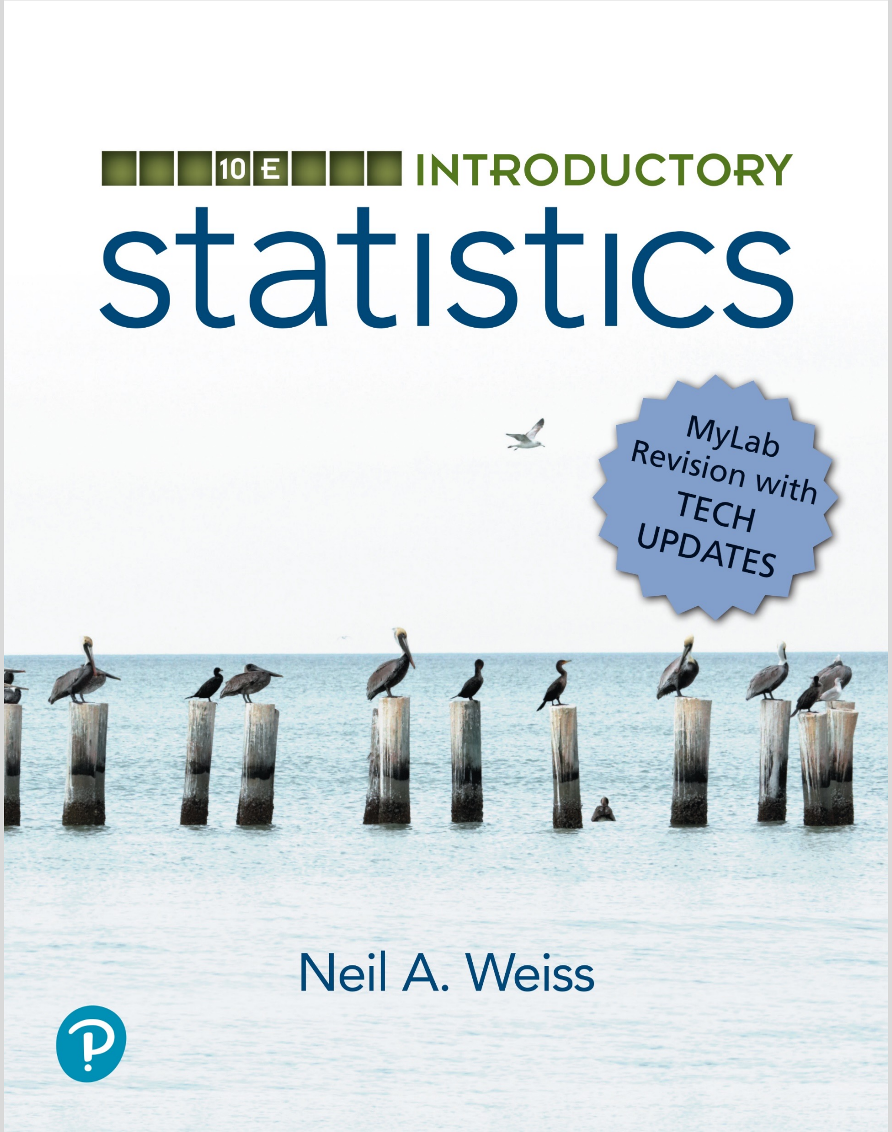
Chapter 13

Chi-Square Procedures

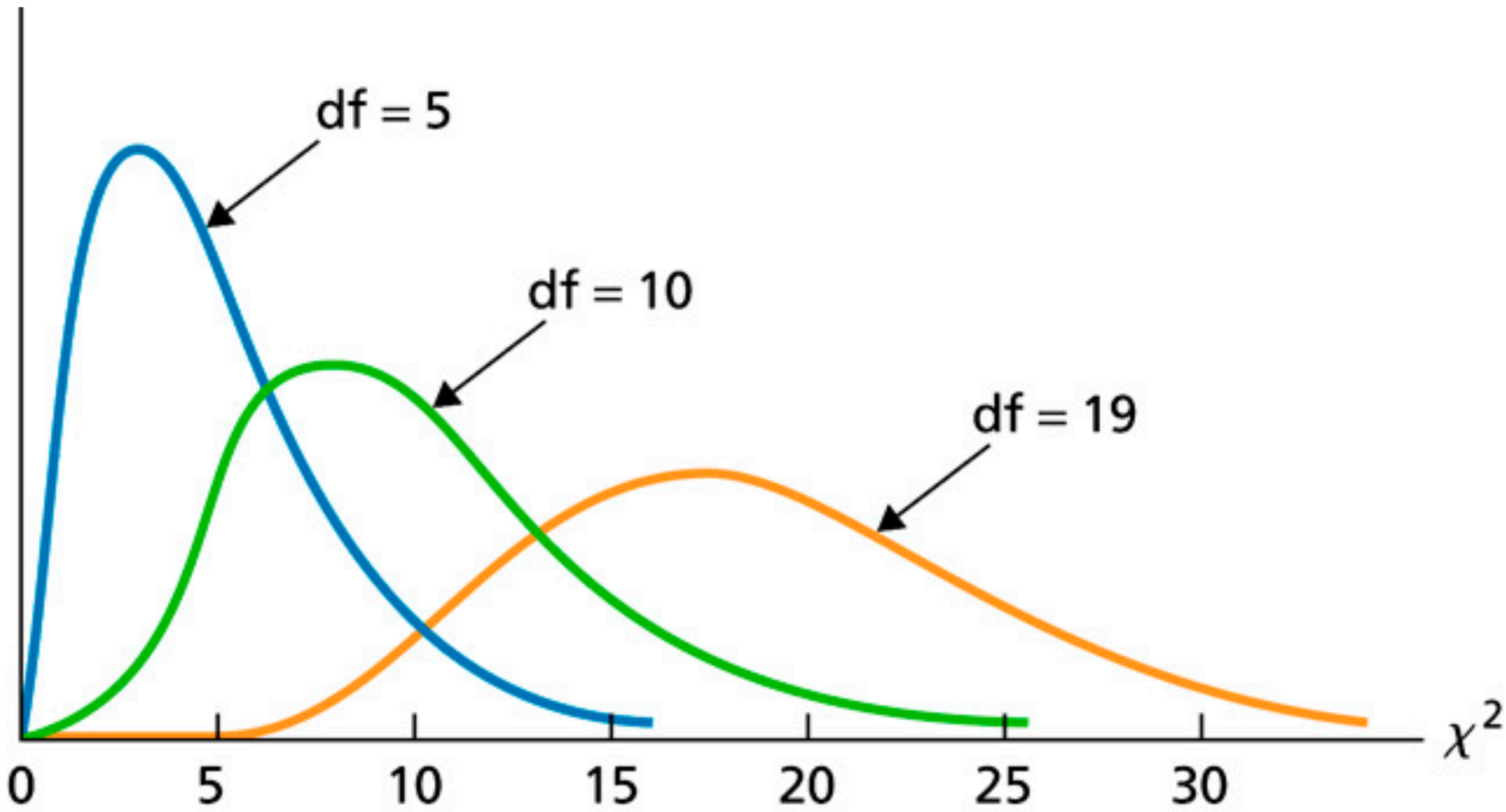


Section 13.1

The Chi-Square Distribution

Figure 13.1

χ^2-curves for df = 5, 10, and 19



Key Fact 13.1

Basic Properties of χ^2-Curves

Property 1: The total area under a χ^2-curve equals 1.

Property 2: A χ^2-curve starts at 0 on the horizontal axis and extends indefinitely to the right, approaching, but never touching, the horizontal axis.

Property 3: A χ^2-curve is right skewed.

Property 4: As the number of degrees of freedom becomes larger, χ^2-curves look increasingly like normal curves.

Section 13.2

Chi-Square Goodness-of-Fit Test

Table 13.3

Expected frequencies if last year's violent-crime distribution is the same as the 2010 distribution

Type of violent crime	Expected frequency
Murder	6.0
Forcible rape	34.0
Robbery	147.5
Agg. assault	312.5

Table 13.4

Calculating the goodness of fit

Type of violent crime x	Observed frequency O	Expected frequency E	Difference $O - E$	Square of difference $(O - E)^2$	Chi-square subtotal $(O - E)^2/E$
Murder	3	6.0	−3.0	9.00	1.500
Forcible rape	36	34.0	2.0	4.00	0.118
Robbery	170	147.5	22.5	506.25	3.432
Agg. assault	291	312.5	−21.5	462.25	1.479
	500	500.0	0		6.529

Formula 13.1

Expected Frequencies for a Goodness-of-Fit Test

In a chi-square goodness-of-fit test, the expected frequency for each possible value of the variable is found by using the formula

$$E = np,$$

where n is the sample size and p is the relative frequency (or probability) given for the value in the null hypothesis.

Key Fact 13.2

Distribution of the χ^2-Statistic for a Goodness-of-Fit Test

For a chi-square goodness-of-fit test, the test statistic

$$\chi^2 = \Sigma(O - E)^2/E$$

has approximately a chi-square distribution if the null hypothesis is true. The number of degrees of freedom is 1 less than the number of possible values for the variable under consideration.

Procedure 13.1

Chi-Square Goodness-of-Fit Test

Purpose To perform a hypothesis test for the distribution of a variable

Assumptions

1. All expected frequencies are 1 or greater
2. At most 20% of the expected frequencies are less than 5
3. Simple random sample

Step 1 **The null and alternative hypotheses are, respectively,**

H_0: The variable has the specified distribution

H_a: The variable does not have the specified distribution.

Step 2 **Decide on the significance level, α.**

Step 3 **Compute the value of the test statistic**

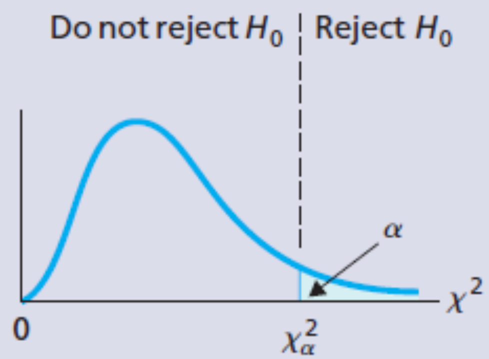
$$\chi^2 = \Sigma(O - E)^2/E,$$

where O and E represent observed and expected frequencies, respectively. Denote the value of the test statistic χ_0^2.

Procedure 13.1 (cont.)

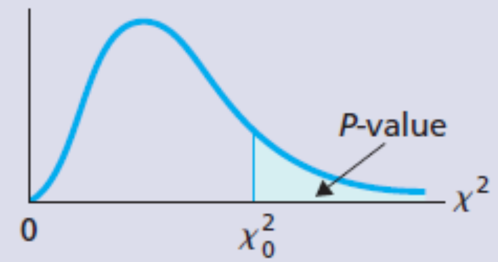
CRITICAL-VALUE APPROACH	OR	P-VALUE APPROACH

Step 4 The critical value is χ^2_α with df $= c - 1$, where c is the number of possible values for the variable. Use Table VII to find the critical value.



Step 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

Step 4 The χ^2-statistic has df $= c - 1$, where c is the number of possible values for the variable. Use Table VII to estimate the P-value, or obtain it exactly by using technology.



Step 5 If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

Section 13.3

Contingency Tables; Association

Table 13.7

Political party affiliation and class level for students in introductory statistics

Student	Political party	Class level	Student	Political party	Class level
1	Democratic	Freshman	21	Democratic	Junior
2	Other	Junior	22	Democratic	Senior
3	Democratic	Senior	23	Republican	Freshman
4	Other	Sophomore	24	Democratic	Sophomore
5	Democratic	Sophomore	25	Democratic	Senior
6	Republican	Sophomore	26	Republican	Sophomore
7	Republican	Junior	27	Republican	Junior
8	Other	Freshman	28	Other	Junior
9	Other	Sophomore	29	Other	Junior
10	Republican	Sophomore	30	Democratic	Sophomore
11	Republican	Sophomore	31	Republican	Sophomore
12	Republican	Junior	32	Democratic	Junior
13	Republican	Sophomore	33	Republican	Junior
14	Democratic	Junior	34	Other	Senior
15	Republican	Sophomore	35	Other	Sophomore
16	Republican	Senior	36	Republican	Freshman
17	Democratic	Sophomore	37	Republican	Freshman
18	Democratic	Junior	38	Republican	Freshman
19	Other	Senior	39	Democratic	Junior
20	Republican	Sophomore	40	Republican	Senior

Table 13.8

Preliminary contingency table for political party affiliation and class level

		Class level				
		Freshman	Sophomore	Junior	Senior	**Total**
Party	Democratic	|	||||	𝍸	|||	
	Republican	||||	𝍸 |||	||||	||	
	Other	|	|||	|||	||	
	Total					

Table 13.9

Contingency table for political party affiliation and class level

		Class level				
		Freshman	Sophomore	Junior	Senior	**Total**
Party	Democratic	1	4	5	3	13
	Republican	4	8	4	2	18
	Other	1	3	3	2	9
	Total	6	15	12	7	40

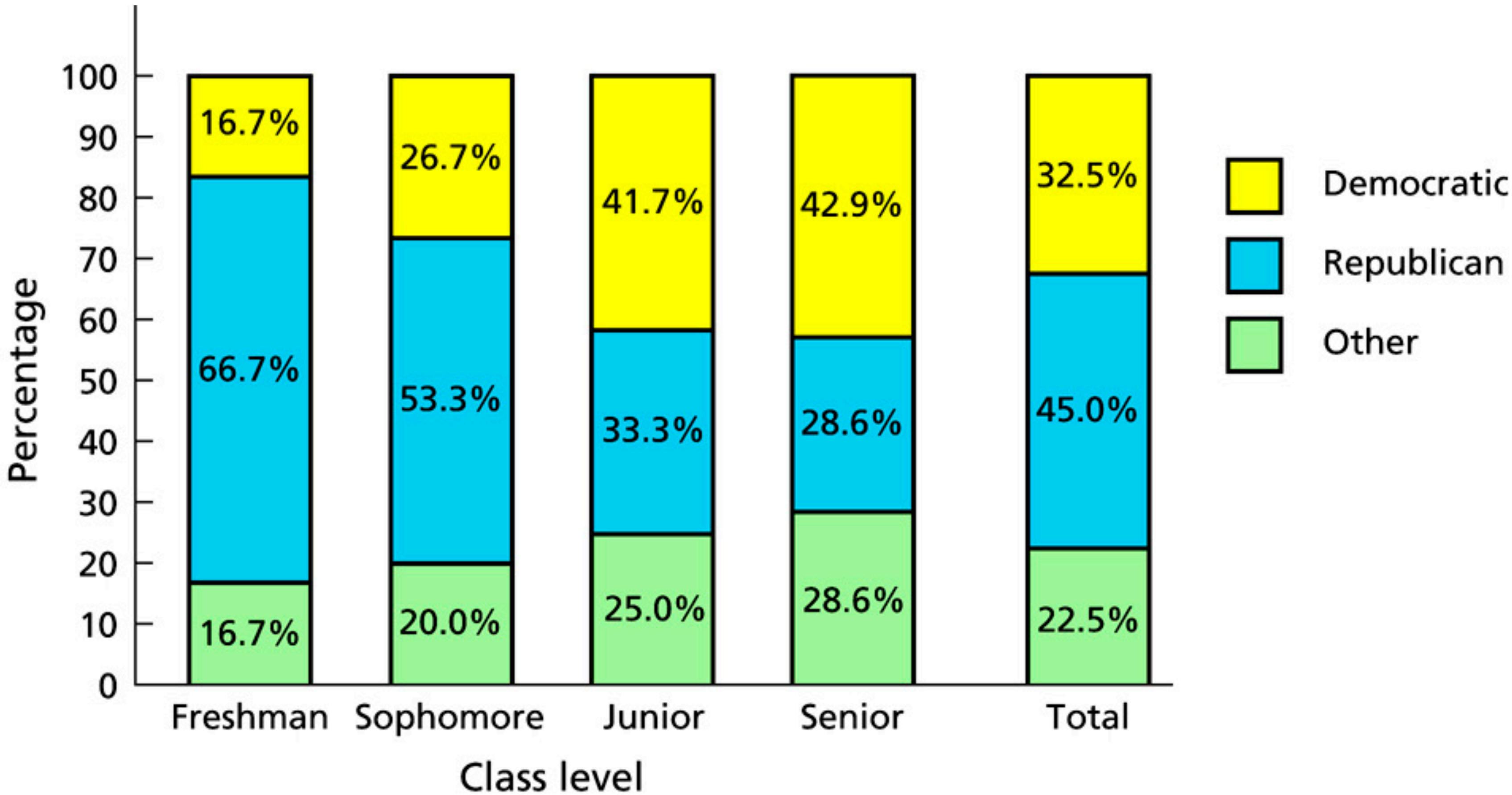
Table 13.10

Conditional distributions of political party affiliation by class level

		Class level				
		Freshman	Sophomore	Junior	Senior	**Total**
Party	Democratic	0.167	0.267	0.417	0.429	0.325
	Republican	0.667	0.533	0.333	0.286	0.450
	Other	0.167	0.200	0.250	0.286	0.225
	Total	1.000	1.000	1.000	1.000	1.000

Figure 13.4

Segmented bar graph for the conditional distributions and marginal distribution of political party affiliation



Definition 13.1

Association between Variables

We say that two variables of a population are **associated** (or that an **association** exists between the two variables) if the conditional distributions of one variable given the other are not identical.

Section 13.4

Chi-Square Independence Test

Table 13.13

Contingency table of marital status and alcohol consumption for 1772 randomly selected U.S. adults

Marital status	Drinks per month: Abstain	1–60	Over 60	**Total**
Single	67	213	74	354
Married	411	633	129	1173
Widowed	85	51	7	143
Divorced	27	60	15	102
Total	590	957	225	1772

Table 13.14

Observed and expected frequencies for marital status and alcohol consumption (expected frequencies printed below observed frequencies)

Marital status	Drinks per month: Abstain	1–60	Over 60	**Total**
Single	67 117.9	213 191.2	74 44.9	354
Married	411 390.6	633 633.5	129 148.9	1173
Widowed	85 47.6	51 77.2	7 18.2	143
Divorced	27 34.0	60 55.1	15 13.0	102
Total	590	957	225	1772

Formula 13.2

Expected Frequencies for an Independence Test

In a chi-square independence test, the expected frequency for each cell is found by using the formula

$$E = \frac{R \cdot C}{n},$$

where R is the row total, C is the column total, and n is the sample size.

Key Fact 13.3

Distribution of the χ^2-Statistic for a Chi-Square Independence Test

For a chi-square independence test, the test statistic

$$\chi^2 = \Sigma(O - E)^2/E$$

has approximately a chi-square distribution if the null hypothesis of non-association is true. The number of degrees of freedom is $(r - 1)(c - 1)$, where r and c are the number of possible values for the two variables under consideration.

Procedure 13.2

Chi-Square Independence Test

Purpose To perform a hypothesis test to decide whether two variables are associated

Assumptions

1. All expected frequencies are 1 or greater
2. At most 20% of the expected frequencies are less than 5
3. Simple random sample

Step 1 **The null and alternative hypotheses are, respectively,**

$$H_0\text{: The two variables are not associated.}$$
$$H_a\text{: The two variables are associated.}$$

Step 2 **Decide on the significance level, α.**

Step 3 **Compute the value of the test statistic**

$$\chi^2 = \Sigma(O - E)^2/E,$$

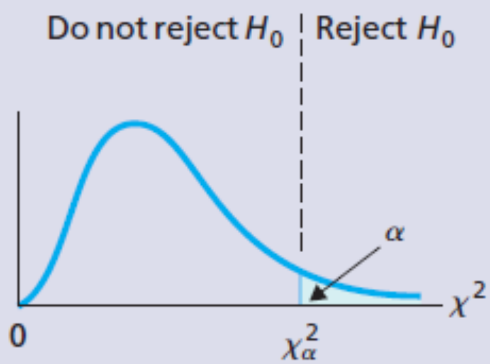
where O and E represent observed and expected frequencies, respectively. Denote the value of the test statistic χ_0^2.

Procedure 13.2 (cont.)

CRITICAL-VALUE APPROACH	OR	P-VALUE APPROACH

CRITICAL-VALUE APPROACH

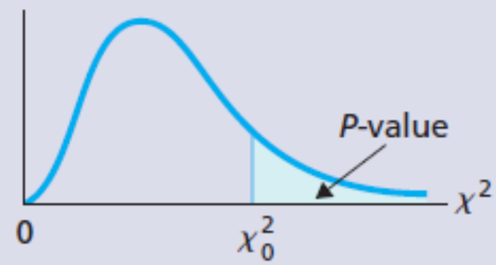
Step 4 The critical value is χ^2_α with df $= (r-1) \times (c-1)$, where r and c are the number of possible values for the two variables. Use Table VII to find the critical value.



Step 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

P-VALUE APPROACH

Step 4 The χ^2-statistic has df $= (r-1)(c-1)$, where r and c are the number of possible values for the two variables. Use Table VII to estimate the P-value, or obtain it exactly by using technology.



Step 5 If $P \leq \alpha$, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.

Section 13.5

Chi-Square Homogeneity Test

Formula 13.3

Expected Frequencies for a Homogeneity Test

In a chi-square homogeneity test, the expected frequency for each cell is found by using the formula

$$E = \frac{R \cdot C}{n},$$

where *R* is the row total, *C* is the column total, and *n* is the sample size.

Key Fact 13.4

Distribution of the χ^2-Statistic for a Chi-Square Homogeneity Test

For a chi-square homogeneity test, the test statistic

$$\chi^2 = \Sigma(O - E)^2/E$$

has approximately a chi-square distribution if the null hypothesis of homogeneity is true. The number of degrees of freedom is $(r - 1)(c - 1)$, where r is the number of populations and c is the number of possible values for the variable under consideration.

Procedure 13.3

Chi-Square Homogeneity Test

Purpose To perform a hypothesis test to compare the distributions of a variable of two or more populations

Assumptions

1. All expected frequencies are 1 or greater
2. At most 20% of the expected frequencies are less than 5
3. Simple random samples
4. Independent samples

Step 1 The null and alternative hypotheses are, respectively,

H_0: The populations are homogeneous with respect to the variable

H_a: The populations are nonhomogeneous with respect to the variable.

Step 2 Decide on the significance level, α.

Step 3 Compute the value of the test statistic

$$\chi^2 = \Sigma(O - E)^2/E,$$

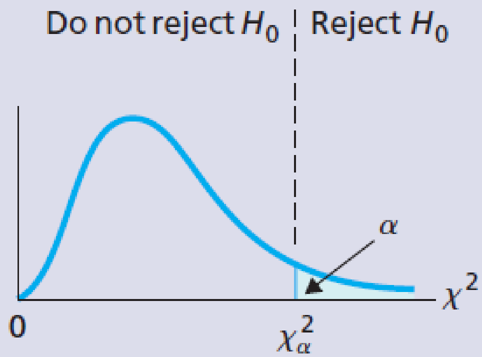
where O and E represent observed and expected frequencies, respectively. Denote the value of the test statistic χ_0^2.

Procedure 13.3 (cont.)

CRITICAL-VALUE APPROACH	OR	*P*-VALUE APPROACH

CRITICAL-VALUE APPROACH

Step 4 The critical value is χ^2_α with df $= (r - 1) \times (c - 1)$, where r is the number of populations and c is the number of possible values for the variable. Use Table VII to find the critical value.

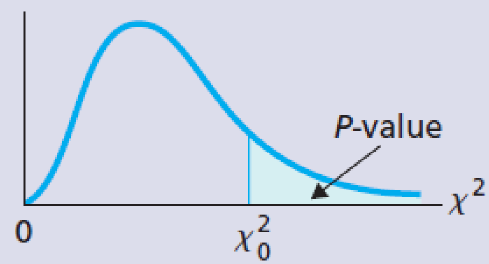


Step 5 If the value of the test statistic falls in the rejection region, reject H_0; otherwise, do not reject H_0.

OR

***P*-VALUE APPROACH**

Step 4 The χ^2-statistic has df $= (r - 1)(c - 1)$, where r is the number of populations and c is the number of possible values for the variable. Use Table VII to estimate the P-value, or obtain it exactly by using technology.



Step 5 If $P \le \alpha$, reject H_0; otherwise, do not reject H_0.

Step 6 Interpret the results of the hypothesis test.