EC201 Macroeconomics

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Class - 1 -

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Happy New Year!

This Week: Outline

1. The Journey of Humatity

2. Review Questions
2.1 Questions 1
2.2 Questions 2

3. Class Questions
3.1 Question 1
3.2 Question 2

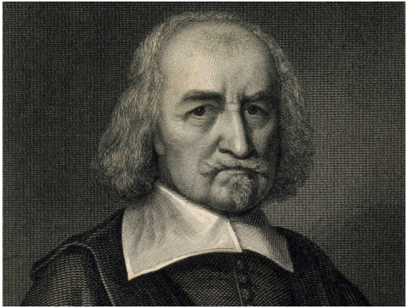
4. Self Study Questions

Thomas Hobbes: nasty, brutish, and short

- For most of history, standards of living were extremely low, not much different from that in the poorest countries of the world today.

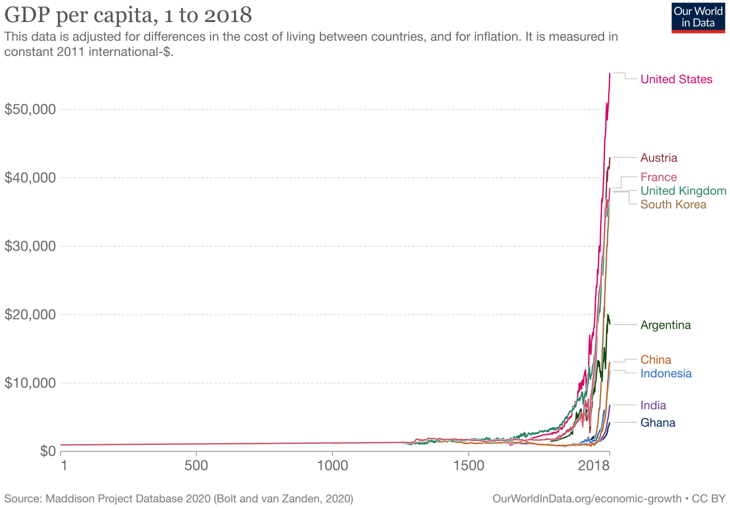
- Evidence suggests, for example, that wages in ancient Greece and Rome were approximately equal to wages in Britain in the fifteenth century or France in the seventeenth.

- In the words of seventeenth-century English philosopher Thomas Hobbes, life was Thomas Hobbes: **"solitary, poor, nasty, brutish, and short"**



THOMAS HOBBES:. 1588 - 1679

A long, long time ago in a galaxy far, far away



Things that didn't exist 20 years ago:

- iPhone
- Facebook
- YouTube
- Instagram
- Twitter
- TikTok
- Android
- Bitcoin
- Tesla
- iPad
- Gmail
- WhatsApp
- Spotify
- Airbnb
- Uber

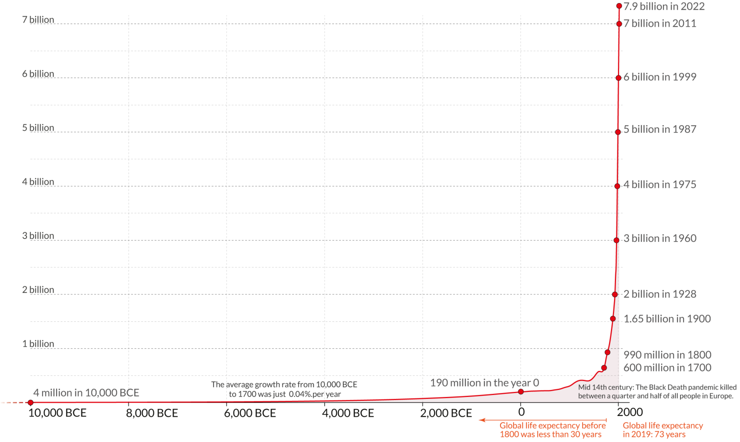
Our lives and habits are completely different than our parents

Not just getting richer but getting crowded too: World Population

Our World in Data

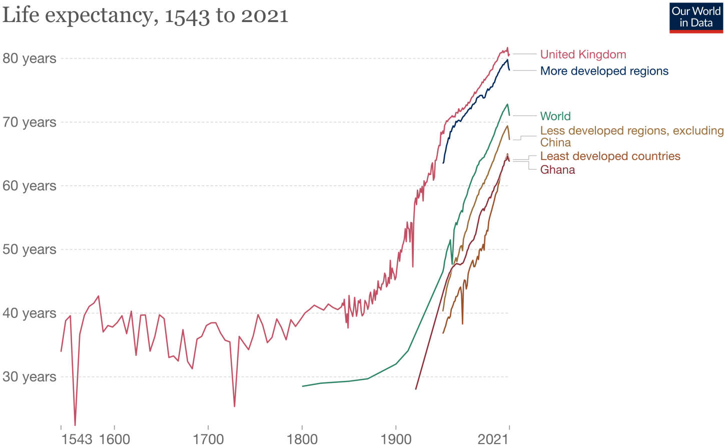
The size of the world population over the last 12.000 years

Demographers expect rapid population growth to end by the end of the 21st century. The UN demographers expect a population of about 11 billion in 2100.



Based on estimates by the History Database of the Global Environment (HYDE) and the United Nations. On OurWorldinData.org you can download the annual data.
This is a visualization from OurWorldinData.org.

Life expectancy: not as in before

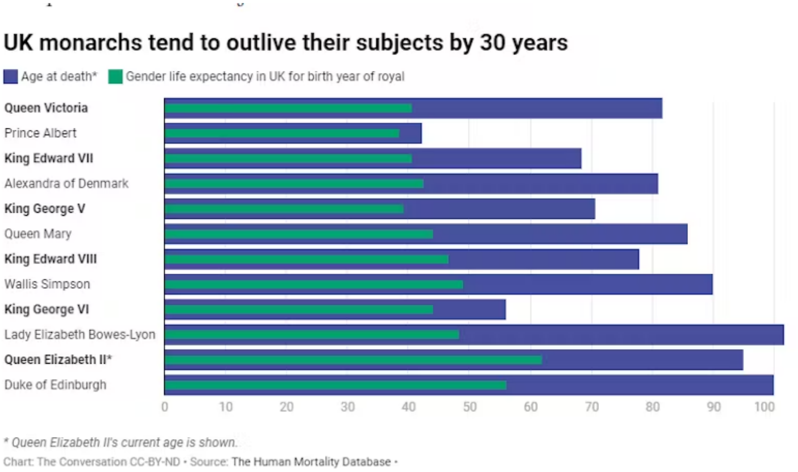


Source: UN WPP (2022); Zijdeman et al. (2015); Riley (2005)

OurWorldInData.org/life-expectancy • CC BY

Note: Shown is the 'period life expectancy'. This is the average number of years a newborn would live if age-specific mortality rates in the current year were to stay the same throughout its life.

Eat well live long

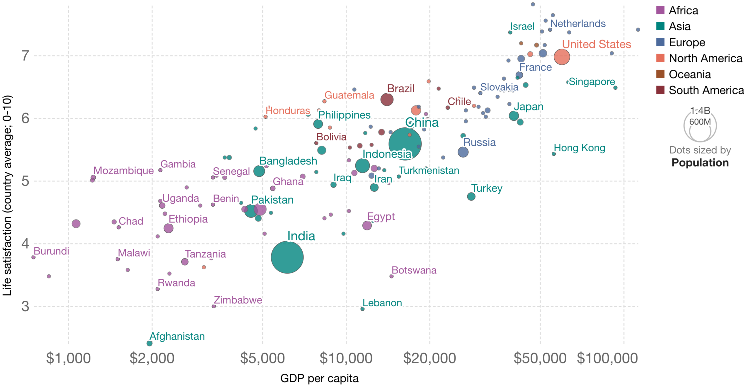


Richer and Happier

Self-reported life satisfaction vs GDP per capita, 2020

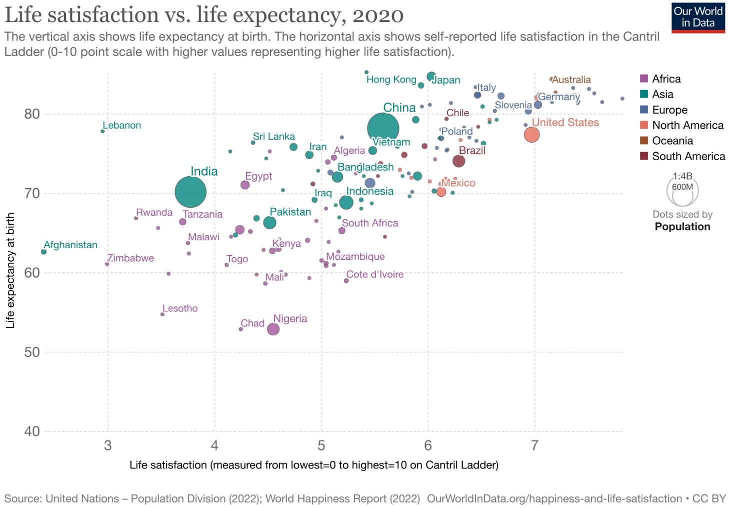
Our World in Data

The vertical axis shows the national average of the self-reported life satisfaction on a scale ranging from 0-10, where 10 is the highest possible life satisfaction. The horizontal axis shows GDP per capita adjusted for inflation and cross-country price differences.

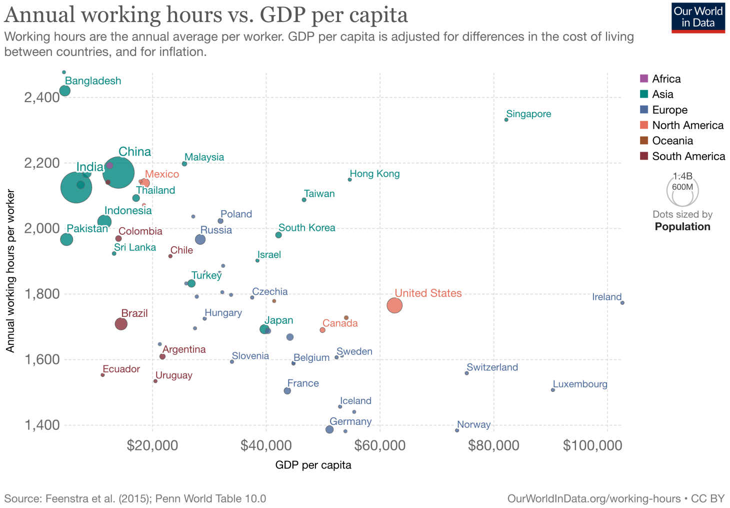


Source: World Happiness Report (2022); Data compiled from multiple sources by World Bank
OurWorldInData.org/happiness-and-life-satisfaction/ • CC BY

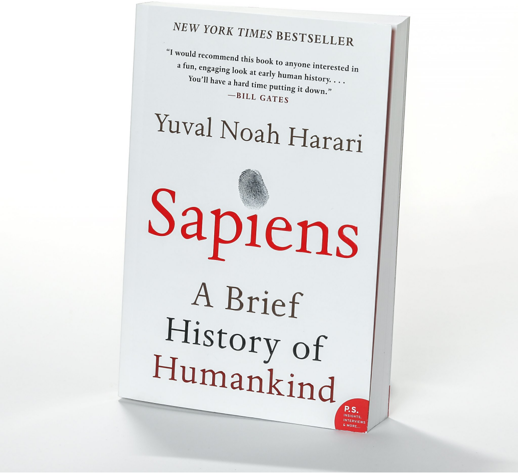
Be happy and live long



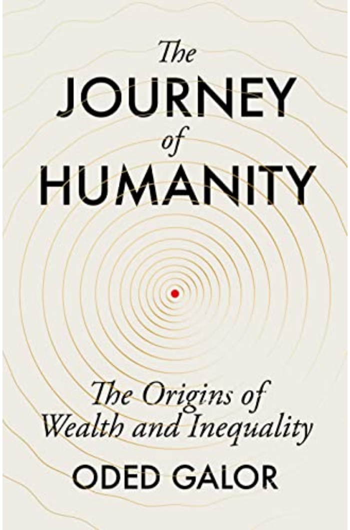
Get richer and work less



Book Advices



YUVAL NOAH HARARI A Historian.



ODED GALOR An Economist.

Review Question 1

- List at least three reasons why higher GDP per capita does not necessarily imply higher **welfare**.
- Does this mean we should not care that much about cross-country income differences?
- Explain why or why not.

Question 1

- What does the GDP include?

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$$Y = C + I + G + NX$$

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$$Y = C + I + G + NX$$

- What does the GDP exlude?
- Many...
  - Life Expectancy
  - Leisure Time / Working Time
  - Inequality
  - Pollution / Weather
  - Education / Health ...

Reason for Welfare $\neq$ GDP

- Higher GDP might be brought about by a higher labour supply and less leisure time
- Higher levels of production might cause higher levels of pollution, which might decrease utility
- Higher GDP might raise individual aspirations, so that a given amount of consumption might generate less utility
- Human welfare depends on many factors which are not directly captured by GDP, such as health, education, life expectancy, weather..

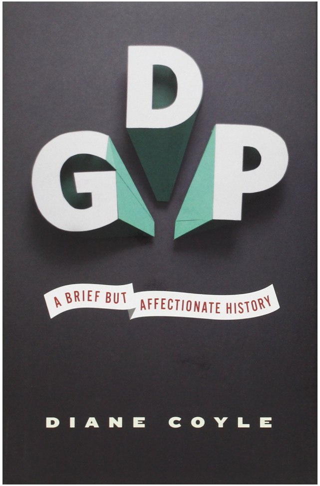
But...

- Still, we care greatly about cross-country income differences since GDP appears to be strongly correlated with alternative measures incorporating some of the factors listed above (while being at the same time easier to measure and compare across countries compared to such measures).
- For example, according to Jones and Klenow (2016) GDP per person is an informative indicator of welfare across a broad range of countries: the two measures have a correlation of **0.98**.
- Moreover, a country's productive capacity is likely to be a key factor in generating improvements in several welfare-relevant domains, such as health and educational opportunities

TABLE 7—WELFARE ACROSS COUNTRIES IN 2007: MACRO DATA

				Decomposition			
Country	Welfare λ	Per capita income	log ratio	LifeExp	C/Y	Leisure	C ineq.
United States	100.0	100.0	0.000	0.000	0.000	0.000	0.000
				77.8	*0.845*	*836*	*0.658*
Sweden	91.2	79.4	0.139	0.181	−0.186	0.010	0.135
				80.9	*0.701*	*807*	*0.404*
France	91.1	70.3	0.259	0.176	−0.085	0.063	0.106
				80.8	*0.776*	*629*	*0.471*
Japan	82.6	71.3	0.147	0.265	−0.154	−0.028	0.063
				82.5	*0.724*	*912*	*0.554*
Norway	81.0	112.8	−0.331	0.148	−0.598	0.019	0.100
				80.4	*0.464*	*780*	*0.483*
Germany	77.4	74.4	0.039	0.098	−0.195	0.047	0.089
				79.5	*0.695*	*687*	*0.506*
Ireland	69.6	96.4	−0.325	0.069	−0.454	−0.022	0.082
				79.0	*0.536*	*896*	*0.519*
Hong Kong	59.0	83.4	−0.345	0.239	−0.433	−0.151	−0.000
				82.4	*0.548*	*1,194*	*0.658*
Singapore	56.7	117.1	−0.726	0.139	−0.685	−0.180	−0.000
				80.4	*0.426*	*1,251*	*0.658*
South Korea	45.3	58.3	−0.252	0.078	−0.290	−0.116	0.076
				79.3	*0.632*	*1,120*	*0.531*
Argentina	21.8	26.2	−0.181	−0.121	−0.108	0.048	−0.000
				75.1	*0.759*	*684*	*0.658*
Chile	19.7	30.9	−0.451	0.029	−0.254	−0.026	−0.199
				78.5	*0.655*	*908*	*0.912*
Thailand	10.9	18.1	−0.507	−0.158	−0.207	−0.043	−0.099
				73.5	*0.687*	*951*	*0.794*
South Africa	4.5	17.4	−1.351	−0.931	−0.053	0.061	−0.427
				51.0	*0.801*	*636*	*1.135*
Botswana	4.3	25.1	−1.767	−0.852	−0.574	−0.008	−0.333
				52.1	*0.476*	*859*	*1.048*
Vietnam	4.0	5.9	−0.378	−0.082	−0.269	−0.020	−0.006
				74.2	*0.645*	*893*	*0.668*
Zimbabwe	3.1	8.3	−0.972	−0.983	0.155	−0.050	−0.094
				[illegible]	[illegible]	[illegible]	[illegible]

Recommendation...



The book ends by making the case that GDP was a good measure for the twentieth century but is increasingly inappropriate for a twenty-first-century economy driven by innovation, services, and intangible goods.

Question?

Review Questions 2

Consider an economy where the production function is

$$Y(K, L) = 10K^{1/2}L^{1/2}$$

Assume that the supply of capital and labor are fixed at $\overline{K} = 100$ and $\overline{L} = 36$.

- Compute the equilibrium levels of
  - output, Y
  - the wage rate, w
  - the rental rate of capital, r

Review Questions 2

By substituting $\overline{K} = 100$ and $\overline{L} = 36$ in the production function, we get

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Factor prices are given by

$$\omega = MPL = 5\frac{\overline{K}^{\frac{1}{2}}}{\overline{L}^{\frac{1}{2}}} = \frac{25}{3}$$

$$r = MPK = 5\frac{\overline{L}^{\frac{1}{2}}}{\overline{K}^{\frac{1}{2}}} = 3$$

Question?

Class Questions

Class Question 1

Consider the model of production discussed in the lecture. Assume that the production function is

$$Y(K, L) = K^{\alpha} L^{1-\alpha}$$

and that the supplies of capital and labour are fixed and exogenous.

(a) Write down an expression for the equilibrium wage and rental rate of capital. Represent graphically the equilibrium in the market for labour and capital.

Question 1a

$$\omega^* = (1-\alpha)\left(\frac{\overline{K}}{\overline{L}}\right)^{\alpha}$$

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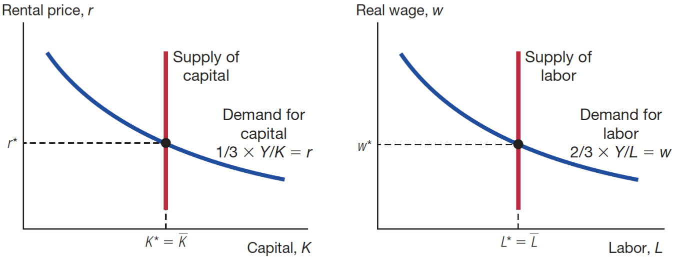
Supply and Demand in the Capital and Labor Markets



(a) The capital market (b) The labor market

Question 1b

Assume now that the government introduces an employment subsidy $s \in (0, 1)$, so that firms pay only a fraction s of the employees' wages. How are the equilibrium levels of labour, capital, the wage rate and the rental rate of capital affected? Show both analytically and graphically.

Question 1b

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$$\omega^{**} = \left(\frac{1-\alpha}{1-s}\right)\left(\frac{\overline{K}}{\overline{L}}\right)^{\alpha}$$

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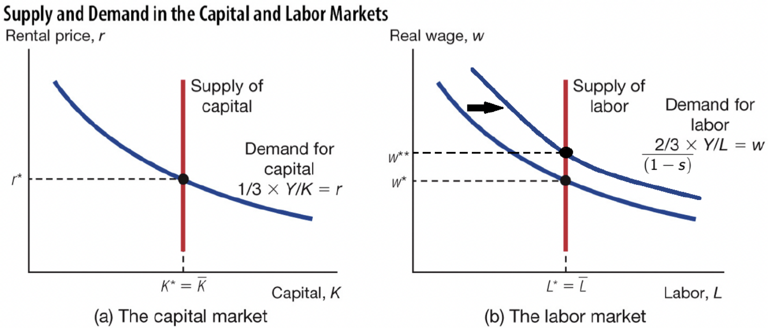
$$\omega^{**} = \left(\frac{1-\alpha}{1-s}\right)\left(\frac{\overline{K}}{\overline{L}}\right)^{\alpha}$$

$$r^{*} = \alpha\left(\frac{\overline{L}}{\overline{K}}\right)^{1-\alpha}$$

The equilibrium levels of capital and labour do not change. The wage increases, while the rental rate of capital is unaffected. **Graph:** the labour demand curve shifts up in the labour market graph; nothing happens in the market for capital.

Question 1b



$$\omega^{**} = \left(\frac{1-\alpha}{1-s}\right)\left(\frac{\overline{K}}{\overline{L}}\right)^{\alpha}$$

$$r^{*} = \alpha\left(\frac{\overline{L}}{\overline{K}}\right)^{1-\alpha}$$

$\omega^{**} > \omega^{*}$.

Question 1c

Assume now instead that the government introduces a tax on revenues $\tau \in (0, 1)$ (payroll and capital costs are not deductible). How are the equilibrium levels of labour, capital, the wage rate and the rental rate of capital affected compared to part (a)? Show both analytically and graphically.

Question 1c

Assume now instead that the government introduces a tax on revenues $\tau \in (0, 1)$ (payroll and capital costs are not deductible). How are the equilibrium levels of labour, capital, the wage rate and the rental rate of capital affected compared to part (a)? Show both analytically and graphically.

$$\omega^* = (1 - \tau)(1 - \alpha)\left(\frac{\overline{K}}{\overline{L}}\right)^{\alpha}$$

Question 1c

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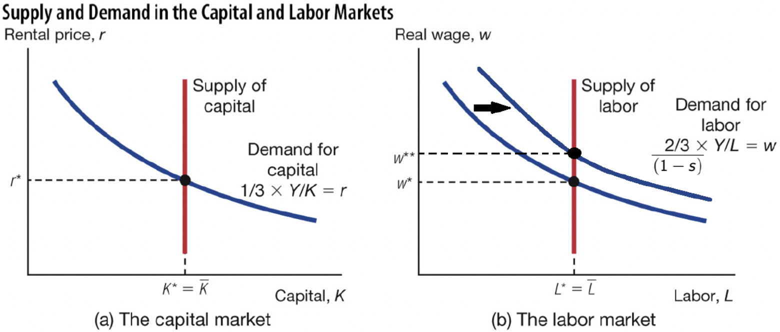
$$\omega^* = (1-\tau)(1-\alpha)\left(\frac{\overline{K}}{\overline{L}}\right)^{\alpha}$$

$$r^{**} = (1-\tau)\alpha\left(\frac{\overline{L}}{\overline{K}}\right)^{1-\alpha}$$

The equilibrium levels of capital and labour do not change. Both the wage and the rental rate of capital decrease.

Graph: the labour demand curve and the capital demand curve both shifts down.

Question 1b



$$\omega^* = (1-\tau)(1-\alpha)\left(\frac{\overline{K}}{\overline{L}}\right)^{\alpha}$$

$$r^{**} = (1-\tau)\alpha\left(\frac{\overline{L}}{\overline{K}}\right)^{1-\alpha}$$

$r^{**} < r^*.$

Question 1.d

Finally, assume that the government implements a lump-sum transfer to firms $T > 0$. How are the equilibrium levels of labour, capital, the wage rate and the rental rate of capital affected compared to part (a)? Show both analytically and graphically.

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All equilibrium quantities are identically to part (a). No changes in the graphs.

Question 1b

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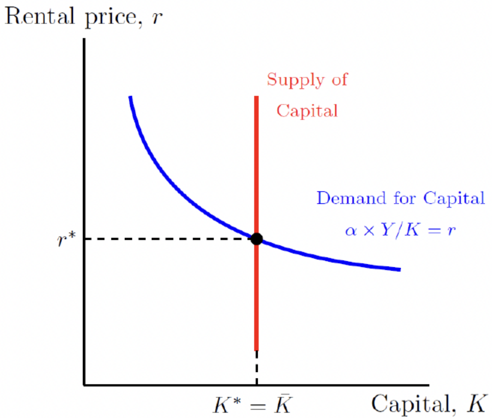
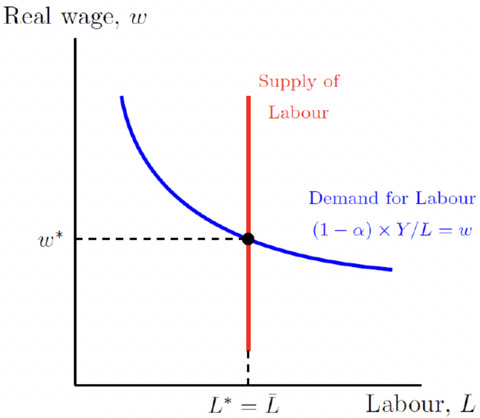


The capital market



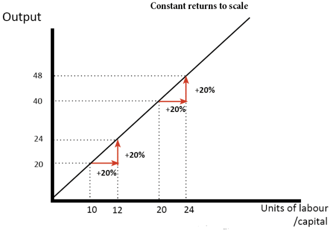
The labour market

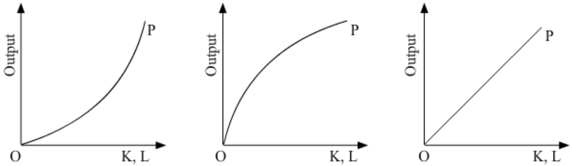
Question?

Question 2

Constant Return to Scale

Graphically;





(a) Increasing Returns to Scale (b) Decreasing Returns to Scale (c) Constant Returns to Scale

Question 2

Do the following production functions display increasing, constant or decreasing returns to scale in K and L? Assume that A is some positive fixed number.

(a) $Y = K^{\frac{1}{2}} L^{\frac{1}{2}}$

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(b) $Y = K^{\frac{2}{3}} L^{\frac{2}{3}}$

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(c) $Y = K^{\frac{1}{3}} L^{\frac{1}{2}}$

Decreasing. $(\gamma K)^{\frac{1}{3}} (\gamma L)^{\frac{1}{2}} = \gamma^{\frac{5}{6}} Y < \gamma Y$ for any $\gamma > 1$

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(e) $Y = K + K^{\frac{1}{3}} L^{\frac{1}{3}}$

Decreasing. $\gamma K + (\gamma K)^{\frac{1}{3}} (\gamma L)^{\frac{1}{3}} = \gamma K + \gamma^{\frac{2}{3}} (K)^{\frac{1}{3}} (L)^{\frac{1}{3}} < \gamma K + \gamma (K)^{\frac{1}{3}} (L)^{\frac{1}{3}} = \gamma Y$ for any $\gamma > 1$

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Self Study Questions

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- Consider the production function

$$Y_c = \overline{A_c} K_c^{\alpha} \left(h_c L_c\right)^{1-\alpha}$$

where h_c is *human capital* per worker, i.e. the productivity of a given worker in country c. The value of h_c might be different across countries due to different levels of educational attainment, health, experience, and so on.

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$$w_c = (1-\alpha)\overline{A_c}k_c^{\alpha}\left(h_c\right)^{1-\alpha}$$

Wages might differ across countries because of differences in productivity, physical capital per worker and human capital per worker.

Self Study Questions

(c) How would you calibrate α?

$\alpha = 1/3$ to match the capital share.

Self Study Questions

(d) In India, physical capital per worker and GDP per worker are respectively 8% and 10% of the corresponding quantity in the US. Some economists argue that the income gap between the two countries might be entirely explained by differences in human capital per worker, while $A_{US} = A_{INDIA}$. How much should h_{INDIA}/h_{US} be for this to be true?

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$$\frac{y_{INDIA}}{y_{US}} = \frac{\overline{A}_{INDIA}}{\overline{A}_{US}} \left(\frac{k_{INDIA}}{k_{US}}\right)^{\alpha} \left(\frac{h_{INDIA}}{h_{US}}\right)^{1-\alpha}$$

$$0.1 = \frac{\overline{A}_{INDIA}}{\overline{A}_{US}} (0.08)^{1/3} \left(\frac{h_{INDIA}}{h_{US}}\right)^{2/3}$$

If $\frac{\overline{A}_{INDIA}}{\overline{A}_{US}} = 1$, then $h_{INDIA}/h_{US} = 0.11$. Human capital per worker should be about 10 times higher in the US.