Chaotic Dynamics: Homework 9

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March 16, 2016

Problem 2

The resulting λ_1 for each data set is as follows: for data set (a) $\lambda_1 = 1.53$ and for data set (b) $\lambda_1 = -0.8$. These results are consistent with what we know of for λ_1 and the system's behavior: If λ_1 is positive, the system is chaotic and if λ_1 is negative, the system's behavior is stable and the trajectory converges to a stable fixed point.

Problem 3

For data set (a), we have picked up $\tau = 105$ as it was the first point at which we could see the local minimum in the figure. From Takens theorem we have picked up $m = 7$. The largest lyapunov exponent λ_1 obtained from this experiment was $\lambda_1 = 0.9$, which is a bit smaller than what we have obtained from using Wolf's algorithm.

Problem 4

(a)

The sequence of λ_is is as follows: $[1.348, -1.387, 0.07]$. The largest lyapunov exponent λ_1 was 1.348. This should not match the value derived in Problem 3, as variational equation uses Jacobian matrix, which is basically linear approximation of changes of slopes. Thus, the result derived by Kantz's algorithm is more reliable than what we gained in this problem.

(b)

We calculated the same quantity as (a), but using data sets of different numbers of points. One data set has 1000-point and the other has 100000-point. The sequences of λ_is are as follows: for 1000-point $[-1.535, 0.027, 0.027]$ and for 100000-point $[14.08, 10.64, 11.65]$. The λ_1 for 1000-point data set is 0.027 and The λ_1 for 100000-point data set is 14.08. If the system is linear, these quantities should be the same even if we change the number of iterations. However, since the system is nonlinear, these quantities change as variational equations evolve over time.