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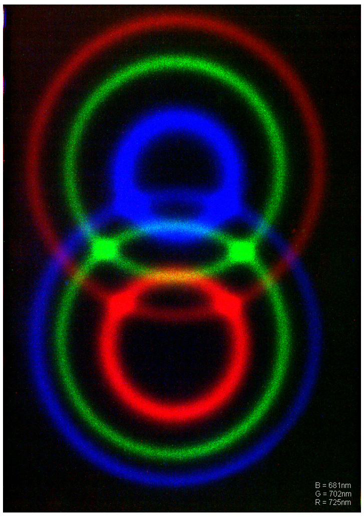
Entanglement and Bell's inequality



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WS 2015/16

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Abstract

The question whether the results of quantum measurements can be understood in an objective and local way as values carried by hidden variables can be decided by correlation measurements on entangled pairs of photons. In this experiment you will use a nonlinear optical source of polarization entangled photon pairs to carry out the measurements required to test Bell's inequality [1].

1 Introduction

Quantum mechanics predicts that non-local correlations between the observable physical properties of distant quantum systems (ex: photons, electrons, ions) are possible and this phenomenon is termed quantum entanglement. Some physicists have argued that quantum mechanics is incomplete and hidden variables have to be added to explain entanglement. The most notable is the Einstein-Podolsky-Rosen-Paradox where the authors proposed a thought experiment and concluded that the quantum mechanical description of physical reality was incomplete [5]. Later John Bell published experimentally detectable inequality relations [1] that were derived assuming that the state of the system was well defined with hidden variables and the measurement on one quantum system does not affect the other. He showed that this inequality could be violated by quantum mechanics. In other words, entangled particles could violate this inequality, while two particles that were assigned local hidden variables could not. The goal of this experiment is to test Bell's inequality using correlation measurements on photon pairs. These polarization entangled photon pairs are generated by a spontaneous parametric down conversion (SPDC) source. A longer introduction and many more details can be found in [8] (in German!).

2 Entanglement

Erwin Schrödinger introduced the term "Entanglement" as a translation to the German word "Verschränkung" in 1935 to describe the non-local quantum correlations between two quantum systems that interact and then separate. Some examples for quantum systems that can be entangled are two level qubit system such as the spins of electrons or nuclei and the polarization of photons. In this text we will focus only on polarization entangled photons.

Let's consider the quantum state of polarization of two photons A and B as a qubit. Then, the orthogonal bases are

$$|H\rangle_A = \begin{pmatrix}1\\0\end{pmatrix}, |V\rangle_A = \begin{pmatrix}0\\1\end{pmatrix} \text{ for photon } A \text{ and } |H\rangle_B = \begin{pmatrix}1\\0\end{pmatrix}, |V\rangle_B = \begin{pmatrix}0\\1\end{pmatrix}$$

for photon B, respectively. H and V refer to horizontal and vertical polarizations of the photons with respect to a suitable frame of reference. The $|H\rangle$ and $|V\rangle$ states correspond to the usual computational basis states $|0\rangle$ and $|1\rangle$, respectively. An entangled state of the two photons A and B can be written as

$$|\psi\rangle = \frac{1}{\sqrt{2}}[|H\rangle_A |H\rangle_B + |V\rangle_A |V\rangle_B]$$

If we make a measurement for the polarization of photon A and if the outcome is H, then the state of the system collapses to

$$|H\rangle_A |H\rangle_B$$

Any subsequent measurement in the same basis for photon B will always have H as the outcome. An entangled state such as the one mentioned above cannot be written as a tensor product of the state of photon A ($|\psi\rangle_A$) and of the state of photon B ($|\phi\rangle_B$), $|\psi\rangle_A \otimes |\phi\rangle_B$ a so-called product state.

States of a composite system that cannot be represented as product states are entangled states:

$$|H\rangle_A |H\rangle_B + |V\rangle_A |V\rangle_B = |H\rangle_A \otimes |H\rangle_B + |V\rangle_A \otimes |V\rangle_B \neq |\psi\rangle_A \otimes |\phi\rangle_B$$

There are four maximally entangled two qubit states, known as Bell states, if we use the polarization of photon to write them, they will have the following form:

$$
\begin{aligned}
|\psi^+\rangle &= \frac{1}{\sqrt{2}}[|HV\rangle + |VH\rangle] & \qquad |\psi^-\rangle &= \frac{1}{\sqrt{2}}[|HV\rangle - |VH\rangle] \\
|\phi^+\rangle &= \frac{1}{\sqrt{2}}[|HH\rangle + |VV\rangle] & \qquad |\phi^-\rangle &= \frac{1}{\sqrt{2}}[|HH\rangle - |VV\rangle]
\end{aligned}
$$

Additionally, we can write the Bell states in the D/A basis, which is the $\pm 45°$ linear polarization basis:

$$|D\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}, \quad |A\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$$

The Bell states in this case are as follows:

$$
\begin{aligned}
|\psi^+\rangle &= \frac{1}{\sqrt{2}}[|DD\rangle - |AA\rangle] & \qquad |\psi^-\rangle &= \frac{1}{\sqrt{2}}[|DA\rangle - |AD\rangle] \\
|\phi^+\rangle &= \frac{1}{\sqrt{2}}[|DD\rangle + |AA\rangle] & \qquad |\phi^-\rangle &= \frac{1}{\sqrt{2}}[|DA\rangle + |AD\rangle]
\end{aligned}
$$

For this basis change, the $|\psi^-\rangle$ and the $|\phi^+\rangle$ state keep the same form. In general, if we change the basis for both subsystems the same way, the $|\psi^-\rangle$ or singlet state will always keep its form, where as the other (triplet) states change into each others form. In any case, no matter how we change bases, even for arbitrary choices in A and B the states always remain entangled.

Let us assume we have a system prepare in the state $|\psi^-\rangle$. If we now measure the polarization of one photon, say A by setting our polarizer to $0°$. Then we will find that half of the photons pass the polarizer and half are reflected (or absorbed). The same will be true for any other angle setting of the polarizer! However, if we also measure polarization of the second photon and look for simultaneous detection the story gets more complicated. If the two polarizers are parallel we will never find the two photons of a pair transmitted. It will always be one transmitted and one reflected. If, on the other hand the two polarizers are oriented orthogonally we find that the two photons of a pair will always be both transmitted or both reflected. This is a strong correlation or better anticorrelation, which appears to hold, no matter how far the two measurements are apart. Clearly if the distance is long enough, there cannot be any relevant communication between the two photons.

3 Bell's inequality

In 1964 J.S. Bell derived an inequality for such correlation measurements, which showed that the results for entangled states, which are predicted by quantum mechanics, could not be reproduced by a local realistic theory based on hidden variables. In other words, entangled particles could violate this inequality, while two particles that were assigned local hidden variables (parameters that the particles carry with them, which determine their measurement results) and acted independently when measured could not. There are many variants of Bell's inequality; In 1969 Clauser, Horne, Shimony, and Holt derived a version [4] that is more suitable for experiments. A slightly modified version of their inequality is given by

$$S = |E(\alpha,\beta) - E(\alpha,\beta')| + |E(\alpha',\beta) + E(\alpha',\beta')| \leq 2, \tag{1}$$

where E(α, β) is the expectation value of polarization correlation measurements made on a two photon system with the polarization measured at the angles α and β respectively. A very good exposition of this version of the inequality was given by John Bell himself in Ref. [2]

Suppose two observers, Alice and Bob, each get one photon from a two-photon system to measure. Alice can measure the polarization of her photon at the angles α and α', while Bob can measure his photon at the angles β and β'. For a particular choice of measurement angle, the measurement device can either tell the observer that the photon was polarized parallel $(+1)$ or perpendicular (-1) to the chosen angle.

For example, Alice measuring a photon at an angle $\alpha = 0°$ (the H/V basis) can either get the result H $(+1)$ or V (-1). We now assume that hidden variables λ from a suitable set control the measurement outcome through measurement functions A and B. In general the outcome on for Photon A would be $a = A(\alpha, \beta, \lambda) = \pm 1$ but we also assume *locality* by defining $a = A(\alpha, \lambda)$ to be independent of the polarizer angle β on the other side. Now let the results for α, α', β and β' be represented by the outcome variables a, a', b and b'. This notation suppresses the hidden variables for brevity. Alternatively we may think of these as the hidden variables (with values ± 1) themselves. The most important assumption here is that irrespective of which measurement is actually carried out, all four variables are assumed to have a defined value. This is the assumption of *realism*.

An interesting quantity is then the product of the two outcomes on the two sides i.e. ab and its expectation value $E(\alpha, \beta) = \langle A(\alpha, \beta) \rangle = \langle ab \rangle$, over all photons or hidden variable values. Generally we call the expectation value of a product of two random variables their covariance and like here, if it is normalized to the widths of the distributions, their *correlation*. The correlation can assume values between -1 (perfectly anti-correlated) and $+1$ (perfectly correlated).

Now consider the equation

$$ab - ab' + a'b + a'b' = a(b - b') + a'(b + b') \tag{2}$$

since $b, b' = \pm 1$, either $(b - b') = 0$ or $(b + b') = 0$ which means that Equation 2 must equal ± 2. Now, if several photon pairs are tested, the j-th pair yields

$$a_j b_j - a_j b'_j + a'_j b_j + a'_j b'_j = \pm 2 \tag{3}$$

Converting this to expectation values by averaging over all the photons (equivalent to averaging over the set of hidden variables λ) and taking the absolute value gives

$$|\langle ab \rangle - \langle ab' \rangle + \langle a'b \rangle + \langle a'b' \rangle| \leq 2 \tag{4}$$

The CHSH inequality of Eq. 1 is slightly different in the position of the absolute bars, but can be proven in a similar way and both versions are valid.

This inequality can be violated, for instance, if the two photons are in any of the Bell states. In particular, if the photons are in the entangled $|\psi^-\rangle$ state, the expectation value according to quantum mechanics is given by

$$E(\alpha, \beta) = -\cos(2(\alpha - \beta)) \tag{5}$$

The other expectation values can be determined similarly. It can be shown that the Bell parameter S has a maximum value of $2\sqrt{2}$ for the angles $\alpha = 0° + \phi$, $\beta = 22.5° + \phi$,$\alpha' = 45° + \phi$ and $\beta' = 67.5° + \phi$ for any constant offset ϕ.

In experiment, the expectation values $E(\alpha,\beta)$ are calculated from the number of events for each set of measurement angles as follows:

$$E(\alpha,\beta)=\frac{1}{N}(C_{++}(\alpha,\beta)+C_{--}(\alpha,\beta)-C_{+-}(\alpha,\beta)-C_{-+}(\alpha,\beta)). \tag{6}$$

where $C_{++}(\alpha,\beta)$ represents the number of events when the polarizer A is set to α and polarizer B is set to β, C_{--} represents the number of events when the polarizer A is set to $\alpha+90°$ and polarizer B is set to $\beta+90°$.

Lastly, N is the total number of events given by

$$N=C_{++}(\alpha,\beta)+C_{--}(\alpha,\beta)+C_{+-}(\alpha,\beta)+C_{-+}(\alpha,\beta) \tag{7}$$

Here we have made a strong assumption that needs some discussion. In the experiment we will find that we detect many photons, but for most of them we will fail to detect its partner photon. However, when we talked about hidden variable theories, we assumed that all measurements yielded results. Only very recently two experiments [7, 3] managed to achieve high enough detection efficiency to be able to violate Bell's inequality without any supplementary assumptions. If the efficiency is small, one can arguably assume that the hidden variables will not influence the detection efficiency and thus that the detected coincidence are a "fair sample" of all the emitted photon pairs. The discussion about this assumption has been going on since the first experiments by Freedman and Clauser [6].

4 Spontaneous parametric down-conversion

There are many sources used to produce entangled photons, such as Quantum Dots (QDs), Spontaneous Parametric Down Conversion (SPDC) crystals, Nitrogen Vacancy Centers (NV centers), single trapped atoms and single trapped ions, and even DNA structures. Here, we use SPDC crystal to produce entangled photons and test the Bell inequality.

Parametric down-conversion is a non-linear optical process which relies on the $\chi^{(2)}$ optical non-linearity of certain media; for example, KDP, $LiIO_3$, $KNbO_3$, $LiNbO_3$ and BBO. For the source used in this experiment, a violet (404 nm) pump laser beam spontaneously down-converts into two infra-red fields, known as the signal and idler beams, inside a BBO (β - BaB_2O_4) crystal. The non-linearity of the crystal leads to photon-pair production and is known as spontaneous parametric down-conversion (SPDC). SPDC is a distinctly quantum mechanical phenomenon, just like the spontaneous emission of an excited atom. Type-II refers to the fact that for the emerging photon pairs, the signal photon has extraordinary polarization (here vertical), while the idler photon has ordinary polarization (here horizontal). Two conservation laws have to be obeyed. The first law is the conservation of energy which requires the frequency of the pump photon to equal the sum of the frequencies of the two generated photons. This process can therefore be considered the splitting of a pump photon into two daughter photons.

$$\hbar\nu_{pump}=\hbar\nu_{signal}+\hbar\nu_{idler} \tag{8}$$

Furthermore, the photons are emitted diametrically opposite to one another around the pump wavevector, which is a consequence of the conservation of momentum. It requires the wavevector of the pump photon to equal the sum of the wavevectors of the two generated photons.

$$\hbar\mathbf{k}_{pump}=\hbar\mathbf{k}_{signal}+\hbar\mathbf{k}_{idler} \tag{9}$$

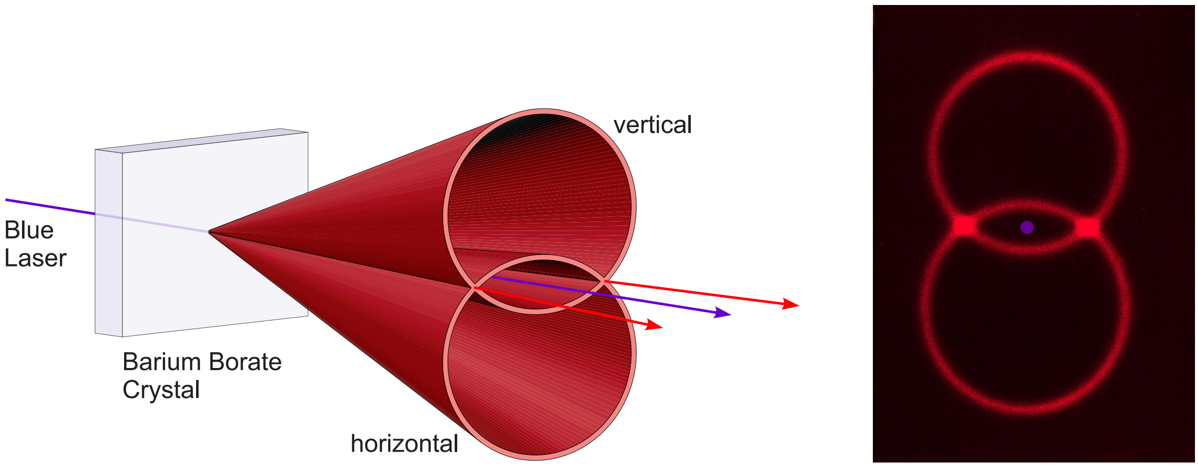

Figure 1: Left: Type-II spontaneous parametric down-conversion produces photon pairs, which emerge on two cones where the vertically polarized photon is on the upper cone and the horizontally polarized photon is on the lower cone. Polarization-entangled photons are observed at the two intersection lines of the cones. Right: False-color photograph of the down-conversion light taken with a bandpass filter at the degenerate wavelength.

In general it is difficult to satisfy the momentum conservation, because all media are dispersive and the wavenumber is proportional to the frequency-dependent refractive index $n(\nu)$, i.e. $k(\nu) = 2\pi\nu n(\nu)/c$. In our setup, we exploit the birefringence of the BBO crystal to arrange the conservation. As a consequence, the SPDC photons are emitted on two cones as can be seen in Figure 1, with one cone being an extraordinary, vertically polarized (signal) photon and the other cone being an ordinary, horizontally polarized (idler) photon.

Polarization entangled photons can be obtained by spatial and spectral filtering of the photons emerging on the intersection lines of the two cones. Energy conservation coupled with the crystal being cut for degenerate wavelength production guarantees that the photons will be indistinguishable according to their frequency. Momentum conservation guarantees that one photon will emerge from each side, but it will be impossible to distinguish which cone each photon came from. However, each of the two photons must be from a different cone, and thus they will always have opposite polarizations. Therefore, two photons will be emitted into two different spatial modes and they will have opposite polarizations but individually will have no definite polarization.

5 Experimental setup

Figure 2 shows the schematic setup of the entangled photon pair source. Via two folding mirrors the blue laser is focused into the BBO crystal, where the entangled photons are produced via spontaneous parametric down-conversion. The directions where the cones intersect are separated by two small prism mirrors and then collected via further lenses into optical fibers to eventually detect the two photons of a pair.

Due to the birefringence of the BBO crystal the propagation directions and velocities of horizontally and vertically polarized light are slightly different as they travel through the

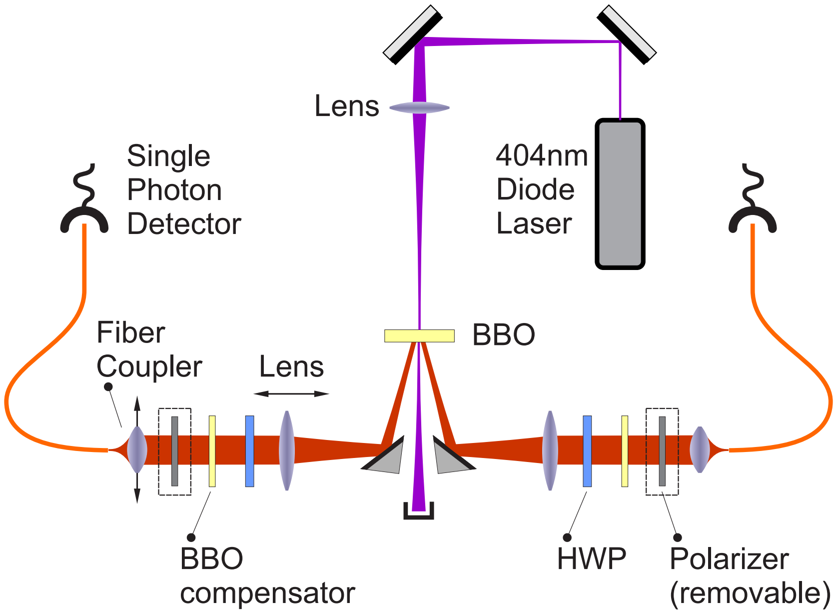

Figure 2: Schematic of the experimental setup. A blue laser pumps the BBO crystal to create photon pairs. The lines at which the H and V cones deviate from the pump beam by about 3°. Small prism mirrors separate the down-conversion beams, which pass a collimating lens, a half-wave-plate, the compensating BBO crystal and a removable polarizer, before getting coupled into a single-mode optical fiber for spatial selection and eventually detection. The collimating lenses can be translated along the beam axis for focusing, whereas the fiber couplers can be translated transverse to the beam direction for selecting the optimal directions. The actual setup includes color glass filters and bandpass filters for spectral selection (not shown here).

crystal. This leads to a transverse walk-off (spatial displacement) and longitudinal walk-off (temporal displacement) of the horizontally polarized photons from the vertically polarized photons. Thus the photons can be distinguished by their position and time-correlation relative to one another and thus do not contribute to the entanglement. To preserve the polarization entanglement, we will have to make sure that there is no information available that can be used to determine the polarization state of the photons. So, it is necessary to compensate the walk-off before the measurements. While the walk-off effects cannot be completely compensated, the distinguishability of the photons can be much reduced by the use of compensator BBO crystals. First, the down-converted photons have their polarizations rotated by 90° using a halfwave plate so that H is exchanged with V and vice versa. Each photon is then passed through a compensator BBO crystal which is half as thick as the original crystal that performs down-conversion. In this fashion, the walk-off effects are reversed by half, which consequently erases all temporal distinguishability and reduces the effects of the transverse walk-off. The temporal compensation can be seen in Figure 3.

In the setup additionally there are filters and polarizers. The filters remove stray pump light and other unwanted components. The polarizers can be inserted to perform the correlation measurements. The fibers are finally connected to single photon detectors (APD - avalanche photodiode), where the photons are counted. The APDs are connected to a logical circuit, to get the coincidence rates. In the computer software it is easy to see the count rates of the first

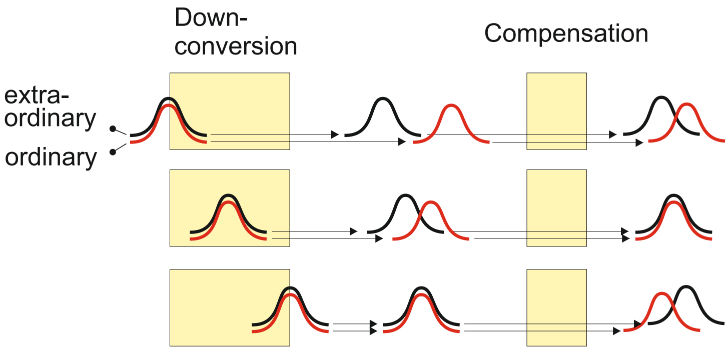

Figure 3: Walk off compensation proceeds by reversing by adding half of the inverse relative time delay between ordinary and extraordinary photons of a pair. In this way the relative time delay reveals no information about the polarization state of a photon.

and the second arm and also the coincidence rates.

6 Measurement

The main task for this experiment is to measure the coincidence rates between the photon pairs for different angles of Polarizer A and Polarizer B. The results from the measurements will be used to test the violation of Bell's inequality. Figure 4 shows such a measurement, where polarizer A was set to a fixed angle and polarizer B was rotated. The coincidence rate was recorded for each setting of polarizer B. When the polarizer A is set to $0°$ or to $90°$ the measurement is in the H/V basis and if it is set in $45°$ or $135°$ the measurement is in the D/A basis.

6.1 Measuring the Visibility

The visibility is defined by:

$$V = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \tag{10}$$

where I_{max} and I_{min} correspond to the expected and unexpected coincidence rates for the particular state being produced. In the case of the $|\psi^-\rangle$ state, the expected coincidences are for the anti-correlated measurements (HV, VH, DA, AD) while the rates of the unexpected events are for the correlated measurements (HH, VV, DD, AA). The names I_{max} and I_{min} are fairly intuitive since I_{max} should be a maximum and I_{min} should theoretically be zero for a

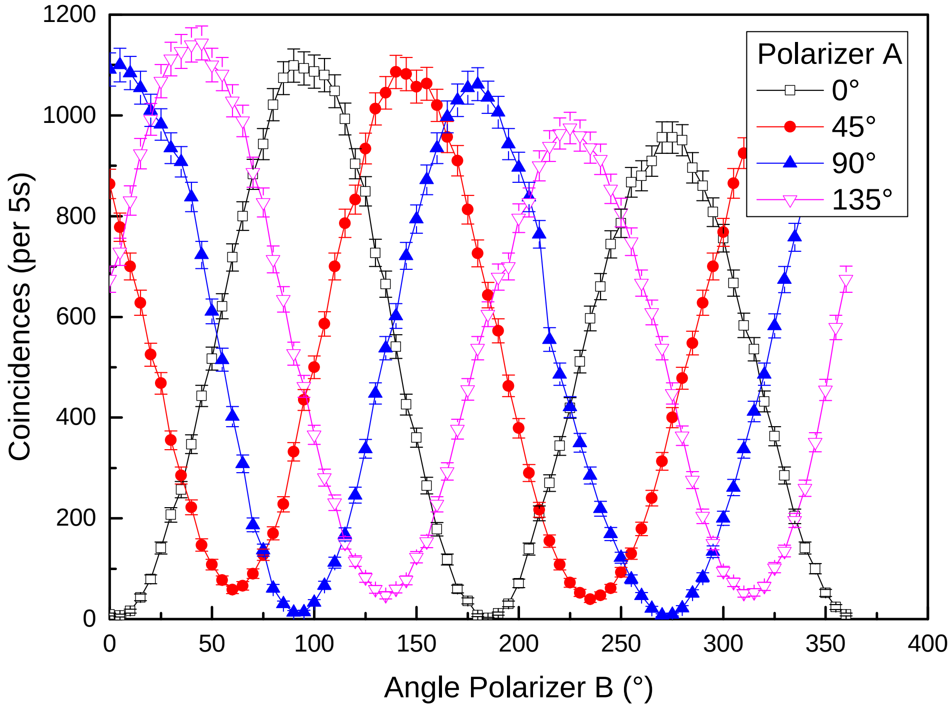

Figure 4: For four different settings of Polarizer A, this figure shows the coincidence count rates as a function of the angle of Polarizer B. The errorbars show the Poissonian standard deviation of the count rates. The lines connecting the symbols are only to guide the eye. From the data we could then evaluate the visibility, and it is apparent that we will obtain different results for the different curves. The visibility is much better for the H/V basis than for the D/A basis.

perfect $|\psi^-\rangle$ source. Therefore you can rewrite the visibility in the following way:

$$V = \frac{N_{\text{HV}} + N_{\text{VH}} - N_{\text{HH}} - N_{\text{VV}}}{N_{\text{HV}} + N_{\text{VH}} + N_{\text{HH}} + N_{\text{VV}}} \tag{11}$$

and

$$V = \frac{N_{\text{DA}} + N_{\text{AD}} - N_{\text{DD}} - N_{\text{AA}}}{N_{\text{DA}} + N_{\text{AD}} + N_{\text{DD}} + N_{\text{AA}}}, \tag{12}$$

respectively, where N_{HV} is the number of coincidences when the polarizer A is in H and the polarizer B is in V. A visibility of 100% is desired.

6.2 Bell Parameter

To calculate the Bell parameter (S) (and thus to see if the Bell inequality is violated or not) you have to measure the coincidence rates for the different angles. Basically you have to fill a table as given in Table 1. From this table you can then calculate the expectation values using equation 6 and then the Bell parameter with equation 1.

Polarizer angle (A/B)	α/β $++$	$\alpha/\beta+90°$ $+-$	$\alpha+90°/\beta$ $-+$	$\alpha+90°/\beta+90°$ $--$
$\alpha=0°/\beta=22.5°$				
$\alpha=0°/\beta=67.5°$				
$\alpha=45°/\beta=22.5°$				
$\alpha=45°/\beta=67.5°$				

Table 1: Matrix of required measurements for testing Bell's inequality.

7 Experiment

7.1 Alignment

The experiment is pre-aligned by the instructor, because it would take too much time to do this from scratch in one afternoon. However, you are expected to optimize the setup, but please proceed with caution and only make small changes, one at a time. The alignment proceeds as follows. Initially we align for the highest single count rate by moving the fiber coupler transversally to the beam while also adjusting the pointing of the fiber coupler and by adjusting the focus of the collimating lens. Second one checks with a polarizer, whether the count rate for H and V polarizations are similar. If they differ by much, more alignment of the coupling lens is required. This may reduce the overall count rate a bit. As a next step we monitor the coincidence rate and try to maximize it again by moving the coupling lenses on both sides. Once we have a useful number of coincidence counts (a few hundred per second) we check how the H-V and V-H coincidence count rates compare. They should be roughly equal. If not, more alignment of the coupling lenses is required. In particular we may need to "walk the beam" by using a small transverse movement on one side (with the appropriate angular adjustment) that is "compensated" by a small movement on the other side. If the H-V and the V-H coincidence count rates are equal to within 20%, we can check the H-H and the V-V coincidence count rates. They should be extremely small. If not, alignment of the HWPs might be necessary. After that the polarizers should be set to D-D and/or A-A to optimize the compensation crystal angle. By adjusting the compensation crystal vertical tilt angle we can set the phase of the state to be $|\psi^-\rangle$ or $|\psi^+\rangle$.

7.2 Correlation measurements

1. Set the Polarizer A to 0° and record coincidence rate by setting Polarizer B from 0° to 200° in steps of 5°.

2. Set the Polarizer A to 90° and record coincidence rate by setting Polarizer B from 0° to 200° in steps of 5°.

3. Set the Polarizer A to 45° and record coincidence rate by setting Polarizer B from 0° to 200° in steps of 5°.

4. Set the Polarizer A to 135° and record coincidence rate by setting Polarizer B from 0° to 200° in steps of 5°.

7.3 Bell measurements

Record the coincidence rates at the Bell angles (16 measurements) as shown in Table 1 for an extended counting period (e.g. 100 s) to get better statistics.

7.4 Data processing and results

The minimum required data to present in your lab report are

1. The visibilities for the HV and DA bases.
2. The correlation curves (coincidence rate vs. angle for the four different settings of α).
3. Variation of the Bell-parameter S with the angle between α and β, but keeping $\alpha' - \alpha = \beta' - \beta = 45°$ constant.
4. Value of S for the extended measurement. Check this result for consistency with the visibilities you report earlier.

Additionally, if you want to improve your report you can

1. Calculate the efficiency of the SPDC process from the laser input power and the ratio of coincidence to single count rates.
2. Check how well the fluctuations of the coincidence count rates taken for the extended measurement period conform to what we would expect for a Poissonian distribution.
3. Try to fit the correlation curves with sinusoidal fit functions.
4. Try to work out the functional dependence of S on the $\alpha - \beta$ and fit this function to the data you plotted earlier. This should yield the position of the maximum, which we expect at 22.5°.

8 Questions for your preparation

1. Why is the polarization of the photon a two-state system, given that the photon is a boson?
2. Calculate the form of $|\psi^+\rangle$ in the D/A basis starting from the state's representation in the H/V Basis
3. What is the quantum mechanical state corresponding to an arbitrary linear polarizer at angle α?
4. Assuming a $|\psi^-\rangle$ state, calculate explicitly the coincidence detection probability for linear polarizer angles α and β.
5. The state of one of the photons, ignoring the other one is mathematically obtained by performing a partial trace, e.g. for photon A, $\rho_A = \mathrm{Tr}_B\{\rho_{AB}\}$, where $\mathrm{Tr}_B\{|a_1\rangle\langle a_2| \otimes |b_1\rangle\langle b_2|\} = |a_1\rangle\langle a_2| \,\mathrm{Tr}\{|b_1\rangle\langle b_2|\}$ and the (partial) trace is linear. Calculate ρ_A for $\rho_{AB} = |\psi^-\rangle\langle\psi^-|$. What is the meaning of the state (density matrix) that you obtain? Try to re-express the result in the AD basis!

6. Can anything interacting with photon B change the reduced state of photon A? Discuss this question in the light of the result obtained for the previous question. What can you conclude about using entanglement for communication?

7. The type-II SPDC source does not produce a perfect state. A good model for the actual state is a mixture of mostly $|\psi^-\rangle$ plus a small $|\psi^+\rangle$ contribution. Write down the density matrix of this state, if the probability of the $|\psi^-\rangle$ state is F. Can you connect the visibility V and the quantity F?

8. If the observed correlations have a visibility V smaller than one, what is the minimum value of V for which the CHSH inequality is violated?

9. The overall collection and detection efficiency of our setup is only a few percent. Formulate a hidden variable model for the polarization of the two photons that exploits this inefficiency to achieve a violation of Bell's inequality.

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