15-312 Assignment 1

Andrew Carnegie
(andrew)

September 4, 2017

1 Syntax

Type	τ ::=		
	`nat`	`nat`	naturals
	`unit`	`unit`	unit
	`bool`	`bool`	boolean
	$\texttt{prod}(\tau_1;\tau_2)$	$\tau_1 \times \tau_2$	product
	$\texttt{arr}(\tau_1;\tau_2)$	$\tau_1 \to \tau_2$	function
	$\texttt{list}(\tau)$	$\tau\,\texttt{list}$	list
Exp	e ::=		
	x	x	variable
	$\texttt{nat}[n]$	$\overline{n}$	number
	`unit`	$()$	unit
	`T`	`T`	true
	`F`	`F`	false
	$\texttt{if}(e;e_1;e_2)$	$\texttt{if}\,e\,\texttt{then}\,e_1\,\texttt{else}\,e_2$	if
	$\texttt{lam}(x:\tau.e)$	$\lambda\, x:\tau.e$	abstraction
	$\texttt{ap}(e_1;e_2)$	$e_1(e_2)$	application
	$\texttt{tpl}(e_1;e_2)$	$\langle e_1, e_2\rangle$	tuple
	$\texttt{fst}(e)$	$e\cdot\texttt{l}$	first projection
	$\texttt{snd}(e)$	$e\cdot\texttt{r}$	second projection
	`nil`	$[]$	nil
	$\texttt{cons}(e_1;e_2)$	$e_1 :: e_2$	cons
	$\texttt{case}\{e\}(e_1;x,xs.e_2)$	$\texttt{case}\,e\,\{\texttt{nil} \hookrightarrow e_1 \mid \texttt{cons}(x;xs) \hookrightarrow e_2\}$	match list
Val	v ::=		
	$\texttt{val}[l](n)$	n^l	numeric value
	$\texttt{val}[l](\texttt{T})$	$\texttt{T}^l$	true value
	$\texttt{val}[l](\texttt{F})$	$\texttt{F}^l$	false value
	$\texttt{val}[l](\texttt{Null})$	$\texttt{Null}^l$	null value
	$\texttt{val}[l](\texttt{cl}(V;x.e))$	$(V,x.e)^l$	function value
	$\texttt{val}[l_2](l_1)$	$l_1^{l_2}$	loc value
	$\texttt{val}[l](\texttt{pair}(v_1;v_2))$	$\langle v_1, v_2\rangle^l$	pair value
Loc	l ::=		
	$\texttt{loc}(l)$	l	location

2 Heap semantics

Model dynamics using judgement of the form:

$$\boxed{V, H, R \vdash e \Downarrow^s v, H'}$$

Where $V : VID \to Val$, $H : Loc \to Val$, and $R : \{Loc\}$. This can be read as: under stack V, heap H, and roots R, the expression e evaluates to v using maximum heap space s, and engenders a new heap H'.

Roots represents the set of locations required to compute the continuation *excluding* the current expression. We can think of roots as the heap allocations necessary to compute the context with a hole that will be filled by the current expression.

Below defines the size of reachable values and space for roots:

$$
\begin{aligned}
&reach_H(n^l) = \{l\} \\
&reach_H(\mathtt{T}^l) = \{l\} \\
&reach_H(\mathtt{F}^l) = \{l\} \\
&reach_H(\mathtt{Null}^l) = \{l\} \\
&reach_H((V, x.e)^l) = \{l\} \cup (\bigcup_{y \in FV(e) \setminus x} reach_H(V(y))) \\
&reach_H(l_1^{l_2}) = \{l_2\} \cup loc_H(H(l_1)) \\
&reach_H(\langle v_1, v_2 \rangle^l) = \{l\} \cup reach_H(v_1) \cup reach_H(v_2)
\end{aligned}
$$

$$loc_H(l) = \{l\} \cup reach_H(H(l))$$

$$space_H(R) = |\bigcup_{l \in R} loc_H(l)|$$

$$locs_{V,H}(e) = \bigcup_{x \in FV(e)} reach_H(V(x))$$

$$\frac{x \in dom(V)}{V, H, R \vdash x \Downarrow^{space_H(R \cup (reach_H(V(x))))} V(x), H}(\mathrm{S}_1) \qquad \frac{(l \text{ fresh}) \qquad H' = H[l \mapsto n^l]}{V, H, R \vdash \overline{n} \Downarrow^{space_{H'}(R \cup (\{l\})} n^l, H'}(\mathrm{S}_2)$$

$$\frac{(l \text{ fresh}) \qquad H' = H[l \mapsto \mathtt{T}^l]}{V, H, R \vdash \mathtt{T} \Downarrow^{space_{H'}(R \cup \{l\})} \mathtt{T}^l, H'}(\mathrm{S}_3) \qquad \frac{(l \text{ fresh}) \qquad H' = H[l \mapsto \mathtt{F}^l]}{V, H, R \vdash \mathtt{F} \Downarrow^{space_{H'}(R \cup \{l\})} \mathtt{F}^l, H'}(\mathrm{S}_4)$$

$$\frac{(l \text{ fresh}) \qquad H' = H[l \mapsto \mathtt{Null}^l]}{V, H, R \vdash () \Downarrow^{space_{H'}(R \cup \{l\})} \mathtt{Null}^l, H'}(\mathrm{S}_5)$$

$$\frac{V, H, R \cup locs_{V,H}(e_1) \cup locs_{V,H}(e_2) \vdash e \Downarrow^{s} \mathtt{T}^l, H' \qquad V, H', R \vdash e_1 \Downarrow^{s_1} v_1, H_1}{V, H, R \vdash \mathtt{if}(e; e_1; e_2) \Downarrow^{\max(s, s_1)} v_1, H_1}(\mathrm{S}_6)$$

$$\frac{V, H, R \cup locs_{V,H}(e_1) \cup locs_{V,H}(e_2) \vdash e \Downarrow^{s} \mathtt{F}^l, H' \qquad V, H', R \vdash e_2 \Downarrow^{s_2} v_2, H_2}{V, H, R \vdash \mathtt{if}(e; e_1; e_2) \Downarrow^{\max(s, s_2)} v_2, H_2}(\mathrm{S}_7)$$

$$\frac{(l \text{ fresh}) \qquad H' = H[l \mapsto (V, x.e)^l]}{V, H, R \vdash \mathtt{lam}(x : \tau.e) \Downarrow^{space_{H'}(R \cup \{l\})} (V, x.e)^l, H'}(\mathrm{S}_8)$$

$$\frac{\begin{array}{c} V, H, R \cup locs_{V,H}(e_1) \vdash e_2 \Downarrow^{s_2} v_2, H_2 \\ V, H_2, R \cup reach_{H_2}(v_2) \vdash e_1 \Downarrow^{s_1} v_1, H_1 \qquad v_1 = (V_1, x.e)^{l_1} \qquad V_1[x \mapsto v_2], H_1, R \vdash e \Downarrow^{s} v, H' \end{array}}{V, H, R \vdash e_1(e_2) \Downarrow^{\max(s_2, s_1, s)} v, H'}(\mathrm{S}_9)$$

$$\frac{\begin{array}{c} V, H, R \cup locs_{V,H}(e_2) \vdash e_1 \Downarrow^{s_1} v_1, H_1 \\ V, H_1, R \cup reach_{H_1}(v_1) \vdash e_2 \Downarrow^{s_2} v_2, H_2 \qquad (l \text{ fresh}) \qquad H' = H_2[l \mapsto \langle v_1, v_2 \rangle^l] \end{array}}{V, H, R \vdash \langle e_1, e_2 \rangle \Downarrow^{\max(s_1, s_2)+1} \langle v_1, v_2 \rangle^l, H'}(\mathrm{S}_{10})$$

$$\frac{V, H, R \vdash e \Downarrow^{s} \langle v_1, v_2 \rangle^l, H'}{V, H, R \vdash e \cdot \mathtt{l} \Downarrow^{s} v_1, H'}(\mathrm{S}_{11}) \qquad \frac{V, H, R \vdash e \Downarrow^{s} \langle v_1, v_2 \rangle^l, H'}{V, H, R \vdash e \cdot \mathtt{r} \Downarrow^{s} v_2, H'}(\mathrm{S}_{12})$$

$$\frac{(l \text{ fresh}) \qquad H' = H[l \mapsto \mathtt{Null}^l]}{V, H, R \vdash \mathtt{nil} \Downarrow^{space_{H'}(R \cup \{l\})} \mathtt{Null}^l, H'}(\mathrm{S}_{13})$$

$$\frac{\begin{array}{c} V, H, R \cup locs_{V,H}(e_2) \vdash e_1 \Downarrow^{s_1} v_1, H_1 \\ V, H_1, R \cup reach_{H_1}(v_1) \vdash e_2 \Downarrow^{s_2} v_2, H_2 \qquad (l \text{ fresh}) \qquad H' = H_2[l \mapsto \langle v_1, v_2 \rangle^l] \end{array}}{V, H, R \vdash \mathtt{cons}(e_1; e_2) \Downarrow^{\max(s_1, s_2)+1} \langle v_1, v_2 \rangle^l, H'}(\mathrm{S}_{14})$$

$$\frac{V, H, R \cup locs_{V,H}(e_1) \cup locs_{V,H}(e_2) \vdash e \Downarrow^{s} \mathtt{Null}^l, H' \qquad V, H', R \vdash e_1 \Downarrow^{s_1} v_1, H_1}{V, H, R \vdash \mathtt{case}\, e\, \{\mathtt{nil} \hookrightarrow e_1 \mid \mathtt{cons}(x; xs) \hookrightarrow e_2\} \Downarrow^{\max(s, s_1)} v_1, H_1}(\mathrm{S}_{15})$$

$$\frac{\begin{array}{c} V, H, R \cup locs_{V,H}(e_1) \cup locs_{V,H}(e_2) \vdash e \Downarrow^{s} \langle v_h, v_t \rangle^l, H' \\ V[x \mapsto v_h, xs \mapsto v_t], H', R \vdash e_2 \Downarrow^{s_2} v_2, H_2 \end{array}}{V, H, R \vdash \mathtt{case}\, e\, \{\mathtt{nil} \hookrightarrow e_1 \mid \mathtt{cons}(x; xs) \hookrightarrow e_2\} \Downarrow^{\max(s, s_2)} v_2, H_2}(\mathrm{S}_{16})$$