give answer to these

Sure! Below are detailed, step-by-step answers to each question from your assignment.

1. Out of 500 car owners investigated, 400 owned car A and 200 owned car B, and 50 owned both A and B cars. Is this data correct?

Let:

- $ n = $ Total number of car owners = **500**
- $ n(A) = $ Number who own car A = **400**
- $ n(B) = $ Number who own car B = **200**
- $ n(A \cap B) = $ Number who own both = **50**

Number who own at least one car = $ n(A \cup B) = n(A) + n(B) - n(A \cap B) = 400 + 200 - 50 = 550 $

This value (550) is **more** than the total number surveyed (500), which is a **contradiction**.
Therefore, the data is not correct.

2. Newspaper Family Survey

Given data:

- Tamil: 275
- English: 150
- Hindi: 45
- Tamil & English: 125
- English & Hindi: 17
- Tamil & Hindi: 5
- All three: 3

Let:

- $ T = $ Tamil, $ E = $ English, $ H = $ Hindi

Let $ n(T) $, $ n(E) $, etc., as per above.

(i) Number of families buying only one newspaper

Families buying only Tamil:
$ = n(T) - n(T \cap E) - n(T \cap H) + n(T \cap E \cap H) $
$ = 275 - 125 - 5 + 3 = 148 $

Families buying only English:
$ = 150 - 125 - 17 + 3 = 11 $

Families buying only Hindi:
$ = 45 - 17 - 5 + 3 = 26 $

Total buying only one: 148 + 11 + 26 = 185

(ii) Number buying at least 2 newspapers

Number buying exactly two:

- Tamil & English only: $ 125 - 3 = 122 $
- English & Hindi only: $ 17 - 3 = 14 $
- Tamil & Hindi only: $ 5 - 3 = 2 $

Number buying all three: **3**

Total buying at least two:
$ = 122 + 14 + 2 + 3 = 141 $

(iii) Total number of families in the colony

By inclusion-exclusion:

$$n(T \cup E \cup H) = n(T) + n(E) + n(H) - n(T \cap E) - n(E \cap H) - n(T \cap H) + n(T \cap E$$
$$= 275 + 150 + 45 - 125 - 17 - 5 + 3 = 326$$

Total = 326

3. Farmer Survey: Number who grew all three crops

Let $ P = $ paddy (600), $ R = $ ragi (350), $ C = $ corn (280)

- $ P \cap R = 120, R \cap C = 100, P \cap C = 80 $

Let $ x = $ number who grew all three.

By inclusion-exclusion:

$$n(P \cup R \cup C) = n(P) + n(R) + n(C) - n(P \cap R) - n(R \cap C) - n(P \cap C) + n(P \cap R$$
$$1000 = 600 + 350 + 280 - 120 - 100 - 80 + x$$
$$1000 = 1230 - 300 + x \implies 1000 = 930 + x \implies x = 70$$

So, 70 farmers grew all three.

4. Find the inverse of $ y = -3x + 7 $

Let $ y = -3x + 7 $

Swap x and y to solve for y:

$$x = -3y + 7$$

$$3y = 7 - x$$

$$y = \frac{7 - x}{3}$$

So,

$$f^{-1}(x) = \frac{7 - x}{3}$$

5. Sets

Given:

- $ A = {1,2,3,4,5},; B = {4,5,6,8},; C = {3,5,9},; U = {1,2,3,4,5,6,7,8,9} $

(I) $ A \cup B \cup $

Assuming this is $ A \cup B \cup C $:

$$= \{1, 2, 3, 4, 5, 6, 8, 9\}$$

(II) $A' \cap C $

$A' = U - A = \{6, 7, 8, 9\}$
$A' \cap C = \{9\}$

(III) $ (A \cup B) - C $

$A \cup B = \{1, 2, 3, 4, 5, 6, 8\}$
So, $ (A \cup B) - C = {1,2,4,6,8} $

(IV) $ A \cap (B \cap C) $

$B \cap C = \{5\}$,
So, $A \cap \{5\} = \{5\}$

(V) $B - (A \cup C)$

$A \cup C = \{1, 2, 3, 4, 5, 9\}$
So, $B - (A \cup C) = \{6, 8\}$

(VI) $ (A \cup B) \cap C $

As above, $A \cup B = \{1, 2, 3, 4, 5, 6, 8\}$
So, intersection with C: $\{3, 5\}$

(VII) $A' \cup B'$

First, $B' = U - B = \{1, 2, 3, 5, 7, 9\}$
So, $A' \cup B' = \{1, 2, 3, 5, 6, 7, 8, 9\}$

(VIII) $ (A \cup B)' \cup C $

$A \cup B = \{1, 2, 3, 4, 5, 6, 8\}$, so complement is $U - (A \cup B) = \{7, 9\}$
Union with C: $ {3,5,7,9}$

6. State and prove De Morgan's law

Statement:
For any two sets A and B,

- $(A \cup B)' = A' \cap B'$
- $(A \cap B)' = A' \cup B'$

Proof:

- $x \in (A \cup B)'$ means $x \notin A \cup B$, so $x \notin A$ and $x \notin B$, thus $x \in A'$ and $x \in B'$, i.e., $x \in A' \cap B'$.
- $x \in (A \cap B)'$ means $x \notin A \cap B$, so $x \notin A$ or $x \notin B$, i.e., $x \in A'$ or $x \in B'$, thus $x \in A' \cup B'$.

7. Example of a relation which is neither symmetric nor anti-symmetric

Let $ A = {1, 2} $. Define relation $ R = {(1,2)} $.

- Not symmetric: $(1, 2) \in R$ but $(2, 1) \notin R$.
- Not anti-symmetric: For $ x \neq y $, if both (x, y) and (y, x) are not in R, anti-symmetry is not violated. But here, only one is present, so the relation is **neither symmetric nor anti-symmetric**.

8. Find the inverse of $ f(x) = 4x - 7 $

Set $ y = 4x - 7 $

Swap and solve:

$$x = 4y - 7 \implies 4y = x + 7 \implies y = \frac{x+7}{4}$$

So,

$$f^{-1}(x) = \frac{x+7}{4}$$

9. $ R $ is symmetric if and only if $ R = R^{-1} $

- If R is symmetric: For all $(x, y) \in R$, since R is symmetric, $(y, x) \in R$ so $R^{-1} \subseteq R$. Similarly, every $(y, x) \in R^{-1}$ implies $(x, y) \in R$, so $R \subseteq R^{-1}$. Thus, $R = R^{-1}$.
- If $R = R^{-1}$: For $(x, y) \in R$, then $(y, x) \in R^{-1} = R$. Hence, R is symmetric.

9b. Composition of functions

Given
$ f(x) = x^2 + 3x + 1 $, $ g(x) = 2x - 3 $

(a) $ f \circ f $

$ f(f(x)) = (x^2 + 3x + 1)^2 + 3(x^2 + 3x + 1) + 1 $

(b) $ f \circ g $

$ f(g(x)) = f(2x-3) = (2x-3)^2 + 3(2x-3) + 1 = 4x^2 -12x + 9 + 6x - 9 + 1 = 4x^2 - 6x + 1 $

(c) $ g \circ f $

$ g(f(x)) = 2(x^2 + 3x + 1) - 3 = 2x^2 + 6x + 2 - 3 = 2x^2 + 6x -1 $

10. Relation $ R = {(x, y): y $ is divisible by $ x} $, $ A = {1,2,3,4,5,6} $

- **Reflexive:** For all $ x $, $ x $ divides $ x $. **Yes, reflexive.**
- **Symmetric:** For $ (x,y) $, does $ (y,x) $ always exist? No (e.g., $2|4$, but 4 does not divide 2), so **not symmetric**.
- **Transitive:** If $ x|y $ and $ y|z $, then $ x|z $. Yes, so **transitive**.

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1. Assignment-1-3.pdf