EC9012 Macroeconomics

Week 4 - Problem Set 2

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Problem Set 2

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Question - 1 -

Question - 1 -

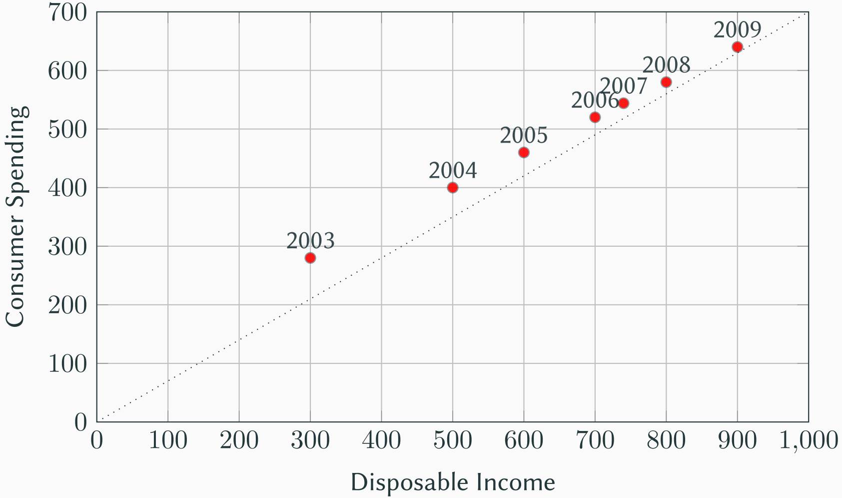
The accompanying table shows the fluctuations in disposable income and consumer spending in Macroeconomia during 2003-2009.

Year	Disposable Income	Consumer Spending
2003	300	280
2004	500	400
2005	600	460
2006	700	520
2007	740	544
2008	800	580
2009	900	640

a) Plot the Keynesian aggregate consumption function.

b) Calculate the marginal propensity to consume and marginal propensity to save.

c) Calculate the aggregate consumption function. Show the autonomous consumption.



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Consumer Spending
Disposable Income
2003
2004
2005
2006
2007
2008
2009

$$c = \alpha + MPC \times yd \tag{1}$$

$$MPC = \frac{\Delta Consumer\ Spending}{\Delta Disposable\ Income}$$

We can calculate the slope of the line, using any two points.

$$= \frac{580 - 544}{800 - 740} = \frac{36}{60}$$ (From 2007-2008)

$$= 0.6$$ (Marginal Propensity of Consumption)

$$\begin{aligned} Marginal\ Propensity\ of\ Saving &= 1 - MPC \\ &= 1 - 0.6 = 0.4 \end{aligned}$$

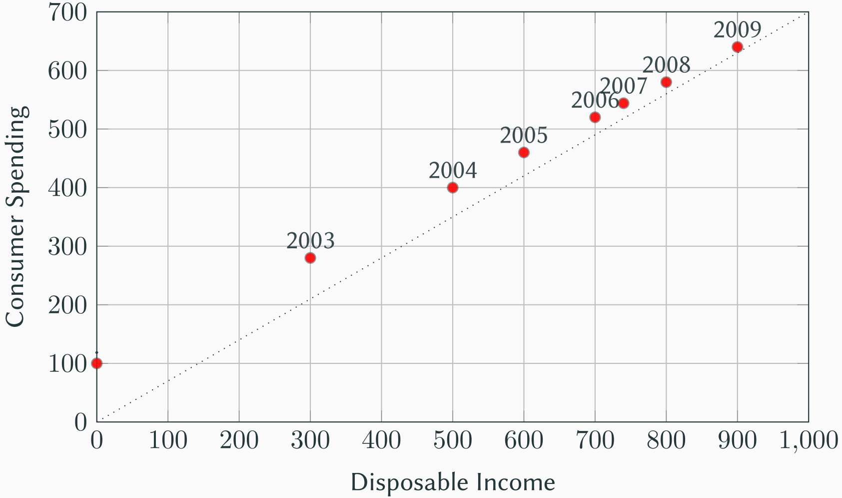
The equation for the aggregate consumption function can be obtained by using the slope (m) and any point on the line

$$
\begin{aligned}
y - y_0 &= m(x - x_0) \\
y - 640 &= 0.6(x - 900) \\
y &= 0.6x + 100 \\
c &= 100 + 0.6yd
\end{aligned}
$$

Autonomous consumption (α) is 100 when $yd = 0$

Remember

Autonomous consumption is defined as the expenditures that consumers must make even when they have no disposable income.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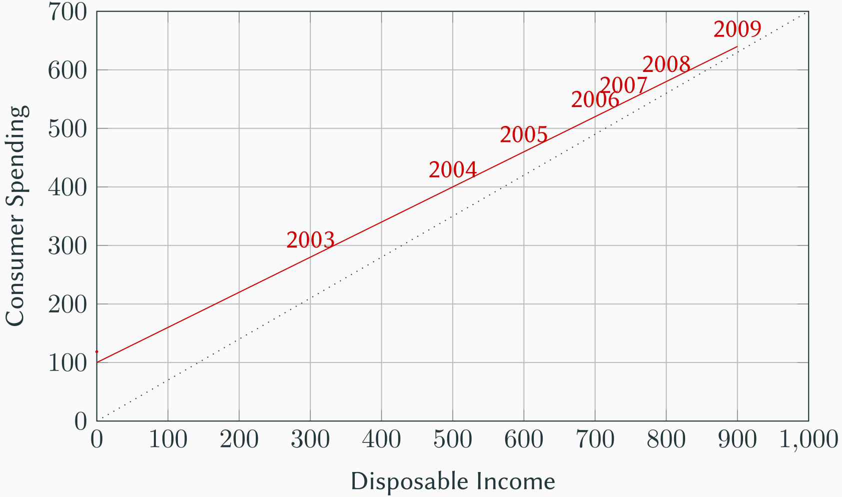
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Question - 2 -

Q2

Ms. Smith obeys the two-period Fisher model of consumption. She has the following utility function: $U(C_1, C_2) = C_1 \times C_2$ where indicate the first and second period consumption. Ms. Smith earns \$200 in the first period and \$330 in the second period. She can borrow or lend at interest rate r.

a) Write down Ms. Smith's inter-temporal budget constraint?

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a) Write down Ms. Smith's inter-temporal budget constraint?

$$C_1 + \frac{C_2}{(1+r)} = Y_1 + \frac{Y_2}{(1+r)}$$
$$C_1 + \frac{C_2}{(1+r)} = 200 + \frac{330}{(1+r)}$$

2B

Calculate the level of period 1 and period 2 consumption that maximizes Ms. Smith's utility subject to the intertemporal budget constraint when the interest rate is 10 percent.

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Calculate the level of period 1 and period 2 consumption that maximizes Ms. Smith's utility subject to the intertemporal budget constraint when the interest rate is 10 percent.

$$Max\, U(C_1, C_2) = C_1 \times C_2$$

$$\text{Subject to} \quad C_1 + \frac{C_2}{(1+0.1)} = 200 + \frac{330}{(1+0.1)}$$

$C_1 + \frac{C_2}{(1.1)} = 500$, then $C_1 = 500 - \frac{C_2}{(1.1)}$

$$Max\, U(C_1, C_2) = \left(500 - \frac{C_2}{(1.1)}\right) \times C_2 = \left(500 \times C_2 - \frac{C_2^2}{(1.1)}\right)$$

Question - 2B -

We can take the first derivative of the utility function with respect to C_2 to find the utility maximizing level of period 2 consumption.

$$\frac{\partial U}{\partial C_2} = 500 - \frac{2C_2}{(1.1)} = 0$$

$550 = 2C_2$ then $C_2 = 275$

Using the budget constraint $C_1 = 500 - \frac{C_2}{(1.1)}$ then $C_1 = 500 - \frac{275}{1.1} = 250$

$$C_1 = 250$$

$$C_2 = 275$$

2C

Draw the results you obtain graphically. Draw the budget constraint and the consumption choices in both periods. Discuss in which period Ms. Smith is a borrower or a saver.

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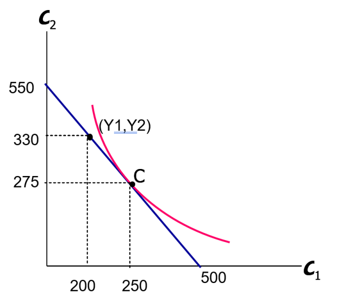
Ms. Smith is a borrower in the first period. She has $200 income but consumes $250 . She is a saver in second period. She has $330 income but consumes only $275 pay back the loan.



Question - 2D -

2D

Calculate the level of period 1 and period 2 consumption, which maximizes Ms Smith's utility subject to the inter-temporal budget constraint when the interest rate is 25 percent. Draw the budget constraint and consumption points. Indicate whether substitution or income effect dominates relative to the consumption choices when interest rate is 10 percent.

2D

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$Max\,U(C_1, C_2) = C_1 \times C_2$ Subject to $C_1 + \frac{C_2}{(1+0.25)} = 200 + \frac{330}{(1+0.25)}$

$C_1 + \frac{C_2}{(1.25)} = 464$, then $C_1 = 464 - \frac{C_2}{(1.25)}$

$$Max\,U(C_1, C_2) = \left(464 - \frac{C_2}{(1.25)}\right) \times C_2 = \left(464 \times C_2 - \frac{C_2^2}{(1.25)}\right)$$

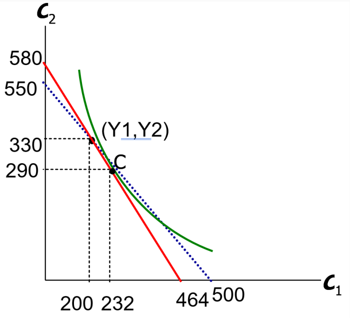
We can take the first derivative of the utility function with respect to C_2 to find the utility maximizing level of period 2 consumption.

$$\frac{\partial U}{\partial C_2} = 464 - \frac{2C_2}{(1.25)} = 0$$

$580 = 2C_2$ then $C_2 = 290$

Using the budget constraint then $C_1 = 464 - \frac{290}{...} = 250$

The budget constaint is the red line that pivoted around the income points. Period 1 consumption declined (from \$250 to \$232) and Period 2 consumption increased (from \$275 to \$290). Saving goes up from -50 to -32. Substitution and income effect go in the same direction. As the interest rate increases, the consumer wants to substitute period 1 consumption with period 2 consumption (i.e. increase C_2 and decrease C_1, substitution effect). At the same time, since the consumer is a borrower, his intertemporal wealth decreases as interest rate goes up (income effect).



2E

Suppose the interest rate stays at 25 percent. This time Ms. Smith found a part-time job that increased her period 1 income to $300. Calculate the consumption in each period and show graphically what happens to budget line and consumption choices.

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$Max\, U(C_1, C_2) = C_1 \times C_2$ Subject to $C_1 + \frac{C_2}{(1+0.25)} = 300 + \frac{330}{(1+0.25)}$

$C_1 + \frac{C_2}{(1.25)} = 564$, then $C_1 = 464 - \frac{C_2}{(1.25)}$

$$Max\, U(C_1, C_2) = \left(564 - \frac{C_2}{(1.25)}\right) \times C_2 = \left(564 \times C_2 - \frac{C_2^2}{(1.25)}\right)$$

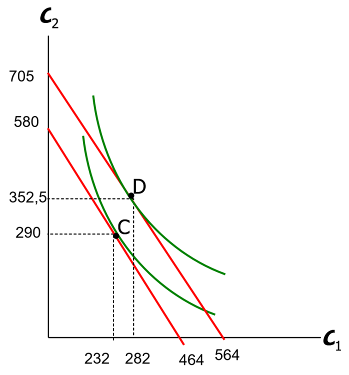
We can take the first derivative of the utility function with respect to C_2 to find the utility maximizing level of period 2 consumption.

$$\frac{\partial U}{\partial C_2} = 564 - \frac{2C_2}{(1.25)} = 0$$

$705 = 2C_2$ then $C_2 = 352.5$

Using the budget constraint then $C_1 = 564 - \frac{352.5}{1.25} = 282$

In this case, Ms. Smith saves in period 1 and then consumes more than her income in period 2.



2F

Using the results from part e, suppose Ms. Smith all of a sudden faces a borrowing constraint of the form $C_2 \leqslant Y_2$, indicating her second period consumption can not exceed the second period income. Show graphically her new budget constraint. Describe if the budget constraint is binding or not. If it is binding, what could be her best consumption choice?

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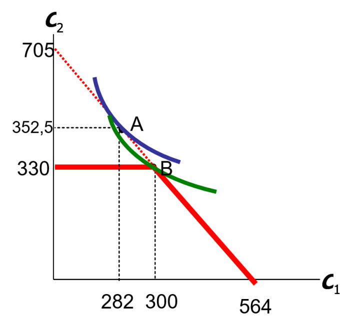
Using the results from part e, suppose Ms. Smith all of a sudden faces a borrowing constraint of the form $C_2 \leqslant Y_2$, indicating her second period consumption can not exceed the second period income. Show graphically her new budget constraint. Describe if the budget constraint is binding or not. If it is binding, what could be her best consumption choice?

A represents the consumption choice Ms. Smith has when there is not any borrowing constraint as in part e. As can be seen, the constraint is binding because Ms. Smith consumes more then her income at point A. The best she can do is to consume exactly her income in period 1 ($C_1 = Y_1 = 300$) and period 2 ($C_2 = Y_2 = 330$) at point B. This however represents a lower utility level than the unconstraint optimization.



Question - 3 -

Equations of Neo-Classical Model of Investment

– $R/P = \alpha A(L/K)^{1-\alpha}$

– $Real\ Cost\ of\ Capital = (P_k/P)(r+\delta)$

– $I = I_n\Big(\alpha A(L/K)^{1-\alpha} - (P_k/P)(r+\delta)\Big) + \delta K$

(a) Assuming prices P remain constant, what would be the impact on the rental price (R) of capital and the level of investment (I) of

i) an influx of immigrant labor
ii) an earthquake that destroys a significant part of the capital stock
iii) introduction of new computer technology that improves the productivity

(b) Going back to the equilibrium position and again assuming that prices P are held constant, what would be the impact on the level of investment (I) of either

i) sizeable inflow of foreign funds into the country lowers the real interest rates
ii) an fall in the depreciation rate
iii) increase in investment tax credit
iv) fall in cost of borrowing

Question - 3 -

$$R/P = \alpha\, A(L/K)^{1-\alpha}$$

Question - 3 -

$$R/P = \alpha\, A\, (\boxed{L}/K)^{1-\alpha}$$

i) an influx of immigrant labor

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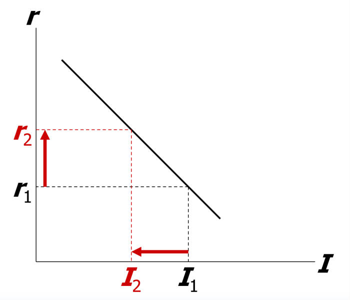
From the marginal product of capital equation, we can see that all of the options increase the marginal product of existing capital stock and therefore the rental price of capital goes up. If the marginal product of capital increases, then the investment goes up as well.

$$I = I_n\Big(\overbrace{\alpha A(L/K)^{1-\alpha}}^{MPK} - \overbrace{(P_k/P)(r+\delta)}^{Real\ Cost\ of\ Capital}\Big) + \delta K$$

An increase in r

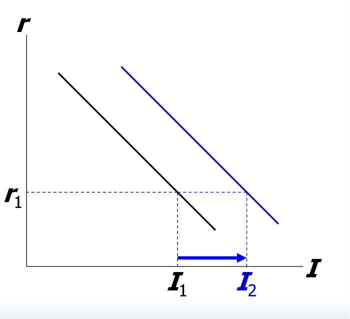
- raises the cost of capital
- reduces the profit rate
- and reduces investment



$$I = I_n\Big(\overbrace{\alpha A(L/K)^{1-\alpha}}^{MPK} - \overbrace{(P_k/P)(r+\delta)}^{Real\ Cost\ of\ Capital}\Big) + \delta K$$

An increase in MPK or decrease in P_k/P

- increases the profit rate
- increases investment at any given interest rate
- shifts I curve to the right.



(b) Going back to the equilibrium position and again assuming that prices P are held constant, what would be the impact on the level of investment (I) of either

$$Real\ Cost\ of\ Capital = (P_k/P)(r+\delta)$$

$$I = I_n\Big(\alpha A(L/K)^{1-\alpha} - (P_k/P)(r+\delta)\Big) + \delta K$$

- **i** sizeable inflow of foreign funds into the country lowers the real interest rates
- **ii** an fall in the depreciation rate
- **iii** increase in investment tax credit
- **iv** fall in cost of borrowing

All of these options lower the real cost of capital and so increase investment

Question - 4 -

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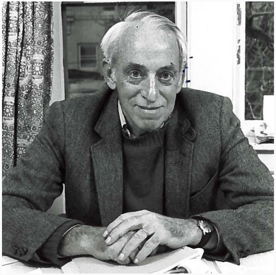
a) Explain how the stock prices can affect the investment in an economy using the simple Tobin's q model.

a) Explain how the stock prices can affect the investment in an economy using the simple Tobin's q model.

James Tobin developed the theory of investment based on the relation between the present value of profits and investment. Tobin reasoned that net investment should depend on whether q is greater or less than 1.



- If q is greater than one, then the stock market values the installed capital at more than it costs to replace. This creates an incentive to invest, because the managers can raise the market value of their firm's stock by buying more capital.

- Conversely, if q is less than one, then the stock market values installed capital at less than its replacement cost. In this case, managers will not replace capital as it wears out. Investors are pessimistic about future asset returns.

Question - 4A -

The theory provides an alternative way to express the neoclassical model of investment. If the marginal product of capital exceeds the cost of capital, for example, then installed capital earns profits. These profits make the firms desirable to own, which raises the market value of these firms' stock, implying a high value of q.

- If $MPK > Cost\ of\ Capital$, then profit rate is high. This drives up the stock market value of the firms, which implies a high value of q. If $q > 1$, firms buy more capital and invest.
- If $MPK < Cost\ of\ Capital$, then firms are incurring losses, so their stock market values fall, so q is low. If $q < 1$, firms do not replace capital as it wears out.

Hence Tobin's q captures the incentive to invest because it reflects the current and expected future profitability of capital.

Question - 4 -

b) Show how persistent increase in the stock prices affects the aggregate demand. How does the economy react in the short-run? What are the long-run effects?

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A wave of optimism about future profitability of capital or technological progress would cause:

- Stock prices to rise a Tobin's Q to rise a shift the investment up a positive aggregate demand shock.
- A positive demand shock will lead to an inflationary gap in the economy.

Question - 5 -

Lucas calculation: first we compute the welfare gain from stabilizing business cycles, then the welfare gain from growing faster. We compare three economies:

A: fluctuations ($f = 0.02$) and growth ($g = 0.015$)

B: no fluctuations ($f = 0$) and growth ($g = 0.015$)

C: fluctuations ($f = 0.02$) and faster growth ($g = 0.025$)

Eliminating business cycles

To begin with, notice that welfare in economy A is defined as follows:

$$W^A = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u\left(c_t^A\right)\right].$$

In the presence of fluctuations and growth, the expected consumption sequence is as follows:

$$W^A = \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} u\left(c_0 (1+g)^t (1+f)\right) + \frac{1}{2} u\left(c_0 (1+g)^t (1-f)\right)\right].$$

Using the utility specification $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$, we get

$$W^A = \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \frac{\left(c_0 (1+g)^t (1+f)\right)^{1-\sigma}}{1-\sigma} + \frac{1}{2} \frac{\left(c_0 (1+g)^t (1-f)\right)^{1-\sigma}}{1-\sigma} \right]$$

$$= \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \frac{c_0^{1-\sigma}}{1-\sigma} (1+f)^{1-\sigma} + \frac{1}{2} \frac{c_0^{1-\sigma}}{1-\sigma} (1-f)^{1-\sigma} \right] \left((1+g)^t \right)^{1-\sigma}.$$

Take the terms independent of t out of the summation and rearrange:

$$W^A = \frac{1}{2} \frac{c_0^{1-\sigma}}{1-\sigma} \left[(1+f)^{1-\sigma} + (1-f)^{1-\sigma} \right] \sum_{t=0}^{\infty} \beta^t \left((1+g)^t \right)^{1-\sigma}$$

$$= \frac{1}{2} \frac{c_0^{1-\sigma}}{1-\sigma} \left[(1+f)^{1-\sigma} + (1-f)^{1-\sigma} \right] \sum_{t=0}^{\infty} \beta^t \left((1+g)^{1-\sigma} \right)^t$$

$$= \frac{1}{2} \frac{c_0^{1-\sigma}}{1-\sigma} \left[(1+f)^{1-\sigma} + (1-f)^{1-\sigma} \right] \sum_{t=0}^{\infty} \left(\beta (1+g)^{1-\sigma} \right)^t.$$

Finally, use the geometric series formula to get rid of the summation:

$$W^A = \frac{1}{2}\frac{c_0^{1-\sigma}}{1-\sigma}\left[(1+f)^{1-\sigma} + (1-f)^{1-\sigma}\right]\frac{1}{1-\beta(1+g)^{1-\sigma}}. \tag{1}$$

In economy B, welfare is defined as follows:

$$W^B = \mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t u\left(c_t^B\right)\right].$$

Since there are no fluctuations in consumption, formula (1) with $f = 0$ becomes

$$W^B = \frac{c_0^{1-\sigma}}{1-\sigma}\frac{1}{1-\beta(1+g)^{1-\sigma}}. \tag{2}$$

It would be incorrect to take the difference between the welfare levels in (1) and (2) as a measure of the welfare gain from eliminating business cycles. Utility functions can only *rank* different consumption plans; they do not reliably *measure* the amount of satisfaction agents get, so they cannot measure differences either. To quantify the welfare differential between these two economies, we must find the change in initial consumption that would make agents in economy B as happy as they are in economy A. In other words, we multiply c_0 by a factor λ_B in (2) and then we equate this formula to (1):

$$\frac{1}{2}\frac{c_0^{1-\sigma}}{1-\sigma}\left[(1+f)^{1-\sigma}+(1-f)^{1-\sigma}\right]\frac{1}{1-\beta(1+g)^{1-\sigma}}=\frac{(\lambda_B c_0)^{1-\sigma}}{1-\sigma}\frac{1}{1-\beta(1+g)^{1-\sigma}}.$$

We solve for λ_B:

$$\lambda_B=\left(\frac{1}{2}\left[(1+f)^{1-\sigma}+(1-f)^{1-\sigma}\right]\right)^{\frac{1}{1-\sigma}}.$$

Plugging in the numbers $\sigma=3$ and $f=0.02$, we find

$$\lambda_B=0.9994.$$

In economy C, welfare is defined as follows:

$$W^C = \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t u\left(c_t^C\right)\right].$$

Fluctuations in consumption exist in this economy, but the rate of growth is higher. Welfare is analogous to equation (1), with g' replacing g:

$$W^C = \frac{1}{2}\frac{c_0^{1-\sigma}}{1-\sigma}\left[(1+f)^{1-\sigma} + (1-f)^{1-\sigma}\right]\frac{1}{1-\beta\left(1+g'\right)^{1-\sigma}}. \tag{3}$$

Again, we look for the consumption variation that would make agents in economy C as happy as they are in economy A. We multiply c_0 by a factor λ_C in (3) and then we equate it to (1):

$$\begin{aligned}
&\frac{1}{2}\frac{c_0^{1-\sigma}}{1-\sigma}\left[(1+f)^{1-\sigma} + (1-f)^{1-\sigma}\right]\frac{1}{1-\beta\left(1+g\right)^{1-\sigma}} \\
&= \frac{1}{2}\frac{\left(\lambda_C c_0\right)^{1-\sigma}}{1-\sigma}\left[(1+f)^{1-\sigma} + (1-f)^{1-\sigma}\right]\frac{1}{1-\beta\left(1+g'\right)^{1-\sigma}}.
\end{aligned}$$

Solve for λ_C and plug in the numbers $\sigma = 3$, $g = 0.015$, $g' = 0.025$ and $\beta = 0.97$:

$$\begin{aligned}\lambda_C &= \left(\frac{1-\beta\left(1+g'\right)^{1-\sigma}}{1-\beta\left(1+g\right)^{1-\sigma}}\right)^{\frac{1}{1-\sigma}} \\ &= \left(\frac{1-0.97\left(1.025\right)^{1-3}}{1-0.97\left(1.015\right)^{1-3}}\right)^{\frac{1}{1-3}} \\ &= 0.873.\end{aligned}$$

Households prefer to be in economy C because faster growth guarantees higher consumption over time. They should give up 12.7% of their initial consumption to experience the welfare level that is achieved by the households of economy A. This is smaller than the number you computed in your lecture because stronger risk aversion makes the utility function flatter at high levels of consumption[1], reducing the welfare gains from further consumption growth.

[1] Because $u'(c_t) = c_t^{-\sigma}$, marginal utility with $\sigma = 3$ is less than marginal utility with $\sigma = 2$ when consumption exceeds 1.