WAVES AND OSCILLATIONS

PROBLEM SET 3 January 19, 2018

1. The simple pendulum swings with a displacement amplitude *a*. If its starting point from rest is $x = a$, find the value of the phase constant ϕ for the solution $x = asin(\omega t + \phi)$. For the value of ϕ calculated, find the value of ωt at which the pendulum swings through the positions $x = +a/\sqrt{2}$, for the first time after release from $x = a$.

2. When the electron in a hydrogen atom bound to the nucleus moves a small distance from its equilibrium position, a restoring force per unit distance is given by $k = e^2/(4\pi\epsilon_0 r^2)$ where $r = 0.05$ nm may be taken as the radius of the atom. Show that the electron can oscillate with a simple harmonic motion with $\omega_0 \sim 4.5 \times 10^{-16}$ rad s^{-1}. If the electron is forced to vibrate at this frequency in which region of the electromagnetic spectrum would its radiation be found? [Use SI units for the values of e, m_e and ε_0].

3. Two argon atoms form the molecule Ar_2 because of a van der Waals interaction with $U_0 = 1.68 \times 10^{-21}$ J and $R_0 = 3.82 \times 10^{-10}$m. Find the frequency of small oscillations of one Ar atom about its equilibrium position.

4. The displacement of a simple harmonic oscillator is given by $x = asin(\omega t + \phi)$. If the oscillation started at time t = 0 from a position x_0 with a velocity $dx/dt = v_0$, show that $tan\phi = \omega x_0/v_0$ and $a = \left(x_0^2 + \frac{v_0^2}{\omega^2}\right)^{1/2}$.

5. A particle oscillates with simple harmonic motion along the x axis with a displacement amplitude *a* and spends a time *dt* in moving from x to x + dx. Show that the probability of finding it between x and x + dx is given by $\frac{dx}{\pi(a^2 - x^2)^{1/2}}$.

6. Many identical simple harmonic oscillators are equally spaced along the x axis of a medium and a photograph shows that the locus of their displacements in the y direction is a sine curve. If the distance λ separates oscillators which differ in phase by 2π radians, what is the phase difference between two oscillators a distance x apart?

7. A mass stands on a platform which vibrates simple harmonically in a vertical direction at a frequency of 5 Hz. Show that the mass loses contact with the platform when the displacement exceeds 10^{-2} m.

8. A mass M is suspended at the end of a spring of length l and stiffness s. If the mass of the spring is m and the velocity of an element dy of its length is proportional to its distance y from the fixed end of the spring, show that the kinetic energy of this element is $\frac{1}{2}\left(\frac{m}{l}dy\right)\left(\frac{y}{l}v\right)^2$. Where v is the velocity of the suspended mass M. Hence, by integrating over the length of the spring, show that its total kinetic energy is $\frac{1}{6}mv^2$ and from the total energy of the oscillating system, show that the frequency of oscillation is given by $\omega^2 = \frac{s}{M + m/3}$.

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