EC916: Topics in Global Finance

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Lecture - 7 -

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This Week: Outline

Which of the following statements does not describe a feature of the RBC model?

(a) Firms cannot influence their total factor productivity.
(b) Households take into account future technology shocks when choosing current consumption.
(c) Capital is accumulated through investment.
(d) When firms maximize profit, they cannot observe their current productivity.
(e) Total income is the sum of capital and labour income.

Review Question 2

In the RBC model, the household maximization problem gives the following optimality conditions:

$$v'(1-h_t) = u'(c_t)w_t$$

$$u'(c_t) = \beta(1 + r_{t+1} - \delta)u'(c_{t+1})$$

Illustrate the intution behind these two equations.

The first equation summarizes the optimal labour supply choice. It says that the marginal cost (in utility terms) of supplying an extra unit of labour, the left-hand side, should be equal to the marginal benefit (in utility terms), the right-hand side. The marginal utility cost is given by the marginal utility of leisure, since an extra unit of time devoted to work implies one unit of time less devoted to leisure. The marginal utility benefit is given by the extra resources one can get by working more, i.e. the wage, times the marginal utility of consumption.

The second equation summarizes the optimal intertemporal consumption-savings decision. It says that the marginal cost (in utility terms) of giving up one unit of consumption today, the left-hand side, should be equal to the marginal benefit (in utility terms), the right-hand side. The marginal utility cost is given by the marginal utility of consumption. The marginal utility benefit is given by the extra resources one can get by saving an extra unit today, i.e. the return (realized tomorrow) of investing this extra unit by buying capital and renting it out to firms, times the discounted marginal utility of consumption tomorrow.

Class Questions - Problem 1

Consider a model equivalent to the RBC except for the fact that technology is constant. The production and utility functions are given by

$$Y_t = AK_t^{\alpha} h_t^{1-\alpha}$$
$$u(c_t) + v(1 - h_t) = \log c_t + \theta \log(1 - h_t)$$

This model has a steady state where all endogenous variables are constant, i.e.

$$Y_t = Y^* \ , \ K_t = K^* \ , \ h_t = h^* \ , \ c_t = c^* \ , \ w_t = w^* \ , \ r_t = r^*$$

(you do not need to show this formally, though this should sound intuitive given what we saw in the Solow model). You can think about this as the long-run average around which the RBC economy fluctuates (because of technology shocks).

(a) Derive the first order conditions of the household's maximization problem on the steady state.

(b) Define the real interest rate as $R^* = r^* - \delta$. Derive the equilibrium value of R^* on the steady state.

(c) Suppose that you want to calibrate β to match the fact that (on average, in the US), $R^* = 0.016$. Which value would you pick for β?

(d) Suppose that you want to calibrate θ to match the fact that (on average, in the US), $h^* = 0.20$, $\frac{c^*}{Y^*} = 0.75$ and $\frac{w^* h^*}{Y^*} = 2/3$. Which value would you pick for θ?

(e) "Since the RBC model aims to explain the evolution of hours worked, calibrating θ to match the fact that $h^* = 0.20$ is not really appropriate." Do you agree with this statement? Explain why or why not.

a Derive the first order conditions of the household's maximization problem on the steady state.

Problem 1.a

$$\max_{\{c_t, h_t, K_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \Big[log c_t + \theta log(1 - h_t) \Big]$$

Subject to

$$K_{t+1} = (1-\delta)K_t + r_t K_t + w_t K_t - c_t, \quad \forall t$$

$$\frac{\partial \mathcal{L}}{\partial c_t} = 0 \Longrightarrow \beta^t \frac{1}{c_t} = \lambda_t$$

$$\frac{\partial \mathcal{L}}{\partial h_t} = 0 \Longrightarrow \beta^t \frac{\theta}{1 - h_t} = \lambda_t w_t$$

$$\frac{\partial \mathcal{L}}{\partial K_{t+1}} = 0 \Longrightarrow \lambda_t = \lambda_{t+1}(1 + r_{t+1} - \delta)$$

Problem 1.a

Substitute out the Lagrangian multipliers to get

$$\frac{\theta}{1-h_t} = \frac{w_t}{c_t}$$
$$\frac{1}{c_t} = \beta(1+r_{t+1}-\delta)\frac{1}{c_{t+1}}$$

Together with the budget constraint, these equations (one for every t) implicitly define the solution to the household problem.

Given that $h_t = h^*$, $c_t = c^*$, $w_t = w^*$ $r_t = r^*$

$$\frac{\theta}{1-h^*} = \frac{w^*}{c^*}$$
$$1 = \beta(1+r^*-\delta)$$

b

Define the real interest rate as $R^* = r^* - \delta$. Derive the equilibrium value of R^* on the steady state.

Problem 1.b

$$1 = \beta(1 + \overbrace{r^* - \delta}^{R^*})$$

$$1 = \beta(1 + R^*)$$

$$R^* = \frac{1 - \beta}{\beta}$$

$$\beta = \frac{1}{1 + R^*}$$

c Suppose that you want to calibrate β to match the fact that (on average, in the US), $R^* = 0.016$. Which value would you pick for β ?

Problem 1.c

Use the fact that:

$$\beta = \frac{1}{1+R^*}$$
$$= \frac{1}{1+0.016}$$
$$= 0.98$$

d Suppose that you want to calibrate θ to match the fact that (on average, in the US), $h^* = 0.20$, $\frac{c^*}{Y^*} = 0.75$ and $\frac{w^* h^*}{Y^*} = 2/3$ Which value would you pick for θ ?

Problem 1.d

What we have:

$$\frac{\theta}{1-h^*} = \frac{w^*}{c^*} \qquad h^*, \frac{c^*}{Y^*} \; and \; \frac{w^* h^*}{Y^*}$$

Rewrite the labour supply optimality condition as:

$$\frac{\theta}{1-h^*} = \frac{w^*}{c^*}$$
$$\frac{\theta h^*}{1-h^*} = \frac{w^* h^*}{Y^*} \frac{Y^*}{c^*}$$

Plug in the above numbers to find:

$$\theta \approx 3.5$$

e:

"Since the RBC model aims to explain the evolution of hours worked, calibrating θ to match the fact that $h^* = 0.2$ is not really appropriate."

Do you agree with this statement? Explain why or why not.

Problem 1.e

The RBC model aims to explain the fluctuations of hours worked over the business cycle, while here θ is calibrated to match the long-run average of hours worked.

These are two different moments, and matching one does not imply that the other is matched as well.

Indeed, we saw that the model fails to reproduce quantitatively the fluctuations in h_t, in spite of matching h^* by construction.

Class Questions - Problem 2

[**Measurement of the Solow Residual**] This question asks you to think about the Solow residual and what it measures.

(a) Briefly explain how the Solow residual is computed and how it is used in the calibration of the RBC model.

Consider a resercher wishing to estimate the Solow residual. As most macroeconomists, he assumes that $Y_t = A_t K_t^{\alpha} (h_t)^{1-\alpha}$, but in reality the production function is $Y_t = A_t K_t^{\alpha} (e_t h_t)^{1-\alpha}$, where e_t represents the average effort exterted by workers in an hour of work.

— —

(b) How does the Solow residual computed by the researcher depend on e_t?

(c) Suppose that e_t is procyclical. Would the Solow residual estimated by the researcher overstate or understate the actual cyclicality of A_t?

— —

Suppose now that the actual production function is $Y_t = A_t (\eta_t K_t)^{\alpha} h_t^{1-\alpha}$, where η_t represents the capital utilization rate (when $\eta_t = 1$ all machines are used in production; when $\eta_t < 1$, some machines are left idle). Our naive researcher still assumes $Y_t = A_t K_t^{\alpha} (h_t)^{1-\alpha}$.

(d) How does the Solow residual computed by the researcher depend on η_t?

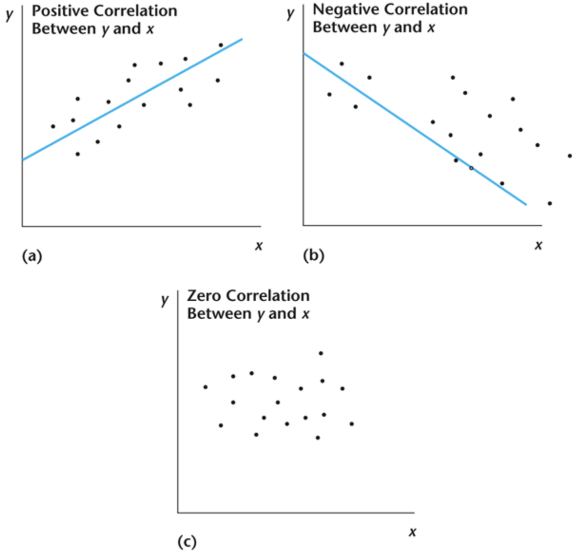
(e) Suppose that η_t is procyclical. Would the Solow residual estimated by the researcher overstate or understate the actual cyclicality of A_t?

a *“Briefly explain how the Solow residual is computed and how it is used in the calibration of the RBC model.*

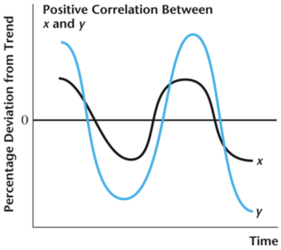
Problem 2.a

- The Solow residual is computed as $\frac{Y_t}{K_t^{\alpha} h_t^{1-\alpha}}$, using data on real GDP, capital and hours worked and a calibrated value for α.
- When calibrating the RBC model, it is used as a measure of technology A_t.
- The researcher feeds the observed time series for technology into the simulation and examines whether the model is able to reproduce business cycle fluctuations similar to the ones in the data.

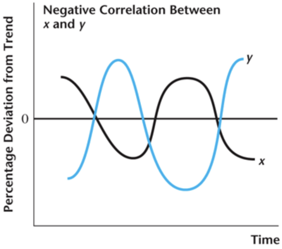
Correlations



Pro- vs Counter-Cyclical



(a)



(b)

- PRO-CYCLICAL

  positive co-movement

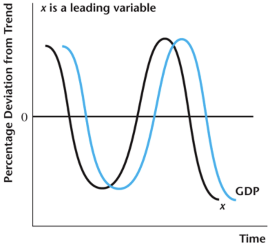
  positive correlation

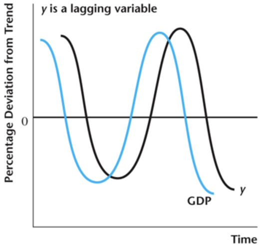
- COUNTER-CYCLICAL

  negative co-movement

  negative correlation

Leading and Lagging Variables





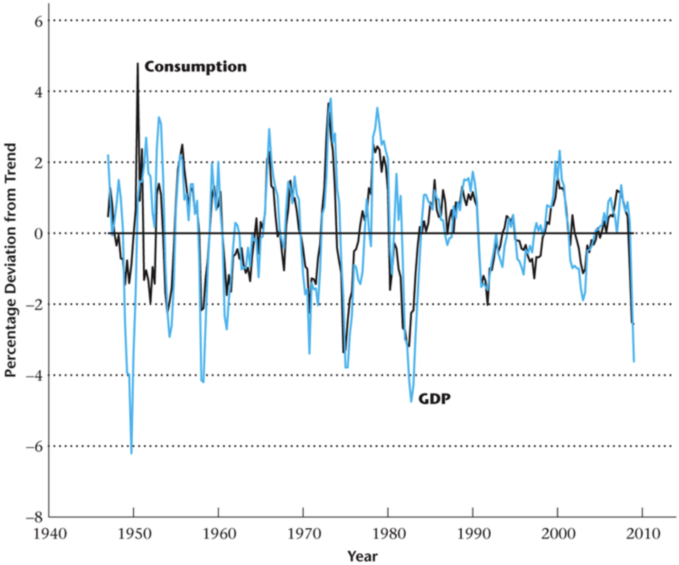
- LEADING

  the peak and trough of the series is **before** the GDP's peak and trough

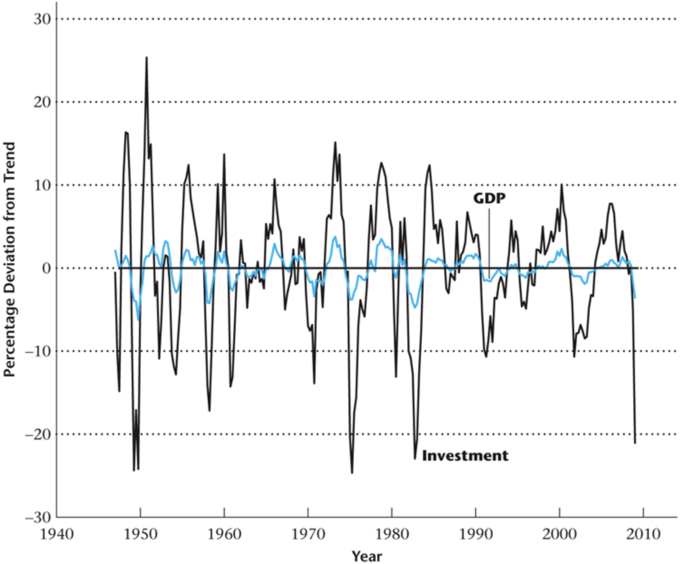
- LAGGING

  the peak and trough of the series is **after** the GDP's peak and trough

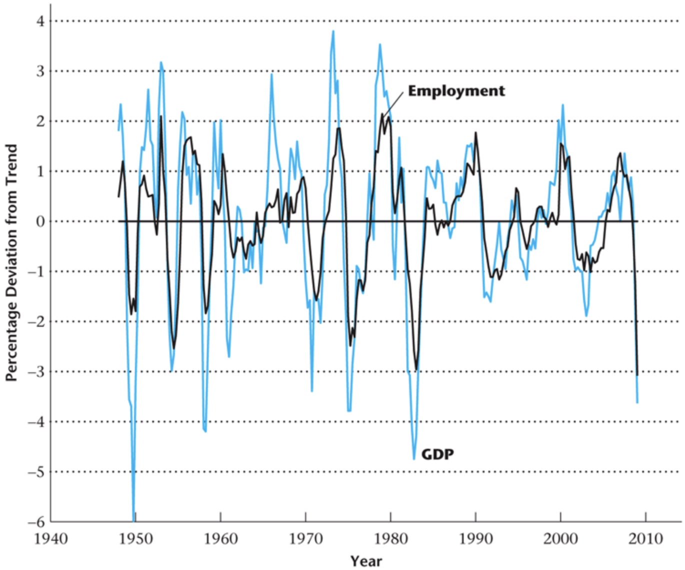
Consumption is Pro-cyclical, Coincident, Less Variable



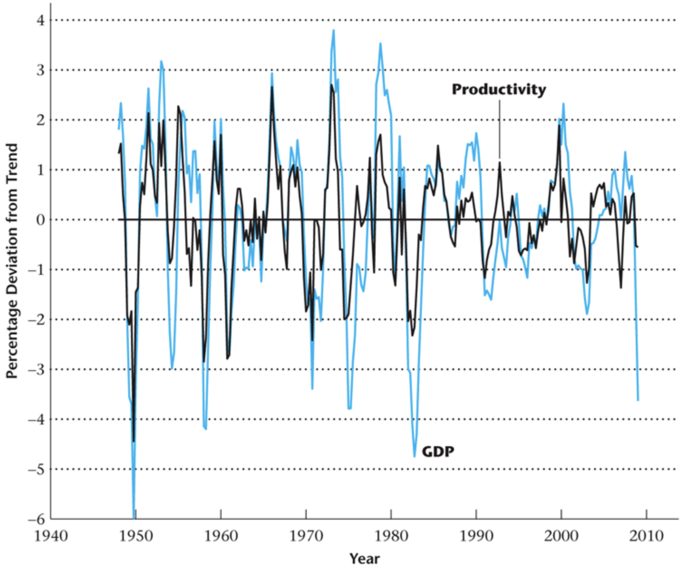
Investment is Pro-cyclical, Coincident, More Variable



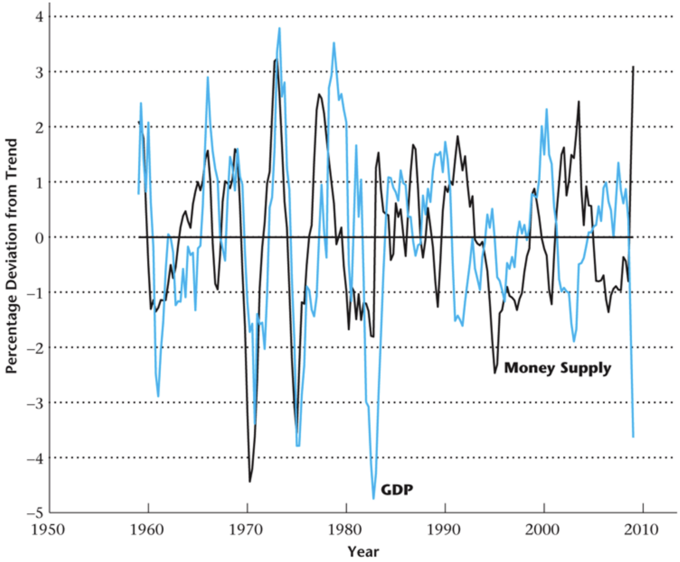
Employment is Pro-cyclical, Lagging, Less Variable



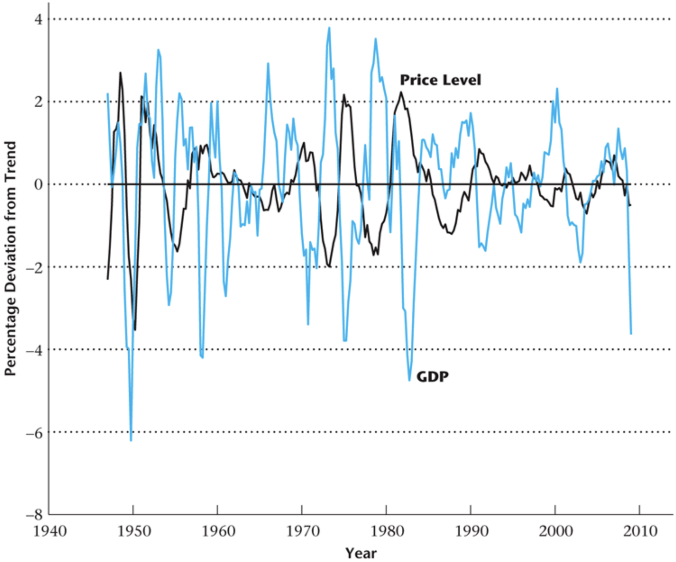
Labour Productivity is Pro-cyclical, Coincident, Less Variable



Money Supply is Pro-Cyclical, Leading, Less Variable



Price Level is Counter-Cyclical, Coincident, Less Variable



Summary of Business Cycle Facts

Table 3.1 Correlation Coefficients and Variability of Percentage Deviations from Trend

	Correlation Coefficient	Standard Deviation (% of S.D. of GDP)
Consumption	0.76	75.9
Investment	0.84	478.9
Price Level	−0.23	57.4
Money Supply	0.26	80.4
Employment	0.80	61.5
Average Labor Productivity	0.81	62.4

Table 3.2 Summary of Business Cycle Facts

	Cyclicality	Lead/Lag	Variation Relative to GDP
Consumption	Procyclical	Coincident	Smaller
Investment	Procyclical	Coincident	Larger
Price Level	Countercyclical	Coincident	Smaller
Money Supply	Procyclical	Leading	Smaller
Employment	Procyclical	Lagging	Smaller
Real Wage	Procyclical	?	?
Average Labor Productivity	Procyclical	Coincident	Smaller

Consider a resercher wishing to estimate the Solow residual. As most macroeconomists, he assumes that $Y_t = A_t K_t^{\alpha} (h_t)^{1-\alpha}$, but in reality the production function is $Y_t = A_t K_t^{\alpha} (e_t h_t)^{1-\alpha}$, where e_t represents the average effort exterted by workers in an hour of work.

b How does the Solow residual computed by the researcher depend on e_t?

Problem 2.b

The Solow residual computed by the researcher is:

$$
\begin{aligned}
\frac{Y_t}{K_t^{\alpha} h_t^{1-\alpha}} &= \frac{A_t K_t^{\alpha} (e_t h_t)^{1-\alpha}}{K_t^{\alpha} h_t^{1-\alpha}} \\
&= A_t e_t^{1-\alpha}
\end{aligned}
$$

Thus, it is increasing in e_t. If workers exert more effort at time t, the researcher would infer that the level of technology is higher at time t.

c Suppose that e_t is procyclical. Would the Solow residual estimated by the researcher overstate or understate the actual cyclicality of A_t?

Problem 2.c

It would overstate the extent to which technology is pro-cyclical.

Part of the co-movement between output and the Solow residual is actually **due to pro-cyclical effort.**

Suppose now that the actual production function is $Y_t = A_t(\eta_t K_t)^\alpha h_t^{1-\alpha}$, where η_t represents the capital utilization rate (when $\eta_t = 1$ all machines are used in production; when $\eta_t < 1$, some machines are left idle). Our naive researcher still assumes $Y_t = A_t K_t^\alpha h_t^{1-\alpha}$

d How does the Solow residual computed by the researcher depend on η_t?

Problem 2.d

The Solow residual computed by the researcher is:

$$\frac{Y_t}{K_t^{\alpha} h_t^{1-\alpha}} = \frac{A_t (\eta_t K_t)^{\alpha} h_t^{1-\alpha}}{K_t^{\alpha} h_t^{1-\alpha}}$$
$$= A_t \eta_t^{\alpha}$$

Thus, it is increasing in η_t. If capital is utilised to a greater extent at time t, the researcher would infer that the level of technology is higher at time t.

e Suppose that η_t is procyclical. Would the Solow residual estimated by the researcher overstate or understate the actual cyclicality of A_t?

Problem 2.e

It would overstate the extent to which technology is pro-cyclical.

Part of the co-movement between output and the Solow residual is actually **due to pro-cyclical capital utilization**.