Simulation and Modeling
Lecture notes
Topic: Introduction to Simulation & Modeling
By

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Introduction

Objectives of this topic: To gain the knowledge about system and its behavior so that a person can transform the physical behavior of a system into a mathematical model that can in turn transform into an efficient algorithm for simulation purpose.

Overview

- Computer simulation is a powerful methodology for design and analysis and complex systems.
- Overall approach in computer simulation is to represent the dynamic characteristics of a real world system in a computer model.
- The model is subjected to experiments to obtain predictive information useful in making informed decision making about the characteristics of the real system.
- Simulations are suitable for problems in which there are no closed-form analytical solutions. Since most dynamic problems in practice cannot be represented and solved fully using mathematical equations, computer simulation is a powerful and flexible methodology in complex systems analysis.
- Simulations can be classified into
    1. Continuous simulation
    2. Discrete simulation
- **In continuous simulations**, the state variables, i.e., the collection of variables needed to describe the system, change continuously over time and the behavior of the system is typically described by differential equations.
- Examples of continuous systems include the modeling of thermal or hydraulic systems.
- **Discrete simulations** are event-driven where the state variables change at discrete time points.
- Examples of discrete-event simulations include service industry applications such as queues in a grocery store and manufacturing applications involving material flow analysis

Modeling is the process of representing a model which includes its construction and working. This model is similar to a real system, which helps the analyst predict the effect of changes to the system. In other words, modelling is creating a model which represents a system including their properties. It is an act of building a model. It’s also the application of methods to analyze complex, real-world problems in order to make predictions about what might happen with various actions.

Simulation of a system is the operation of a model in terms of time or space, which helps analyze the performance of an existing or a proposed system. In other words, simulation is the process of using a model to study the performance of a system. It is an act of using a model for simulation.

- Briefly we can say that Simulation is
    - Simulated system imitates operation of actual system over time
    - Artificial history of system can be generated and observed
    - Internal (perhaps unobservable) behavior of system can be studied
    - Time scale can be altered as needed
    - Conclusions about actual system characteristics can be inferred

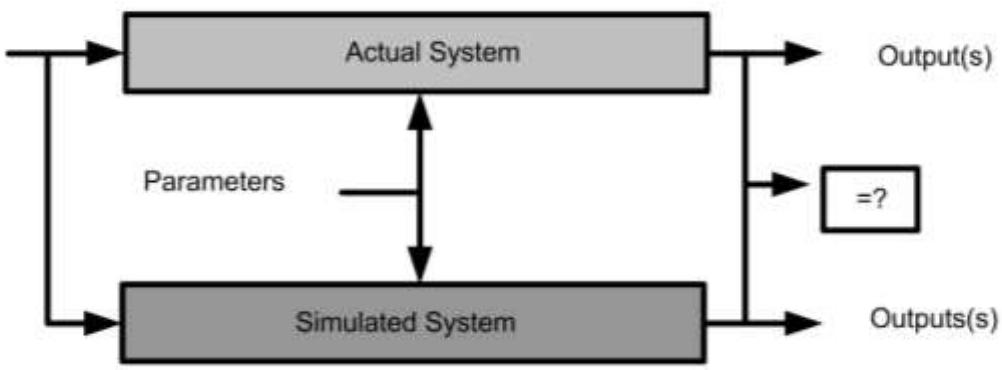


Fig: Simulation vs Actual system

Simulation can be broadly *defined as a technique for studying real-world dynamical systems by imitating their behavior using a mathematical model of the system implemented on a digital computer*.

Simulation can also be viewed as a *numerical technique for solving complicated probability models*, ordinary differential equation and partial differential equation, analogously to the way in which we can use a computer to numerically evaluate the integral of a complicated function. That's why science of simulation is considered as an interdisciplinary subject.

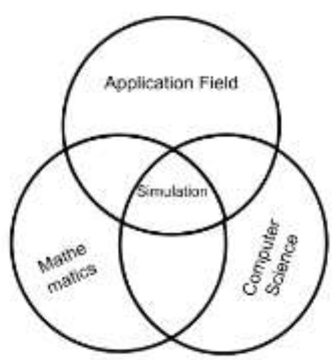


Fig: Simulation is Interdisciplinary

History of Simulation

The historical perspective of simulation is as enumerated in a chronological order.

- **1940** − A method named ‘Monte Carlo’ was developed by researchers (John von Neumann, Stanislaw Ulan, Edward Teller, Herman Kahn) and physicists working on a Manhattan project to study neutron scattering.
- **1960** − The first special-purpose simulation languages were developed, such as SIMSCRIPT by Harry Markowitz at the RAND Corporation.
- **1970** − During this period, research was initiated on mathematical foundations of simulation.
- **1980** − During this period, PC-based simulation software, graphical user interfaces and object-oriented programming were developed.
- **1990** − During this period, web-based simulation, fancy animated graphics, simulation-based optimization, Markov-chain Monte Carlo methods were developed. Became ever more prominent as a method for studying complex systems in which uncertainty is present. In various surveys, simulation has been found to be the most frequently used tool of Operation Research practitioners. Simulation is an interdisciplinary subject, using ideas and techniques from Statistics, Probability, Number Theory, and Computer Science.

Developing Simulation Models

Simulation models consist of the following components: system entities, input variables, performance measures, and functional relationships. Following are the steps to develop a simulation model.

- **Step 1** − Identify the problem with an existing system or set requirements of a proposed system.
- **Step 2** − Design the problem while taking care of the existing system factors and limitations.
- **Step 3** − Collect and start processing the system data, observing its performance and result.
- **Step 4** − Develop the model using network diagrams and verify it using various verifications techniques.
- **Step 5** − Validate the model by comparing its performance under various conditions with the real system.
- **Step 6** − Create a document of the model for future use, which includes objectives, assumptions, input variables and performance in detail.
- **Step 7** − Select an appropriate experimental design as per requirement.
- **Step 8** − Induce experimental conditions on the model and observe the result.

Performing Simulation Analysis

Following are the steps to perform simulation analysis.

- **Step 1** − Prepare a problem statement.

- **Step 2** − Choose input variables and create entities for the simulation process. There are two types of variables - decision variables and uncontrollable variables. Decision variables are controlled by the programmer, whereas uncontrollable variables are the random variables.
- **Step 3** − Create constraints on the decision variables by assigning it to the simulation process.
- **Step 4** − Determine the output variables.
- **Step 5** − Collect data from the real-life system to input into the simulation.
- **Step 6** − Develop a flowchart showing the progress of the simulation process.
- **Step 7** − Choose an appropriate simulation software to run the model.
- **Step 8** − Verify the simulation model by comparing its result with the real-time system.
- **Step 9** − Perform an experiment on the model by changing the variable values to find the best solution.
- **Step 10** − Finally, apply these results into the real-time system.

Modelling & Simulation ─ Advantages

Following are the advantages of using Modelling and Simulation −

- Simulation arbitrary model complexity, circumvents analytically intractable models, facilitates what-if and sensitivity analyses, building a model can lead to system improvements and greater understanding can be used to verify analytic solutions
- **Easy to understand** − Allows to understand how the system really operates without working on real-time systems.
- **Easy to test** − Allows to make changes into the system and their effect on the output without working on real-time systems.
- **Easy to upgrade** − Allows to determine the system requirements by applying different configurations.
- **Easy to identifying constraints** − Allows to perform bottleneck analysis that causes delay in the work process, information, etc.
- **Easy to diagnose problems** − Certain systems are so complex that it is not easy to understand their interaction at a time. However, Modelling & Simulation allows to understand all the interactions and analyze their effect. Additionally, new policies, operations, and procedures can be explored without affecting the real system.

Modelling & Simulation ─ Disadvantages

Following are the disadvantages of using Modelling and Simulation −

- Simulation provides only estimates of solution, only solves one parameter at a time, can take a large amount of development and / or computer time ("simulation as a last resort"). Don't use computer simulation if a common-sense or analytical solution is available, or if resources are insufficient, or if simulation costs outweigh benefits.
- Designing a model is an art which requires domain knowledge, training and experience.
- Operations are performed on the system using random number, hence difficult to predict the result.

- Simulation requires manpower and it is a time-consuming process.
- Simulation results are difficult to translate. It requires experts to understand.
- Simulation process is expensive.

Modelling & Simulation ─ Application Areas

Modelling & Simulation can be applied to the following areas − Military applications, training & support, designing semiconductors, telecommunications, computer systems; civil engineering designs & presentations; finance; and E-business models.

Additionally, it is used to study the internal structure of a complex system such as the biological system. It is used while optimizing the system design such as routing algorithm, assembly line, etc. It is used to test new designs and policies. It is used to verify analytic solutions.

Difficulties of Simulation

- Provides only individual, not general solutions
- Manpower and time-consuming
- Computing memory and time-intensive
- Difficult so experts are required
- Hard to interpret results
- Expensive

When to use Simulation?

- Study internals of a complex system e.g. biological system
- Optimise an existing design e.g. routing algorithms, assembly line
- Examine effect of environmental changes e.g. weather forecasting
- System is dangerous or destructive e.g. atom bomb, atomic reactor, missile launching
- Study importance of variables
- Verify analytic solutions (theories)
- Test new designs or policies
- Impossible to observe/influence/build the system
- When it allows inspection of system internals that might not otherwise be observable
- Observation of the simulation gives insights into system behavior
- System parameters can be adjusted in the simulation model allowing assessment of their sensitivity (scale of impact on overall system behavior)
- Simulation verifies analysis of a complex system, or can be used as a teaching tool to provide insight into analytical techniques
- A simulator can be used for instruction, avoiding tying up or damaging an expensive, actual system (e.g., a flight simulation vs. use of multimillion dollar aircraft)

Modeling Concepts & Classifications

- There are various concepts underlying simulation and classification of Modelling. Some of important concepts include system and model, events, system state variable, entities and attributes, list processing, activities and delays, and finally the definition of discrete-event simulation.

- The process of making and testing hypotheses about models and then revising designs or theories has its foundation in the experimental sciences.
- Similarly, Computational scientists use modeling to analyze complex, real-world problems in order to predict what might happen with some course of action. For example, Dr. Jerrold Marsden, a computational physicist at CalTech, models space mission trajectory design (Marsden). Dr. Julianne Collins, a genetic epidemiologist (statistical genetics) at the Greenwood Genetics Center, runs genetic analysis programs and analyzes epidemiological studies using the Statistical Analysis Software (SAS) (Greenwood Genetics Center). Some of the projects on which she has worked involve analyzing data from a genome scan of Alzheimer's disease, performing linkage analyses of X-linked mental retardation families, determining the recurrence risk in nonsyndromic mental retardation, analyzing folic acid levels from a nutritional survey of Honduran women, and researching new methods to detect genes or risk factors involved in autism. Scientists in areas such as cognitive psychology and social psychology at the Human-Technology Interaction Center of The University of Oklahoma perform research on the interaction of people with modern technologies (Human-Technology Interaction Center). Some of the studies involve "strategic planning in air traffic control" and "designing interfaces for effective information retrieval from collections of multimedia." Buried land mines are a serious danger in many areas of the world (Weldon et al. 2001). Scientists are using a combination of mathematics, signal processing, and scientific visualization to model, image, and discover land mines. Lourdes Esteva, Cristobal Vargas, and Jorge Velasco- Hernandez have modeled the oscillating patterns of the disease dengue fever, for which an estimated 50 to 100 million cases occur globally each year (Esteva and Vargas 1999).

System, Models & Events

Following are the basic concepts of Modelling & Simulation.

- **Object** is an entity which exists in the real world to study the behavior of a model. It can exhibit widely varying behaviour depending on the context in which it is studied, as well as the aspects of its behaviours which are under study.
- **Base Model** is a hypothetical, abstract representation, explanation of object's properties and its behavior, which is valid in ***all*** possible context, and describes all the object's facets.
- Model **is a representation of an actual system**. It is an **abstraction** of the real system. Involves simplifying assumptions are used to capture (only) important behaviors. Such as, Linearization, time-bound behaviors, etc., may make analysis tractable.
- **System** is the articulate (well defined) object under definite conditions, which exists in the real world.
- **Experimental Frame** (EF) is used to study a system in the real world, such as experimental conditions (context), aspects, objectives, etc within which that system and corresponding models will be used. EF reflects the *objectives* of the experimenter who performs experiments on real system or, through simulation, on a model. Basic Experimental Frame consists of two sets of variables − the Frame Input Variables & the Frame Output Variables, which matches the system or model terminals. The Frame input variable is responsible for matching the inputs applied to the system or a model. The

Frame output variable is responsible for matching the output values to the system or a model.

- **Lumped Model** is an exact explanation of a system which follows the specified conditions of a given Experimental Frame.
- **Verification** is the process of comparing two or more items to ensure their accuracy. In Modelling & Simulation, verification can be done by comparing the consistency of a simulation program and the lumped model to ensure their performance. There are various ways to perform validation process, which we will cover in a separate chapter.
- **Validation** is the process of comparing two results. In Modelling & Simulation, validation is performed by comparing experiment measurements with the simulation results within the context of an Experimental Frame. The model is invalid, if the results mismatch. There are various ways to perform validation process, which we will cover in separate chapter.

Discrete-event simulation models are contrasted with other types of models such as mathematical models, descriptive models, statistical models, and input-output models. A discrete-event model attempts to represent the components of a system and their interactions to such an extent that the objectives of the study are met. Most mathematical, statistical, and input output models represent a system's inputs and outputs explicitly, but represent the internals of the model with mathematical or statistical relationships. An example is the mathematical model from physics,

Force = Mass × Acceleration

based on theory. Discrete-event simulation models include a detailed representation of the actual internals. Discrete-event models are dynamic, i.e., the passage of time plays a crucial role. Most mathematical and statistical models are static in that they represent a system at a fixed point of time. Consider the annual budget of a firm. This budget resides in a spreadsheet. Changes can be made in the budget and the spreadsheet can be recalculated, but the passage of time is usually not a critical issue. Further comments will be made about discrete-event models after several additional concepts are presented.

Models have Many Uses, Typically

- To understand the behaviour of an existing system (why does my network performance die when more than 10 people are at work?)
- To predict the effect of changes or upgrades to the system (will spending 100,000 on a new switch cure the problem?)
- To study new or imaginary systems (let’s bin the Ethernet and design our own scalable custom routing network)

System State Variables

The system state variables are a set (collection) of data (all information), required to define the internal process within the system at a given point of time to a sufficient level (i.e. to attain the desired output). The determination of system state variables is a function of the purposes of the investigation, so what may be the system state variables in one case may not be the same in another case even though the physical system is the same. Determining the system state variables is as much an art as a science. However, during the modeling process, any omissions will readily come to light. (And, on the other hand, unnecessary state variables may be eliminated.) Having defined system state variables, a contrast can be made between discrete-event models and continuous models based on the variables needed to track the system state. The system state variables in a discrete-event model remain constant over intervals of time and change value only at certain well-defined points called event times. Continuous models have system state variables defined by differential or difference equations giving rise to variables that may change continuously over time.

- In a **discrete-event model**, the system state variables remain constant over intervals of time and the values change at defined points called event times.
- In **continuous-event model**, the system state variables are defined by differential equation results whose value changes continuously over time.

Following are some of the system state variables −

- **Entities & Attributes** − An entity represents an object whose value can be static or dynamic, depending upon the process with other entities---example customer is a dynamic entity, whereas bank teller is a static entity. Attributes are the local values used by the entity. An entity may have attributes pertain to that entity alone.
- **Resources** − A resource is an entity that provides service to one or more dynamic entities at a time i.e. operates like a parallel server. The dynamic entity can request one or more units of a resource; if accepted then the entity can use the resource and release when completed. If rejected, the entity can join a queue, or takes some other action (i.e. diverted to another resource, ejected from the system). If permitted to capture the resource, the entity remains for a time, then releases the resource. Possible states of of the resources include – idle, busy, failed, blocked, or starved.
- **List Processing** − Lists are used to represent the queues used by the entities and resources placing them in an ordered list. Lists are often processed according to various possibilities of queues such as LIFO, FIFO, etc. depending upon the process, according to the value of an attribute, or randomly, to mention a few. The entities are ordered according to the value of that attribute with the lowest value at the head or front of the queue.
- **Activities and Delay** – An **Activity** is duration of time whose duration is known prior to commencement of the activity. Thus, when duration begins, its end can be scheduled. The duration can be a constant, a random value from a statistical distribution, the result of an equation, input from a file, or computed based on event state. While a **Delay** is an indefinite duration that is caused by some combination of system conditions. When an entity joins a queue for a resource, the time that it will remain in the queue may be unknown initially since that time may depend on other events that may occur. Discrete event simulations contain activities that cause time to advance. Most discrete-event simulations also contain delays as entities wait. The beginning and ending of an activity or delay is an event.

Classifications of Model

A system can be classified into the following categories. Modeling a system exhibits **Probabilistic** or **Stochastic** behaviour if an element of chance exists.

A **probabilistic** or **stochastic** model exhibits random effects, while a deterministic model does not. The results of a deterministic model depend on the initial conditions; and in the case of computer implementation with particular input, the output is the same for each program execution. As we studied this and other modules, we can have a probabilistic model for a deterministic situation, such as a model that uses random numbers to estimate the area under a curve.

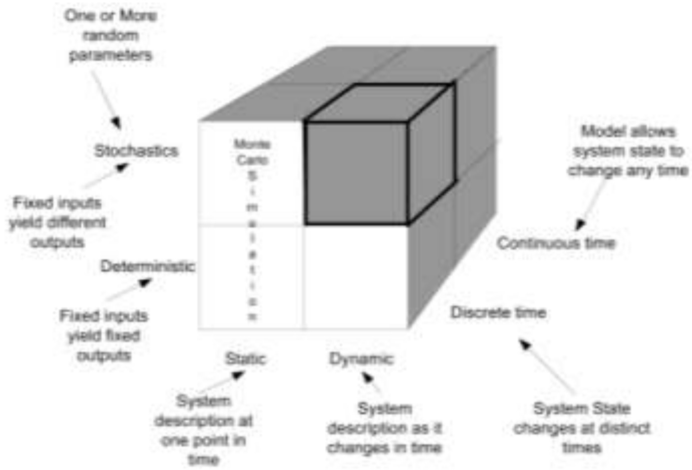
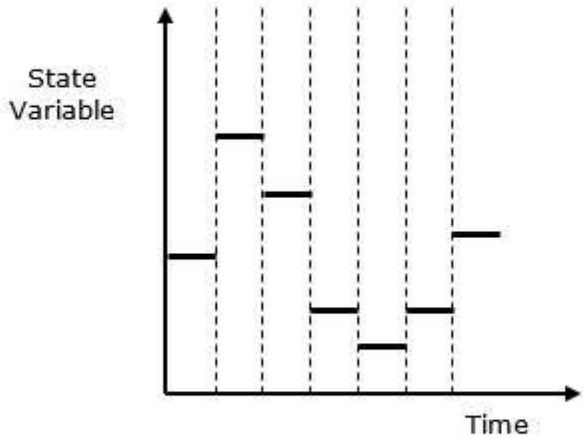
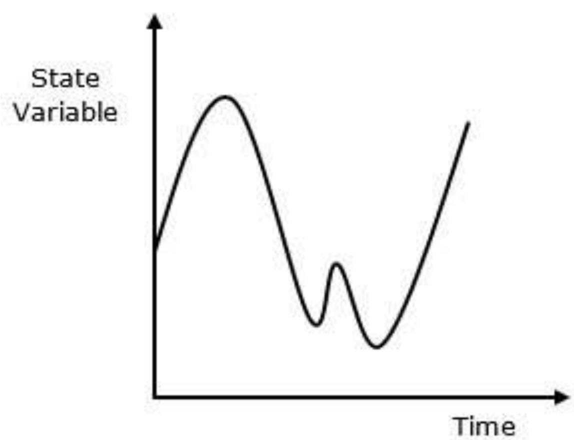


Fig: Classification of different types of models

- **Discrete-Event Simulation Model** − In this model, the state variable values change only at some discrete points in time where the events occur. Events will only occur at the defined activity time and delays. Events occur as a consequence of activity times and delays. Entities may compete for system resources, possibly joining queues while waiting for an available resource. Activity and delay times may "hold" entities for durations of time. A discrete-event simulation model is conducted over time ("run") by a mechanism that moves simulated time forward. The system state is updated at each event along with capturing and freeing of resources that may occur at that time.
- **Stochastic vs. Deterministic Systems** − **Stochastic** systems are not affected by randomness and their output is not a random variable, whereas **deterministic** systems are affected by randomness and their output is a random variable. A **probabilistic** or **stochastic** model exhibits random effects, while a **deterministic** model does not.
- **Static vs. Dynamic Simulation** − **Static** simulation include models which are not affected with time (does not consider time), and the model is comparable to a snapshot or a map. For example: Monte Carlo Model. **Dynamic** Simulation include models which are affected with time (time changes), so that such model is comparable to an animated cartoon or a movie.----**Static** a simulation of a system at one specific time, where time is not a relevant parameter e.g. Monte Carlo & Steady-state simulations – Dynamic representing a system evolving over time.
- **Discrete vs. Continuous Systems** − **Discrete** system is affected by the state variable changes at a discrete point of time. Where time changes in incremental steps, the model is **discrete**. **Discrete** model is analogous to a movie, where a sequence of frames moves so quickly that the viewer perceives motion—the action is continuous--. Its behavior is depicted in the following graphical representation.



Whereas **Continuous** system is affected by the state variable, which changes continuously as a function with time. The time changes continuously and smoothly, the model is **continuous**. Its behavior is depicted in the following graphical representation.



Continuous: **State** variables change continuously as a function of time and generally analytical method like deductive mathematical reasoning is used to define and solve the system.

State Variable (S.V.) = f(*t*)

Discrete: **State** variables change at discrete points in time and generally numerical method like computational procedure is used to solve mathematical models.

State Variable (S.V.) = f(*nt*)

Examples of Different Systems

- Queue length at a cash machine: Stochastic, Discrete Time, Discrete System
- The motion of the planets: Deterministic, Continuous Time, Discrete System

- Logic circuit in a computer: Deterministic, Discrete Time, Discrete System
- Flow of air around a car: Deterministic, Continuous Time, Continuous System
- Closing prices of the 30 DAX shares: Stochastic, Discrete Time, Discrete System

A Classic Example of Queue at Bank Counter

We see queues at everywhere. Queues are buffers to smooth out differences in arrival rates and service times. Queue Theory is well understood. Closed-form queue-theoretic models can be used to speed up simulations. Deriving results from such models requires simulation. Here are we given a example of queue formed at bank counter (Concept of queue is discussed in more detail in unit IV) .At bank counter customers arrive at random intervals and suppose there is only one cashier .Customers must wait in a queue. Service times at the cashier are also random Measured inter-arrival times (seconds):25, 111, 56, 232, 97, 452, 153, 45,...Measured service times (seconds): 45, 32, 11, 61, 93, 56, 30,...

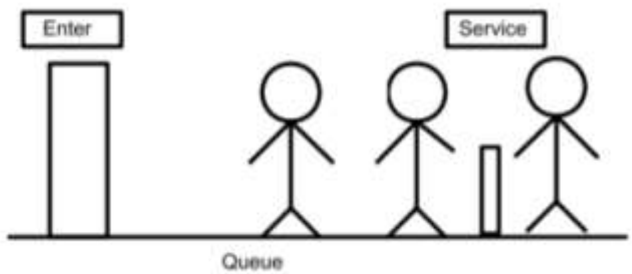


Fig: Classical example of Queue

Now compute the average length of the queue and the probability that the cashier is busy.

Modelling Process

Modelling process is cyclic and closely parallels the scientific method and the software development life cycle (SDLC) for development of major software projects. The process includes the following steps.

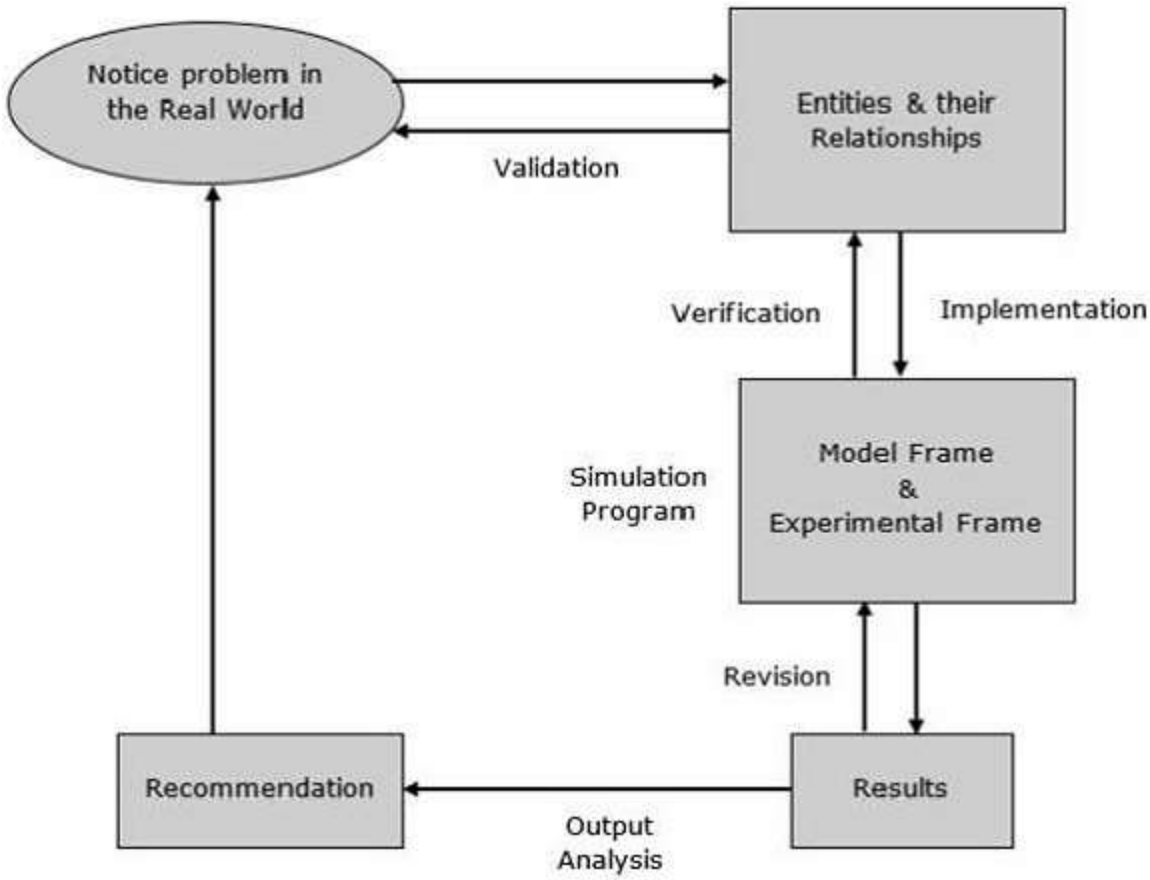


Fig: Phases of Computer Simulation Development and Analysis

Step 1 − Examine the problem (Analyse the problem). In this stage, we must understand the problem and choose its classification accordingly, such as deterministic or stochastic. We determine the problem's objectives and decide problem classification….translate problem into mathematical symbols and develop and solve the model.

Step 2 − Design (formulate) a model. The goals could be for what-if analysis of a system being designed or for evaluation of variety of prototypical scenarios of an existing system. In this stage, we have to perform the following simple tasks which help us design a model-

- Collect data as per the system behavior and future requirements.
- Analyze the system features, its assumptions and necessary actions to be taken to make the model successful.
- Determine the variable names, functions, its units, relationships, and their applications used in the model.
- Solve the model using a suitable technique and verify the result using verification methods. Next, validate the result.
- Prepare a report which includes results, interpretations, conclusion, and suggestions.

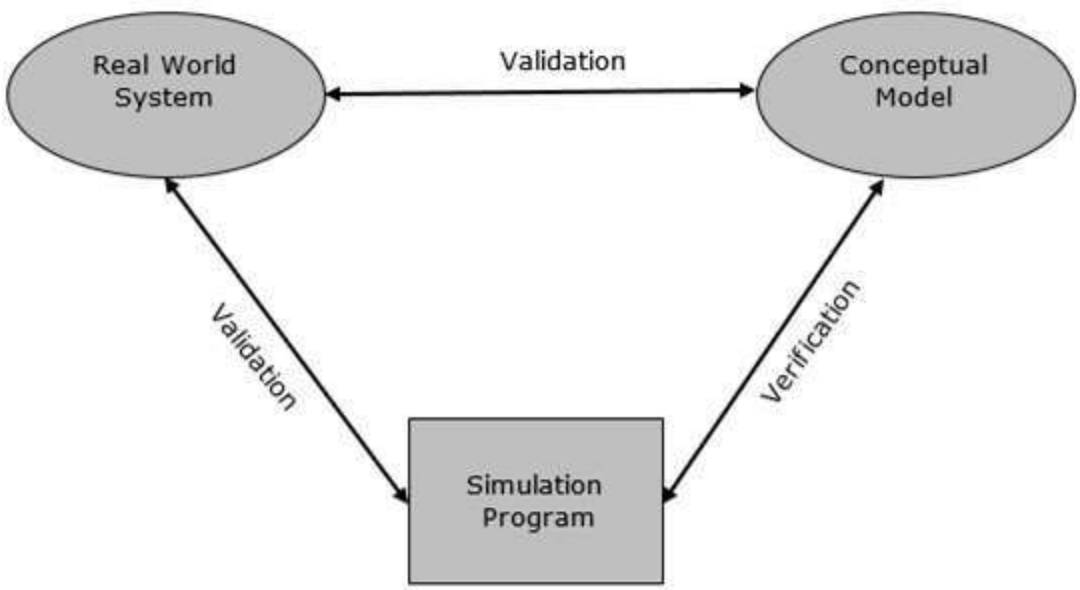
Step 3 − Provide recommendations after completing the entire process related to the model. It includes investment, resources, algorithms, techniques, etc.

Verification and Validation of Models

One of the real problems that the simulation analyst faces is to validate the model. The simulation model is valid only if the model is an accurate representation of the actual system, else it is invalid.

Validation and verification are the two steps in any simulation project to validate a model.

- **Validation** is the process of comparing two results. In this process, we need to compare the representation of a conceptual model to the real system. If the comparison is true, then it is valid, else invalid.
- **Verification** is the process of comparing two or more results to ensure its accuracy. In this process, we have to compare the model’s implementation and its associated data with the developer's conceptual description and specifications.



Verification & Validation Techniques

There are various techniques used to perform Verification & Validation of Simulation Model. Following are some of the common techniques −

Techniques to Perform Verification of Simulation Model

Following are the ways to perform verification of simulation model −

- By using programming skills to write and debug the program in sub-programs.
- By using “Structured Walk-through” policy in which more than one person is to read the program.
- By tracing the intermediate results and comparing them with observed outcomes.
- By checking the simulation model output using various input combinations.

- By comparing final simulation result with analytic results.

Techniques to Perform Validation of Simulation Model

Step 1 − Design a model with high validity. This can be achieved using the following steps −

- The model must be discussed with the system experts while designing.
- The model must interact with the client throughout the process.
- The output must supervised by system experts.

Step 2 − Test the model at assumptions data. This can be achieved by applying the assumption data into the model and testing it quantitatively. Sensitive analysis can also be performed to observe the effect of change in the result when significant changes are made in the input data.

Step 3 − Determine the representative output of the Simulation model. This can be achieved using the following steps −

- Determine how close is the simulation output with the real system output
- Comparison can be performed using the Turing Test. It presents the data in the system format, which can be explained by experts only.
- Statistical method can be used for compare the model output with the real system output.

Model Data Comparison with Real Data

After model development, we have to perform comparison of its output data with real system data. Following are the two approaches to perform this comparison.

Validating the Existing System

In this approach, we use real-world inputs of the model to compare its output with that of the real-world inputs of the real system. This process of validation is straightforward, however, it may present some difficulties when carried out, such as if the output is to be compared to average length, waiting time, idle time, etc. it can be compared using statistical tests and hypothesis testing. Some of the statistical tests are chi-square test, Kolmogorov-Smirnov test, Cramer-von Mises test, and the Moments test.

Validating the First Time Model

Consider we have to describe a proposed system which doesn't exist at the present nor has existed in the past. Therefore, there is no historical data available to compare its performance with. Hence, we have to use a hypothetical system based on assumptions. Following useful pointers will help in making it efficient.

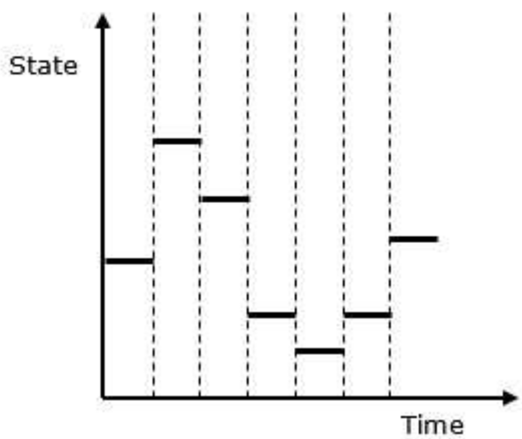
- **Subsystem Validity** − A model itself may not have any existing system to compare it with, but it may consist of a known subsystem. Each of that validity can be tested separately.
- **Internal Validity** − A model with high degree of internal variance will be rejected as a stochastic system with high variance due to its internal processes will hide the changes in the output due to input changes.

- **Sensitivity Analysis** − It provides the information about the sensitive parameter in the system to which we need to pay higher attention.
- **Face Validity** − When the model performs on opposite logics, then it should be rejected even if it behaves like the real system.

Discrete System Simulation

In discrete systems, the changes in the system state are discontinuous and each change in the state of the system is called an **event**. The model used in a discrete system simulation has a set of numbers to represent the state of the system, called as a **state descriptor**. In this chapter, we will also learn about queuing simulation, which is a very important aspect in discrete event simulation along with simulation of time-sharing system.

Following is the graphical representation of the behavior of a discrete system simulation.



Discrete Event Simulation ─ Key Features

Discrete event simulation is generally carried out by a software designed in high level programming languages such as Pascal, C++, or any specialized simulation language. Following are the five key features −

- **Entities** − These are the representation of real elements like the parts of machines.
- **Relationships** − It means to link entities together.
- **Simulation Executive** − It is responsible for controlling the advance time and executing discrete events.
- **Random Number Generator** − It helps to simulate different data coming into the simulation model.
- **Results & Statistics** − It validates the model and provides its performance measures.

Time Graph Representation

Every system depends on a time parameter. In a graphical representation it is referred to as clock time or time counter and initially it is set to zero. Time is updated based on the following two factors −

- **Time Slicing** − It is the time defined by a model for each event until the absence of any event.
- **Next Event** − It is the event defined by the model for the next event to be executed instead of a time interval. It is more efficient than Time Slicing.

Simulation of a Queuing System

A queue is the combination of all entities in the system being served and those waiting for their turn.

Parameters

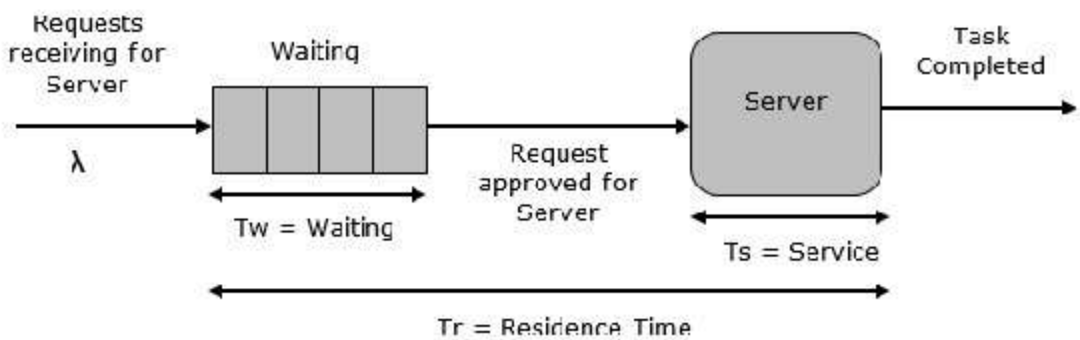
Following is the list of parameters used in the Queuing System.

Symbol	Description
λ	Denotes the arrival rate which is the number of arrivals per second
Ts	Denotes the mean service time for each arrival excluding the waiting time in the queue
σTs	Denotes the standard deviation of service time
ρ	Denotes the server time utilization, both when it was idle and busy
u	Denotes traffic intensity
r	Denotes the mean of items in the system
R	Denotes the total number of items in the system
Tr	Denotes the mean time of an item in the system
TR	Denotes the total time of an item in the system
σr	Denotes the standard deviation of r
σTr	Denotes the standard deviation of Tr
w	Denotes the mean number of items waiting in the queue
σw	Denotes the standard deviation of w
Tw	Denotes the mean waiting time of all items
Td	Denotes the mean waiting time of the items waiting in the queue

N	Denotes the number of servers in a system
mx(y)	Denotes the y^{th} percentile which means the value of y below which x occurs y percent of the time

Single Server Queue

This is the simplest queuing system as represented in the following figure. The central element of the system is a server, which provides service to the connected devices or items. Items request to the system to be served, if the server is idle. Then, it is served immediately, else it joins a waiting queue. After the task is completed by the server, the item departs.

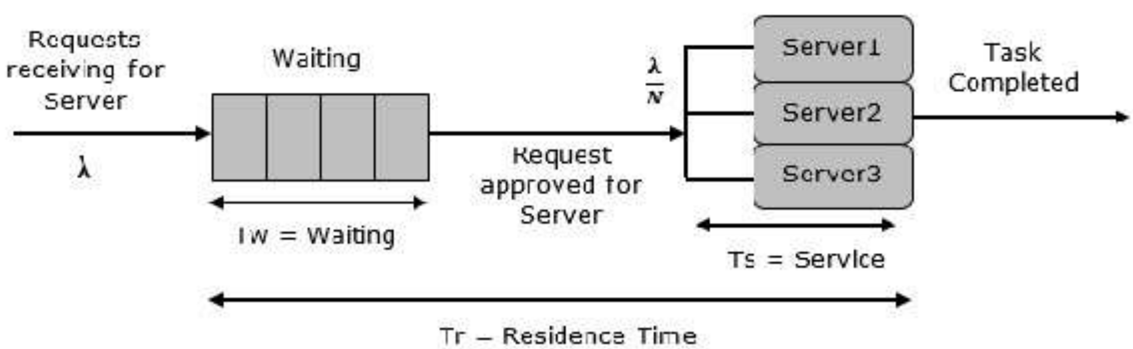


Multi Server Queue

As the name suggests, the system consists of multiple servers and a common queue for all items. When any item requests for the server, it is allocated if at-least one server is available. Else the queue begins to start until the server is free. In this system, we assume that all servers are identical, i.e. there is no difference which server is chosen for which item.

There is an exception of utilization. Let **N** be the identical servers, then **ρ** is the utilization of each server. Consider **Nρ** to be the utilization of the entire system; then the maximum utilization is **N*100%**, and the maximum input rate is −

$$\lambda max = \frac{\text{N}}{\text{T}s}$$



Queuing Relationships

The following table shows some basic queuing relationships.

General Terms	Single Server	Multi server
$r = \lambda Tr$ Little's formula	$\rho = \lambda Ts$	$\rho = \lambda Ts/N$
$w = \lambda Tw$ Little's formula	$r = w + \rho$	$u = \lambda Ts = \rho N$
$Tr = Tw + Ts$		$r = w + N\rho$

Simulation of Time-Sharing System

Time-sharing system is designed in such a manner that each user uses a small portion of time shared on a system, which results in multiple users sharing the system simultaneously. The switching of each user is so rapid that each user feels like using their own system. It is based on the concept of CPU scheduling and multi-programming where multiple resources can be utilized effectively by executing multiple jobs simultaneously on a system.

Example − SimOS Simulation System.

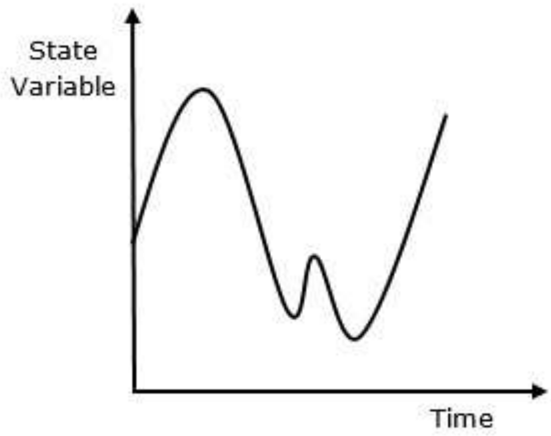
It is designed by Stanford University to study the complex computer hardware designs, to analyze application performance, and to study the operating systems. SimOS contains software simulation of all the hardware components of the modern computer systems, i.e. processors, Memory Management Units (MMU), caches, etc.

Continuous Simulation

A continuous system is one in which important activities of the system completes smoothly without any delay, i.e. no queue of events, no sorting of time simulation, etc. When a continuous system is modeled mathematically, its variables representing the attributes are controlled by continuous functions.

What is Continuous Simulation?

Continuous simulation is a type of simulation in which state variables change continuously with respect to time. Following is the graphical representation of its behavior.



Why Use Continuous Simulation?

We have to use continuous simulation as it depends on differential equation of various parameters associated with the system and their estimated results known to us.

Application Areas

Continuous simulation is used in the following sectors. In civil engineering for the construction of dam embankment and tunnel constructions. In military applications for simulation of missile trajectory, simulation of fighter aircraft training, and designing & testing of intelligent controller for underwater vehicles.

In logistics for designing of toll plaza, passenger flow analysis at the airport terminal, and proactive flight schedule evaluation. In business development for product development planning, staff management planning, and market study analysis.

Monte Carlo Simulation

Monte Carlo simulation is a computerized mathematical technique to generate random sample data based on some known distribution for numerical experiments. This method is applied to risk quantitative analysis and decision making problems. This method is used by the professionals of various profiles such as finance, project management, energy, manufacturing, engineering, research & development, insurance, oil & gas, transportation, etc.

This method was first used by scientists working on the atom bomb in 1940. This method can be used in those situations where we need to make an estimate and uncertain decisions such as weather forecast predictions.

Monte Carlo Simulation ─ Important Characteristics

Following are the three important characteristics of Monte-Carlo method −

- Its output must generate random samples.
- Its input distribution must be known.
- Its result must be known while performing an experiment.

Monte Carlo Simulation ─ Advantages

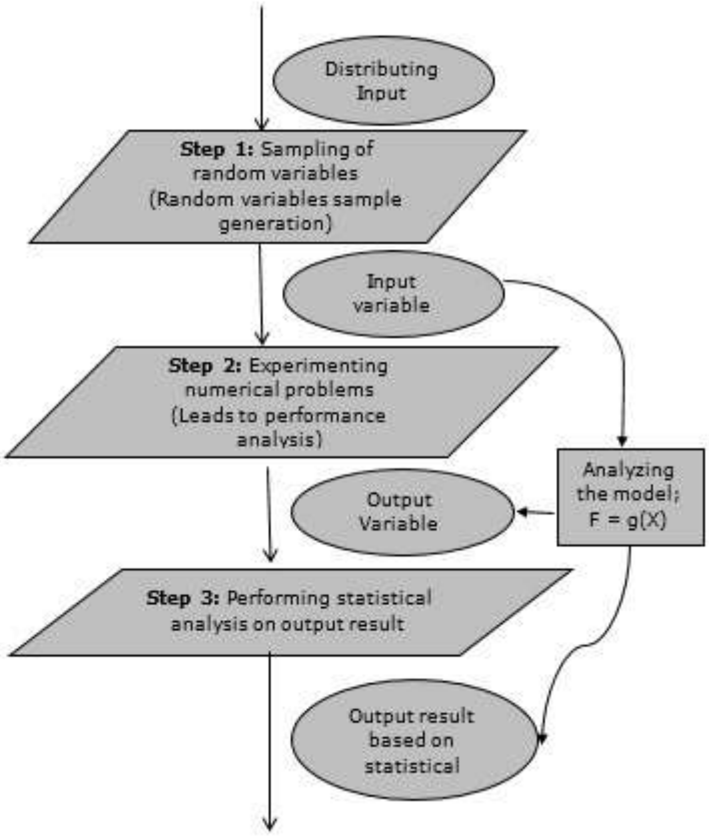
- Easy to implement.
- Provides statistical sampling for numerical experiments using the computer.
- Provides approximate solution to mathematical problems.
- Can be used for both stochastic and deterministic problems.

Monte Carlo Simulation ─ Disadvantages

- Time consuming as there is a need to generate large number of sampling to get the desired output.
- The results of this method are only the approximation of true values, not the exact.

Monte Carlo Simulation Method ─ Flow Diagram

The following illustration shows a generalized flowchart of Monte Carlo simulation.



Database in Simulation and Modeling

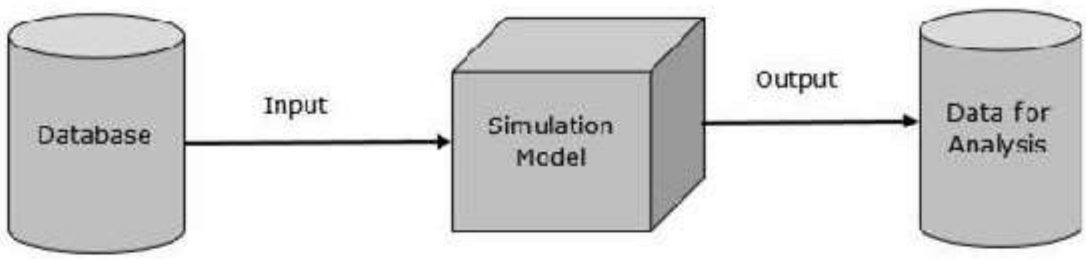
The objective of the database in Modelling & Simulation is to provide data representation and its relationship for analysis and testing purposes. The first data model was introduced in 1980 by Edgar Codd. Following were the salient features of the model.

- Database is the collection of different data objects that defines the information and their relationships.
- Rules are for defining the constraints on data in the objects.
- Operations can be applied to objects for retrieving information.

Initially, Data Modelling was based on the concept of entities & relationships in which the entities are types of information of data, and relationships represent the associations between the entities.

The latest concept for data modeling is the object-oriented design in which entities are represented as classes, which are used as templates in computer programming. A class having its name, attributes, constraints, and relationships with objects of other classes.

Its basic representation looks like –



Data Representation

Data Representation for Events

A simulation event has its attributes such as the event name and its associated time information. It represents the execution of a provided simulation using a set of input data associated with the input file parameter and provides its result as a set of output data, stored in multiple files associated with data files.

Data Representation for Input Files

Every simulation process requires a different set of input data and its associated parameter values, which are represented in the input data file. The input file is associated with the software which processes the simulation. The data model represents the referenced files by an association with a data file.

Data Representation for Output Files

When the simulation process is completed, it produces various output files and each output file is represented as a data file. Each file has its name, description and a universal factor. A data file is classified into two files. The first file contains the numerical values and the second file contains the descriptive information for the contents of the numerical file.

Neural Networks in Modelling & Simulation

Neural network is the branch of artificial intelligence. Neural network is a network of many processors named as units, each unit having its small local memory. Each unit is connected by unidirectional communication channels named as connections, which carry the numeric data. Each unit works only on their local data and on the inputs they receive from the connections.

History

The historical perspective of simulation is as enumerated in a chronological order.

The first neural model was developed in **1940** by McCulloch & Pitts.

In **1949**, Donald Hebb wrote a book “The Organization of Behavior”, which pointed to the concept of neurons.

In **1950**, with the computers being advanced, it became possible to make a model on these theories. It was done by IBM research laboratories. However, the effort failed and later attempts were successful.

In **1959**, Bernard Widrow and Marcian Hoff, developed models called ADALINE and MADALINE. These models have Multiple ADAptive LINear Elements. MADALINE was the first neural network to be applied to a real-world problem.

In **1962**, the perceptron model was developed by Rosenblatt, having the ability to solve simple pattern classification problems.

In **1969**, Minsky & Papert provided mathematical proof of the limitations of the perceptron model in computation. It was said that the perceptron model cannot solve X-OR problem. Such drawbacks led to the temporary decline of the neural networks.

In **1982**, John Hopfield of Caltech presented his ideas on paper to the National Academy of Sciences to create machines using bidirectional lines. Previously, unidirectional lines were used.

When traditional artificial intelligence techniques involving symbolic methods failed, then arises the need to use neural networks. Neural networks have its massive parallelism techniques, which provide the computing power needed to solve such problems.

Application Areas

Neural network can be used in speech synthesis machines, for pattern recognition, to detect diagnostic problems, in robotic control boards and medical equipments.

Fuzzy Set in Modelling & Simulation

As discussed earlier, each process of continuous simulation depends on differential equations and their parameters such as a, b, c, d > 0. Generally, point estimates are calculated and used in the model. However, sometimes these estimates are uncertain so we need fuzzy numbers in differential equations, which provide the estimates of the unknown parameters.

What is a Fuzzy Set?

In a classical set, an element is either a member of the set or not. Fuzzy sets are defined in terms of classical sets **X** as −

$$\mathbf{A = \{(x,\mu A(x)) \mid x \in X\}}$$

Case 1 − The function **μA(x)** has the following properties −

$$\mathbf{\forall x \in X \ \mu A(x) \geq 0}$$

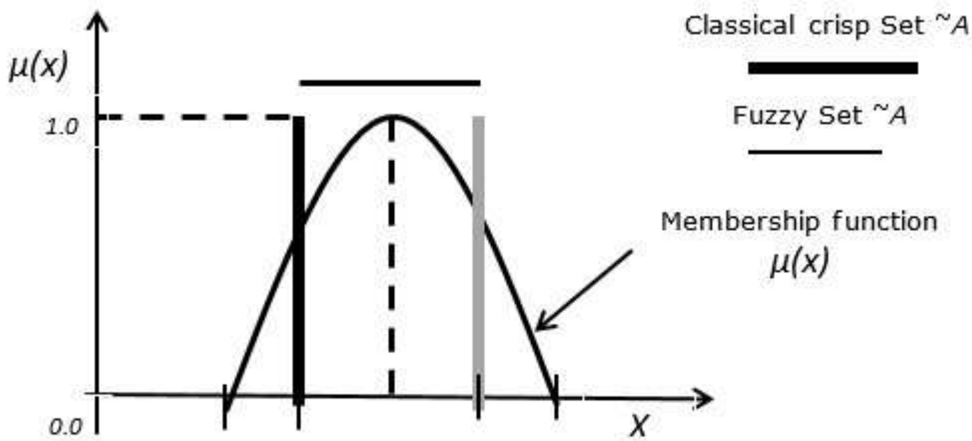
$$\mathbf{\sup x \in X \{\mu A(x)\} = 1}$$

Case 2 − Let fuzzy set **B** be defined as **A = {(3, 0.3), (4, 0.7), (5, 1), (6, 0.4)}**, then its standard fuzzy notation is written as **A = {0.3/3, 0.7/4, 1/5, 0.4/6}**

Any value with a membership grade of zero doesn’t appear in the expression of the set.

Case 3 − Relationship between fuzzy set and classical crisp set.

The following figure depicts the relationship between a fuzzy set and a classical crisp set.



μ(x)
1.0
0.0
X
Classical crisp Set ~A
Fuzzy Set ~A
Membership function
μ(x)