IMSc – 2nd Semester
Waves, Oscillations and Optics – PY151
Minor – 3

April 12, 2018 @ 3 PM **Max. Marks: 20**

Answer ONLY the questions written in 'BOLD' ***All questions carry equal marks (2 each)***

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1. A scientist wants to suspend an atomic force microscope (AFM) of mass 5.4 kg (including the mass of the platform) using a rubber bungee cord of equilibrium length 1.2 m. If the scientist wants $\omega_0 = 10$ rad/s, what is the required diameter for the rubber cord? (Assume the mass of the cord is negligible compared to that of the AFM, Young's modulus for rubber is 0.002×10^9 N/m^2).
   Q: (i) Mention at least <u>two</u> the scientific / technical reasons for suspending the AFM on a rubber cord. (ii) At what frequency (ω) will the AFM be protected from the surrounding vibrations?

2. A massless spring with no mass attached to it hangs from the ceiling. Its length is 20 cm. A mass M is now hung on the lower end of the spring. Support the mass with your hand so that the spring remains relaxed, then suddenly remove your supporting hand. The mass and spring oscillate. The lowest position of the mass during the oscillation is 10 cm below the place it was resting when you supported it. (a) What is the frequency of oscillation? (b) What is the velocity when the mass is 5 cm below its original resting place?
   Q: A second mass of 300 g is added to the first mass, making a total mass of M + 300 g. When this system oscillates, it has half the frequency of the system with mass M alone. Then, (i) what is the value of M? and (ii) where is the new equilibrium position?

3. The CO_2 molecule can be crudely and classically modeled as a system with a central mass $m_2 = 12$ AMU connected by equal springs of spring constant k to two masses $m_1 = m_3 = 16$ AMU, constrained to move only along the line joining their centers. Set up and solve the equations for the two normal modes in which the masses oscillate along that line.
   Q: (i) There are three masses but only two oscillatory normal modes. What is the third mode? (ii) What are the ratio of frequencies of the two normal modes of oscillation?

4. Two simple harmonic motions of same angular frequency ω, $x_1 = a_1 sin\omega t$ and $x_2 = a_2 sin(\omega t + \phi)$ act on a particle along the x-axis simultaneously. Find amplitude, phase angle and hence the displacement of the resultant motion.
   Q: Describe briefly the resultant motion when (i) $\boldsymbol{\phi = \pm 2n\pi; n = 0, 1, 2, \ldots}$ and (ii) $\boldsymbol{\phi = \pm(2n+1)\pi; n = 0, 1, 2, \ldots}$

5. Consider a mass subjected to a restoring spring force $F_S = -kx$, a damping force $F_d = -b\dot{x}$, an oscillating drive force $F_D = F_0 sin\omega_D t$. Setup the differential equation of motion that describes the system and the general steady-state solution.
   Q: (i) What is the frequency of the oscillator in the steady-state; (ii) For what value of $\boldsymbol{\omega_D/\omega_0}$ are the oscillations due to $\boldsymbol{F_D}$ are in phase with the natural oscillations ($\boldsymbol{\omega_0}$) of the system.

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nkv

M3-1

* To isolate sensitive scientific equipments from building vibrations.

* Economical & effective way to place the equipment on a platform suspended by soft springs.

* effectiveness of vibration isolation is best when vibration freq. of the system $\omega_0 = \sqrt{k/m}$ is as low as possible.

$\cancel{\boxed{\omega < \omega_0}}$ $\boxed{\omega_0 < \omega}$

M(3-2)

i) $\omega_0 = \sqrt{\frac{k}{M}} = 2 \times \sqrt{\frac{k}{M + 300g}}$

(or) $M + 300g = 4M$

(or) $\underline{M = 100g}$.

$\omega_1 = \sqrt{\frac{k}{M}}$; $\omega_2 = \sqrt{\frac{k}{M + 300g}} = \frac{1}{2}\omega_1 \rightarrow \omega_1 = 2\sqrt{\frac{k}{M+300}}$

(or) $k/M = 4 \frac{k}{(M+300)} \rightarrow M+300 = 4M$

ii) if mass increases by a factor of 4,
the equilibrium point is $5 \times 4 = \underline{20\ cm}$.
below the starting point.

M3-3

i) ~~for~~ 3rd eigen vector & eigen value is

$$\psi_0 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} ; \quad \omega_0^2 = 0.$$

This is not a proper mode of oscillatory motion, but describes uniform center of mass motion of the molecule along its axis, because all components are moving ~~at~~ at the same velocity, although with 'zero' freq.

ii) Ratio of the two freq. modes is

$$\sqrt{\frac{11}{48}} \times \sqrt{16} = \sqrt{\frac{11}{3}} \simeq \underline{\underline{1.91}}$$

M-3-4

i) for $\phi = \pm 2n\pi$,

the two SHMs x_1 & x_2 are in phase

& $R = a_1 + a_2$

ii) for $\phi = \pm (2n+1)\pi$,

the two SHMs x_1 & x_2 are in opposite phase

& $R = a_1 \sim a_2$

& when $a_1 = a_2$, resultant amplitude is zero.

$\Rightarrow$ one motion is destroyed by the other.

M-3

i) In the steady state, the osc. moves with the same angular freq. as the driving force. : ω_D.

ii) $\omega_D/\omega_0 \leq 1$ in ideal case.