UNIVERSITY OF HYDERABAD

College of Integrated Studies

I.M.Sc.-Semester IV (2017)

PY 251 : Modern Physics [Part 2: STR]

Assignment: 1

Due date for optional submission: Mar. 05, 2019

Total Marks/Grade: N/A

Please be precise for answering correctly.

N.B.: Symbols have their usual meaning.

- 1. At what speed does a clock move to run at a rate of half of that at rest?

- 2 A. At what speed does a meter-stick move to shrink to $\mathbf{0.5}$ m?

  B. A rod of length $\boldsymbol{L'}$ in an inertial frame $\boldsymbol{S'}$ frame makes an angle $\boldsymbol{\theta'}$ with the $\boldsymbol{x'}$ axis. The frame $\boldsymbol{S'}$ is moving along $\boldsymbol{x}$ axis with velocity $\boldsymbol{v\hat{\imath}}$ with respect to a rest observer in $\boldsymbol{S}$. Determine the length ($\boldsymbol{L}$) of the rod and the angle ($\boldsymbol{\theta}$) it makes with the $\boldsymbol{x}$ axis as measured by the rest observer in $\boldsymbol{S}$.

- 3. Two ordinary forces $\boldsymbol{F\hat{\jmath}}$ and $\boldsymbol{-F\hat{\jmath}}$ are acting respectively at the points $\boldsymbol{x = 0}$ and $\boldsymbol{x = a}$ simultaneously at time $\boldsymbol{t = 0}$ in an inertial frame $\boldsymbol{S}$. This is the observation of a rest observer in $\boldsymbol{S}$ though his/her clock was moving along $\boldsymbol{x}$ axis with velocity $\mathbf{v} = \boldsymbol{v\hat{\imath}}$ with respect to him/her. What would be the observation from the clock's frame on the same events?

- 4 A. Show that two successive Lorentz transformations ($\boldsymbol{x^\mu \to x'^\mu \to x''^\mu}$) with the two velocities $\boldsymbol{\vec{v}_1 = v\hat{\imath}}$ and $\boldsymbol{\vec{v}_2 = v\hat{\imath}}$ respectively in the same direction are equivalent to a single Lorentz transformation with the velocity $\boldsymbol{\vec{v} = \frac{\vec{v}_1+\vec{v}_2}{1+\frac{\vec{v}_2\cdot\vec{v}_2}{c^2}}}$.

  B. Show that two successive Lorentz transformations ($\boldsymbol{x^\mu \to x'^\mu \to x''^\mu}$) with the two velocities $\boldsymbol{\vec{v}_1 = v\hat{\imath}}$ and $\boldsymbol{\vec{v}_2 = v\hat{\jmath}}$ respectively in the two perpendicular direction results the single Lorentz transformation matrix $\boldsymbol{\Lambda} = \begin{pmatrix} \gamma^2 & -\gamma^2\beta & -\gamma\beta & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ -\gamma^2\beta & \gamma^2\beta^2 & \gamma & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

  where $\boldsymbol{\gamma = 1/\sqrt{1-\beta^2}}$ and $\boldsymbol{\beta = v/c}$.

  C. Comment on your result on B if $\boldsymbol{\vec{v}_1}$ and $\boldsymbol{\vec{v}_2}$ were interchanged.

  D. A particle is moving along $\boldsymbol{x}$ axis. Draw the locus of its time-like interval $\boldsymbol{I = x^2 - c^2t^2 < 0}$ and space-like interval $\boldsymbol{I' = x^2 - c^2t^2 > 0}$ in the 2-D space-time diagram.

  E. Show that the loci in D are invariant under Lorentz transformations.

- 5. A light source is moving away from a stationary observer with velocity $\boldsymbol{\vec{v}}$. The light source is emitting a yellow light signal from its own rest frame with frequency $\boldsymbol{\nu' = 1/T'}$ where $\boldsymbol{T'}$ is the time period of sending signal in the body-fixed frame. Now solve the following problems on the relativistic Doppler effect.

A. Show that if frequency of the signal received by the observer is ν with the time period T, then the wavelength of the signal received by the observer is $\lambda = cT + vT$ and $\nu = \frac{c}{T(c+v)}$.

B. Using the time dilation formula $T = \gamma T'$, show that $\nu = \frac{\sqrt{c-v}}{\sqrt{c+v}}\nu'$.

C. Show that the frequency shifts to that of a red light. Think of a situation for observing blue shift of light.

- 6 A. In a laboratory experiment a muon is observed to travel 800m before it spontaneously decays (disintegrates) into an electron and two neutrinos. Life-time of a muon in its own frame of reference is 2μs. Find the speed of the muon in the laboratory.

  B. A pion at rest decays into a muon and a neutrino. Find the ordinary momentum of the outgoing muon in terms of the two rest masses m_π and m_μ ($m_\nu \to 0$).

  C. A body of rest mass m_0 collides perfectly inelastically at a speed of $0.5c$ with another body of equal rest mass kept at rest. Calculate the common speed of the bodies after the collision and the rest mass of the combined body.

- 7 A. Explain the difference between a conserved quantity and an invariant quantity in special theory of relativity (STR).

  B. Show that the Minkowski norm squared ($\sum_{\mu=0}^{3} p^\mu p_\mu$) of the 4-momentum ($p^\mu = (p^0, \vec{p})$) of a particle is a Lorentz invariant.

  C. A relativistic particle of rest mass m_0 is moving with a speed v. Find the value of v at which it's (relativistic) kinetic energy is equal to it's rest mass energy (m_0c^2).

  D. Show that conservation of relativistic energy of a free particle is consistent with the principle of relativity.

  E. Write down Hamiltonian of a system in the non relativistic limit if its relativistic energy-momentum relation is given by $[E - V(r)]^2 = p^2c^2 + m_0^2c^4$.

  F. Comment on whether the relativistic momentum ($\vec{p}$) can be called generalized momentum.

- 8. A particle is kept at the origin. A constant ordinary force $\vec{F}$ stars acting on it at time $t = 0$. Find the speed of the particle at time t.

Refs.: R. Resnick, *Introduction to Special Relativity*; D. J. Griffiths, *Introduction to Electrodynamics*; E. F. Taylor and J. A. Wheelar, *Spacetime Physics*