EC201 Macroeconomics 2

Week 8 - Seminar 6

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This Week

1. Review Questions
2. Class Questions

Review Questions

R1 Consider a two-period small open economy populated by a large number of identical households with preferences described by the utility function:

$$\ln C_1^T + \ln C_1^N + \ln C_2^T + \ln C_2^N,$$

where C_1^T and C_2^T denote consumption of tradables in periods 1 and 2, respectively, and C_1^N and C_2^N denote consumption of nontradables in periods 1 and 2. Households are born in period 1 with no debts or assets and are endowed with $L_1 = 1$ units of labor in period and $L_2 = 1$ units of labor in period 2. Households offer their labor to firms, for which they get paid the wage rate w_1 in period 1 and w_2 in period 2. The wage rate is expressed in terms of tradable goods. Households can borrow or lend in the international financial market at the world interest rate r^*. Let p_1^N and p_2^N denote the relative price of nontradable goods in terms of tradable goods in periods 1 and 2, respectively. Firms in the traded sector produce output with the technology $Q_1^T = a_T L_1^T$ in period 1 and $Q_2^T = a_T L_2^T$ in period 2, where Q_t^T denotes output in period $t = 1, 2$ and L_t^T denotes employment in the traded sector in period $t = 1, 2$. Similarly, production in the nontraded sector in periods 1 and 2 is given by $Q_1^N = a_N L_1^N$ and $Q_2^N = a_N L_2^N$.

R-1A Write down the budget constraint of the household in periods 1 and 2.

Review Q1-Budget Constraints

R1-A As labour income is $W_t \cdot Lt$ then budget constraints in period 1 and 2 then given by

$$P_1^T C_1^T + P_1^N C_1^N + P_1^T B_1^* = (1+r_0) P_1^T B_0^* + W_1 L_1$$

$$P_2^T C_2^T + P_2^N C_2^N + P_2^T B_2^* = (1+r_1) P_2^T B_1^* + W_2 L_2$$

In order to express in terms of tradable goods divide the period-t budget constraints by P_t^T and remember that $w_t = \frac{W_t}{P_t^T}$, $r_1 = r^*$, and $L_1 = L_2 = 1$. Therefore, the budget constraints in period 1 and 2 written in terms of tradable goods are

$$C_1^T + p_1^N C_1^N + B_1^* - B_0^* = r_0 B_0^* + w_1,$$

$$C_2^T + p_2^N C_2^N + B_2^* - B_1^* = r_1 B_1^* + w_2.$$

Since $B_0^* = 0$ by assumption, $B_2^* = 0$ by the usual no-Ponzi-game condition then these can be rewritten as follows:

$$C_1^T + p_1^N C_1^N + B_1^* = w_1,$$

$$C_2^T + p_2^N C_2^N = (1 + r^*)\, B_1^* + w_2.$$

R-1B Write down the intertemporal budget constraint of the household.

R-1C State the household's utility maximization problem.

R1-B **Intertemporal Budget Constraint**

Solve the second constraint for B_1^*:

$$B_1^* = \frac{C_2^T + p_2^N C_2^N - w_2}{1 + r^*}.$$

Plug this into the first constraint and rearrange to get the intertemporal budget constraint:

$$C_1^T + p_1^N C_1^N + \frac{C_2^T + p_2^N C_2^N}{1 + r^*} = w_1 + \frac{w_2}{1 + r^*}.$$

R1-C **Household's Problem**

$$\max_{C_1^T, C_1^N, C_2^T, C_2^N} U(C) = \ln C_1^T + \ln C_1^N + \ln C_2^T + \ln C_2^N$$

$$\text{s.t.} \quad C_1^T + p_1^N C_1^N + \frac{C_2^T + p_2^N C_2^N}{1 + r^*} = w_1 + \frac{w_2}{1 + r^*}.$$

R-1D Derive the optimality conditions associated with the household's maximization problem.

R1-D We take the Lagrange multipliers approach to constrained optimization. The Lagrangean function of the household's problem is

$$\mathcal{L} = \ln C_1^T + \ln C_1^N + \ln C_2^T + \ln C_2^N + \\ + \lambda \left[w_1 + \frac{w_2}{1+r^*} - C_1^T - p_1^N C_1^N - \left(\frac{C_2^T + p_2^N C_2^N}{1+r^*} \right) \right]$$

The first-order conditions with respect to each argument of the utility function are as follows:

$$\frac{\partial \mathcal{L}}{\partial C_1^T} = \frac{1}{C_1^T} - \lambda = 0,$$

$$\frac{\partial \mathcal{L}}{\partial C_1^N} = \frac{1}{C_1^N} - \lambda p_1^N = 0,$$

$$\frac{\partial \mathcal{L}}{\partial C_2^T} = \frac{1}{C_2^T} - \lambda \left(\frac{1}{1+r^*} \right) = 0,$$

$$\frac{\partial \mathcal{L}}{\partial C_2^N} = \frac{1}{C_2^N} - \lambda p_2^N \left(\frac{1}{1+r^*} \right) = 0.$$

Rearrange them into

$$\lambda = \frac{1}{C_1^T},$$

$$\lambda = \frac{1}{p_1^N C_1^N},$$

$$\lambda = (1 + r^*) \frac{1}{C_2^T},$$

$$\lambda = (1 + r^*) \frac{1}{p_2^N C_2^N}.$$

Combine the first with the second and the third with the fourth to obtain the allocation of consumption across tradables and nontradables in each period:

$$C_1^T = p_1^N C_1^N, \qquad C_2^T = p_2^N C_2^N. \tag{1}$$

Also combine the first with the third and the second with the fourth to find the intertemporal allocation of consumption for each category of goods:

$$C_1^T = \frac{C_2^T}{1+r^*}, \qquad C_1^N = \frac{C_2^N}{1+r^*}\frac{p_2^N}{p_1^N}. \tag{2}$$

Class Questions

Question - B - Continue from the First Review Question

a) Derive an expression for the optimal levels of consumption of tradables and nontradables in periods 1 and 2 ($C_1^T, C_2^T, C_1^N, C_2^N$) as functions of r^*, w_1, w_2, p_1^N , and p_2^N .

b) Using the zero-profit conditions on firms, derive expressions for the real wage and the relative price of nontradables (w_t and p_t^N ,$t = 1, 2$), in terms of the parameters a_T and a_N .

c) Write down the market clearing condition for nontradables.

d) Write down the market clearing condition for labor.

e) Using the above results, derive the equilibrium levels of consumption , the trade balance, and sectoral employment ($C_1^T, C_2^T, C_1^N, C_2^N, TB_1, TB_2, L_1^T, L_2^T$) in terms of the structural parameters R^*, a_T and a_N .

f) Is there any sectoral labor reallocation over time? If so, explain the intuition behind it.

C-1A Derive an expression for the optimal levels of consumption of tradables and nontradables in periods 1 and 2 ($C_1^T, C_2^T, C_1^N, C_2^N$) as functions of r^*, w_1, w_2, p_1^N , and p_2^N .

C1-A

Use (1) to eliminate nontradables in the intertemporal budget constraint:

$$C_1^T + C_1^T + \frac{C_2^T + C_2^T}{1+r^*} = w_1 + \frac{w_2}{1+r^*}.$$

Then use (2) to get rid of period 2 consumption too:

$$2C_1^T + 2C_1^T = w_1 + \frac{w_2}{1+r^*}.$$

Solve this equation for the level of consumption of tradables in period 1:

$$C_1^T = \frac{1}{4}\left(w_1 + \frac{w_2}{1+r^*}\right).$$

From the answer above:

$$C_1^T = \frac{1}{4}\left(w_1 + \frac{w_2}{1+r^*}\right). \tag{3}$$

$$C_1^T = p_1^N C_1^N, \qquad C_2^T = p_2^N C_2^N. \tag{4}$$

$$C_1^T = \frac{C_2^T}{1+r^*}, \qquad C_1^N = \frac{C_2^N}{1+r^*}\frac{p_2^N}{p_1^N}. \tag{5}$$

Now use equations ((3), (4), and (5)) to compute the remaining consumption levels as functions of wages, prices and the interest rate as well:

$$C_2^T = (1+r^*)\,\frac{1}{4}\left(w_1 + \frac{w_2}{1+r^*}\right),$$

$$C_1^N = \left(\frac{1}{p_1^N}\right)\frac{1}{4}\left(w_1 + \frac{w_2}{1+r^*}\right),$$

$$C_2^N = (1+r^*)\left(\frac{1}{p_2^N}\right)\frac{1}{4}\left(w_1 + \frac{w_2}{1+r^*}\right).$$

C-1B Using the zero-profit conditions on firms, derive expressions for the real wage and the relative price of nontradables (w_t and p_t^N ,$t = 1, 2$), in terms of the parameters a_T and a_N

C1-B If firms are perfectly competitive, the following zero-profit conditions apply:

$$Q_1^T = w_1 L_1^T, \qquad p_1^N Q_1^N = w_1 L_1^N,$$

$$Q_2^T = w_2 L_2^T, \qquad p_2^N Q_2^N = w_2 L_2^N.$$

Plug in the following production functions:

$$Q_1^T = a_T L_1^T, \qquad Q_1^N = a_N L_1^N,$$

$$Q_2^T = a_T L_2^T, \qquad Q_2^N = a_N L_2^N.$$

The following relations emerge between wages and labor productivities:

$$w_1 = a_T, \qquad w_1 = p_1^N a_N,$$

$$w_2 = a_T, \qquad w_2 = p_2^N a_N.$$

Combining these equations we get the relative prices of nontradables:

$$p_1^N = \frac{a_T}{a_N},$$

$$p_2^N = \frac{a_T}{a_N}.$$

C-1C Write down the market clearing condition for nontradables.

C1-C

$$Q_1^N = C_1^N,$$

$$Q_2^N = C_2^N.$$

C-1D Write down the market clearing condition for labor.

C1-D Combine the labor market resource constraints

$$L_1^T + L_1^N = 1, \qquad L_2^T + L_2^N = 1$$

with the production functions shown above to get the following market clearing conditions:

$$\frac{Q_1^T}{a_T} + \frac{Q_1^N}{a_N} = 1, \qquad \frac{Q_2^T}{a_T} + \frac{Q_2^N}{a_N} = 1.$$

C-1E Using the above results, derive the equilibrium levels of consumption , the trade balance, and sectoral employment $(C_1^T, C_2^T, C_1^N, C_2^N, TB_1, TB_2, L_1^T, L_2^T)$ in terms of the structural parameters R^*, a_T and a_N

C1-E First use the wage/productivity relations found above to eliminate wages from our four consumption equations:

$$C_1^T = \frac{1}{4}\left(a_T + \frac{a_T}{1+r^*}\right) = \frac{a_T}{4}\left(\frac{2+r^*}{1+r^*}\right),$$

$$C_2^T = (1+r^*)\,\frac{1}{4}\left(a_T + \frac{a_T}{1+r^*}\right) = \frac{a_T}{4}\left(2+r^*\right),$$

$$C_1^N = \left(\frac{1}{p_1^N}\right)\frac{1}{4}\left(a_T + \frac{a_T}{1+r^*}\right) = \left(\frac{1}{p_1^N}\right)\frac{a_T}{4}\left(\frac{2+r^*}{1+r^*}\right),$$

$$C_2^N = (1+r^*)\left(\frac{1}{p_2^N}\right)\frac{1}{4}\left(a_T + \frac{a_T}{1+r^*}\right) = \left(\frac{1}{p_2^N}\right)\frac{a_T}{4}\left(2+r^*\right).$$

Then use the relative price formulae obtained before to rewrite the last two equations in terms of productivity and the interest rate only:

$$C_1^N = \left(\frac{a_N}{a_T}\right)\frac{a_T}{4}\left(\frac{2+r^*}{1+r^*}\right) = \frac{a_N}{4}\left(\frac{2+r^*}{1+r^*}\right),$$

$$C_2^N = \left(\frac{a_N}{a_T}\right)\frac{a_T}{4}\left(2+r^*\right) = \frac{a_N}{4}\left(2+r^*\right).$$

To find the sectoral employment levels, start from the equilibrium level of labour in the nontraded goods sector: combine the last two equations and the appropriate production function with the market clearing condition for non-traded goods in each period, to get

$$\begin{aligned} Q_1^N &= C_1^N \\ a_N L_1^N &= \frac{a_N}{4}\left(\frac{2+r^*}{1+r^*}\right) \\ L_1^N &= \frac{1}{4}\left(\frac{2+r^*}{1+r^*}\right), \end{aligned}$$

$$\begin{aligned} Q_2^N &= C_2^N \\ a_N L_2^N &= \frac{a_N}{4}(2+r^*) \\ L_2^N &= \frac{1}{4}(2+r^*). \end{aligned}$$

Now plug these results into the labor market resource constraints to recover the employment levels in the traded goods sector:

$$L_1^T = 1 - L_1^N = 1 - \frac{1}{4}\left(\frac{2+r^*}{1+r^*}\right),$$

$$L_2^T = 1 - L_2^N = 1 - \frac{1}{4}(2+r^*).$$

With these hours worked, the period 1 and period 2 levels of output in the tradable sector are

$$Q_1^T = a_T L_1^T = a_T \left[1 - \frac{1}{4}\left(\frac{2+r^*}{1+r^*}\right)\right],$$

$$Q_2^T = a_T L_2^T = a_T \left[1 - \frac{1}{4}(2+r^*)\right].$$

From these we can obtain the trade balances:

$$TB_1 = Q_1^T - C_1^T = a_T \left[1 - \frac{1}{2}\left(\frac{2+r^*}{1+r^*}\right)\right].$$

$$TB_2 = Q_2^T - C_2^T = a_T \left[1 - \frac{1}{2}(2+r^*)\right].$$

C-1F Is there any sectoral labor reallocation over time? If so, explain the intuition behind it.

Although the relative price of tradables and nontradables is constant across the two periods because there is no productivity gain in either sector, some reallocation of labor from the former to the latter does occur: notice that $L_1^T > L_2^T$ because $r^* > 0$.

This reallocation allows the production of nontradables to be scaled up in period 2, so that the hoursehold's desire for a growing consumption of those goods can be met. Because there is a fixed supply of labor, this comes at the cost of a decline in the production of tradables.

Notice that the desired consumption path for tradable goods is growing as well, however. Therefore, the reduction in tradable output must be reflected in a deterioration of the trade balance. Observe that indeed $TB_1 > TB_2$ as $r^* > 0$.