Generation and manipulation of entangled photon pairs in coupled resonators

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July 19, 2017

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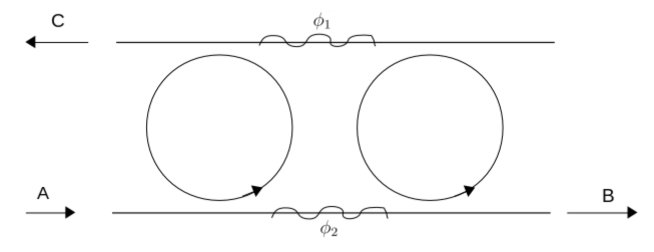
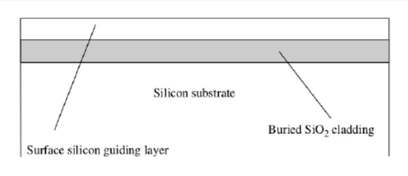
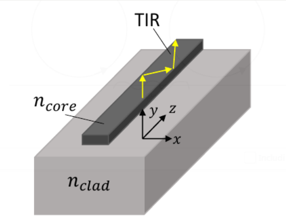
Aim of the thesis



- Input coherent state
- Field Enhancement
- Spontaneous Four Wave Mixing $\implies$ photons generation
- Heaters can change the phases $\implies$ manipulation
- Output states

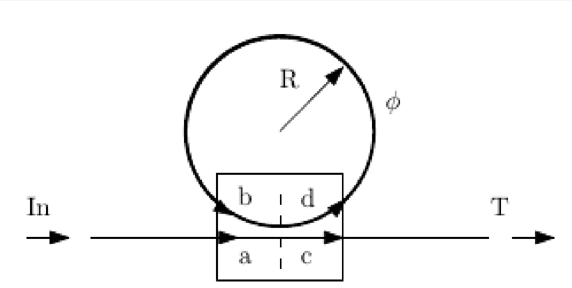
Silicon photonic





Typical dimension:	substrate	$\simeq 700\mu m$
	cladding	$\simeq \mu m$
	core	$\simeq 10^2$ nm

Resonator APF



Constructive interference $n_{eff} L = m\lambda_m \implies$ field enhancement
$\implies$ non linear effects

Coupling

Gap in the order of $\simeq 100$ nm $\implies$ evanescent field
In quantum mechanics $\implies$ quantum tunneling

Mathematically can be treated as a Quad port beam splitter

$$\begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} r & ik \\ ik & r \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \tag{1}$$

r reflection coefficient, k transmission coefficient $k^2 + r^2 = 1$

Resonator APF

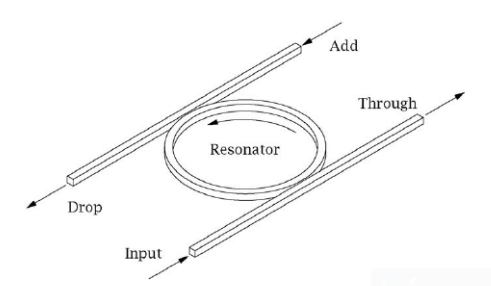
with the roundtrip phase condition

$$b = e^{-\alpha 2\pi R} e^{-i\beta 2\pi R} d \equiv \tau e^{-i\phi(\lambda)} d$$

the Transfer function can be found

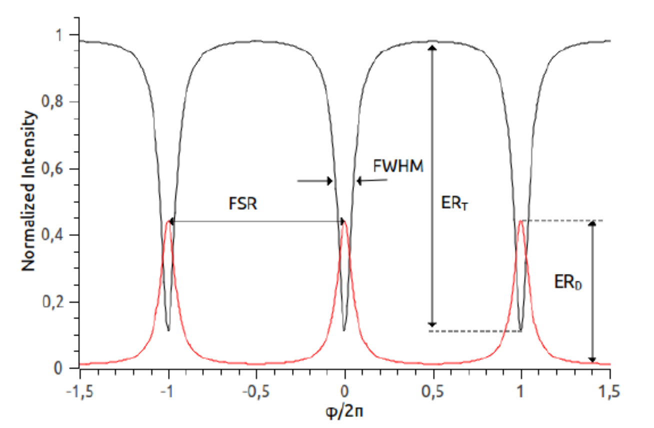
$$H_{AP} = \frac{c}{a} = \frac{\tau - re^{i\phi(\lambda)}}{r\tau - e^{i\phi(\lambda)}} \tag{2}$$

Resonator ADF

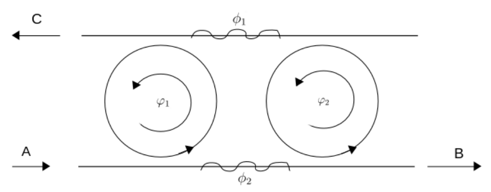


$$H_{AD}^{T} = \frac{k^2\sqrt{\tau}e^{i\phi/2}}{r^2\tau - e^{i\phi}} \qquad H_{AD}^{D} = \frac{r(e^{i\phi} - \tau)}{e^{i\phi} - r^2\tau} \tag{3}$$

Transfer function



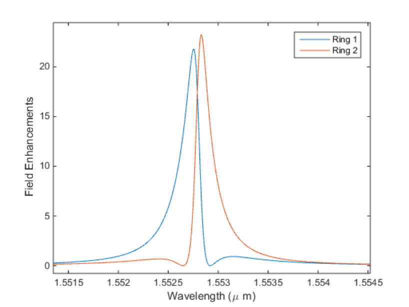
Coupled resonators



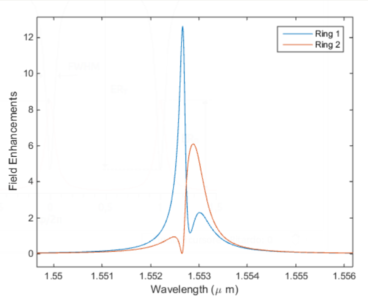
The most important quantities are the field enhancements for the two rings, they need to be equal for both ring

$$\phi_1 + \phi_2 = 2m\pi$$

Field enhancements



$\phi_1 + \phi_2 = 2\pi$



$\phi_1 + \phi_2 = -\pi$

Full expressions

$$FE_1 = \frac{ie^{i(\varphi_1+\phi_2+\phi_1)}k(e^{i\varphi_2} - r^2\tau_2) - ie^{\frac{i}{2}(\varphi_1+\varphi_2)}k^3(k^2+r^2)\sqrt{\tau_1\tau_2}\tau L^2}{e^{i(\phi_1+\phi_2)}(e^{i\varphi_1} - r^2\tau_1)(e^{i\varphi_2} - r^2\tau_2) - e^{\frac{i}{2}(\varphi_1+\varphi_2)}k^4\sqrt{\tau_1\tau_2}\tau L^2}$$

$$FE_2 = \frac{ie^{i(\varphi_2+\phi_1)}kr(-e^{i\varphi_1} + (r^2+k^2)\tau_1)\tau L}{e^{i(\phi_1+\phi_2)}(e^{i\varphi_1} - r^2\tau_1)(-e^{i\varphi_2} + r^2\tau_2) + e^{\frac{i}{2}(\varphi_1+\varphi_2)}k^4\sqrt{\tau_1\tau_2}\tau L^2}$$

NON LINEAR OPTICS

Polarization

In first approximation

$$\mathbf{P} = \varepsilon_0 \chi \mathbf{E} \tag{4}$$

which can be expandend in

$$P_i = \varepsilon_0 \Big(\sum_j \chi_{ij}^{(1)} E_j + \sum_{j,k} x_{ijk}^{(2)} E_j E_k + \sum_{j,k,l} \chi_{ijkl}^{(3)} E_j E_k E_l + \dots \Big) \tag{5}$$

silicon is a centrosymmetric crystal $\implies \chi^{(2)} = 0$, the first non-linear order is the third one: Kerr nonlinearity

Polarization

Wave equation can be written as

$$\left(\nabla^2 - \frac{n^2}{c^2}\frac{\partial^2}{\partial t^2}\right)E = \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2} \tag{6}$$

If we take

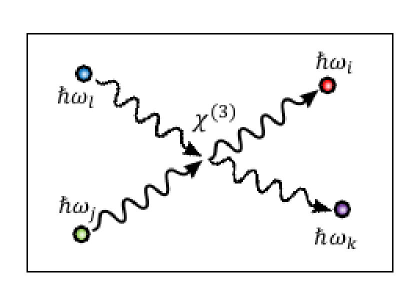
$$E = \sum_{j=1}^{4} \frac{1}{2}(A_j(z)e^{i(\omega_j t - k_j z)} + \text{c.c})$$

in $P_{NL} = \varepsilon_0 \chi^{(3)} E^3$ there are terms

$$\sum_{l,j,m=\pm1,\pm2,\pm3,\pm4} \left(\frac{1}{8}A_l A_j A_l e^{i(\omega_l+\omega_j+\omega_m)t} e^{-i(k_l+k_j+k_m)z}\right)$$

Non-linear polarization act as a source $\implies$ new frequencies arise

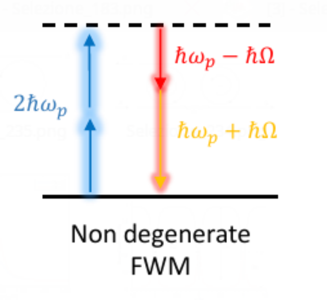
Four Wave Mixing



Frequency of Four Wave Mixing $\omega_l = \omega_i - \omega_j + \omega_k$ phase matching condition $k_l = k_i - k_j + k_k$

Can be seen as a simultaneous creation of two photons and an annhilation of two photons

non degenerate Four Wave Mixing



$\omega_j = \omega_l \equiv \omega_p$ pump; $\omega_i \equiv \omega_s$; signal $\omega_k \equiv \omega_i$ idler
In the classical description the process is stimulated, in the quantum figure is spontaneous

Displacement field

in a medium **D** is divergenceless $\implies$ it is more convenient to take **D** as our fundamental field

$$P^i = \Gamma_1^{ij} D^j + \Gamma_2^{ijk} D^j D^k + \Gamma_3^{ijkl} D^j D^k D^l + \dots \tag{7}$$

summation implied

$$\Gamma_3^{ijkl} = \frac{\chi_{ijk}^{(3)}}{\varepsilon_0 n^2(\omega_l) n^2(\omega_j) n^2(\omega_i) n^2(\omega_k)} \tag{8}$$

Field quantization

The displacement field is quantizied and written in terms of the annhilation operator a_k and the creation operator $a_k^\dagger$

$$\mathbf{D}(\mathbf{r}) = \sqrt{\frac{\hbar\omega_k}{2}} a_k \mathbf{D}_k(\mathbf{r}) + \sqrt{\frac{\hbar\omega_k}{2}} a_k^\dagger \mathbf{D}_k^*(\mathbf{r}) \tag{9}$$

Linear Hamiltonian

$$H_L = \int dk \hbar\omega_k a_k^\dagger a_k \tag{10}$$

Coherent state

Laser input state is a coherent state

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{+\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle \tag{11}$$

$|\alpha|^2$ is the average photon number of the field.

$$|\alpha\rangle = \hat{D}(\alpha)\,|vac\rangle = e^{\alpha a_k^\dagger - \mathrm{H.c}}\,|vac\rangle \tag{12}$$

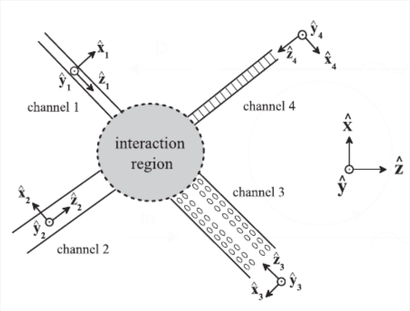
generalized

$$\hat{D}(\alpha) = e^{\alpha A_P^\dagger - \mathrm{H.c}} \tag{13}$$

$A_P^\dagger = \int dk \phi_P(k) a_k^\dagger$, where $|\phi(k)|^2$ is the probability of finding a photon with wavevector k

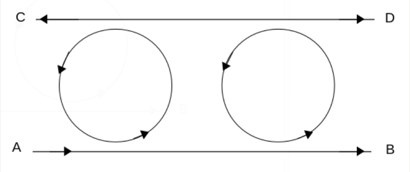
$$\int dk |\phi_P(k)|^2 = 1$$

Asymptotic field



The idea behind this theory is to provide an easier way to study complex stuctures.
An asymptotic-in and an asymptotic-out state are introduced

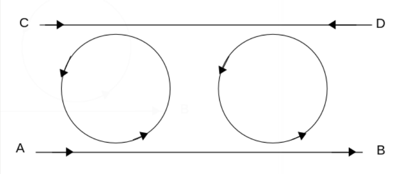
Asymptotic-in



An asymptotic-in far from the interaction region corresponds to a wave incoming in channel n and outgoing waves in every other channel

$$\mathbf{D}_{n,k}^{asy-in} \sim \mathbf{D}_{n,k}(\mathbf{r}_n) + \sum_{n'} \int_0^{+\infty} dk' \, T_{n,n'}^{out}(k,k') \mathbf{D}_{n',-k'}(r_{n'}) \quad (14)$$

Asymptotic-out



An asymptotic-out far from the interaction region corresponds to a wave outgoing in channel n and incoming waves in every other channel

$$\mathbf{D}_{n,k}^{asy-out} \sim \mathbf{D}_{n,-k}(\mathbf{r}_n) + \sum_{n'} \int_0^{+\infty} dk' \, T_{n,n'}^{in}(k,k') \mathbf{D}_{n',k'}(r_{n'}) \quad (15)$$

Field operator

we can write the field operator as a superposition of asymptotic-out fields

$$\mathbf{D}(\mathbf{r}) = \sum_n \int_0^{+\infty} dk \sqrt{\frac{\hbar\omega_{n,k}}{2}} a_{n,k} \mathbf{D}_{n,k}^{asy-in}(\mathbf{r}) + \text{H.c} \tag{16}$$

$$[a_{n,k}, a_{n',k'}] = 0 \qquad [a_{n,k}, a^\dagger_{n',k'}] = \delta_{nn'}\delta(k-k') \tag{17}$$

or asymptotic-out fields

$$\mathbf{D}(\mathbf{r}) = \sum_n \int_0^{+\infty} dk \sqrt{\frac{\hbar\omega_{n,k}}{2}} b_{n,k} \mathbf{D}_{n,k}^{asy-out}(\mathbf{r}) + \text{H.c} \tag{18}$$

$$[b_{n,k}, b_{n',k'}] = 0 \qquad [b_{n,k}, b^\dagger_{n',k'}] = \delta_{nn'}\delta(k-k') \tag{19}$$

With this approach the linear Hamiltonian can be written with the state of the whole structure, hence the linear Hamiltonian is diagonal

Non linear Hamiltonian

The non linear Hamiltonian is

$$H_{NL} = -\frac{1}{3\varepsilon_0} \int \Gamma_3^{ijkl}(\mathbf{r}) D^i D^j D^k D^l \, d\mathbf{r} \tag{20}$$

Since the asymptotic states are a complete basis we can expand D either on the asympotic-out states, or on the asymptotic-in, we can choose to expand two fields of this equation in terms of the asymptotic-in and the other two in terms of the asympotic-out.

Non linear Hamiltonian

$$
\begin{aligned}
H_{NL} = &- \int dk_1 dk_2 dk_3 dk_4 S_{bb}(k_1, k_2, k_3, k_4) a_{k_1} a_{k_2} b^\dagger_{b,k_3} b^\dagger_{b,k_4} \\
&- 2 \int dk_1 dk_2 dk_3 dk_4 S_{bc}(k_1, k_2, k_3, k_4) a_{k_1} a_{k_2} b^\dagger_{b,k_3} b^\dagger_{c,k_4} \\
&- \int dk_1 dk_2 dk_3 dk_4 S_{cc}(k_1, k_2, k_3, k_4) a_{k_1} a_{k_2} b^\dagger_{c,k_3} b^\dagger_{c,k_4} + \text{H.c.}
\end{aligned}
\tag{21}
$$

where a_k is the annihilation operator associated with channel A, $b_{b,k}$ is for channel B and $b_{c,k}$ refers to channel C

$$
\begin{aligned}
S_{xy}(k_1, k_2, k_3, k_4) = \frac{3}{2\varepsilon_0} \sqrt{\frac{(\hbar\omega_{k_1})(\hbar\omega_{k_2})(\hbar\omega_{k_3})(\hbar\omega_{k_4})}{16}} \cdot \\
\int d\mathbf{r} \Gamma_3^{ijkl} D^{i,asy-in}_{a,k_1}(\mathbf{r}) D^{j,asy-in}_{a,k_2}(\mathbf{r}) \left[D^{k,asy-out}_{x,k_3}(\mathbf{r}) \right]^* \left[D^{l,asy-out}_{y,k_4}(\mathbf{r}) \right]^*
\end{aligned}
\tag{22}
$$

Dynamics

Full evolution of the state subject to the full Hamiltonian $H = H_L + H_{NL}$

$$|\psi(t_1)\rangle = e^{-\frac{i}{\hbar}H(t_1 - t_0)}\,|\psi(t_0)\rangle \tag{23}$$

we can introduce an asymptotic-in state

$$|\psi_{in}\rangle = e^{-\frac{i}{\hbar}H_L(0 - t_0)}\,|\psi(t_0)\rangle = e^{\frac{i}{\hbar}H_L t_0}\,|\psi(t_0)\rangle \tag{24}$$

and an asymptotic-out

$$|\psi(t_1)\rangle = e^{-\frac{i}{\hbar}H_L t_1}\,|\psi_{out}\rangle \tag{25}$$

with the relationship between them

$$|\psi_{out}\rangle = U(t_1, t_0)\,|\psi_{in}\rangle \tag{26}$$

$$U(t', t) = e^{\frac{i}{\hbar}H_L t'}\,e^{-\frac{i}{\hbar}H(t' - t)}e^{-\frac{i}{\hbar}H_L t} \tag{27}$$

Backward Heisenberg approach

the non-linear scattering problem is contained only in the transition $|\psi_{in}\rangle \to |\psi_{out}\rangle$ which can be solved with the Backward Heisenberg approach: if we assume

$$|\psi_{in}\rangle = e^{O} |vac\rangle \tag{28}$$

the asymptotic-out state is given by

$$|\psi_{out}\rangle = e^{\overline{O}(t_0)} |vac\rangle \tag{29}$$

$$\overline{O}(t) = U(t_1, t) O U^{\dagger}(t_1, t) \tag{30}$$