Due date: 2025/10/9

Physical Constants: g = 9.80 m/s^2 (gravitational acceleration)

Kinematics

1. The vector position of a particle varies in time according to the expression $\vec{r} = 3.00t\hat{i} - 6.00t^3\hat{j}$, where $\vec{r}$ is in meters and t is in seconds. (a) Find an expression for the velocity of the particle as a function of time. (b) Determine the acceleration of the particle as a function of time. (c) Calculate the particle's position and velocity at $t = 1.00$ s.

(a) $\vec{v} = \frac{d\vec{r}}{dt} = 3.00\hat{i} - 18.0t^2\hat{j}$ (m/s)

(b) $\vec{a} = \frac{d\vec{v}}{dt} = -36.0t\hat{j}$ (m/s²)

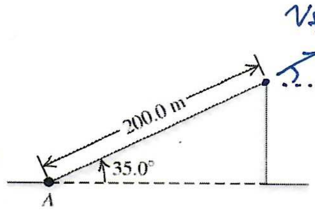
(c) $\vec{r}(t=1.00s) = 3.00\hat{i} - 6.00\hat{j}$ (m)

$\vec{v}(t=1.00s) = 3.00\hat{i} - 18.0\hat{j}$ (m/s)

#

2. A test rocket starting from rest at point A is launched by accelerating it along a 200.0 m incline at 1.90 m/s^2 (Fig. 2.1). The incline rises at 35.0° above the horizontal, and at the instant the leaves it, the engines turn off and the rocket is subject to gravity only (ignore air resistance). Find (a) the maximum height above the ground that the rocket reaches, and (b) the rocket's greatest horizontal range beyond point A.

Fig 2.1



$V_f^2 = 2a\Delta S = 2(1.9\ m/s^2)(200m)$

$V_f = 27.6$ m/s

$V_{f,x} = V_f \cos 35° = 22.6$ m/s

$V_{f,y} = V_f \sin 35° = 15.8$ m/s

(a) Maximum height $= 200\sin(35°) + \frac{0 - V_{f,y}^2}{-2\cdot g} = 127$ m

#

(b) The time in air is t, initial height $y_0 = 200\sin(35°)$

$0 = y_0 + V_{f,y}t - \frac{1}{2}gt^2 \Rightarrow t = \frac{V_{f,y}}{g} + \sqrt{\frac{V_{f,y}^2}{g^2} + \frac{2y_0}{g}} = 6.71$ s

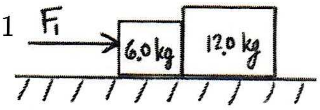
$t^2 - \frac{2V_{f,y}}{g} - \frac{2y_0}{g} = 0$

Horizotal range $= 200\cos(35°) + V_{f,x}t = 315$ m

#

Dynamics

3. A constant force $F_1 = 72.0$ N is applied to a 6.00-kg block, which contacts a 12.0-kg block. They accelerate together on a horizontal frictionless surface. (a) How large is their common acceleration? (b) What is the magnitude of the force on the 6.0-kg block due to the contact with the 12.0-kg block? (c) What is the magnitude of the net force on the 6.0-kg block?

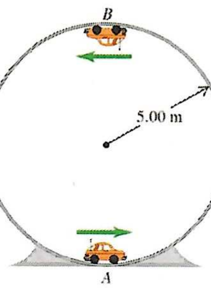
Fig 3.1


(a) $a = \frac{F_1}{m_1+m_2} = \frac{72.0}{6.0+12.0} = 4.0 \ m/s^2$ #

(b) Let F_2 be the contact force, $F_2 = 12.0 \times a = 48.0$ N #

(c) The net force $= F_1 - F_2 = 24.0$ N #

4. A small car with mass 0.800 kg travels at constant speed on the inside of a track that is a vertical circle with radius 5.00 m (Fig. 4.1). If the normal force exerted by the track on the car when it is at the top of the track (point B) is 6.00 N, what is the normal force on the car when it is at the bottom of the track (point A)?

Fig 4.1


A constant circular motion

$a = v^2/R$

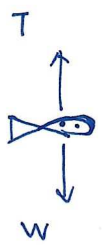
At point B, $\Sigma F = mg + N_B = ma$

At point A, $\Sigma F = N_A - mg = ma$

$\therefore N_A = 2mg + N_B = 2(0.800)(9.8) + 6.00 = 21.68 \approx 21.7$ N #

5. A 2.2-kg fish is being pulled out of the water, in such a way that the tension in the fishing line is 3.0 times its weight. Draw the forces acting on the fish (free body diagram) and find the acceleration of the fish (magnitude and direction).

Free-body diagram



$T = 3W$, $W = mg = (2.2)(9.8) = 21.56$ N

$\Sigma F = T - W = 2W = 2mg = ma$

$\therefore a = 2g = 19.6 \ m/s^2$ #

6. Two ropes are connected to a steel cable that supports a hanging weight (Fig. 6.1). (a) Draw a free-body diagram showing all of the forces acting at the knot that connects the two ropes to the steel cable. Based on your diagram, which of the two ropes will have the greater tension? (b) If the maximum tension either rope can sustain without breaking is 5000 N, determine the maximum value of the hanging weight that these ropes can safely support. Ignore the weight of the ropes and of the steel cable.

(a) Free-body diagram

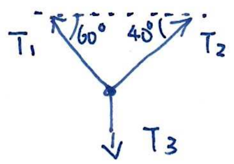
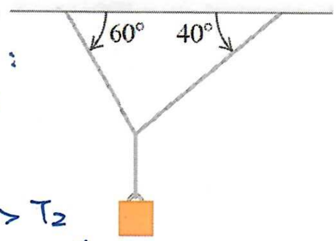


Fig 6.1



In horizontal direction :

$$T_1 \cos 60^\circ = T_2 \cos 40^\circ$$

$$\frac{T_1}{T_2} = \frac{\cos 40^\circ}{\cos 60^\circ} > 1 \ , \ T_1 > T_2$$

#

(b) In vertical direction :

$$T_3 = T_1 \sin 60^\circ + T_2 \sin 40^\circ$$

$$= T_1 \left(\sin 60^\circ + \frac{\cos 60^\circ}{\cos 40^\circ} \sin 40^\circ \right) = 1.286\, T_1$$

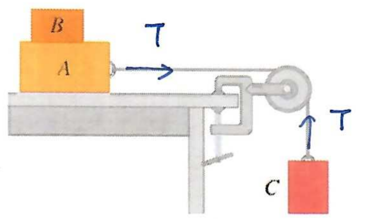
Set $T_1 = 5000\,N$, $T_3 = 6430\ N$

#

7. Block B, with mass 5.00 kg, rests on block A, with mass 8.00 kg, which in turn is on a horizontal tabletop (Fig. 7.1). There is no friction between block A and the tabletop, but the coefficient of static friction between blocks A and B is 0.750. A light string attached to block A passes over a frictionless, massless pulley, and block C is suspended from the other end of the string. What is the largest mass that block C can have so that blocks A and B still slide together when the system is released from rest?

Let T be the tension force

Fig 7.1

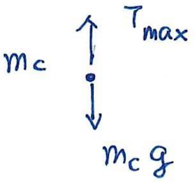


$\mu_{s,AB} = 0.750$

$m_B \longrightarrow f_s \qquad \Sigma F_B = m_B g \mu_{s,AB} = m_B a$

Here a is the acceleration of block A+B (& block C) , $a = \dfrac{T}{m_A + m_B}$

$$\therefore\ T_{max} = (m_A + m_B)\, g\, \mu_{s,AB} = 95.55\ N$$



$$\Sigma F_c = m_c g - T_{max} = m_c a = \frac{m_c T_{max}}{m_A + m_B} = m_c g \mu_{s,AB}$$

$$\Rightarrow m_c = \frac{T_{max}}{g(1 - \mu_{s,AB})} = \frac{(m_A + m_B)\mu_{s,AB}}{1 - \mu_{s,AB}} = 39.0\ kg$$

#

8. (*) An object of mass m is dropped from rest in air. The drag force acting on it is proportional to the square of its speed, $f_d = cv^2$ where c is the a positive drag coefficent. (a) Write down the equation of motion for the object. (b) Derive the expression of the terminal velocity v_t. (c) Solve for the velocity $v(t)$ as a function of time. (d) If the object is a human body of 60.0 kg and $c = 0.430$. Determine how long it takes for the object to reach 90% of its terminal velocity. (Hint: use hyperbolic function $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x}-1}{e^{2x}+1}$) -※ (d) in additional page

(a) f_d, m, mg

$$\Sigma F = mg - f_d = ma$$

Equation of motion:

$$m\frac{dv}{dt} = mg - cv^2$$

(b) At terminal velocity V_t, $\Sigma F = 0$

$$mg - cV_t^2 = 0$$

$$V_t = \sqrt{mg/c}$$

(c) Solve $m\frac{dv}{dt} = mg - cV^2$

Rearrange:

$$dt = m\frac{dv}{mg - cV^2} = -\frac{m}{c}\cdot\frac{dv}{V^2 - V_t^2}$$

Integral both sides

$$\int_0^t dt' = -\frac{m}{c}\int_0^V \frac{dV'}{V'^2 - V_t^2}$$

use integral formula:

$$\int_0^V \frac{dV'}{V'^2 - V_t^2} = \frac{1}{2V_t}\ln\left|\frac{V - V_t}{V + V_t}\right| = \frac{1}{2V_t}\ln\left(\frac{V_t - V}{V_t + V}\right)$$

$\because V < V_t$

$$\Rightarrow -\frac{2cV_t}{m}t = \ln\left(\frac{V_t - V}{V_t + V}\right)$$

let $\alpha = cV_t/m$,

$$e^{-2\alpha t} = \frac{V_t - V}{V_t + V}$$

Rearrange: $V = V_t\left(\frac{1 - e^{-2\alpha t}}{1 + e^{-2\alpha t}}\right)$

use tanh: $V(t) = \tanh(\alpha t)\cdot V_t$

$$\boxed{V(t) = V_t \tanh\left(\frac{cV_t}{m}t\right)}$$ #

9. A 1400-kg car initially has a velocity of 33.3 m/s due south. It brakes to a stop over a 180 m distance. (a) What is the magnitude of the car's acceleration, in m/s^2? (b) What average net force magnitude was necessary to stop the car? (c) Assuming the tires do not skid, what coefficient of static friction between tires and pavement is needed?

(a) $V_i = 33.3\,m/s$ $\longrightarrow$ $V_f = 0$

$$V_f^2 - V_i^2 = 2a\Delta X$$

$$a = \frac{-V_i^2}{2\Delta X} = -3.08$$

$\Rightarrow$ magnitude is $3.08\ m/s^2$ #

(b) $\Sigma\vec{F} = m\vec{a}$

$$F_{net} = 1400 \cdot 3.08 = 4312\ N$$ #

(c) The static friction causes the deceleration

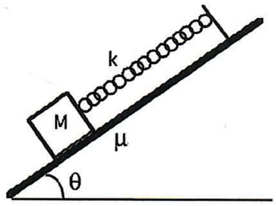
$$F_{net} = f_s = mg\mu_s$$

$$\mu_s = \frac{F_{net}}{mg} = 0.314$$ #

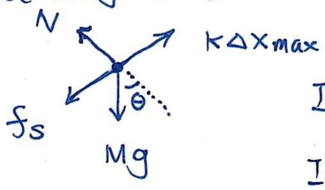
10. A box of mass M sits on an incline elevated an angle θ above the horizontal. The mass is held in place by a spring (of spring constant k) as shown. Assume that the coefficient of static friction between the box and the incline is μ. (a) Find the maximum amount that the spring can be **stretched** if the mass is to remain stationary on the incline. (b) Find the maximum amount that the spring can be **compressed** if the mass is to remain stationary on the incline.

(a) Stretched

Fig 10.1



Draw free-body diagram (at maximum stretch)

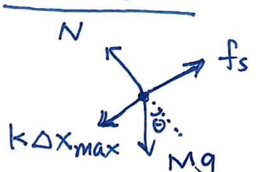


In normal direction: $N = Mg\cos\theta$

In incline direction: $f_s + Mg\sin\theta = k\Delta x_{max}$

$\Rightarrow \Delta x_{max} = \frac{Mg}{k}(\mu\cos\theta + \sin\theta)$ #

(b) Compressed



In incline direction: $f_s = Mg\sin\theta + k\Delta x_{max}$

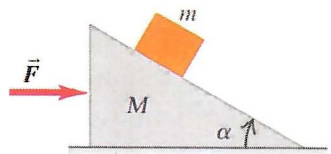
$\Rightarrow \Delta x_{max} = \frac{Mg}{k}(\mu\cos\theta - \sin\theta)$ #

11. A wedge with mass M rests on a frictionless, horizontal tabletop. A block with mass m is placed on the wedge, and a horizontal force $\vec{F}$ is applied to the wedge (Fig. 11.1). (a) What must the magnitude of $\vec{F}$ be if the block is to remain at a constant height above the tabletop? (b) If there is friction between mass M and m (the coefficient of static friction μ), what is the condition that the block is to remain at a constant height.

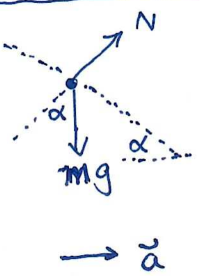
※ (b) in additional page

(a)

Fig 11.1



(i) In lab frame



Net force for mass m

$\Sigma \vec{F}_m = m\vec{a}$

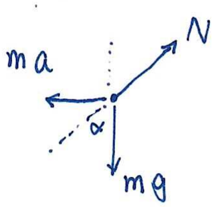
where $\vec{a} = \frac{\vec{F}}{M+m}$ because the block remained at a constant height

$\therefore$ In vertical direction: $\Sigma \vec{F}_{m,\perp} = N\cos\alpha - mg = 0$, $N = \frac{mg}{\cos\alpha}$

In $\vec{F}$ direction: $\Sigma \vec{F}_{m,//} = N\sin\alpha = ma = \frac{mF}{M+m}$

$\Rightarrow F = (M+m)g\tan\alpha$ #

(ii) In acceleration frame



$\Sigma \vec{F}_m = 0 \Rightarrow \begin{cases} N\cos\alpha = mg \\ N\sin\alpha = ma = \frac{mF}{m+M} \end{cases}$

$\therefore F = (M+m)g\tan\alpha$ #

Solution : Assignment #2 PH1031-C

Additional page :

8(d) The drag coefficient c has dimension of $\left[\frac{\text{mass}}{\text{length}}\right]$

Set $c = 0.430\ \text{kg/m}$ and $m = 60.0\ \text{kg}$

then $v_t = \sqrt{\frac{mg}{c}} = 36.98 \approx 37.0\ \text{m/s}$, $\alpha = \frac{c v_t}{m} = 0.265$

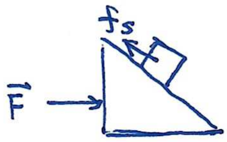
At time T, $\frac{v(T)}{v_t} = 0.9 = \tanh(0.265\,T)$

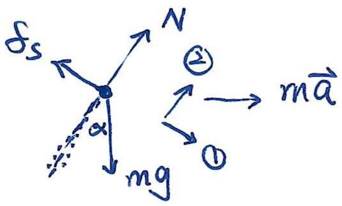
$0.265\,T = \tanh^{-1}(0.9) = 1.4722$ $\therefore T = 5.56\ \text{s}$ #

Note : 此題不計分，因 c 沒給單位

11.(b) The critical force $F = (m+M)\,g\tan\alpha$

If F is smaller than critical force, m tends to slide down

so situation is like  , we first find smallest F



Consider direction ①

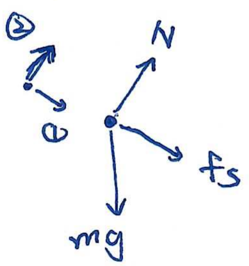
$$ma\cos\alpha = mg\sin\alpha - N\mu$$

and ② : $ma\sin\alpha = N - mg\cos\alpha$

$$N = ma\sin\alpha + mg\cos\alpha \quad \text{— sub to ①}$$

and we have $ma = \frac{mF}{M+m} \Rightarrow F_{min} = g(M+m)\left(\frac{\sin\alpha - \mu\cos\alpha}{\cos\alpha + \mu\sin\alpha}\right)$

Next, we want to find the largest F, the F_{max} will happen as block m tends to move upward on the wedge, the friction force should be in the opposite direction, so



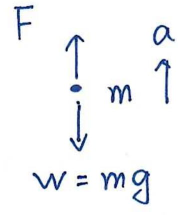
Consider again direction ① & ②

$$ma\cos\alpha = mg\sin\alpha + N\mu$$

$$N = ma\sin\alpha + mg\cos\alpha$$

then $F_{max} = g(M+m)\left(\frac{\sin\alpha + \mu\cos\alpha}{\cos\alpha - \mu\sin\alpha}\right)$ #

12. A small rocket with mass 20.0 kg is moving in free fall toward the earth. Air resistance can be neglected. When the rocket is 80.0 m above the surface of the earth, it is moving downward with a speed of 30.0 m/s. At that instant the rocket engines start to fire and produce a constant upward force F on the rocket. Assume the change in the rocket's mass is negligible. What is the value of F if the rocket's speed becomes zero just as it reaches the surface of the earth, for a soft landing?



$\Sigma \vec{F} = m\vec{a} = F - mg = ma$, $F = m(g+a)$

Find the value of a for a soft landing

$$a = \frac{v_f^2 - v_i^2}{2\Delta x} = \frac{30^2}{2(80)} = 5.63\ m/s^2$$

$$F = 20 \cdot (9.8 + 5.63) = 309\ N \quad \#$$

13. (*) A spring with a uniform mass density ρ has a spring constant k. When the spring is placed horizontally on a table without being stretched, its length is L. What is its length when it is hung vertically and stretched by the gravitational force? Note that when the spring is hung vertically, its mass density is no longer uniform, because the upper part is stretched more than the lower part.

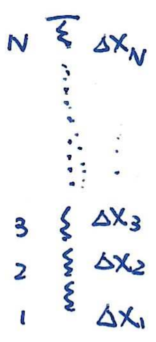
Method 1

Imagine the spring consists of N small springs linking in series

→ No.
→ N identical spring

each small spring has effective spring constant $k' = Nk$, mass $\rho L / N$

hung vertically



Consider the first spring

$k'\Delta x_1 = 0$, $\Delta x_1 = 0$

$k'\Delta x_2 = \frac{\rho L g}{N}$, $\Delta x_2 = \frac{\rho L g}{N^2 k}$

$k'\Delta x_3 = \frac{2\rho L g}{N}$, $\Delta x_3 = \frac{2\rho L g}{N^2 k}$

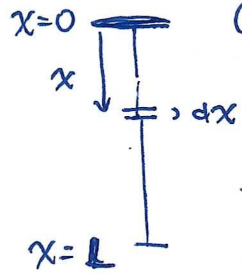
$\vdots$

$k'\Delta x_N = \frac{(N-1)\rho L g}{N}$, $\Delta x_N = \frac{(N-1)\rho L g}{N^2 k}$

$$\Delta x = \Delta x_1 + \Delta x_2 + \cdots + \Delta x_N = \frac{\rho L g}{N^2 k}[0+1+2+\cdots+(N-1)]$$

$$= \frac{\rho L g}{N^2 k} \cdot \frac{(N-1)N}{2} = \frac{\rho L g}{2k}\left(\frac{N-1}{N}\right) , \quad \lim_{N\to\infty} \Delta x = \frac{\rho L g}{2k} \quad \#$$

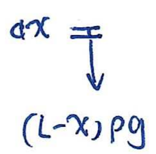
Method 2 (Integration)



Consider a small section at x , the length is dx ,

The effective spring constant $k' = \frac{L}{dx} k$

The weight hung by this section is $(L-x)\rho g$



$$d(\Delta x) k' = (L-x)\rho g$$

$$d(\Delta x) = \rho g \frac{L-x}{Lk} dx$$

$$\Delta_{total} = \int_0^L d(\Delta x) = \frac{\rho g}{Lk}\int_0^L (L-x)dx$$

$$= \frac{\rho g}{Lk}\left(L^2 - \frac{L^2}{2}\right) = \frac{\rho g L}{2k} \quad \#$$

$\therefore$ Total length $= L + \frac{\rho g L}{2k}$

Work and Energy

14. A soccer ball with mass 0.420 kg is initially moving with speed 2.00 m/s. A soccer player kicks the ball, exerting a constant force of magnitude 40.0 N in the same direction as the ball's motion. Over what distance must the player's foot be in contact with the ball to increase the ball's speed to 6.00 m/s?

Work - energy theorem

$$W = \Delta K = \frac{1}{2} m (V_f^2 - V_i^2) = 0.5\,(0.420\,kg)\,(6^2 - 2^2\ m^2/s^2) = 6.72\ J$$

$$W = F \Delta x \ , \quad \therefore\ \Delta x = \frac{W}{F} = \frac{6.72 J}{40.0 N} = 0.168\ m$$

#

15. A force in the $+x$-direction with magnitude $F(x) = 18.0\text{ N} - (0.530\text{ N/m})\,x$ is applied to a 6.00 kg box that is sitting on the horizontal, frictionless surface of a frozen lake. $F(x)$ is the only horizontal force on the box. If the box is initially at rest at $x = 0$, what is its speed after it has traveled 14.0 m?

$$W = \int_0^x F(x')\,dx' = 18.0x - 0.265x^2 = \Delta K = \frac{1}{2} m V_f^2$$

$$V_f = \sqrt{\frac{2W}{m}} = \left\{\frac{2[(18.0)(14) - 0.265(14)^2]}{6}\right\}^{\frac{1}{2}} = 8.17\ m/s$$

#

16. It is 5.0 km from your home to the physics lab. As part of your physical fitness program, you could run that distance at 10 km/h (which uses up energy at the rate of 700 W), or you could walk it leisurely at 3.0 km/h (which uses energy at 290 W). Which choice would burn up more energy, and how much energy (in joules) would it burn? Why does the more intense exercise burn up less energy than the less intense exercise?

$$E_R\ (\text{energy by running}) = 700\,W \cdot \left(\frac{5\ km}{10\ km/h}\right) = 350\ W{\cdot}h$$

$$E_W\ (\text{energy by walking}) = 290\,W \cdot \left(\frac{5\ km}{3.0\ km/h}\right) = 483\ W{\cdot}h$$

∴ Walking burns more energy

Time and power both matter.