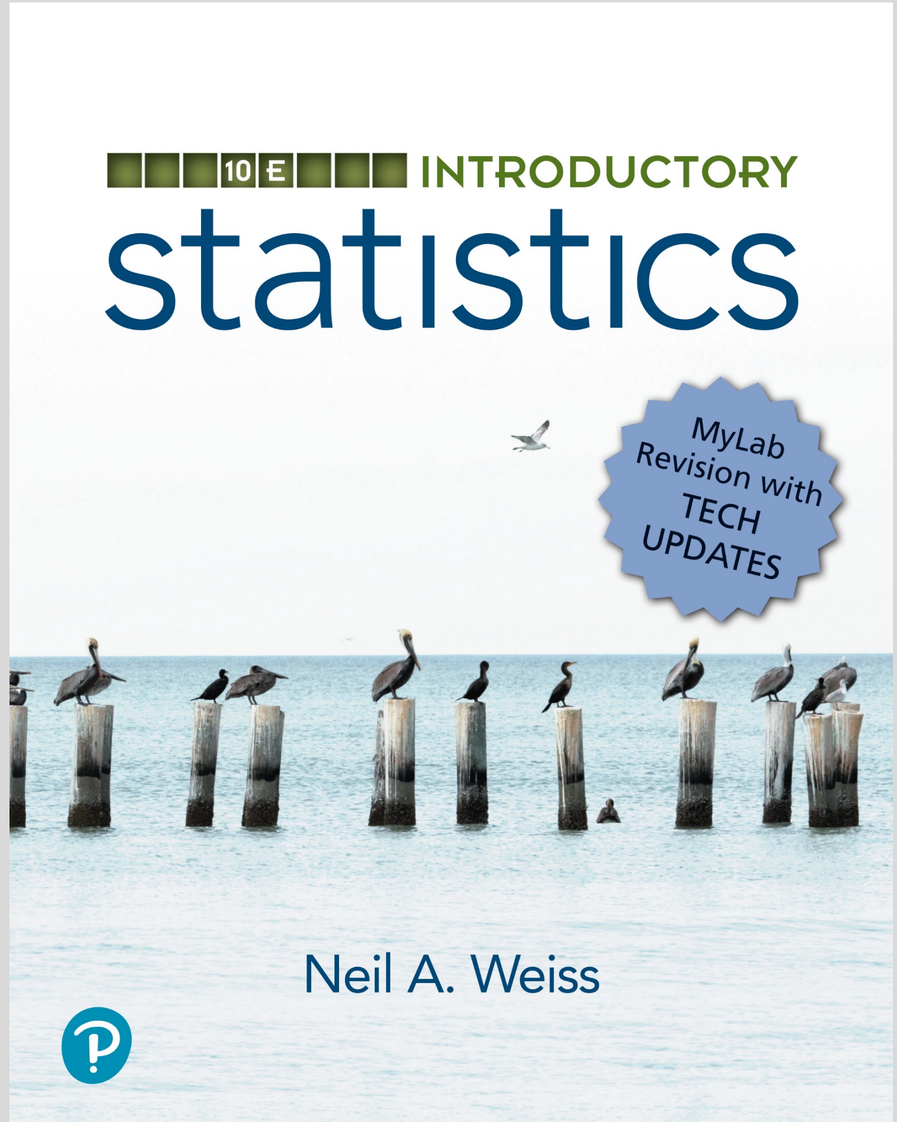
Chapter 7

The Sampling Distribution of the Sample Mean



Chapter 7

The Sampling Distribution of the Sample Mean

Section 7.1

Sampling Error; the Need for Sampling Distributions

Definition 7.1

Sampling Error

Sampling error is the error resulting from using a sample to estimate a population characteristic.

Definition 7.2

Sampling Distribution of the Sample Mean

For a variable x and a given sample size, the distribution of the variable $\bar{x}$ is called the **sampling distribution of the sample mean.**

Table 7.2

Possible samples and sample means for samples of size 2

Sample	Heights	$\bar{x}$
A, B	76, 78	77.0
A, C	76, 79	77.5
A, D	76, 81	78.5
A, E	76, 86	81.0
B, C	78, 79	78.5
B, D	78, 81	79.5
B, E	78, 86	82.0
C, D	79, 81	80.0
C, E	79, 86	82.5
D, E	81, 86	83.5

Figure 7.1

Dotplot for the sampling distribution of the sample mean for samples of size 2 (n = 2)

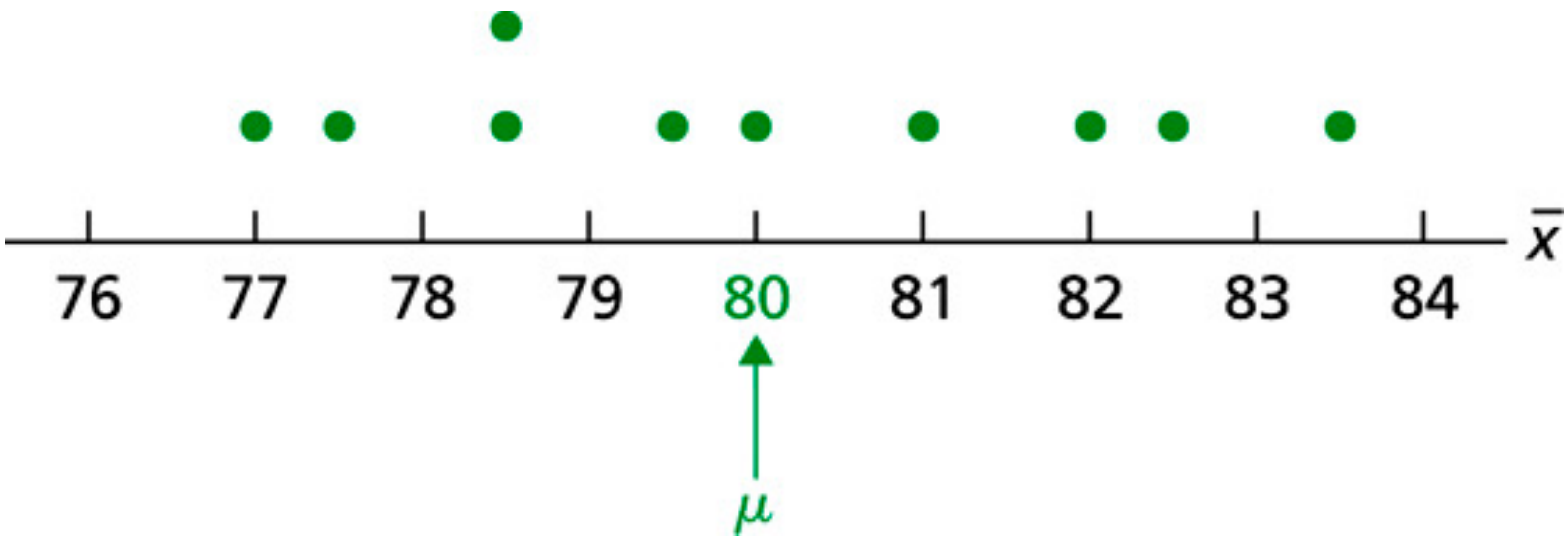


Figure 7.3

Dotplots for the sampling distributions of the sample mean for the heights of the five starting players for samples of sizes 1, 2, 3, 4, and 5

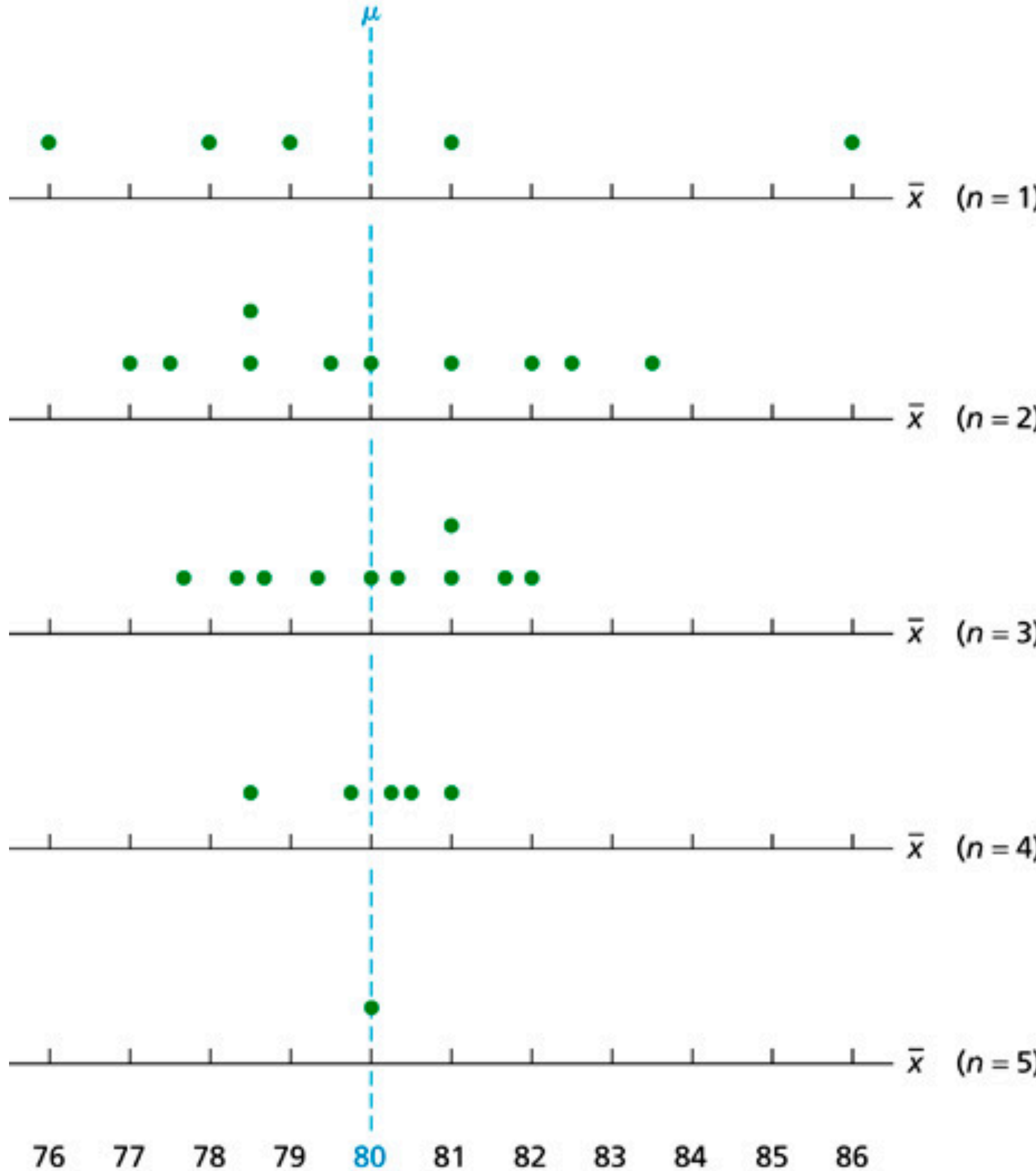


Table 7.4

Sample size and sampling error illustrations for the heights of the basketball players ("No." is an abbreviation of "Number")

Sample size n	No. possible samples	No. within $1''$ of μ	% within $1''$ of μ	No. within $0.5''$ of μ	% within $0.5''$ of μ
1	5	2	40%	0	0%
2	10	3	30%	2	20%
3	10	5	50%	2	20%
4	5	4	80%	3	60%
5	1	1	100%	1	100%

Key Fact 7.1

Sample Size and Sampling Error

The larger the sample size, the smaller the sampling error tends to be in estimating a population mean, μ, by a sample mean, $\bar{x}$.

Section 7.2

The Mean and Standard Deviation of the Sample Mean

Formula 7.1

Mean of the Sample Mean

For samples of size n, the mean of the variable $\bar{x}$ equals the mean of the variable under consideration. In symbols,

$$\mu_{\bar{x}} = \mu.$$

Formula 7.2

Standard Deviation of the Sample Mean

For samples of size n, the standard deviation of the variable $\bar{x}$ equals the standard deviation of the variable under consideration divided by the square root of the sample size. In symbols,

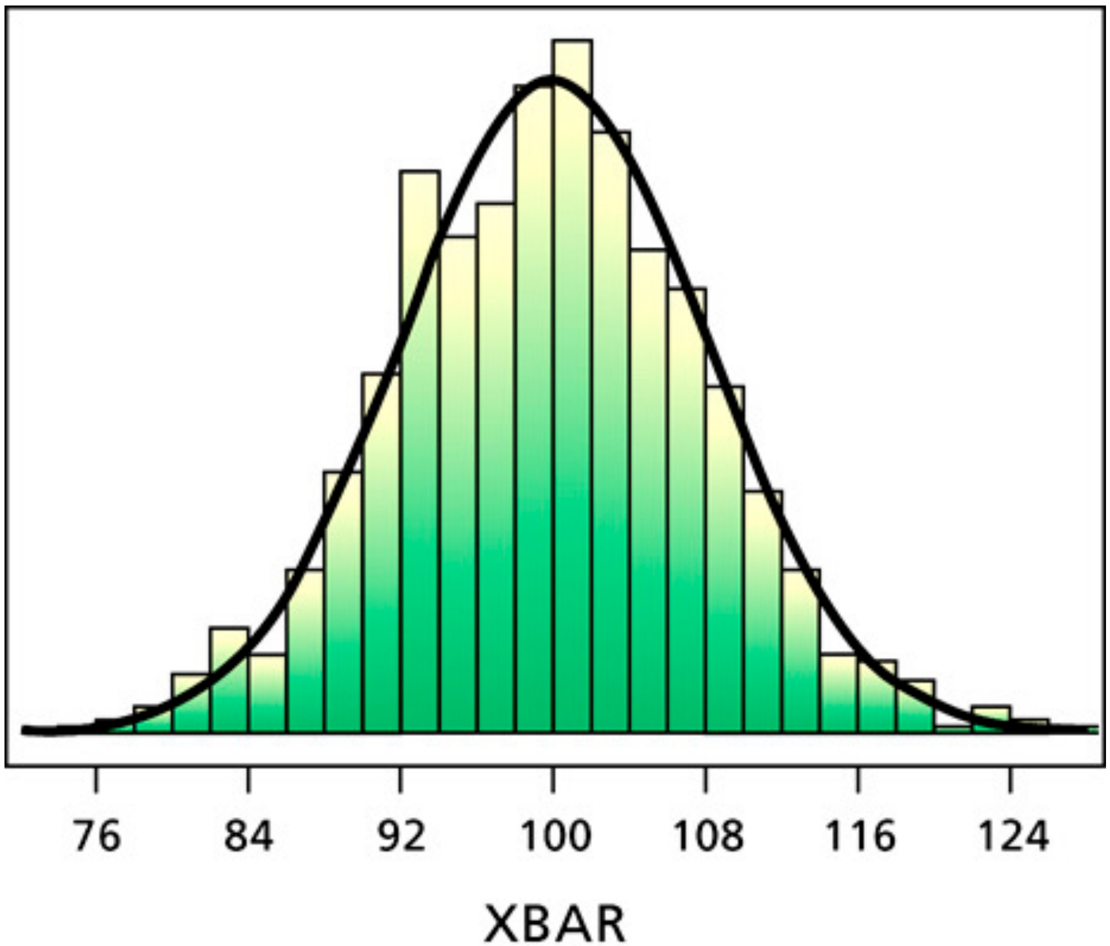
$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}.$$

Section 7.3

The Sampling Distribution of the Sample Mean

Output 7.1

Histogram of the sample means for 1000 samples of four IQs with superimposed normal curve



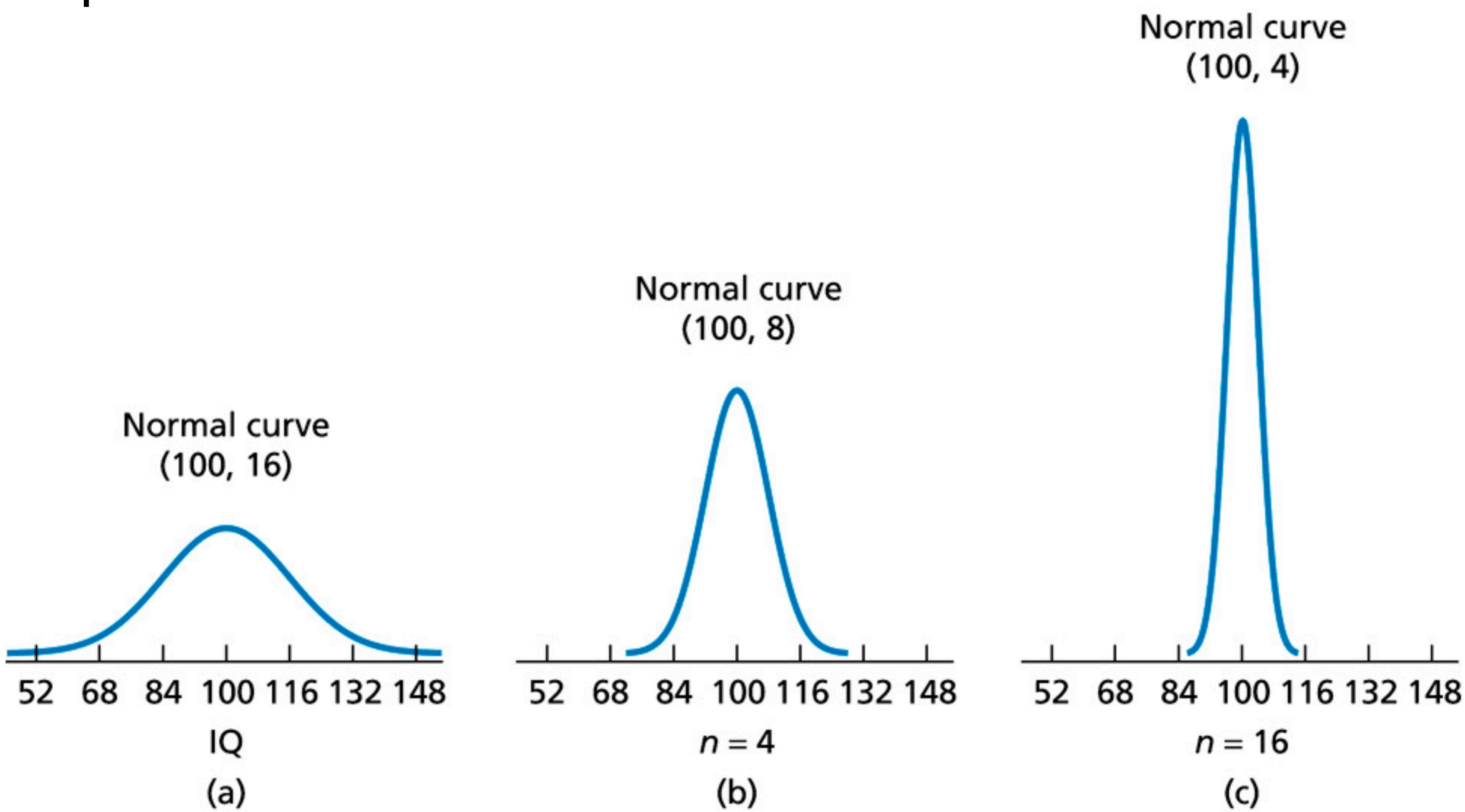
Key Fact 7.2

Sampling Distribution of the Sample Mean for a Normally Distributed Variable

Suppose that a variable x of a population is normally distributed with mean μ and standard deviation σ. Then, for samples of size n, the variable $\bar{x}$ is also normally distributed and has mean μ and standard deviation $\sigma/\sqrt{n}$.

Figure 7.4

(a) Normal distribution for IQs; (b) sampling distribution of the sample mean for n = 4; (c) sampling distribution of the sample mean for n = 16



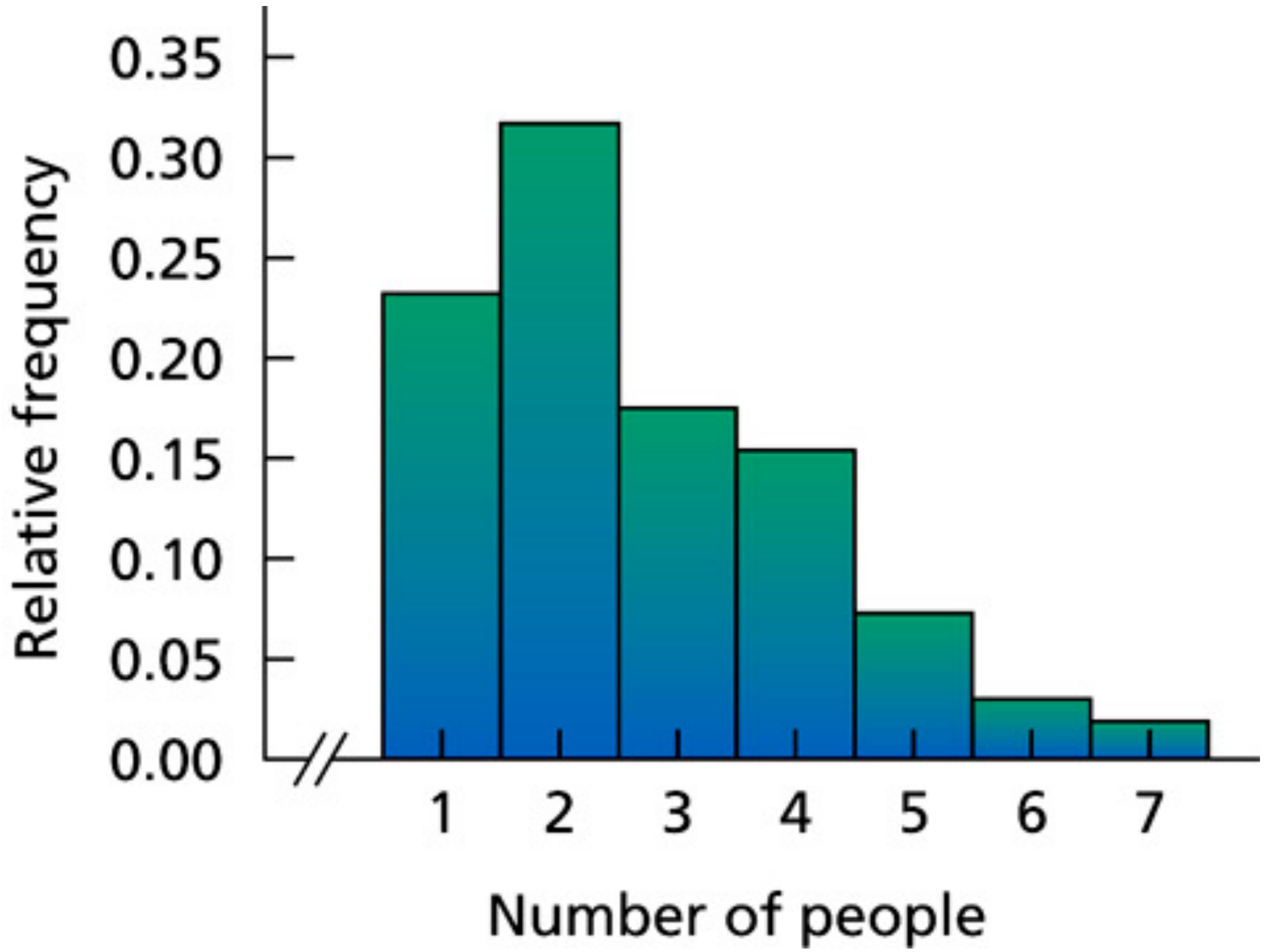
Key Fact 7.3

The Central Limit Theorem (CLT)

For a relatively large sample size, the variable $\bar{x}$ is approximately normally distributed, regardless of the distribution of the variable under consideration. The approximation becomes better with increasing sample size.

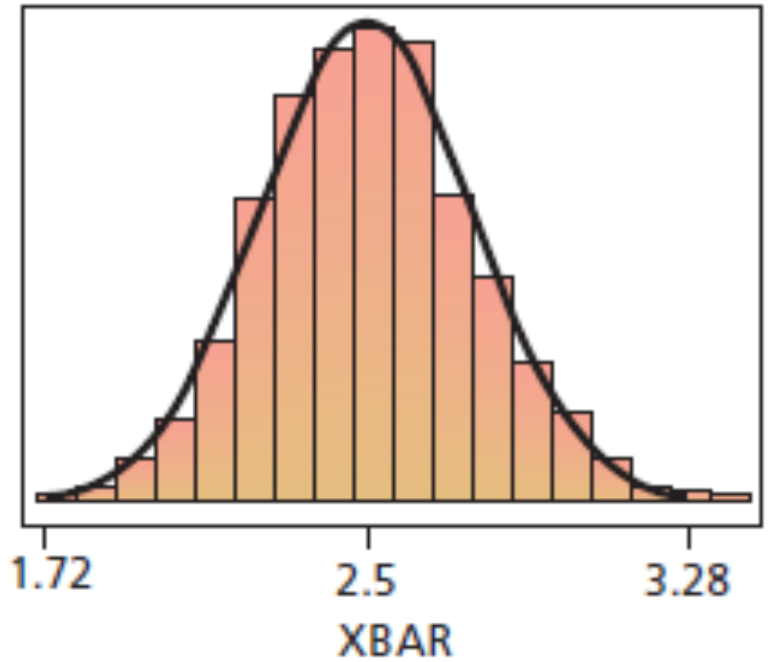
Figure 7.5

Relative-frequency histogram for household size



Output 7.2

Histogram of the sample means for 1000 samples of 30 household sizes with superimposed normal curve



Key Fact 7.4

Sampling Distribution of the Sample Mean

Suppose that a variable x of a population has mean μ and standard deviation σ. Then, for samples of size n,

- the mean of $\bar{x}$ equals the population mean, or $\mu_{\bar{x}} = \mu$;
- the standard deviation of $\bar{x}$ equals the population standard deviation divided by the square root of the sample size, or $\sigma_{\bar{x}} = \sigma/\sqrt{n}$;
- if x is normally distributed, so is $\bar{x}$, regardless of sample size; and
- if the sample size is large, $\bar{x}$ is approximately normally distributed, regardless of the distribution of x.

Figure 7.6

Sampling distributions of the sample mean for (a) normal, (b) reverse-J-shaped, and (c) uniform variables