

Part 3 Question 1

$$\frac{\partial L}{\partial \hat{y}} = \frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}}$$

$$\begin{aligned} \frac{\partial L}{\partial c} &= \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial \hat{y}_{input}} \cdot \frac{\partial \hat{y}_{input}}{\partial c} = \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right) \cdot \frac{\gamma(h_{inout} \cdot v_1 + h_{2out} \cdot v_2 + h_{3out} \cdot v_3 + h_{4out} \cdot v_4 + c)}{\gamma c} \cdot \hat{y}(1-\hat{y}) \\ &= \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right) \cdot \hat{y}(1-\hat{y}) \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial v_1} &= \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial \hat{y}_{input}} \cdot \frac{\partial \hat{y}_{input}}{\partial v_1} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\gamma(h_{inout} \cdot v_1 + h_{2out} \cdot v_2 + h_{3out} \cdot v_3 + h_{4out} \cdot v_4 + c)}{\gamma v_1} \cdot \hat{y}(1-\hat{y}) \\ &= h_{inout} \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right) \cdot \hat{y}(1-\hat{y}) \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial v_2} &= \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial \hat{y}_{input}} \cdot \frac{\partial \hat{y}_{input}}{\partial v_2} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\gamma(h_{inout} \cdot v_1 + h_{2out} \cdot v_2 + h_{3out} \cdot v_3 + h_{4out} \cdot v_4 + c)}{\gamma v_2} \cdot \hat{y}(1-\hat{y}) \\ &= h_{2out} \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right) \cdot \hat{y}(1-\hat{y}) \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial v_3} &= \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial \hat{y}_{input}} \cdot \frac{\partial \hat{y}_{input}}{\partial v_3} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\gamma(h_{inout} \cdot v_1 + h_{2out} \cdot v_2 + h_{3out} \cdot v_3 + h_{4out} \cdot v_4 + c)}{\gamma v_3} \cdot \hat{y}(1-\hat{y}) \\ &= h_{3out} \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right) \cdot \hat{y}(1-\hat{y}) \end{aligned}$$

$$\begin{aligned} \frac{\partial L}{\partial v_4} &= \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial \hat{y}_{input}} \cdot \frac{\partial \hat{y}_{input}}{\partial v_4} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\gamma(h_{inout} \cdot v_1 + h_{2out} \cdot v_2 + h_{3out} \cdot v_3 + h_{4out} \cdot v_4 + c)}{\gamma v_4} \cdot \hat{y}(1-\hat{y}) \\ &= h_{4out} \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right) \cdot \hat{y}(1-\hat{y}) \end{aligned}$$

$$\begin{aligned}
 \frac{\partial L}{\partial b_1} &= \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial \hat{y}_{input}} \frac{\partial \hat{y}_{input}}{\partial h_{out}} \frac{\partial h_{out}}{\partial h_{input}} \frac{\partial h_{input}}{\partial b_1} \\
 &= \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right) v_1 \cdot \frac{\partial (w_{11}x_1 + w_{21}x_2 + b_1)}{\partial b_1} \cdot \hat{y}(1-\hat{y}) \\
 &= \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right) \cdot v_1 \cdot \hat{y}(1-\hat{y})
 \end{aligned}$$

$$\frac{\partial L}{\partial b_2} = \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right) \cdot v_2 \cdot \hat{y}(1-\hat{y})$$

$$\frac{\partial L}{\partial b_3} = \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right) \cdot v_3 \cdot \hat{y}(1-\hat{y})$$

$$\frac{\partial L}{\partial b_4} = \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right) \cdot v_4 \cdot \hat{y}(1-\hat{y})$$

$$\begin{aligned}
 \frac{\partial L}{\partial w_{11}} &= \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial \hat{y}_{input}} \frac{\partial \hat{y}_{input}}{\partial h_{out}} \frac{\partial h_{out}}{\partial h_{input}} \frac{\partial h_{input}}{\partial w_{11}} \\
 &= \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right) \cdot v_1 \cdot \frac{\partial (w_{11}x_1 + w_{21}x_2 + b_1)}{\partial w_{11}} \cdot \hat{y}(1-\hat{y}) \\
 &= \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right) \cdot v_1 \cdot x_1 \cdot \hat{y}(1-\hat{y})
 \end{aligned}$$

$$\frac{\partial L}{\partial w_{12}} = \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right) v_2 \cdot x_1 \cdot \hat{y}(1-\hat{y})$$

$$\frac{\partial L}{\partial w_{13}} = \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right) v_3 \cdot x_1 \cdot \hat{y}(1-\hat{y})$$

$$\frac{\partial L}{\partial w_{14}} = \left(\frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right) \cdot v_4 \cdot x_1 \cdot \hat{y}(1-\hat{y})$$

$$\frac{\partial L}{\partial w_{21}} = \left(\frac{y}{\hat{y}} - \frac{1-y}{(1-\hat{y})} \right) \cdot V_1 \cdot X_2 \cdot \hat{y} (1-\hat{y})$$

$$\frac{\partial L}{\partial w_{22}} = \left(\frac{y}{\hat{y}} - \frac{1-y}{(1-\hat{y})} \right) \cdot V_2 \cdot X_2 \cdot \hat{y} (1-\hat{y})$$

$$\frac{\partial L}{\partial w_{23}} = \left(\frac{y}{\hat{y}} - \frac{1-y}{(1-\hat{y})} \right) \cdot V_3 \cdot X_2 \cdot \hat{y} (1-\hat{y})$$

$$\frac{\partial L}{\partial w_{24}} = \left(\frac{y}{\hat{y}} - \frac{1-y}{(1-\hat{y})} \right) \cdot V_4 \cdot X_2 \cdot \hat{y} (1-\hat{y})$$