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New experimental methods for determining the optical parameters of elliptic retarders

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Abstract. A set consisting of linear optical retarders, three-dimensional photoelastic models, some crystals, twisted fibres, etc. is optically equivalent to an elliptic retarder, and can be described by its three parameters. In this paper, new experimental methods for determining the optical parameters of an elliptic retarder are presented with a brief review of the existing methods. The Stokes vector representation of polarized light is used in developing these methods. It is shown that it is possible to find the integral order of the retardation (connected with the elliptic retarder) only in certain cases where this retardation can be gradually increased from zero. Some experimental results are presented as supporting evidence for these methods. The relative accuracies of these methods are discussed. A combination of different methods is recommended for obtaining maximum experimental accuracy.

1. Introduction

It has been established that a system of linear optical retarders (or a system having both birefringence and optical activity) is equivalent to an elliptic retarder or a system (which we shall call the optically equivalent system) containing a linear retarder and a pure rotator (Ramachandran and Ramaseshan 1961, Aben 1966). Using the Poincaré sphere (Ramachandran and Ramaseshan 1961), the effect of this system can be represented by a rotation of the sphere about one of its diameters through a particular angle. A set consisting of two-dimensional photoelastic models, three-dimensional photoelastic models, some crystals, twisted fibres, etc. belong to this category of elliptic retarders.

1.1. Elliptic retarder

The elliptic retarder is characterized by the two states of orthogonal elliptical polarization that pass through the elliptic retarder unaltered, and the retardation Δ that is introduced. These two states of orthogonal polarization are characterized by the orientation λ_0 of the major axis of these elliptic polarizations with respect to some reference axis and the ellipticity tan ω_0. Δ is the angle through which the Poincaré sphere is to be rotated to represent the effect of this system (or elliptic retarder) about a particular diameter and the above two orthogonal elliptical polarizations are the points on the sphere lying on this diameter. In figure 1 the point M represents one such elliptical polarization of an elliptic retarder. OR is the reference axis and OM is the axis of the elliptic retarder. The effect

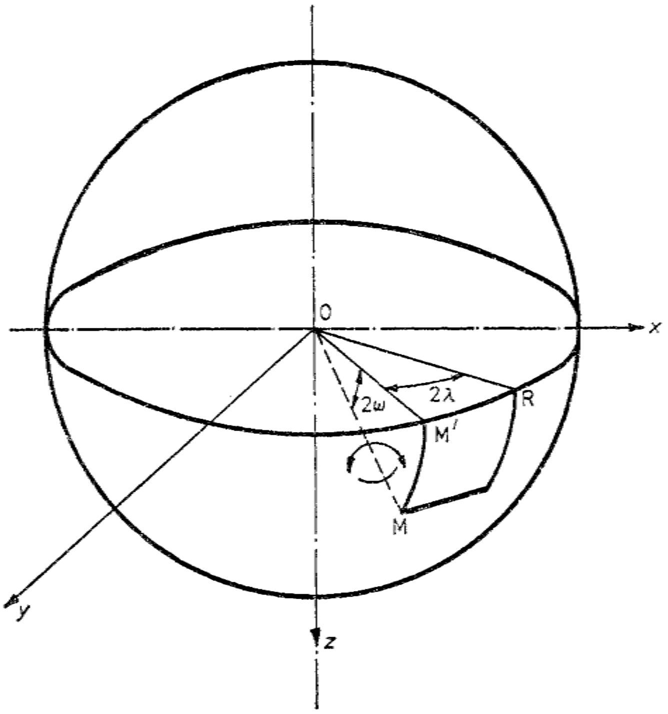

Figure 1. Poincaré sphere representation with right-handed Cartesian coordinate system: OR, reference axis; OM, elliptical polarization that characterizes the elliptical retarder.

of this elliptic retarder on a particular incident polarization represented by a point P on the Poincaré Sphere is obtained by rotating the line OP about the axis OM through an angle Δ. ω_0 and λ_0 are the other parameters of the elliptic retarder. λ_0 is the azimuth of the elliptic retarder and it is equal to the angle which the meridional plane containing the point M makes with the xOz (reference) plane.

Two methods of finding these parameters are presented by El-Hosseiny (1975): one the so-called generalized Sénarmont method; and the second the flux method. These methods require a set of experimental results from which the required parameters are to be computed. In what follows, new methods of directly finding these parameters by experiment are derived and their relative accuracies are discussed.

1.2. Optically equivalent system

The optically equivalent system consisting of a linear retarder followed by a pure rotator is characterized by the orientation ϕ of the fast axis of the linear retarder, its retardation δ and the rotatory power ψ_1 of the pure rotator. Aben (1966) termed this retardation the characteristic retardation, the orientation of the (fast and slow) axes of the retarder the primary characteristic directions, and the directions which make an angle equal to the rotatory power ψ_1 with these, the secondary characteristic directions. The orientations of these primary and secondary characteristic directions are measured with respect to some orthogonal reference axes. These orientations and the retardation are referred to as the characteristic parameters. Two simple experimental methods have been described (Sarma and Srinath 1972, Srinath and Sarma 1974) for determining these parameters. Another method for finding these parameters is by using a spinning analyser (Ramachandran and Ramaseshan 1961, Robert 1967). This method consists of measuring the alternating and constant components of the intensity of light coming from the analyser and relating these to the ellipticity of the light. These methods will now be briefly explained using the Stokes vector, and relations between the parameters of the optically equivalent system and the elliptic retarder will be derived.

1.3. Stokes vector and its equivalent forms

Any state of polarized light can be represented by the Stokes vector (Ramachandran and Ramaseshan 1961) with its four parameters S_0, S_1, S_2 and S_3. Only three of these are independent, since they are related by the identity $S_0^2 = S_1^2 + S_2^2 + S_3^2$. In the present discussion we assume that S_0 (corresponding to the total intensity of light) equals unity and that there is no absorption of light and the light is completely polarized. So, we deal only with the three parameters S_1, S_2 and S_3. These parameters correspond to the x, y and z coordinates on the Poincaré sphere. These are referred to a particular set of axes, say x and y. The Stokes vector is expressed in the following forms:

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} \equiv \begin{bmatrix} a_x^2 - a_y^2 \\ 2a_x a_y \cos E \\ 2a_x a_y \sin E \end{bmatrix} \equiv \begin{bmatrix} \cos 2\theta \\ \sin 2\theta \cos E \\ \sin 2\theta \sin E \end{bmatrix}$$

$$\equiv \begin{bmatrix} \cos 2\omega \cos 2\lambda \\ \cos 2\omega \sin 2\lambda \\ \sin 2\omega \end{bmatrix} \tag{1}$$

$$a_x^2 + a_y^2 = 1 \tag{2}$$

where
a_x, a_y are the amplitudes of the components of the light vector along the reference axes, E is the phase difference between these two components, $\tan\omega$ is the ellipticity, and λ is the azimuth or the inclination of the major axis of the light ellipse with the reference axis measured in the clockwise direction.

$$\tan\theta = a_y/a_x = \text{amplitude ratio.}$$

1.4. Effect of an optically equivalent system

The equations of transformation for the change of phase difference along a particular set of axes and rotation of the reference axis can be directly derived from equation (1). For example, for the incident Stokes vector (S_1, S_2, S_3), for a rotation of the reference axes by an angle θ, the azimuth decreases by an amount θ and the ellipticity remains unaltered i.e. the new Stokes vector is given by

$$\begin{bmatrix} \bar{S}_1 \\ \bar{S}_2 \\ \bar{S}_3 \end{bmatrix} \equiv \begin{bmatrix} \cos 2\omega \cos 2(\lambda - \theta) \\ \cos 2\omega \sin 2(\lambda - \theta) \\ \sin 2\omega \end{bmatrix} \equiv \begin{bmatrix} S_1 \cos 2\theta + S_2 \sin 2\theta \\ S_2 \cos 2\theta - S_1 \sin 2\theta \\ S_3 \end{bmatrix}.$$

Similarly, for an addition of a retardation of $\bar{E}$ along the reference axes, the amplitude ratio (a_y/a_x) remains unaltered and the phase difference increases by $\bar{E}$. In this case, the new Stokes vector is given by

$$\begin{bmatrix} \bar{S}_1 \\ \bar{S}_2 \\ \bar{S}_3 \end{bmatrix} \equiv \begin{bmatrix} a_x^2 - a_y^2 \\ 2a_xa_y \cos(E+\bar{E}) \\ 2a_xa_y \sin(E+\bar{E}) \end{bmatrix} \equiv \begin{bmatrix} S_1 \\ S_2 \cos \bar{E} - S_3 \sin \bar{E} \\ S_3 \cos \bar{E} + S_2 \sin \bar{E} \end{bmatrix}$$

Now, consider the case when the light represented by the Stokes vector (S_1, S_2, S_3) is incident on a system consisting of a linear retarder (aximuth ϕ and retardation δ) followed by a pure rotator (rotatory power ψ_1). This vector is referred to a particular set of orthogonal reference axes with which the axes of the retarder make an angle of ϕ. At the exit of this system, the Stokes vector with respect to the same set of reference axes is given by

$$\begin{bmatrix} \bar{S}_1 \\ \\ \bar{S}_2 \\ \\ \bar{S}_3 \end{bmatrix} \equiv \begin{bmatrix} \cos 2\psi\,(S_1 \cos 2\phi + S_2 \sin 2\phi) - \sin 2\psi\,[\cos\delta\,(S_2 \cos 2\phi \\ \quad - S_1 \sin 2\phi) - S_3 \sin\delta] \\ \cos 2\psi\,[\cos\delta\,(S_2 \cos s\phi - S_1 \sin 2\phi) - S_3 \sin]\,\delta \\ \quad + \sin 2\psi\,(S_1 \cos 2\phi + S_2 \sin 2\phi) \\ S_3 \cos\delta + \sin\delta\,(S_2 \cos 2\phi - S_1 \sin 2\phi) \end{bmatrix} \tag{3}$$

where

$$\psi = \phi + \psi_1. \tag{4}$$

Equation (3) is obtained by using the above transformation equations in three steps: (1) rotating the reference axes through an angle ϕ; (2) adding a retardation of δ, and (3) rotating through an angle $(-\phi - \psi_1)$.

2. Experimental procedure

2.1. Experimental determination of the parameters of the optically equivalent system

We briefly describe some of the existing methods for finding the characteristic parameters. These methods essentially vary from one another in the determination of one of the parameters. After knowing this parameter, the other two can be found by the usual procedures like that of Sénarmont etc.

2.1.1. Method 1. In this method (Sarma and Srinath 1972, Srinath and Sarma 1974), the secondary characteristic directions and the characteristic retardation are first found. For this, consider the incident light to be circularly polarized; that is, $S_1 = S_2 = 0$, $S_3 = 1$. From equations (3)

$$\bar{S}_2/\bar{S}_1 = \tan 2\lambda = -\cot 2\psi$$

or

$$\psi = \lambda + \pi/4. \tag{5}$$

The quantity ψ represents the orientation of the secondary characteristic direction with the reference axis. Equation (5) shows that these secondary characteristic directions are at 45° to the major and minor axes of the light at the exit (which can be directly found by rotating the analyser to obtain maximum and minimum intensities of light). Also

$$\bar{S}_3 = \sin 2\omega = \cos\delta$$

i.e.

$$\delta = \pi/2 - 2\omega. \tag{6}$$

The ellipticity tan ω of the light at the exit can be found by Sénarmont's method and the characteristic retardation δ can be found using equation (6). Now, the analyser is kept along one of the secondary characteristic directions and the polarizer is rotated to obtain the maximum intensity of light. The final orientation of the polarizer corresponds to the primary characteristic direction (corresponding to that particular secondary characteristic direction). The angle between this and the reference axis is ϕ.

2.1.2. Method 2. Consider the light represented by the vector (S_1, S_2, S_3) to be incident on a system consisting of a quarter-wave plate and analyser with the angle between their axes being 45°. Let the Stokes vector of light referred to the axes of the analyser, at the exit be $(\bar{S}_1, \bar{S}_2, \bar{S}_3)$. Then, putting $\phi = 0$, $\psi_1 = \pi/4$ and $\delta = \pi/2$ we obtain

$$\bar{S}_1 = S_3 = -\sin 2\omega. \tag{7}$$

(The Stokes vector (S_1, S_2, S_3) is referred with respect to the axes of the quarter-wave plate.)

$$I = Ka_x{}^2 = K(1 + \bar{S}_1)/2 \tag{8}$$

where I is the intensity of light from the analyser, K is the proportionality constant (see below) and a_x is the amplitude of the component of the light vector at the exit along the axis of the analyser. So

$$I = K(1 - \sin 2\omega)/2 \tag{9}$$

i.e. the intensity at the exit of the system is dependent only on the ellipticity (tan ω) of the incident light.

In this method, the constant K is determined first by a calibration experiment. This constant takes into account the absorption of light in various elements of the experimental set-up. When the incident light is plane polarized, $\omega = 0$ and $I = K/2$. The polarizer is rotated (in the setup consisting of polarizer, model, quarter-wave plate-analyser combination with the angle between their axes being 45°) until the intensity of light at the exit equals $K/2$. This final orientation of the polarizer corresponds to one of the primary characteristic directions.

2.1.3. Method 3 (spinning polarizer method). We have

$$I = K(1 + S_1)/2 = K(1 + \cos 2\omega \cos 2\lambda)/2 \tag{10}$$

where I is the intensity of light, tan ω is the ellipticity, and λ is the azimuth (angle between the axis of the analyser and the major axis) of the light from the analyser. If the analyser is rotated at an angular velocity of Ω, then $\lambda = \Omega t$ (t is the time from some arbitrary instant of time), the ratio of the alternating to the constant components of this intensity is given by

$$r = \cos 2\omega. \tag{11}$$

This ratio r is measured experimentally. The position of the polarizer for which $r = 1$ corresponds to one of the primary characteristic directions.

3. Parameters of the elliptic retarder

3.1. Incident light which does not get altered by the elliptic retarder

Consider the light (S_1, S_2, S_3) (referred to the axes of the retarder of the optically equivalent system) to be incident on the elliptic retarder. The light $(\bar{S}_1, \bar{S}_2, \bar{S}_3)$ at the exit is obtained by putting $\phi=0$ and $\psi=\psi_1$ in equation (3). To find the parameters of the incident light which does not get altered by the elliptic retarder, we equate the parameters of the light at the entrance and exit of the elliptic retarder. Thus

$$S_1=\bar{S}_1=S_1 \cos 2\psi_1-S_2 \cos \delta \sin 2\psi_1+S_3 \sin \delta \sin 2\psi_1 \tag{12}$$

$$S_2=\bar{S}_2=S_2 \cos \delta \cos 2\psi_1-S_3 \sin \delta \cos 2\psi_1+S_1 \sin 2\psi_1 \tag{13}$$

$$S_3=\bar{S}_3=S_3 \cos \delta+S_2 \sin \delta \tag{14}$$

$$S_3/S_2=\tan E_0 \text{ (say)}=\cot (\delta/2)$$

therefore

$$E_0=\pi/2-\delta/2 \tag{15}$$

$$S_2/S_1=\tan 2\lambda_0 \text{ (say)}=\tan \psi_1$$

or

$$\lambda_0=\psi_1/2. \tag{16}$$

That is, the major axis of the incident light makes an angle of $(\psi_1/2)$ with the fast axis of the retarder of the optically equivalent system. These axes are shown in figure 2,

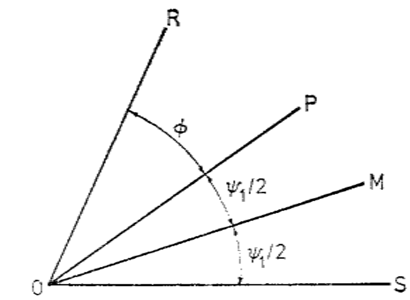

Figure 2. Orientations of various axes of the elliptic retarder and the optically equivalent system: OR, reference axis; OP and OS, primary and secondary characteristic directions; OM elliptical polarization that characterizes the elliptic retarder.

where OR is the reference axis, OP and OS are the primary and secondary characteristic directions (fast axes) respectively, and OM is the direction of the major axis of the incident light ellipse which does not get altered by the elliptic retarder. Also,

$$S_3/S_2=\tan E_0=\tan 2\omega_0/\sin 2\lambda_0 \text{ from equation (1)}$$

$$\tan 2\omega_0=\cot (\delta/2) \sin \psi_1 \tag{17}$$

using equations (15) and (16) where $\tan \omega_0$ is the ellipticity of this incident light.

3.2. Relations between the parameters of the optically equivalent system and the elliptic retarder

Since simple experimental methods exist for the determination of the parameters of the optically equivalent system, the parameters of the elliptic retarder can be computed from these if the relations between these two sets of parameters are established. Equations (16) and (17) are two such relations connecting λ_0 and ω_0 of the elliptic retarder with δ and ψ_1 of the optically equivalent system. Now, the relation connecting Δ, the retardation of the elliptic retarder, with the other parameters ϕ, δ and ψ_1 is to be derived.

For this, consider a point P (x, y, z) on the Poincaré sphere in the cartesian coordinate system, shown in figure 3. The y axis is perpendicular to the plane of the paper. The

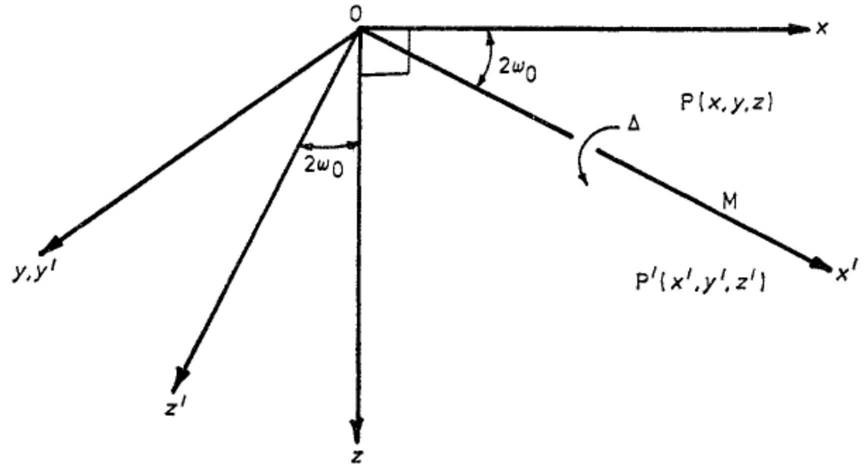

Figure 3. Representation of the effect of elliptic retarder: M, elliptic polarization that characterizes the elliptic retarder; Δ and $2\omega_0$, parameters of the elliptic retarder.

Stokes parameters S_1, S_2 and S_3 of this point P are equal to the coordinates x, y and z respectively. OM, the axis of the elliptic retarder is assumed to be in the reference plane $x0z$ for simplicity, but without the loss of any generality, i.e. the azimuth λ of the point M is zero. The point M represents one of the two orthogonal elliptical polarizations which characterize the elliptical retarder. The effect of the elliptic retarder on the incident polarization P (x, y, z) is obtained by rotating the axis OP about OM through an angle Δ as shown in the figure. If the new location of this point is $\mathrm{P}'(x', y', z') = \mathrm{P}'(\bar{S}_1, \bar{S}_2, \bar{S}_3)$, then the Stokes vector $(\bar{S}_1, \bar{S}_2, \bar{S}_3)$ represents the outcoming light. The expressions for the parameters $\bar{S}_1$, $\bar{S}_2$ and $\bar{S}_3$ in terms of ω_0, Δ, S_1, S_2 and S_3 can be obtained by a three-step process: (1) obtain the coordinates of P with respect to the new axes $0x'$, $0y'$ and $0z'$ by rotating the coordinate axes through an angle $2\omega_0$ about y axis, (2) rotate the new system of axes through an angle $-\Delta$ and obtain the coordinates for the point P′ and (3) refer these coordinates of P′ with the respect to the old axes x, y and z. These new coordinates x', y' and z' thus obtained are equal to $\bar{S}_1$, $\bar{S}_2$ and $\bar{S}_3$ respectively. For the rotation of the coordinate axes, the following relations are used. (x, y, z) and (x', y', z') are the old and new coordinates of a point and $a_1 = \cos(x, x')$, $a_2 = \cos(x, y')$, $a_3 = \cos(x, z')$, $b_1 = \cos(y, x')$, $b_2 = \cos(y, y')$, $b_3 = \cos(y, z')$ $c_1 = \cos(z, x')$, $c_2 = \cos(z, y')$ and $c_3 = \cos(z, z')$. The argument of the cosine in the brackets is equal to the angle between the corresponding axes, e.g. (x, x') is the angle between x and x' axes. Thus

$$x' = a_1x + b_1y + c_1z$$

$$y' = a_2x + b_2y + c_2z$$

and

$$z' = a_3 + b_3 y + c_3 z.$$

Also

$$x' = \bar{S}_1 = S_1 (\cos^2 2\omega_0 + \cos\Delta \sin^2 2\omega_0) + S_2 \sin\Delta \sin 2\omega_0 + S_3 \sin 2\omega_0 \cos 2\omega_0 + (1 - \cos\Delta) \tag{18}$$

$$y' = \bar{S}_2 = -S_1 \sin\Delta \sin 2\omega_0 + S_2 \cos\Delta + S_3 \sin\Delta \cos 2\omega_0 \tag{19}$$

$$z' = \bar{S}_3 = S_1 \cos 2\omega_0 \sin 2\omega_0 (1 - \cos\Delta) - S_2 \sin\Delta \cos 2\omega_0 + S_3 (\cos\Delta \cos^2 2\omega_0 + \sin^2 2\omega_0) \tag{20}$$

where x', y' and z' are the coordinates of the point P′ (after rotation about OE) referred to the orthogonal axes $0x'$, $0y'$ and $0z'$ shown in figure 3. So, the parameters $\bar{S}_1$, $\bar{S}_2$ and $\bar{S}_3$ refer to the light at the exit referred to the axes of the elliptic retarder. To obtain the relations between the parameters of the optically equivalent system and the elliptic retarder we compare the set of equations (18)–(20) with the set of equation (3) after substituting $\phi = -\psi_1/2$. We then obtain

$$\tan 2\omega_0 = \sin\delta \sin\psi_1/(1 - \cos\delta) \text{ (the same as equation (17))}$$

and

$$\cos\Delta = \cos\delta \cos^2\psi_1 - \sin^2\psi_1. \tag{21}$$

4. New experimental methods for the determination of the parameters ω_0 and Δ of the elliptic retarder

It is known that the orientation of the major axis of the light ellipse which does not get altered by the elliptic retarder can be obtained by rotating the elliptic retarder between crossed polarizers until the intensity is minimum. The orientations of the polarizers (with respect to the model) correspond to the axes of this incident light ellipse. Now, the other two parameters, namely the ellipticity $\tan\omega_0$ and Δ, the retardation of the elliptic retarder, can also be found directly by experiment as described below.

Keep the polarizer at $-45°$ to the axis of the elliptic retarder (i.e. the major axis of the incident light which will not get altered by the elliptic retarder), i.e. the input light has Stokes parameters (1, 0, 0) with respect to the axis of the polarizer.

From equation (3) with $\phi = \pi/4 - \psi_1/2$ and $\psi = \pi/4 + \psi_1/2$ and with respect to the axis of the polarizer, we have

$$\bar{S}_1 = -\sin^2\psi_1 + \cos\delta \cos^2\psi_1 = \cos\Delta \text{ (from equation (21))} \tag{22}$$

$$\bar{S}_2 = (1 + \cos\delta) \cos\psi_1 \sin\psi_1 \tag{23}$$

$$\bar{S}_3 = -\sin\delta \cos\psi_1. \tag{24}$$

If $\bar{S}_3/\bar{S}_2 = \tan E$, then from equations (21), (23) and (24)

$$2\omega_0 = E + \pi/2. \tag{25}$$

Keep a compensator (say a Babinet-Soleil type) behind the model with its axis parallel to the polarizer axis. Let the axis of the analyser be parallel to the (major) axis of the elliptic retarder. Change the retardation in the compensator until the intensity is minis mum. The retardation introduced by the compensator will be $-E$ or $-E + \pi$. The parameter ω_0 can be calculated using equation (25). The reasoning for this is as follows.

Let the light before the analyser and with respect to the axes of the compensator be represented by

$$S_1 = \cos 2\theta \quad \text{or} \quad \theta = \Delta/2 \text{ from equation (22)} \tag{26}$$

$$S_2 = \sin 2\theta \cos (E + \Delta E) \tag{27}$$

$$S_3 = \sin 2\theta \sin (E + \Delta E) \tag{28}$$

where ΔE is the retardation introduced by the compensator.

If the Stokes vector is referred to the axis of the analyser,

$$\bar{S}_1 = \sin 2\theta \cos (E + \Delta E), \text{ etc.} \tag{29}$$

$$\mathrm{d}I/\mathrm{d}(\Delta E) = \mathrm{d}\bar{S}_1/\mathrm{d}(\Delta E) = -\sin 2\theta \sin (E + \Delta E) = 0$$

when $\Delta E = -E$ for maximum intensity and $\Delta E = -E + \pi$ for minimum intensity (assuming that $\sin 2\theta > 0$).

Rotate the analyser to obtain the minimum intensity of light. If the angle between the axes of the analyser (in the final position) and the polarizer is θ', then $\Delta = 2\theta'$. The reasoning is as follows: with respect to the axis of the polarizer, $S_1 = \cos 2\theta$. With respect to the axis of the analyser

$$\bar{S}_1 = \cos 2\theta \cos 2\theta' + \sin 2\theta \sin 2\theta' \tag{30}$$

where θ' is the angle between the axes of the analyser and polarizer.

$$\mathrm{d}\bar{S}_1/\mathrm{d}(2\theta') = 0 \text{ when } \theta = \theta' = (\Delta/2) \text{ from equation (26).} \qquad (31$$

The parameter Δ can also be obtained from the value of the intensity of light. The intensity of light from the analyser, when the analyser is kept parallel to the polarizer is given (from equations (10) and (29)) by

$$I = K(1 + S_1)/2 = K \cos^2 \theta = K \cos^2 (\Delta/2) \tag{32}$$

The quantity K is found by a calibration experiment as described earlier. The value of Δ can be computed from equation (32).

5. Ambiguity in the determination of the parameters

Experimental methods have been described to determine the parameters δ, ψ_1, θ and ω_0 directly. There is no ambiguity in finding the parameters ϕ and ψ_1 since this only involves finding the orientation of the analyser or polarizer to obtain minimum (or maximum) intensity. However, it is to be noted that the pairs of primary and secondary characteristic directions have mutual correspondence. If plane-polarized light is incident along one of the primary characteristic directions, the direction (at the exit of the model) along which the light emerges as plane polarized is to be taken as the corresponding (or say conjugate) secondary characteristic direction. This correspondence is to be borne in mind in finding the other parameters δ, ω_0 and Δ for which the method of compensation (either using a quarter-wave plate or a Babinet-Soleil type compensator) is used. The methods again involve the rotation of the analyser or the change of retardation in the compensator until minimum (or maximum) intensity is obtained. No ambiguity arises if the following are taken care of: (i) there is correspondence for the characteristic directions, (ii) axis for the retarder always means the fast axis, (iii) axis of the polarizer or analyser is always the transmission axis, (iv) rotation is always measured in a particular direction (say clockwise direction when looking towards the source of light), and (v) the extremum of intensity means always the maximum intensity.

6. Determination of the integral part in δ or Δ

It is in general not possible to find the integral part of either Δ of the elliptic retarder or δ of the optically equivalent system. This is so since there exists no simple relation between these parameters and the wavelength of light as in the case of a linear retarder (where a relation exists). As an example, consider the simple case of a combination of two retarders each of equal retardance ρ. Let the angle between their axes be 45°. Then

$$\cos\delta = \cos^2\rho \tag{33}$$

and

$$\cos\Delta = (\cos^2\rho + \cos\rho - 1)/2 \tag{34}$$

It can be seen from these relations that even for this simple case, the relations connecting δ and Δ with ρ are nonlinear (relations between ρ and $\bar{\lambda}$, the wavelength, can be determined experimentally for any material). So, the variation of δ or Δ with $\bar{\lambda}$, the wavelength of light, cannot be used in general to find the integral fringe order. Only in cases where the retardation is induced by mechanical loading can the integral part be found by increasing the load gradually from zero to maximum and simultaneously measuring the quantities Δ and δ.

7. Discussion on the accuracy of the experimental methods

The methods described for direct measurement of the parameters of the elliptic retarder and the optically equivalent system are of two types: (i) through the intensity measurement, and (ii) through rotation of the optical elements of the system to obtain minimum or maximum intensity of light.

The accuracy of methods (i) essentially depends on the accuracy with which the instrument (photometer) used can measure the intensity. The accuracy of the methods (ii) however depends also on the particular method used. The present discussion will be restricted to methods (ii). As stated earlier, in these methods, one of the optical elements is rotated, say through an angle α from some arbitrary position, to obtain minimum or maximum intensity. So the accuracy of these methods is directly dependent on the magnitude of the quantity $\mathrm{d}^2I/\mathrm{d}\alpha^2$, where I is the intensity of light coming from the analyser. The higher the magnitude of this quantity, the better will be the accuracy of the method.

The accuracy of these methods can be compared as follows:

(*a*) Rotation of the elliptic retarder between the crossed polarizers to find the orientation of the axis of the elliptic retarder:

$$\mathrm{d}^2I/\mathrm{d}(2\phi)^2 = K\sin^2(\delta/2) \tag{35}$$

where K is a constant which takes into account the absorption of light in the various optical elements.

(*b*) Finding the orientation of the major axis of the light at the exit (when the incident light is circularly polarized):

$$\mathrm{d}^2I/\mathrm{d}(2\lambda)^2 = (K/2)\sin\delta. \tag{36}$$

(*c*) Changing the retardation in the compensator to determine the parameter $2\omega_0$ of the elliptic retarder:

$$d^2I/d(\Delta E)^2 = K \sin \Delta. \tag{37}$$

(*d*) Rotation of the analyser to find the parameter Δ of the elliptic retarder:

$$d^2I/d(2\theta')^2 = K/2. \tag{38}$$

From equations (35)–(37) we note that the accuracy of the methods represented by the term $d^2I/d\alpha^2$ depends on the parameters themselves. For example, if methods (*a*) and (*b*) are compared,

$$[d^2I/d(2\phi)^2]/[d^2I/d(2\lambda)^2] = \tan(\delta/2).$$

So method (*a*) is better when $|\tan(\delta/2)| > 1$ and method (*b*) is better when $|\tan(\delta/2)| < 1$. It is to be noted that methods (*c*) and (*d*) can be used only after finding the parameter λ_0. In method (*d*), the accuracy is independent of the parameters of the system.

8. Experimental results

The parameters ϕ, δ, and ψ were found experimentally for a photoelastic model using the above described methods. These values, along with the calculated values of the parameters ω_0 and Δ are shown in table 1. The photoelastic model taken is a rectangular one

Table 1. Experimental values of the parameters for three wavelengths of light.

Wavelength of light (Å)	Material fringe constant (kg m^{-1} per fringe)	Values of the parameters in degrees (E = experimental, T = theoretical)									
		ϕ		ψ		δ (total)		ω_0		Δ fractional part	
		E	T	E	T	E	T	E	T	E	T
5950	2821	36	34	−38	−34	465	451	−18	−21	19	30
5520	2625	29	28	−26	−28	478	473	−13	−14	34	36
5004	2286	0	1	2	−1	499	490	0	0	41	50

($0{\cdot}0254 \times 0{\cdot}0135 \times 0{\cdot}1651$ m^3) under combined torsion ($0{\cdot}219$ kg m) and tension ($49{\cdot}46$ kg). The length of the optical path in the model is $0{\cdot}0135$ m and is through the middle point and perpendicular to one of the faces of the model. The results are presented for three wavelengths of light. The integral part of δ is found to be 1 by increasing the load from zero to maximum. The experimental results were checked with the theoretical values obtained by numerically integrating the three-dimensional photoelasticity equations for the present loading conditions, and the agreement is good. For the present experiment, since $|\tan(\delta/2)| > 1$, the method of rotating the crossed polarizers to obtain the value of ϕ is more accurate than the method of using circularly polarized light for finding the parameter ψ. However, these parameters ϕ and ψ are found by both these methods and these are matching within 1°. The accuracy of the measurement is limited by the photometric errors like drift etc in the photometer used. For the type of loading considered (combined axial tension and torsion), the quantity $\phi + \psi$ should be zero from theoretical considerations. But in the experiment, this is obtained as $-2°$, $3°$ and $2°$ for the three wavelengths of light. This is probably due to a small amount of residual birefringence rather than the errors in the measurement techniques. The experimental results are presented as supporting evidence for the applicability of the experimental methods described and the possibility of finding the integral order of δ (or Δ) in such cases. From these experimental results it can also be seen that the values of δ (total fringe orders) are not proportional to either the wavelength of light or the material fringe constant. This shows that the data for various wavelengths of light cannot be used to find the integral order of the retardation δ.

9. Conclusions

New, simple and direct experimental methods for the determination of the parameters of an elliptic retarder are described. The equivalence relations between these parameters and the parameters of an optically equivalent system are derived. Discussion regarding the relative accuracies of these experimental methods is presented. The accuracy of most of these methods is dependent on the parameters also. It would be better to use all these methods simultaneously to achieve maximum accuracy. It is emphasized that, in general it is not possible to find the integral order of the retardation δ of the optically equivalent system or Δ of the elliptic retarder. Only in cases where these retardations are induced by mechanical loading can these be found. Some experimental results for a three-dimensional photoelastic model are presented as supporting evidence.

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