EC201 Macroeconomics 2

Week 4 - Seminar 2

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This Week

1. Review Questions - A
2. Class Questions - B -
3. Self Study Questions - C

Review Questions - A

Sustainability

Indicate whether the following statements are true, false, or uncertain and explain why.

1. An economy that starts with a positive net international investment position will run a trade balance deficit at some point.
2. The fact that over the past quarter century the United States has run larger and larger current account deficits is proof that American household savings have been shrinking.

Read again***

Appendix: Perpetual TB and CA Deficits in In/finite-Horizon Economies

The statement is true. Repeated use of the law of motion of foreign assets in the absence of valuation changes, i.e. equation.

$CA_1 = rB_0^* + TB_1$	From Balance of Payments	(1)
$CA_1 = B_1^* - B_0^*$	Change in NIIP	(2)
$B_1^* = (1+r)B_0^* + TB_1$	Combine 1 and 2	(3)
$B_2^* = (1+r)B_1^* + TB_2$	For the next period	(4)
$B_t^* = (1+r)B_{t-1}^* + TB_t$	General case	(5)

Combining the Eq-3 and 4 to eliminate B_1^* do iterate again we obtain the condition

$$B_0^* = \frac{B_T^*}{(1+r)^T} - \frac{TB_1}{(1+r)} - \frac{TB_2}{(1+r)^2} - \dots - \frac{TB_T}{(1+r)^T}$$

In order for the rest of the world not to perpetually roll over its debt to our economy, it must be the case that

$$\lim_{T\to\infty} \frac{B_T^*}{(1+r)^T} \leq 0.$$

In order for our economy not to run Ponzi schemes either, it must also be the case that

$$\lim_{T\to\infty} \frac{B_T^*}{(1+r)^T} \geq 0.$$

Therefore, the net international investment position must be zero in the limit (so-called transversality condition) and we are left with the net present value of future trade balances:

$$B_0^* = -\frac{TB_1}{(1+r)} - \frac{TB_2}{(1+r)^2} - \dots$$

Since the initial NIIP of our economy is positive, its trade balance will have to be in deficit at some point in the future.

Read again

The Current Account As The Gap Between Savings and Investment

The statement is false: in and of themselves, growing CA deficits are not a proof of a decline in household saving. All they show is a widening gap between national savings and investment. We can understand this if we look at the current account from the following angle:

$$\begin{aligned} CA_t &= rB^*_{t-1} + TB_t \\ &= rB^*_{t-1} + Q_t - C_t - G_t - I_t \\ &= Y^*_t - C_t - G_t - I_t \\ &= S_t - I_t, \end{aligned}$$

where the first equality is only valid in the absence of international employee compensation and unilateral transfers, the second line uses the trade balance identity $TB_t = Q_t - C_t - G_t - I_t$, the third line uses the definition of national income $Y^*_t = rB^*_{t-1} + Q_t$ and the last line uses that of national savings $S_t = Y^*_t - C_t - G_t$ Whether rising CA deficits are due to falling private or public saving (or even rising investment) remains to be seen.

2. **[CA and TB Determination]** Consider a two-period small open endowment economy populated by a large number of households with preferences described by the lifetime utility function:

$$U(C_1, C_2) = C_1^{\frac{1}{10}} C_2^{\frac{1}{11}}$$

where C_1 and C_2 denote, respectively, consumption in periods 1 and 2. Suppose that households receive exogenous endowments of goods given by $Q_1 = Q_2 = 10$ in periods 1 and 2, respectively. Every household enters period 1 with some debt, denoted B_0^*, inherited from the past. Let B_0^* be equal to -5. The interest rate on these liabilities, denoted r_0, is 20 percent. Finally, suppose that the country enjoys free capital mobility and that the world interest rate on assets held between periods 1 and 2, denoted r^*, is 10 percent.

(a) Compute the equilibrium levels of consumption, the trade balance, and the current account in periods 1 and 2.

(b) Assume now that the endowment in period 2 is expected to increase from 10 to 15. Calculate the effect of this anticipated output increase on consumption, the trade balance, and the current account in both periods. Compare your answer to that you gave for item (a) and provide intuition.

(c) Finally, suppose now that foreign lenders decide to forgive all of the countryï¿½s initial external debt. How does this decision affect the country's levels of consumption, trade balance, and current account in periods 1 and 2. (For this question, assume that $Q_1 = Q_2 = 10$.) Compare your answer to the one you gave for item (a) and explain.

(a) We are told that $Q_1 = Q_2 = 10$, $B_0^* = -5$, $r_0 = 0.20$ and $r_1 = r^* = 0.10$ because of free capital mobility. With the given utility function, the marginal utilities of consumption in period 1 and period 2 are respectively

$$U_1(C_1, C_2) = \frac{1}{10}(C_1)^{-\frac{9}{10}}(C_2)^{\frac{1}{11}},$$

$$U_2(C_1, C_2) = \frac{1}{11}(C_1)^{\frac{1}{10}}(C_2)^{-\frac{10}{11}}.$$

We compute the optimal consumption plan by equating the marginal rate of intertemporal substitution of consumption to the slope of the intertemporal budget constraint:

$$-\frac{U_1(C_1, C_2)}{U_2(C_1, C_2)} = -(1 + r_1) \tag{6}$$

$$-\frac{11}{10}(C_1)^{-\frac{10}{10}}(C_2)^{\frac{11}{11}} = -(1 + 0.10)$$

$$\frac{11}{10}\frac{1}{1.10} = \frac{C_1}{C_2}$$

$$C_1 = C_2.$$

We plug this condition into the intertemporal budget constraint (IBC)

$$C_1 + \frac{C_2}{1+r_1} = (1+r_0)\,B_0^* + Q_1 + \frac{Q_2}{1+r_1} \tag{7}$$

to find the optimal consumption levels for periods 1 and 2:

$$C_1 + \frac{C_1}{1.10} = 1.20\,(-5) + 10 + \frac{10}{1.10}$$
$$C_1 = 6.86 = C_2.$$

We compare these with the endowments to compute the trade balances in each period:

$$TB_1 = Q_1 - C_1 = 10 - 6.86 = 3.14,$$
$$TB_2 = Q_2 - C_2 = 10 - 6.86 = 3.14.$$

Therefore, the net foreign asset position at the end of period 1 is

$$\begin{aligned} B_1^* &= (1+r_0)\,B_0^* + TB_1 \\ &= 1.2\,(-5) + 3.14 \\ &= -2.86. \end{aligned}$$

Now we are in a position to calculate the current account balances:

$$CA_1 = r_0 B_0^* + TB_1 = 0.20\,(-5) + 3.14 = 2.14,$$

$$CA_2 = r_1 B_1^* + TB_2 = 0.10\,(-2.86) + 3.14 = 2.85.$$

(b)

Now we have $Q_1 = 10$ and $Q_2 = 15$. We combine conditions (6) and (7) again:

$$C_1 + \frac{C_1}{1.10} = 1.20\,(-5) + 10 + \frac{15}{1.10}$$
$$C_1 = 9.24 = C_2.$$

We calculate the trade balance, net foreign assets and the current account balance just like before:

$$TB_1 = Q_1 - C_1 = 10 - 9.24 = 0.76,$$
$$TB_2 = Q_2 - C_2 = 15 - 9.24 = 5.76.$$

Therefore, the net foreign asset position at the end of period 1 is

$$\begin{aligned} B_1^* &= (1 + r_0)\,B_0^* + TB_1 \\ &= 1.2\,(-5) + 0.76 \\ &= -5.24. \end{aligned}$$

Now we are in a position to calculate the current account balances:

$$CA_1 = r_0 B_0^* + TB_1 = 0.20\,(-5) + 0.76 = -0.24,$$

$$CA_2 = r_1 B_1^* + TB_2 = 0.10\,(-5.24) + 5.76 = 5.24.$$

The repayment of the initial debt is postponed under this endowment stream: the country runs a smaller trade surplus in period 1 and a larger trade surplus in period 2. Accordingly, it runs a current account deficit in period 1 and a much larger surplus in period 2.

(c)

We plug B_0^* into (7) and solve:

$$C_1 + \frac{C_1}{1.10} = 0 + 10 + \frac{10}{1.10}.$$

We find

$$C_1 = C_2 = 10,$$

$$TB_1 = TB_2 = 0,$$

$$CA_1 = CA_2 = 0.$$

Since the desired consumption plan coincides with the endowment now, there is no need for intertemporal trade with the rest of the world: the country runs trade and current account balances in both periods.

Class Questions - B -

Consider a two-period model of a small open economy with a single good each period and no investment. Let preferences of the representative household be described by the utility function:

$$U(C_1, C_2) = \sqrt{C_1} + \beta\sqrt{C_2}.$$

The parameter β is known as the subjective discount factor. It measures the consumer's degree of impatience in the sense that the smaller is β, the higher is the weight the consumer assigns to present consumption relative to future consumption. Assume that β = 1/1.1. The representative household has initial net foreign wealth of $(1 + r_0)B_0^* = 1$, with $r_0 = 0.1$, and is endowed with $Q_1 = 5$ units of goods in period 1 and $Q_2 = 10$ units in period 2. The world interest rate paid on assets held from period 1 to period 2, r^*, equals 10% (i.e., $r^* = 0.1$) and there is free international capital mobility.

B-a

Calculate the equilibrium levels of consumption in period 1, C_1, consumption in period 2, C_2, the trade balance in period 1, TB_1, and the current account balance in period 1, CA_1.

B-a

Calculate the equilibrium levels of consumption in period 1, C_1, consumption in period 2, C_2, the trade balance in period 1, TB_1, and the current account balance in period 1, CA_1.

We are told that $(1+r_0)\,B_0^* = 1$ and $r_0 = 0.10$, which imply an initial net foreign asset position equal to

$$B_0^* = \frac{1}{1.10}.$$

We also know that $\beta = \frac{1}{1.1}$, $Q_1 = 5$, $Q_2 = 10$ and $r_1 = r^* = 0.1$ because of perfect capital mobility. The marginal utilities of consumption are

$$U_1\left(C_1, C_2\right) = \frac{1}{2}\left(C_1\right)^{-\frac{1}{2}}, \quad U_2\left(C_1, C_2\right) = \beta\frac{1}{2}\left(C_2\right)^{-\frac{1}{2}}.$$

Using the optimality condition (6), we get

$$-\frac{1}{\beta}\left(\frac{C_2}{C_1}\right)^{\frac{1}{2}} = -\left(1+0.10\right)$$

$$C_1 = C_2.$$

Plugging it into the intertemporal budget constraint (7) we get

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$$C_1 + \frac{C_1}{1.10} = 1 + 5 + \frac{10}{1.10}$$

$$C_1 = 7.9 = C_2.$$

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$$C_1 + \frac{C_1}{1.10} = 1 + 5 + \frac{10}{1.10}$$

$$C_1 = 7.9 = C_2.$$

We then compute the trade and CA balances as usual:

$$TB_1 = Q_1 - C_1$$

$$TB_2 = Q_2 - C_2$$

$$B_1^* = (1 + r_0)\, B_0^* + TB_1$$

$$CA_1 = r_0 B_0^* + TB_1$$

$$CA_2 = r_1 B_1^* + TB_2$$

Equilibrium Levels of Consumption

Plugging it into the intertemporal budget constraint (7) we get

$$C_1 + \frac{C_1}{1.10} = 1 + 5 + \frac{10}{1.10}$$

$$C_1 = 7.9 = C_2.$$

We then compute the trade and CA balances as usual:

$$TB_1 = Q_1 - C_1 = 5 - 7.9 = -2.9$$

$$TB_2 = Q_2 - C_2 = 10 - 7.9 = 2.1$$

$$B_1^* = (1 + r_0)\, B_0^* + TB_1 = 1 - 2.9 = -1.9$$

$$CA_1 = r_0 B_0^* + TB_1 = 0.1 \times \frac{1}{1.1} - 2.9 = -2.81$$

$$CA_2 = r_1 B_1^* + TB_2 = 0.1\,(-1.9) + 2.1 = 1.91$$

Parameters	Free Capital Mobility	Capital Control
C_1	7.9	
C_2	7.9	
TB_1	-2.9	
TB_2	2.1	
CA_1	-2.81	
CA_2	1.91	
B_1^*	-1.9	
$U(C_1, C_2$	5.37	

B-b

Suppose now that the government imposes capital controls that require that the country's net foreign asset position at the end of period 1 be non-negative $B_1^* \geq 0$. Compute the equilibrium value of the domestic interest rate, r_1, consumption in periods 1 and 2, and the trade and current account balances in period 1.

B-b

Suppose now that the government imposes capital controls that require that the country's net foreign asset position at the end of period 1 be non-negative $B_1^* \geq 0$. Compute the equilibrium value of the domestic interest rate, r_1, consumption in periods 1 and 2, and the trade and current account balances in period 1.

The constraint that $B_1^* \geq 0$ is binding because the desired B_1^* calculated above is negative. With this restriction, the most that can be consumed in period 1 is the sum of the current endowment, the initial savings and the associated interest income:

$$C_1 = Q_1 + (1 + r_0)\, B_0^* = 5 + 1 = 6.$$

Because no further savings are left for the future, consumption in period 2 is

$$C_2 = Q_2 = 10.$$

This implies that

$$TB_1 = -\left(1 + r_0\right) B_0^* = -1,$$

$$TB_2 = 0.$$

$$CA_1 = r_0 B_0^* + TB_1 = -B_0^* = -\frac{1}{1.1} = -0.91.$$

The equilibrium value of the domestic interest rate can be computed from the following equilibrium condition:

$$U_1\left(C_1, C_2\right) = \left(1 + r_1\right) U_2\left(C_1, C_2\right).$$

We get

$$\frac{1}{2}\left(6\right)^{-\frac{1}{2}} = \left(1 + r_1\right)\beta\frac{1}{2}\left(10\right)^{-\frac{1}{2}}$$

$$\left(\frac{10}{6}\right)^{\frac{1}{2}} \times 1.1 = 1 + r_1$$

$$r_1 = 0.42.$$

Capital Controls

$$B_1^* \geq 0$$

Parameters	Free Capital Mobility	Capital Control
C_1	7.9	6
C_2	7.9	10
TB_1	-2.9	-1
TB_2	2.1	0
CA_1	-2.81	-0.91
CA_2	1.91	0
B_1^*	-1.9	0
$U(C_1, C_2$	5.37	5.32

B-C

Evaluate the effect of capital controls on welfare. Specifically, find the level of utility under capital controls and compare it to the level of utility obtained under free capital mobility.

B-C

Evaluate the effect of capital controls on welfare. Specifically, find the level of utility under capital controls and compare it to the level of utility obtained under free capital mobility.

With capital controls, the household achieves the following level of utility:

$$U(C_1, C_2) = \sqrt{6} + \frac{1}{1.1}\sqrt{10} = 5.32.$$

With free capital mobility, the household achieves instead

$$U(C_1, C_2) = \sqrt{7.9} + \frac{1}{1.1}\sqrt{7.9} = 5.37.$$

In an economic environment like the one under consideration (i.e without market failures of any sort) capital controls are welfare-reducing.

To see this, compute the permanent consumption loss that equates welfare across the two policies, i.e. solve for the μ that satisfies $\sqrt{(1-\mu)\,7.9} + \frac{1}{1.1}\sqrt{(1-\mu)\,7.9} = \sqrt{6} + \frac{1}{1.1}\sqrt{10}$

B-d

For this question and the next, suppose that the country experiences a temporary increase in the endowment of period 1 to $Q_1 = 9$, with period 2 endowment unchanged. Calculate the effect of this output shock on C_1, C_2, TB_1, CA_1, and r_1 in the case that capital is freely mobile across countries.

B-d

For this question and the next, suppose that the country experiences a temporary increase in the endowment of period 1 to $Q_1 = 9$, with period 2 endowment unchanged. Calculate the effect of this output shock on C_1, C_2, TB_1, CA_1, and r_1 in the case that capital is freely mobile across countries.

Now we are told that $Q_1 = 9$ and $Q_2 = 10$. Using the optimality condition (6) and the intertemporal budget constraint (7) once more we get

$$C_1 + \frac{C_1}{1.10} = 1 + 9 + \frac{10}{1.10},$$

which implies

$$C_1 = 10 = C_2,$$

$$TB_1 = 9 - 10 = -1,$$

$$CA_1 = r_0 B_0^* + TB_1 = 0.1 \times \frac{1}{1.1} - 1 = -0.91.$$

B-e

Finally, suppose that the capital controls described in part (b) are in place. Will they still be binding (i.e., affect household behaviour)?

B-e

Finally, suppose that the capital controls described in part (b) are in place. Will they still be binding (i.e., affect household behaviour)?

Since

$$B_1^* = (1 + r_0)\, B_0^* + TB_1 = 1 - 1 = 0$$

with these endowments, capital controls are not binding anymore and household behavior is unaffected.

Self Study Questions - C

Consider a two-period economy that has at the beginning of period 1 a net foreign asset position of -100. In period 1, the country runs a current account deficit of 5 percent of GDP, and GDP in both periods is 120. Assume the interest rate in periods 1 and 2 is 10 percent.

(a) Find the trade balance in period 1 (TB_1), the current account balance in period 1 (CA_1), and the country's net foreign asset position at the beginning of period 2 (B_1^*).

(b) Is the country living beyond its means? To answer this question find the country's current account balance in period 2 and the associated trade balance in period 2. Is this value for the trade balance feasible? [Hint: Keep in mind that the trade balance cannot exceed GDP.]

(c) Now assume that in period 1, the country runs instead a much larger current account deficit of 10 percent of GDP. Find the country's net foreign asset position at the end of period 1, B_1^*. Is the country living beyond its means? If so, show why.

(A) We are told that $B_0^* = -100$, $CA_1 = -0.05Q_1$, $Q_1 = Q_2 = 120$ and $r = 0.10$. Therefore, the current account in period 1 is

$$CA_1 = -0.05 \times 120 = -6.$$

We also know that in a two-period economy with no international employee compensation and zero net unilateral transfers

$$CA_1 = rB_0^* + TB_1, \tag{8}$$

so we can find the trade balance in period 1:

$$\begin{aligned} -6 &= -100 \times 0.10 + TB_1 \\ TB_1 &= 4. \end{aligned}$$

Now we are in a position to compute net foreign assets:

$$\begin{aligned} B_1^* &= (1+r)\,B_0^* + TB_1 \\ &= -100 \times 1.10 + 4 \\ &= -106. \end{aligned}$$

(B) A transversality condition rules out non-zero external assets at the end of period 2, so

$$B_1^* = -\frac{TB_2}{1+r}. \tag{9}$$

Knowing that $B_1^* = -106$ and $r = 0.10$, this implies that the trade balance in period 2 is

$$TB_2 = 116.60.$$

We apply equation (8) again to find the current account in period 2:

$$\begin{aligned} CA_2 &= rB_1^* + TB_2 \\ &= -106 \times 0.10 + 116.60 \\ &= 106. \end{aligned}$$

As $Q_2 = 120$, this state of affairs is sustainable: the trade balance does not exceed gross domestic product.

(C) We start from the current account balance in period 1 again:

$$CA_1 = -0.10 \times 120 = -12.$$

We use equation (8) once more to find the trade balance in period 1:

$$\begin{aligned} -12 &= -100 \times 0.10 + TB_1 \\ TB_1 &= -2. \end{aligned}$$

This trade deficit is not sustainable, because it must be offset by a trade surplus that is not feasible in period 2. To see this, compute end-of-period 1 net foreign assets again

$$\begin{aligned} B_1^* &= (1+r)\, B_0^* + TB_1 \\ &= -100 \times 1.10 - 2 \\ &= -112 \end{aligned}$$

and apply the intertemporal condition (9) one more time to find the trade balance needed in period 2:

$$TB_2 = 123.20.$$

This is not possible because it exceeds the country's GDP.

Question 2: Saving a lot in a Growing Economy

Consider a two-period model of a small open endowment economy populated by households with preferences given by:

$$U(C_1, C_2) = \sqrt{C_1} + \sqrt{C_2}.$$

where C_1 and C_2 denote consumption in periods 1 and 2, respectively. Households' endowments are 5 units of goods in period 1 and 10 units of goods in period 2. Households start period 1 with a zero net asset position and the world interest rate is zero, $r^* = 0$.

1. Find consumption in periods 1 and 2, the country's net foreign asset position at the end of period 1, and the trade balance in periods 1 and 2. Provide intuition for your findings.
2. Find the level of welfare in the economy.
3. Now assume that the government announces in period 1 that a strong nation is one with a positive net foreign asset position and that therefore the country must save more. In particular, the government enforces that the net foreign asset position at the end of period 1 is positive and equal to 2. That is, the government imposes capital controls of the form $B_1^* \geq 2$. Find the domestic interest rate that supports this allocation.
4. Find the level of welfare under capital controls and compare it to the level of welfare under free capital mobility.

(A)

With this preference specification, the marginal utilities of consumption are as follows:

$$U_1\left(C_1, C_2\right) = \frac{1}{2}\left(C_1\right)^{-\frac{1}{2}}, \quad U_2\left(C_1, C_2\right) = \frac{1}{2}\left(C_2\right)^{-\frac{1}{2}}.$$

In this case the intertemporal optimality condition (6) reads

$$\left(\frac{C_2}{C_1}\right)^{\frac{1}{2}} = 1 + r_1.$$

With perfect capital mobility and $r^* = 0$, this condition implies constant consumption:

$$C_1 = C_2.$$

Since $B_0^* = 0$, $Q_1 = 5$ and $Q_2 = 10$, the intertemporal budget constraint (7) reads

$$C_1 + \frac{C_2}{1+0} = 5 + \frac{10}{1+0}.$$

Therefore

$$C_1 = C_2 = 7.5,$$

$$TB_1 = 5 - 7.5 = -2.5,$$

$$TB_2 = 10 - 7.5 = 2.5,$$

$$B_1^* = (1 + r_0)\,0 + TB_1 = -2.5.$$

Since consumption is valued equally in periods 1 and 2, the household consumes a half of its wealth in each period. As initial foreign assets are zero, the household smooths consumption by running a trade deficit in the period with a smaller endowment and a surplus in the period with a larger endowment. This intertemporal allocation of consumption is achieved by borrowing from the rest of the world in period 1.

(B)

$$U\left(C_1, C_2\right) = \sqrt{7.5} + \sqrt{7.5} = 5.48$$

(C)

The constraint that $B_1^* \geq 2$ is binding because the country would have a negative NFA position in period 1 without capital controls, as we have seen before. Hence, the constrained allocation cannot optimally feature a NFA position larger than 2: the constraint on capital mobility must hold with strict equality in equilibrium. This observation allows us to calculate the trade balance in period 1:

$$B_1^* = (1 + r_0)\,0 + TB_1$$

$$2 = TB_1.$$

To find the domestic interest rate that supports this constrained allocation, we must compute the consumption levels first. We gauge them from the trade balance:

$$TB_1 = Q_1 - C_1$$

$$2 = 5 - C_1$$

$$C_1 = 3.$$

Using the intertemporal budget constraint, we can also find

$$C_2 = 12.$$

Now we are in a position to identify the domestic interest rate: it is 100 percent.

$$U_1\left(C_1, C_2\right) = \left(1 + r_1\right) U_2\left(C_1, C_2\right)$$
$$\frac{1}{2}\left(3\right)^{-\frac{1}{2}} = \left(1 + r_1\right)\frac{1}{2}\left(12\right)^{-\frac{1}{2}}$$
$$r_1 = 1.$$

(D)

Capital controls impose a sub-optimal consumption sequence in this environment. Social welfare is below the level achieved under free capital mobility:

$$U\left(C_1, C_2\right) = \sqrt{3} + \sqrt{12} = 5.20.$$