

Advanced laboratory class 3

## FP3 - SQUID

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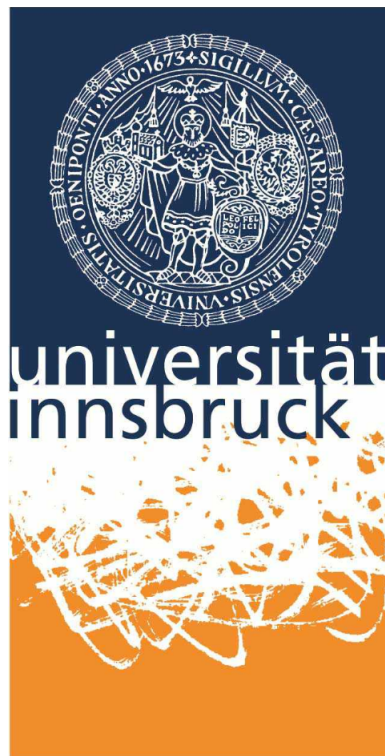
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### Abstract

In the course of this experiment a look was taken at some basic properties of superconductors and SQUIDs, like the current-voltage and the current-flux characteristics. Additionally, Shapiro-steps were observed from which the  $e/h$ -ratio can be calculated. Finally, the resistance of the SQUID was measured, dependent on the temperature.



# 1 Introduction

In most solid body physics courses one learns that the resistance of a substance is caused by electron scattering on obstacles like phonons, impurities in the lattice, or other electrons.[2] This model predicts that the resistance decreases with decreasing temperature, but ultimately remains at a finite, non-zero value due to impurities in the lattice. However, in 1911 Heike Onnes discovered a drastic and unforeseen decrease in resistance of mercury at 4.2 K, later terming the observed phenomenon superconduction.

Mercury has passed into a new state which, on account of its extraordinary electrical properties, may be called the superconductive state.

– HEIKE KAMERLINGH ONNES, according to [2]

In 1933 Walther Meißner and Robert Ochsenfeld discovered that in addition to being perfect conductors, superconducting materials also are perfect diamagnets. Today, applications of superconduction range from Magnetic Resonance Imaging (MRI) in medicine to particle accelerators, where superconducting coils are used to generate powerful magnetic fields.[2]

## 2 Theoretical Background

The theoretical introduction in this report is split into two parts, one being a general introduction to superconductivity, while the other one is about the Josephson Effect and its application in this experiment. The presented information is based on the “Mr. SQUID”-manual [1] as well as the book “*Festkörperphysik*” [2].

### 2.1 Superconductors

### 2.2 Josephson Effect and SQUID

#### DC Josephson Effect

The Josephson effect is caused by the long range quantum interaction in superconductors. The junction itself can be seen as a gap or barrier between two superconductors as is sketched in Fig. 1. In the Mr. SQUID [1] device, the junctions are grain boundary weak-link junctions.

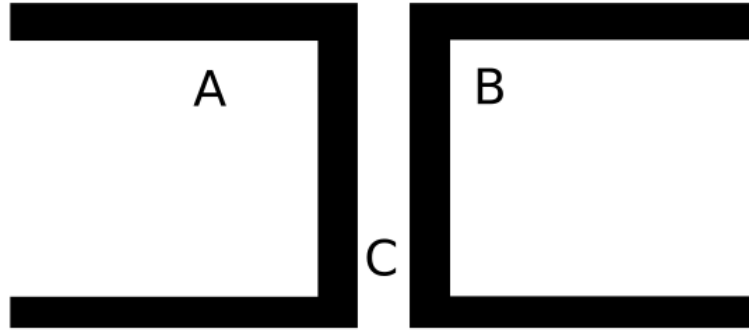


Figure 1: Sketch of a Josephson junction. The wavefunctions with respective phases  $\varphi_A$  and  $\varphi_B$  interfere over the barrier C.

The supercurrent flowing through the junction is dependent on the density of electron-electron pairs or Cooper pairs  $n_{S,A}$  and  $n_{S,B}$ , the phases of the two macroscopic wave functions  $\Psi_A$  and  $\Psi_B$  and the properties of the junction. Assuming that the coupling is small, it follows that  $|\Psi_A|^2 = n_{S,A}$  and  $|\Psi_B|^2 = n_{S,B}$  remain approximately constant and should have the same value  $n_S$ . According to [2], the most general expression for the supercurrent density is then

$$J_S = q_S n_S(\vec{r}, t) \left[ \frac{\hbar}{m_S} \nabla \Theta(\vec{r}, t) - \frac{q_S}{m_S} \vec{A}(\vec{r}, t) \right] = \frac{q_S n_S \hbar}{m_S} \vec{\gamma}(\vec{r}, t) \quad (1)$$

with  $\vec{\gamma}(\vec{r}, t) = \nabla \Theta - \frac{2\pi}{\Phi_0} \vec{A}(\vec{r}, t)$ . It is fair to assume that the phase gradient in the barrier is big compared to the gradient in the superconductor. Assuming further that the phase gradient  $\vec{\gamma}$  is constant over the small junction,

one can introduce a gauge invariant phase difference  $\phi(\vec{r}, t)$  as

$$\begin{aligned}\varphi(\vec{r}, t) &= \int_A^B \vec{\gamma}(\vec{r}, t) = \int_A^B \left[ \nabla \Theta - \frac{2\pi}{\Phi_0} \vec{A}(\vec{r}, t) \right] d\vec{l} \\ &= \Theta_B(\vec{r}, t) - \Theta_A(\vec{r}, t) - \frac{2\pi}{\Phi_0} \int_A^B \vec{A}(\vec{r}, t) d\vec{l}\end{aligned}\quad (2)$$

According to [1], the supercurrent  $I_S$  solely depends on the phase difference  $\varphi$ . Furthermore, the wave functions  $\Psi_A$  and  $\Psi_B$  have to be invariant under a phase change of a multiple of  $2\pi$ .

$$I_S(\varphi) = I_S(\varphi + n2\pi) \quad (3)$$

Also, if there is no supercurrent in the superconductor, the phase difference has to be 0. Hence, one can write

$$I_S(\varphi = 0) = 0 = I_S(n2\pi). \quad (4)$$

Combining these results leads to sinusoidal dependence of the supercurrent  $I_S$  on the phase difference  $\phi$ .

$$I_S(\varphi) = I_C \sin(\varphi) + \underbrace{\sum_{m=2}^{\infty} I_m \sin(m\varphi)}_{\approx 0} \quad (5)$$

Eq. 5 is often referred to as the first Josephson equation. The higher order terms are often ignored as they are usually very small.

The second Josephson equation can be calculated from the time derivative of Eq. 2, which is

$$\frac{\partial \varphi}{\partial t} = \frac{\partial \Theta_B}{\partial t} - \frac{\partial \Theta_A}{\partial t} - \frac{2\pi}{\Phi_0} \frac{\partial}{\partial t} \int_A^B \vec{A}(\vec{r}, t) d\vec{l}. \quad (6)$$

Using the energy-phase relation form [2], which is given as

$$-\hbar \frac{\partial \Theta}{\partial t} = \frac{1}{2n_S} \Lambda \vec{J}_S^2 + q_S \phi + \mu, \quad (7)$$

where  $\Lambda = \frac{m_S}{n_S q_S^2}$  is the London coefficient,  $q_S = -2e$  is the charge of a Cooper pair,  $\phi$  is a gauge scalar field and  $\mu$  is the chemical potential. This yields the following expression:

$$\frac{\partial \varphi}{\partial t} = -\frac{1}{\hbar} \left( \frac{\Lambda}{2n_S} \underbrace{[\vec{J}_S^2(B) - \vec{J}_S^2(A)]}_{=0} + q_S [\phi(B) - \phi(A)] + [\mu(B) - \mu(A)] \right) - \frac{2\pi}{\Phi_0} \frac{\partial}{\partial t} \int_A^B \vec{A}(\vec{r}, t) d\vec{l} \quad (8)$$

Continuity requires that the currents at both sides of the junction are equal in size. This then leads to the second Josephson equation:

$$\frac{\partial \varphi}{\partial t} = \frac{2\pi}{\Phi_0} \int_A^B \left[ -\nabla \tilde{\phi} - \frac{\partial \vec{A}}{\partial t} \right] d\vec{l} = \frac{2\pi}{\Phi_0} \int_A^B \vec{E}(\vec{r}, t) d\vec{l} = \frac{2\pi}{\Phi_0} U, \quad (9)$$

where  $U$  is the voltage over the junction. It follows that the phase increases with time, resulting in

$$\varphi(t) = \varphi_0 + \frac{2\pi}{\Phi_0} U t. \quad (10)$$

## AC Josephson Effect

If one now adds an alternating current to the DC voltage  $U = U_0 + U_1 \sin(\omega_1 t)$  in Eq. 10, one gets

$$\varphi(t) = \varphi(0) + \frac{2eU_0}{\hbar} t + \frac{2eU_1}{\hbar\omega_1} \sin(\omega_1 t). \quad (11)$$

This expression can now be inserted into the first Josephson equation, which yields

$$I_S(\varphi) = I_C \sin \left( \varphi(0) + \frac{2eU_0}{\hbar} t + \frac{2eU_1}{\hbar\omega_1} \sin(\omega_1 t) \right). \quad (12)$$

Using the first order Bessel function  $\mathcal{J}$ , one can write

$$J_S(t) = J_C \sum_{n=0}^{\infty} \mathcal{J}_n \left[ \frac{2eU_1}{\hbar\omega_1} \right] \underbrace{\sin \left( \varphi(0) + \frac{2eU_0}{\hbar} t \pm n\omega_1 t \right)}_{=const \Leftrightarrow U_0 = n \frac{\hbar\omega_1}{2e}}. \quad (13)$$

Since one usually measures the DC components in the experiment, one has to find the DC voltage at which the sine becomes time-independent. This is true for  $U_0 = n \frac{\hbar\omega_1}{2e}$ . If one measures the  $I$ - $V$  characteristic of a Josephson junction, one will see steps in the curve. The height of the steps is determined by the amplitude of the alternating current  $U_1$  and the corresponding Bessel function  $\mathcal{J}_n$ . They are often called Shapiro steps.

## Superconducting QUantum Interference Device - SQUID

A simple circuit diagram of a SQUID is displayed in Fig. 2. It shows that the Josephson junctions are set up in a parallel way and are connected with superconducting material. Assuming that the critical current  $I_C$  is equal for both junctions, one can use the phase-current relation from [2], which says that the currents in then junctions can be expressed as  $I_{S,1} = I_C \sin(\varphi_1)$  and  $I_{S,2} = I_C \sin(\varphi_2)$ . Using Kirchhoff's law and the addition

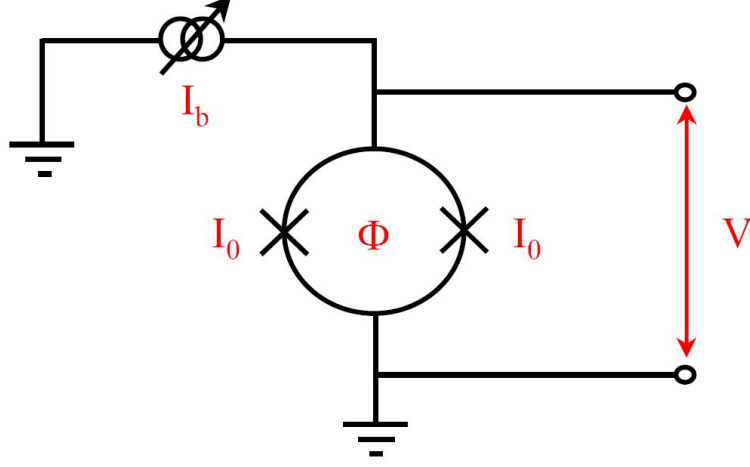


Figure 2: Schematic of a DC SQUID, taken from [3]. An input bias current  $I_b$  produces a voltage over the two parallel arranged Josephson junctions, through which a current  $I_0$  is passing. If one now adds a magnetic flux  $\Phi$  through the SQUID, an additional current  $I_S$  is induced, which is running circular in the SQUID, adding to the bias current in the Josephson junctions. This induced current reduces the critical current of the SQUID.

theorems, one can write the total current as

$$I_S = I_{S,1} + I_{S,2} = 2I_C \cos\left(\frac{\varphi_2 - \varphi_1}{2}\right) \sin\left(\frac{\varphi_1 + \varphi_2}{2}\right) \quad (14)$$

According to [2], one can further argue that in the total phase change across the ring, the phase change in the superconductor is negligible compared to the change in the Josephson junctions. Therefore, one can reach the following equation using Eq. 2.

$$\varphi_1 - \varphi_2 = -\frac{2\pi}{\Phi_0} \oint \underbrace{\vec{A} d\vec{l}}_{=\Phi} \quad (15)$$

Here  $\Phi$  is the flux through the SQUID. Inserting this into Eq. 14 leads to

$$I_S^{\max} = 2I_C \cos\left(\frac{\pi\Phi}{\Phi_0}\right) \sin\left(\varphi_1 + \frac{\pi\Phi}{\Phi_0}\right) \stackrel{\Phi=\Phi_{\text{ext}}}{=} 2I_C \left| \cos\left(\frac{\pi\Phi_{\text{ext}}}{\Phi_0}\right) \right|. \quad (16)$$

This equation finally shows the critical maximum the supercurrent can reach when a magnetic field is applied on the SQUID.  $I_C$  in this equation means the critical current per junction, which is half the critical current of the SQUID.

## 3 Experiment setup

## 4 Analysis

## References

- [1] STAR Cryoelectronics, *Mr. SQUID User's Guide*. RANDY W. SIMON, MICHAEL J. BURNS, MARK S. COLCLOUGH, GREG ZAHARCHUK, ROBIN CANTOR.
- [2] De Gruyter Studium, *Festkörperphysik*. RUDOLF GROSS, ACHIM MARX. 3. Edition.
- [3] Wikimedia, JAN OLAF. [https://upload.wikimedia.org/wikipedia/commons/7/71/SQUID\\_IV.jpg](https://upload.wikimedia.org/wikipedia/commons/7/71/SQUID_IV.jpg).