# STyLuS*: A Temporal Logic Optimal Control Synthesis Tool for Large-Scale Multi-Robot Systems

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## I. Setting Up STyLuS*

### A. Dependencies

STyLuS* has been tested on Windows 7 on MATLab R2016b and MATLab R2018. To use STyLuS*, the ltl2ba toolbox needs to be installed that is available online on http://www.lsv.fr/~gastin/ltl2ba/. Once the executable file of the ltl2ba is generated (follow the instructions on previously mentioned link), it should be placed in the folder `functions`. Executable files generated on Windows 7 (ltl2baWin7) and macOS HighSierra (ltl2ba) are already included there. The user is encouraged to generate new executable files to avoid any inconsistencies. The ltl2ba toolbox is used in the function `create_buchi` to translate LTL formulas into Non-deterministic Bucchi automata (NBA). Note that the current MATLAB folder should be `.../functions/` (i.e., where the executable file is located), otherwise `create_buchi` will not be able to generate a NBA.[1]

### B. Inputs

The user-specified inputs to STyLuS* are determined in the function `DefineInputs.m`:

- a weighted Transition System (TS) modeling robot mobility; see also Figs. 1-2. STyLuS has currently been implemented assuming that all robots have the same model, i.e., the same TS. A TS is represented by a structure `T`. The user needs to define the (i) number of TS states (`numStates`) and the TS state-space defined as $\texttt{T.Q} = [1 : \texttt{numStates})]$, (ii) the physical locations of the TS states, i.e., the x (vector `T.x` with dimensions $\texttt{numStates} \times 1$) and y (vector `T.y` with dimensions $\texttt{numStates} \times 1$) coordinates of each state, and (iii) the adjacency matrix of the TS ( `T.adj` which is a $\texttt{numStates} \times \texttt{numStates}$ matrix) where the entry $\texttt{T.adj}(i, j)$ is 0 if a direct transition from state $i$ to state $j$ is infeasible and 1 otherwise. By default, STyLuS* generates these three inputs randomly using the command `T=createFTS(numOfStates,degree/2)`. If a user-specified TS is needed, the command `T=createFTS(numOfStates,degree/2)` should be commented out. The field `T.Dist` is a $\texttt{numStates} \times \texttt{numStates}$ capturing the cost for each TS transition. By default, this field is generated automatically using the command `T.Dist = DistanceMatrixA(T)`. As per this command, `T.Dist` is defined so that it captures the Euclidean distance from the TS state $i$ to the TS state $j$ if it holds $\texttt{T.adj}(i, j) = 1$; for all other transitions for which $\texttt{T.adj}(i, j) = 0$, it holds that $\texttt{T.Dist}(i, j) = \texttt{Inf}$. The user can comment out the command `T.Dist = DistanceMatrixA(T)` and assign manually new weights to each transition.

```
%% ---------------------Define transition system---------------------------
numOfStates    = 1000;                          % number of states in the TS
degree         = 28;                            % average degree per node in the TS
T              = createFTS(numOfStates,degree/2); % generates a graph-based representation of a random transition system with <numOfStates>
                                                % number of states and average degree per node <degree>
T              = adjMatrix2adjList(T);          % adjacency list of the TS
T.Dist         = DistanceMatrixA(T);            % weight/cost matrix for each transition in the TS; selected to be the Euclidean distance between two TS states
```

Fig. 1. A weighted Transition System as an input to STyLuS*.

```
T =

  struct with fields:

          Q: [1×1000 double]
          x: [1000×1 double]
          y: [1000×1 double]
        adj: [1000×1000 double]
    adjList: [1000×1000 double]
       Dist: [1000×1000 double]
```

Fig. 2. Structure of a weighted Transition System as an input to STyLuS*.

- number `N` of robots and their initial state `x0`; see also Fig. 3. `x0` is an $\texttt{N} \times 1$ vector where the $i$-th entry captures the initial state of robot $i$ and has to be an integer within the interval $[1, \texttt{numStates}]$.

[1] STyLuS* has also been tested on macOS HighSierra. We have noticed that the ltl2ba toolbox often generates incorrect NBA when executed on macOS; STyLuS* generates correct paths with respect to the NBA on macOS as well

```
%% ----------------------------Team of robots-------------------------------
N             = 100;                              % number of robots
x0            = (1:100);                          % initial states of robots within their TS
%  -------------------------------------------------------------------------
```

Fig. 3. Initialization of a team of $N$ robots.

- an LTL mission captured in the string $phi$; see also Fig. 4. To define the LTL formula $phi$, the following notation is used: $G, F, U, X$ stand for the 'always', 'eventually', 'until', and 'next' operator, and $\&, |, !$ stand for the logical 'and', 'or', and 'not' operators. The number of atomic predicates $p$ that appear in $phi$ needs to be defined as well (see `N_p`). For instance, consider the LTL task

$$phi = F(p1)\ \&\ G(!p2) \tag{1}$$

which requires that eventually $p1$ to be satisfied and that $p2$ should never be satisfied. In (1), we have that N_p $= 2$. The definition of where the predicates (e.g., $p1$ and $p2$) in $phi$ are satisfied is captured by the cell array $AP$; see also Fig. 5. Let $pk$ denote the $k$-th predicate which is captured by $AP(k, :)$. Specifically, $AP(k, r)$ is an empty cell if robot $r$ does not play any role in satisfying $pk$ and a non-empty cell array otherwise. If it is a non-empty cell, then $AP(k, r)$ is defined as $AP(k, r) = \{[\text{state } 1, \text{state } 2, \ldots, \text{state m}]\}$ collecting all states that if at least one of them is visited by robot $r$ then $AP(k, r)$ is true. Note that this definition allows for more than one robot to contribute to satisfaction of $pk$. For instance, if $pk$ is defined to be true if either robot $r1$ or robot $r2$, or robot $r3$ are in state $m$, then $pk$ is defined as follows $AP(k, r1) = \{m\}$, $AP(k, r2) = \{m\}$, $AP(k, r3) = \{m\}$. Also, to capture the logical 'or' in the definition of $pk$, we set $AP(k, N + 3) = \{1\}$. Otherwise, if $AP(k, N + 3) = \{0\}$, it means that all three robots have to be in state $m$ so that $pk$ is true.

```
%% --------------------------Buchi Automaton-----------------------------
phi           = 'G(p1-> X(!p1 U p2)) & GF(p1) & GF(p3) & GF(p4) & (!p1 U p5) & G(!p6) & F(p7 | p8) & GF (p5)'; % LTL formula
N_p           = 8;                                      % number of atomic prositions that appear in phi
alphabet      = alphabet_set(obtainAlphabet(N_p));      % Generating the powerset (alphabet)
                                                        % (e.g., if N_p=3 then: alphabet = p1, p2, p3, p1p2, p1p3, p2p3, p1p2p3, {empty word} )
disp('Translation of the LTL formula into a NBA has started')
tic
B1            = create_buchi(phi,alphabet);
time = toc;
textOut = ['The LTL formula was successfully translated into a NBA after ', num2str(time), 'secs'];
disp(textOut)
```

Fig. 4. An LTL formula as an input to STyLuS*.

```
AP(1,4)       = {[720,340]}; %robot 4 has to be in either region 720 or 340 (boolean formula: b_1^1)
AP(1,3)       = {[110,330]}; %robot 3 has to be in either region 110 or 330 (boolean formula: b_2^1)
AP(1,1)       = {[110,980]}; %robot 1 has to be in either region 110 or 980 (boolean formula: b_3^1)
AP(1,2)       = {210};       %robot 2 has to be in region 210 (boolean formula: b_4^1) ...
AP(1,5)       = {172};       %...
AP(1,6)       = {10};
AP(1,7)       = {107};
AP(1,8)       = {501};
AP(1,9)       = {104};
AP(1,10)      = {71};
AP(1,11)      = {[900, 800]};
AP(1,12)      = {11};
```

Fig. 5. Determining when $p1$ is satisfied within STyLuS*.

- Termination criterion (`Termination`); see also Fig. 6. The user needs to define if STyLuS* should terminate once the first feasible solution is found (`Termination.MaxIter=0`) or after a maximum number of iterations (`Termination.MaxIter=1`). In the later case, the user needs to determine the maximum number of iterations for the construction of the prefix (`Termination.nMaxPre`) and suffix (`Termination.nMaxSuf`) plan.
- a parameter `w` $\in [0, 1]$ required to determine the cost of a prefix suffix plan; see also Fig. 7. In particular the cost of a prefix-suffix plan is $(1 - w)J(\texttt{prefix}) + wJ(\texttt{suffix})$, where the cost of the prefix and suffix (i.e., $J(\texttt{prefix})$ and $J(\texttt{suffix})$) is determined using the weights in `T.Dist`.

### C. Outputs

The output of STyLuS* is generated by the function `MainSTyLuS_star` and includes the following:

```
%% ======================== Termination Criterion ========================
Termination.MaxIter = 0; % if the tree construction has to stop after maximum number of iterations, then set it to 1.
% If the tree construction should stop once the first feasible (prefix/suffix) path is detected, then set it to 0
Termination.nMaxPre = 5000; % maximum number of iterations for the construction of the prefix tree
Termination.nMaxSuf = 5000; % maximum number of iterations for the construction of the suffix trees
% ========================================================================
```

Fig. 6. Determining the termination criterion of STyLuS*.

```
%% =========================== Cost Function ============================
w = 0.9;                    % cost of prefix-suffix plan is (1-w)* costPrefix + w*(costSuffix)
% ======================================================================
```

Fig. 7. Determining the cost-related parameter $w$ within STyLuS*.

- the best found prefix-suffix plan (`BestPrefix`, `BestSuffix`). The prefix part is defined as an array with dimensions $H \times N$ where $H$ captures the number or waypoints (or in other words TS states) that the multi-robot system has to visit. To get the full prefix path, type in the command window `BestPrefix`. To get $k$-th waypoint of robot $j$, type in the command window `BestPrefix(k,j)`. The same holds for the corresponding suffix part `BestSuffix`.
- a cell array containing all the detected prefix parts (`ListOfPrefix`) including the best one. To get the $m$-th prefix part, type in the command window: `ListOfPrefixm` . To get $k$-th waypoint of robot $j$ type in the $m$-th prefix part, type in the command window `ListOfPrefixm(k,j)` . The same holds for the cell array containing all the detected suffix parts (`ListOfSuffix`)
- a vector (`wrongPre`) containing the indices of the detected prefix parts that did not pass sanity checks (e.g., a prefix path has to respect the transition rule of the TS). If any prefix part does not pass the sanity check a warning message will appear on the command window and the index to these prefix parts will be included in this vector. A corresponding vector for the suffix parts is also returned (`wrongSuf`).

### D. Execution

To execute STyLuS*, first add the folder where `STyLuS_star.m` is saved to the search folder by typing in the command window `addpath("path")`, where `path` is the path to `STyLuS_star.m`. Second, type `STyLuS_star` in the MATLab command window. The output of STyLuS* for a case study with $N = 100$ robots where each robot is modeled as TS with 1000 states and collaborative LTL formula corresponding to an NBA with 21 states is shown in Fig. 8. This is the default input case study for STyLuS*.

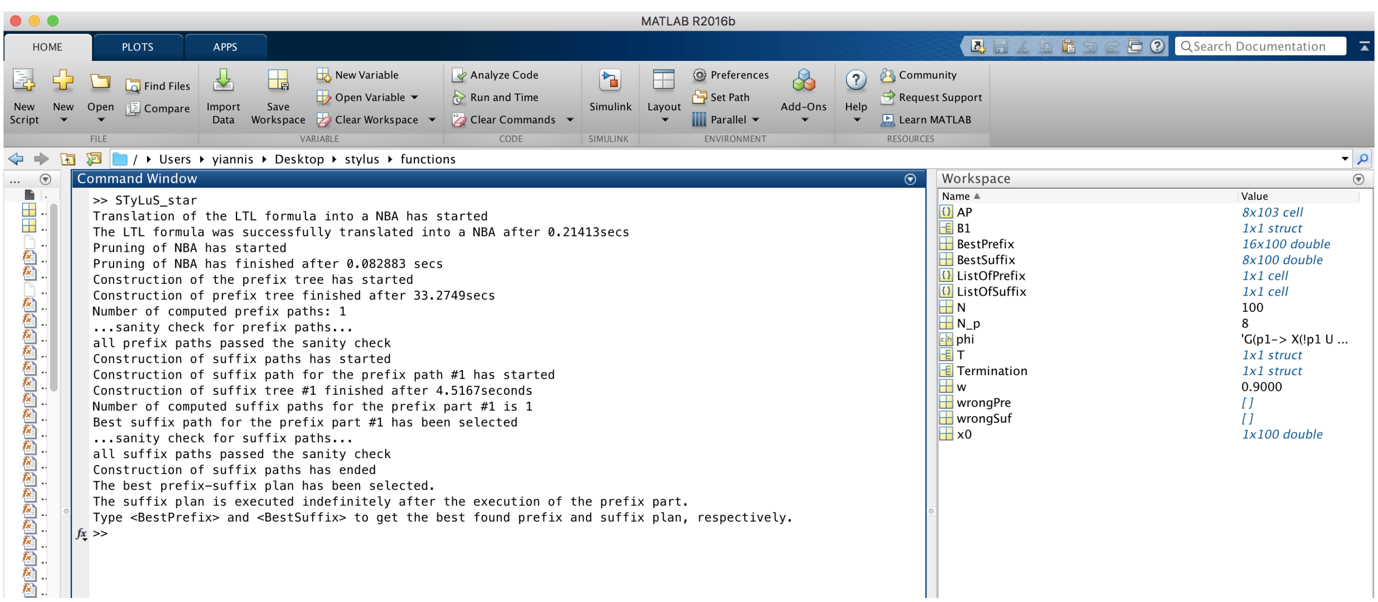


Fig. 8. Output of STyLuS* for a case study with (i) $N = 100$ robots, (ii) TS with 1000 states, and (iii) an LTL formula corresponding to an NBA with 21 states.

## II. Example: Setting Up a Single-Agent Reachability Avoidance Task

In this section, we discuss how STyLuS* can be configured to set up a simple single-agent reachability avoidance task. Here, we consider a randomly generated transition system with $1000$ states as discussed in the previous section. Also, we assume that the initial state of the robot is $10 \in [1, 1000]$. Thus, we define `N=1`, and $\texttt{x0} = 10$; see also Fig. 9.

```
%% ---------------------------Team of robots-------------------------------
N              = 1;                              % number of robots
x0             = 10;                          % initial states of robots within their TS
%  -----------------------------------------------------------------------
```

Fig. 9. Initializing the single-robot system.

The robot mission requires the robot to eventually reach either the state/region $720$ or $340$ and always avoid the regions $191, 20, 1000, 500, 600$. This mission can be defined as an LTL formula that has the same structure as in (1). This mission is given as an input to STyLuS* as shown in Fig. 10 where $p1$ and $p2$ are defined as in Fig. 11.

```
%% --------------------------Buchi Automaton----------------------------
phi           = 'F (p1) & G (! p2)'; % LTL formula
N_p           = 2;                                    % number of atomic prositions that appear in phi
alphabet      = alphabet_set(obtainAlphabet(N_p));          % Generating the powerset (alphabet)
                                                      % (e.g., if N_p=3 then: alphabet = p1, p2, p3, p1p2, p1p3, p2p3, p1p2p3, {empty word} )
disp('Translation of the LTL formula into a NBA has started')
tic
B1            = create_buchi(phi,alphabet);
time = toc;
textOut = ['The LTL formula was successfully translated into a NBA after ', num2str(time), 'secs'];
disp(textOut)
%numOfBuchiEdges =length(find(~cellfun(@isempty,B1.trans))); %number of edges in the NBA B1

% ----------------------------------------------------------------------
```

Fig. 10. Specifying a reachability-avoidance task as an LTL formula as an input to STyLuS*.

```
%% ---------------------------Atomic Propositions------------------------
AP            = cell(N_p,N+2);
for i = 1 : N_p
    AP(i,N+1) = {2^(i-1)}; % atomic proposition p_i corresponds to the 2^(i-1) entry in "alphabet"
end

%pi_k (or xi__k in the IJRR paper) is defined as either the conjuction (AND) or
%disjuction (OR) of Boolean formulas b_i^k. Specifically, it is AND if
%AP(k,N+3)={0} and it is OR if AP(k,N+3)={0}.

%atomic proposition p1 (\xi_1 in the paper)
AP(1,1)       = {[720,340]}; %robot 1 has to be in either region 720 or 340 (boolean formula: b_1^1)


%atomic proposition p2 (\xi_2 in the paper)
AP(2,1) = {[191, 20, 1000, 500, 600]}; % p2 is satisfied if robot 1 is in one the states included in [191, 20, 1000, 500, 600]

AP = findRobots(AP, N, N_p); % computes which robots are involved in \xi_k for all k (see: AP{k,N+2})

%if 0=> "AND". If "1"=> OR
AP(1,N+3) = {0}; AP(2,N+3) = {0};
%-------------------------------------------------------------------------
```

Fig. 11. Defining variables $p1$ and $p2$ for the reachability-avoidance task (1).

To run STyLuS* for this case study, open `STyLuS_star.m` (type `open STyLuS_star.m`), replace `DefineInputs()` with `DefineInputs4ReachabilityAvoidance()`, save the changes, and then run STyLuS* as discussed in the previous section. The output of STyLuS* for this case study is shown in Fig. 12.

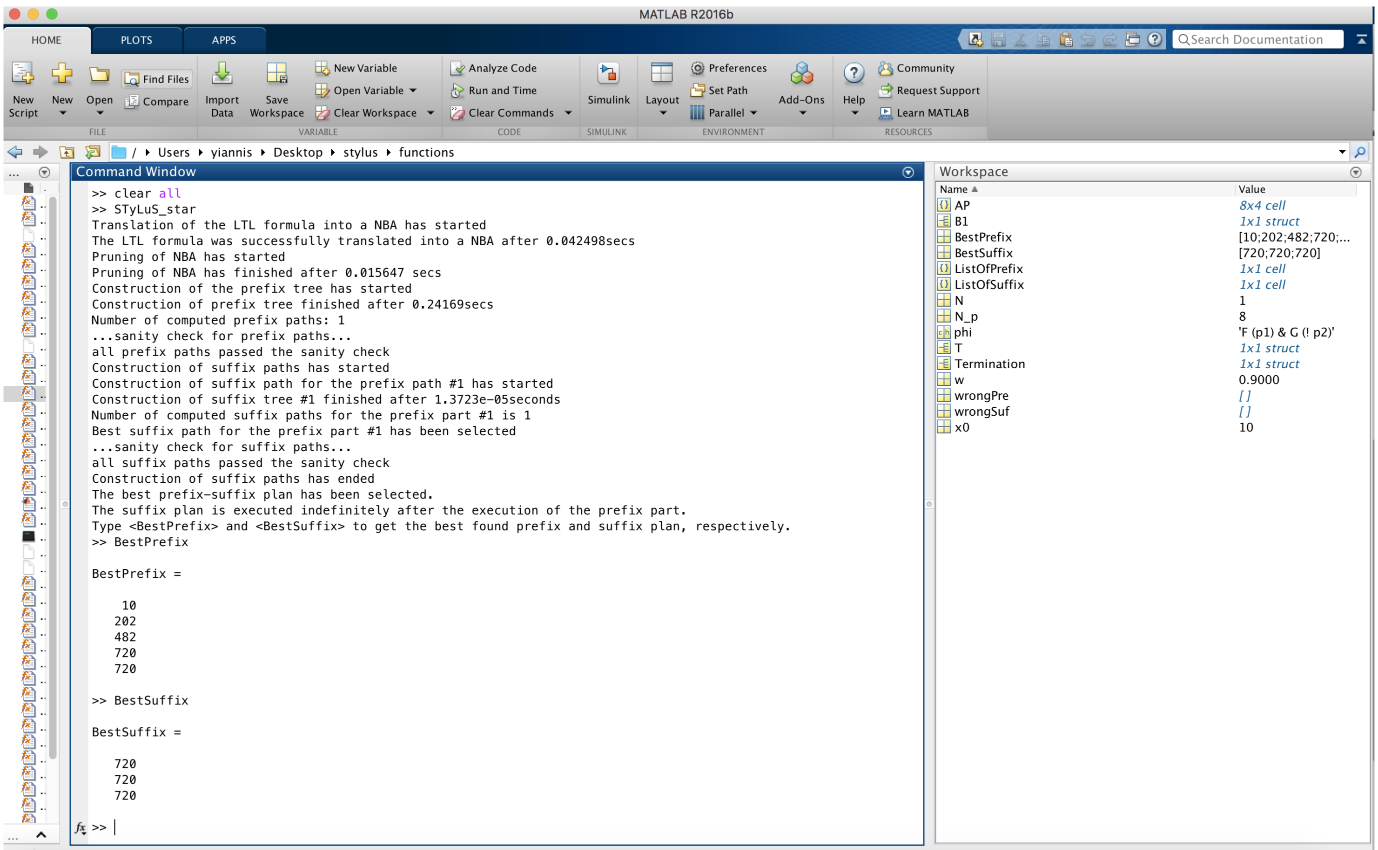


Fig. 12. Output of STyLuS$^*$ for a case study with (i) $N = 1$ robots, (ii) TS with 1000 states, and (iii) an LTL formula corresponding to the task (1).