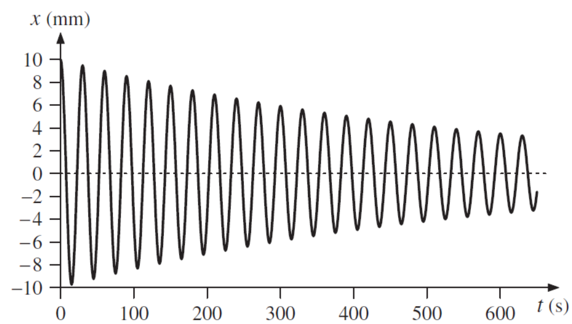
WAVES AND OSCILLATIONS

PROBLEM SET 4 February 02, 2018

1. A spring balance consists of a pan that hangs from a spring. A damping force $F_d = -bv$ is applied to the balance so that when an object is placed in the pan it comes to rest in the minimum time without overshoot. Determine the required value of b for an object of mass 2.5 kg that extends the spring by 6.0 cm. (Assume $g = 9.81\ \mathrm{m\ s^{-2}}$.)

2. A mass of 0.50 kg hangs from the end of a light spring. The system is damped by a light sail attached to the mass so that the ratio of amplitudes of consecutive oscillations is equal to 0.90. It is found that 10 complete oscillations takes 25 s. Obtain a quantitative expression for the damping force and determine the damping factor γ of the system.



3. The figure above shows a graph of displacement x against time t for a damped harmonic oscillator. Deduce the quality factor Q of the oscillator.

4. The energy of a damped harmonic oscillator is observed to reduce by a factor of 2 after 10 complete cycles. By what factor will it reduce after 50 complete cycles?

5. An undamped oscillator has a natural frequency ω_o of π rad s−1. Various amounts of damping are added to the system to give values of the damping factor γ equal to 0.01, 0.30 and 1.0 $\mathrm{s^{-1}}$, respectively. (a) For each value of γ find the corresponding Q-value and frequency ω of the damped oscillations. Comment on the change in ω over this range of γ. (b) For each of the values of Q, plot $x = A_0 \exp(-\gamma\ t/2) \cos \omega t$ over the time period t = 0 to 10 s, using a value of 10 mm for A_0. (c) Obtain an expression for x for the case of critical damping with the initial conditions, x = 10 mm and dx/dt = 0. Plot x over the time-period t = 0 to 10 s.

6. When damping is applied to a simple harmonic oscillator its frequency of oscillation changes from ω_o to a different frequency ω. Show for a very lightly damped harmonic oscillator of quality factor Q that the fractional change in frequency is equal to $1/8Q^2$ to a good approximation.

7. A simple pendulum is constructed from an aluminum sphere attached to a light rod. A second pendulum is constructed of the same length but with a brass sphere. The diameters of the two spheres are the same. The two pendulums are set in motion at the same time with the same amplitude of oscillation. After 10 min the amplitude of oscillation of the aluminum pendulum has

decreased to one-half its initial value. By what factor has the amplitude of oscillation of the brass pendulum decreased at this time? (Assume that the damping force acting on a pendulum is directly proportional to the velocity of the sphere. The densities of aluminum and brass are 2.7×10^3 kg m^{-3} and 8.5×103 kg m^{-3}, respectively.)

8. According to classical electromagnetic theory, an accelerating electron radiates energy at a rate Ke^2a^2/c^3, where a is the acceleration, e is the electronic charge, c is the velocity of light and K is a constant with a value of 6×109 N m^2 C^{-2}. Suppose that the motion of the electron can be represented by the expression $x = A\sin \omega t$ during one cycle of its motion. (a) Show that the energy radiated during one cycle is $Ke^2\pi\omega^3A^2/c^3$. (b) Recalling that the total energy of a harmonic oscillator is $1/2\ (m\omega^2A^2)$ where m is the mass, show that the quality factor Q is $mc^3/Ke^2\omega$. (c) Using a typical value of ω for a visible photon, estimate the 'lifetime' of the radiating system ($e = 1.6 \times 10^{-19}$ C, mass of electron $= 9.1 \times 10^{-31}$ kg).

--*nkv*