Lecture 12 - More on Parallel/Processor Domain Decomposition Techniques

- **In our last lecture, we discussed general ways to perform parallel/processor domain decomposition for different types of problems**
- **Again, in engineering simulations, we usually generate multiple blocks of structured- or unstructured-grids to solve a (set of) ODE's or PDE's that govern the problem physics**
- **Assuming we have a computer code that will solve these governing equations on a parallel computer, the next step is to map the blocks of grids onto P processors of the parallel computer**

Mapping Techniques

- **There are numerous techniques to map blocks (nodes, or cells) onto processors**
  - First-come-first-serve (i.e. in order of stack) or even distribution (naïve)
    - Trouble is that all blocks may not be the same! Some blocks may have more nodes (elements) than others. (naïve)
  - Geometrically constrained distribution of blocks
    - Distribute blocks such that we attempt to evenly distribute the number of nodes (elements) to the processors. (much better!)
  - Communication and geometrically constrained distribution of blocks
    - Distribute blocks such that we not only attempt an even distribution of nodes (elements) but also attempt to evenly distribute the inter-processor communication cost. (even better!)
- **Let's assume we are mapping blocks as we will do in the class project**

First-Come-First-Serve or Even Distribution

- **If we distribute N blocks over P processors, then the average number of blocks per processor will be N/P**
  - There are different possible distributions (some better than others)
    - First-come-first-serve: Take the first block and give it to the first processor, the second block to the second processor, and so on….. (more random, naïve)
      - This approach may be OK in some situations where a random distribution would more evenly load-balance the processors (but not for our project)
    - Even distribution: Take the first N/P blocks and assign them to the first processor, the second N/P blocks to the second processor, and so on. Take the remainder and sequentially distribute them over some part of P processors. (better!)
      - This approach would be much better in our project since our block list is contiguous and it would result in less inter-processor communication.
    - A measurement of the "load-balance" of a processor will be (M*P)/N where M is the actual number of blocks on a given processor, P is the total number of processors, and N is the number of blocks

First-Come-First-Serve or Even Distribution

- **This is equivalent to creating a task-dependency graph with no weights associated with the tasks**
  - All blocks are assumed to have the same number of nodes (cells) and therefore, have the same operation count and weight
  - Grouping of blocks onto processors would be performed in a way to evenly distribute the number of blocks amongst the processors
  - No accounting of communication cost is made

Geometrically Constrained

- **Here, we could distribute the blocks such that an attempt is made to give each processor approximately the same number of nodes or cells**
- **This can also be done in a number of ways:**
  - Pre-sorting:
    - Determine the “perfect load-balance”,PLB= (total number of nodes (cells)/P)
    - Sort the blocks in terms of number of nodes (cells) from maximum to minimum.
    - Then distribute the sorted blocks sequentially to the processors with the least number of nodes (cells) (i.e. take the next block in the sorted list and assign it to the processor that has the least number of nodes (cells) thus far)
    - A measurement of the “load-balance” of a processor will be M/PLB where M is the actual number of nodes (cells) on a given processor and PLB is the “perfect load-balance”
  - Non-pre-sorted:
    - You could do the same steps without pre-sorting but you are more likely to end up with a worse “load-balanced” system

Geometrically Constrained

- **Geometrically constrained mapping would be equivalent to again creating a task-dependency graph. But now, the weight of each task would depend on the number of nodes (cells).**
  - Grouping of blocks onto processors would be performed in a way to evenly distribute the “weight” amongst the processors
  - Again, no accounting of communication cost is made

Example Code for Geometrically Constrained Mapping

```
C    ******************************************************************
C    NAME=LOADBAL
     SUBROUTINE LOADBAL(NTCELL,NBL,JPROC,NPROC,NGDOM,NBLIST,NBPROC)
C    ******************************************************************
C    *                                                                *
C    *  ASSIGNS BLOCKS TO PROCESSORS FOR OPTIMUM LOAD BALANCE  *
C    *                                                                *
C    ******************************************************************
     IMPLICIT REAL*8 (A-H,O-Z)
     INTEGER   NTCELL, NPROC, NGDOM
     DIMENSION NBL(3,*), JPROC(*), NBPROC(*)
     DIMENSION NBLIST(NPROC,*)
C   NTCELL    =  TOTAL NUMBER OF CELLS
C   NBL          =  ARRAY OF DIMENSIONS PER BLOCK (1,NB) = IDIMENSION
C                    (2,NB) = JDIMENSION, (3,NB) = KDIMENSION
C   NPROC     =  NUMBER OF PROCESSORS
C   NGDOM   =  NUMBER OF BLOCKS IN GLOBAL DOMAIN
C
C    LOCAL SPACE
     INTEGER ISIZE(NGDOM), ICLAIM(NGDOM), ISORT(NGDOM)
     INTEGER NCELL(NPROC)
C
C    FIND THE SIZE OF EACH BLOCK AND MARK BLOCKS UNCLAIMED
     DO NB=1,NGDOM
C    CALCULATE THE NUMBER OF CELLS FOR THIS BLOCK (2 GHOST CELL LAYERS)
      ISIZE(NB)  = (NBL(1,NB)+3)*(NBL(2,NB)+3)*(NBL(3,NB)+3)
      ICLAIM(NB) = 0
     END DO
     DO N=1,NPROC
      NCELL(N) = 0
     END DO
C
C    FIND SIZE OF PERFECT LOAD BALANCE
     FNTEVEN = FLOAT(NTCELL)/FLOAT(NPROC)
C
C    SORT BLOCKS IN ORDER OF DECREASING SIZE
     DO NB=1,NGDOM
      IMAXSIZE = 0
      DO I=1,NGDOM
       IF (ISIZE(I).GT.IMAXSIZE .AND. ICLAIM(I).EQ.0) THEN
        IMAXSIZE = ISIZE(I)
        IMAX     = I
       END IF
      END DO
      ICLAIM(IMAX) = 1
      ISORT(NB)    = IMAX
     END DO
C
C    ASSIGN BLOCKS TO PROCESSORS IN ORDER TO KEEP MAXIMUM TO A MINIMUM
     DO NB=1,NGDOM
      IMINSIZE = 10000000
C    FIND THE PROCESSOR WITH THE MINIMUM NUMBER OF CELLS
      DO I=1,NPROC
       IF(NCELL(I).LT.IMINSIZE) THEN
        IMIN = I
        IMINSIZE = NCELL(I)
       END IF
      END DO
C    ASSIGN THE CELLS OF THIS NB BLOCK TO THAT PROCESSOR
      NCELL(IMIN)     = NCELL(IMIN) +ISIZE(ISORT(NB))
      JPROC(ISORT(NB)) = IMIN
     END DO
```

Example Code for Geometrically Constrained Mapping (cont)

```
C
C   OUTPUT LOAD BALANCE INFORMATION
      WRITE(*,*)
      WRITE(*,*)'LOAD BALANCE INFORMATION'
      WRITE(*,*)
C
      FLBMAX = -1.E10
      FLBMIN = +1.E10
      DO NP=1,NPROC
        FLB = FLOAT(NCELL(NP))/FNTEVEN
        IF (FLB.GT.FLBMAX) FLBMAX = FLB
        IF (FLB.LT.FLBMIN) FLBMIN = FLB
        WRITE(*,*)'PROCESSOR',NP,'   LOAD PERCENTAGE=',FLB
      END DO
      WRITE(*,*)
      WRITE(*,*) 'MAX/MIN LOAD = ',FLBMAX,FLBMIN
      WRITE(*,*) 'LOAD BALANCE RATIO:',FLBMAX/FLBMIN
      WRITE(*,*)
      IF (FLBMAX/FLBMIN .GT. 1.2) THEN
        WRITE(*,*)
        WRITE(*,*) 'WARNING: WARNING: WARNING: WARNING: WARNING:'
        WRITE(*,*) 'The load is not balanced well with the'
        WRITE(*,*) 'current decomposition and # of processors.'
        WRITE(*,*) 'Consider re-decomposition or reduce number'
        WRITE(*,*) 'of processors.'
        WRITE(*,*)
      END IF
C
C   WRITE OUT BLOCKS FOR EACH PROCESSOR
      DO NP=1,NPROC
        NN = 0
        DO I=1,NGDOM
          IF (JPROC(I).EQ.NP) THEN
            NN = NN +1
            NBLIST(NP,NN) = I
          END IF
        END DO
        NBPROC(NP) = NN
        WRITE(*,*)'PROCESSOR',NP,'BLOCKS',(NBLIST(NP,I),I=1,NN)
      END DO
      WRITE(*,*)
C
      DO NP=1,NPROC
        WRITE(*,*)'PROCESSOR',NP,'NCELLS',NCELL(NP)
      ENDDO
C
      RETURN
      END
```

Communication and Geometrically Constrained

- **Here, we could distribute the blocks such that an attempt is made to give each processor approximately the same communication cost as well as the number of nodes or cells**
- **This can also be done in a number of ways (depending on how you wish to account for communication costs):**
  - Pre-sorting:
    - Sort the blocks in terms of a “weight” from maximum to minimum where the “weight” is a blended formula of the number of nodes (cells) and communication cost. Communication cost could be approximated as the number of nodes along the block faces.
    - Then distribute the sorted blocks sequentially to the processors with the least “weight” (i.e. take the next block in the sorted list and assign it to the processor that has the least accumulated “weight” thus far)
    - Determine the “perfect load-balance”,PLB= (total “weight”/P)
    - A measurement of the “load-balance” of a processor will be M/PLB where M is the actual accumulated “weight” on a given processor and PLB is the “perfect load-balance”
- **You can see that this is a generalization of the geometrically constrained mapping**

Communication and Geometrically Constrained

- **Communication and geometrically constrained mapping would be equivalent to creating both task-dependency and task-interaction graphs. Now, the weight of each task would depend on the number of nodes (cells) and the weight of each path between interactions would depend on the communication cost.**
  - Grouping of blocks onto processors would be performed in a way to evenly distribute the total “weight” based upon the number of nodes (cells) and estimate of communication cost across edges amongst the processors

Example Code for Communication and Geometrically Constrained Mapping

```
C     *********************************************************************
C     NAME=LOADBAL_JCV
      SUBROUTINE LOADBAL_JCV(NTCELL,NBL,ITYPE,NEIGHB,INTRF,JPROC,
     &                        NPROC,NGDOM,NBLIST,NBPROC)
C     *********************************************************************
C     *  ASSIGNS BLOCKS TO DIFFERENT PROCESSORS IN ORDER TO          *
C     *   OPTIMIZE LOAD BALANCE, INCLUDING COMMUNICATION INFLUENCES    *
C     *********************************************************************
C     MODIFIED BY:  JC VASSBERG  16 APR 1997  TO INCLUDE COMM INFLUENCE.
      IMPLICIT REAL*8 (A-H,O-Z)
C    NTCELL    =  TOTAL NUMBER OF CELLS
C    NBL          =  ARRAY OF DIMENSIONS PER BLOCK (1,NB) = IDIMENSION
C                    (2,NB) = JDIMENSION, (3,NB) = KDIMENSION
C    NPROC     =  NUMBER OF PROCESSORS
C    NGDOM   =  NUMBER OF BLOCKS IN GLOBAL DOMAIN
C
      INTEGER   NTCELL,NPROC,NGDOM
      DIMENSION NBL(3,*),JPROC(*),NBPROC(*)
      DIMENSION ITYPE(6,*),NEIGHB(6,*),INTRF(6,*)
      DIMENSION NBLIST(NPROC,*)
C
C     LOCAL SPACE
      INTEGER ITABLE(NPROC,NPROC),ICLAIM(NGDOM),ISORT(NGDOM)
      REAL*8  SIZE(NGDOM),FNCELL(NPROC)
C
C     *********************************************************************
```

```
C     FIND THE SIZE OF EACH BLOCK AND MARK BLOCKS UNCLAIMED
C     ADD TO THIS SIZE A VIRTUAL SIZE RELATED TO COMMUNICATIONS
C        FNOPS DEPENDS ON CODE, FLOPS, BANDWIDTH, AND LATENCY DEPEND
C        ON COMPUTER SYSTEM
C     FNOPS     - NUMBER OF FLOATING POINT OPERATIONS PER POINT REQUIRED
C                 TO COMPLETE A SINGLE ITERATION
C     FLOPS     - FLOPS RATING FOR THE PROCESSORS (REALISTIC FLOATING
C                 POINT OPERATIONS PER SECOND)
C     BANDWIDTH - BYTES PER SECOND FOR MPI COMMUNICATION
C     LATENCY   - MPI TIME FOR A MESSAGE OF ZERO LENGTH
C     BANDWIDTH AND LATENCY CAN BE FOUND USING BOUNCE
C
C     SP2:      FLOPS     = 50.0E+6 (FLOATING POINT OPS. / SEC)
C               BANDWIDTH = 35.0E+6 (BYTES / SEC)
C               FLATENCY  = 43.0E-6 (SEC)
C
C     ETHERNET:  FLOPS     = 50.0E+6 (FLOATING POINT OPS. / SEC)
C               BANDWIDTH = 0.1E+6  (BYTES / SEC)
C               FLATENCY  = 800.E-6 (SEC)
C
      FNOPS     = 2000.0
      FLOPS     = 50.0E+6
      BANDWIDTH = 35.E+6
      FLATENCY  = 45.E-6
      CBNDWD    = (120. *FLOPS)/(BANDWIDTH *FNOPS)
      CLATNC    = (5.0 *FLATENCY *FLOPS)/FNOPS
C
```

Example Code for Communication and Geometrically Constrained Mapping

```
C
 WRITE(*,*)
    WRITE(*,*) 'THE CURRENT COMMUNICATION PARAMETERS ARE FOR SP2.'
    WRITE(*,*)
    WRITE(*,*)
    WRITE(*,*) 'LOADBAL WEIGHTS'
    WRITE(*,*)
    WRITE(*,*) 'WEIGHT ON NUMBER OF CELLS =',1.0
    WRITE(*,*) 'WEIGHT ON BANDWIDTH      =',CBNDWD
    WRITE(*,*) 'WEIGHT ON LATENCY        =',CLATNC
    WRITE(*,*)
C
C   ASSIGN A SIZE TO EACH DOMAIN BASED ON ITS FLOP & BANDWIDTH
C     REQUIREMENTS.
    DO NB=1,NGDOM
C     COMPUTE DIMENSIONS OF EACH BLOCK (2 GHOST CELL LAYERS)
      IDIM     = NBL(1,NB) +3
      JDIM     = NBL(2,NB) +3
      KDIM     = NBL(3,NB) +3
C    SIZE IS NOW A FUNCTION OF NUMBER OF CELLS+BANDWIDTH WEIGHT
C     ASSOCIATED WITH FACE COMMUNICATION  (NOTE THAT YOU CAN MAKE
C     SIZE A FUNCTION  OF DIFFERENT PARAMETERS
      SIZE(NB)   = DBLE(IDIM*JDIM*KDIM)
   &          +4.*CBNDWD*
   &          DBLE(IDIM*JDIM +IDIM*KDIM +JDIM*KDIM)
      ICLAIM(NB) = 0
      JPROC (NB) = 0
    END DO
```

```
C
C   INITIALIZE ADDITIONAL ARRAYS
    DO N=1,NPROC
     FNCELL(N)     = 0.0
     DO I=1,NPROC
      ITABLE(I,N) = 0
     END DO
     ITABLE(N,N)   = 1
    END DO
C
C   FIND SIZE OF PERFECT LOAD BALANCE W/O COMMUNICATION OVERHEAD
    FNTEVEN = DBLE(NTCELL)/DBLE(NPROC)
C
C   SORT BLOCKS IN ORDER OF DECREASING SIZE
    DO NB=1,NGDOM
     AMAXSIZE = 0.0
     DO I=1,NGDOM
      IF (SIZE(I).GT.AMAXSIZE .AND. ICLAIM(I).EQ.0) THEN
       AMAXSIZE = SIZE(I)
       IMAX     = I
      END IF
     END DO
     ICLAIM(IMAX) = 1
     ISORT(NB)    = IMAX
    END DO
```

Example Code for Communication and Geometrically Constrained Mapping

```
C
C   ASSIGN BLOCKS TO PROCESSORS IN ORDER TO KEEP THE RESULTING
C    MAX-LOADED PROCESSOR TO A MINIMUM
   DO NB=1,NGDOM
    NBLK     = ISORT(NB)
    IDIM    = NBL(1,NBLK) +3
    JDIM    = NBL(2,NBLK) +3
    KDIM     = NBL(3,NBLK) +3
    IJKD    = IDIM*JDIM*KDIM
    IMIN    = 1
    AMINSIZE = 1.0E10
    DO I=1,NPROC
     ATMPSIZE = FNCELL(I) +SIZE(NBLK)
     DO LSIDE = 1,6
      LDIR = (LSIDE +1)/2
      NABR = NEIGHB(LSIDE,NBLK) +INTRF(LSIDE,NBLK)
      IF (NABR .EQ. 0) THEN
       ATMPSIZE = ATMPSIZE
  .              -2.*CBNDWD*DBLE(IJKD/(NBL(LDIR,NBLK) +3))
      ELSE
       JPNABR = JPROC(NABR)
       IF (JPNABR .EQ. I) THEN
        ATMPSIZE = ATMPSIZE
  .         -4.*CBNDWD*DBLE(IJKD/(NBL(LDIR,NBLK) +3))
       ELSE
        IF (ITABLE(I,JPNABR) .EQ. 0) THEN
          ATMPSIZE = ATMPSIZE + CLATNC
        ENDIF
       ENDIF
      ENDIF
     END DO
C
     IF (ATMPSIZE .LT. AMINSIZE) THEN
      IMIN    = I
      AMINSIZE = ATMPSIZE
     END IF
    END DO
C
    FNCELL(IMIN) = AMINSIZE
    JPROC (NBLK) = IMIN
C
C   FLAG THAT WE ARE COMMUNICATING BETWEEN THE JPNBLK & JPNABR
C     PROCESSORS.
    JPNBLK   = JPROC(NBLK)
    DO LSIDE = 1,6
     NABR   = NEIGHB(LSIDE,NBLK)
     IF (NABR .GT. 0) THEN
      JPNABR   = JPROC(NABR)
      IF (JPNABR .GT. 0) THEN
       IF (ITABLE(JPNBLK,JPNABR) .EQ. 0) THEN
         FNCELL(JPNABR) = FNCELL(JPNABR) +CLATNC
       ENDIF
       ITABLE(JPNBLK,JPNABR) = 1
       ITABLE(JPNABR,JPNBLK) = 1
      ENDIF
     ENDIF
    END DO
   END DO
```

Example Code for Communication and Geometrically Constrained Mapping

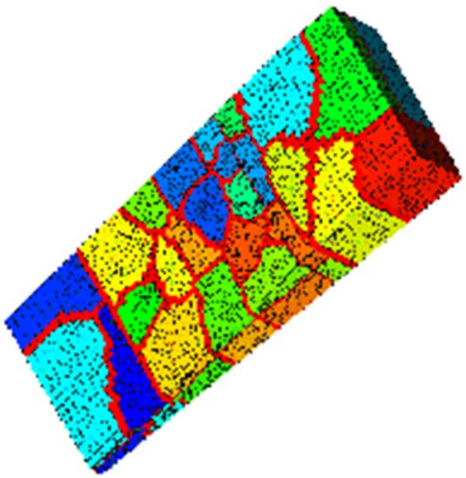
```
C
C    OUTPUT LOAD BALANCE INFORMATION
     WRITE(*,*)
     WRITE(*,*)'LOAD BALANCE INFORMATION'
     WRITE(*,*)
C
     FLBMAX = -1.E10
     FLBMIN = +1.E10
     DO NP=1,NPROC
       FLB = FNCELL(NP)/FNTEVEN
       IF (FLB.GT.FLBMAX) FLBMAX = FLB
       IF (FLB.LT.FLBMIN) FLBMIN = FLB
       WRITE(*,*)'PROCESSOR',NP,'   LOAD PERCENTAGE=',FLB
     END DO
     WRITE(*,*)
     IF (FLBMAX/FLBMIN .GT. 1.2) THEN
       WRITE(*,*)
       WRITE(*,*) 'WARNING: WARNING: WARNING: WARNING: WARNING:'
       WRITE(*,*) 'The load is not balanced well with the'
       WRITE(*,*) 'current decomposition and # of processors.'
       WRITE(*,*) 'Consider re-decomposition or reduce number'
       WRITE(*,*) 'of processors.'
       WRITE(*,*)
     END IF
C
C
C    WRITE OUT BLOCKS FOR EACH PROCESSOR
     DO NP=1,NPROC
       NN = 0
       NC = 0
       DO I=1,NGDOM
         IF (JPROC(I).EQ.NP) THEN
           NN = NN +1
           NBLIST(NP,NN) = I
           IDIM = NBL(1,I) +3
           JDIM = NBL(2,I) +3
           KDIM = NBL(3,I) +3
           IJKD = IDIM*JDIM*KDIM
           NC   = NC + IJKD
         END IF
       END DO
       FNCELL(NP)  = NC
       NBPROC(NP) = NN
       WRITE(*,*)'PROCESSOR',NP,'BLOCKS',(NBLIST(NP,I),I=1,NN)
     END DO
c
     WRITE(*,*)'FNTEVEN',FNTEVEN
     DO NP=1,NPROC
       NC = FNCELL(NP)
       WRITE(*,*)'PROCESSOR',NP,'NCELLS',NC,(FLOAT(NC)/FNTEVEN)
     END DO
     RETURN
     END
```

Other Mapping Techniques

- **The mapping of blocks, nodes, or cells onto processors is a subject of research that has been ongoing since the advent of parallel computing**
- **Several researchers have developed open-source libraries of static, graph-partitioning schemes:**
  - Chaco: B. Hendrickson and R. Leland (Sandia National Lab)
    http://www.cs.sandia.gov/CRF/chac.html
  - Metis/Parmetis:G. Karypis and V. Kumar (University of Minnesota)
    http://glaros.dtc.umn.edu/gkhome/views/metis/
  - Many others (do a web-search under “graph partitioning”)

Chaco

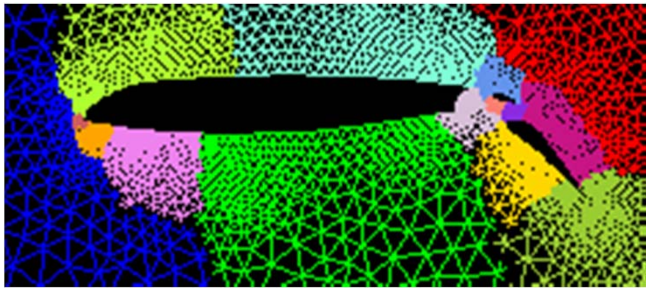
- **Chaco is a graph partitioning system**
  – From Sandia National Lab
  – See http://www3.cs.stonybrook.edu/~algorith/implement/chaco/implement.shtml
  – Allows for recursive application of several methods for finding small edge separators in weighted graphs.
    - "These methods include inertial, spectral, Kernighan-Lin and multilevel methods in addition to several simpler strategies."*
    - "Each of these approaches can be used to partition the graph into two, four or eight pieces at each level of recursion. In addition, the Kernighan-Lin (optimization) method can be used to improve partitions generated by any of the other algorithms."*



*From Chaco users' manual

Jostle

- **Jostle is an open-source library for static and dynamic graph partitioning that are both serial and parallel (similar to Metis/Parmetis)**
  - Combination of a graph contraction algorithm (similar to grid-sequencing) with a local optimization method which refines the partition at each graph level
  - Kernighan-Lin partition optimization which incorporates load-balancing
  - Evolutionary search algorithms
  - See http://staffweb.cms.gre.ac.uk/~wc06/jostle/



Metis/Parmetis

- **Metis and Parmetis are open-source codes/libraries that also perform graph partitioning**
- **I have downloaded Metis and Parmetis and put it in the "Codes" folder in smartsite for your information**
- **Metis and Parmetis (parallel version of Metis) can be run as a separate program or has subroutines that may be linked and called from your program (Metislib).**
- **Metis may be used to partition structured- or unstructured nodes, cells, or blocks**
  - Blocks may be treated as cells with weighted nodes and/or edges
  - Minimizes either the number of edges that straddle partitions (edgecut) or the total communication volume (totalv)
  - See http://glaros.dtc.umn.edu/gkhome/metis/metis/overview or http://glaros.dtc.umn.edu/gkhome/metis/parmetis/overview

Metis/Parmetis

- **Multi-Constraint Partitioning**
  - Each vertex or edge (of a cell or block) has a vector of weights of size *m*
  - “The objective of the partitioning algorithm is to minimize the edgecut subject to the constraints that each one of the *m* weights is equally distributed among the domains.”*
- **Minimizing the Total Communication Volume**
  - Objective of traditional graph partitioning is to compute a balanced k-way partitioning such that the number of edges (or the sum of their weights) that straddle different partitions is minimized. The objective of minimizing the edgecut is only an approximation of the true communication cost.
  - Metis can also directly minimize the communication cost as defined by the total communication volume.

Metis/Parmetis

- **Minimizing the Maximum Connectivity of the Subdomains:**
  - Communication costs generally depends on:
    - The total communication volume
    - The maximum amount of data that any particular processor needs to send and receive
    - The number of messages a processor needs to send and receive
  - Metis attempts to reduce all three factors
- **Reducing the Number of Non-Contiguous Subdomains**
  - A k-way partitioning of a continuous graph can often lead to some sub-domains being assigned non-contiguous portions of the graph (i.e. certain domains are broken needlessly)
  - Metis attempts to eliminate these non-contiguous subdomains

Homework 6 (cont)

- **Read Chapter 5 of Introduction to Parallel Computing by Grama et. al. if you purchased book**