9/30 普物作業 114601015 陳柏勝

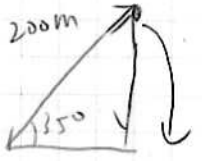
1. $\vec{r} = 3t\hat{i} - 6t^3\hat{j}$

(a) 速度 $= \frac{d\vec{r}}{dt} = 3\hat{i} - 18t^2\hat{j}$

(b) 加速度 $= \frac{d^2\vec{r}}{dt^2} = -36t\hat{j}$

(c) $V = 3\hat{i} - 18t^2\hat{j}$, $t=1$, $\Rightarrow V = 3\hat{i} - 18\hat{j}$

2.



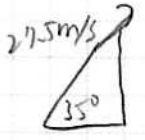
加速度 $= 1.9\ m/s^2$

$S = \frac{1}{2}gt^2$

$t \fallingdotseq 14.51$

代公式 $V^2 = V_0^2 + 2aS$

$V^2 = 0^2 + 1.9 \cdot 2 \cdot 200 \Rightarrow V \fallingdotseq 27.57\ m/s$



水平 $= 27.57 \times \cos 35° \fallingdotseq 22.58\ m/s$

鉛直 $= 27.57 \times \sin 35° \fallingdotseq 15.81\ m/s$

(c) 單純看鉛直 (拿來加速得到的)

$V^2 = V_0^2 + 2aS$

$0^2 = (15.81)^2 + 2(-9.8)(S)$

$S = 12.77$, 最大高度 $= S + 200\sin 35°$

$\fallingdotseq 127.49\ m$ #

(b) 加速到最後一刻後, $V = V_0 + at$ (15.81), $t \fallingdotseq 1.613$ ┐ 這些時間等於連

從最高點到地面(看鉛直) $S = \frac{1}{2}gt^2$, $t \fallingdotseq 5.100$ ┘

水平射程 $= (6.713) \times (22.58) + 200 \times (\cos 35°)$ (6.713)

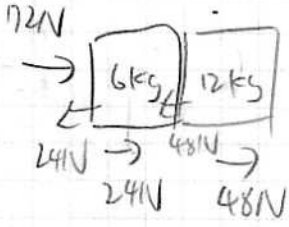
$\fallingdotseq 315.40\ m$ #

3.

(a) $F = ma$

$\Rightarrow 72 = 18(a)$, $a = 4\ m/s^2$

(b)



48N #

(c) $F = ma = 6 \cdot 4 = 24N$ #

等速圓周走8個=

$V = \frac{2\pi r}{t}$ 位移0

$w = \frac{2\pi}{t}$

$V = rw$

$a = Vw$

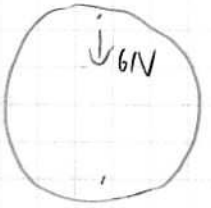
$= \frac{V^2}{R}$

$= w^2 R$

$= \frac{2\pi V}{t}$

$= \frac{4\pi^2 R}{t^2}$

4.



$F = ma$

$6 = (0.8)\frac{V^2}{5}$ $V^2 = \frac{75}{2}$

$\frac{1}{2}mv^2 = \frac{1}{2} \cdot \frac{4}{5} \cdot \frac{75}{2} = 15$

由 B→A，力場能守恆 減少U ⇒ 增加動K

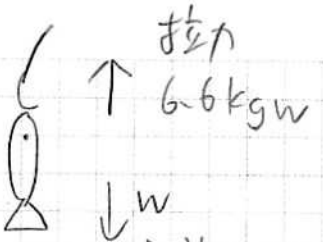
$mgh = \frac{1}{2}mv^2$

$0.8 \cdot 9.8 \cdot 10 = \frac{1}{2}mv^2$ $V^2 = 2gh = 196$

$F = ma$

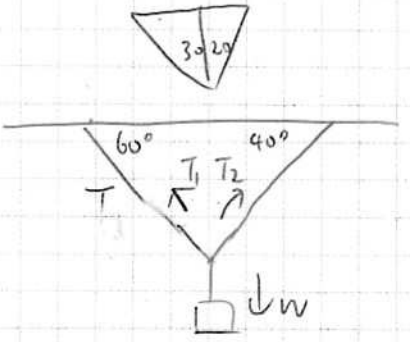
$F = 0.8\frac{196}{5}$, $F = 31.36N$ #

5、



$F = ma$, $a = \frac{F}{m} = \frac{4.4 \times 9.8}{2.2} = 19.6\ m/s^2$ #

6、



水平：$T_1 \sin 30° + T_2 \sin 20° = 0$ (平衡)

鉛直：$T_1 \cos 30° + T_2 \cos 20° = W$

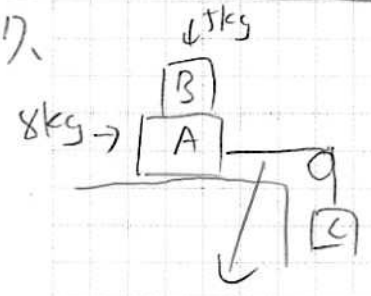
$\Rightarrow T_1 = T_2 \fallingdotseq 1 : 1.46$

(a) 因 $T_1 : T_2 \fallingdotseq 1 : 1.46$，所以右邊繩子拉力較大

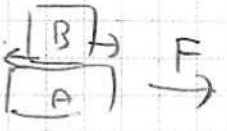
(b) 看鉛直 $T_1 \cos 30° + T_2 \cos 20° = W$ ↓最重

$W \fallingdotseq 9028.59\ N$，$\div 9.8 \Rightarrow 921.28\ kg$ #

7、



設 C = x kg，A 和 B 之間 $f_{max} = \mu_s N = 0.75 \times 5 \times 9.8 = 36.75$



$a \fallingdotseq 2.83\ m/s^2$

如果 拉力 > 36.75N，AB 間會滑動

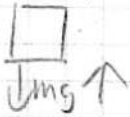
拉力 = C的重量 × g

$\frac{36.75}{9.8} = 3.75$ C最重 3.75kg #

8、(a) 變加速度運動

$$ma = m\frac{dv}{dt} = mg - cv^2.$$

(b)



$$mg = f_{max} = cv_t^2$$

$$v_t = \sqrt{\frac{mg}{c}}$$

(c) 微分方程

$$m\frac{dv}{dt} = mg - cv^2$$

$$\frac{dv}{dt} = g - \frac{c}{m}v^2$$

$\frac{dv}{g - \frac{c}{m}v^2} = dt$ 可分離變分程

令 $v_t = \sqrt{\frac{mg}{c}}$

$$\Rightarrow \frac{dv}{g\left(1 - \frac{v^2}{v_t^2}\right)} = dt$$

$$\int \frac{dv}{g\left(1 - \frac{v^2}{v_t^2}\right)} = \int dt$$ 牛頓分

$$\frac{v_t}{2}\ln\left(\frac{v_t + v}{v_t - v}\right) = gt + C$$

$$v(0) = 0 \Rightarrow C = 0$$

$$\frac{v_t + v}{v_t - v} = e^{\frac{2g}{v_t}t}$$

$$\Rightarrow v(t) = v_t \tanh\left(\frac{g}{v_t}t\right) \#$$

(d) $v_t = \sqrt{\frac{mg}{c}} \fallingdotseq 37.1\ m/s$

要到 $0.9\,v_t$

$$v(t) = v_t \tanh\left(\frac{g}{v_t}t\right) = 0.9\,v_t$$

$$\tanh\left(\frac{g}{v_t}t\right) = 0.9$$

取反雙曲

$$\frac{g}{v_t}t = \tanh^{-1}(0.9)$$

$$\Rightarrow \tanh^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$$

$$\tanh^{-1}(0.9) = \frac{1}{2}\ln\left(\frac{1.9}{0.1}\right) = \frac{1}{2}\ln(19)$$

$$\ln(19) \fallingdotseq 2.944 \Rightarrow \tanh^{-1}(0.9) \fallingdotseq 1.472$$

$$\Rightarrow t = \frac{v_t}{g} \times 1.472$$

$t \fallingdotseq 5.5$ 秒 #

9. (a)

代公式 $V^2 = V_0^2 + 2as$ $a \fallingdotseq 3.08\ m/s^2$ #

(b) $F = ma = 1400 \cdot 3.08 = 4312\ N$ # $N =$

(c) $f = \mu_s \cdot N = 4312$

$= \mu_s \cdot mg = 4312$ $\mu_s \fallingdotseq 0.314$

10.

(a)

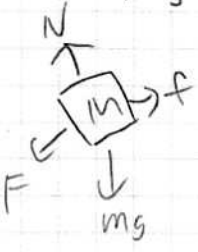
$F = kx$，分析 M 的情況

$kx_{max} - Mg\sin\theta - \mu Mg\cos\theta = 0$

$kx_{max} = Mg(\sin\theta + \mu\cos\theta)$

$x_{max} = \dfrac{Mg(\sin\theta + \mu\cos\theta)}{k}$

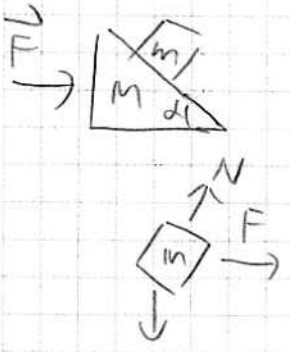
(b)



$kx_{min} + Mg\sin\theta - \mu Mg\cos\theta = 0$

$x_{min} = \dfrac{Mg(\mu\cos\theta - \sin\theta)}{k}$

11. (a)



$ma\cos\theta - mg\sin\theta = 0$

$a = g\tan\theta$

$F = ma$

$= (M+m)g\tan\theta$

(b)

$ma\cos\theta - mg\sin\theta + mg\sin\theta - ma\cos\theta$

$N = mg\cos\theta + ma\sin\theta$

$|f| \le \mu N$

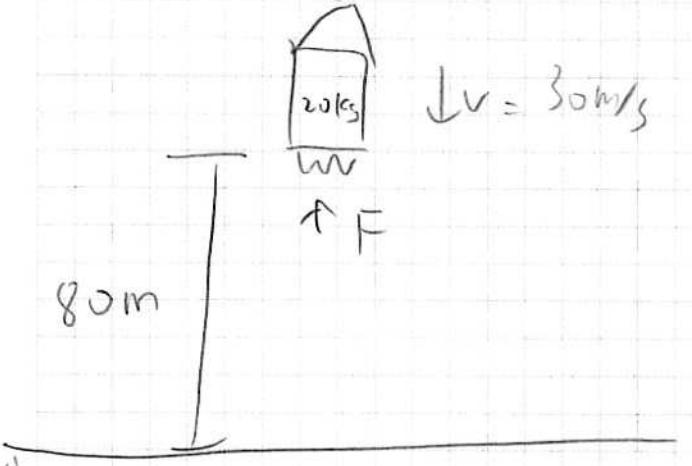
$\Rightarrow |g\sin\theta - a\cos\theta| \le \mu(g\cos\theta + a\sin\theta)$

$\Rightarrow \frac{g(\sin\theta - \mu\cos\theta)}{\cos\theta + \mu\sin\theta} \le a \le \frac{g(\sin\theta + \mu\cos\theta)}{\cos\theta - \mu\sin\theta}$

$F = (m+M)a$

$\Rightarrow (m+M)\left(\frac{g(\sin\theta - \mu\cos\theta)}{\cos\theta + \mu\sin\theta}\right) \le F \le (m+M)\left(\frac{g(\sin\theta + \mu\cos\theta)}{\cos\theta - \mu\sin\theta}\right)$ #

12.



代公式

$V^2 = V_0^2 + 2as$

$0 = 900 + 2 \cdot a \cdot 80$

$a = 5$

$F = ma$

$= 20 \times 5$

$= 100\ N$

13.

$m = \rho l$

$F = k'\Delta x$

$(l-x)\rho g = F_g$

$k' = k\frac{l}{dx}$

$F_{spring} = k\frac{l}{dx}\Delta x = (l-x)\rho g = F_g$

$\Delta x = \frac{(l-x)}{kl}dx$

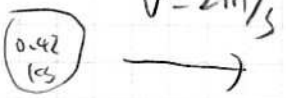
$\int_0^l \Delta x\,dx = \frac{\rho g}{kl}\int_0^l (l-x)\,dx$

$= \frac{\rho g}{kl}\left(lx - \frac{1}{2}x^2\right)\Big|_0^l = \frac{\rho g l}{2k}$

$L' = l + \frac{\rho g l}{2k}$

$= l + \frac{mg}{2k}$

14.

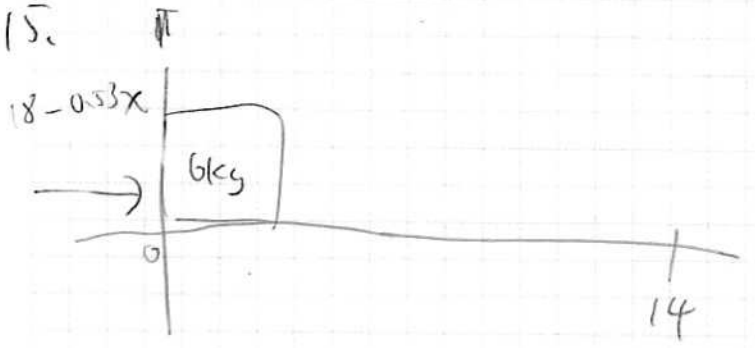


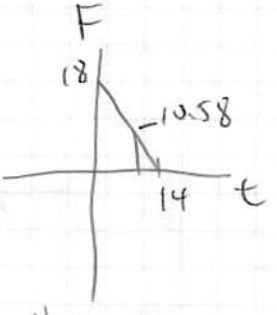
$V^2 = V_0^2 + 2as$

$36 = 4 + 2\frac{F}{m}s$ $\quad s = 0.168\ m$ #

$32 + 2\frac{40}{0.42}s$

15.





F-t 圖下為 W

$\frac{(18+10.58)\cdot 14}{2} = 200.06$

在此題目 $\frac{1}{2}mv^2 = 200.06$

W = 動能 $\quad v = 8.166\ m/s$