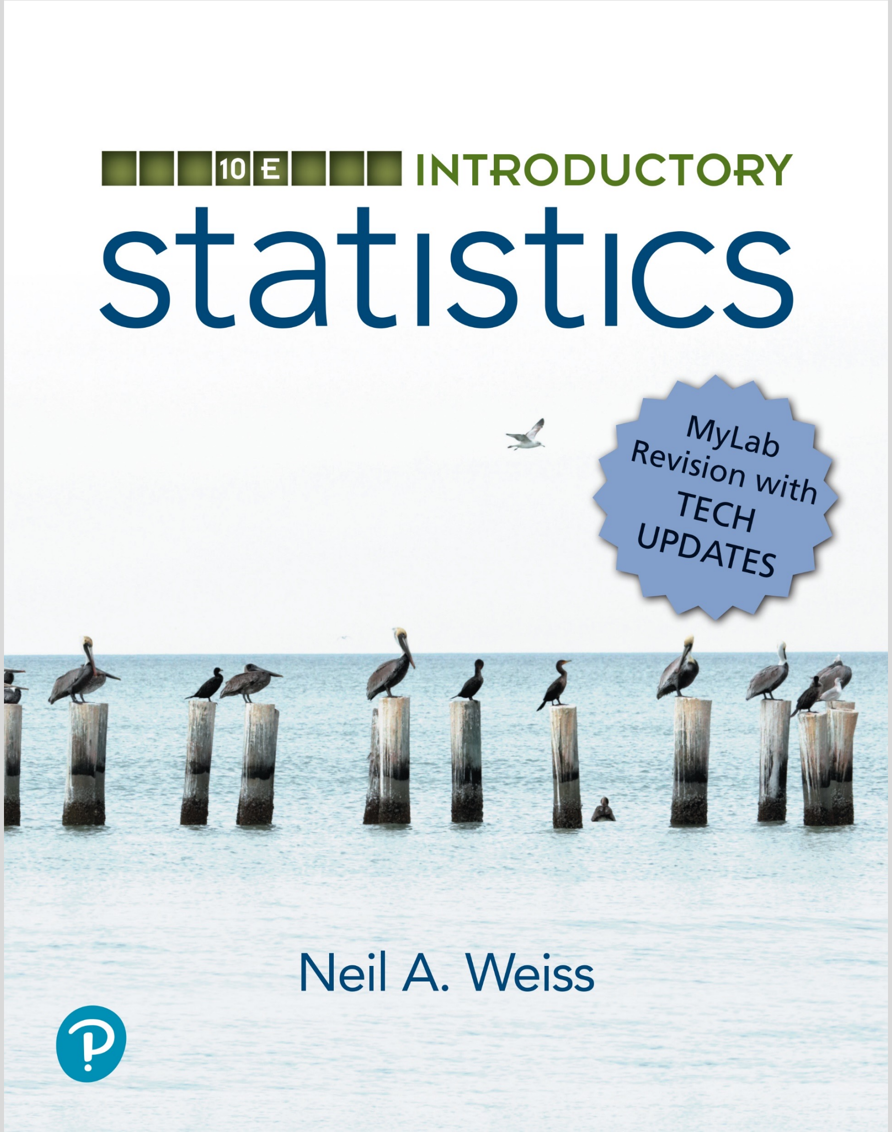
Chapter 11

Inferences for Population Standard Deviations

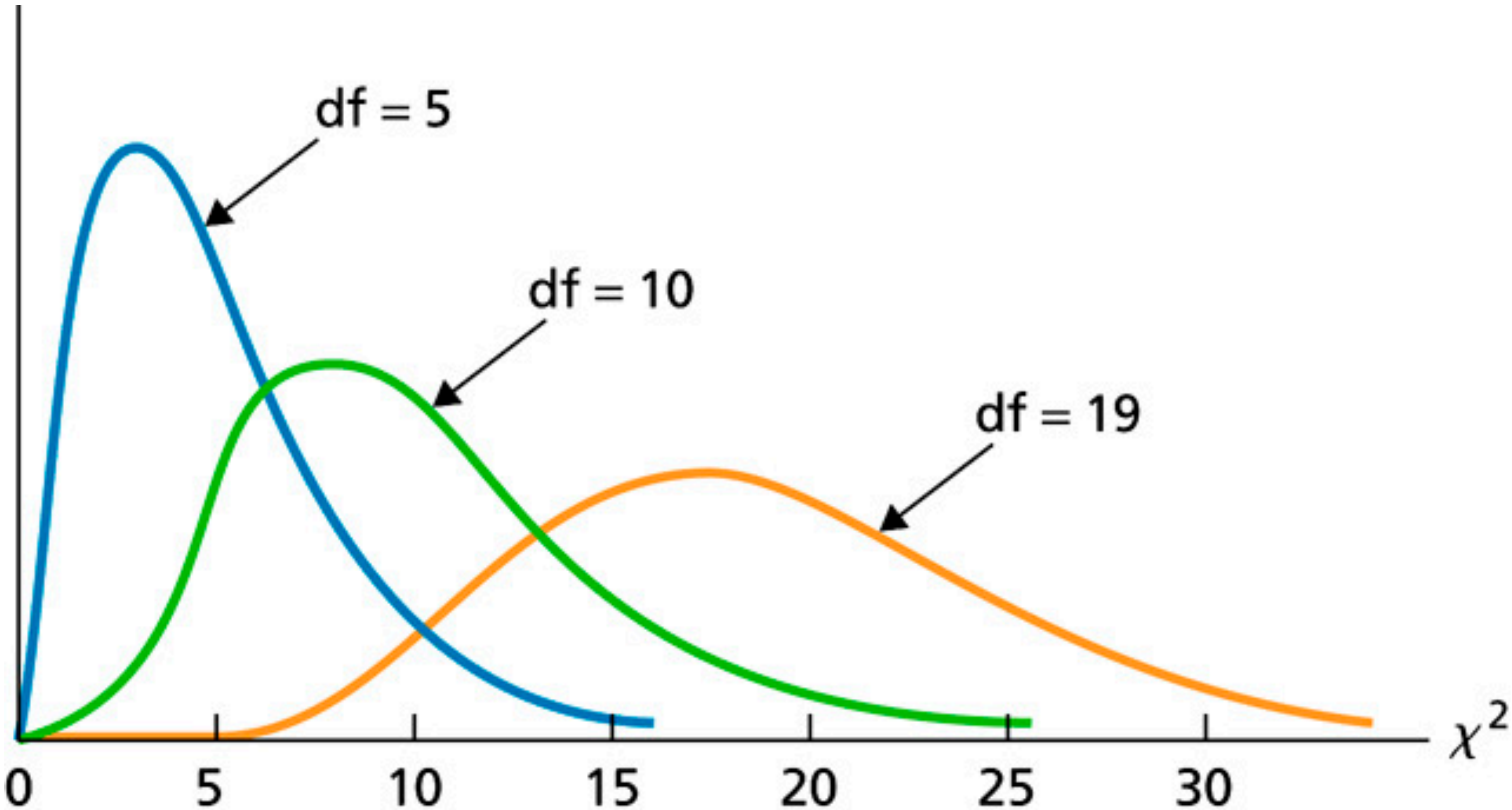


Section 11.1

Inferences for One Population Standard Deviation

Figure 11.1

χ^2-curves for df = 5, 10, and 19



Key Fact 11.1

Basic Properties of χ^2-Curves

Property 1: The total area under a χ^2-curve equals 1.

Property 2: A χ^2-curve starts at 0 on the horizontal axis and extends indefinitely to the right, approaching, but never touching, the horizontal axis as it does so.

Property 3: A χ^2-curve is right skewed.

Property 4: As the number of degrees of freedom becomes larger, χ^2-curves look increasingly like normal curves.

Key Fact 11.2

The Sampling Distribution of the Sample Standard Deviation[†]

Suppose that a variable of a population is normally distributed with standard deviation σ. Then, for samples of size n, the variable

$$\chi^2 = \frac{n-1}{\sigma^2} s^2$$

has the chi-square distribution with $n - 1$ degrees of freedom.

Procedure 11.1

One-Standard-Deviation χ^2-Test

Purpose To perform a hypothesis test for a population standard deviation, σ

Assumptions

1. Simple random sample
2. Normal population

Step 1 **The null hypothesis is H_0: $\sigma = \sigma_0$, and the alternative hypothesis is**

H_a: $\sigma \neq \sigma_0$	or	H_a: $\sigma < \sigma_0$	or	H_a: $\sigma > \sigma_0$
(Two tailed)		**(Left tailed)**		**(Right tailed)**

Step 2 **Decide on the significance level, α.**

Step 3 **Compute the value of the test statistic**

$$\chi^2 = \frac{n-1}{\sigma_0^2} s^2$$

and denote that value χ_0^2.

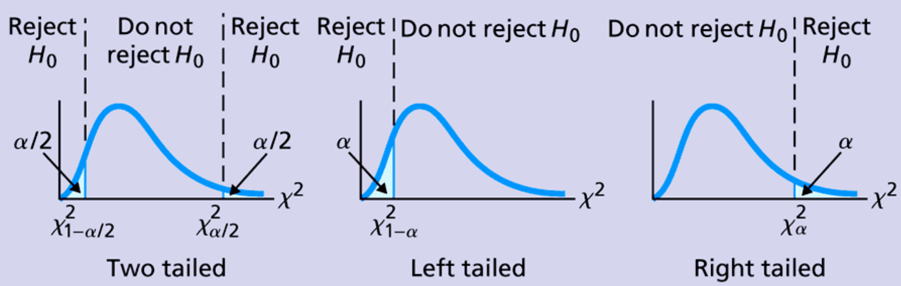
Procedure 11.1 (cont.)

CRITICAL-VALUE APPROACH	OR	*P*-VALUE APPROACH

Step 4 The critical value(s) are

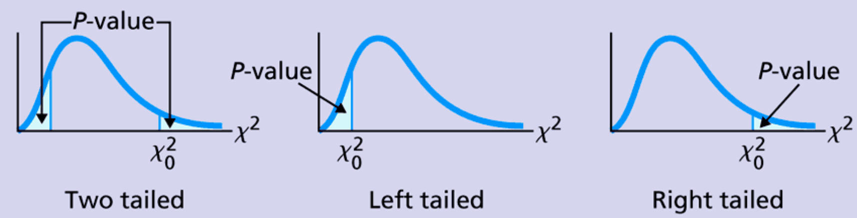
$\chi^2_{1-\alpha/2}$ and $\chi^2_{\alpha/2}$ (Two tailed) or $\chi^2_{1-\alpha}$ (Left tailed) or χ^2_{α} (Right tailed)

with df $= n - 1$. Use Table VII to find the critical value(s).



Step 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

Step 4 The χ^2-statistic has df $= n - 1$. Obtain the *P*-value by using technology.



Step 5 If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

Procedure 11.2

One-Standard-Deviation χ^2-Interval Procedure

Purpose To find a confidence interval for a population standard deviation, σ

Assumptions

1. Simple random sample
2. Normal population

Step 1 **For a confidence level of $1 - \alpha$, use Table VII to find $\chi^2_{1-\alpha/2}$ and $\chi^2_{\alpha/2}$ with df $= n - 1$.**

Step 2 **The confidence interval for σ is from**

$$\sqrt{\frac{n-1}{\chi^2_{\alpha/2}}} \cdot s \quad \text{to} \quad \sqrt{\frac{n-1}{\chi^2_{1-\alpha/2}}} \cdot s,$$

where $\chi^2_{1-\alpha/2}$ and $\chi^2_{\alpha/2}$ are found in Step 1, n is the sample size, and s is computed from the sample data obtained.

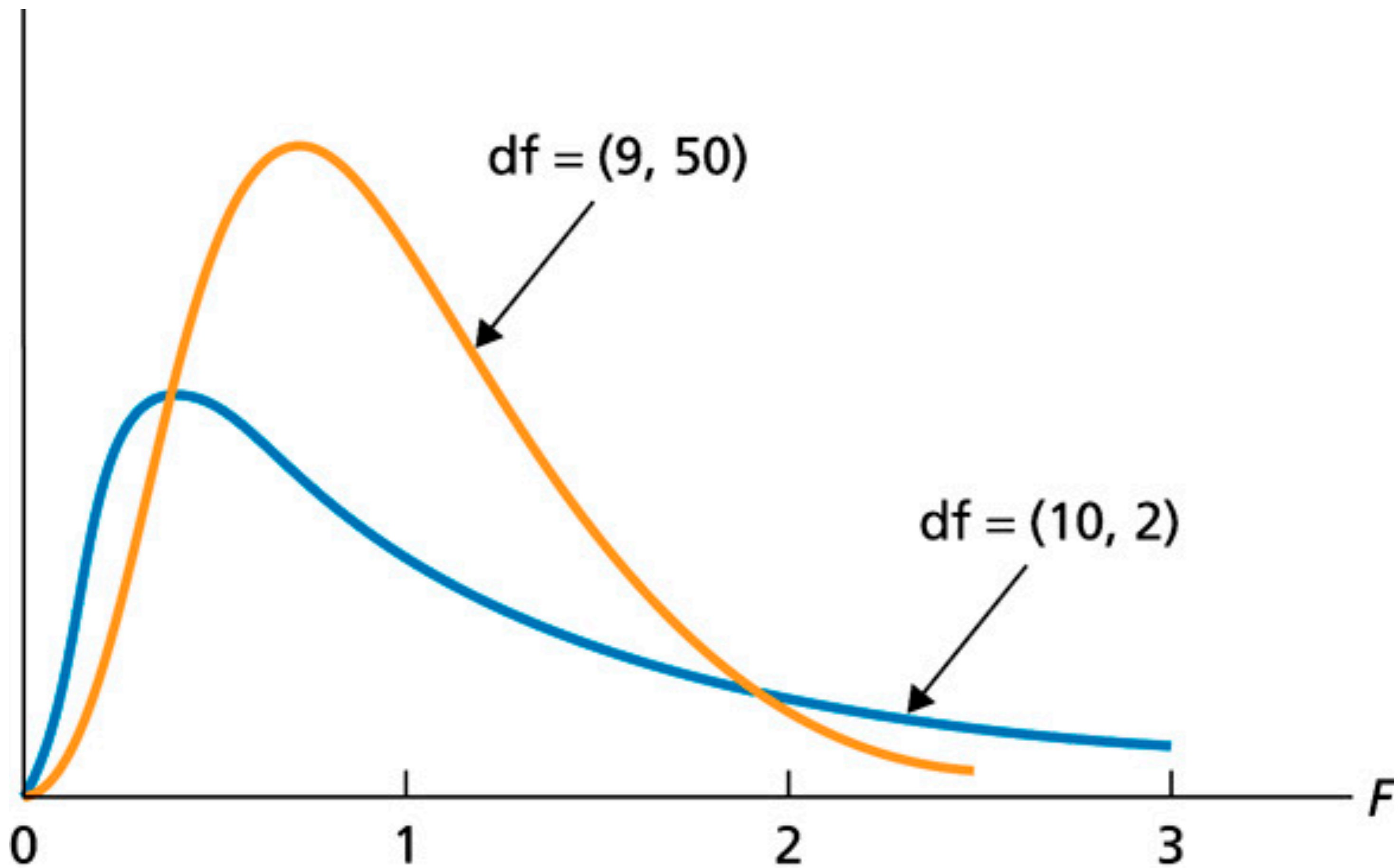
Step 3 **Interpret the confidence interval.**

Section 11.2

Inferences for Two Population Standard Deviations, Using Independent Samples

Figure 11.7

Two different *F*-curves



Key Fact 11.3

Basic Properties of *F*-Curves

Property 1: The total area under an F-curve equals 1.

Property 2: An F-curve starts at 0 on the horizontal axis and extends indefinitely to the right, approaching, but never touching, the horizontal axis as it does so.

Property 3: An F-curve is right skewed.

Key Fact 11.4

Reciprocal Property of F-Curves

For an F-curve with df $= (\nu_1, \nu_2)$, the F-value having area α to its left equals the reciprocal of the F-value having area α to its right for an F-curve with df $= (\nu_2, \nu_1)$.

Key Fact 11.5

Distribution of the *F*-Statistic for Comparing Two Population Standard Deviations

Suppose that the variable under consideration is normally distributed on each of two populations. Then, for independent samples of sizes n_1 and n_2 from the two populations, the variable

$$F = \frac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2}$$

has the F-distribution with $\text{df} = (n_1 - 1, n_2 - 1)$.

Procedure 11.3

Two-Standard-Deviations *F*-Test

Purpose To perform a hypothesis test to compare two population standard deviations, σ_1 and σ_2

Assumptions

1. Simple random samples
2. Independent samples
3. Normal populations

Step 1 **The null hypothesis is H_0: $\sigma_1 = \sigma_2$, and the alternative hypothesis is**

H_a: $\sigma_1 \neq \sigma_2$ (Two tailed)	**or**	**H_a: $\sigma_1 < \sigma_2$ (Left tailed)**	**or**	**H_a: $\sigma_1 > \sigma_2$ (Right tailed)**

Step 2 **Decide on the significance level, α.**

Step 3 **Compute the value of the test statistic**

$$F = \frac{s_1^2}{s_2^2}$$

and denote that value F_0.

Procedure 11.3 (cont.)

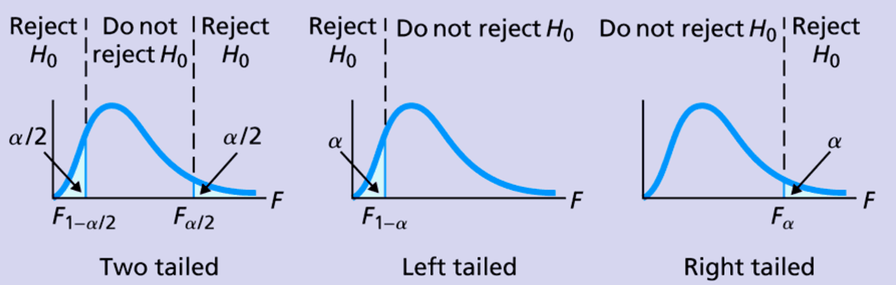
CRITICAL-VALUE APPROACH	OR	*P*-VALUE APPROACH

CRITICAL-VALUE APPROACH

Step 4 The critical value(s) are

$F_{1-\alpha/2}$ and $F_{\alpha/2}$ (Two tailed) or $F_{1-\alpha}$ (Left tailed) or F_{α} (Right tailed)

with df $= (n_1 - 1, n_2 - 1)$. Use Table VIII to find the critical value(s).

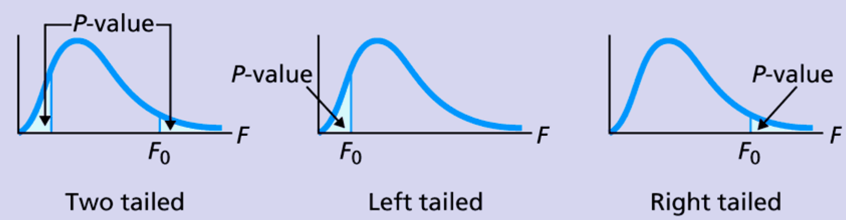


Step 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

OR

***P*-VALUE APPROACH**

Step 4 The F-statistic has df $= (n_1 - 1, n_2 - 1)$. Obtain the P-value by using technology.



Step 5 If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.