

Image Classification

A Core Task in Computer Vision

Today:

- The image classification task
- Two basic data-driven approaches to image classification
 - K-nearest neighbor and linear classifier

Image Classification: A core task in Computer Vision



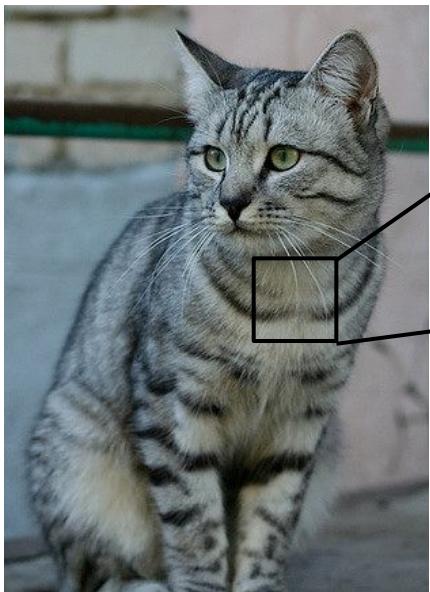
(assume given a set of possible labels)
{dog, cat, truck, plane, ...}



cat

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The Problem: Semantic Gap



[105 112 108 111 104 99 106 99 96 103 112 119 104 97 93 87]
[91 98 102 106 104 79 98 103 99 105 123 136 110 105 94 85]
[76 85 90 105 128 105 87 96 95 99 115 112 106 103 99 85]
[99 81 93 120 131 127 100 95 98 102 99 96 93 101 94]
[106 91 61 64 69 91 88 85 101 107 109 98 75 84 96 95]
[114 108 85 55 55 69 64 54 64 87 112 129 98 74 84 91]
[133 137 147 103 65 81 80 65 52 54 74 84 102 93 85 82]
[128 137 144 140 109 95 86 70 62 65 63 63 60 73 86 101]
[125 133 148 137 119 121 117 94 65 79 80 65 54 64 72 98]
[127 125 131 147 133 127 126 131 111 96 89 75 61 64 72 84]
[115 114 109 123 150 148 131 118 113 109 100 92 74 65 72 78]
[89 93 90 97 108 147 131 118 113 114 113 109 106 95 77 80]
[63 77 86 81 77 79 102 123 117 115 117 125 125 130 115 87]
[62 65 82 89 78 71 80 101 124 126 119 101 107 114 131 119]
[63 65 75 88 89 71 62 81 128 138 135 105 81 98 110 118]
[87 65 71 87 106 95 69 45 76 130 126 107 92 94 105 112]
[118 97 82 86 117 123 116 66 41 51 95 93 89 95 102 107]
[164 146 112 88 82 120 124 104 76 48 45 66 88 101 102 109]
[157 170 157 128 93 86 114 132 112 97 69 55 70 82 99 94]
[130 128 134 161 139 100 109 118 121 134 114 87 65 53 69 86]
[128 112 96 117 150 144 128 115 104 107 102 93 87 81 72 79]
[123 107 96 86 83 112 153 149 122 109 104 75 80 107 112 99]
[122 121 102 88 82 86 94 117 145 148 153 102 58 78 92 107]
[122 164 148 103 71 56 78 83 93 103 119 139 102 61 69 84]]

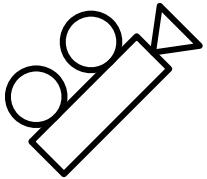
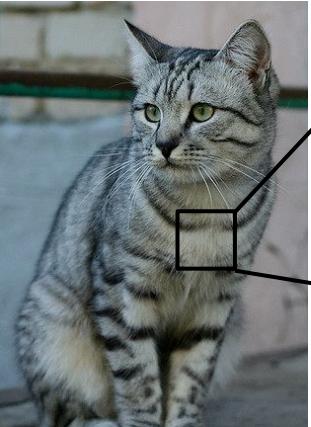
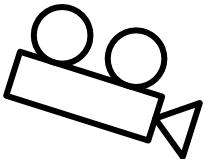
What the computer sees

An image is a tensor of integers between [0, 255]:

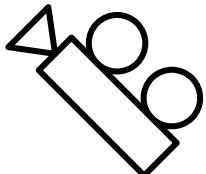
e.g. 800 x 600 x 3
(3 channels RGB)

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Challenges: Viewpoint variation



```
[1185 112 188 111 184 99 186 99 96 183 112 119 184 97 93 87]  
[ 91 98 182 106 184 79 98 183 99 185 123 136 118 185 94 85]  
[ 76 85 98 185 128 105 87 96 95 99 115 112 106 183 99 85]  
[ 99 80 81 100 128 103 127 98 98 101 108 109 98 75 84 94]  
[104 91 86 84 60 91 68 85 101 108 109 98 75 84 94 95]  
[114 108 85 55 55 69 64 54 64 87 112 129 98 74 84 94 91]  
[133 137 147 183 65 81 80 65 52 54 74 84 102 93 85 82]  
[128 137 144 148 105 95 86 78 62 65 63 68 68 73 86 101]  
[102 125 131 147 133 127 116 131 111 98 89 75 61 64 72 80]  
[127 125 131 147 133 127 116 131 111 98 89 75 61 64 72 84]  
[115 115 189 123 150 148 131 118 113 109 108 92 74 65 72 78]  
[ 89 93 98 97 108 147 131 118 113 114 113 108 106 95 77 80]  
[ 63 77 86 81 77 79 182 123 137 115 111 125 125 130 115 87]  
[ 62 85 88 89 73 62 81 128 138 135 105 81 98 118 118]  
[ 63 65 75 88 89 73 62 81 128 138 135 105 81 98 118 118]  
[ 87 65 71 87 100 95 69 45 76 126 126 107 92 94 105 112]  
[118 97 82 86 117 123 116 66 41 51 95 93 89 89 95 102 107]  
[164 147 112 88 100 128 126 184 78 48 66 70 101 102 108]  
[157 98 100 118 103 92 86 104 105 108 109 95 70 84 85 94]  
[138 128 134 161 139 180 109 118 121 134 114 87 65 53 69 86]  
[128 112 96 117 150 144 120 115 104 107 102 93 87 81 72 79]  
[123 107 96 86 83 112 153 149 122 189 104 75 88 107 112 99]  
[122 121 102 88 82 86 94 117 145 148 153 105 58 78 92 107]  
[122 164 148 103 71 56 78 83 93 103 119 139 102 61 69 84]
```



All pixels change when
the camera moves!

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Challenges: Illumination



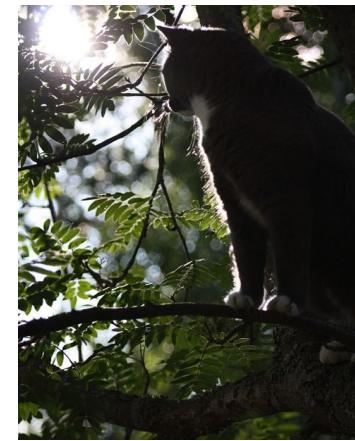
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Challenges: Background Clutter



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Challenges: Occlusion



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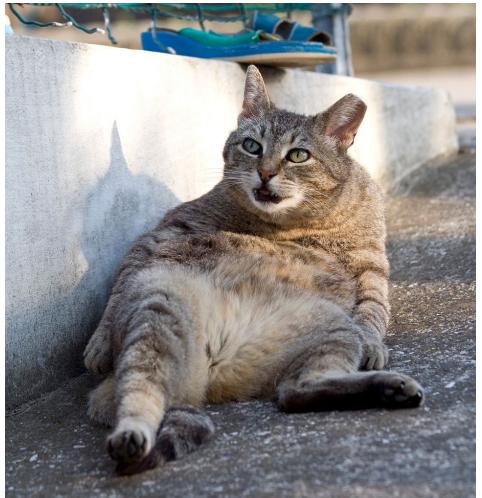


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Challenges: Deformation



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Challenges: Intraclass variation



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Challenges: Context

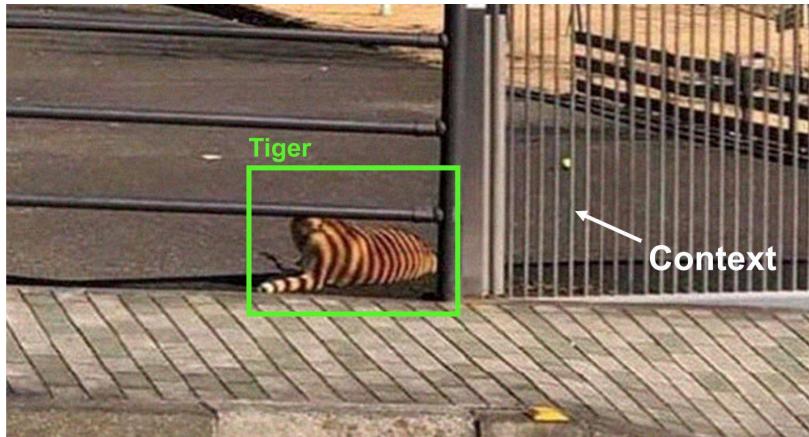
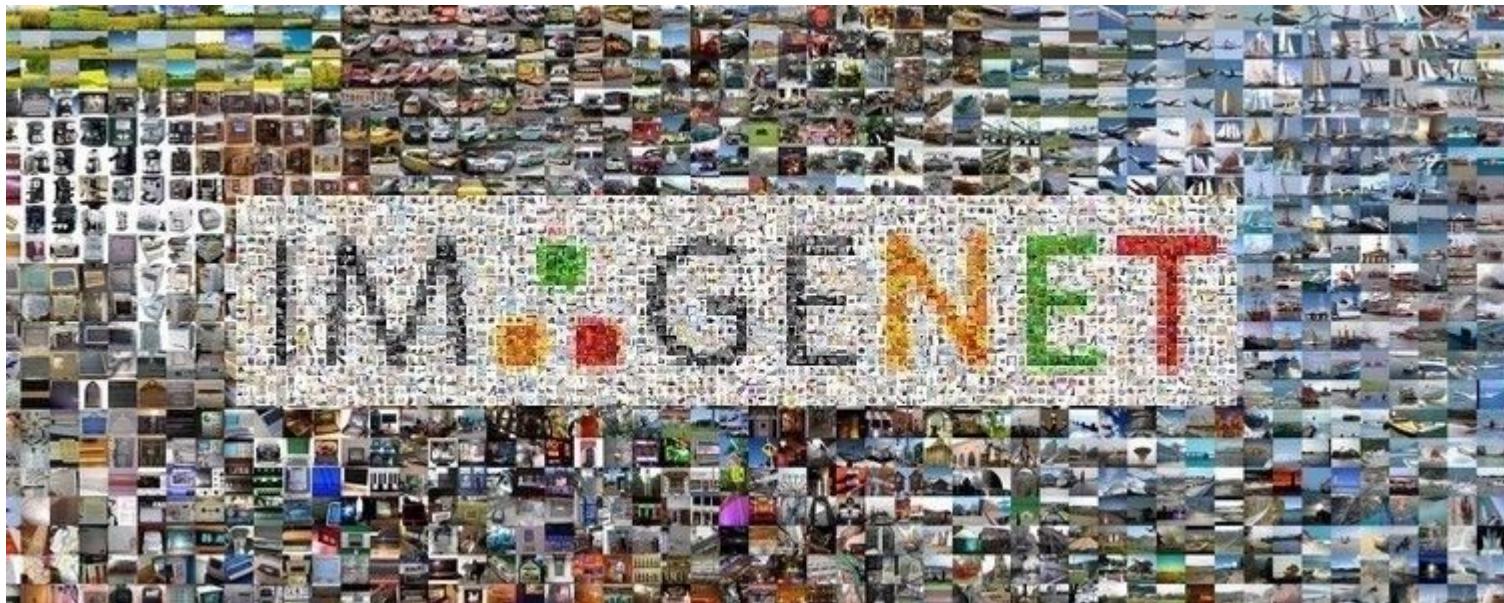


Image source:

https://www.linkedin.com/posts/ralph-aboujaoude-diaz-40838313_technology-artificialintelligence-computervision-activity-6912446088364875776-h-lq/?utm_source=linkedin_share&utm_medium=member_desktop_web

Modern computer vision algorithms



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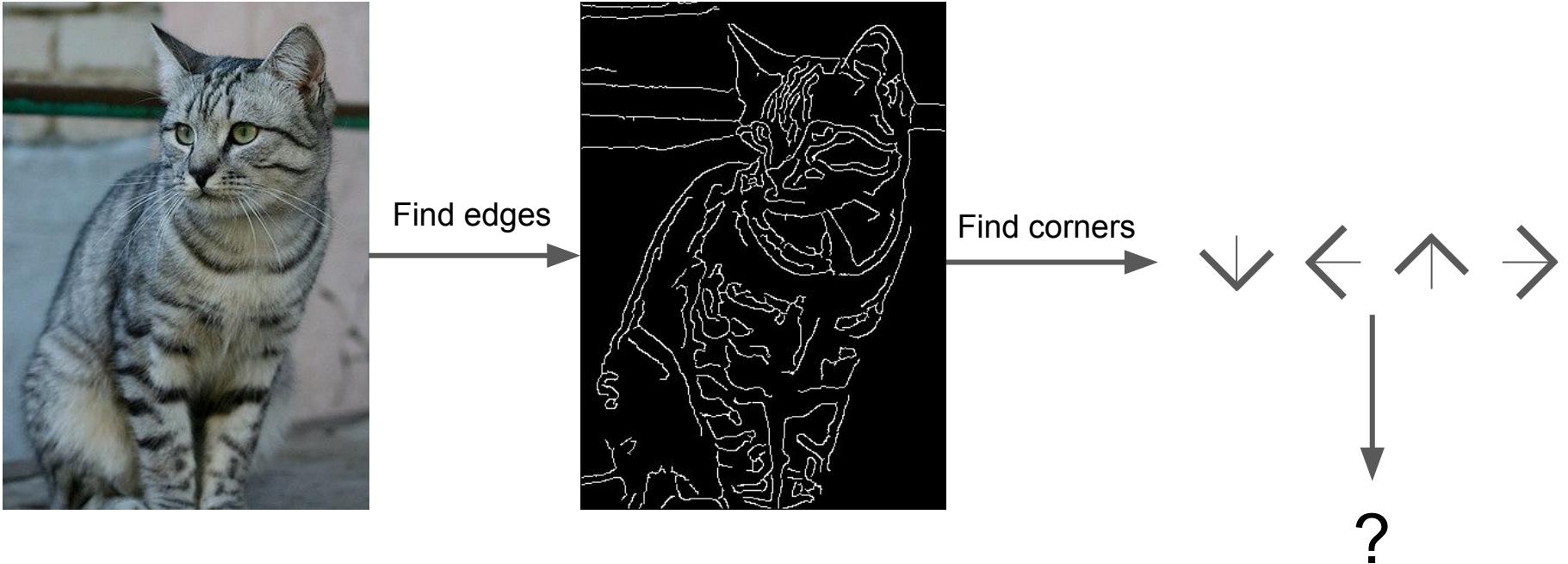
An image classifier

```
def classify_image(image):  
    # Some magic here?  
    return class_label
```

Unlike e.g. sorting a list of numbers,

no obvious way to hard-code the algorithm for
recognizing a cat, or other classes.

Attempts have been made



John Canny, "A Computational Approach to Edge Detection", IEEE TPAMI 1986

Machine Learning: Data-Driven Approach

1. Collect a dataset of images and labels
2. Use Machine Learning algorithms to train a classifier
3. Evaluate the classifier on new images

Example training set

```
def train(images, labels):  
    # Machine learning!  
    return model
```

```
def predict(model, test_images):  
    # Use model to predict labels  
    return test_labels
```

airplane



automobile



bird



cat



deer



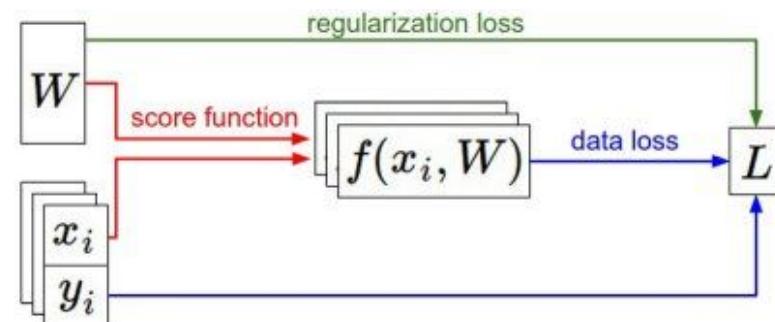
Recap

- We have some dataset of (x, y)
- We have a **score function**: $s = f(x; W) = Wx$ e.g.
- We have a **loss function**:

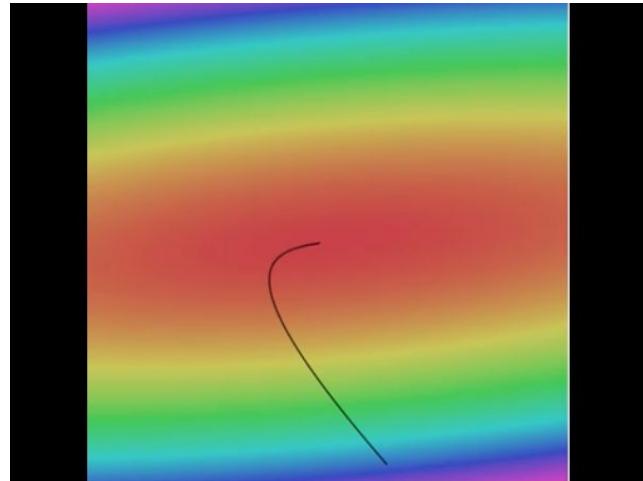
$$L_i = -\log\left(\frac{e^{sy_i}}{\sum_j e^{sj}}\right) \text{ Softmax}$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1) \text{ SVM}$$

$$L = \frac{1}{N} \sum_{i=1}^N L_i + R(W) \text{ Full loss}$$



Finding the best W: Optimize with Gradient Descent



```
# Vanilla Gradient Descent

while True:
    weights_grad = evaluate_gradient(loss_fun, data, weights)
    weights += - step_size * weights_grad # perform parameter update
```

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Walking man image is [CC0 1.0](#) public domain

Gradient descent

$$\frac{df(x)}{dx} = \lim_{h \rightarrow 0} \frac{f(x + h) - f(x)}{h}$$

Numerical gradient: slow :, approximate :, easy to write :)

Analytic gradient: fast :), exact :), error-prone :(

In practice: Derive analytic gradient, check your implementation with numerical gradient

Stochastic Gradient Descent (SGD)

$$L(W) = \frac{1}{N} \sum_{i=1}^N L_i(x_i, y_i, W) + \lambda R(W)$$

$$\nabla_W L(W) = \frac{1}{N} \sum_{i=1}^N \nabla_W L_i(x_i, y_i, W) + \lambda \nabla_W R(W)$$

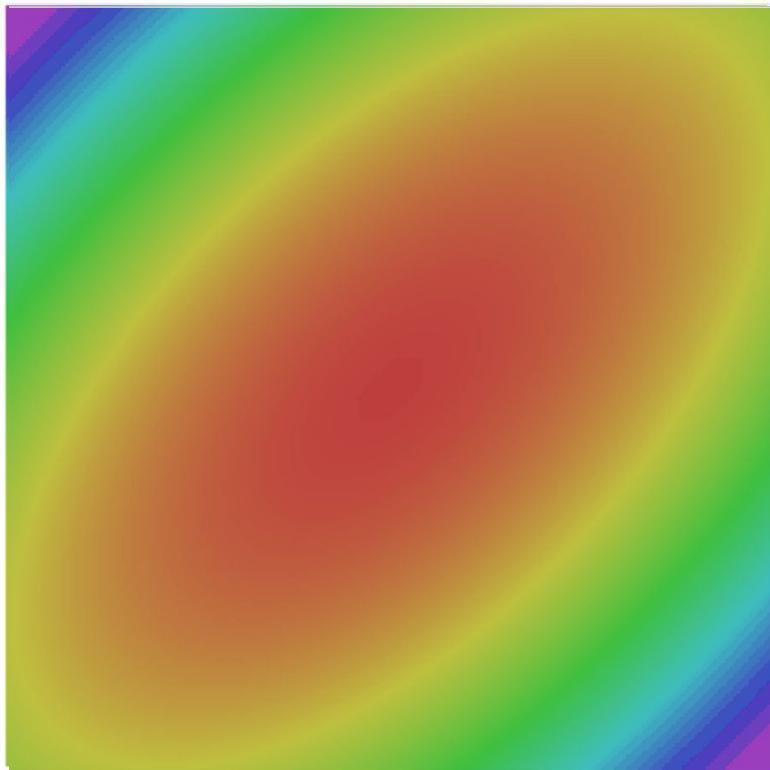
Full sum expensive
when N is large!

Approximate sum
using a **minibatch** of
examples
32 / 64 / 128 common

```
# Vanilla Minibatch Gradient Descent
```

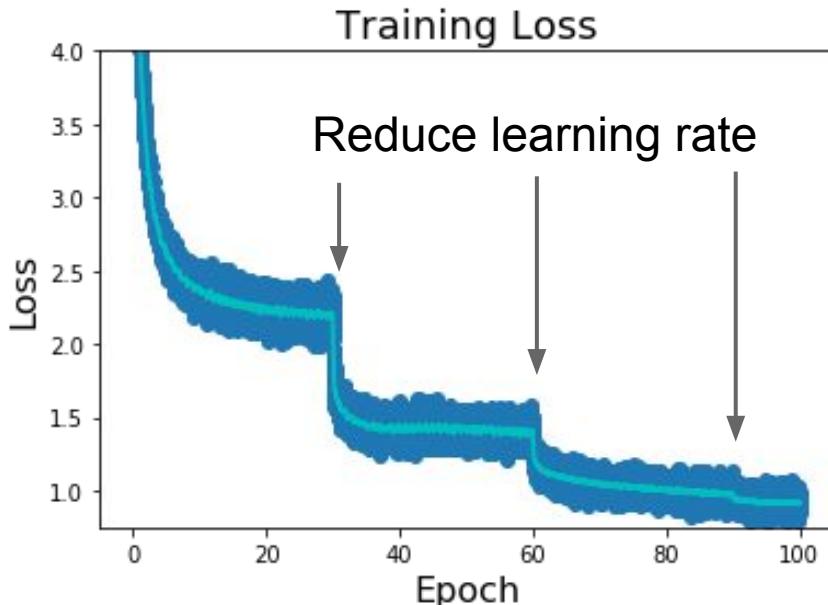
```
while True:  
    data_batch = sample_training_data(data, 256) # sample 256 examples  
    weights_grad = evaluate_gradient(loss_fun, data_batch, weights)  
    weights += - step_size * weights_grad # perform parameter update
```

Last time: fancy optimizers



- SGD
- SGD+Momentum
- RMSProp
- Adam

Last time: learning rate scheduling



Step: Reduce learning rate at a few fixed points. E.g. for ResNets, multiply LR by 0.1 after epochs 30, 60, and 90.

Cosine: $\alpha_t = \frac{1}{2}\alpha_0 (1 + \cos(t\pi/T))$

Linear: $\alpha_t = \alpha_0(1 - t/T)$

Inverse sqrt: $\alpha_t = \alpha_0/\sqrt{t}$

α_0 : Initial learning rate

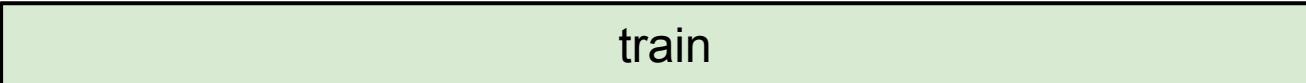
α_t : Learning rate at epoch t

T : Total number of epochs

Setting Hyperparameters

Idea #1: Choose hyperparameters that work best on the **training data**

BAD: $K = 1$ always works perfectly on training data



train

Idea #2: choose hyperparameters that work best on **test data**

BAD: No idea how algorithm will perform on new data



train

test

Idea #3: Split data into **train**, **val**; choose hyperparameters on val and evaluate on test

Better!



train

validation

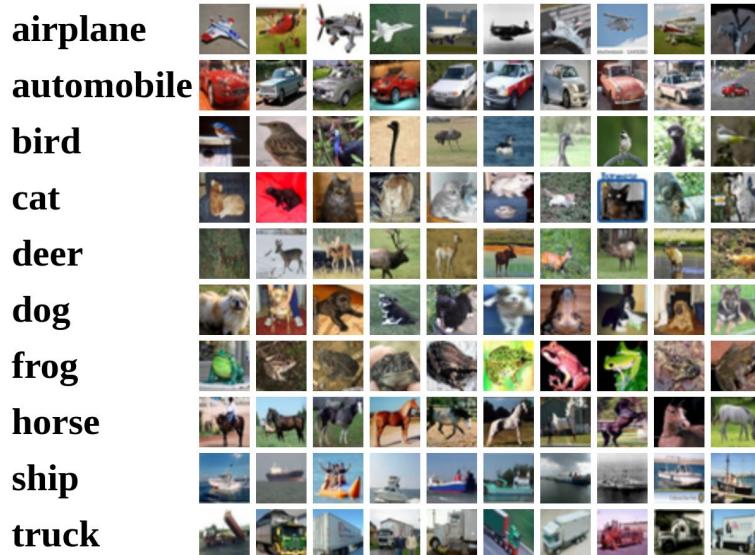
test

Example Dataset: CIFAR10

10 classes

50,000 training images

10,000 testing images



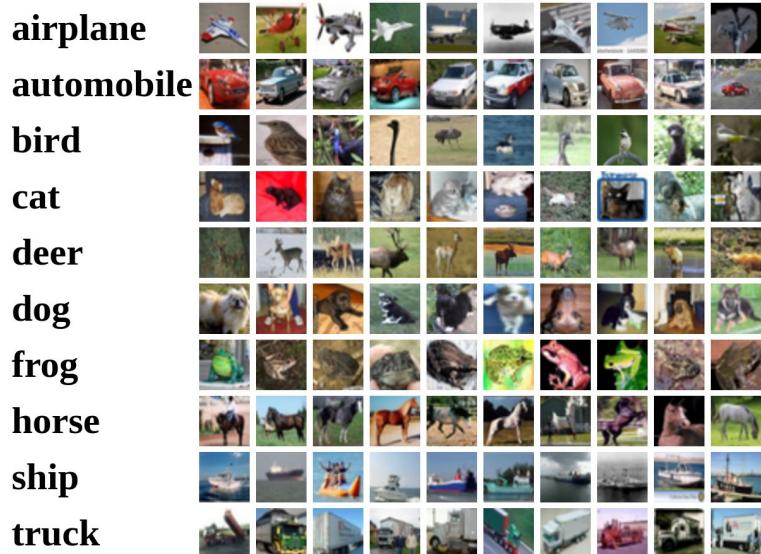
Alex Krizhevsky, "Learning Multiple Layers of Features from Tiny Images", Technical Report, 2009.

Example Dataset: CIFAR10

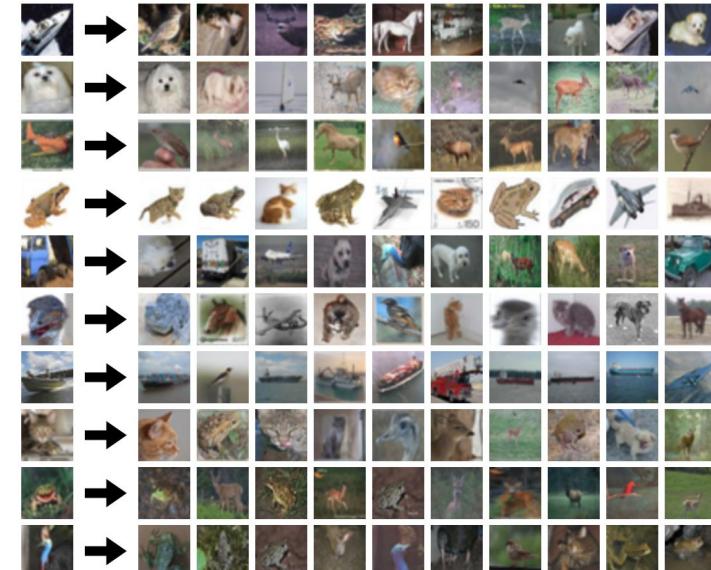
10 classes

50,000 training images

10,000 testing images



Test images and nearest neighbors



Alex Krizhevsky, "Learning Multiple Layers of Features from Tiny Images", Technical Report, 2009.

What does this look like?



What does this look like?



Neural Networks

Neural networks: the original linear classifier

(Before) Linear score function: $f = Wx$

$$x \in \mathbb{R}^D, W \in \mathbb{R}^{C \times D}$$

Neural networks: 2 layers

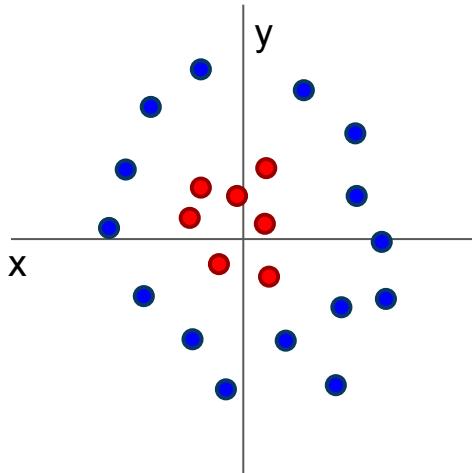
(Before) Linear score function: $f = Wx$

(Now) 2-layer Neural Network $f = W_2 \max(0, W_1 x)$

$$x \in \mathbb{R}^D, W_1 \in \mathbb{R}^{H \times D}, W_2 \in \mathbb{R}^{C \times H}$$

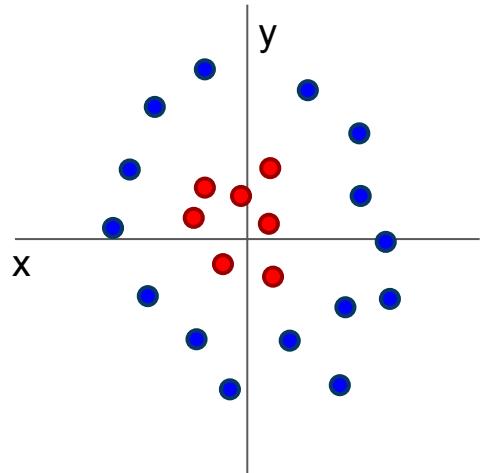
(In practice we will usually add a learnable bias at each layer as well)

Why do we want non-linearity?



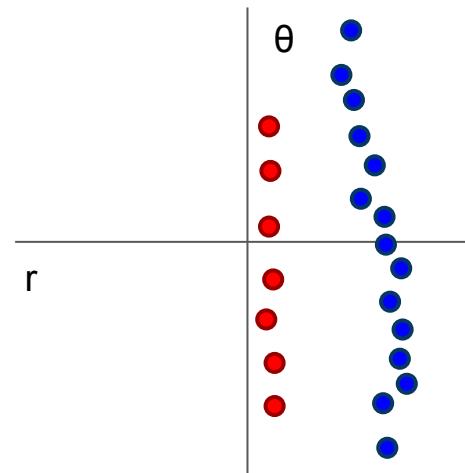
Cannot separate red
and blue points with
linear classifier

Why do we want non-linearity?



Cannot separate red
and blue points with
linear classifier

$$f(x, y) = (r(x, y), \theta(x, y))$$



After applying feature
transform, points can
be separated by linear
classifier

Neural networks: also called fully connected network

(Before) Linear score function: $f = Wx$

(Now) 2-layer Neural Network $f = W_2 \max(0, W_1 x)$

$$x \in \mathbb{R}^D, W_1 \in \mathbb{R}^{H \times D}, W_2 \in \mathbb{R}^{C \times H}$$

“Neural Network” is a very broad term; these are more accurately called “fully-connected networks” or sometimes “multi-layer perceptrons” (MLP)

(In practice we will usually add a learnable bias at each layer as well)

Neural networks: 3 layers

(Before) Linear score function: $f = Wx$

(Now) 2-layer Neural Network $f = W_2 \max(0, W_1 x)$
or 3-layer Neural Network
$$f = W_3 \max(0, W_2 \max(0, W_1 x))$$

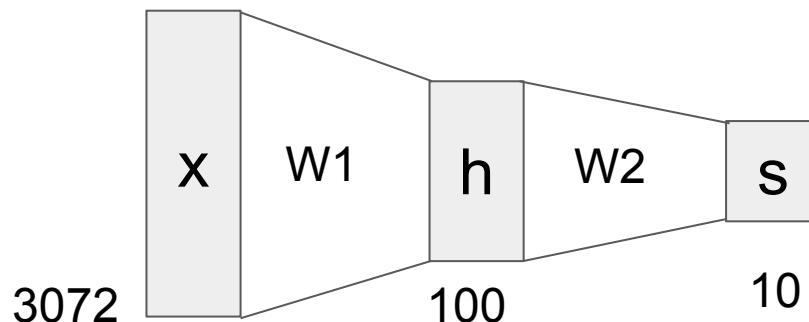
$$x \in \mathbb{R}^D, W_1 \in \mathbb{R}^{H_1 \times D}, W_2 \in \mathbb{R}^{H_2 \times H_1}, W_3 \in \mathbb{R}^{C \times H_2}$$

(In practice we will usually add a learnable bias at each layer as well)

Neural networks: hierarchical computation

(Before) Linear score function: $f = Wx$

(Now) 2-layer Neural Network $f = W_2 \max(0, W_1 x)$

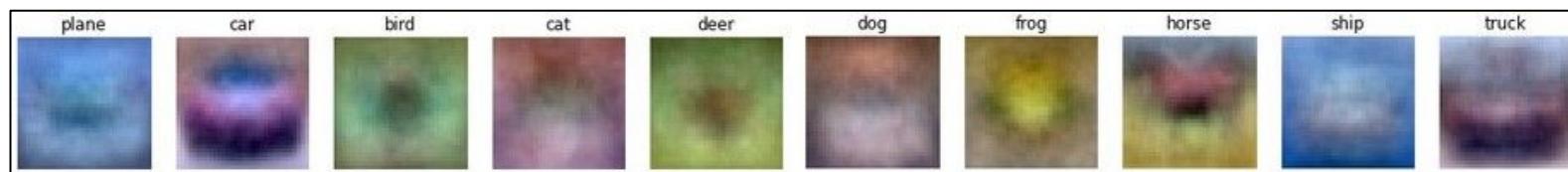
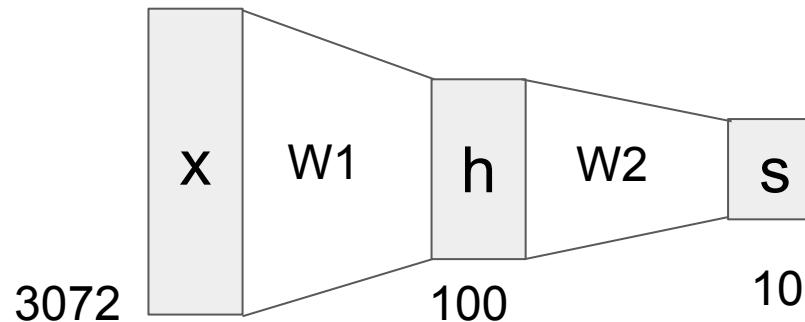


$$x \in \mathbb{R}^D, W_1 \in \mathbb{R}^{H \times D}, W_2 \in \mathbb{R}^{C \times H}$$

Neural networks: learning 100s of templates

(Before) Linear score function: $f = Wx$

(Now) 2-layer Neural Network $f = W_2 \max(0, W_1 x)$



Learn 100 templates instead of 10.

Share templates between classes

Neural networks: why is max operator important?

(Before) Linear score function: $f = Wx$

(Now) 2-layer Neural Network $f = W_2 \max(0, W_1 x)$

The function $\max(0, z)$ is called the **activation function**.

Q: What if we try to build a neural network without one?

$$f = W_2 W_1 x$$

Neural networks: why is max operator important?

(Before) Linear score function: $f = Wx$

(Now) 2-layer Neural Network $f = W_2 \max(0, W_1 x)$

The function $\max(0, z)$ is called the **activation function**.

Q: What if we try to build a neural network without one?

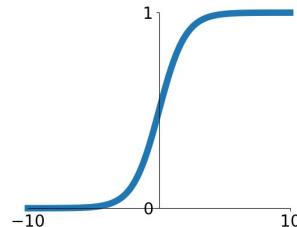
$$f = W_2 W_1 x \quad W_3 = W_2 W_1 \in \mathbb{R}^{C \times H}, f = W_3 x$$

A: We end up with a linear classifier again!

Activation functions

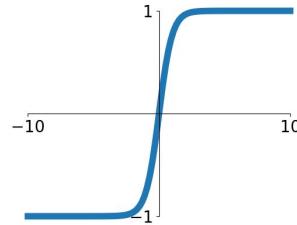
Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



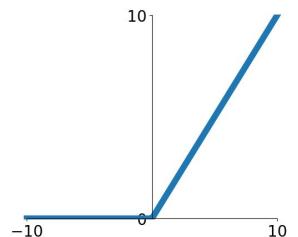
tanh

$$\tanh(x)$$



ReLU

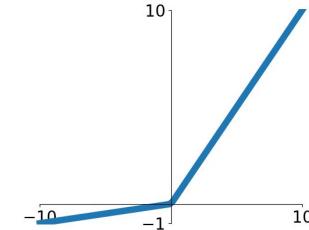
$$\max(0, x)$$



ReLU is a good default choice for most problems

Leaky ReLU

$$\max(0.1x, x)$$

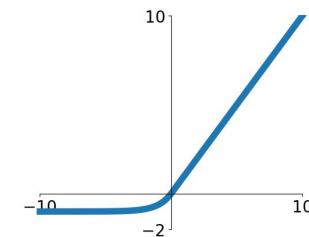


Maxout

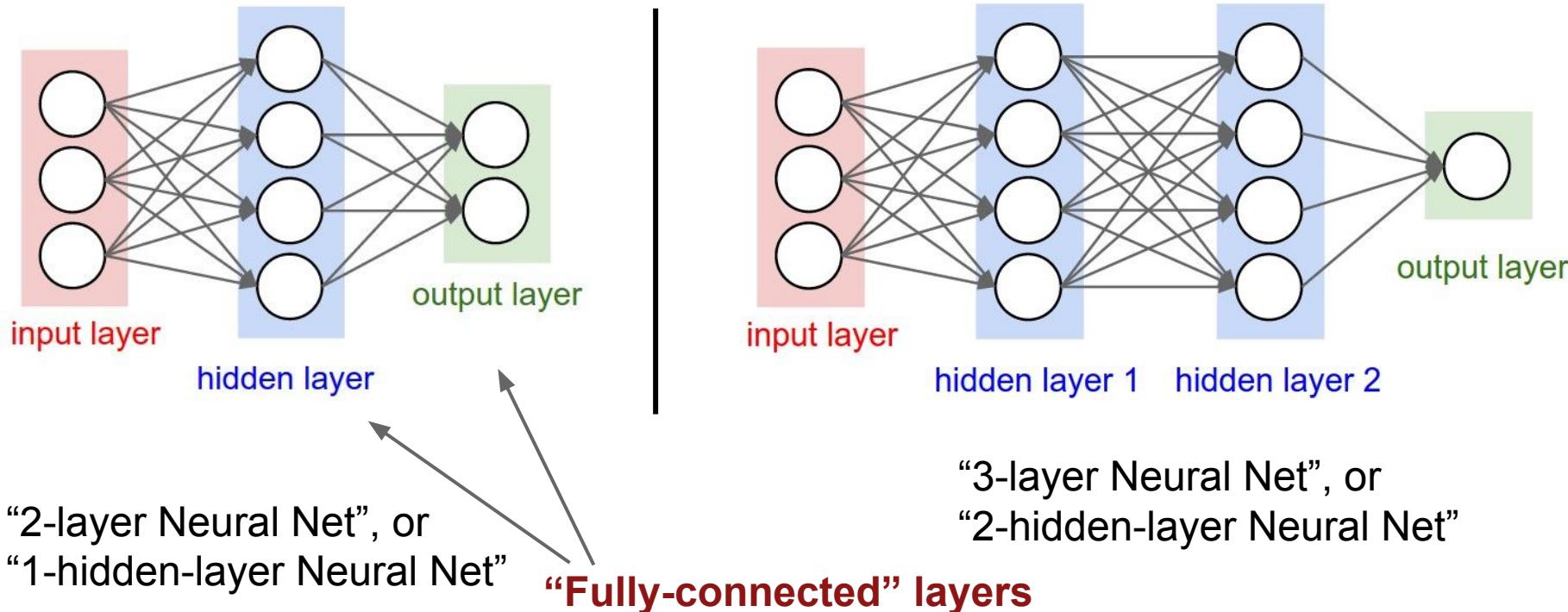
$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

ELU

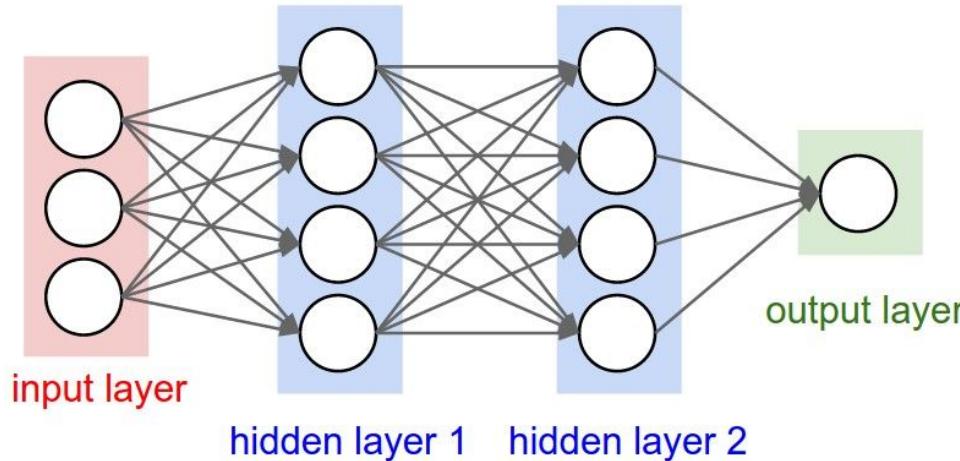
$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



Neural networks: Architectures



Example feed-forward computation of a neural network



```
# forward-pass of a 3-layer neural network:  
f = lambda x: 1.0/(1.0 + np.exp(-x)) # activation function (use sigmoid)  
x = np.random.randn(3, 1) # random input vector of three numbers (3x1)  
h1 = f(np.dot(W1, x) + b1) # calculate first hidden layer activations (4x1)  
h2 = f(np.dot(W2, h1) + b2) # calculate second hidden layer activations (4x1)  
out = np.dot(W3, h2) + b3 # output neuron (1x1)
```

Full implementation of training a 2-layer Neural Network needs ~20 lines:

```
1 import numpy as np
2 from numpy.random import randn
3
4 N, D_in, H, D_out = 64, 1000, 100, 10
5 x, y = randn(N, D_in), randn(N, D_out)
6 w1, w2 = randn(D_in, H), randn(H, D_out)
7
8 for t in range(2000):
9     h = 1 / (1 + np.exp(-x.dot(w1)))
10    y_pred = h.dot(w2)
11    loss = np.square(y_pred - y).sum()
12    print(t, loss)
13
14    grad_y_pred = 2.0 * (y_pred - y)
15    grad_w2 = h.T.dot(grad_y_pred)
16    grad_h = grad_y_pred.dot(w2.T)
17    grad_w1 = x.T.dot(grad_h * h * (1 - h))
18
19    w1 -= 1e-4 * grad_w1
20    w2 -= 1e-4 * grad_w2
```

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Define the network

Full implementation of training a 2-layer Neural Network needs ~20 lines:

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6 w1, w2 = randn(D_in, H), randn(H, D_out)
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8 for t in range(2000):
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19    w1 -= 1e-4 * grad_w1
20    w2 -= 1e-4 * grad_w2
```

Define the network

Forward pass

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5 x, y = randn(N, D_in), randn(N, D_out)
6 w1, w2 = randn(D_in, H), randn(H, D_out)
7
8 for t in range(2000):
9     h = 1 / (1 + np.exp(-x.dot(w1)))
10    y_pred = h.dot(w2)
11    loss = np.square(y_pred - y).sum()
12    print(t, loss)
13
14    grad_y_pred = 2.0 * (y_pred - y)
15    grad_w2 = h.T.dot(grad_y_pred)
16    grad_h = grad_y_pred.dot(w2.T)
17    grad_w1 = x.T.dot(grad_h * h * (1 - h))
18
19    w1 -= 1e-4 * grad_w1
20    w2 -= 1e-4 * grad_w2
```

Define the network

Forward pass

Calculate the analytical gradients

Full implementation of training a 2-layer Neural Network needs ~20 lines:

```
1 import numpy as np
2 from numpy.random import randn
3
4 N, D_in, H, D_out = 64, 1000, 100, 10
5 x, y = randn(N, D_in), randn(N, D_out)
6 w1, w2 = randn(D_in, H), randn(H, D_out)
7
8 for t in range(2000):
9     h = 1 / (1 + np.exp(-x.dot(w1)))
10    y_pred = h.dot(w2)
11    loss = np.square(y_pred - y).sum()
12    print(t, loss)
13
14    grad_y_pred = 2.0 * (y_pred - y)
15    grad_w2 = h.T.dot(grad_y_pred)
16    grad_h = grad_y_pred.dot(w2.T)
17    grad_w1 = x.T.dot(grad_h * h * (1 - h))
18
19    w1 -= 1e-4 * grad_w1
20    w2 -= 1e-4 * grad_w2
```

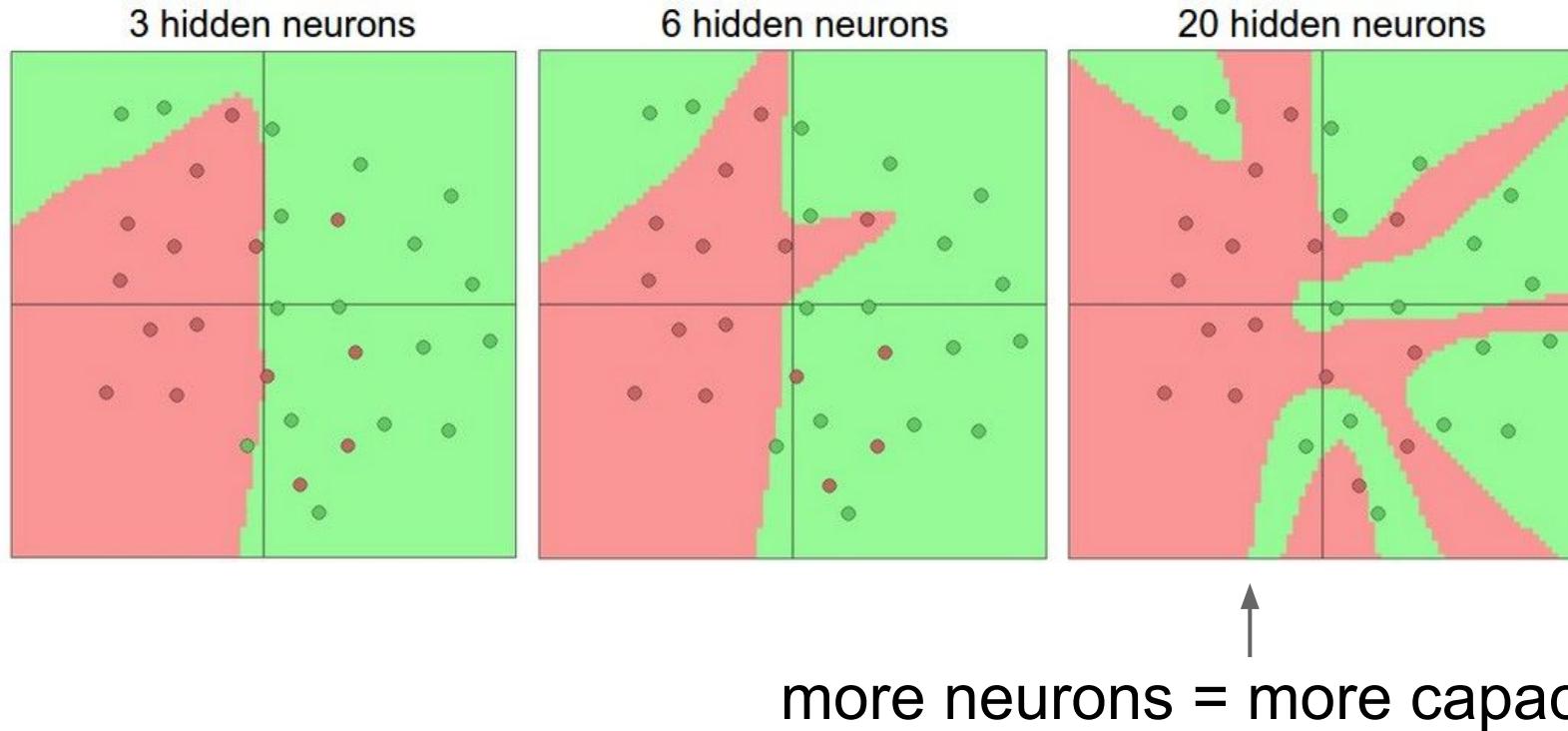
Define the network

Forward pass

Calculate the analytical gradients

Gradient descent

Setting the number of layers and their sizes

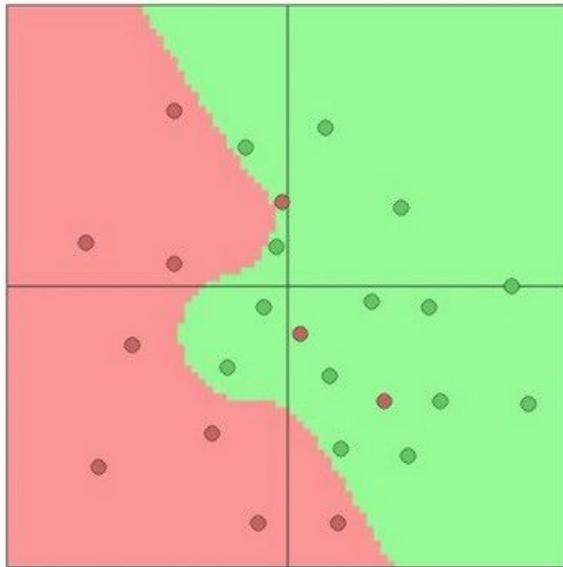


Do not use size of neural network as a regularizer. Use stronger regularization instead:

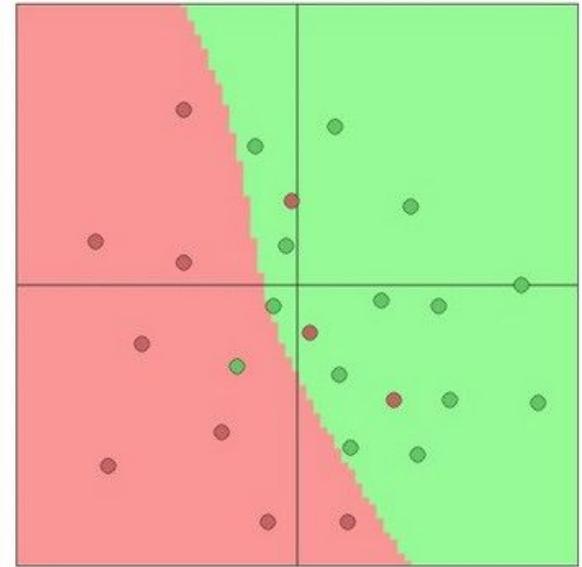
$\lambda = 0.001$



$\lambda = 0.01$



$\lambda = 0.1$



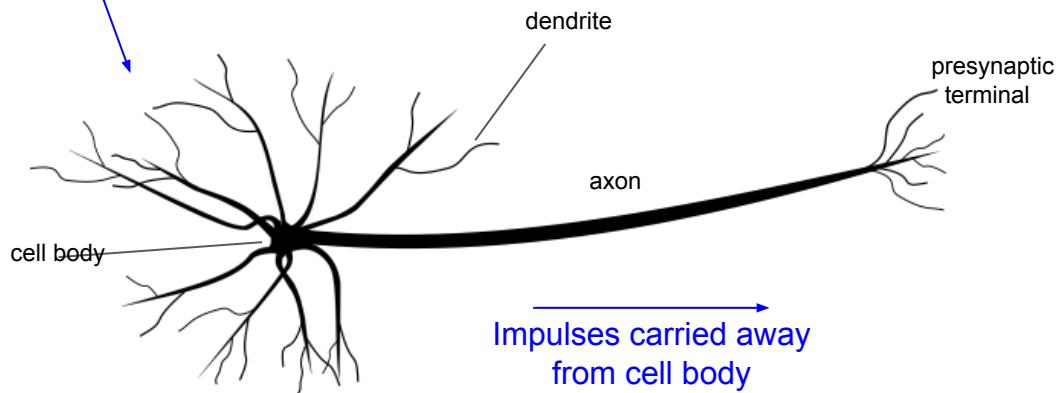
(Web demo with ConvNetJS:
[http://cs.stanford.edu/people/karpathy/convnetjs/demo
/classify2d.html](http://cs.stanford.edu/people/karpathy/convnetjs/demo/classify2d.html))

$$L(W) = \frac{1}{N} \sum_{i=1}^N L_i(f(x_i, W), y_i) + \lambda R(W)$$



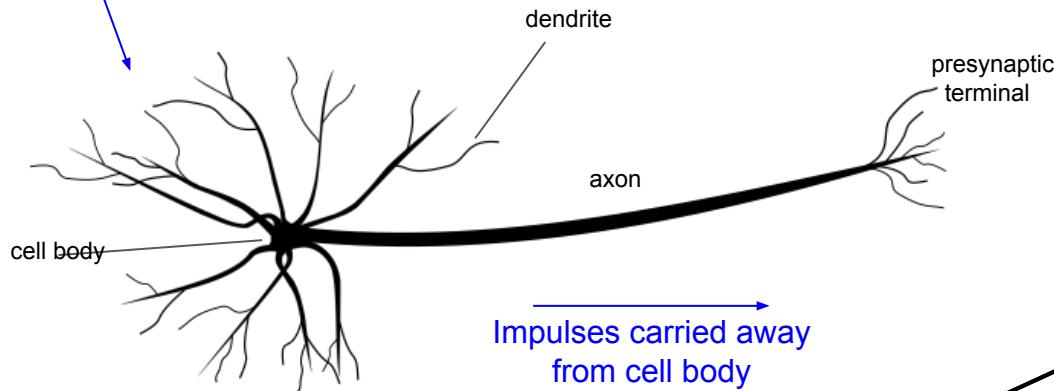
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Impulses carried toward cell body



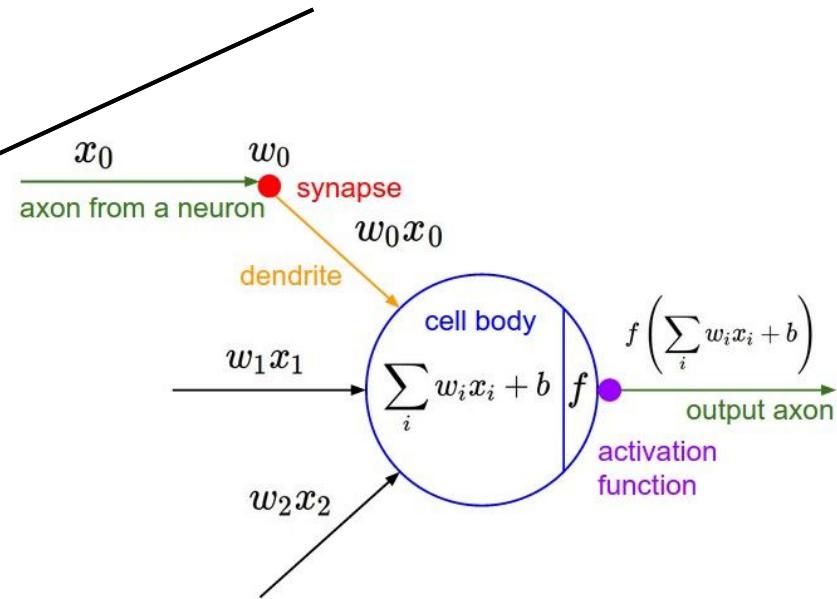
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Impulses carried toward cell body

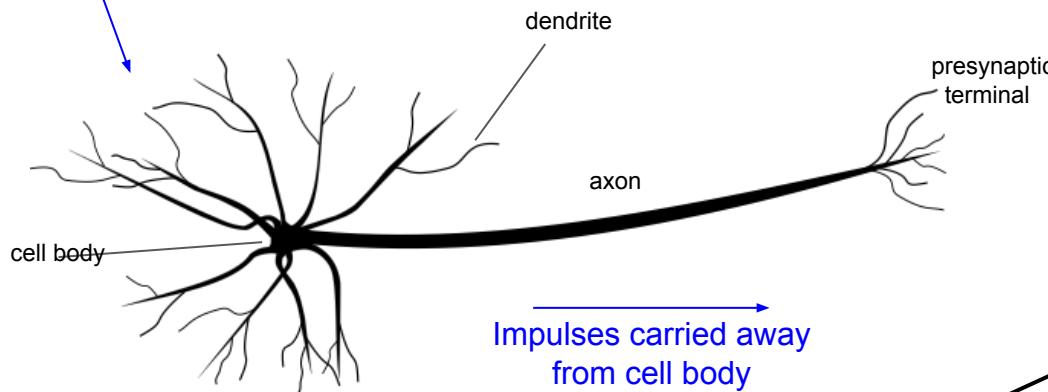


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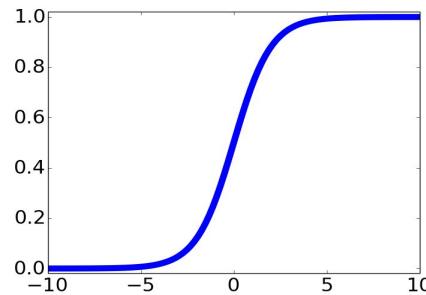
Impulses carried away
from cell body



Impulses carried toward cell body

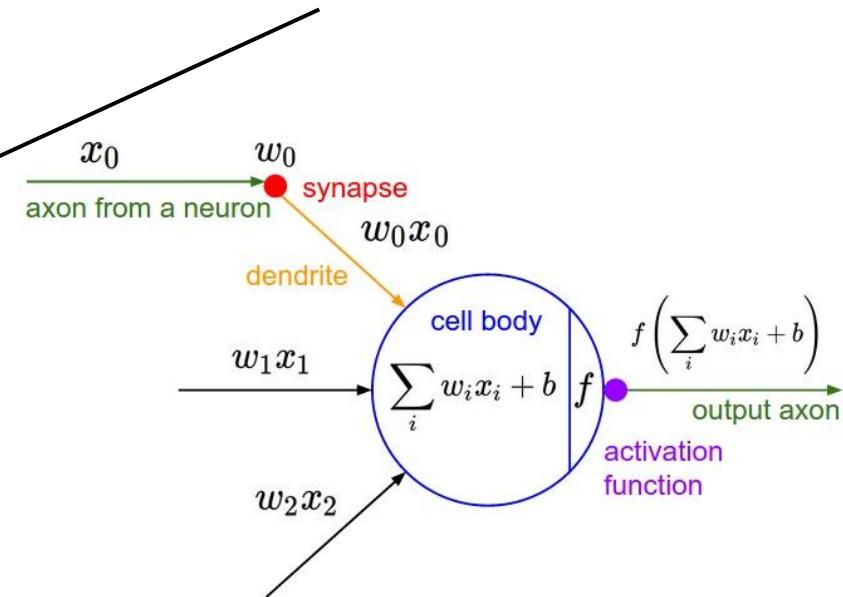


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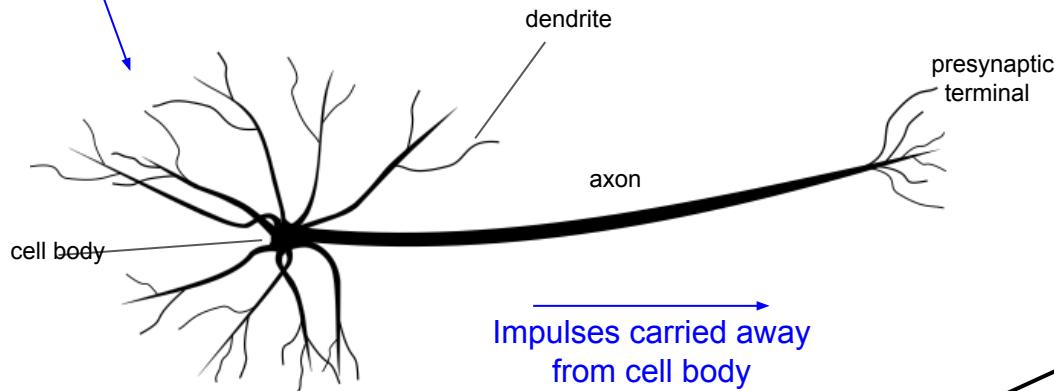


sigmoid activation function

$$\frac{1}{1 + e^{-x}}$$

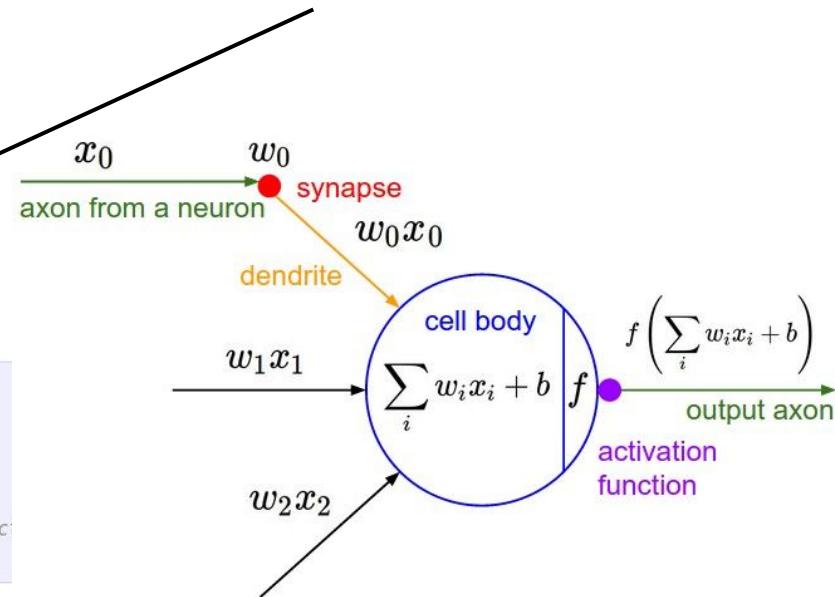


Impulses carried toward cell body

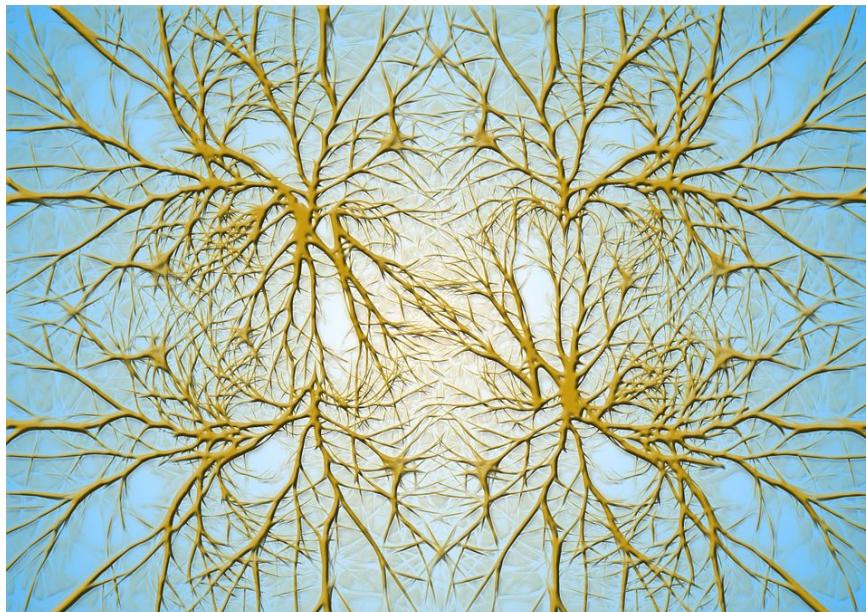


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is licensed under CC-BY 3.0

```
class Neuron:  
    ...  
    def neuron_tick(inputs):  
        """ assume inputs and weights are 1-D numpy arrays and bias is a number """  
        cell_body_sum = np.sum(inputs * self.weights) + self.bias  
        firing_rate = 1.0 / (1.0 + math.exp(-cell_body_sum)) # sigmoid activation function  
        return firing_rate
```

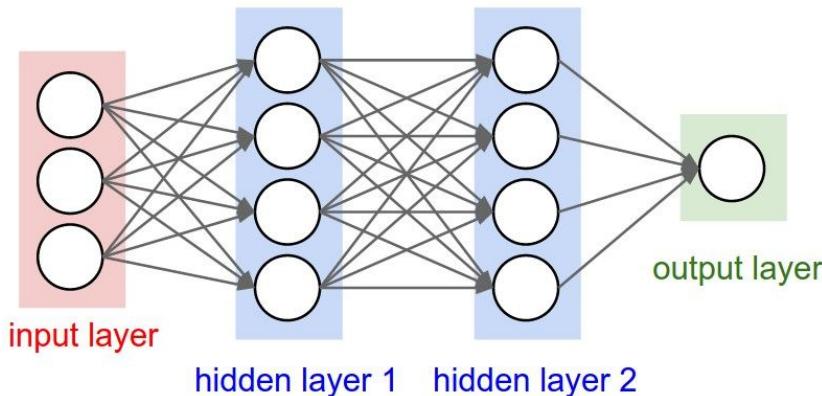


Biological Neurons: Complex connectivity patterns

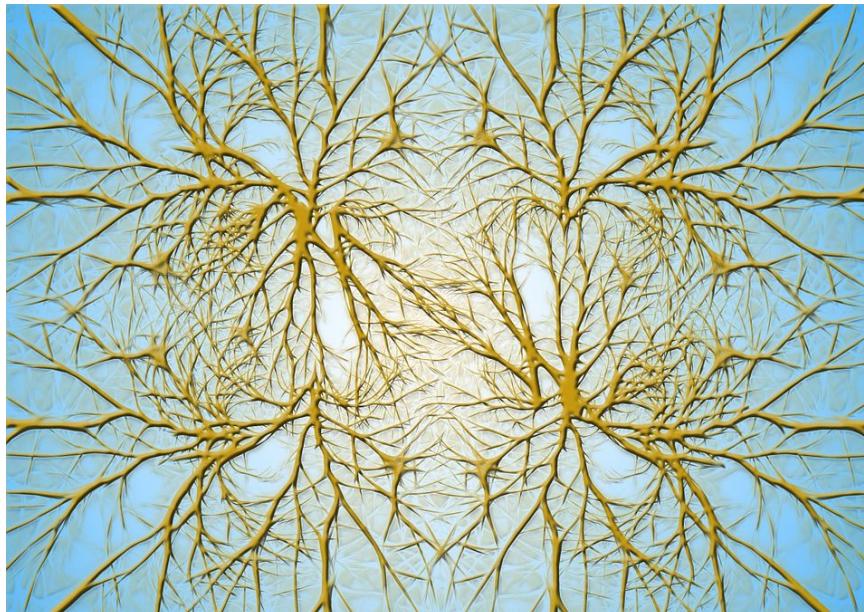


[This image is CC0 Public Domain](#)

Neurons in a neural network:
Organized into regular layers for
computational efficiency

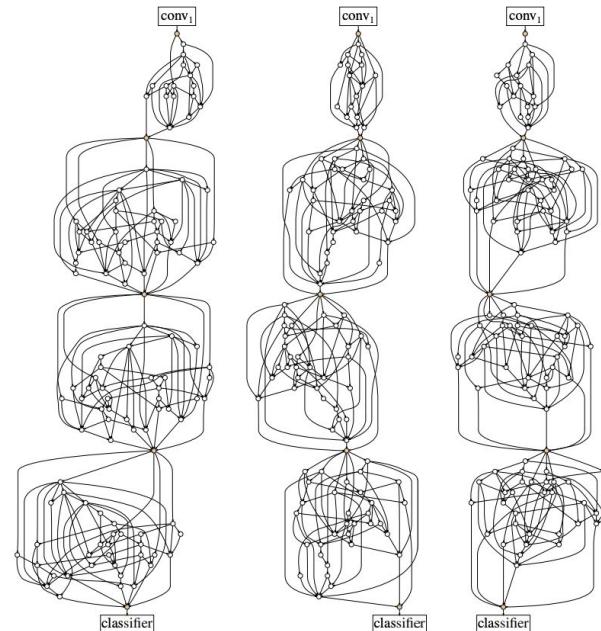


Biological Neurons: Complex connectivity patterns



[This image is CC0 Public Domain](#)

But neural networks with random connections can work too!



Xie et al, "Exploring Randomly Wired Neural Networks for Image Recognition", arXiv 2019

Be very careful with your brain analogies!

Biological Neurons:

- Many different types
- Dendrites can perform complex non-linear computations
- Synapses are not a single weight but a complex non-linear dynamical system

[Dendritic Computation. London and Häusser]

Softmax classifier

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**



cat	3.2
car	5.1
frog	-1.7

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**



$$s = f(x_i; W)$$

$$P(Y = k | X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax
Function

cat	3.2
car	5.1
frog	-1.7

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**



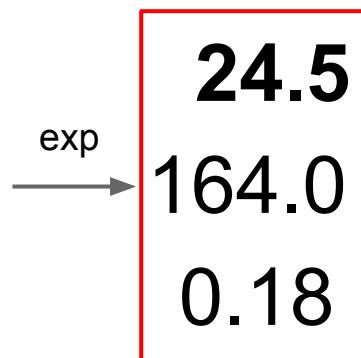
$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax
Function

Probabilities
must be ≥ 0

cat	3.2
car	5.1
frog	-1.7



unnormalized
probabilities

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**



$$s = f(x_i; W)$$

Probabilities
must be ≥ 0

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax
Function

cat	3.2
car	5.1
frog	-1.7

exp

24.5
164.0
0.18

unnormalized
probabilities

normalize

0.13
0.87
0.00

probabilities

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**



$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

Probabilities
must be ≥ 0

Probabilities
must sum to 1

cat
car
frog

3.2
5.1
-1.7

Unnormalized
log-probabilities / logits

exp

24.5
164.0
0.18

unnormalized
probabilities

normalize

0.13
0.87
0.00

probabilities

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Softmax Function

Probabilities
must be ≥ 0

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must sum to 1

cat
car
frog

3.2
5.1
-1.7

exp

24.5
164.0
0.18

normalize

0.13
0.87
0.00

$$\rightarrow L_i = -\log(0.13) = 2.04$$

Unnormalized
log-probabilities / logits

unnormalized
probabilities

probabilities

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**



$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$
 Softmax Function

Probabilities
must be ≥ 0

Probabilities
must sum to 1

cat
car
frog

3.2
5.1
-1.7

Unnormalized
log-probabilities / logits

exp

24.5
164.0
0.18

unnormalized
probabilities

normalize

0.13
0.87
0.00

probabilities

$$L_i = -\log P(Y = y_i|X = x_i)$$

$$\rightarrow L_i = -\log(0.13) \\ = 2.04$$

Maximum Likelihood Estimation
Choose weights to maximize the likelihood of the observed data
(See CS 229 for details)

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**



$$s = f(x_i; W)$$

Probabilities
must be ≥ 0

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax
Function

cat
car
frog

3.2
5.1
-1.7

Unnormalized
log-probabilities / logits

exp

24.5
164.0
0.18

unnormalized
probabilities

normalize

0.13
0.87
0.00

probabilities

compare

1.00
0.00
0.00

Correct
probs

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**



$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

cat
car
frog

3.2
5.1
-1.7

Unnormalized
log-probabilities / logits

exp

24.5
164.0
0.18

unnormalized
probabilities

normalize

0.13
0.87
0.00

probabilities

compare

1.00

0.00

0.00

Probabilities
must sum to 1

$$L_i = -\log P(Y = y_i|X = x_i)$$

Kullback–Leibler
divergence

$$D_{KL}(P||Q) = \sum_y P(y) \log \frac{P(y)}{Q(y)}$$

Correct
probs

Softmax Classifier (Multinomial Logistic Regression)



Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

Probabilities
must be ≥ 0

cat
car
frog

3.2
5.1
-1.7

Unnormalized
log-probabilities / logits

exp

24.5
164.0
0.18

unnormalized
probabilities

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax
Function

Probabilities
must sum to 1

0.13
0.87
0.00

probabilities

$$L_i = -\log P(Y = y_i|X = x_i)$$

compare

1.00

0.00

0.00

Cross Entropy

$$H(P, Q) = H(p) + D_{KL}(P||Q)$$

Correct
probs

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**



$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax
Function

Maximize probability of correct class

cat	3.2
car	5.1
frog	-1.7

Putting it all together:

$$L_i = -\log P(Y = y_i|X = x_i)$$

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

Softmax Classifier (Multinomial Logistic Regression)

Want to interpret raw classifier scores as **probabilities**



$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax
Function

Maximize probability of correct class

Putting it all together:

$$L_i = -\log P(Y = y_i|X = x_i)$$

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

cat	3.2
car	5.1
frog	-1.7

Q1: What is the min/max possible softmax loss L_i ?

Q2: At initialization all s_j will be approximately equal; what is the softmax loss L_i , assuming C classes?

Softmax Classifier (Multinomial Logistic Regression)



Want to interpret raw classifier scores as **probabilities**

$$s = f(x_i; W)$$

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

Softmax Function

Maximize probability of correct class

Putting it all together:

$$L_i = -\log P(Y = y_i|X = x_i)$$

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

cat	3.2
car	5.1
frog	-1.7

Q2: At initialization all s will be approximately equal; what is the loss?
A: $-\log(1/C) = \log(C)$,
If $C = 10$, then $L_i = \log(10) \approx 2.3$

Next Time: Convolutional Neural Networks!

