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Senarmont compensator for elliptically birefringent media

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Abstract

The application of the Senarmont compensator setup for measuring of elliptically birefringent media properties is presented. The measurement procedure is carried out in two steps. In the first one, the medium is treated as a linearly birefringent, while, after a little modification of the measurement setup, is treated as a circularly one. Formulae for the phase shift introduced by the medium and the ellipticity angle of the first eigenvector of the medium's Mueller matrix are presented, also in the case when the quarterwave plate does not introduce the phase shift that is equal exactly 90°. © 2001 Published by Elsevier Science B.V.

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1. Introduction

There are many methods of determining the optical birefringence media properties. The most popular are ellipsometric [1–3], polarimetric [4–10] and interferometric [11] ones. The Senarmont setup [12], being of a polarimetric type, is one of the azimuthal compensators, which are presented in Refs. [13–15]. The idea of this compensation is as follows: a quarterwave plate (one of the elements of the compensator) transforms the ellipticity angle of the light passing through the linearly birefringent medium into the azimuth angle change of the light, which can be simply measured by rotation of the analyzer. Due to specific orientation of the medium in the setup the ellipticity angle above mentioned is equal to the phase shift introduced by the medium. Most of optical media are linearly birefringent, so the reduction of applying the Senarmont method only to these ones is not a major disadvantage of this method. In this paper, a simple modification of this method allowing the phase difference measurement in elliptically birefringent media is presented. It enables the measurement of not only the phase difference but also the ellipticity angle of the first eigenvector of the medium's Mueller matrix. Measurements of these properties are made in two steps. Generally speaking, in the first step, a medium is treated as linearly birefringent, while, after a little modification of the measurement setup, it is treated as a circularly one. The description of these measurements and simple formulae for calculation of desired parameters of the medium are presented in Section 2.

The quarterwave plate is one of the elements of the Senarmont setup. It can be a weak point of

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measurements. The incorrectness of the quarterwave plate's realization is typically <9°. An additional deviation can take place when using the plate in the measurement setup with a source of the wave with the wavelength different from the nominal one for which the phase difference is equal to 90°. The way of avoiding this problem is presented in Section 3.

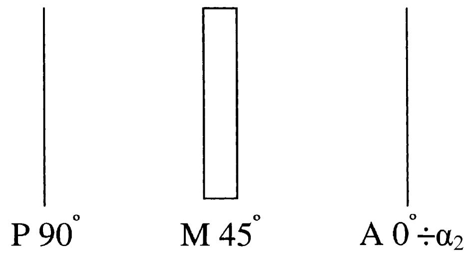

Fig. 2. Scheme of the measurement setup—the second step.

2. The measurement setup

The procedure of determining the polarization properties of the medium, such as the ellipticity angle ϑ_f of the first eigenvector of the medium's Mueller matrix and the phase shift γ introduced by the medium, is made in two steps.

The measurement setup for the first step is presented in Fig. 1. The polarizer P and the analyzer A are crossed and the medium M is placed between them with the azimuth angle 45° with regard to the azimuths of the polarizer and the analyzer, respectively. It is possible to do it in a simple way because the intensity I of the light at the output of the system takes the maximum value for the above mentioned angle, even if the medium is elliptically birefringent (see Appendix A). Next, the quarterwave plate, which generally introduces the phase shift γ_p, is placed between the medium and the analyzer, with the same azimuth as the one of the polarizer. Then, as in the classical Senarmont compensator method, the analyzer is rotated to the position for which the intensity I of the light takes minimum value. It is easy to show that the intensity I depends on the azimuth angle α_1 of the analyzer as (see Appendix A)

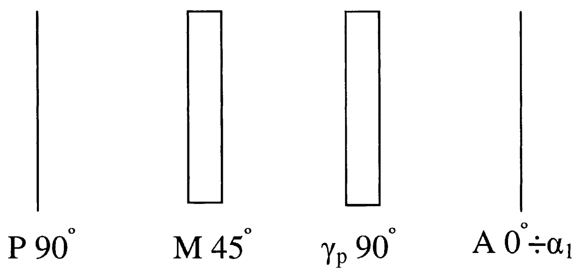

Fig. 1. Scheme of the measurement setup—the first step.

$$I \propto 1 - \cos 2\alpha_1 \cos\gamma + \sin 2\alpha_1 \sin\gamma \cos 2\vartheta_f. \tag{1}$$

Thus the change of the analyzer's position $\Delta\alpha_1$ from its initial value into the position for which the intensity I of the light takes minimum value could be simply described as

$$\mathrm{tg}\, 2\Delta\alpha_1 = \cos 2\vartheta_f \,\mathrm{tg}\,\gamma. \tag{2}$$

In the second step, the quarterwave plate is simply removed from the system (see Fig. 2) and the analyzer is rotated to find the position for which the intensity I takes minimum value, as in the first step. This is, of course, a typical setup for the measurement of the optical activity of media. Similarly to Eq. (1) the intensity I depends on the azimuth angle α_2 of the analyzer as described below:

$$I \propto 1 - \cos 2\alpha_2 \cos\gamma + \sin 2\alpha_2 \sin\gamma \sin 2\vartheta_f. \tag{3}$$

So the change of the analyzer's position $\Delta\alpha_2$ into the position for which the intensity I of the light takes minimum value is described by

$$\mathrm{tg}\, 2\Delta\alpha_2 = \sin 2\vartheta_f \,\mathrm{tg}\,\gamma. \tag{4}$$

The desired formulae for the ellipticity angle ϑ_f of the first eigenvector of the medium's Mueller matrix and the phase shift γ introduced by the medium follow immediately from Eqs. (2) and (4):

$$\mathrm{tg}\, 2\vartheta_f = \frac{\mathrm{tg}\, 2\Delta\alpha_2}{\mathrm{tg}\, 2\Delta\alpha_1}, \tag{5}$$

$$\mathrm{tg}^2\gamma = \mathrm{tg}^2\, 2\Delta\alpha_1 + \mathrm{tg}^2\, 2\Delta\alpha_2. \tag{6}$$

For pure linearly or circularly birefringent media Eq. (6) reduces to a well known formula

$$\gamma = 2\Delta\alpha, \tag{7}$$

applied for the first step of the presented procedure if the medium is linearly birefringent, and for the second one if the medium is circularly birefringent.

3. The setup with imperfect quarterwave plate

Sometimes, the quarterwave plate does not introduce the phase shift that is equal to 90°. It can be unintentional and follows from improper workmanship of the phase plate or intentional if we want to measure spectral properties of the medium and we do not have adjusted quarterwave plate. For example when in the classical Senarmont setup the phase plate with $\gamma = 45°$ is measured using the quarterwave plate with $\gamma_p = 60°$, the measured value of the phase shift is 49°. It is possible to apply the two-step method with a slight modification taking into account the real value γ_p of the quarterwave plate or/and its spectral properties. More detailed calculation of the intensity I of the light at the output of the setup presented in Fig. 1 is shown in Appendix A. It follows from this analysis that for the first step

$$\operatorname{tg} 2\Delta\alpha_1 = \sin(\gamma_p - 2\vartheta_f)\operatorname{tg}\gamma. \tag{8}$$

Let us note that Eq. (8) reduces to Eq. (2) when $\gamma_p = 90°$.

For the second step the expression for $\Delta\alpha_2$ is the same as in the Section 2:

$$\operatorname{tg} 2\Delta\alpha_2 = \sin 2\vartheta_f \operatorname{tg}\gamma. \tag{9}$$

This is a system of the two equations with two unknown values ϑ_f and γ with two measured values $\Delta\alpha_1$ and $\Delta\alpha_2$ and one known γ_p. The solution to the system of these two equations is a little bit more complicated than in the case of the perfect quarterwave plate and as a result it gives the following equation for the ellipticity angle ϑ_f:

$$\operatorname{ctg} 2\vartheta_f = \frac{1}{\sin\gamma_p}\frac{\operatorname{tg} 2\Delta\alpha_1}{\operatorname{tg} 2\Delta\alpha_2} - \operatorname{ctg}\gamma_p. \tag{10}$$

After calculating the ellipticity angle ϑ_f, the phase difference γ could be calculated from Eq. (9).

4. Conclusions

Presented above method is simple and fast. Two measurements (or three if taking into account the adjustment of the examined plate with the azimuth angle 45°) are required for the determination birefringence properties of the optical medium. It does not require any advanced measurement apparatus and also allows measuring spectral properties of media if only spectral properties of the quarterwave plate are known. Of course since the Senarmont compensator belongs to the group of azimuthal type compensators, measurements are reduced to the first order of the phase shift in the described method.

One can make the presented method objective, for example choosing three positions of the analyzer (setup with quarterwave plate), measuring three light intensities after the analyzer and solving three equations with three wanted birefringence parameters of the medium. This modification is similar to the method presented in Ref. [4], but fewer measurements are needed. In the presented paper this methodology has not been shown because in the author's opinion such a method could be hardly treated as a classical Senarmont one.

Appendix A.

Mueller matrix M of elliptically birefringent medium with the azimuth angle α_f and the ellipticity angle ϑ_f of its first eigenvector:

$$M(\alpha_f) = \begin{bmatrix} 1 & 0 & * & * \\ 0 & M^2Z + X & * & * \\ 0 & MCZ - YS & * & * \\ 0 & MSZ + YC & * & * \end{bmatrix}, \tag{A.1}$$

where $M = \cos 2a_f \cos 2\vartheta_f$, $C = \sin 2\alpha_f \cos 2\vartheta_f$, $S = \sin 2\vartheta_f$, $X = \cos\gamma$, $Y = \sin\gamma$, $Z = 1 - X$, and * denotes unimportant elements.

Mueller matrix P of the phase plate with the azimuth angle $\alpha = 90°$:

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos\gamma_p & -\sin\gamma_p \\ 0 & 0 & \sin\gamma_p & \cos\gamma_p \end{bmatrix}. \tag{A.2}$$

Mueller matrix A of the analyzer with variable azimuth angle α:

$$A(\alpha) = \begin{bmatrix} 1 & \cos 2\alpha & \sin 2\alpha & 0 \\ * & * & * & * \\ * & * & * & * \\ * & * & * & * \end{bmatrix}. \tag{A.3}$$

Due to the simplicity of calculations transmission coefficients of media were omitted.

The input light is linearly polarized with the azimuth angle $\alpha = 90°$ and its Stokes vector S_0 is given by:

$$S_0 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}. \tag{A.4}$$

So the Stokes vector S_{out} of the light at the output of the nonadjusted setup (α and α_f are treated as arbitrary) is given by:

$$S_{out} = A(\alpha)PM(\alpha_f)S_0. \tag{A.5}$$

The intensity I of the output light is a first element of the Stokes vector S_{out} and it is proportional to:

$$\begin{aligned} I \propto\ & 1 - \cos 2\alpha(M^2 Z + X) \\ & + \sin 2\alpha[MZ(S\sin\gamma_p - C\cos\gamma_p) \\ & + Y(S\cos\gamma_p + C\sin\gamma_p)]. \end{aligned} \tag{A.6}$$

When the adjusting procedure of the medium placed between the polarizer and the analyzer is taken into account, one should insert $\alpha = 0$ into Eq. (A.6) and differentiate Eq. (A.6) with regard to the angle α_f, which leads to the condition:

$$\partial I/\partial\alpha_f = 0 \iff \sin(4\alpha_f)Z = 0 \tag{A.7}$$

and it means that the intensity I takes the maximum value for the angle $\alpha_f = 45°$.

When the setup with quarterwave plate is considered (Fig. 1) one should insert $\alpha_f = 45°$ and $\gamma_p = 90°$ into Eq. (A.6) if the phase plate is a perfect quarterwave plate or generally γ_p if it is a nonperfect one. Similarly, when the second step of measurements is taken into account (Fig. 2) one should insert $\alpha_f = 45°$ and $\gamma_p = 0°$ into Eq. (A.6). Differentiating Eq. (A.6) with regard to the azimuth angle α and looking for minimum of the intensity I, it is possible to obtain Eqs. (2), (4) and (8).

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