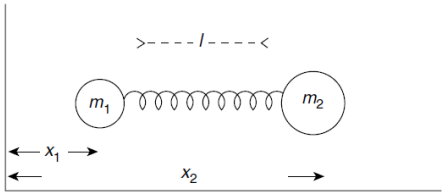
WAVES AND OSCILLATIONS

PROBLEM SET 6 March 9, 2018

1. Figures 1 and 2 (given below) shows how the pendulum configurations x = 2a; y = 0 and x = 0; y = 2a result from the superposition of the normal modes X and Y. Using the initial conditions x = 2a; y = 0; $\dot{x} = \dot{y} = 0$ draw similar sketches to show how X and Y superpose to produce x = -2a; y = 0 and x = 0; y = -2a.

2. In the figure given two masses m_1 and m_2 are coupled by a spring of stiffness k and natural length l. If x is the extension of the spring show that equations of motion along the x axis are $m_1\ddot{x}_1 = kx$ and $m_2\ddot{x}_2 = -kx$, and combine these to show that the system oscillates with a frequency $\omega^2 = \frac{k}{\mu}$, where $\mu = \frac{m_1 m_2}{m_1 + m_2}$ is the reduced mass.



Let the figure represent a diatomic molecule as a harmonic oscillator with an effective mass equal to its reduced mass. If a sodium chloride molecule has a natural vibration frequency = 1.14 x 10^{13} Hz (in the infrared region of the electromagnetic spectrum) show that the interatomic force constant k = 120 Nm^{-1}. (Note: this simple model gives a higher value for s than more refined methods which account for other interactions within the salt crystal lattice)
Mass of Na atom = 23 a.m.u.; Mass of Cl atom = 35 a.m.u.; 1 a.m.u. = 1.67x10^{-27} kg

3. The steady state displacement of a particle acted on by a force $F\cos\omega t$ is $x = a\cos\omega t + b\sin\omega t$. Calculate the average power. Hence interpret the amplitudes a and b physically.
4. Consider a symmetrical linear triatomic molecule such as carbon dioxide, which has the chemical structure O-C-O. Find the normal mode frequencies of the longitudinal and transverse vibrations of the molecule considering only the nearest neighbor interactions.
5. The equation of motion of two coupled oscillators are $\ddot{q}_1 + \sqrt{2}q_2 + 4q_1 = 0$ and $\ddot{q}_2 + \sqrt{2}q_1 + 5q_2 = 0$. Find the normal mode frequencies and the ratio of amplitudes of the normal modes.
6. A set of coupled pendula has m = 0.10 kg for both masses, but k and l are unknown. At t = 0, the left mass is held at x_1 = 2.0 cm and the right mass is held at x_2 = 0 cm. The masses are then released. At first, the left mass oscillates with a period of about 1.1 sec, and the right mass is motionless. At t = 10 sec, the two masses are oscillating with approximately equal amplitudes. At t = 20 sec, the right mass is oscillating strongly and the left mass is nearly motionless. At t = 30 sec, the two masses are oscillating with approximately equal amplitude. At t = 40 sec, the left mass is oscillating strongly and the right mass is nearly motionless. Find approximate values of k and l.

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