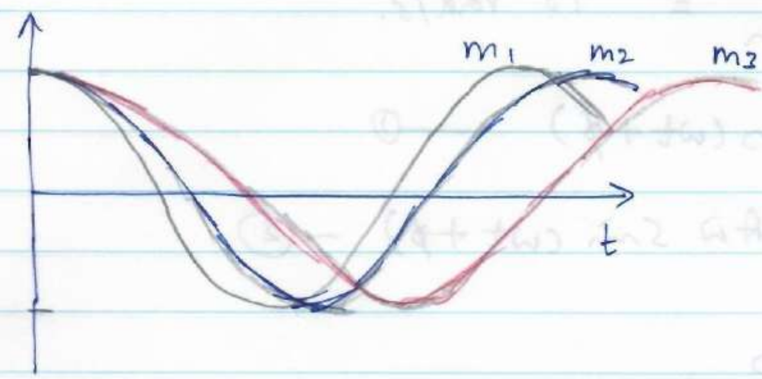
Answers. Minor-1

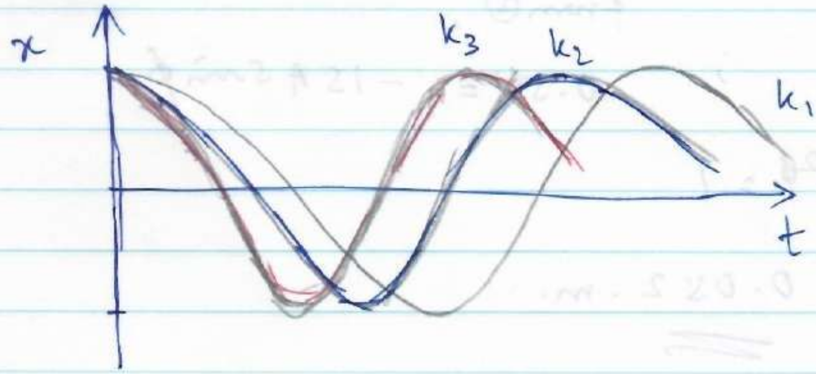
Q-1.

(a)



Increasing 'm' alone increases period of oscillations.

(b)



Increasing 'k' alone decreases the period of osc.

Q-2 a) P.E is 25% of total energy & hence K.E. is 75% of total energy.

b) i) Total energy is quadrupled.

ii) Max. velocity is doubled

iii) Max. acceleration is doubled.

Q.3 $\omega = \sqrt{k/m} = 15$ rad/s.

from $x = A\cos(\omega t + \phi)$ —①

$v = \dot{x} = -A\omega \sin(\omega t + \phi)$ —②

substitute initial values, $t = 0$,

from ① $0.04 = A\cos\phi$; from ②, $0.50 = -15A\sin\phi$

using $\sin^2\phi + \cos^2\phi = 1$

get $A = 0.052$ m.

using this in ① & ②,

$\cos\phi = 0.04/0.052 \longrightarrow \phi = 39.8^\circ$ or 320°

$\sin\phi = -0.50/(15 \times 0.052) \longrightarrow \phi = -39.8$ or 320°

$\because$ ϕ must satisfy both eqns., $\phi = 320^\circ$

(or) $\phi = (\pi/180) \times 320$ rad $= 5.59$ rad.

$\therefore x = 0.052 \cos(15t + 5.59)$ m.

Q.4 Total extension $= 2x$

& tension in the spring $= 2kx$

$\therefore$ eqn. of motion : $m \dfrac{d^2x}{dt^2} = -2kx$

(or) $\underbrace{\dfrac{m}{2}}_{\text{reduced mass.}} \dfrac{d^2x}{dt^2} = -kx.$

using

& $\omega = \sqrt{2k/m}.$

$\nu = \omega/2\pi$ & $m = 1.67\times10^{-27}$ kg.

$\therefore k = 4\pi^2\nu^2\dfrac{m}{2} = \dfrac{4\pi^2\times(1.32\times10^{14})^2\times1.67\times10^{-27}}{2}$

$k = 574$ N/m.

Q.5 From conservation of energy

$\frac{1}{2}mv^2 + U(x) = \text{constant} = U(A).$

a) $\therefore v = \sqrt{2[U(A) - U(x)]/m}.$

b) $v = dx/dt$ or $dt = dx/v$

period $T = \displaystyle\int_{\text{Period}} dt = 2\int_{-A}^{+A} \frac{dx}{v} = 4\int_0^A \frac{dx}{v}$ (for symm. potential)

$= 4\sqrt{\dfrac{m}{2U(A)}} \displaystyle\int_0^A \frac{dx}{\sqrt{1 - U(x)/U(A)}}$

Q.6 $$F = -\frac{dV}{dx} = -\frac{6a}{x^7} + \frac{12b}{x^{13}}$$

at equilibrium, $F = 0 \Rightarrow x_0 = (2b/a)^{1/6}$.

Q.7 For critical damping, $\gamma^2/4 = b^2/4m^2 = \omega_0^2 = k/m$.

$\therefore b = \sqrt{4mk} = \sqrt{4 \times 2.5 \times 600} = 77.5$ kg/s

General soln. for critical damping,

$$x = A \exp(-\gamma t/2) + Bt \exp(-\gamma t/2)$$

$$v = \dot{x} = \exp(-\gamma t/2)\left(B - \gamma Bt/2 - \gamma A/2\right)$$

with initial conditions, $x = 0$ & $v = v_i$ at $t = 0$

gives $A = 0$ & $B = v_i$.

$\therefore x(t) = v_i t \exp(-\gamma t/2)$.

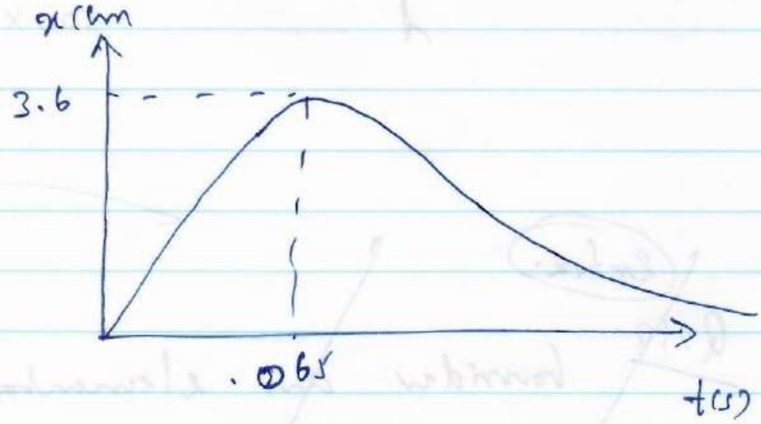
Max. displ. occurs when $dx/dt = 0 \longrightarrow$

$$v_i \exp(-\gamma t/2)(1 - \gamma t/2) = 0.$$

Hence, $t = \frac{2}{\lambda} = \frac{2m}{b} = \frac{2 \times 2.5}{77.5} = 6.5 \times 10^{-2}$ s.

& $x = \frac{2 v_i}{e \lambda} = \frac{2 m v_i}{e b} = \frac{2 \times 2.5 \times 1.5}{e \times 77.5}$

$= 3.6 \times 10^{-2}$ m.



Q.8 Sound intensity $\propto$ energy of osc.

$$E(t) = E_0 \exp(-t/\tau)$$

$$\rightarrow \tau = \frac{t}{\ln[E_0/E(t)]} = \frac{4}{\ln 2} = 5.77 \text{ s.}$$

$$Q = \omega_0/\lambda = \omega_0 \tau = 2\pi \times 330 \times 5.77 = 1.2 \times 10^4$$

$$\frac{\Delta E}{E} = \frac{2\pi}{Q} = 5.3 \times 10^{-4}$$

Q.9 life time $\tau = 1/\lambda$

$\omega_0 = 2\pi c/\lambda$

$$Q = \frac{\omega_0}{\lambda} = \frac{2\pi \times 3\times10^{8} \times 10^{-8}}{500\times10^{-9}} \simeq 4\times10^{7}$$

extra.

Q.10 Consider an elemental length 'dl' of spring at a dist. 'l' from support.

Mass of element $= m\,dl/l_0$ $\quad l_0$ = eq. length of spring

Velocity of the element $= vl/l_0$

$\therefore$ K.E of the spring $= \dfrac{1}{2}\dfrac{mv^2}{l_0^3}\displaystyle\int_0^{l_0} l^2\,dl = \dfrac{1}{6}mv^2$

extra.

Q. 10 At equilibrium each spring exerts tension

$T_0 = k(a - a_0)$.

In config shown in Fig (b), each spring has a length l and tension $T = (l - a_0)$, exerted along dir. CA or CB. y-component of the force is $-T\sin\theta$. & each spring ~~exert~~ contributes a return force $T\sin\theta$ in $-$ve y-dir.

eqn. of motion: $m\ddot{y} = -2T\sin\theta$

$$= -2k(l - a_0)\frac{y}{l}$$

x-components of force due to 2 springs balance each other $\Rightarrow$ no motion along x-dir.

$$\therefore m\ddot{y} = -2ky\left[1 - \frac{a_0}{\sqrt{a^2+y^2}}\right]$$

Small osc. appr. $y \ll a$,

$$\therefore \frac{a_0}{\sqrt{a^2+y^2}} \simeq \frac{a}{a_0}\left[1 - \frac{y^2}{2a^2}\right]$$

$$\therefore\ m\ddot{y} = -2ky\left[1 - \frac{a_0}{a} + \frac{a_0 y^2}{2a^3}\right]$$

neglect $(y/a)^3$ term,

$$\ddot{y} = \frac{2ky}{ma}(a-a_0) = -\frac{2T_0}{ma}y$$

$$\therefore\ \omega_{Tr}^2 = \frac{2T_0}{ma} = \frac{2k}{ma}(a-a_0) = \frac{2k}{m}\left(1-\frac{a_0}{a}\right)$$

Period of osc. $= \dfrac{2\pi\sqrt{m/2k}}{\sqrt{1-a_0/a}} = 2\pi\sqrt{ma/2T_0}$

1 (a) $T = 2\pi\sqrt{\frac{m}{k}}$ (a) 

(b) 

(1+1) marks

2 (a) $PE = \frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{a}{2}\right)^2 = \frac{1}{4}(E) = 25\%$ ∴ $KE = 75\%$

(b) (i) $\frac{1}{2}ka^2 \rightarrow \frac{1}{2}k(2a)^2 = 4E$ (0.4 × 5) marks

(ii) $\frac{1}{2}mv_1^2 = \frac{1}{2}ka^2 = E \Rightarrow v_1 = \sqrt{\frac{2E}{m}}$

$\frac{1}{2}mv_2^2 = 4E \Rightarrow v_2 = 2\sqrt{\frac{2E}{m}}$ ∴ $v_2 = 2v_1$

(iii) $F = -kx$ $F_{max1} = -ka$ $F_{max2} = -2ka$ ∴ $a_2 = 2a_1$

$T = 2\pi\sqrt{\frac{m}{k}}$ → not changed

3 $\omega = \sqrt{\frac{k}{m}}$ → find $\omega = \sqrt{\frac{180}{0.8}}$ rad s^{-1} = 15 rad/s

$\frac{1}{2}kx^2 + \frac{1}{2}mv^2 = E = \frac{1}{2}kA^2$ → find A ∴ $\omega^2x^2 + v^2 = \omega^2A^2 \Rightarrow A = \sqrt{x^2 + \frac{v^2}{\omega^2}}$
$= 0.052$ m

$x = A\cos(\omega t + \phi)$ → find ϕ.
$v = -A\omega\sin(\omega t + \phi)$ (3 marks)

$0.04 = 0.052\cos\phi \Rightarrow \phi = \pm 39.7°$

$0.5 = -0.052 \times 15 \times \sin\phi \Rightarrow \phi = -39.8° = -0.698$ rad
≈ -0.7 rad
or 5.59 rad

4 
$\omega = \sqrt{\frac{2k}{m}} \Rightarrow k = \frac{1}{2}m\omega^2$ (2 marks)
$= \frac{1}{2} \times 1.67 \times 10^{-27} \times 4\pi^2 \times 1.32^2 \times 10^{28}$ N m^{-1}
$= 574$ N m^{-1}

5 (a) $\frac{1}{2}mv^2 + U(x) = E \Rightarrow v = \sqrt{\frac{2}{m}(E - U(x))}$ (1+1) marks

(b) $T = \oint dt = \oint \frac{dx}{v} = 2\int_{-A}^{A}\frac{dx}{v} = 4\int_{0}^{A}\frac{dx}{v} = 4\int_{0}^{A}\frac{dx}{\sqrt{\frac{2}{m}(E - U(x))}}$

6 $U(x) = -\frac{a}{x^6} + \frac{b}{x^{12}}$ $F = -\frac{\partial U}{\partial x} = -\left((-a)\frac{(-6)}{x^7} + (b)\frac{(-12)}{x^{13}}\right) = -\frac{6a}{x^7} + \frac{12b}{x^{13}}$ (1+1) marks

$F = 0$ at $x = x_0$ ∴ $-6a + \frac{12b}{x_0^6} = 0 \Rightarrow x_0^6 = \frac{12b}{6a}$ ∴ $x_0 = \sqrt[6]{\frac{2b}{a}}$

7
$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$
$\Rightarrow \frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \omega_0^2x = 0$
$\Rightarrow \frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega_0^2x = 0$

$\frac{d^2x}{dt^2} + 2\left(\frac{\gamma}{2}\right)\frac{dx}{dt} + \omega_0^2x = 0$
∴ $\alpha = -\frac{\gamma}{2} \pm \sqrt{\frac{\gamma^2}{4} - \omega_0^2}$
$x_{critical} = (A + Bt)e^{-\frac{\gamma}{2}t}$ $b = \gamma m$

$b = \gamma m = 2\omega_0 m = 2\sqrt{\frac{k}{m}}\, m = \sqrt{4\frac{k}{m}m^2} = \sqrt{4km}$

$= \sqrt{4\times 2.5\times 600} = \sqrt{6000} = \sqrt{100\times 60} = 10\sqrt{4\times 15} = 20\sqrt{15}$

$= 77.5$ kg/s $\qquad \frac{MLT^{-2}}{LT^{-1}} = MT^{-1}$

$x = e^{-\frac{\gamma t}{2}}(A+Bt)$

$\dot{x} = Be^{-\frac{\gamma t}{2}} + (A+Bt)\left(-\frac{\gamma}{2}\right)e^{-\frac{\gamma t}{2}}$

$= e^{-\frac{\gamma t}{2}}\left[B - \frac{A\gamma}{2} - \frac{B\gamma t}{2}\right]$

$t=0 \to x=0,\ V=v_0$

$\therefore V_0 = B - \frac{A\gamma}{2}$

$0 = A \Rightarrow A=0$

$\therefore B = v_0$

$\therefore x = v_0 t e^{-\frac{\gamma t}{2}}$ $\qquad$ (1+2 marks)

$\dot{x} = v_0 e^{-\frac{\gamma t}{2}}\left[1 - \frac{\gamma t}{2}\right]$

for max x, $\dot{x}=0 \Rightarrow 1-\frac{\gamma t}{2}=0 \Rightarrow t = \frac{2}{\gamma} = \frac{2m}{b} = \frac{2\times 2.5}{77.5} = 0.065$ s

$x = v_0 \frac{2}{\gamma} e^{-\frac{\gamma}{2}\frac{2}{\gamma}} = \frac{2v_0}{e\gamma} = \frac{1.5\times 0.065}{e} = 0.036$ m

8 (i) $E(t) = E_0 e^{-t/\tau} \Rightarrow \ln\left(\frac{E_0}{E}\right) = \frac{t}{\tau} \Rightarrow \tau = \frac{t}{\ln(E_0/E)} = \frac{4}{\ln 2} = 5.77$ s

(ii) $Q = \omega_0\tau = 2\pi\times 330\times 5.77 = 11964 \approx 1.2\times 10^4$

(iii) $\frac{\Delta E}{E} = \frac{2\pi}{Q} = 5.25\times 10^{-4}$ $\qquad$ (1+1+1 marks)

9 $Q = \omega_0\tau = \frac{2\pi c}{\lambda}\tau = \frac{2\pi\times 3\times 10^8\times 10^{-8}}{500\times 10^{-9}} = \frac{6\pi}{5}\times 10^7 = 3.77\times 10^7$ $\qquad$ (1 mark)

10 $F_y = -2k(l-a_0)\sin\theta = -2k\frac{l-a_0}{l}y$

$= -2k\left(1-\frac{a_0}{l}\right)y = -2k\left(1-\frac{a_0}{\sqrt{a^2+y^2}}\right)y$

$= -2k\left[1-\frac{a_0}{a}\left(1+\frac{y^2}{a^2}\right)^{-\frac{1}{2}}\right]y = -2k\left[1-\frac{a_0}{a}\left(1-\frac{y^2}{2a^2}\right)\right]y$

$= -2k\left(1-\frac{a_0}{a}\right)y$

$\Rightarrow m\ddot{y} = -2k\left(1-\frac{a_0}{a}\right)y$

$\Rightarrow \ddot{y} = -\left(\frac{2k(a-a_0)}{ma}\right)y = -\omega^2 y \qquad \therefore \omega = \sqrt{\frac{2k}{m}\left(1-\frac{a_0}{a}\right)}$

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{ma}{2k(a-a_0)}}$ $\qquad$ (4+1 marks)

Anirban Debnath