

Probability Final Report

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Introduction

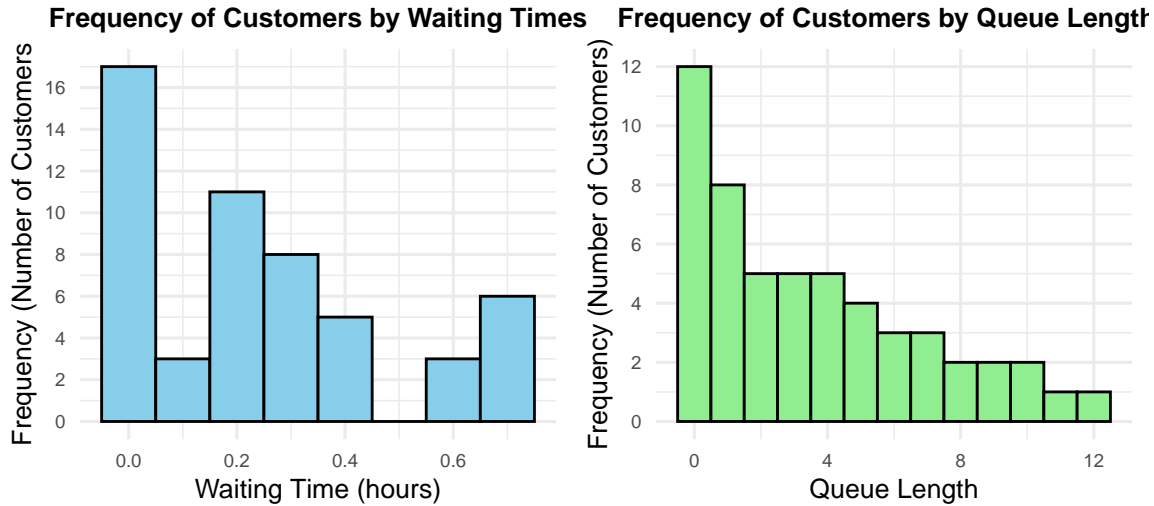
The goal of this analysis is to use probabilistic modelling in order to determine the best setup for our restaurant (Restaurant X) to maximize the satisfaction of our patrons as well as the profit. To do so, we will model customer behavior in order to find a balance between wait time and profit. After careful modeling and consideration, we will take the decision of how many dining tables and chefs to include, as well as the operating hours of the restaurant. Over the course of the analysis, we will try to be as comprehensive as possible, yet we cannot test every single scenario. Therefore, using the scenarios that we test, we will try to determine the optimal scenario given our information. Some additional challenges that were hard to account for were unpredictable customer behavior, staffing constraints, menu complexity (longer time to make certain dishes), and customer patience. Undergoing this analysis with the information that we have and being as thorough as possible is crucial for the opening of our restaurant. Modeling crucial metrics that are essential in measuring the performance of our restaurant (waiting times, queue lengths, down-times, and profits) is of utmost interest to us (the managers of the restaurant) and investors. We must know how to maximize efficiency and profits so that we can start our business in the smartest way possible, and present results to investors that affirm the profitability and satisfaction that our restaurant generates.

Scenario 1

In the first scenario, the restaurant operates with one dining table and one chef from 10 AM to 10 PM (12 hours). Customer arrivals follow a Poisson process with a rate of 5 customers per hour, and the service times follow an exponential distribution with a rate of 6. If the table was occupied when a customer arrived, they would wait in line until the table became vacant.

##	Metric	Value
## 1	Mean Waiting (min)	14.346861
## 2	Median Waiting (min)	12.011621
## 3	Mean Queue Length	3.566038
## 4	Downtime (hours)	3.661892

The average waiting time of 14.35 minutes and the median waiting time of 12.01 minutes show that customers experience moderate waiting times. This indicates that demand for the restaurant is steady, but wait times are still reasonable. The mean queue length of 3.57 customers suggests that the line grows consistently but does not become unmanageable. The 3.66 hours of downtime provide the chef with sufficient breaks, ensuring they are not overworked and that the restaurant maintains operational efficiency.

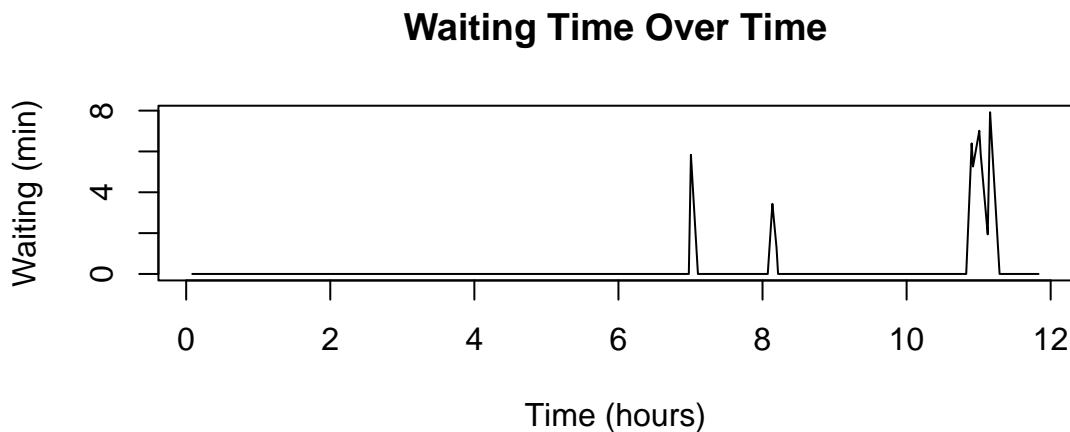


The histogram of waiting times reveals that most customers experience minimal to moderate waits, with a significant portion of waiting times falling near 0 hours. This is a positive indicator, as short wait times are more likely to retain customers while also creating the impression of consistent demand. However, a noticeable tail in the distribution shows that a smaller portion of customers experience longer waits, which is expected since there is only one table and one chef.

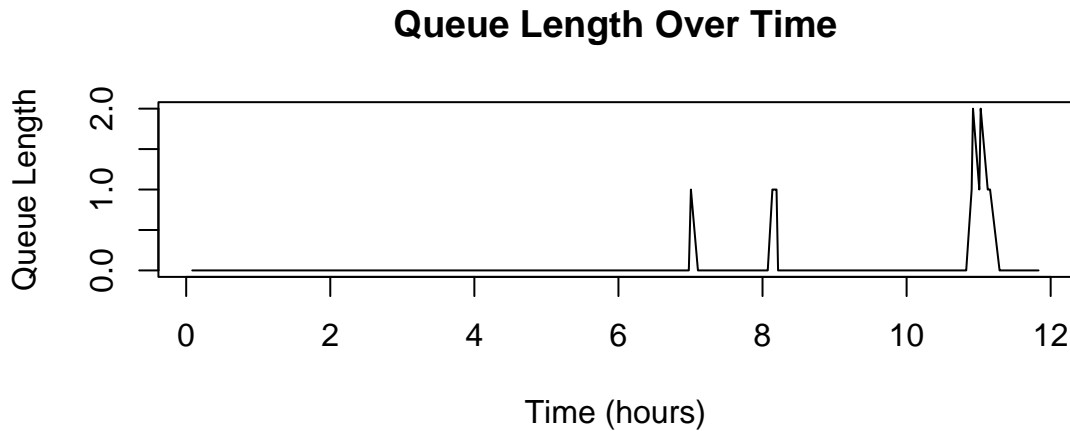
The histogram of queue lengths shows that queues are frequently short, with most queue lengths ranging between 0 and 5 customers. This aligns with the manageable level of demand observed in this setting. The queue occasionally reaches longer lengths, peaking at 12 customers, but this remains rare. Peaks can contribute to the perception of high demand for the restaurant without overwhelming the single chef.

Scenario 2

Scenario 2 For the second scenario, we examine the performance metrics of a restaurant with 5 dining tables and L chefs, operating daily from 10:00 AM to 10:00 PM. Customers arrive according to a Poisson process with an average arrival rate of 10 customers per hour. Once a customer is seated, their total service time (including ordering, cooking, and eating) follows an exponential distribution with a rate of 3, where L represents the number of chefs employed. Each customer spends 50 per meal, and each chef earns 40 per hour. The plot below shows how the waiting time varies throughout the 12 hour period. The X-axis represents time in hours, while the y-axis shows waiting time in minutes.



The waiting time remains relatively stable and low throughout most of the day. The peak waiting times occur during the busiest times of the day when customer arrivals are at their highest, such as around 7 to 8 hours into the 12 hour period (5:00 PM to 6:00 PM) and 11 hours into the 12 hour period (9:00 PM). This plot confirms that for most of the day, customers are experiencing little to no wait, on average and the restaurant is able to handle customer arrivals efficiently. However, surges in waiting times suggest that staffing adjustments during peak hours could further optimize customer flow and reduce these spikes.

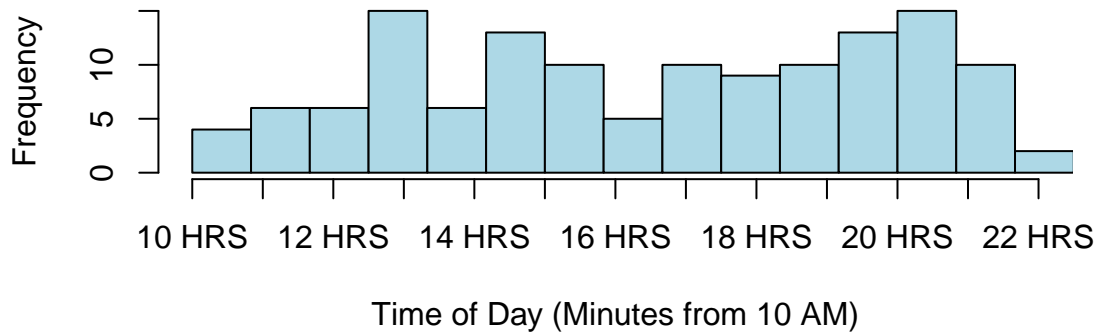


This plot shows the number of customers waiting in line over the course of the operating day. The x-axis represents time in hours, and the y-axis shows the number of customers in the queue. The queue length tends to remain very low, with only increases at times when customer arrivals are higher. At certain points, the queue grows slightly but does not exceed more than 8 minutes, indicating that customers are being seated fairly quickly. This indicates that the restaurant is operating efficiently, with sufficient capacity to serve customers quickly. The low queue length during off-peak hours also indicates that there are periods of underutilization, which might lead to potential staffing adjustments.

Scenario 3

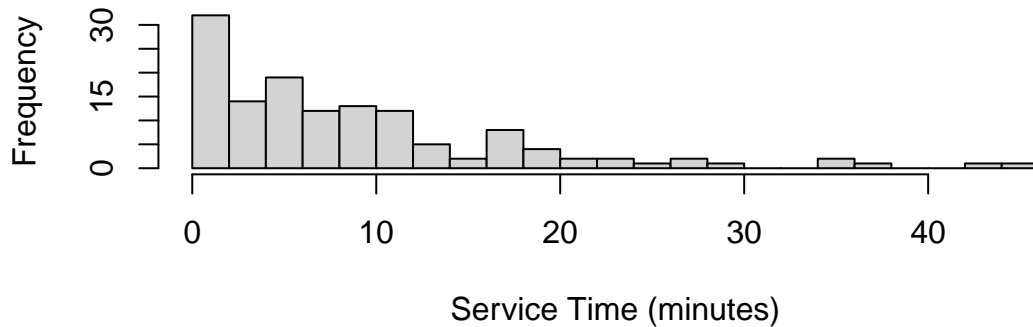
For the third scenario, we laid out the following situation: the restaurant still operates from 10AM-10PM with 5 tables, but with differing arrival times (modeled by a poisson process) and service times to more realistically model the restaurant experience. From 10AM-12PM, the arrival rate is 4 customers/hour (not prime meal time), from 12-3PM it is 12 customers/hour (more popular lunch time), from 3-5PM it is 5 customers/hour, and from 5-10PM it is 15 customers/hour (busy dinner time). The distribution of the arrival times is shown below. We can see that more people are arriving during lunch and especially the busier dinner hours in which the arrival rate increases:

Histogram of Arrival Times (10 AM to 10 PM)



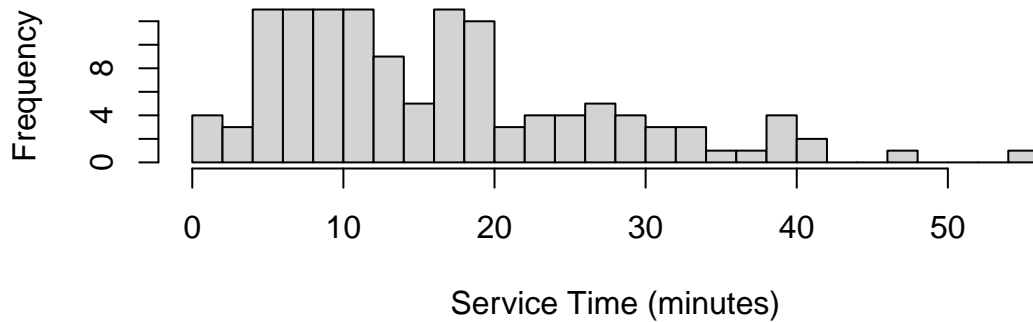
The service times were modeled with a rate 6 from 10AM-5PM and with rate 9 from 5PM-10PM to reflect the restaurant going from 2 chefs to 3. At first, we considered modelling the rate with an exponential distribution (as before), but then considered the following visualization:

Service Time Distribution – Exponential



The exponential distribution made it so that there were a few service times that were very small just by the nature of the distribution. This was unrealistic, so we landed on a gamma distribution of the service times instead:

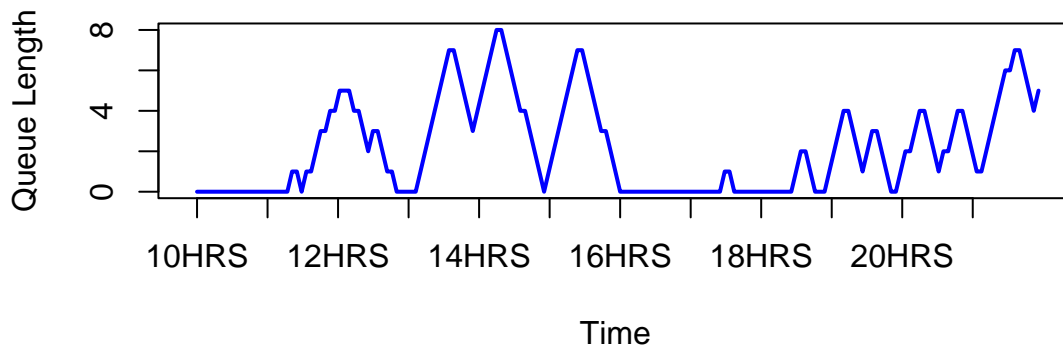
Service Time Distribution – Gamma



We considered this distribution to be more realistic, and we can see that the mean and median lie around 15 minutes of service time.

Then, we modeled the queue length over time. The visualization shows that lines reach peaks at around 12, 2PM and around 10PM. These are the busiest times for Restaurant X and the longest the line gets is 8 people. This is a good thing, considering that we want to create perceived demand for the restaurant, but not have a line that is so long that people leave or don't even try to wait. Still, there are less popular times when the line is 0 and therefore easy to get a table if anyone was desperate to try Restaurant X. Therefore, this is a good dynamic.

Queue Length Over Time



After conduction more modelling, it was determined that the average waiting time across all customers that attend the restaurant is **3.88 minutes**, the average waiting time once in the queue is **19.23 minutes**, the total downtime is **51.95 minutes**. Overall we conclude that these are good numbers that not only increase the perception that our restaurant is in demand while overall not having overwhelming wait times, but also provides enough downtime to keep staff working productively. This scenario was important, because it was more realistic not only to the rates of arrival based on the hours, but also to the distribution of service rates. Also, assuming like in scenario 2 that each customer spends \$50 and each chef is paid \$40 an hour, it was concluded that one day results in a profit of \$5540, and after accounting for buying around \$1500 worth of food a day, the profit can be thought of as **\$4040**. This is a large profit that will allow Restaurant X a lot of opportunities for growth and investment. The profitability of the restaurant will enable it to expand its operations, improve its facilities, and potentially hire additional staff to accommodate more customers.

Project Analysis/Results/Conclusion

Through detailed probabilistic modeling of various scenarios, we identified key insights into the operations of Restaurant X. Each scenario provided a valuable perspective on balancing efficiency, customer satisfaction, and profitability.

Scenario 1, with one table and one chef, demonstrated a moderate queue length (peaking at 12) and reasonable waiting times, as seen in the waiting time histogram and queue length histogram. While manageable, the constraints of limited capacity resulted in frequent queues and downtime, indicating the need for expanded operations to improve throughput.

Scenario 2, with five tables and two chefs, showcased significantly improved efficiency, evidenced by low and stable waiting times throughout the day (Waiting Time Over Time plot). The queue length remained minimal, peaking briefly during busy hours but never becoming unmanageable. These results confirm that increasing both tables and staff optimizes customer flow, reduces waiting times, and minimizes customer frustration, especially during peak periods like dinner hours.

Scenario 3 introduced realistic variations in customer arrivals and service times. The arrival time histogram revealed clear peaks during lunch and dinner hours, aligning with typical dining patterns. Service times modeled with a gamma distribution (as opposed to the exponential distribution) reflected more practical scenarios, eliminating unrealistically short service durations. The queue length over time plot highlighted demand surges at peak hours, with the queue peaking at 8 customers—creating a perception of popularity without deterring potential customers. Waiting times remained low on average, reinforcing the efficiency of the system during both peak and off-peak hours.

The results across all scenarios clearly demonstrate that the five-table layout with adjusted staffing during peak times (Scenario 3) strikes the ideal balance between demand, efficiency, and profitability. By ensuring low average waiting times (3.88 minutes) and manageable queues, Restaurant X can deliver a satisfying customer experience while maintaining operational stability. Furthermore, the daily net profit of \$4040 underscores the financial viability of this configuration, supporting future investments in growth, improved facilities, and staffing flexibility.

In conclusion, Scenario 3 is the optimal setup for Restaurant X, combining a dynamic model of realistic arrivals and service times with strong operational and financial performance. Future analyses could incorporate dynamic staffing adjustments and external variables, such as seasonality or reservation systems, to further refine operations and enhance profitability. This analysis confirms that Restaurant X is a sound and scalable investment with the potential for sustained success.