A Lecture on Topological Defect Operators

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Table of contents

1 What is this

This is a lecture note prepared for two sets of "intensive lectures":[1]

- at Tohoku University, Oct. 11-13, 2023, and
- at Yukawa Insititute for Theoretical Physics, Kyoto University, Nov. 29-1, 2023.

In this lecture I will try to explain the constructions of topological defects corresponding to generalized symmetries. Due to lack of time and (more significantly) my understanding, the lecture will focus on bosonic systems, and the generalization to fermionic systems is left for the readers/audiences.

1.1 Prerequisite

- Basic knowledge about scalar field theory and (abelian) gauge theory in path-integral formalism, and
- Knowledge about renoarmalization group (RG) flows to understand motivations.

1.2 What is contained and what is not

1.3 Other Lectures/Reviews

Recently there has been a surge of lecture notes/ review articles on generalized symmetries. The ones I have noticed are [1–6]. Because this lecture will focus on the fundamental aspects of the topic and will not connect very well with the existent literature (so sorry about that), readers/audiences are strongly encouraged to refer to at least one of them, or something similar.

Also, about conventional symmetries and their anomalies, there are nice old lectures. The one I would particularly recommend is [7].

[1] In Japan, an "intensive lecture" is a format of a lecture course where a lecturer (usually from another university) gives lectures in consecutive days filling 7-9 slots in usually 3 days.

2 Introduction

2.1 Symmetry

Symmetry plays a fundamental role in theoretical physics. In this lecture we consider them in *quantum field theories* (QFTs). The fundamental fact about symmetry in QFTs is that is it preserved along the renormalization group flow. More precisely, when an ultraviolet (UV) theory $\mathcal{T}_{\mathrm{UV}}$ flows into an infrared theory $\mathcal{T}_{\mathrm{IR}}$, there is a canonical homomorphism f_{RG} from the UV symmetry group G_{UV} to the IR symmetry group G_{IR}:[1]

RG flow homomorphism from UV symmetry to IR symmetry

$$f_{\mathrm{RG}} : G_{\mathrm{UV}} \to G_{\mathrm{IR}}. \tag{2.1}$$

Given this relation, there are two ways of applying symmetry in QFT:

- UV to IR: Given a microscopic model (e.g. a model of elementary particles or electrons in a matter), constrain/guess what happens in the macroscopic scale.
- IR to UV: Given some macroscopic phenomena, constrain/guess what could be the microscopic origin of it (e.g. guessing QCD lagrangian from hadron spectrum).

⚠ Warning

In this lecture, whenever something is called "symmetry", it means a *global* symmetry. Here *global* means that the symmetry operation acts on the entire space. In addition, in most contents we exclude symmetries acting on the spacetime out of consideration for simplicity.

[1] If the UV theory is a fixed point, G_{UV} should be understood as the one preserved by the perturbation triggering the RG flow. If the RG flow is to a lower nonzero energy, and if one retains all the (even very massive) degrees of freedom in the description of $\mathcal{T}_{\mathrm{IR}}$, the map f_{RG} is an isomorphism. However, typically one integrates out massive dofs in the description of $\mathcal{T}_{\mathrm{RG}}$, in which case some symmetry can decouple and thus f_{RG} can be non-surjective. Also, if one also drops some higher-order interaction terms, or runs the flow to the zero energy, there can be an *emergent* symmetry, in which case f_{RG} can be non-injective.

2.2 Locality

The second important keyword in this lecture is **locality**. By the word quantum field theory, in most cases we implicitly mean *local* quantum field theory. This can roughly be explain that the action is written as the integration of Lagrangian density, which is a local functional of fields over the spacetime.

Locality of QFT has a concequence with respect to symmetry: that is, in most situations we only consider symmetries that *preserves locality.* In terms of fields, this means that the symmetry transformation is local:

$$\phi(x) \mapsto F(\phi(x)),$$

where $F(\phi(x))$ is a function depends on the *local* value a field (or a collection of fields and its derivatives) at the point x. If the symmetry involves a spacetime transformation, the point x in the right hand side should be replaced by the image of x under the symmetry. This preservation of locality of a symmetry is the reason for the symmetry relation in Equation 2.1.

3 Summary

In summary, this book has no content whatsoever.

References

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[3] P. R. S. Gomes, “An introduction to higher-form symmetries”, SciPost Phys. Lect. Notes **74**, 1 (2023), arXiv:2303.01817 [hep-th].

[4] L. Bhardwaj, L. E. Bottini, L. Fraser-Taliente, L. Gladden, D. S. W. Gould, A. Platschorre, and H. Tillim, “Lectures on Generalized Symmetries”, (2023), arXiv:2307.07547 [hep-th].

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[6] S.-H. Shao, “What’s Done Cannot Be Undone: TASI Lectures on Non-Invertible Symmetry”, (2023), arXiv:2308.00747 [hep-th].

[7] T. Yuji, “Lecture on anomalies and topological phases”, (2019), https://member.ipmu.jp/yuji.tachikawa/lectures/2019-top-anom/.