

Due date: 2025/10/9

Physical Constants:  $g = 9.80 \text{ m/s}^2$  (gravitational acceleration)

## Kinematics

1. The vector position of a particle varies in time according to the expression  $\vec{r} = 3.00t\hat{i} - 6.00t^3\hat{j}$ , where  $\vec{r}$  is in meters and  $t$  is in seconds. (a) Find an expression for the velocity of the particle as a function of time. (b) Determine the acceleration of the particle as a function of time. (c) Calculate the particle's position and velocity at  $t = 1.00 \text{ s}$ .

$$(a) \vec{v}(t) = \frac{d\vec{r}}{dt} = 3\hat{i} - 18t^2\hat{j} \text{ (m/s)}$$

$$(b) \vec{a}(t) = \frac{d\vec{v}}{dt} = -36t\hat{j} \text{ (m/s}^2\text{)}$$

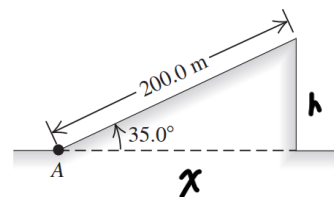
$$(c) \vec{r}(1) = 3\hat{i} - 6\hat{j} \text{ m}$$

$$\vec{v}(1) = 3\hat{i} - 18\hat{j} \text{ m/s}$$

2. A test rocket starting from rest at point A is launched by accelerating it along a 200.0 m incline at  $1.90 \text{ m/s}^2$  (Fig. 2.1). The incline rises at  $35.0^\circ$  above the horizontal, and at the instant it leaves it, the engines turn off and the rocket is subject to gravity only (ignore air resistance). Find (a) the maximum height above the ground that the rocket reaches, and (b) the rocket's greatest horizontal range beyond point A.

$$\text{末端速 } v = \sqrt{2as} = \sqrt{2 \times 1.9 \times 200} \\ \approx 27.6 \text{ m/s}$$

Fig 2.1



$$h = s \cdot \sin 35^\circ = 200 \sin 35^\circ \approx 114.7 \text{ m}$$

$$x = s \cdot \cos 35^\circ = 200 \cos 35^\circ \approx 163.8 \text{ m}$$

$$\text{離坡瞬間 } v_x = v \cos 35^\circ \approx 22.6 \text{ m/s} \quad v_y = v \sin 35^\circ \approx 15.8 \text{ m/s}$$

$$(a) H_{\max} = h + \frac{v_y^2}{2g} = 114.7 + \frac{(15.8)^2}{2 \times 9.8} \approx 127.4 \text{ m}$$

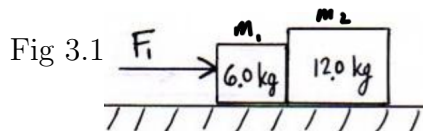
$$(b) \text{ (落地 } t) \quad 0 = h + v_y t - \frac{1}{2} g t^2 \rightarrow t = \frac{v_y + \sqrt{v_y^2 + 2gh}}{g} = \frac{15.8 + \sqrt{(15.8)^2 + 2 \times 9.8 \times 114.7}}{9.8} \\ = 6.7 \text{ s}$$

$$x + v_x t = 163.8 + 22.6 \times 6.7 \approx 315.4 \text{ m}$$

## Dynamics

3. A constant force  $F_1 = 72.0$  N is applied to a 6.00-kg block, which contacts a 12.0-kg block. They accelerate together on a horizontal frictionless surface. (a) How large is their common acceleration? (b) What is the magnitude of the force on the 6.0-kg block due to the contact with the 12.0-kg block? (c) What is the magnitude of the net force on the 6.0-kg block?

$$(a) \quad a = \frac{F_1}{m_1 + m_2} = \frac{72}{6 + 12} = 4 \text{ m/s}^2$$



$$(b) \quad F = m_2 a = 12 \times 4 = 48 \text{ N}$$

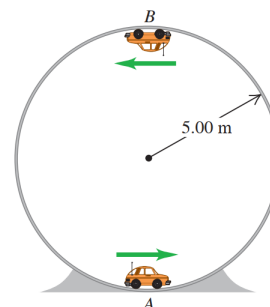
$$(c) \quad F_1 - F = 72 - 48 = 24 \text{ N}$$

4. A small car with mass 0.800 kg travels at constant speed on the inside of a track that is a vertical circle with radius 5.00 m (Fig. 4.1). If the normal force exerted by the track on the car when it is at the top of the track (point B) is 6.00 N, what is the normal force on the car when it is at the bottom of the track (point A)?

$$\text{頂端 } N_B + mg = \frac{mv^2}{r} = 6 + 0.8 \times 9.8 = 13.84 \text{ N} \quad \text{Fig 4.1}$$

$$\text{底端 } N_A - mg = \frac{mv^2}{r} = 13.84 \text{ N}$$

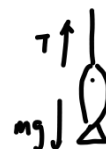
$$N_A = 13.84 + mg = 13.84 + 7.84 = 21.68 \text{ N}$$



5. A 2.2-kg fish is being pulled out of the water, in such a way that the tension in the fishing line is 3.0 times its weight. Draw the forces acting on the fish (free body diagram) and find the acceleration of the fish (magnitude and direction).

$$F = T - mg = 3mg - mg = 2mg$$

$$a = \frac{F}{m} = \frac{2mg}{m} = 2g = 19.6 \text{ m/s}^2$$



6. Two ropes are connected to a steel cable that supports a hanging weight (Fig. 6.1). (a) Draw a free-body diagram showing all of the forces acting at the knot that connects the two ropes to the steel cable. Based on your diagram, which of the two ropes will have the greater tension? (b) If the maximum tension either rope can sustain without breaking is 5000 N, determine the maximum value of the hanging weight that these ropes can safely support. Ignore the weight of the ropes and of the steel cable.

$$T_1 \cos 60^\circ = T_2 \cos 40^\circ \rightarrow T_1 = \frac{\cos 40^\circ}{\cos 60^\circ} T_2$$

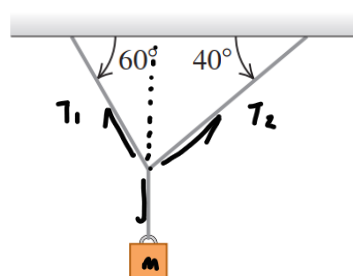
$$T_1 \approx 1.5 T_2$$

(a)  $T_1 \approx 1.5 T_2$ , 左邊較大

$$(b) T_2 = \frac{W}{1.5 \sin 60^\circ + \sin 40^\circ} \approx 0.508 W \quad T_1 \approx 0.778 W$$

$$0.778 W \leq 5000 \rightarrow W \approx 6430 \text{ N}$$

Fig 6.1



7. Block B, with mass 5.00 kg, rests on block A, with mass 8.00 kg, which in turn is on a horizontal tabletop (Fig. 7.1). There is no friction between block A and the tabletop, but the coefficient of static friction between blocks A and B is 0.750. A light string attached to block A passes over a frictionless, massless pulley, and block C is suspended from the other end of the string. What is the largest mass that block C can have so that blocks A and B still slide together when the system is released from rest?

$$f = \mu_s m_B g = m_B a$$

$$\rightarrow a = \mu_s g = 0.75 \times 9.8 = 7.35 \text{ m/s}^2$$

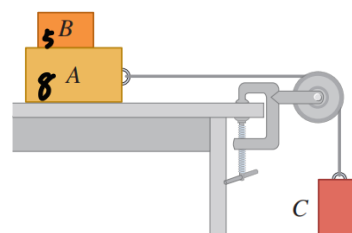
$$\begin{cases} T = (m_A + m_B) a \\ m_C g - T = m_C a \end{cases} \rightarrow a = \frac{m_C g}{m_A + m_B + m_C}$$

$$\frac{m_C}{m_A + m_B + m_C} \leq \mu_s \rightarrow m_C (1 - \mu_s) \leq \mu_s (m_A + m_B)$$

$$\rightarrow m_C \leq \frac{\mu_s}{1 - \mu_s} (m_A + m_B)$$

$$= \frac{0.75}{0.25} (5 + 8) = 39 \text{ kg}$$

Fig 7.1



8. (\*) An object of mass  $m$  is dropped from rest in air. The drag force acting on it is proportional to the square of its speed,  $f_d = cv^2$  where  $c$  is a positive drag coefficient. (a) Write down the equation of motion for the object. (b) Derive the expression of the terminal velocity  $v_t$ . (c) Solve for the velocity  $v(t)$  as a function of time. (d) If the object is a human body of 60.0 kg and  $c = 0.430$ . Determine how long it takes for the object to reach 90% of its terminal velocity. (Hint: use hyperbolic function  $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}$ )

$$(a) \quad m \frac{dv}{dt} = mg - cv^2$$

$$(b) \quad \frac{dv}{dt} = 0, \quad mg - cv_t^2 = 0 \rightarrow v_t = \sqrt{\frac{mg}{c}}$$

$$(c) \quad \frac{dv}{dt} = g \left(1 - \frac{v^2}{v_t^2}\right) \quad \int \frac{\frac{dv}{v_t}}{1 - \left(\frac{v}{v_t}\right)^2} = \int \frac{g}{v_t} dt$$

$$\rightarrow \tanh^{-1}\left(\frac{v}{v_t}\right) = \frac{g}{v_t} t$$

$$v(t) = v_t \tanh\left(\frac{gt}{v_t}\right)$$

$$(d) \quad v_t = \sqrt{\frac{mg}{c}} = \sqrt{\frac{60 \times 9.8}{0.43}} \approx 37 \text{ m/s}$$

$$t = \frac{v_t}{g} \times 1.47$$

$$\nearrow = \frac{37}{9.8} \times 1.47 \approx 5.6 \text{ s}$$

$$0.9 = \tanh\left(\frac{gt}{v_t}\right) \quad \text{arctanh}(0.9) = \frac{1}{2} \ln\left(\frac{1+0.9}{1-0.9}\right) \approx 1.47$$

9. A 1400-kg car initially has a velocity of 33.3 m/s due south. It brakes to a stop over a 180 m distance. (a) What is the magnitude of the car's acceleration, in m/s<sup>2</sup>? (b) What average net force magnitude was necessary to stop the car? (c) Assuming the tires do not skid, what coefficient of static friction between tires and pavement is needed?

$$(a) \quad v^2 = v_0^2 + 2ad$$

$$0 = (33.3)^2 + 2a \times 180$$

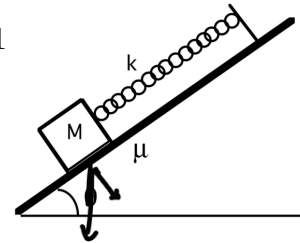
$$|a| \approx 3.08 \text{ m/s}^2$$

$$(b) \quad F = ma = 1400 \times 3.08 = 4310 \text{ N}$$

$$(c) \quad \mu_s = \frac{a}{g} = \frac{3.08}{9.8} = 0.314$$

10. A box of mass  $M$  sits on an incline elevated an angle  $\theta$  above the horizontal. The mass is held in place by a spring (of spring constant  $k$ ) as shown. Assume that the coefficient of static friction between the box and the incline is  $\mu$ . (a) Find the maximum amount that the spring can be **stretched** if the mass is to remain stationary on the incline. (b) Find the maximum amount that the spring can be **compressed** if the mass is to remain stationary on the incline.

Fig 10.1



$$\text{法 } N = Mg \cos \theta$$

$$(a) f = \mu N = \mu Mg \cos \theta$$

$$kx - Mg \sin \theta - \mu Mg \cos \theta = 0$$

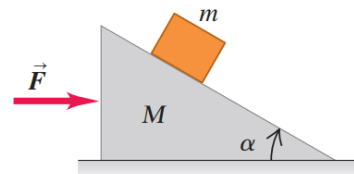
$$x = \frac{Mg}{k} (\sin \theta + \mu \cos \theta)$$

$$(b) \mu Mg \cos \theta - Mg \sin \theta - kx = 0$$

$$x = \frac{Mg}{k} (\mu \cos \theta - \sin \theta)$$

11. A wedge with mass  $M$  rests on a frictionless, horizontal tabletop. A block with mass  $m$  is placed on the wedge, and a horizontal force  $\vec{F}$  is applied to the wedge (Fig. 11.1). (a) What must the magnitude of  $\vec{F}$  be if the block is to remain at a constant height above the tabletop? (b) If there is friction between mass  $M$  and  $m$  (the coefficient of static friction  $\mu$ ), what is the condition that the block is to remain at a constant height.

Fig 11.1



$$(a) ma \cos \alpha + mg \sin \alpha = 0$$

$$a = g \tan \alpha$$

$$a = \frac{F}{M+m} \rightarrow F = (M+m)g \tan \alpha$$

$$(b) f = m(a \cos \alpha + g \sin \alpha)$$

$$N = m(g \cos \alpha - a \sin \alpha)$$

$$a(\cos \alpha + \mu \sin \alpha) \leq g(\mu \cos \alpha - \sin \alpha)$$

$$\rightarrow 0 \leq a \leq \min \left( g \cot \alpha, g \frac{\mu \cos \alpha - \sin \alpha}{\cos \alpha + \mu \sin \alpha} \right)$$

12. A small rocket with mass 20.0 kg is moving in free fall toward the earth. Air resistance can be neglected. When the rocket is 80.0 m above the surface of the earth, it is moving downward with a speed of 30.0 m/s. At that instant the rocket engines start to fire and produce a constant upward force  $F$  on the rocket. Assume the change in the rocket's mass is negligible. What is the value of  $F$  if the rocket's speed becomes zero just as it reaches the surface of the earth, for a soft landing?

$$W_{\text{net}} = \Delta K = K_f - K_i = 0 - \frac{1}{2} m v_0^2$$

$$mg\Delta s - F\Delta s = -\frac{1}{2} m v_0^2 \rightarrow F = mg + \frac{\frac{1}{2} m v_0^2}{\Delta s}$$

$$= 20 \times 9.8 + \frac{0.5 \times 20 \times 30^2}{80}$$

$$= 196 + 112.5 = 308.5 \text{ N}$$

13. (\*) A spring with a uniform mass density  $\rho$  has a spring constant  $k$ . When the spring is placed horizontally on a table without being stretched, its length is  $L$ . What is its length when it is hung vertically and stretched by the gravitational force? Note that when the spring is hung vertically, its mass density is no longer uniform, because the upper part is stretched more than the lower part.

$$T(y) = \rho g y$$

$$d\ell = \frac{T(y)}{k y} = \frac{T(y) dy}{k L} = \frac{\rho g y}{k L} dy$$

$$\Delta L = \int_0^L \frac{\rho g y}{k L} dy = \frac{\rho g}{k L} \cdot \frac{L^2}{2}$$

$$= \frac{\rho g L}{2k} = \frac{mg}{2k}$$

$$A: L + \frac{mg}{2k} = L + \frac{\rho g L}{2k}$$

## Work and Energy

14. A soccer ball with mass 0.420 kg is initially moving with speed 2.00 m/s. A soccer player kicks the ball, exerting a constant force of magnitude 40.0 N in the same direction as the ball's motion. Over what distance must the player's foot be in contact with the ball to increase the ball's speed to 6.00 m/s?

$$W = \Delta K = \frac{1}{2} m (v_f^2 - v_i^2)$$

$$Fs = \frac{1}{2} m (v_f^2 - v_i^2)$$

$$40s = \frac{1}{2} \times 0.42 \times (6^2 - 2^2)$$

$$= 0.21 \times 32$$

$$s = 0.168 \text{ m}$$

15. A force in the  $+x$ -direction with magnitude  $F(x) = 18.0 \text{ N} - (0.530 \text{ N/m})x$  is applied to a 6.00 kg box that is sitting on the horizontal, frictionless surface of a frozen lake.  $F(x)$  is the only horizontal force on the box. If the box is initially at rest at  $x = 0$ , what is its speed after it has traveled 14.0 m?

$$W = \Delta K = \frac{1}{2} m v^2$$

$$W = \int_0^{14} F(x) dx = \int_0^{14} (18 - 0.53x) dx$$

$$= 18x - 0.5 \times 0.530 x^2 \Big|_0^{14} \approx 200$$

$$200 = \frac{1}{2} \times 6 \times v^2 \rightarrow v^2 = \frac{400}{6} \rightarrow v \approx 8.17$$

16. It is 5.0 km from your home to the physics lab. As part of your physical fitness program, you could run that distance at 10 km/h (which uses up energy at the rate of 700 W), or you could walk it leisurely at 3.0 km/h (which uses energy at 290 W). Which choice would burn up more energy, and how much energy (in joules) would it burn? Why does the more intense exercise burn up less energy than the less intense exercise?

$$\text{跑 } t = \frac{5}{10} = 0.5 \text{ h} = 1800 \text{ s}$$

$$\text{走 } t = \frac{5}{3} = 1.6 \text{ h} = 6000 \text{ s}$$

$$\text{跑 } E = 700 \times 1800 = 1.26 \times 10^6$$

$$\text{走 } E = 290 \times 6000 = 1.74 \times 10^6$$

走路更多, 因跑步  $t$  小很多,  $E = Pt$