EC201 Macroeconomics 2

Week 4 - Seminar 2

Fatih Kansoy

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Email: f.kansoy@warwick.ac.uk

Web: warwick.ac.uk/fatihkansoy

This Week

1. Review Questions - A
2. Class Questions - B -
3. Self Study Questions - C

Review Questions - A

Information Effects

Indicate whether the following statement is true, false, or uncertain and explain why.

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Indicate whether the following statement is true, false, or uncertain and explain why.

- *Good news about future productivity leads to a trade deficit today.*

The statement is uncertain. In a small open economy with production, the trade balance is

$$TB_1 = Q_1 - C_1 - I_1.$$

News about future productivity leave output unchanged because Q_1 depends on investment in the previous period, I_0, which is predetermined. Investment rises because the investment schedule $I_1 = I\left(r_1, A_2\right)$ is increasing in its second argument. Consumption rises as well, due to income effects caused by the jump in future profits $\Pi_2 = A_2 F\left(I_1\right) - \left(1 + r_1\right) D_1^f$ (see the discussion of technological improvements in the textbook). Whether this leads to a trade deficit or not depends on the output level Q_1 and on the size of the jump in C_1 and I_1

A2 An Economy With Investment

Consider a two-period model of a small open economy with a single good each period. Let preferences of the representative household be described by the utility function:

$$U(C_1, C_2) = \ln(C_1) + \ln(C_2),$$

where C_1 and C_2 denote, respectively, consumption in periods 1 and 2 and $\ln$ denotes the natural logarithm. In period 1, the household receives an endowment of $Q_1 = 10$. In period 2, the household receives profits, denoted by Π_2, from the firms it owns. Households and firms have access to financial markets where they can borrow or lend at the interest rate r_1. (r_1 is the interest rate on assets held between periods 1 and 2.) Firms invest in period 1 to be able to produce goods in period 2. The production technology in period 2 is given by:

$$Q_2 = \sqrt{I_1},$$

where Q_2 and I_1 denote, respectively, output in period 2 and investment in period 1. Assume that there exists free international capital mobility and that the world interest rate, r^*, is 10% per period (i.e., $r^* = 0.1$). Finally, assume that the economy's initial net foreign asset position is zero ($B_0^* = 0$).

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Given the production function $Q_2 = \sqrt{I_1}$, the marginal product of capital is $MPK = \frac{1}{2}(I_1)^{-\frac{1}{2}}$. Equating this with the cost of capital $1 + r_1 = 1 + r^* = 1.10$, we find the optimal level of investment in period 1:

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Remember

$$\Pi_2 = A_2 F(I_2) - (1 + r_1) D_1^f$$
$$\Pi_2 = Q_2 - (1 + r_1) I_1$$

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Remember

$$\Pi_2 = A_2 F(I_2) - (1 + r_1) D_1^f$$
$$\Pi_2 = Q_2 - (1 + r_1) I_1$$

The implied level of output and profits in period 2 are

$$Q_2 = \sqrt{0.21},$$

$$\Pi_2 = \sqrt{0.21} - 1.10 \times 0.21 = 0.23.$$

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The household maximizes its lifetime utility

$$U\left(C_1, C_2\right) = \ln\left(C_1\right) + \ln\left(C_2\right)$$

subject to the intertemporal budget constraint

$$C_1 + \frac{C_2}{1+r_1} = \left(1+r_1\right) B_0^* + Q_1 + \frac{\Pi_2}{1+r_1}. \tag{1}$$

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The marginal utilities of consumption are

$$U_1\left(C_1, C_2\right) = \frac{1}{C_1}, \quad U_2\left(C_1, C_2\right) = \frac{1}{C_2}.$$

With free capital mobility and $r^* = 0.10$, the intertemporal optimality condition (1) reads

$$-\frac{1/C_1}{1/C_2} = -\left(1+0.10\right),$$

which implies

$$C_2 = 1.10 \times C_1.$$

Since $B_0^* = 0, Q_1 = 10$ and $\Pi_2 = 0.23$, the budget constraint requires that

$$C_1 + \frac{C_2}{1 + r_1} = 10.21.$$

Combining the last two conditions we obtain the optimal consumption plan:

$$C_1 + \frac{1.1 \times C_1}{1.1} = 10.21.$$
$$C_1 \approx 5.11, \quad C_2 = 5.62.$$

- **(C)** Find the country's net foreign asset position at the end of period 1, the trade balance in periods 1 and 2, and the current account in periods 1 and 2.

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Since $I_2 = 0$ in a two-period economy, the trade balances are

$$TB_1 = Q_1 - C_1 - I_1 = 10 - 5.11 - 0.21 = 4.68,$$

$$TB_2 = Q_2 - C_2 - I_2 = 0.46 - 5.62 - 0 = -5.16.$$

The implied net foreign assets position is

$$B_1^* = (1 + r_0)\, B_0^* + TB_1 = 4.68.$$

The current account balances are then

$$CA_1 = r_0 B_0^* + TB_1 = 4.68,$$

$$CA_2 = r_1 B_1^* + TB_2 = 0.1 \times 4.68 - 5.16 = -4.69.$$

[▶] Now consider an investment surge. Specifically, assume that as a result of a technological improvement, the production technology becomes $Q_2 = 2\sqrt{I_1}$. Find the equilibrium levels of savings, investment, the trade balance, the current account, and the country's net foreign asset position in period 1. Compare your results with those obtained in items 1-3. providing interpretation and intuition.

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The production function is now $Q_2 = 2\sqrt{I_1}$, so the marginal product of capital becomes $MPK = (I_1)^{-\frac{1}{2}}$. Equating this with the cost of capital again, we find a new investment level:

$$I_1 = 0.83.$$

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Output and profits in period 2 are larger:

$$Q_2 = \sqrt{0.83} = 1.82,$$

$$\Pi_2 = 1.82 - 1.10 \times 0.83 = 0.91.$$

With this level of income, the intertemporal budget constraint requires that

$$C_1 + \frac{C_2}{1 + r_1} = 10.83.$$

Combined with the intertemporal optimality condition $C_2 = 1.10 \times C_1$ again, this implies the new consumption plan

$$C_1 = 5.415, \quad C_2 = 5.957,$$

which implies the following trade balances:

$$TB_1 = 10 - 5.415 - 0.83 = 3.755,$$

$$TB_2 = 1.82 - 5.957 = -4.137.$$

The net foreign asset position and the current account balances are computed through the usual steps:

$$B_1^* = (1 + r_0)\, B_0^* + TB_1 = 3.76,$$

$$CA_1 = r_0 B_0^* + TB_1 = 3.76,$$

$$CA_2 = r_1 B_1^* + TB_2 = 0.1 \times 3.755 - 4.14 = -3.76.$$

Higher productivity in period 2 raises both C_1 and C_2.

The household reduces its savings in period 1 to bring forward a portion of this productivity gain: the NFA is 3.76 instead of 4.68 in period 1. Because of this consumption smoothing, the country runs smaller trade and CA surpluses in period 1.

Class Questions - B -

Consider a two-period model of a small open economy with a single good each period. Let preferences of the representative household be described by the utility function:

$$U(C_1, C_2) = \sqrt{C_1 C_2}$$

where C_1 and C_2 denote, respectively, consumption in periods 1 and 2. In period 1, the household receives an endowment of $Q_1 = 10$. In period 2, the household receives profits, denoted by Π_2, from the firms it owns. Households and firms have access to financial markets where they can borrow or lend at the interest rate r_1. Firms borrow in period 1 to invest in physical capital. They are subject to a collateral constraint of the form $D_1^f \leq \kappa_1$ where D_1^f denotes the amount of debt assumed by the firm in period 1 and κ_1 denotes the value of the firm's collateral. Suppose that $\kappa_1 = 4$. In turn, firms use the physical capital purchased in period 1 to produce final goods in period 2. The production technology in period 2 is given by $Q_2 = 6I_1^{\frac{1}{3}}$ where Q_2 and I_1 denote, respectively, output in period 2 and investment in period 1. Assume that there exists free international capital mobility and that the world interest rate, r^*, is 10% per period (i.e., $r^* = 0.1$). Finally, assume that the economy's initial net foreign asset position is zero ($B_0^* = 0$).

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- **(A)** Compute the firm's optimal levels of period-1 investment and period-2 profits. Is the collateral constraint binding in period 1? Explain.

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With the production function specification $Q_2 = 6\sqrt[3]{I_1}$, the marginal product of capital is $MPK = \frac{1}{3} \times 6 \times (I_1)^{-\frac{2}{3}}$. Equated with the cost of capital $r^* = 0.10$ under perfect capital mobility, this implies the following investment level:

$$I_1 = 2.45.$$

Output and profits in period 2 are as follows:

$$Q_2 = 6\sqrt[3]{2.45} = 8.09,$$

$$\Pi_2 = 8.09 - 1.10 \times 2.45 = 5.40.$$

The collateral constraint $D_1^f \leq 4$ is not binding. With the given cost of capital and productivity level, the firm maximizes its profits with a level of investment that is smaller than 4.

- **(B)** State the maximization problem of the representative household and derive the associated optimality condition.

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The household maximizes its lifetime utility

$$U\left(C_1, C_2\right) = \sqrt{C_1 C_2}$$

subject to the intertemporal budget constraint

$$C_1 + \frac{C_2}{1+r_1} = \left(1+r_0\right) B_0^* + Q_1 + \frac{\Pi_2}{1+r_1}.$$

The marginal utilities of consumption are

$$U_1\left(C_1, C_2\right) = \frac{1}{2}\left(C_1\right)^{-\frac{1}{2}}\left(C_2\right)^{\frac{1}{2}},$$

$$U_2\left(C_1, C_2\right) = \frac{1}{2}\left(C_1\right)^{\frac{1}{2}}\left(C_2\right)^{-\frac{1}{2}}.$$

The intertemporal optimality condition with free capital mobility and $r^* = 0.10$ is

$$-\frac{C_2}{C_1} = -1.10.$$

- **(C)** Solve for the equilibrium levels of period 1 consumption, the country's net foreign asset position (B_1^*), the trade balance, and the current account.

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With $B_0^* = 0, Q_1 = 10$ and $\Pi_2 = 5.40$, the budget constraint is

$$C_1 + \frac{C_2}{1 + r_1} = 14.90.$$

Combined with the intertemporal optimality condition found above, this implies

$$C_1 = 7.45, \quad C_2 = 8.20.$$

The associated trade balances are

$$TB_1 = Q_1 - C_1 - I_1 = 10 - 7.45 - 2.45 = 0.10,$$

$$TB_2 = Q_2 - C_2 - I_2 = 8.09 - 8.20 - 0 = -0.11.$$

The net foreign assets position and the current account balances are

$$B_1^* = (1 + r_0)\, B_0^* + TB_1 = 0.10,$$

- **(D)** Now suppose that a financial panic causes banks to lower their assessment of the value of firms' collateral. Specifically, suppose that $\boldsymbol{\kappa_1}$ **falls from** 4 to 1. Solve for the equilibrium levels of investment, consumption, the trade balance, the current account, and the country's net asset position in period 1, and output and profits in period 2. Provide intuition.

- **(D)** Now suppose that a financial panic causes banks to lower their assessment of the value of firms' collateral. Specifically, suppose that $\boldsymbol{\kappa_1}$ **falls from** 4 to 1. Solve for the equilibrium levels of investment, consumption, the trade balance, the current account, and the country's net asset position in period 1, and output and profits in period 2. Provide intuition.

The collateral constraint $\boldsymbol{D_1^f \leq 1}$ is binding, because it imposes a level of investment that is below its unconstrained counterpart. We find the following quantities:

$$I_1 = 1,$$

$$Q_2 = 6\sqrt[3]{1} = 6,$$

$$\Pi_2 = 6 - 1.10 \times 1 = 4.90.$$

The household's budget constraint becomes

$$C_1 + \frac{C_2}{1 + r_1} = 14.45,$$

which implies

$$C_1 = 7.23, \quad C_2 = 7.95.$$

As a consequence

$$TB_1 = 10 - 7.23 - 1 = 1.77,$$

$$TB_2 = 6 - 7.95 - 0 = -1.95,$$

$$B_1^* = (1 + r_0)\, B_0^* + TB_1 = 1.77,$$

$$CA_1 = r_0 B_0^* + TB_1 = 1.77,$$

$$CA_2 = r_1 B_1^* + TB_2 = 0.10 \times 1.77 - 1.95 = -1.77.$$

The financial panic reduces investment in period 1 and output in period 2. Net foreign assets improve as the household saves more in period 1 to offset the decline in period 2 profits. Although the fall in investment improves the trade and CA balances in period 1, welfare declines due to the reduction in lifetime consumption.

- **(E) [A Bailout]**. Suppose that as a way to mitigate the financial crisis, in period 1 the government levies a tax on households, denoted T_1, and lends the proceeds to firms at the world interest rate. Let $T_1 = 0.5$, and let D^{fG} denote the debt that firms owe to the government and D^{fB} the debt that firms owe to private banks. Continue to assume that lending of private banks to firms is limited by the collateral constraint $D^{fB} \leq \kappa_1$ and that $\kappa_1 = 1$. In period 2, the government collects loan payments from firms and rebates the whole amount (including interest) to households in the form of a subsidy. State the household's and firm's optimization problems. Compute the equilibrium levels of investment, consumption, the trade balance, the current account, and the country's net foreign asset position in period 1 and output and profits in period 2.

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Firms borrow 1 from the banks and 0.5 from the government. Therefore

$$I_1 = 1.5,$$

$$Q_2 = 6\sqrt[3]{1.5} = 6.87,$$

$$\Pi_2 = 6.87 - 1.10 \times 1.5 = 5.22.$$

In the presence of lump-sum taxes and subsidies, the budget constraint of the household reads

$$C_1 + \frac{C_2}{1+r_1} = (1+r_0)\,B_0^* + Q_1 - T_1 + \frac{\Pi_2}{1+r_1} + \frac{(1+r_1)\,T_1}{1+r_1}.$$

Since the two terms in T_1 cancel out on the right-hand side, we are left with

$$C_1 + \frac{C_2}{1+r_1} = Q_1 + \frac{\Pi_2}{1+r_1} = 14.75.$$

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$$C_1 + \frac{C_2}{1+r_1} = Q_1 + \frac{\Pi_2}{1+r_1} = 14.75.$$

Under the given preference specification, we find

$$C_1 = 7.37, \quad C_2 = 8.11.$$

Therefore

$$TB_1 = 10 - 7.37 - 1.5 = 1.13,$$

$$TB_2 = 6.87 - 8.11 - 0 = -1.24,$$

$$B_1^* = (1 + r_1)\, B_0^* + TB_1 = 1.13,$$

$$CA_1 = r_0 B_0^* + TB_1 = 1.13,$$

$$CA_2 = r_1 B_1^* + TB_2 = 0.10 \times 1.13 - 1.24 = -1.13.$$

- **(F)** Is the bailout welfare improving? Answer this question by computing the lifetime welfare of the representative household with and without bailout. Discuss your result.

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The bailout must be welfare improving, because it effectively relaxes the borrowing constraint on firms and brings investment closer to its unconstrained counterpart. The larger investment enabled by the government's intervention in period 1 raises output in period 2 and makes it possible to achieve a higher level of consumption in both periods. Indeed, with the bailout

$$U(C_1, C_2) = \sqrt{7.37 \times 8.11} = 7.73,$$

whereas without it

$$U(C_1, C_2) = \sqrt{7.23 \times 7.95} = 7.58.$$

Self Study Questions - C

[▶] Everything is the same but with different utility function.