WAVES AND OSCILLATIONS

PROBLEM SET 5 February 23, 2018

1. A periodic external force acts on a 3 kg mass suspended from the lower end of a vertical spring having spring constant 75 N/m. The damping force is proportional to the instantaneous speed of the mass and is 20 N when the speed is 1 m/s. Find the frequency at which amplitude resonance occurs.

2. A machine of total mass 90 kg is supported by a spring resting on the floor and its motion is constrained to be in the vertical direction only. The system is lightly damped with a damping constant 900 Ns/m. The machine contains an eccentrically mounted shaft which, when rotating at an angular frequency p, produces a vertical force on the system of $Fp^2 \sin(pt)$ where F is a constant. It is found that resonance occurs at 1200 r.p.m. (revolutions per minute) and the amplitude of vibration in the steady state is then 1 cm. Find the amplitude of vibration in the steady state when the driving frequency is (a) 2400 r.p.m. (b) 3000 r.p.m. (c) very large. Find also the quality factor Q at resonance. Assume that the gravity has a negligible effect on the motion.

3. Two bodies of masses m_1 and m_2 connected by a spring of spring constant k, can move along a horizontal line (axis of the spring). A periodic force $F \cos \omega t$ is exerted on the body of mass m_1 along the line. Find expression for the displacements of the two masses and indicate by a sketch graph the dependence of the amplitude of motion of m_1 on frequency ω.

4. A vertical spring has a spring constant $50\ N/m$. At $t = 0$ a force given in newtons by $F(t) = 48 \cos 7t,\ t \geq 0$ is applied to a $20\ N$ weight which hangs in equilibrium at the end of the spring. Neglecting damping find the position of the weight at any later time t.

5. Show that for a forced vibration, the total energy of the vibrating system is not a constant. Prove that for such a case, (average potential energy / average kinetic energy) = ω_0^2 / ω^2.

6. An object of mass $2\ kg$ hangs from a spring of negligible mass. The spring is extended by $2.5\ cm$ when the object is attached. The top end of the spring is oscillated up and down in SHM with an amplitude of $2\ mm$. the Q of the system is 20. If $g = 10\ m/s^2$, (i) what is the angular frequency ω_0 of the free undamped oscillations, (ii) what is the amplitude of forced oscillations at $\omega = \omega_0$, (iii) what is the mean power input to maintain the forced oscillations at an angular frequency ω one percent greater than ω_0.

7. If ω_1 and ω_2 are the half power frequencies and ω_0 is the resonant frequency of a forced system show that $\omega_0^2 = \omega_1 \omega_2$.

8. In a forced mechanical oscillator show that the following are frequency independent (a) the displacement amplitude at low frequencies (b) the velocity amplitude at velocity resonance and (c) the acceleration amplitude at high frequencies, ($\omega \rightarrow \infty$).

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