Chaotic Dynamics: Homework 8

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Problem 1

Fig.1 shows a state-space trajectory of data1 constructed by divided difference. In the process of reconstruction, data was *undersampled* with the interval of 50 data points. If the interval was smaller, however, the reconstructed trajectory would look like having several lines horizontally.

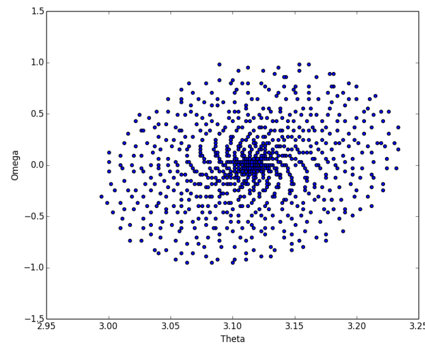


Figure 1: Constructed state-space trajectory by using divided difference.

Problem 2

(a)

Fig.2 shows the constructed embedding of data2 with the conditions defined in (a). This embedding displays a chaotic attractor of a driven pendulum.

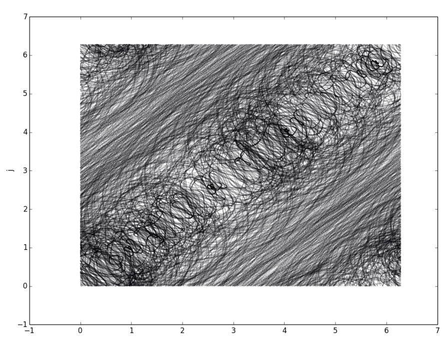
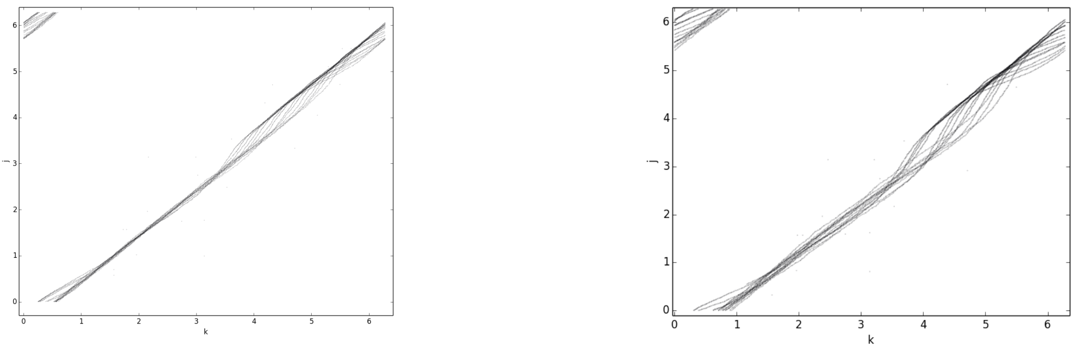


Figure 2: Reconstructed chaotic trajectory



(a) $\tau = 0.01$ sec (b) $\tau = 0.15$ sec

Figure 3: Reconstructed state-space trajectory

(b)

Figs.3a and 3b show the reconstructed state-space trajectories. As τ increases, the underlying dynamics gets unfolded as seen in fig.3b. The attractor is a periodic orbit.

Problem 3

(a)

Takens theorem states that in order to successfully reconstruct a topologically conjugate trajectory, one needs to satisfy the following:

$$m > 2d,$$

where m is the dimension of the reconstructed state-space and d is the true dimension of the dynamics one is trying to reconstruct. Since driven pendulum has 3 dimensions in its dynamics: angle, angular velocity and angular velocity of driving force, we need $m = 7$ for successful embedding. For undriven pendulum, however, we only need $m = 5$ since its true dimension is 2: angle and angular velocity.

(b)

If $m = 2$, the resulting trajectory would look like a squashed form of the original trajectory since the dimension is too low and it has more points than one in a higher dimension. If $m = 25$, the trajectory would show its true form since we have a enough dimension to retain the original shape, but fewer points on the trajectory.

(c)

If τ was small, the reconstructed trajectory would basically be diagonal since sampled points are highly correlated. If τ was large, the trajectory would look like totally random, as sampled points are too far away from each other, resulting in no correlation.

Problem 4

The first minimum of the curve of mutual information with respect to τ happens to be at the step 155, which can be converted to 0.31 second. The command line used to produce the output is as follows: `tisean-mutual ps8data/data2.first250sec -D 200 -o out.txt`

Problem 5

Fig.5 shows the change of the ratio of false neighbors versus embedding dimension m. The first ratio which gets below 10% (0.1) is at the dimension of 8. The command line used to produce the output is as follows: `tisean-false_nearest ps8data/data2.first250sec -M 1,10 -d 155 -o false_nearest.txt`

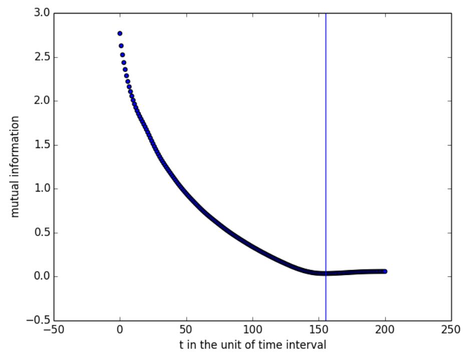


Figure 4: Change of mutual information with respect to time delay

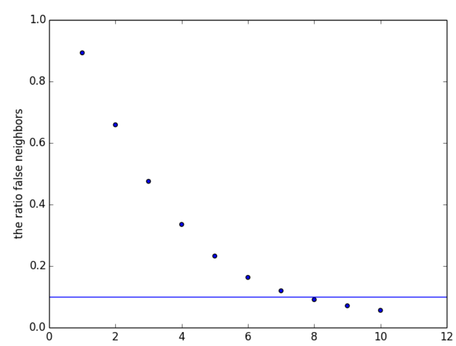


Figure 5: The ratio of false neighbors versus embedding dimension m