CSC903MODELLING AND SIMULATION

Based on Stewart Robinson (2004). Simulation: The Practice of Model Development and Use

Overview of modeling and simulation

Session topics

What, why, when, Time, Variability, Distributions and the related issues with modeling and simulation.

What is modeling and simulation ? - recap

Simulation is the *experimentation* with a *simplified imitation* (on a computer) of an *operations system* as it progresses through time, for the purpose of *better understanding and/or improving* that system [Robinson Stewart, 2004].

Why should simulation be necessary?

To gain the insight necessary for making some decisions: eg for example in a simulation of a port it may be necessary to model the tidal and weather conditions for purposes of advising ships not to enter the port.

To make use of models in understanding, changing, managing and controlling reality. In particular, this involves understanding and/or identifying ways of improving a system.

Inform decision-making on the real system regarding the future items.

To enable the prediction of the performance of an operations system under a specific set of inputs. For example, it might predict the average waiting time for telephone customers at a call centre when a specific number of operators are employed.

Why should simulation be necessary?

To allow a “what-if” analysis as the user enters a scenario and the model predicts the outcome. The alternative scenarios may be explored until the experimenter has obtained sufficient understanding or identified how to improve the real system. It thus acts as a decision support system [Robinson, 2004].

To handle a problem that is too complicated to solve analytically.

To handle tractable problem *(Easily solved or worked)* whose level of detail provided by the analytical answers is insufficient for the required needs.

To put a new concept into practice on an experimental basis and see if it produces the desired results;

Why should simulation be necessary?

Provide a higher level of detail than other techniques.
Provide (approximate) *answers at a lesser cost (or effort)* to some problems which are fully tractable mathematically but whose solution may be cumbersome and time-consuming.

Permit modification or design of systems by *trial and error*.

Allows for easy *exploration of the system's sensitivity* to changes in the input parameters, and provides *a highly controllable environment* for experiments.

Why should simulation be necessary?

***To test the applicability and validity of mathematical models and expressions*.**

To gain insight and predictions in a system that is complex and has variability and several component interconnections.

To predict the performance of systems that have interconnected components and have both combinatorial and dynamic complexity.

Advantages of simulation

Simulation is better than working with the *real system:-*

-Simulation is less costly, **while experimentation with the real system can be very expensive. Consider for example interrupting day-to-day operations in order to try out new ideas. The shut downs may lead to loss of customers or customer dissatisfaction.**

-Simulation takes less time, **while an experiment with a real system may take many weeks or months before a true reflection of the performance of the system can be obtained. With simulation the, results on system performance can be obtained in a matter of minutes, maybe hours. The faster experimentation also enables many ideas to be explored in a short time frame.**

Advantages of simulation

Simulation is better than working with the *real system:-*

-Simulation enables easier control of the experimental conditions, **which is useful in comparing alternatives. This can be very difficult when experimenting with the real system. For example consider the difficulty of controlling the arrival of patients at a hospital.**

-Simulation can be used even where the real system does not exist, **such as the case of a new yet to be built school, football stadium or hospital.**

Advantages of simulation

-Simulation is better than other modeling approaches (simple paper calculations, spreadsheet models, heuristic methods, linear programming, dynamic programming and genetic algorithms):

-Simulations can cope with modeling variability, **other methods that are mentioned are not able to do so. This is often due to increases in their complexity. Some systems cannot be modeled analytically.**

Advantages of simulation

-Simulations have fewer restrictive assumptions **compared to other methods for example queuing theory, often assumes particular distributions for arrival and service times while for many processes these distributions are not appropriate. In simulation, any distribution can be selected.**

***-Simulations provides more transparency* to the manager** **than other methods because it is more intuitive more so if the display is animated any non-expert greater understanding of, and confidence in, the model.**

Advantages of simulation

Simulation gives managers particular benefits:

-Simulation fosters creativity **by allowing trials without fear of failure.**

-Simulation leads to the creation of knowledge and understanding **as it forces the management to take time examining the problem given 'problem specified is half solved'.**

-Simulation with visualization and communication **enable easier demonstration of ideas to management.**

-Simulation enables consensus building **as it provides a powerful tool for sharing concerns and testing ideas. Sometimes opinions are at variance for example in a factory, managers and workers may not agree over working hours and shifts.**

Some issues with simulation

-Simulation software can be expensive, **especially where consultants have to be employed.**

-Simulation can be time consuming **since sometimes it needs much time to get useful results.**

-Simulation can be data hungry **and this can present a problem where significant amount of data is needed and it is not immediately available.**

-Simulation requires expertise **as it is beyond the development of a computer program or the use of a software package. It requires skills in, among other things, conceptual modeling, validation and statistics, as well as skills in working with people and project management.**

Some issues with simulation

-Overconfidence in simulation can be a problem **especially when anything produced on a computer is seen to be right. Usually interpreting the results from a simulation, requires checking the validity of the underlying model and the assumptions and simplifications that have been made [Robinson, 2004].**

-Developing cause-and-effect relationships **through simulation can be difficult, especially when the system under consideration requires the specification of many input parameters and involves complex interactions.**

-The statistical analysis of simulation results is difficult **especially determining the effect of the starting conditions of the simulation on the final results.**

When to use simulation

Where features involve the entities that are being processed through a series of stages, such as queuing systems. There are very many areas where simulation can be used and they include:

- **Manufacturing systems;**
- **Public systems: health care, military, natural resources, agriculture;**
- **Transportation systems; Construction systems**
- **Restaurant and entertainment systems**
- **Business process reengineering/management;**
- **Food processing;**
- **Computer system performance**
- **Service and retail systems**
- [Banks et al, 1996]

Some basic principles in Simulation

Tools: specialist software do not require programing from scratch.

Key elements in simulation software:-

- modeling the progress of time - present in all dynamic simulations

- modeling variability - present in the majority of simulation software.

Some basic principles in Simulation

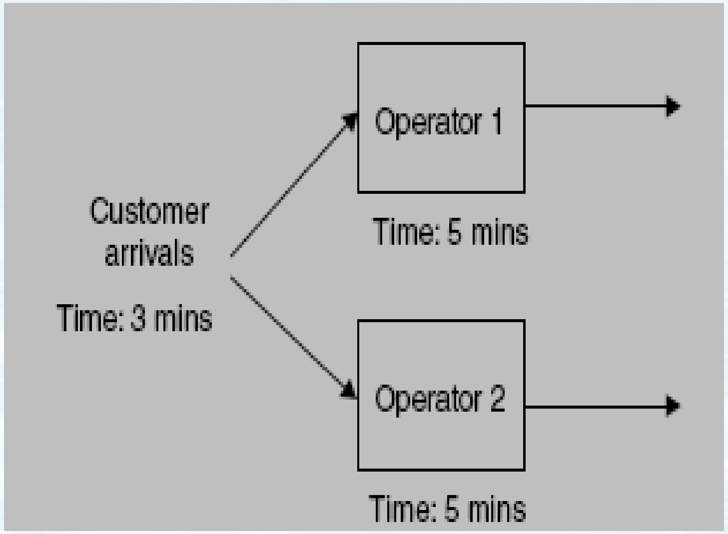
Modeling the progress of time using time-slicing

The progress of time is modeled using a constant time-step (Δ_t). *Example [Robinson, 2004].*

$$t_{n+1} = t_n + \Delta_t$$

Some basic principles in Simulation

Modeling the progress of time using time-slicing- call center example



Time	Call arrival	Operator 1	Operator 2
0	3		
1	2		
2	1		
3	3	5	
4	2	4	
5	1	3	
6	3	2	5
7	2	1	4
8	1		3
9	3	5	2
10	2	4	1
11	1	3	
12	3	2	5
13	2	1	4
14	1		3
15	3	5	2
16	2	4	1
17	1	3	
18	3	2	5
19	2	1	4
20	1		3
21	3	5	2
22	2	4	1
23	1	3	
24	3	2	5
Completed calls		3	3

Call center, source: [Robinson, 2004 pg.15]

Some basic principles in Simulation

Limitations of the Time-slicing approach

Inefficiency. For example consider the many time-steps where there is no change in the system-state making the many computations are unnecessary. The only points of interest are when a call arrives, when an operator takes a call and when an operator completes a call and there are 22 points in total. However 72 (24×3) calculations are performed.

Non obvious value of Δ_t. The second problem is determining the value of Δ_t. In the example above time can be counted in whole numbers. However, there can be a wide variation in activity times within a model from possibly seconds (or less) through to hours, days, weeks or more.

Some basic principles in Simulation

Modeling the Progress of Time Using the Discrete-Event Simulation Approach

Only the points in time at which the state of the system changes are represented.

The system is modeled as a series of events, which are the, instants in time when a state-change occurs.

Examples of events: a customer arrives, a customer starts receiving service and a machine is repaired. Each of these occurs at an instant in time.

Some basic principles in Simulation

Modeling the Progress of Time Using the Discrete-Event Simulation Approach

Time	Event
3	Customer arrives
	Operator 1 starts service
6	Customer arrives
	Operator 2 starts service
8	Operator 1 completes service
9	Customer arrives
	Operator 1 starts service
11	Operator 2 completes service
12	Customer arrives
	Operator 2 starts service
14	Operator 1 completes service
15	Customer arrives
	Operator 1 starts service
17	Operator 2 completes service
18	Customer arrives
	Operator 2 starts service
20	Operator 1 completes service
21	Customer arrives
	Operator 1 starts service
23	Operator 2 completes service
24	Customer arrives
	Operator 2 starts service

Discrete Event [Robinson, pg. 16]

Some basic principles in Simulation

The three-phase simulation approach for discrete events

The events are classified into two types: the bound or booked events (B) and the conditional (C) events.

B (bound or booked) events**:** are state changes that are scheduled to occur at a point in time.

Examples: call arrivals in the call centre model that occur every 3 minutes. Usually the B-events are related to the arrivals or the completion of activities.

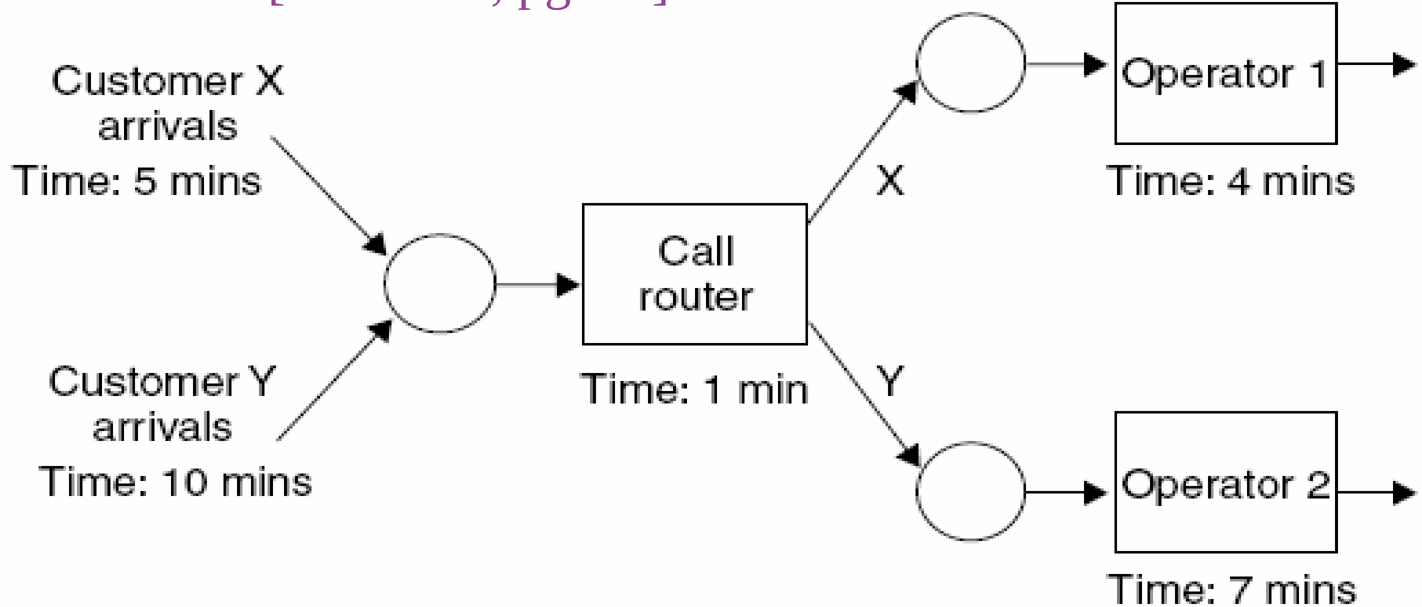
C (conditional) events**:** are state changes that are dependent on the conditions in the model.

Examples: an operator that can only start serving a customer *if* there is a customer waiting to be served and the operator is not busy. Usually the C-events are related to the start of some activities.

Some basic principles in Simulation

The three-phase simulation approach for discrete events

Call Center-b [Robinson, pg. 17]



Customers: X, Y -make calls. X calls arrive every 5 minutes; Y calls arrive every 10 minutes. A queue (denoted by a circle) is used to hold the calls before the call router directs the call to the right operator. The routing activity takes 1 minute. Operator 1 the first takes all customer X calls, operator 2 takes customer Y calls. Operator 1 takes exactly 4 minutes to deal with a call and operator 2 exactly 7 minutes.

Some basic principles in Simulation

The three-phase simulation approach for discrete events

Event	Type	Change in state	Future events to schedule
B1	Arrival	Customer X arrives and enters router queue	B1
B2	Arrival	Customer Y arrives and enters router queue	B2
B3	Finish activity	Router completes work and outputs X to operator 1 queue, Y to operator 2 queue	
B4	Finish activity	Operator 1 completes work and outputs to world (increment result work complete X by 1)	
B5	Finish activity	Operator 2 completes work and outputs to world (increment result work complete Y by 1)	

Event	Type	Condition	Change in state	Future events to schedule
C1	Start activity	Call in router queue and router is idle	Router takes call from router queue and starts work	B3
C2	Start activity	Call is in operator 1 queue and operator 1 is idle	Operator 1 takes call from operator 1 queue and starts work	B4
C3	Start activity	Call is in operator 2 queue and operator 1 is idle	Operator 2 takes call from operator 2 queue and starts work	B5

B-and C-Events: Call Center-b [Robinson, pg. 18]

Some basic principles in Simulation

The three-phase simulation approach for discrete events

The Process

Initialization process: **the initial B-events are scheduled eg. the arrival of the first customers. The event list that keeps a record of all future events that have been scheduled is set up. The simulation then moves into three phases that are continuously repeated:**

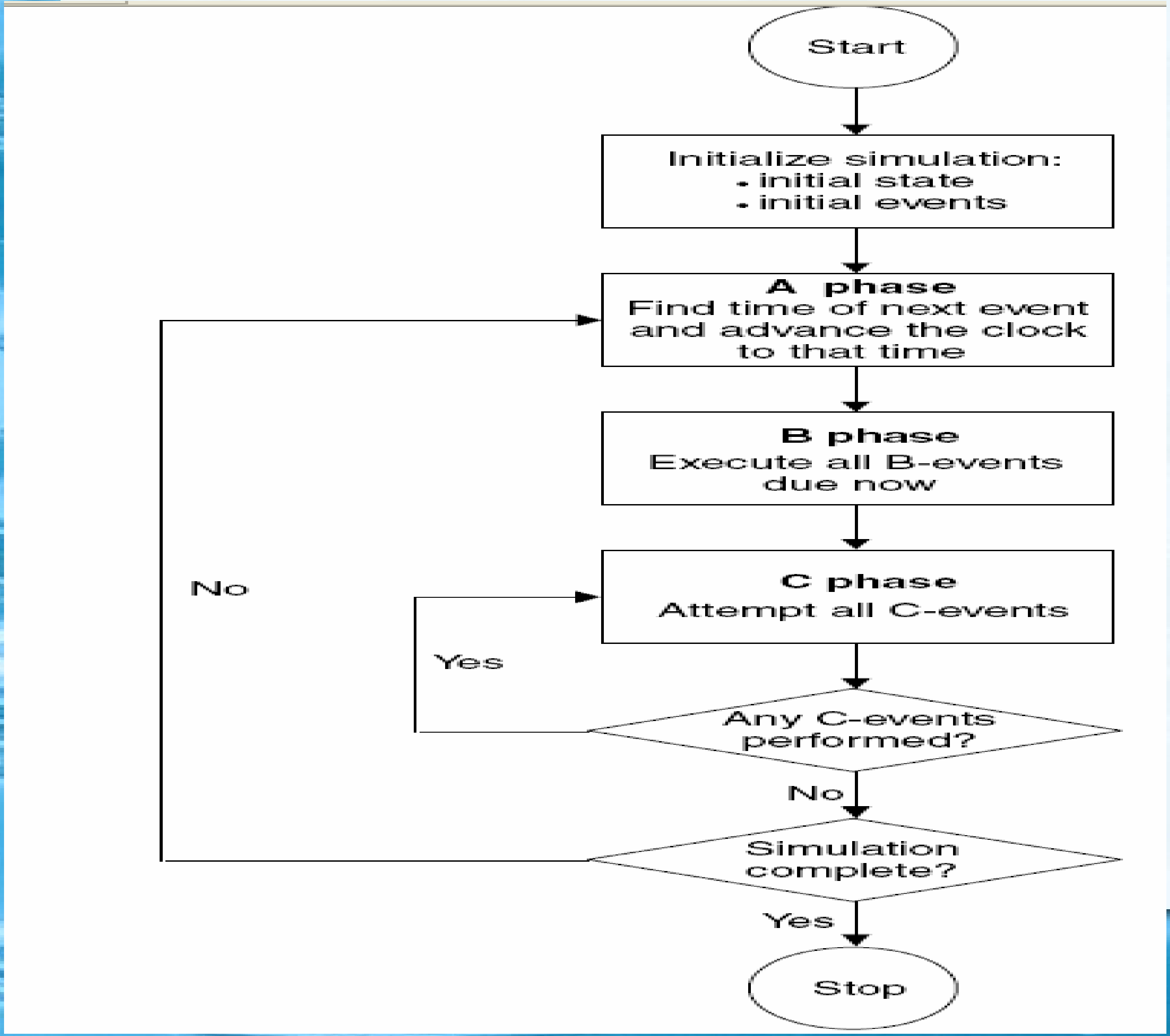
The A-phase, **also called the simulation executive, is where the time of the next event is determined by inspecting the event list. The simulation clock is then advanced to the time of the next event.**

The B-phase, **is where all B-events that are due at the appropriate clock time are executed.**

The C-phase, **is where all C-events are attempted and those for which the conditions are met are executed. The simulation continues to attempt C-events until no further events can be executed. The simulation then repeats from the *A-phase* unless it is deemed that the simulation is complete.**

Some basic principles in Simulation

The three-phase simulation approach for discrete events



The Three-Phase Simulation Approach- Call Center-b [Robinson, pg. 19]

Some basic principles in Simulation

The three-phase simulation approach for discrete events

Model Status

Phase	Router queue	Router	Oper. 1 queue	Oper. 1	Oper. 2 queue	Oper. 2
	Empty	Idle	Empty	Idle	Empty	Idle

Event List

Event	Time
B1	5
B2	10

Results

Work complete	
X	0
Y	0

Call Centre Simulation: Clock = 0 (Initialize Simulation).

Some basic principles in Simulation

The three-phase simulation approach for discrete events

Model Status

Phase	Router queue	Router	Oper. 1 queue	Oper. 1	Oper. 2 queue	Oper. 2
B	X1	Idle	Empty	Idle	Empty	Idle
C	Empty	X1	Empty	Idle	Empty	Idle

Event List

Event	Time
B3	6
B2	10
B1	10

Results

	Work complete
X	0
Y	0

Call Centre Simulation: Clock = 5 (Event B1).

Some basic principles in Simulation

The three-phase simulation approach for discrete events

Model Status

Phase	Router queue	Router	Oper. 1 queue	Oper. 1	Oper. 2 queue	Oper. 2
B	Empty	Idle	X1	Idle	Empty	Idle
C	Empty	Idle	Empty	X1	Empty	Idle

Event List

Event	Time
B2	10
B1	10
B4	10

Results

Work complete	
X	0
Y	0

Call Centre Simulation: Clock = 6 (Event B3).

Some basic principles in Simulation

The three-phase simulation approach for discrete events

Table 2.12 Call Centre Simulation: Clock = 16 (Events B4, B3).

Model Status

Phase	Router queue	Router	Oper. 1 queue	Oper. 1	Oper. 2 queue	Oper. 2
B	Empty	Idle	X3	Idle	Empty	Y1
C	Empty	Idle	Empty	X3	Empty	Y1

Event List

Event	Time
B5	18
B2	20
B1	20
B4	20

Results

	Work complete
X	2
Y	0

Call Centre Simulation: Clock = 16 (Event B3, B4).

Some basic principles in Simulation

The three-phase simulation approach for discrete events

Model Status

Phase	Router queue	Router	Oper. 1 queue	Oper. 1	Oper. 2 queue	Oper. 2
B	Empty	Idle	Empty	X3	Empty	Idle
C	Empty	Idle	Empty	X3	Empty	Idle

Event List

Event	Time
B2	20
B1	20
B4	20

Results

Work complete	
X	2
Y	1

Call Centre Simulation: Clock = 18 (Event B5).

Some basic principles in Simulation

MODELING VARIABILITY

Variability - the changes that occur in the components of the system as time goes on. Variability can be predictable or unpredictable.

We focus on modeling unpredictable variability as it presents the key challenge.

Modeling unpredictable variability

Call center example: variability may arise from calls arrivals, the time it takes the calls to be routed, and the time that the operators take to process the calls.

Unpredictable variability is handled using the random numbers.

Some basic principles in Simulation

MODELING VARIABILITY

Random numbers

Are a sequence of numbers that appear in some random order.

They can be integers (whole) numbers on a scale of say 0 to 99 or 0 to 999, or as real numbers (with decimal places) numbers on a scale of 0 to 1.

Some basic principles in Simulation

MODELING VARIABILITY

Important properties of random numbers

Uniformity **- each number has the same probability of occurring**

Independence **- the occurrence of any number is not influenced by other numbers.**

Most of the development software environments today including spreadsheets have facilities for generating random numbers. During earlier times there were some random number tables that could be used.

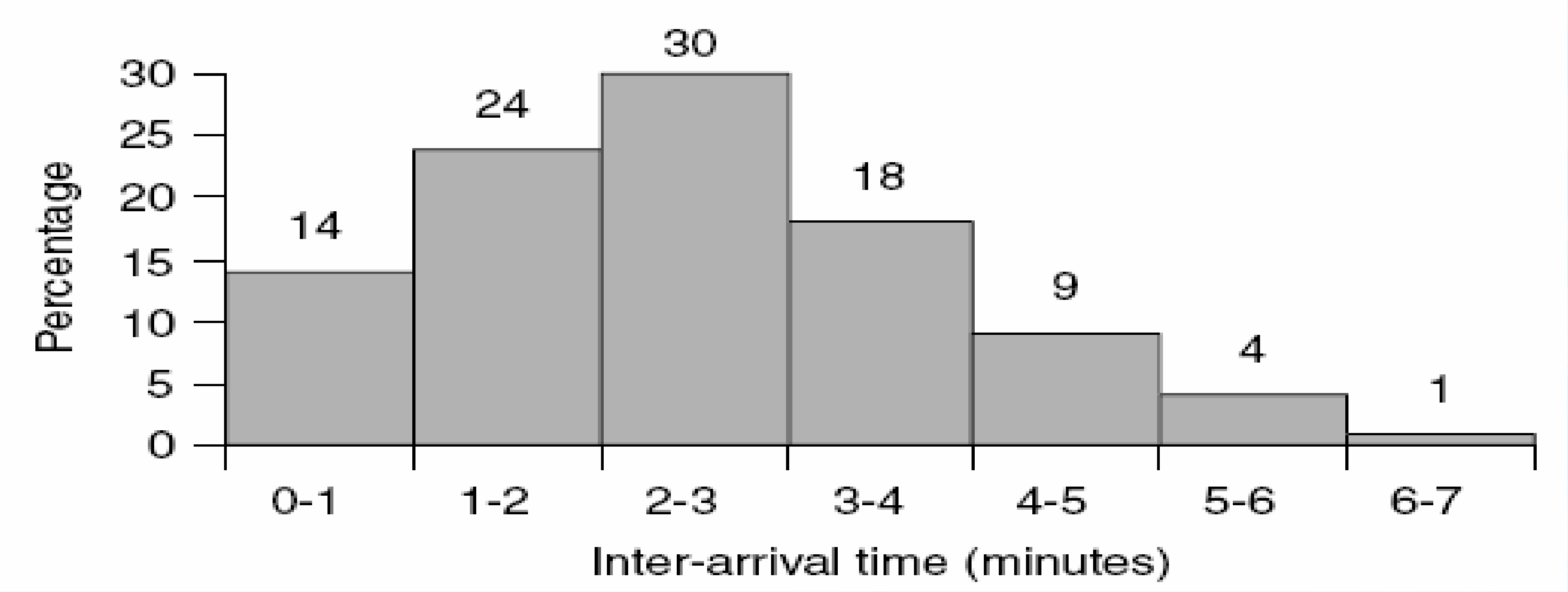
Variability in proportions can be modeled by direct size comparisons such as in a space of 0-999 how many are below 400? For types we can have 0-60 of type X and 61-99 of type Y.

Some basic principles in Simulation

MODELING VARIABILITY

Modeling variability that are related to time

Examine the frequency of occurrence over a given period of time.
Consider the time it takes between two calls to arrive and a situation where it varies from 0 -7. This can be as shown on the figure below.

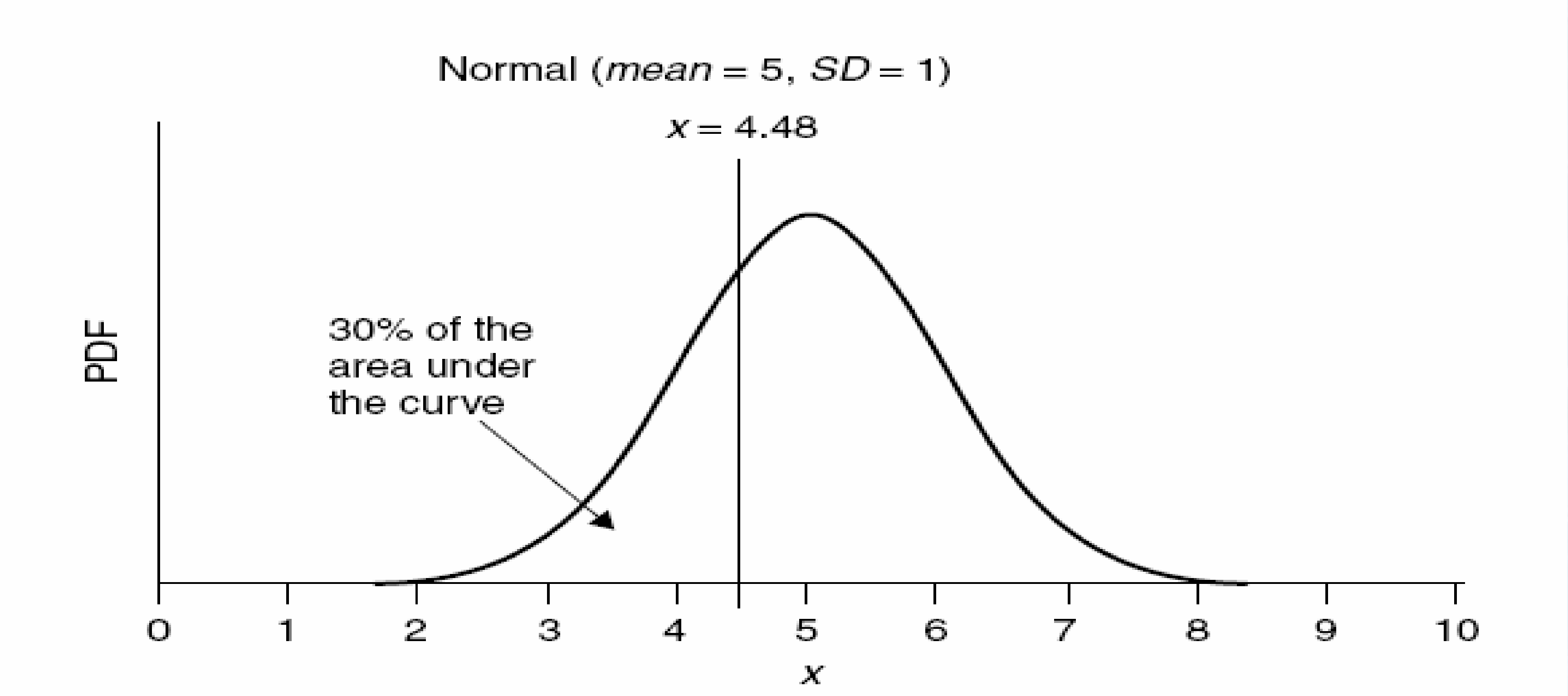


Frequency Distribution for Inter-Arrival Time of Calls [Robinson, p.28]

Some basic principles in Simulation

MODELING VARIABILITY

Sampling from standard statistical distributions

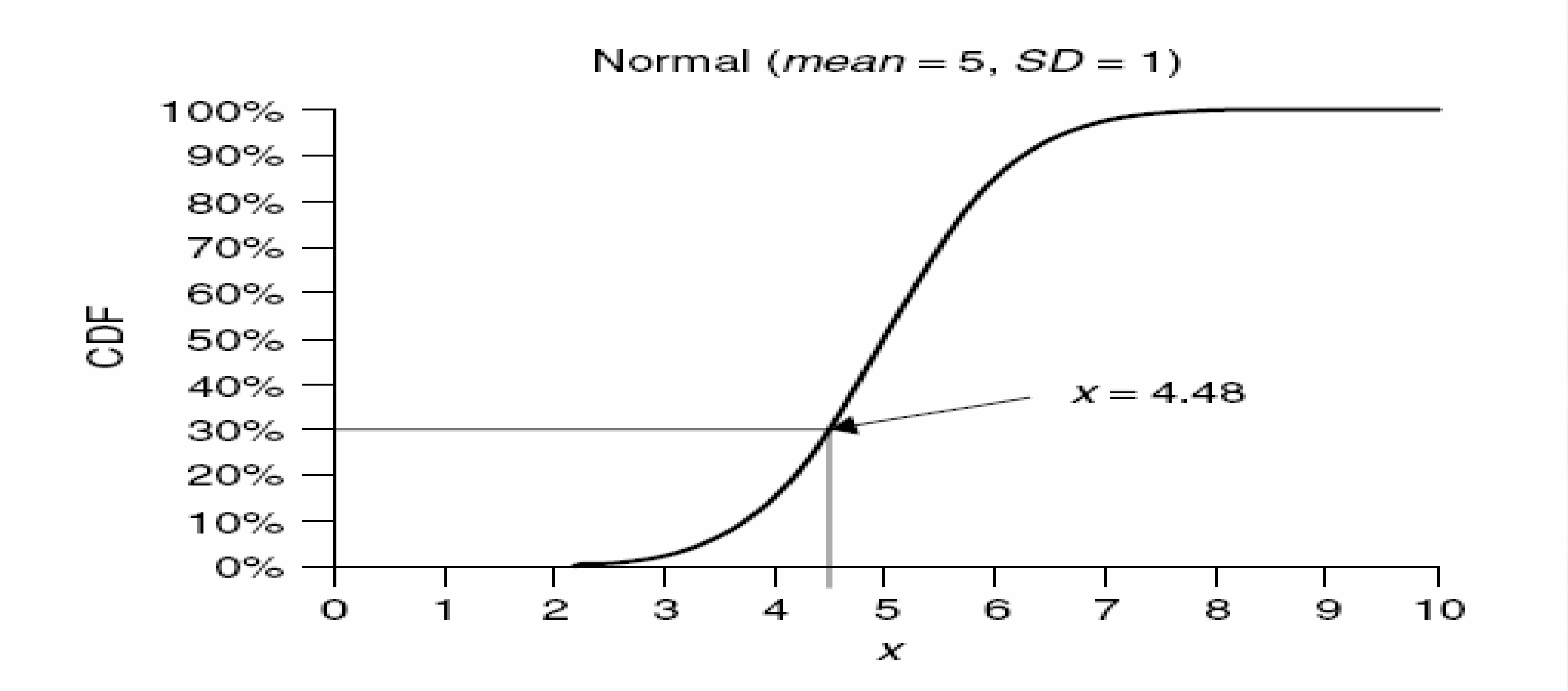


A normal distribution [Robinson, p.31]

Some basic principles in Simulation

MODELING VARIABILITY

Sampling from standard statistical distributions



Cumulative distribution functions from the Normal Distribution- to avoid areas as in the previous slide

Some basic principles in Simulation

MODELING VARIABILITY

Sampling from standard statistical distributions

Computer generated random numbers

Needed to meet large demand of thousands or even millions of random numbers.

The random numbers that are generated are known as *pseudo random numbers*.

An algorithm for generating such random numbers is as follows:

$$X_{i+1} = [aX_i + c\,](mod\ m), \text{ where:}$$

X_i **: stream of random numbers (integer) on the interval** $(0, m-1)$

a **: multiplier constant;** ***c*** **: additive constant**

m **: modulus;** ***mod m*** **means take the remainder having divided by** ***m***

Some basic principles in Simulation

MODELING VARIABILITY

Sampling from standard statistical distributions

Computer generated random numbers

$$X_{i+1} = [aX_i + c\,](mod\ m)\text{, where:}$$

X_i : stream of random numbers (integer) on the interval $(0, m - 1)$

a : multiplier constant; c : additive constant

m : modulus; *mod m* means take the remainder having divided by m

Example

When $X_0 = 8$, $a = 4$, $c = 0$ and $m = 25$. This gives random numbers on a range of 0 to 24, the maximum always being one less than the value of m. Note that the stream repeats itself after $i = 9$. Carefully select the constants.

Some basic principles in Simulation

MODELING VARIABILITY

Some common statistical distributions

Beta ($Shape_1$, $Shape_2$)

Potential applications: **time to complete a task; proportions (e.g. defects in a batch of items); useful as an approximation in the absence of data, task times in Pert networks.**

***Mean*:** $(shape_1) / (shape_1 + shape_2)$

***Standard deviation*:** $\sqrt{[(shape_1 \times shape_2) / (shape_1 + shape_2)^2 (shape_1 + shape_2 + 1)]}$

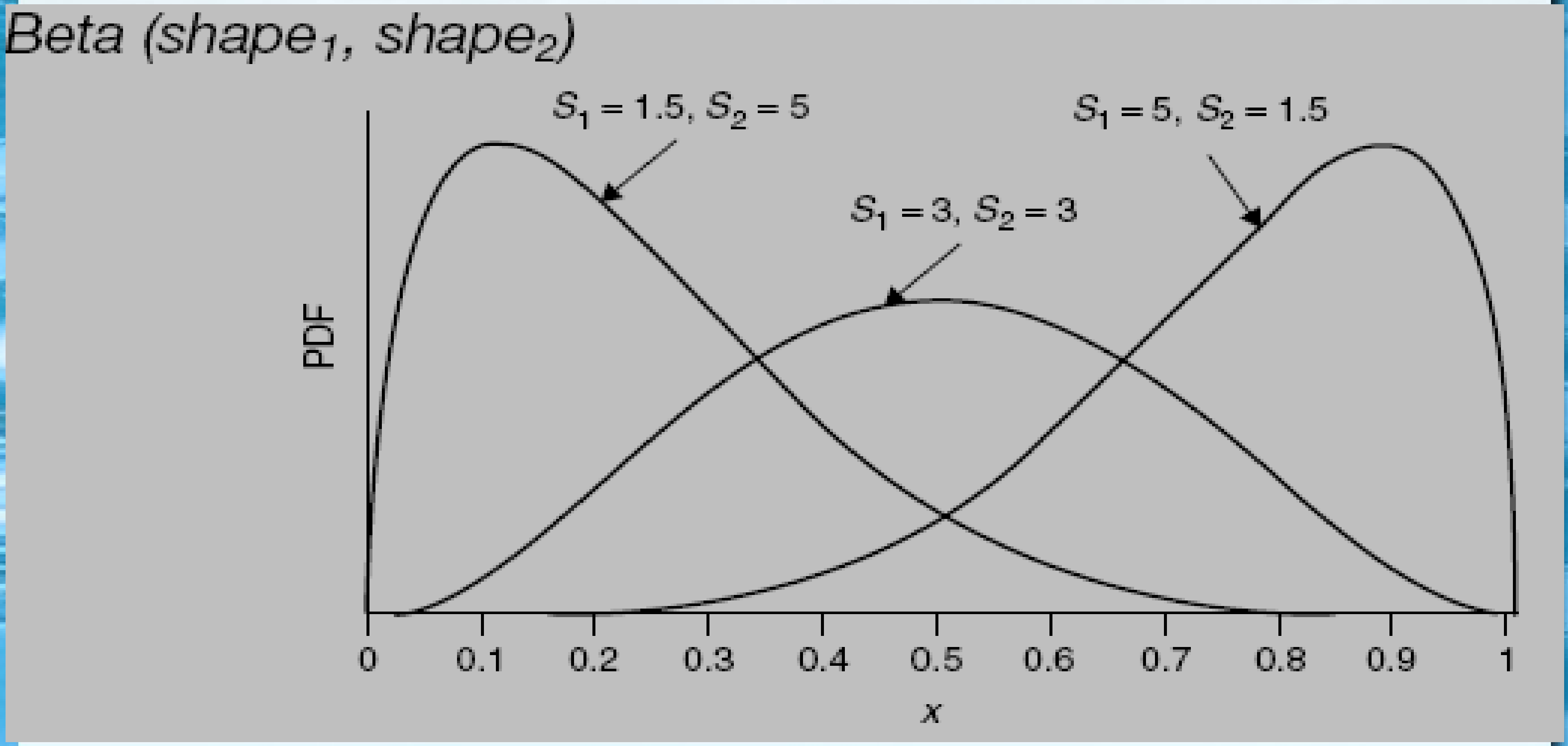
***Range of values*:** **$0 < x < 1$ (use a multiplier to extend the range)**

Some basic principles in Simulation

MODELING VARIABILITY

Some common statistical distributions

Beta ($Shape_1$, $Shape_2$):



Some basic principles in Simulation

MODELING VARIABILITY: Some common statistical distributions

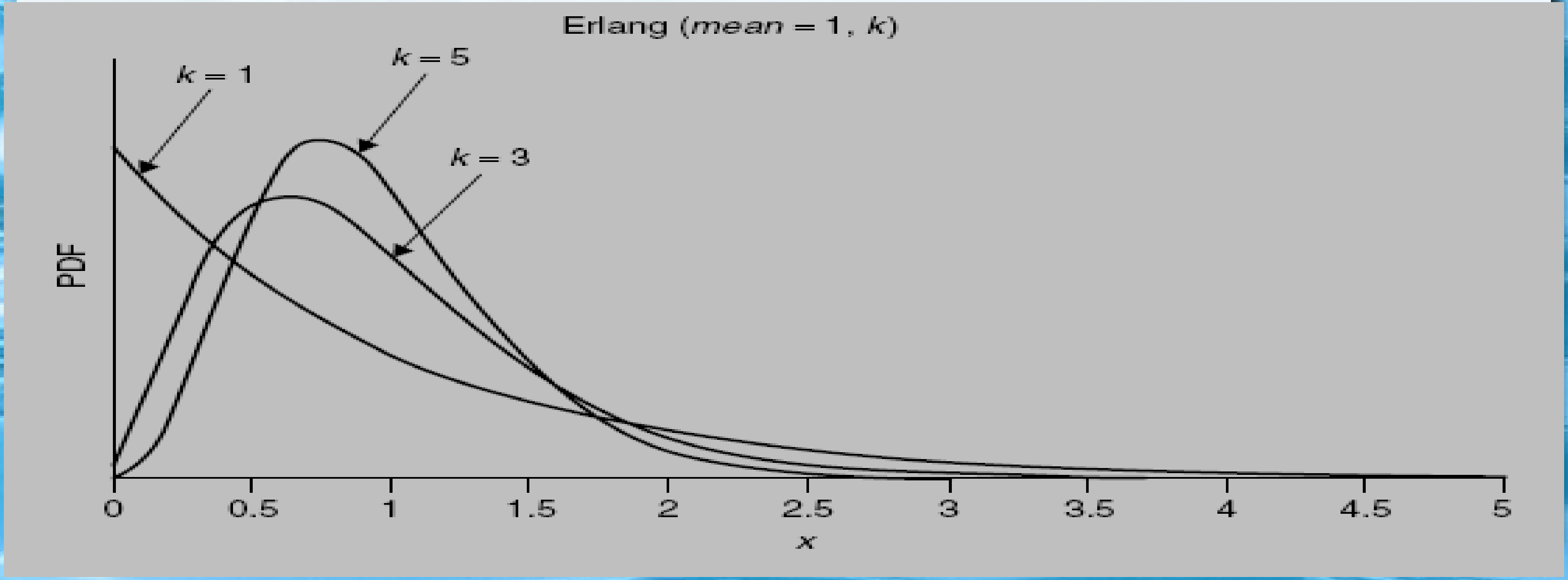
Erlang (mean k)- special form of Gamma distribution

***Potential applications*: time to complete a task; inter-arrival times (e.g. customer arrivals); time between failure, queuing theory**

***Mean*: mean=k**

***Standard deviation:* (mean)/ (√k)**

***Range of values:* 0< x < ∞**



Some basic principles in Simulation

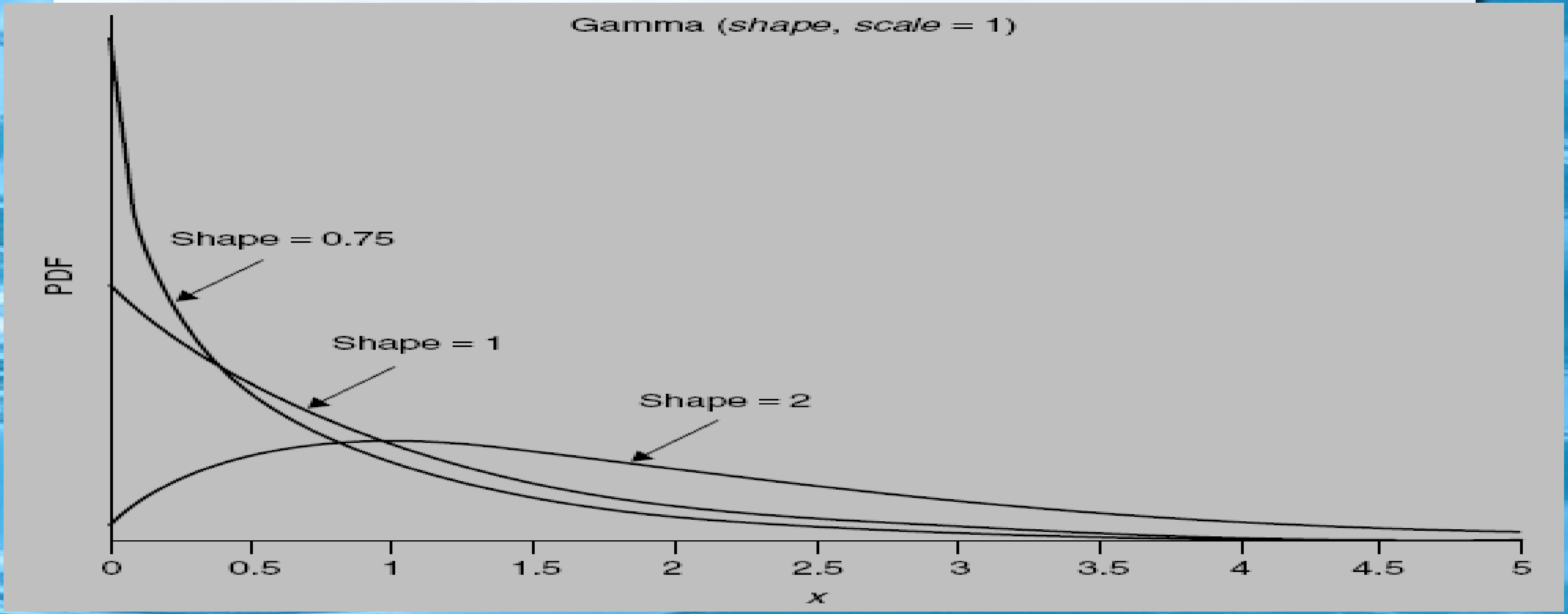
MODELING VARIABILITY: **Some common statistical distributions**

Gamma (shape, scale)

Potential applications: **time to complete a task; inter-arrival times (e.g. customer arrivals, time between failure)**

Mean*: shape × scale;** ***Standard deviation: **√(shape × scale);**

***Range of values*: 0< x < ∞**



Some basic principles in Simulation

MODELING VARIABILITY: **Some common statistical distributions**

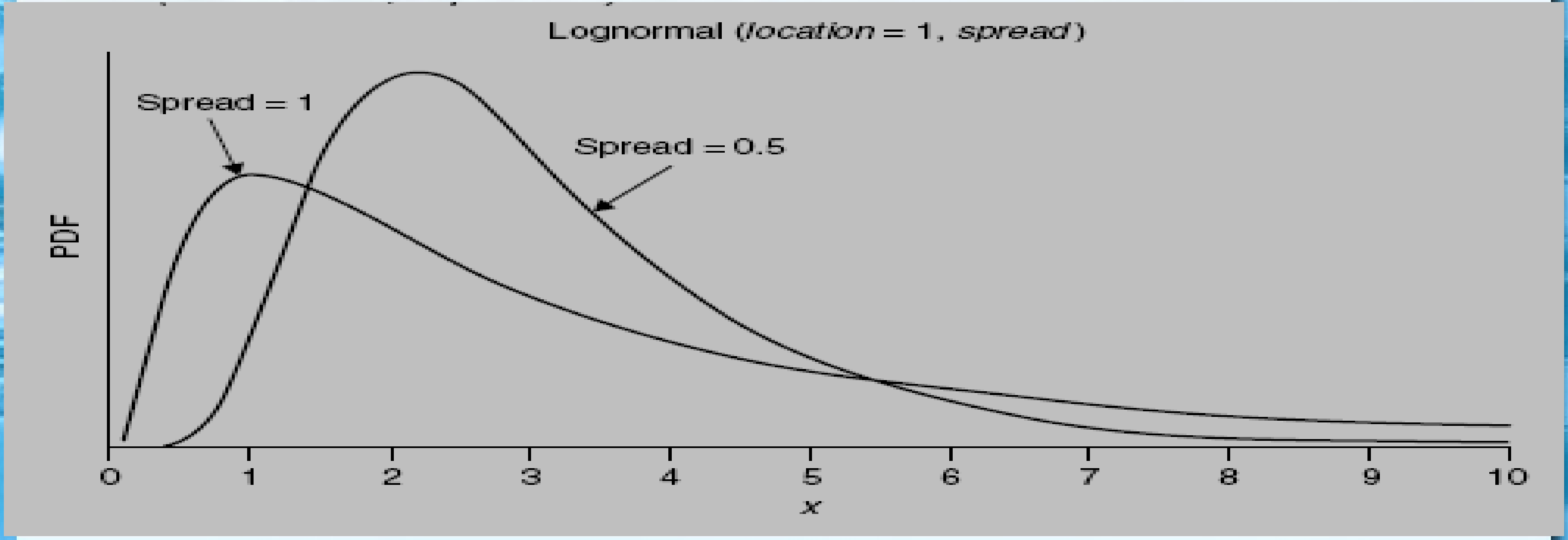
Lognormal (location, spread)

***Potential applications*: time to complete a task;**

***Mean*: $e^{location+spread/2}$**

***Standard deviation*: $\sqrt{(e^{2location+spread}(e^{spread} - 1))}$**

***Range of values*: $0 < x < \infty$**



Some basic principles in Simulation

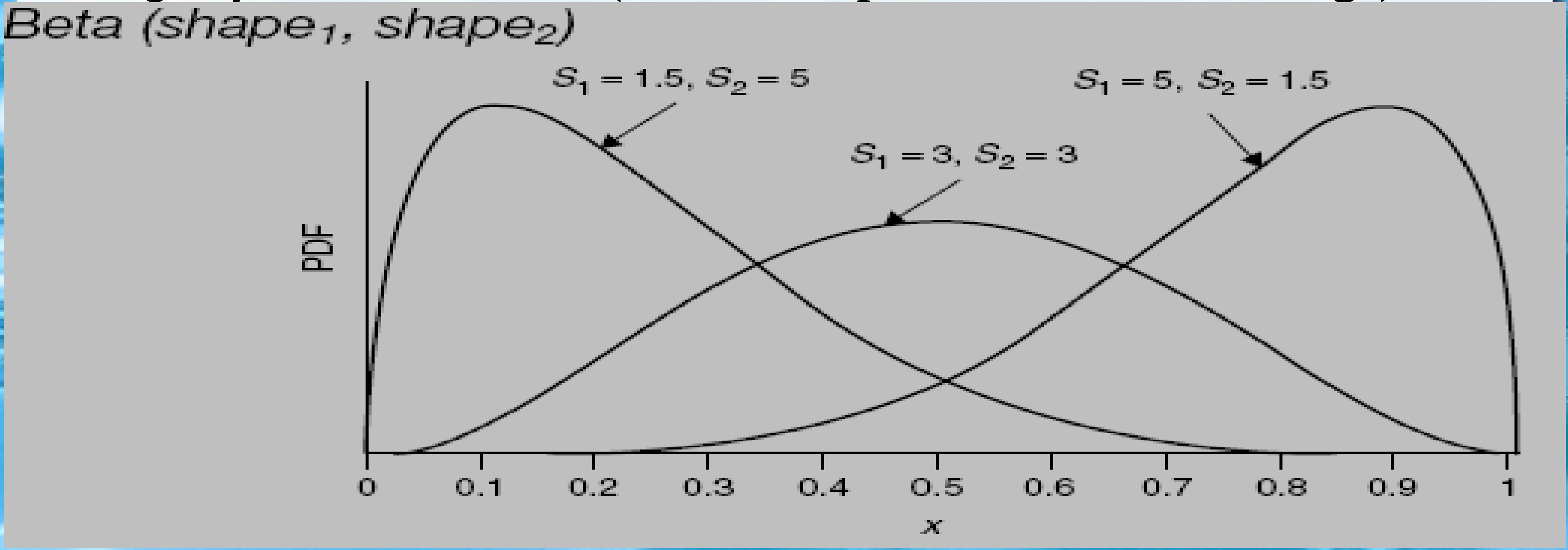
MODELING VARIABILITY: Some common statistical distributions

Beta ($Shape_1$, $Shape_2$): ***Potential applications:*** **time to complete a task; proportions (e.g. defects in a batch of items); useful as an approximation in the absence of data, task times in Pert networks.**

***Mean*: $(shape_1) / (shape_1 + shape_2)$**

***Standard deviation*: $\sqrt{[(shape_1 \times shape_2) / (shape_1 + shape_2)^2 (shape_1 + shape_2 + 1)]}$**

***Range of values*: $0 < x < 1$ (use a multiplier to extend the range)**



Some basic principles in Simulation

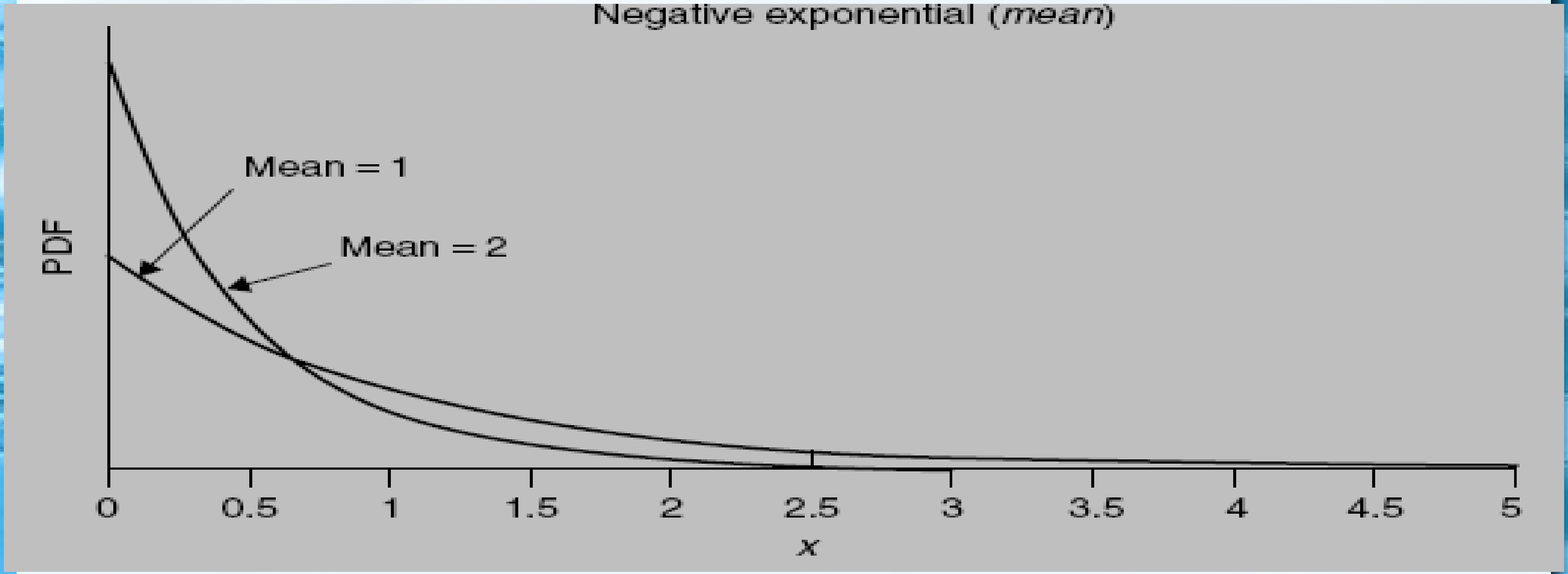
MODELING VARIABILITY:Some common statistical distributions

Negative exponential (mean)-exponential distribution

***Potential applications*: inter-arrival times (e.g. customer arrivals, time between failure); time to complete a task;**

***Mean*: mean;**

***Standard deviation*: mean; Range of values: 0<= x < ∞**



Some basic principles in Simulation

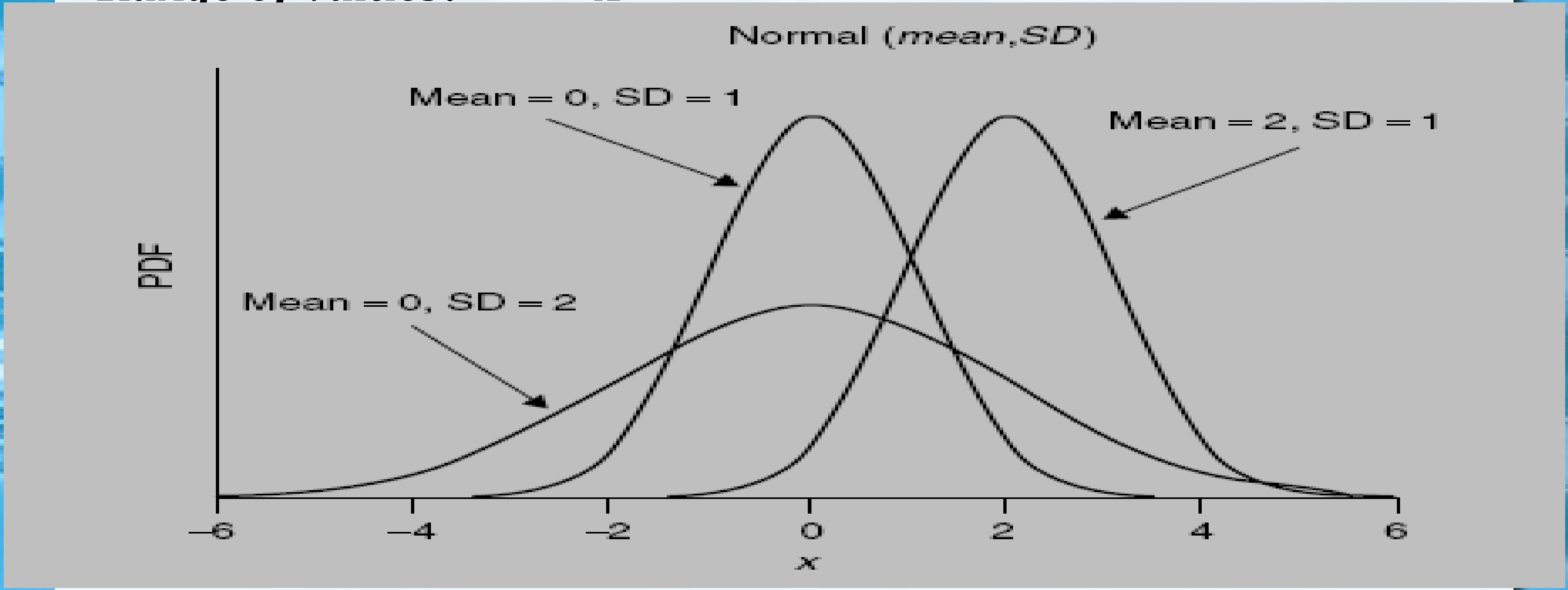
MODELING VARIABILITY: **Some common statistical distributions**

Normal (mean, standard deviation)

Potential applications: **errors (e.g. in weight or dimension of components)**

Mean: **mean;** ***Standard deviation***: **SD;**

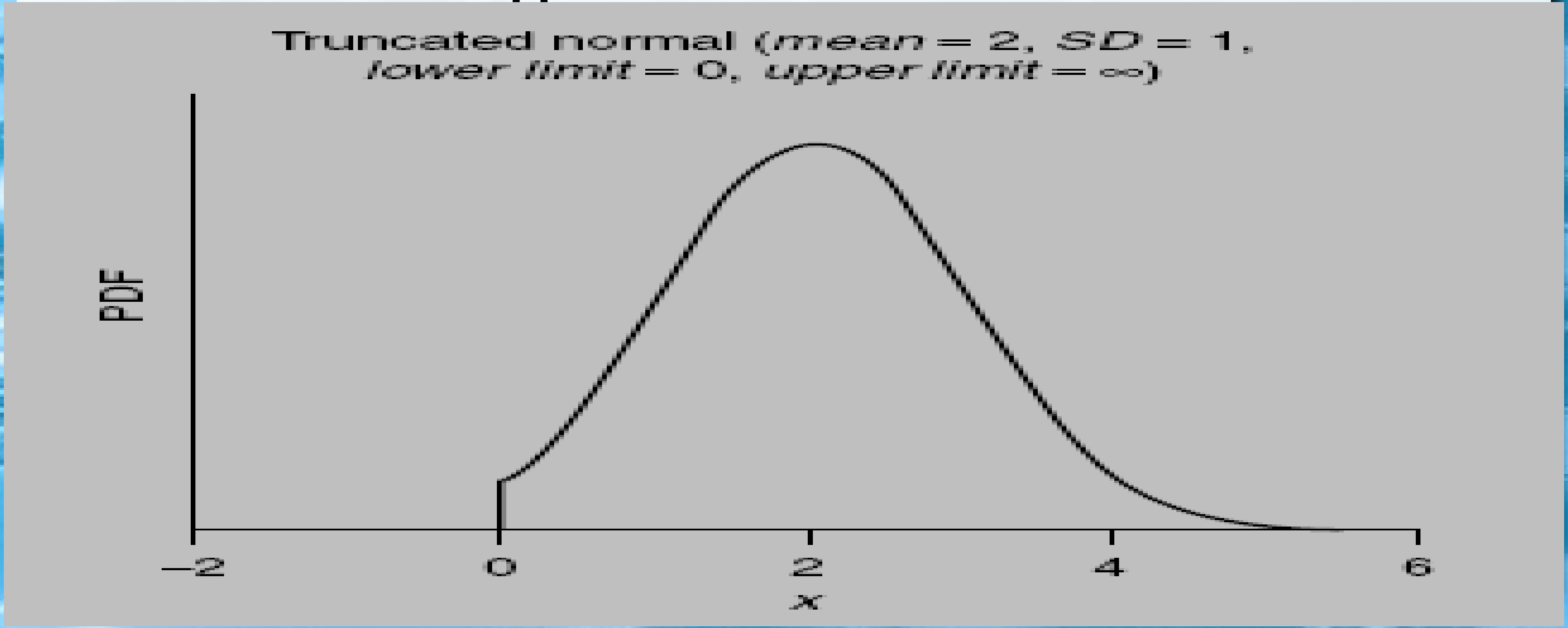
Range of values: **$-\infty < x < \infty$**



Some basic principles in Simulation

MODELING VARIABILITY:Some common statistical distributions

Truncated normal (mean, standard deviation, lower limit, upper limit): ***Potential applications*: similar to normal distribution but avoids problem of extreme values (e.g. negative values)**
***Range of values*: if lower limit specified: lower limit <= x < ∞ if upper limit specified: −∞ < x <= upper limit *if both limits specified*: lower limit <= x <= upper limit**



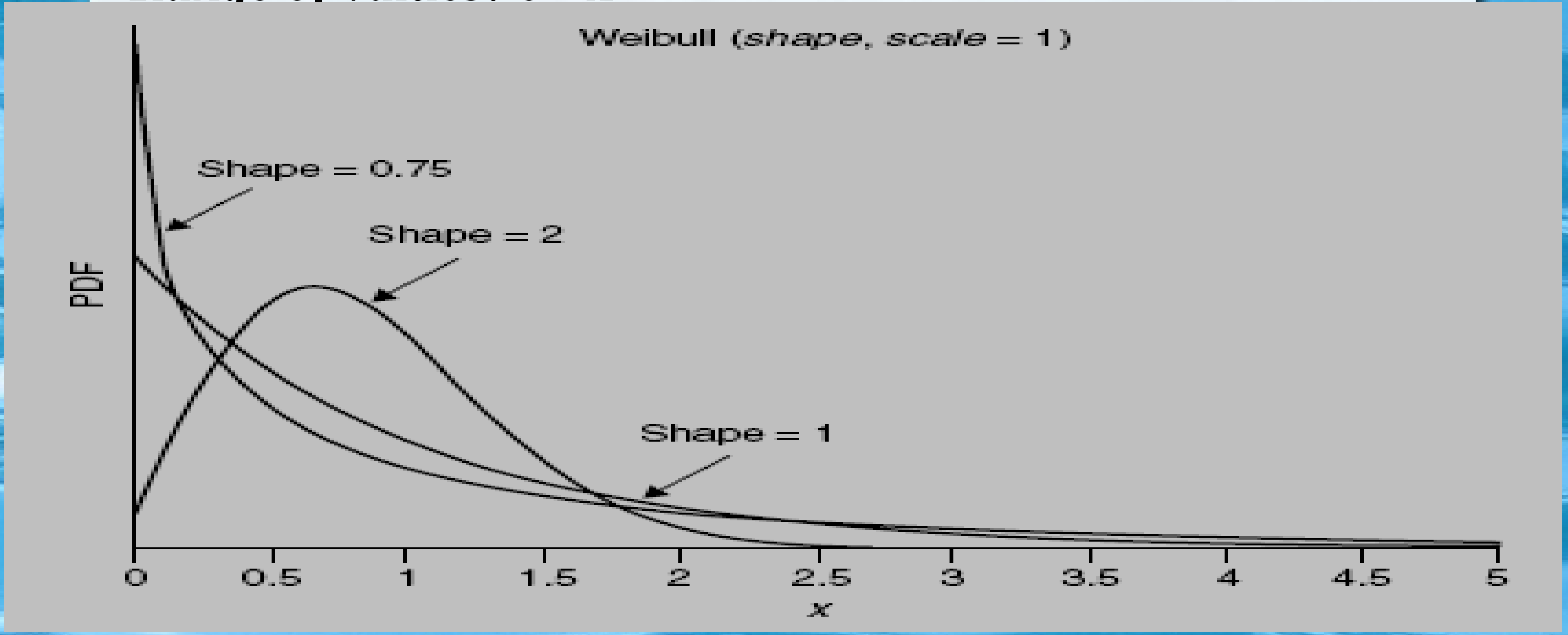
Some basic principles in Simulation

MODELING VARIABILITY: **Some common statistical distributions**

Weibull (shape, scale)

Potential applications**: time between failure; time to complete a task; model equipment failures;**

Range of values**: 0< x < ∞**



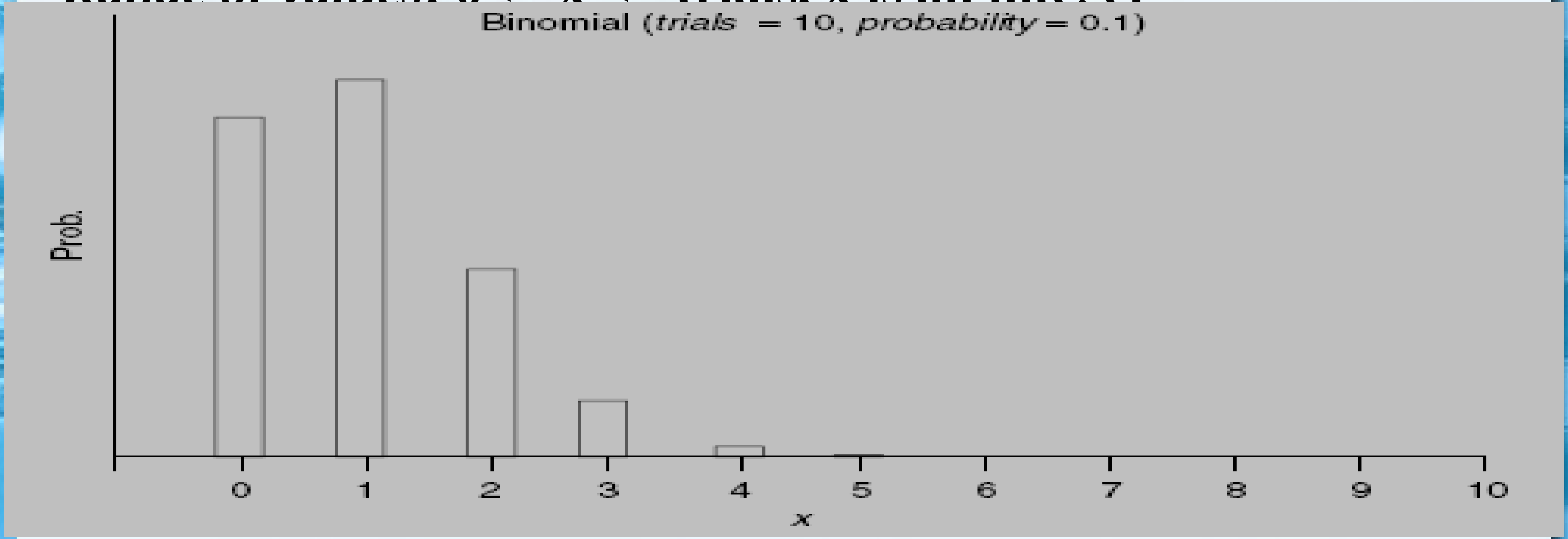
Binomial (trials, probability):

Potential applications: total “successes” in a number of trials (e.g. number of defective items in a batch); number of items in a batch (e.g. size of an order)

Mean: trials × probability;

Standard deviation: √(trials × probability (1 − probability))

Range of values: 0<= x <= trials, x is an integer



Some basic principles in Simulation

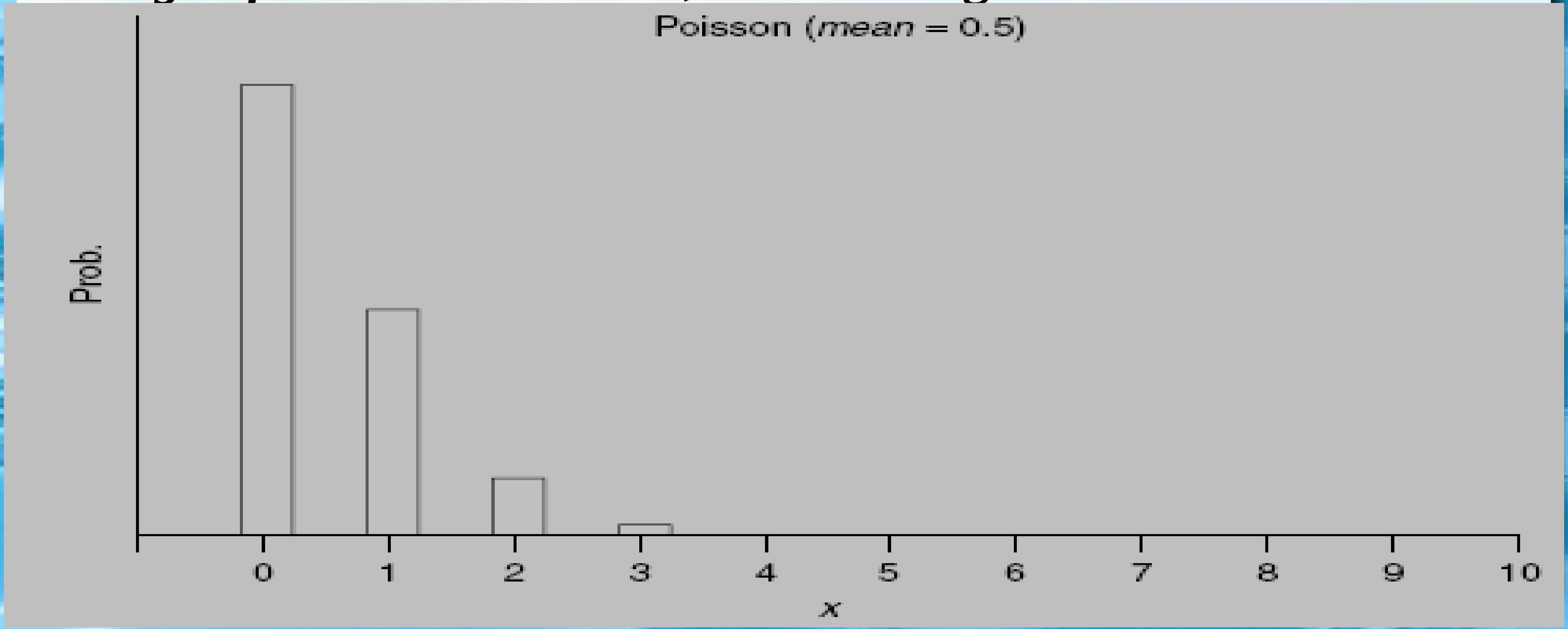
MODELING VARIABILITY: Some common statistical distributions

Poisson (mean)

***Potential applications*: number of events in a period of time (e.g. customer arrivals in an hour); number of items in a batch (e.g. size of an order)**

***Mean*: mean; *Standard deviation*: √mean;**

***Range of values*: 0<= x < ∞, x is an integer**



MODELING VARIABILITY: **Some common statistical distributions**

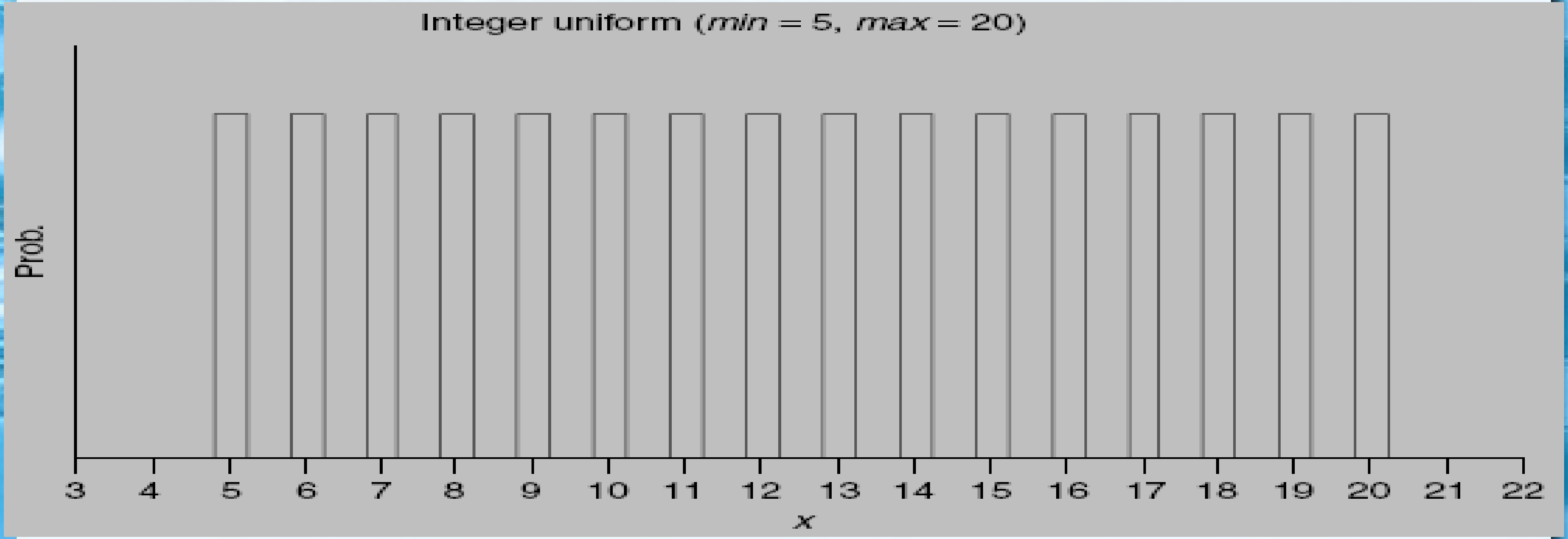
Integer Uniform (min, max)

***Potential applications*: useful as an approximation when little is known other than the likely range of values;**

Mean: (min+max)/2;

***Standard deviation*: $\sqrt{[((max-min+1)^2 - 1)/12]}$;**

***Range*: min <= x <= max, x is an integer**



Some basic principles in Simulation

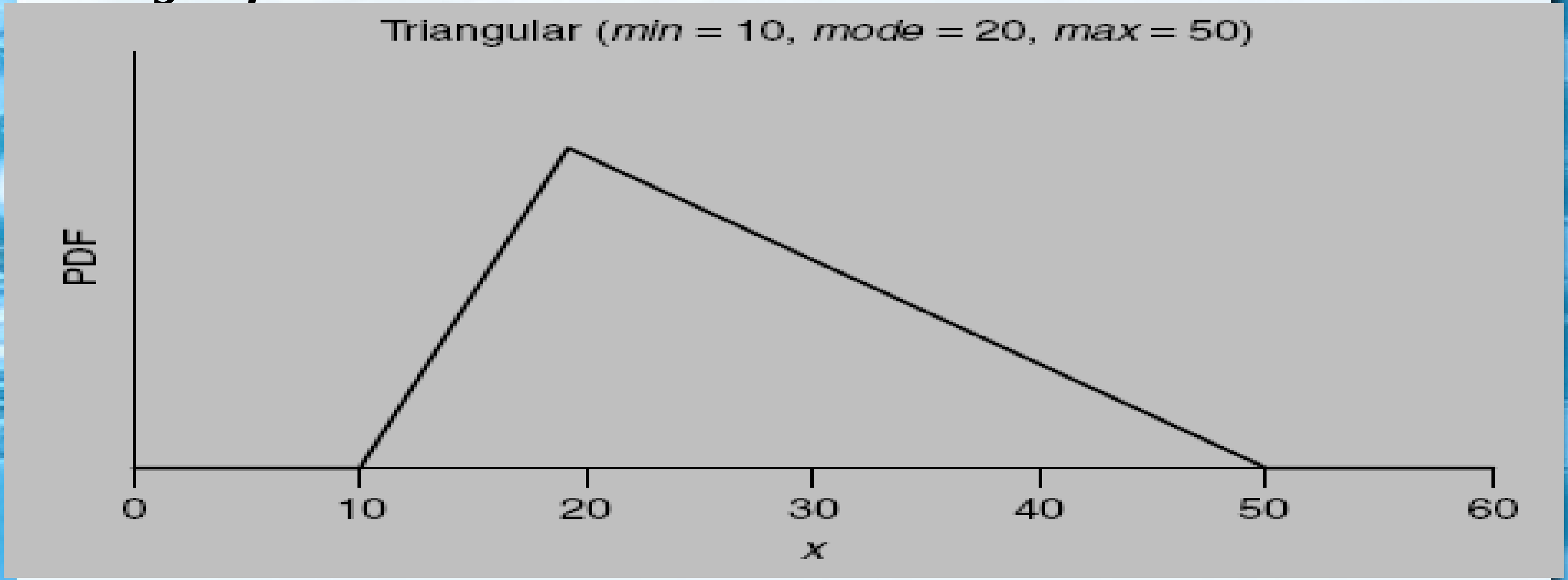
MODELING VARIABILITY: **Some common statistical distributions**

Triangular (min, mode, max)

Potential applications: **useful as an approximation when little is known other than the likely range of values and the most likely value (mode);** ***Mean*****: (min+mode + max)/3**

Standard deviation**:** $\sqrt{[(min^2 + mode^2 + max^2 - (min \times mode + min \times max + mode \times max)/18]}$;

Range of values**: min <= x <= max**



Exercises

1. Discuss why simulation may be necessary.
2. Discuss advantages of simulation.
3. Discuss how simulation may be having advantages over other problem solving approaches.
4. Discuss when it may be necessary to use simulation.
5. Describe the three phase approach of discrete event simulation.
6. Discuss how time may be managed in simulation.
7. Show how variability may be handled in simulation.
8. Discuss how random numbers may be generated by hand (top of hat) and by a computer.
9. Discuss how distributions may be used as sources of random numbers.
10. Discuss other useful distributions in simulations.