Part 3 Question 1

$$\frac{\partial L}{\partial \hat{y}} = \frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}}$$

$$\frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial c} = \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right) \cdot \frac{\partial (h_{1out} \cdot v_1 + h_{2out} \cdot v_2 + h_{3out} \cdot v_3 + h_{4out} \cdot v_4 + c)}{\partial c}$$

$$= \frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}}$$

$$\frac{\partial L}{\partial v_1} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial v_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial (h_{1out} \cdot v_1 + h_{2out} \cdot v_2 + h_{3out} \cdot v_3 + h_{4out} \cdot v_4 + c)}{\partial v_1}$$

$$= h_{1out} \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right)$$

$$\frac{\partial L}{\partial v_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial v_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial (h_{1out} \cdot v_1 + h_{2out} \cdot v_2 + h_{3out} \cdot v_3 + h_{4out} \cdot v_4 + c)}{\partial v_2}$$

$$= h_{2out} \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right)$$

$$\frac{\partial L}{\partial v_3} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial v_3} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial (h_{1out} \cdot v_1 + h_{2out} \cdot v_2 + h_{3out} \cdot v_3 + h_{4out} \cdot v_4 + c)}{\partial v_3}$$

$$= h_{3out} \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right)$$

$$\frac{\partial L}{\partial v_4} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial v_4} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial (h_{1out} \cdot v_1 + h_{2out} \cdot v_2 + h_{3out} \cdot v_3 + h_{4out} \cdot v_4 + c)}{\partial v_4}$$

$$= h_{4out} \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right)$$

$$\frac{\partial L}{\partial b_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial h_{1,out}} \frac{\partial h_{1,out}}{\partial h_{1,input}} \frac{\partial h_{1,input}}{\partial b_1}$$

$$= \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}}\right) v_1 \cdot \frac{\partial (w_{11}x_1 + w_{21}x_2 + b_1)}{\partial b_1}$$

$$= \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}}\right) \cdot v_1$$

$$\frac{\partial L}{\partial b_2} = \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}}\right) \cdot v_2$$

$$\frac{\partial L}{\partial b_3} = \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}}\right) \cdot v_3$$

$$\frac{\partial L}{\partial b_4} = \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}}\right) \cdot v_4$$

$$\frac{\partial L}{\partial w_{11}} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial h_{1,out}} \frac{\partial h_{1,out}}{\partial h_{1,input}} \frac{\partial h_{1,input}}{\partial w_{11}}$$

$$= \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}}\right) \cdot v_1 \cdot \frac{\partial (w_{11}x_1 + w_{21}x_2 + b_1)}{\partial w_{11}}$$

$$= \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}}\right) \cdot v_1 \cdot x_1$$

$$\frac{\partial L}{\partial w_{12}} = \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}}\right) v_2 \cdot x_1$$

$$\frac{\partial L}{\partial w_{13}} = \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}}\right) v_3 \cdot x_1$$

$$\frac{\partial L}{\partial w_{14}} = \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}}\right) \cdot v_4 \cdot x_1$$

$$\frac{\partial L}{\partial w_{21}} = \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}}\right) v_1 x_2$$

$$\frac{\partial L}{\partial w_{22}} = \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}}\right) \cdot v_2 x_2$$

$$\frac{\partial L}{\partial w_{23}} = \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}}\right) \cdot v_3 \cdot x_2$$

$$\frac{\partial L}{\partial w_{24}} = \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}}\right) \cdot v_4 \cdot x_2$$