock across the two countries.

## Problem 1.c

The shock implies that at time  $t + 1$  in both countries  $\bar{o}_t > 0$ . This causes an upward shift of the AS curve of the same magnitude across the two countries (given that the underlying shock is the same).

Figure III: Doveland

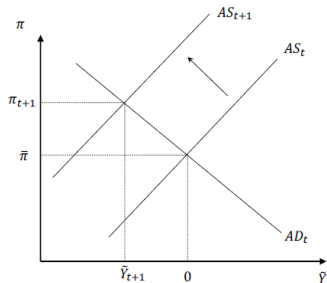
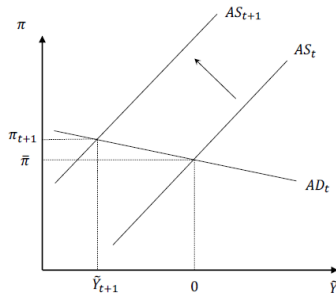


Figure IV: Hawkland



The shock generates an increase in inflation and a negative output gap in both countries. In Doveland, the increase in inflation is larger and the decline in short-term output is smaller compared to Hawkland. Intuitively, **the Central Bank in Hawkland is more aggressive in fighting inflation**, and when the gas price shock hits it raises the real interest rate more compared to the Central Bank in Doveland. This does moderate the increase in inflation but generates a larger recession.

**d** Suppose that at time  $t + 2$  the shock has dissipated, and gas prices are back to the normal level. Compare graphically and intuitively what happens in the two economies.



## Problem 1.d

In both countries, the AS curve shifts back towards the original position. The shift is not complete, since inflation expectation at time  $t + 2$  are based on inflation at time  $t + 1$  and are therefore above their long-run level  $\bar{\pi}$ . Inflation at time  $t + 2$  is still higher in Doveland compared to Hawkland, while output is lower in Hawkland compared to Doveland.

# Problem 1.d

In both countries, the AS curve shifts back towards the original position. The shift is not complete, since inflation expectation at time  $t + 2$  are based on inflation at time  $t + 1$  and are therefore above their long-run level  $\bar{\pi}$ . Inflation at time  $t + 2$  is still higher in Doveland compared to Hawkland, while output is lower in Hawkland compared to Doveland.

Figure V: Doveland

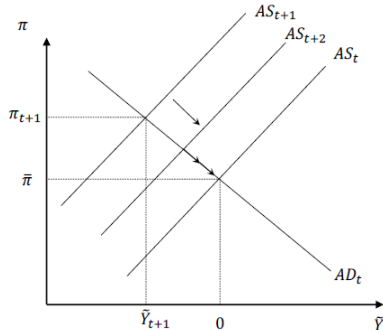
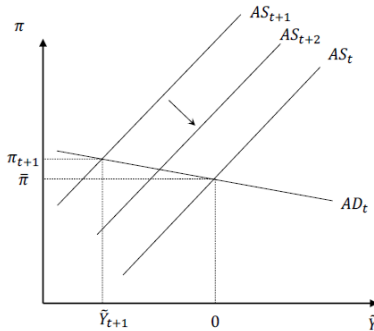


Figure VI: Hawkland



# Summary

- ▶ At  $t + 1$ , the shock  $o_{t+1}$  shifts up the AS curve by the same magnitude in both countries (given that the shock is the same).

# Summary

- ▶ At  $t + 1$ , the shock  $o_{t+1}$  shifts up the AS curve by the same magnitude in both countries (given that the shock is the same).
- ▶ The shock  $o_{t+1}$  generates an increase in inflation and a negative output gap in both countries.

# Summary

- ▶ At  $t + 1$ , the shock  $o_{t+1}$  shifts up the AS curve by the same magnitude in both countries (given that the shock is the same).
- ▶ The shock  $o_{t+1}$  generates an increase in inflation and a negative output gap in both countries.
- ▶ In Doveland, the increase in inflation is larger and the decline in output is smaller compared to Hawkland.

# Summary

- ▶ At  $t + 1$ , the shock  $o_{t+1}$  shifts up the AS curve by the same magnitude in both countries (given that the shock is the same).
- ▶ The shock  $o_{t+1}$  generates an increase in inflation and a negative output gap in both countries.
- ▶ In Doveland, the increase in inflation is larger and the decline in output is smaller compared to Hawkland.
- ▶ Why? Because the Central Bank in Hawkland is more aggressive in fighting inflation: it raises the real interest rate more compared to the Central Bank in Doveland.

# Summary

- ▶ At  $t + 1$ , the shock  $o_{t+1}$  shifts up the AS curve by the same magnitude in both countries (given that the shock is the same).
- ▶ The shock  $o_{t+1}$  generates an increase in inflation and a negative output gap in both countries.
- ▶ In Doveland, the increase in inflation is larger and the decline in output is smaller compared to Hawkland.
- ▶ Why? Because the Central Bank in Hawkland is more aggressive in fighting inflation: it raises the real interest rate more compared to the Central Bank in Doveland.
- ▶ Hawkland's monetary policy moderates the increase in inflation but generates a larger recession.

## Class Questions Problem 2

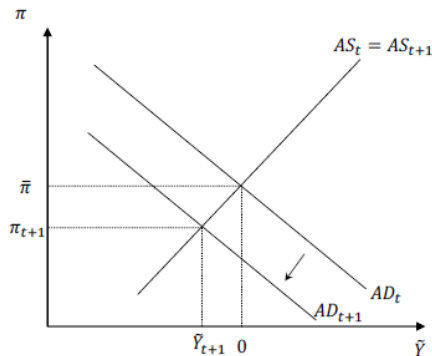
**[Demand Shock]** This question guides you through the analytical derivation of the effect of a demand shock. Consider the standard Keynesian AS-AD model described in the question above. Assume that the economy is at its long-run equilibrium at time  $t$ .

- (a) What are the levels of inflation and short-run output at time  $t$ ?
- (b) At time  $t + 1$  a negative demand shock hits, i.e.  $a_{t+1} = -a$  where  $a$  is a positive number. Write down the AS and AD equations for time  $t + 1$ , and use those to write inflation and short-run output as functions of exogenous parameters only.
- (c) At time  $t + 2$  the demand shock has dissipated, i.e.  $a_{t+2} = 0$  where  $a$  is a positive number. Write down the AS and AD equations for time  $t + 1$ , and use those to write inflation and short-run output as functions of exogenous parameters only. Verify that  $\pi_{t+2} > \pi_{t+1}$ , as stated in the lecture slides.
- (d) Repeat the same analysis for time  $t + 3$ , where  $a_{t+3} = 0$ . Can you recognize a pattern? What is the inflation rate at time  $t + \tau$ , where  $\tau > 0$ ? What happens as  $\tau$  becomes bigger?



# A Demand-Driven Business Cycle

- ▶ At time  $t$ , the economy is on its long-run equilibrium:  
 $\pi_t = \bar{\pi}$ ,  $\tilde{Y}_t = 0$ ,  $R_t = \bar{R}$
- ▶ Suppose that at time  $t + 1$  a wave of pessimism hits the economy, depressing the confidence of consumers and investors:  $a_{c,t+1} < 0$ ,  $a_{i,t+1} < 0 \implies a_{t+1} < 0$
- ▶ The shock is temporary and lasts only one period:  
 $a_{c,t+2} = a_{i,t+2} = a_{t+2} = 0$
- ▶ At time  $t + 1$ , the AD curve shifts downwards
  - ▶  $\pi_{t+1} < \bar{\pi}$ , lower demand decreases prices
  - ▶  $R_{t+1} < \bar{R}$ , the CB reacts to lower inflation through an expansionary policy



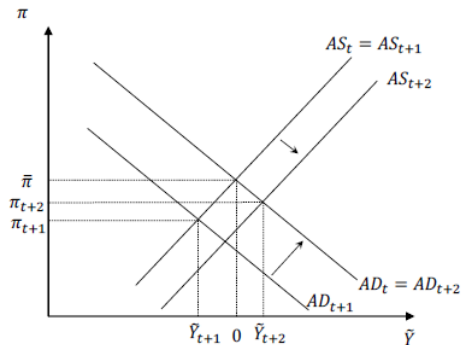
# A Demand-Driven Business Cycle

- ▶ At time  $t + 2$ , the demand shock has disappeared and the AD curve shifts back to the original position
- ▶ However, this is not the end of the story
- ▶ The AS curve at  $t + 2$  is

$$\pi_{t+2} = \pi_{t+1} + \bar{v}\tilde{Y}_{t+2}$$

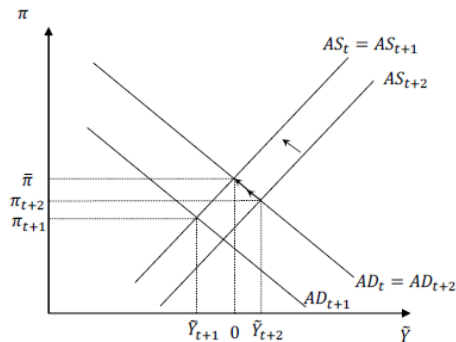
where  $\pi_{t+1} < \bar{\pi}$ !

- ▶ The AS curve shifts down because of lower inflation expectation at time  $t + 2$  (recall we assumed  $\pi_t^E = \pi_{t-1}$ )
- ▶ Notice:  $\pi_{t+2} > \pi_{t+1}$



# A Demand-Driven Business Cycle

- ▶ At time  $t + 3$ , expected inflation ( $=\pi_{t+2}$ ) is higher than at  $t + 2$  ( $=\pi_{t+1}$ )  $\rightarrow$  the AS curve shifts up
  - ▶ This process continues gradually until the AS curve is back to the original position
  - ▶ During the transition, as inflation increases the Central Bank tightens monetary policy (raising  $R_t$  and lowering  $\tilde{Y}_t$ )
- $\rightarrow$  Notice that a negative demand shock creates a recession followed by a small expansion
- ▶ This is due to the sluggish adjustment of inflation expectations
- $\rightarrow$  Overall, the model tells us that if the business cycle is driven by demand-shocks, inflation should be procyclical (lower inflation or deflation during recessions)



**a** What are the levels of inflation and short-run output at time  $t$ ?

## Problem 2.a

Given that the economy is at its long-run equilibrium at time  $t$ ,  $\pi_t = \bar{p}$  and  $\tilde{Y}_t = 0$ .

**b** At time  $t + 1$  a negative demand shock hits, i.e.  $a_{t+1} = -a$  where  $a$  is a positive number. Write down the AS and AD equations for time  $t + 1$ , and use those to write inflation and short-run output as functions of exogenous parameters only.

## Problem 2.b

AD:

$$\pi_{t+1} = -\frac{a}{\bar{b}\bar{m}} - \frac{1}{\bar{b}\bar{m}}\tilde{Y}_{t+1} + \bar{\pi} \quad (4)$$

AS:

$$\pi_{t+1} = \bar{\pi} + \bar{v}\tilde{Y}_{t+1} \quad (5)$$

The two can be combined to get:

$$\pi_{t+1} = \bar{\pi} - \frac{a\bar{v}}{1 + \bar{b}\bar{m}\bar{v}} < \bar{\pi} \quad (6)$$

$$\tilde{Y}_{t+1} = -a\left(1 - \frac{\bar{b}\bar{m}\bar{v}}{1 + \bar{b}\bar{m}\bar{v}}\right) < 0 \quad (7)$$

**c** At time  $t + 2$  the demand shock has dissipated, i.e.  $a_{t+2} = 0$  where  $a$  is a positive number. Write down the AS and AD equations for time  $t + 1$  and use those to write inflation and short-run output as functions of exogenous parameters only. Verify that  $\pi_{t+2} \Rightarrow \pi_{t+1}$  as stated in the lecture slides.



## Problem 2.c

AD:

$$\pi_{t+2} = -\frac{1}{\bar{b}\bar{m}}\tilde{Y}_{t+2} + \bar{\pi} \quad (8)$$

AS:

$$\pi_{t+2} = \pi_{t+1} + \bar{v}\tilde{Y}_{t+2} \quad (9)$$

The two can be combined to get:

$$\pi_{t+2} = \bar{\pi} - \frac{a\bar{v}}{(1 + \bar{b}\bar{m}\bar{v})^2} \quad (10)$$

$$\tilde{Y}_{t+2} = a \frac{\bar{b}\bar{m}\bar{v}}{(1 + \bar{b}\bar{m}\bar{v})^2} \quad (11)$$

**d** Repeat the same analysis for time  $t + 3$  where  $a_{t+3} = 0$ . Can you recognise a pattern? What is the inflation rate at time  $t + \tau$ , where  $\tau > 0$ ? What happens as  $\tau$  becomes bigger?

## Problem 2.d

The result for  $t + 3$  is

$$\pi_{t+3} = \bar{\pi} - \frac{a\bar{v}}{(1 + \bar{b}\bar{m}\bar{v})^3} \quad (12)$$

$$\tilde{Y}_{t+3} = a \frac{\bar{b}\bar{m}\bar{v}}{(1 + \bar{b}\bar{m}\bar{v})^3} \quad (13)$$

## Problem 2.d

The result for  $t + 3$  is

$$\pi_{t+3} = \bar{\pi} - \frac{a\bar{v}}{(1 + \bar{b}\bar{m}\bar{v})^3} \quad (12)$$

$$\tilde{Y}_{t+3} = a \frac{\bar{b}\bar{m}\bar{v}}{(1 + \bar{b}\bar{m}\bar{v})^3} \quad (13)$$

and for a generic  $t + \tau$ ,

$$\pi_{t+\tau} = \bar{\pi} - \frac{a\bar{v}}{(1 + \bar{b}\bar{m}\bar{v})^\tau} \quad (14)$$

$$\tilde{Y}_{t+\tau} = a \frac{\bar{b}\bar{m}\bar{v}}{(1 + \bar{b}\bar{m}\bar{v})^\tau} \quad (15)$$

As  $\tau$  becomes bigger,  $\pi_{t+\tau}$  and  $\tilde{Y}_{t+\tau} \rightarrow 0$

As  $\tau$  becomes bigger,  $\pi_{t+\tau}$  and  $\tilde{Y}_{t+\tau} \rightarrow 0$

$$\tilde{Y}_{t+\tau} = a \frac{\bar{b}\bar{m}\bar{v}}{(1 + \bar{b}\bar{m}\bar{v})^\tau}$$

$$\pi_{t+\tau} = \bar{\pi} - \frac{a\bar{v}}{(1 + \bar{b}\bar{m}\bar{v})^\tau}$$

