

# Chaotic Dynamics: Homework 7

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February 28, 2016

## Problem 1

$$D_{\vec{x}}\vec{F} = \begin{pmatrix} -a & a & 0 \\ r-z & -1 & -x \\ y & x & -b \end{pmatrix}$$

## Problem 2

$$\begin{aligned} \dot{\delta} &= \begin{pmatrix} -a & a & 0 \\ r-z & -1 & -x \\ y & x & -b \end{pmatrix} \begin{pmatrix} \delta_{xx} & \delta_{yx} & \delta_{zx} \\ \delta_{xy} & \delta_{yy} & \delta_{zy} \\ \delta_{xz} & \delta_{yz} & \delta_{zz} \end{pmatrix} \\ &= \begin{pmatrix} a(\delta_{xy} - \delta_{xx}) & a(\delta_{yy} - \delta_{yx}) & a(\delta_{zy} - \delta_{zx}) \\ (r-z)\delta_{xx} - \delta_{xy} - x\delta_{xz} & (r-z)\delta_{yx} - \delta_{yy} - x\delta_{yz} & (r-z)\delta_{zx} - \delta_{zy} - x\delta_{zz} \\ y\delta_{xx} + x\delta_{xy} - b\delta_{xz} & y\delta_{yx} + x\delta_{yy} - b\delta_{yz} & y\delta_{zx} + x\delta_{zy} - b\delta_{zz} \end{pmatrix} \end{aligned}$$

## Problem 3

All the values are rounded after the decimal point.

(a)

1.92, 4.22, 1.53, 2.40, 5.18, 0.48, 1.92, 4.21, 0.37, -0.03, -0.09, 0.67

(b)

14.02, 28.38, 11.12, 2.13, 3.86, 3.12, 1.64, 3.01, 2.53, -0.49, -1.11, -0.02

(c)

-1.92, -4.22, 1.53, 2.40, 5.18, -0.48, 1.92, 4.21, -0.37, 0.03, 0.09, 0.67

(d)

The growth of state-space volume can be calculated by summing variation of each direction,  $x, y$ , and  $z$ . The initial condition in (a) is the fastest (and largest in the grown volume) in this measure. The fastest growing direction is  $x$ . The resulting variation matrices for (a) and (c) are quite symmetric as their initial condition around  $y$ -axis and the symmetry of Lorenz system.