Chaotic Dynamics: Homework 1

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January 16, 2016

In the sections below, we explain the phenomenon observed in each plot.

$R = 2, x_0 = 0.2$

When we have the coefficient $R = 2$ and the initial condition $x_0 = 0.2$, we observe the pattern as shown in the fig.1. Note that initial 100 steps are truncated in order to exclude the transient phase. What we observe is that the evolution of x_n settles down to a fixed point, around 0.5.

$R = 3.3, x_0 = 0.2$

With the coefficient $R = 2$ and the initial condition $x_0 = 0.2$, we observe the 2-periodic cycle. In the fig.3 displaying the relationship between x_{n+2} and x_n we see two points sit on fixed points, which makes sense since the system is 2-periodic cycle, its state in the next 2 steps should be the same as the current state.

$R = 4.0, x_0 = 0.2$ **and** $x_0 = 0.200001$

In order to see the chaotic behavior, we plot two trajectories with $R = 4.0$ for two different initial conditions, namely $x_0 = 0.2$ and $x_0 = 0.200001$. Only a difference of 0.000001, two trajectories display very distinct behavior after $n = 15$.

$R > 4.0$

When $R > 4.0$, the value of x_n diverges rapidly. In fig.4, the output of logistic map with $R = 5$ and $x_0 = 0.2$ is displayed on the standard output. Analytically, we can easily see that with these conditions, x_n converges to a fixed point 0.8. However, due to the precision of floating point in Python, after x_2, x_n gradually deviates from 0.8, and it diverges rapidly after a certain point. This indicates that when $R > 4.0$, fixed points are unstable, meaning that even a tiny deviation from them results in rapid divergence.

$R = 2.5$

Any initial condition value $0 < x_0 < 1$ eventually converges to 0.6. This set of initial conditions is called the basin of an attractor, which in this case $x_n = 0.6$.

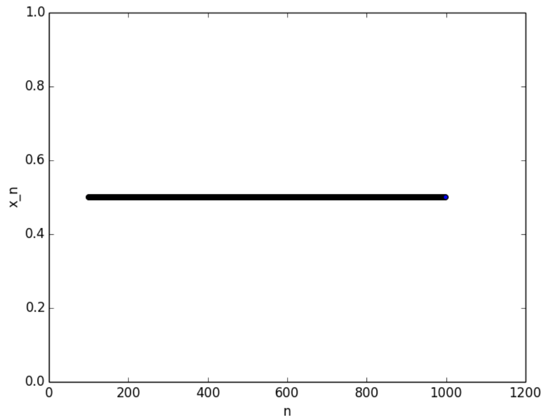

Figure 1: The evolution of x_n with $R = 2, x_0 = 0.2$

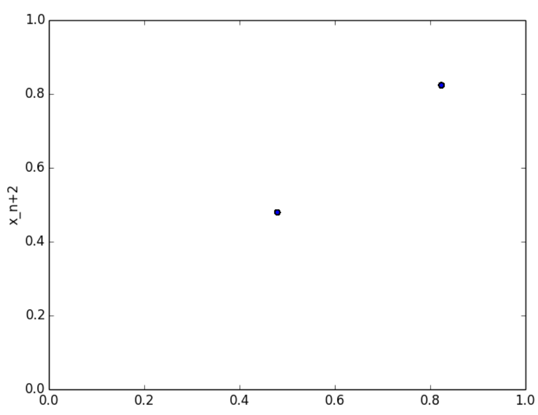

Figure 2: x_{n+2} versus x_n

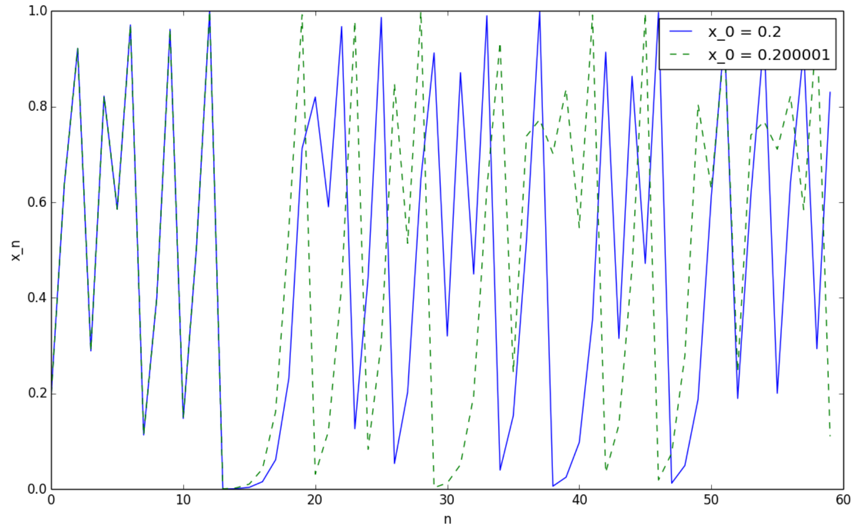

Figure 3: Two trajectories for two different initial conditions.

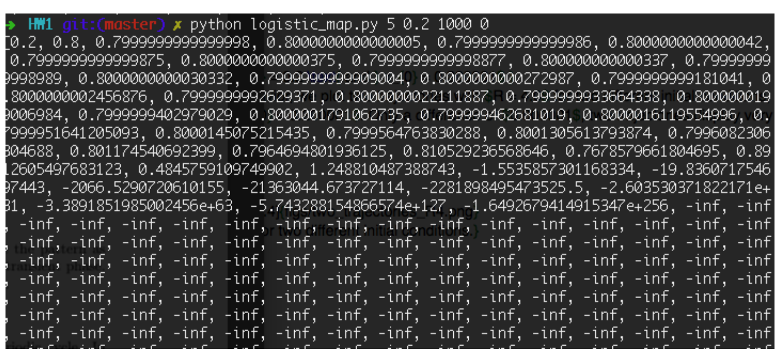

Figure 4: Divergence of x_n when $R = 5$ and $x_0 = 0.2$.