# Chaotic Dynamics: Homework 7

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## Problem 1

$$D_{\vec{x}}\vec{F} = \begin{pmatrix} -a & a & 0 \\ r-z & -1 & -x \\ y & x & -b \end{pmatrix}$$

## Problem 2

$$\dot{\delta} = \begin{pmatrix} -a & a & 0 \\ r-z & -1 & -x \\ y & x & -b \end{pmatrix} \begin{pmatrix} \delta_{xx} & \delta_{yx} & \delta_{zx} \\ \delta_{xy} & \delta_{yy} & \delta_{zy} \\ \delta_{xz} & \delta_{yz} & \delta_{zz} \end{pmatrix}$$

$$= \begin{pmatrix} a(\delta_{xy}-\delta_{xx}) & a(\delta_{yy}-\delta_{yx}) & a(\delta_{zy}-\delta_{zx}) \\ (r-z)\delta_{xx}-\delta_{xy}-x\delta_{xz} & (r-z)\delta_{yx}-\delta_{yy}-x\delta_{yz} & (r-z)\delta_{zx}-\delta_{zy}-x\delta_{zz} \\ y\delta_{xx}+x\delta_{xy}-b\delta_{xz} & y\delta_{yx}+x\delta_{yy}-b\delta_{yz} & y\delta_{zx}+x\delta_{zy}-b\delta_{zz} \end{pmatrix}$$

## Problem 3

All the values are rounded after the decimal point.

### (a)

1.92, 4.22, 1.53, 2.40, 5.18, 0.48, 1.92, 4.21, 0.37, −0.03, −0.09, 0.67

### (b)

14.02, 28.38, 11.12, 2.13, 3.86, 3.12, 1.64, 3.01, 2.53, −0.49, −1.11, −0.02

### (c)

−1.92, −4.22, 1.53, 2.40, 5.18, −0.48, 1.92, 4.21, −0.37, 0.03, 0.09, 0.67

### (d)

$$\delta_a = \begin{pmatrix} 2.40 & 1.92 & -0.03 \\ 5.18 & 4.21 & -0.09 \\ 0.48 & 0.37 & 0.67 \end{pmatrix} \delta_b = \begin{pmatrix} 2.13 & 1.64 & -0.49 \\ 3.86 & 3.01 & -1.11 \\ 3.12 & 2.53 & -0.02 \end{pmatrix} \delta_c = \begin{pmatrix} 2.40 & 1.92 & 0.03 \\ 5.18 & 4.21 & 0.09 \\ -0.48 & -0.37 & 0.67 \end{pmatrix}$$

$\delta_a$,$\delta_b$ and $\delta_c$ above are the resulting variation matrices for initial conditions in problems (a), (b) and (c) respectively. The growth of state-space volume can be calculated by summing variation of each direction, $x$,$y$, and $z$. The initial condition in (a) has grown fastest (and the largest in the volume) in this measure (15.11, as opposed to 14.67 for (b) and 13.65 for (c)). The fastest growing direction is $x$. The resulting variation matrices for (a) and (c) are quite symmetric as their initial condition around $y$-axis and the symmetry of Lorenz system.