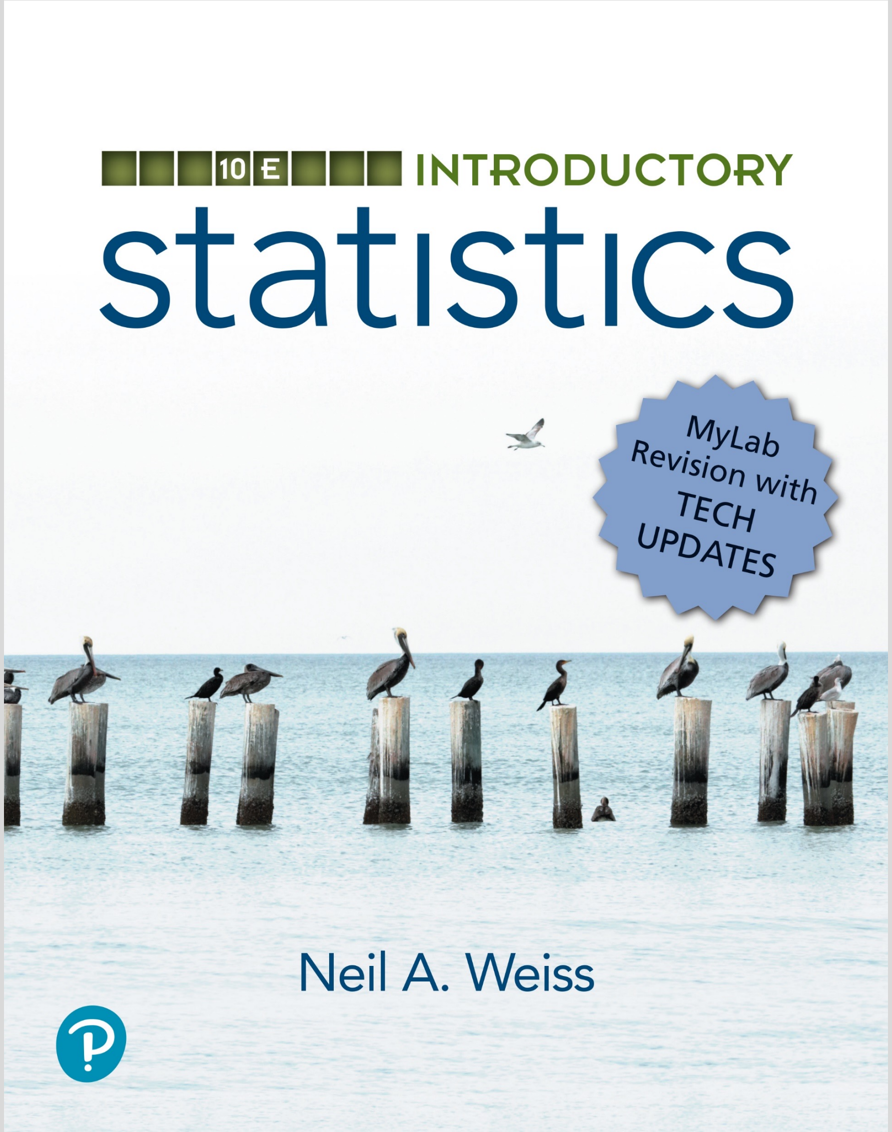
# Chapter 8

## Confidence Intervals for One Population Mean



# Chapter 8

## Confidence Intervals for One Population Mean

# Section 8.1

## Estimating a Population Mean

# Definition 8.1

**Point Estimate**

A **point estimate** of a parameter is the value of a statistic used to estimate the parameter.

# Definition 8.2

### Confidence-Interval Estimate

**Confidence interval (CI):** An interval of numbers obtained from a point estimate of a parameter.

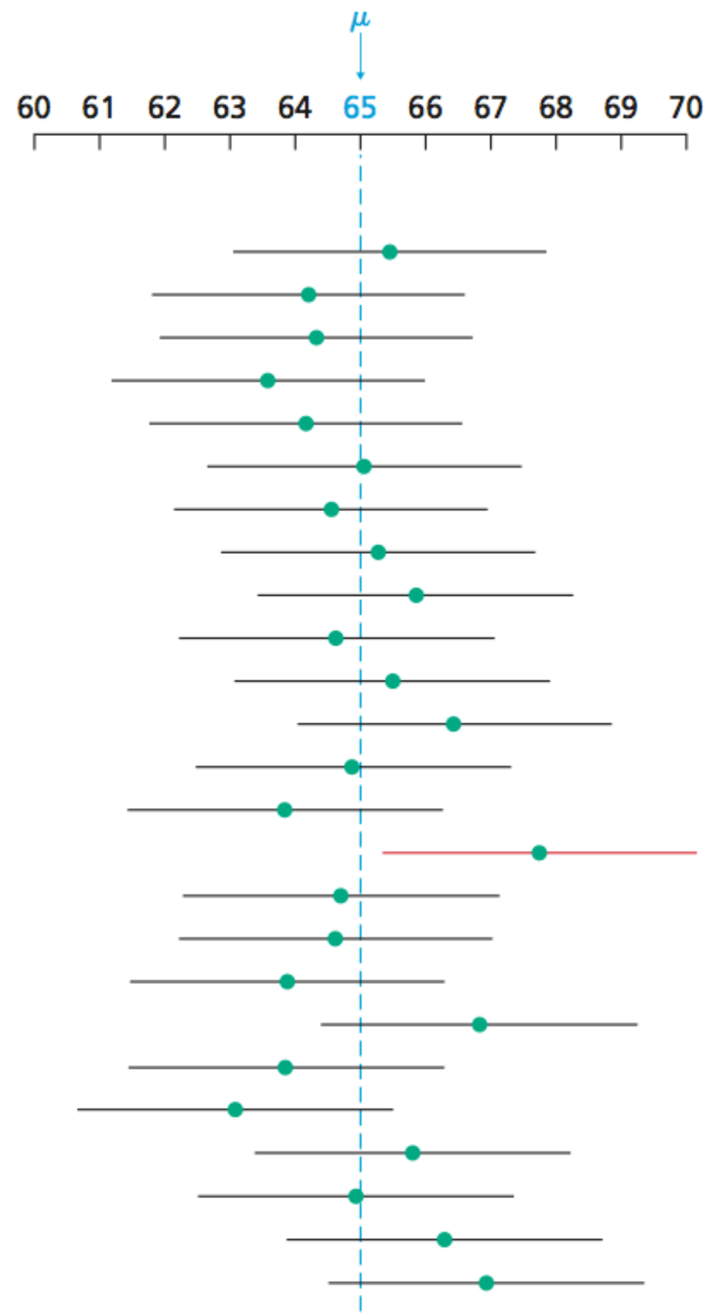
**Confidence level:** The confidence we have that the parameter lies in the confidence interval (i.e., that the confidence interval contains the parameter).

**Confidence-interval estimate:** The confidence level and confidence interval.

# Figure 8.2

Twenty-five confidence intervals for the mean price of all new mobile homes, each based on a sample of 36 new mobile homes

| Sample | $\bar{x}$ | 95% CI | $\mu$ in CI? |
|---|---|---|---|
| 1 | 65.45 | 63.05 to 67.85 | yes |
| 2 | 64.21 | 61.81 to 66.61 | yes |
| 3 | 64.33 | 61.93 to 66.73 | yes |
| 4 | 63.59 | 61.19 to 65.99 | yes |
| 5 | 64.17 | 61.77 to 66.57 | yes |
| 6 | 65.07 | 62.67 to 67.47 | yes |
| 7 | 64.56 | 62.16 to 66.96 | yes |
| 8 | 65.28 | 62.88 to 67.68 | yes |
| 9 | 65.87 | 63.47 to 68.27 | yes |
| 10 | 64.61 | 62.21 to 67.01 | yes |
| 11 | 65.51 | 63.11 to 67.91 | yes |
| 12 | 66.45 | 64.05 to 68.85 | yes |
| 13 | 64.88 | 62.48 to 67.28 | yes |
| 14 | 63.85 | 61.45 to 66.25 | yes |
| 15 | 67.73 | 65.33 to 70.13 | no |
| 16 | 64.70 | 62.30 to 67.10 | yes |
| 17 | 64.60 | 62.20 to 67.00 | yes |
| 18 | 63.88 | 61.48 to 66.28 | yes |
| 19 | 66.82 | 64.42 to 69.22 | yes |
| 20 | 63.84 | 61.44 to 66.24 | yes |
| 21 | 63.08 | 60.68 to 65.48 | yes |
| 22 | 65.80 | 63.40 to 68.20 | yes |
| 23 | 64.93 | 62.53 to 67.33 | yes |
| 24 | 66.30 | 63.90 to 68.70 | yes |
| 25 | 66.93 | 64.53 to 69.33 | yes |

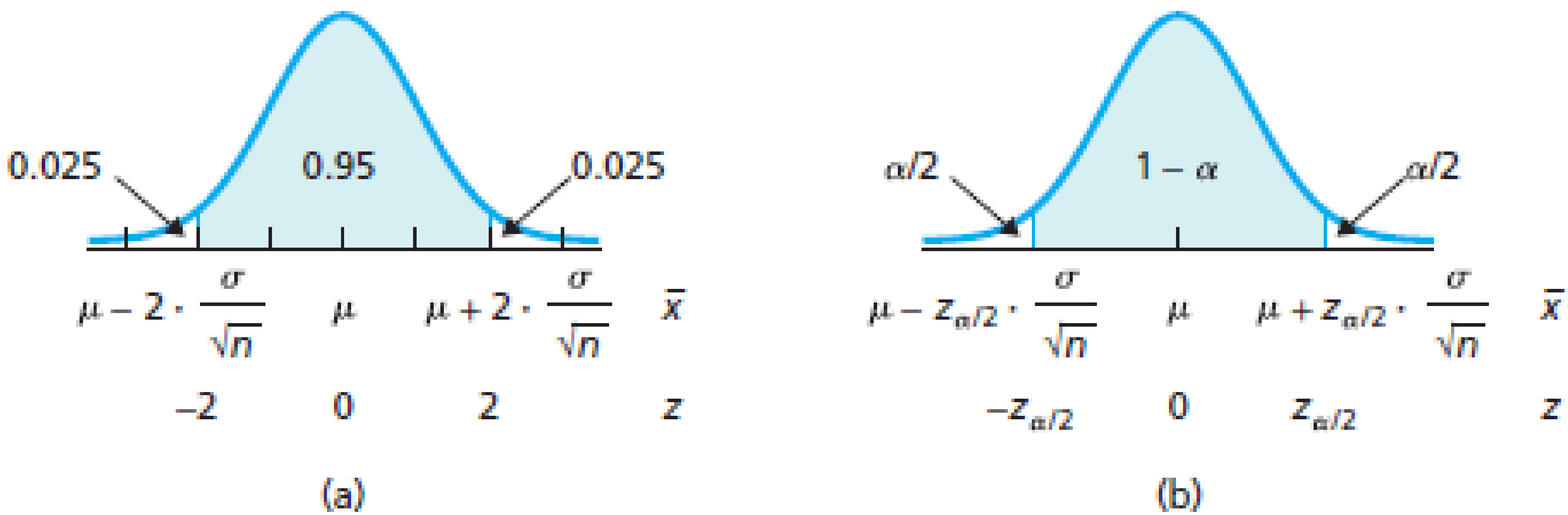


# Section 8.2

## Confidence Intervals for One Population Mean when $\sigma$ Is Known

# Figure 8.3

(a) Approximately 95% of all samples have means within 2 standard deviations of $\mu$; (b) 100(1-$\alpha$)% of all samples have means within $z_{\alpha/2}$ standard deviations of $\mu$.



# Procedure 8.1

## One-Mean z-Interval Procedure

***Purpose*** To find a confidence interval for a population mean, $\mu$

***Assumptions***

1. Simple random sample
2. Normal population or large sample
3. $\sigma$ known

**Step 1** **For a confidence level of $1 - \alpha$, use Table II to find $z_{\alpha/2}$.**

**Step 2** **The confidence interval for $\mu$ is from**

$$\bar{x} - z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \quad \text{to} \quad \bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}},$$

**where $z_{\alpha/2}$ is found in Step 1, $n$ is the sample size, and $\bar{x}$ is computed from the sample data.**

**Step 3** **Interpret the confidence interval.**

*Note:* The confidence interval is exact for normal populations and is approximately correct for large samples from nonnormal populations.

# Key Fact 8.1

## When to Use the One-Mean z-Interval Procedure

- For small samples – say, of size less than 15– the *z*-interval procedure should be used only when the variable under consideration is normally distributed or very close to being so.
- For samples of moderate size—say, between 15 and 30—the *z*-interval procedure can be used unless the data contain outliers or the variable under consideration is far from being normally distributed.
- For large samples—say, of size 30 or more—the *z*-interval procedure can be used essentially without restriction. However, if outliers are present and their removal is not justified, you should compare the confidence intervals obtained with and without the outliers to see what effect the outliers have. If the effect is substantial, use a different procedure or take another sample, if possible.
- If outliers are present but their removal is justified and results in a data set for which the *z*-interval procedure is appropriate (as previously stated), the procedure can be used.

# Key Fact 8.2

**A Fundamental Principle of Data Analysis**

Before performing a statistical-inference procedure, examine the sample data. If any of the conditions required for using the procedure appear to be violated, do not apply the procedure. Instead use a different, more appropriate procedure, if one exists.

# Table 8.4

Ages, in years, of 50 randomly selected people in the civilian labor force

| | | | | |
|---|---|---|---|---|
| 16 | 37 | 52 | 65 | 36 |
| 50 | 47 | 51 | 34 | 45 |
| 40 | 37 | 61 | 46 | 62 |
| 39 | 40 | 19 | 33 | 59 |
| 24 | 47 | 45 | 48 | 26 |
| 60 | 42 | 46 | 33 | 20 |
| 24 | 31 | 38 | 22 | 61 |
| 30 | 34 | 70 | 34 | 58 |
| 61 | 39 | 49 | 41 | 21 |
| 32 | 60 | 45 | 32 | 27 |

# Formula 8.1

**Margin of Error for the Estimate of $\mu$**

The margin of error for the estimate of $\mu$ is $z_{\alpha/2} \cdot \sigma/\sqrt{n}$, which is denoted by the letter *E*. Thus,

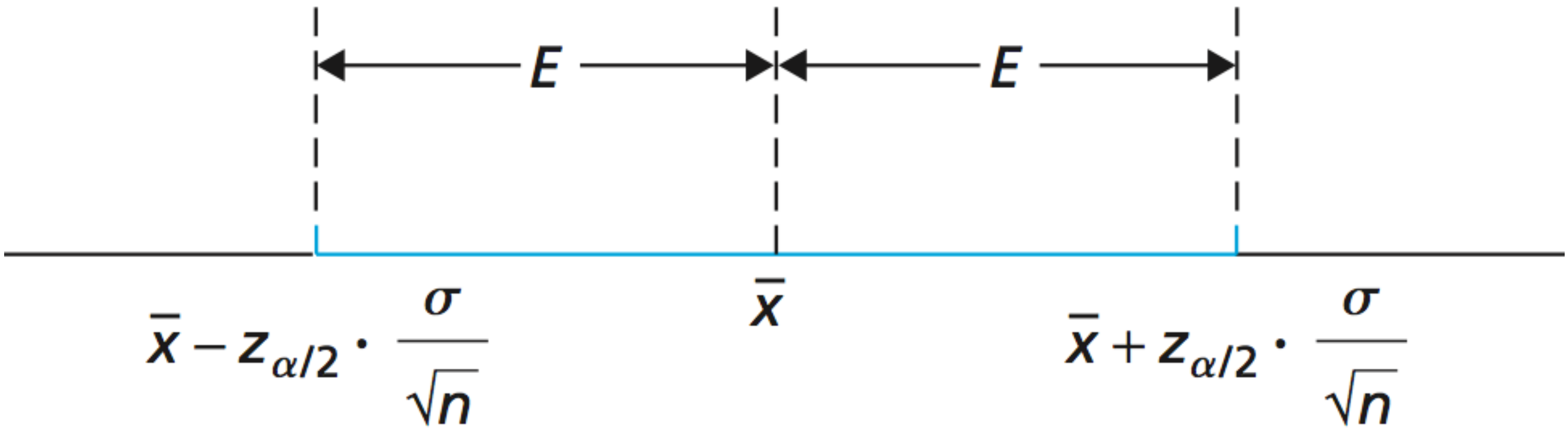
$$E = z_{\alpha/2} \cdot \sigma/\sqrt{n}$$

See Fig. 8.5

# Figure 8.5

Margin of error, *E*



# Key Fact 8.3

**Confidence and Accuracy**

For a fixed sample size, decreasing the confidence level decreases the margin of error and, hence, improves the accuracy of a confidence-interval estimate.

# Key Fact 8.4

**Sample Size and Accuracy**

For a fixed confidence level, increasing the sample size decreases the margin of error and, hence, improves the accuracy of a confidence-interval estimate.

# Formula 8.2

## Sample Size for Estimating $\mu$

The sample size required for a $(1-\alpha)$-level confidence interval for $\mu$ with a specified margin of error, $E$, is given by the formula

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2,$$

rounded up to the nearest whole number.

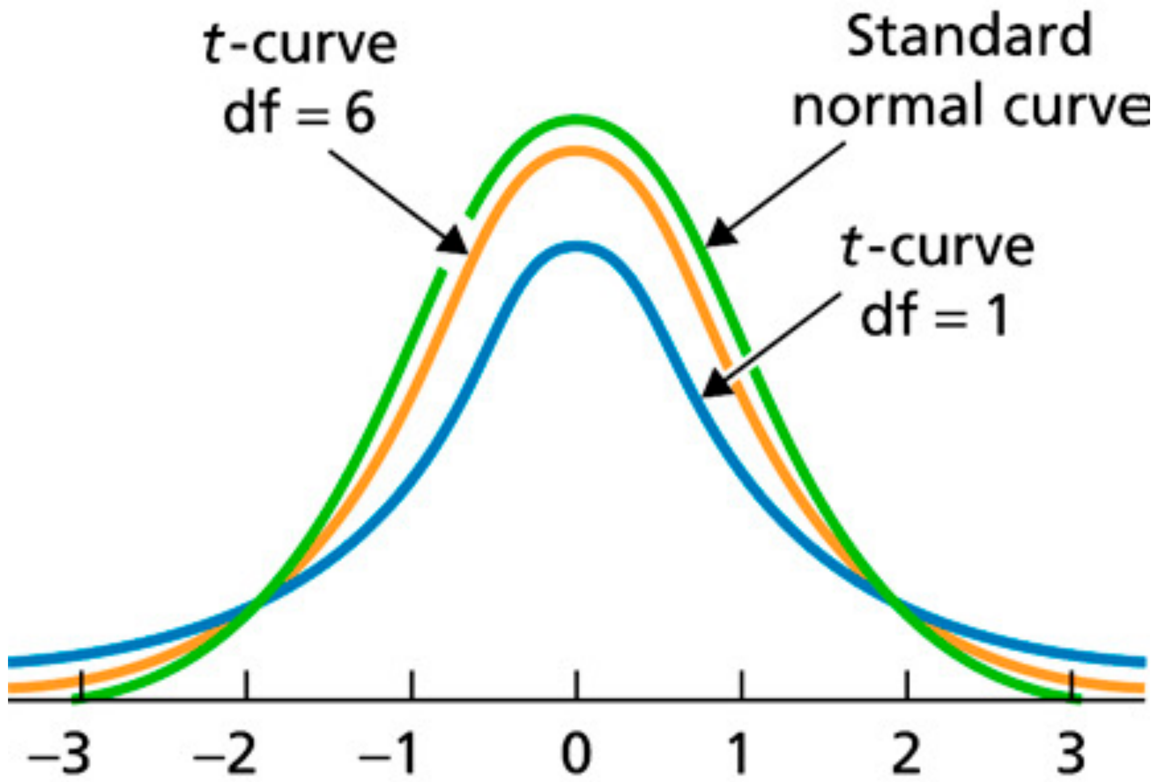
# Section 8.3

## Confidence Intervals for One Population Mean When $\sigma$ Is Unknown

# Figure 8.8

Standard normal curve and two t-curves



# Key Fact 8.5

**Studentized Version of the Sample Mean**

Suppose that a variable $x$ of a population is normally distributed with mean $\mu$. Then, for samples of size $n$, the variable

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

has the $t$-distribution with $n - 1$ degrees of freedom.

# Key Fact 8.6

**Basic Properties of *t*-Curves**

**Property 1:** The total area under a *t*-curve equals 1.

**Property 2:** A *t*-curve extends indefinitely in both directions, approaching, but never touching, the horizontal axis as it does so.

**Property 3:** A *t*-curve is symmetric about 0.

**Property 4:** As the number of degrees of freedom becomes larger, *t*-curves look increasingly like the standard normal curve.

# Table 8.5

Values of $t_\alpha$

| df | $t_{0.10}$ | $t_{0.05}$ | $t_{0.025}$ | $t_{0.01}$ | $t_{0.005}$ | df |
|---|---|---|---|---|---|---|
| . | . | . | . | . | . | . |
| . | . | . | . | . | . | . |
| . | . | . | . | . | . | . |
| *12* | 1.356 | 1.782 | 2.179 | 2.681 | 3.055 | *12* |
| *13* | 1.350 | 1.771 | 2.160 | 2.650 | 3.012 | *13* |
| *14* | 1.345 | 1.761 | 2.145 | 2.624 | 2.977 | *14* |
| *15* | 1.341 | 1.753 | 2.131 | 2.602 | 2.947 | *15* |
| . | . | . | . | . | . | . |
| . | . | . | . | . | . | . |
| . | . | . | . | . | . | . |

# Procedure 8.2

## One-Mean *t*-Interval Procedure

***Purpose*** To find a confidence interval for a population mean, $\mu$

***Assumptions***

1. Simple random sample
2. Normal population or large sample
3. $\sigma$ unknown

**Step 1** For a confidence level of $1 - \alpha$, use Table IV to find $t_{\alpha/2}$ with $df = n - 1$, where $n$ is the sample size.

**Step 2** The confidence interval for $\mu$ is from

$$\bar{x} - t_{\alpha/2} \cdot \frac{s}{\sqrt{n}} \quad \text{to} \quad \bar{x} + t_{\alpha/2} \cdot \frac{s}{\sqrt{n}},$$

where $t_{\alpha/2}$ is found in Step 1 and $\bar{x}$ and $s$ are computed from the sample data.

**Step 3** Interpret the confidence interval.

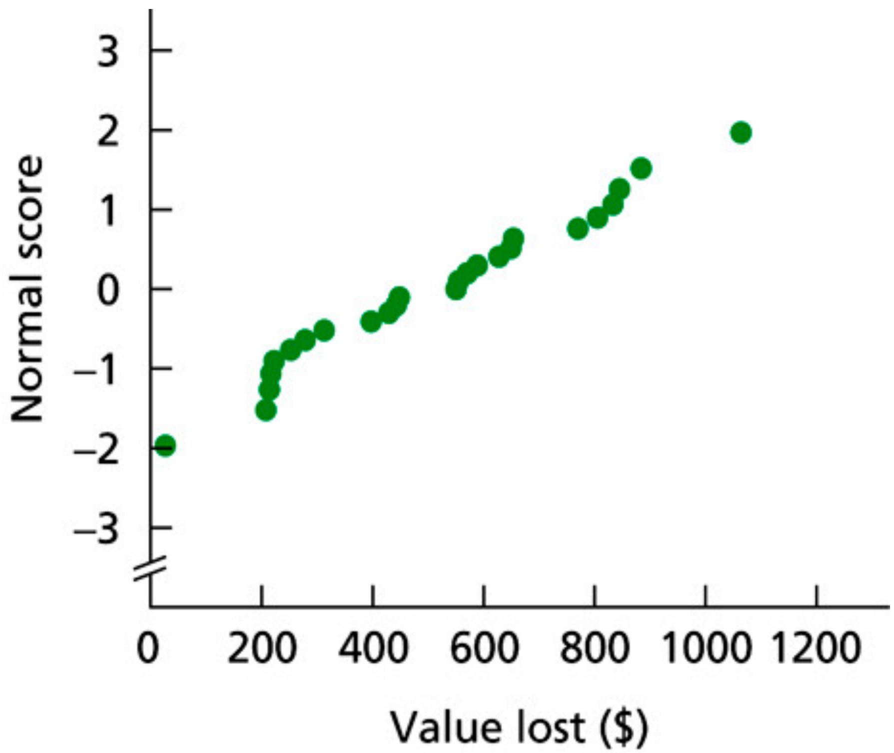
*Note:* The confidence interval is exact for normal populations and is approximately correct for large samples from nonnormal populations.

# Table 8.6 & Figure 8.10

Losses ($) for a sample of 25 pickpocket offenses

| | | | | |
|---|---|---|---|---|
| 447 | 207 | 627 | 430 | 883 |
| 313 | 844 | 253 | 397 | 214 |
| 217 | 768 | 1064 | 26 | 587 |
| 833 | 277 | 805 | 653 | 549 |
| 649 | 554 | 570 | 223 | 443 |

Normal probability plot of the loss data in Table 8.5



# Figure 8.11

Normal probability plots for chicken consumption: (a) original data and (b) data with outlier removed