15-312 Assignment 1

Andrew Carnegie
(andrew)

September 4, 2017

1 Syntax

Type	τ ::=		
	$\texttt{nat}$	$\texttt{nat}$	naturals
	$\texttt{unit}$	$\texttt{unit}$	unit
	$\texttt{bool}$	$\texttt{bool}$	boolean
	$\texttt{prod}(\tau_1; \tau_2)$	$\tau_1 \times \tau_2$	product
	$\texttt{arr}(\tau_1; \tau_2)$	$\tau_1 \to \tau_2$	function
	$\texttt{list}(\tau)$	$\tau\, \texttt{list}$	list
Exp	e ::=		
	x	x	variable
	$\texttt{nat}[n]$	$\overline{n}$	number
	$\texttt{unit}$	$()$	unit
	$\texttt{T}$	$\texttt{T}$	true
	$\texttt{F}$	$\texttt{F}$	false
	$\texttt{if}(x; e_1; e_2)$	$\texttt{if}\, x\, \texttt{then}\, e_1\, \texttt{else}\, e_2$	if
	$\texttt{lam}(x : \tau.e)$	$\lambda\, x : \tau.e$	abstraction
	$\texttt{ap}(f; x)$	$f(x)$	application
	$\texttt{tpl}(x_1; x_2)$	$\langle x_1, x_2 \rangle$	tuple
	$\texttt{fst}(x)$	$x \cdot \texttt{l}$	first projection
	$\texttt{snd}(x)$	$x \cdot \texttt{r}$	second projection
	$\texttt{nil}$	$[]$	nil
	$\texttt{cons}(x_1; x_2)$	$x_1 :: x_2$	cons
	$\texttt{case}\{l\}(e_1; x, xs.e_2)$	$\texttt{case}\, l\, \{\texttt{nil} \hookrightarrow e_1 \mid \texttt{cons}(x; xs) \hookrightarrow e_2\}$	match list
	$\texttt{let}(e_1; x : \tau.e_2)$	$\texttt{let}\ x = e_1\ \texttt{in}\ e_2$	let
Val	v ::=		
	$\texttt{val}[l](n)$	n^l	numeric value
	$\texttt{val}[l](\texttt{T})$	$\texttt{T}^l$	true value
	$\texttt{val}[l](\texttt{F})$	$\texttt{F}^l$	false value
	$\texttt{val}[l](\texttt{Null})$	$\texttt{Null}^l$	null value
	$\texttt{val}[l](\texttt{cl}(V; x.e))$	$(V, x.e)^l$	function value
	$\texttt{val}[l_2](l_1)$	$l_1^{l_2}$	loc value
	$\texttt{val}[l](\texttt{pair}(v_1; v_2))$	$\langle v_1, v_2 \rangle^l$	pair value
Loc	l ::=		
	$\texttt{loc}(l)$	l	location

2 Garbage collection semantics

Model dynamics using judgement of the form:

$$\boxed{V, H, \vdash e \Downarrow^s v, H'}$$

Where $V : VID \to Val$, $H : Loc \to Val$, and $R : \{Loc\}$. This can be read as: under stack V, heap H, and roots R, the expression e evaluates to v using maximum heap space s, and engenders a new heap H'.

Roots represents the set of locations required to compute the continuation *excluding* the current expression. We can think of roots as the heap allocations necessary to compute the context with a hole that will be filled by the current expression.

Below defines the size of reachable values and space for roots:

$$
\begin{aligned}
&reach_H(n^l) = \{l\} \\
&reach_H(\mathtt{T}^l) = \{l\} \\
&reach_H(\mathtt{F}^l) = \{l\} \\
&reach_H(\mathtt{Null}^l) = \{l\} \\
&reach_H((V, x.e)^l) = \{l\} \cup (\bigcup_{y \in FV(e) \setminus x} reach_H(V(y))) \\
&reach_H(l_1^{l_2}) = \{l_2\} \cup loc_H(H(l_1)) \\
&reach_H(\langle v_1, v_2 \rangle^l) = \{l\} \cup reach_H(v_1) \cup reach_H(v_2)
\end{aligned}
$$

$$loc(val[l](\_)) = l$$

$$locs_{V,H}(e) = \bigcup_{x \in FV(e)} reach_H(V(x))$$

$$\frac{x \in dom(V)}{V, H, \vdash x \Downarrow^0 V(x), H}(\mathrm{S}_1) \qquad \frac{(l \text{ fresh}) \qquad H' = H[l \mapsto n^l]}{V, H, \vdash \overline{n} \Downarrow^1 n^l, H'}(\mathrm{S}_2)$$

$$\frac{(l \text{ fresh}) \qquad H' = H[l \mapsto \mathtt{T}^l]}{V, H, \vdash \mathtt{T} \Downarrow^1 \mathtt{T}^l, H'}(\mathrm{S}_3) \qquad \frac{(l \text{ fresh}) \qquad H' = H[l \mapsto \mathtt{F}^l]}{V, H, \vdash \mathtt{F} \Downarrow^1 \mathtt{F}^l, H'}(\mathrm{S}_4)$$

$$\frac{(l \text{ fresh}) \qquad H' = H[l \mapsto \mathtt{Null}^l]}{V, H, \vdash () \Downarrow^1 \mathtt{Null}^l, H'}(\mathrm{S}_5) \qquad \frac{V(x) = \mathtt{T}^l \qquad V, H, \vdash e_1 \Downarrow^{s_1} v_1, H_1}{V, H, \vdash \mathtt{if}(x; e_1; e_2) \Downarrow^{s_1} v_1, H_1}(\mathrm{S}_6)$$

$$\frac{V(x) = \mathtt{F}^l \qquad V, H, \vdash e_2 \Downarrow^{s_2} v_2, H_2}{V, H, \vdash \mathtt{if}(x; e_1; e_2) \Downarrow^{s_2} v_2, H_2}(\mathrm{S}_7) \qquad \frac{(l \text{ fresh}) \qquad H' = H[l \mapsto (V, x.e)^l]}{V, H, \vdash \mathtt{lam}(x : \tau.e) \Downarrow^1 (V, x.e)^l, H'}(\mathrm{S}_8)$$

$$\frac{V(f) = (V_1, x.e)^{l_1} \qquad V(x) = v_1 \qquad V_1[x \mapsto v_1], H, \vdash e \Downarrow^s v, H'}{V, H, \vdash f(x) \Downarrow^s v, H'}(\mathrm{S}_9)$$

$$\frac{V(x_1) = v_1 \qquad V(x_2) = v_2 \qquad (l \text{ fresh}) \qquad H' = H[l \mapsto \langle v_1, v_2 \rangle^l]}{V, H, \vdash \langle x_1, x_2 \rangle \Downarrow^1 \langle v_1, v_2 \rangle^l, H'}(\mathrm{S}_{10})$$

$$\frac{V(x) = \langle v_1, v_2 \rangle^l}{V, H, \vdash x \cdot \mathtt{l} \Downarrow^0 v_1, H}(\mathrm{S}_{11}) \qquad \frac{V(x) = \langle v_1, v_2 \rangle^l}{V, H, \vdash x \cdot \mathtt{r} \Downarrow^0 v_2, H'}(\mathrm{S}_{12})$$

$$\frac{(l \text{ fresh}) \qquad H' = H[l \mapsto \mathtt{Null}^l]}{V, H, \vdash \mathtt{nil} \Downarrow^1 \mathtt{Null}^l, H'}(\mathrm{S}_{13})$$

$$\frac{V(x_1) = v_1 \qquad V(x_2) = v_2 \qquad (l \text{ fresh}) \qquad H' = H[l \mapsto \langle v_1, v_2 \rangle^l]}{V, H, \vdash \mathtt{cons}(x_1; x_2) \Downarrow^1 \langle v_1, v_2 \rangle^l, H'}(\mathrm{S}_{14})$$

$$\frac{V(z) = \mathtt{Null}^l \qquad V, H, \vdash e_1 \Downarrow^{s_1} v_1, H_1}{V, H, \vdash \mathtt{case}\, z \,\{\mathtt{nil} \hookrightarrow e_1 \mid \mathtt{cons}(x; xs) \hookrightarrow e_2\} \Downarrow^{s_1} v_1, H_1}(\mathrm{S}_{15})$$

$$\frac{V(z) = \langle v_h, v_t \rangle^l \qquad V[x \mapsto v_h, xs \mapsto v_t], H, \vdash e_2 \Downarrow^{s_2} v_2, H_2}{V, H, \vdash \mathtt{case}\, z \,\{\mathtt{nil} \hookrightarrow e_1 \mid \mathtt{cons}(x; xs) \hookrightarrow e_2\} \Downarrow^{s_2} v_2, H_2}(\mathrm{S}_{16})$$

$$\frac{V, H \vdash e_1 \Downarrow^{s_1} v_1, H' \qquad V[x \mapsto v_1], H' \vdash e_2 \Downarrow^{s_2} v_2, H_2 \qquad loc(v_1) \in locs_{V,H}(e_2)}{V, H \vdash \mathtt{let}(e_1; x : \tau.e_2) \Downarrow^{s_1 + s_2} v_2, H_2}(\mathrm{S}_{17})$$

$$\frac{V, H \vdash e_1 \Downarrow^{s_1} v_1, H' \qquad V[x \mapsto v_1], H' \vdash e_2 \Downarrow^{s_2} v_2, H_2 \qquad loc(v_1) \notin locs_{V,H}(e_2)}{V, H \vdash \mathtt{let}(e_1; x : \tau.e_2) \Downarrow^{\max(s1, s2)} v_2, H_2}(\mathrm{S}_{18})$$