11460151l 李威志 大氣一

1.

(a)

$$v(t) = \frac{d\vec{r}}{dt} = \frac{d}{dt}(3.00t\hat{i} - 6.00t^3\hat{j}) = 3.00\hat{i} - 18.00t^2\hat{j}$$

(b)

$$a(t) = \frac{d\vec{v}}{dt} = \frac{d}{dt}(3.00\hat{i} - 18.00t^2\hat{j}) = 0\hat{i} - 36.00t\hat{j}$$

(c)

$$\vec{r}(1.00) = 3.00\hat{i} - 6.00\hat{j} \ (m)$$

$$\vec{v}(1.00) = 3.00\hat{i} - 18.00t^2\hat{j} \ (m/s)$$

2.

離開斜坡時的 v：

$V^2 = V_0^2 + 2aS$ $\quad V^2 = 0 + 2 \cdot 1.9 \cdot 200 = 760$ $\quad V = \sqrt{760} \approx 27.57\ m/s$

$$\begin{cases} V_{x_0} = V\cos(35^\circ) \approx 27.57 \cdot \cos(35^\circ) \approx 22.59\ m/s \\ V_{y_0} = V\sin(35^\circ) \approx 27.57 \cdot \sin(35^\circ) \approx 15.81\ m/s \end{cases}$$

火箭最高高度：

$V_y^2 = V_{y_0}^2 - 2gh$ $\quad 0 = 15.81^2 - 2 \cdot 9.8h$ $\quad h \approx 12.76m$

$200.0\sin 35^\circ \approx 114.7\ m$ $\quad$ 總高：$12.76 + 114.7 \approx 127.5\ m$

A點起最大水平距離：

垂直運動：$y(t) = V_{y_0}t - \frac{1}{2}gt^2$

$-114.7 = 15.81t - 4.9t^2$ $\quad t = \frac{15.81 \pm 49.76}{9.8}$ (負不合) $\quad t \approx 6.71\ s$

$x = 22.59 \cdot 6.71 \approx 151.7\ m$

3\.

(a)

$F = ma$

$72 = (12.0 + 6.0)\,a \qquad a = 4\ m/s^2$

(b)

$F_1 - F_{21} = 6.0 \times 4 = 24$

$72 - F_{21} = 24 \qquad F_{21} = 48\ N$

(c)

$72 - 48 = 24\ (N)$

4\.

軌道施的法向力

$F_c = N_B + mg = ma_c$

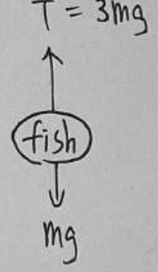
向心力

$6.00 + 0.800 \cdot 9.8 = 0.800\,a_c$

$13.84 = 0.800\,a_c \qquad a_c = 17.3\ m/s^2$

$F_c = N_A - mg = ma_c \qquad N_A = 0.800 \cdot 17.3 + 0.800 \cdot 9.8 = 21.68\ (N)$

5\.



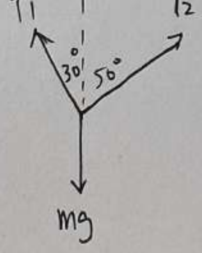
$T - mg = ma$

$2mg = ma$

$a = 2g = 2 \cdot 9.8 = 19.6\ m/s^2$ (方向向上)

6\.

(a)



在X分量 → $T_1 \sin 30^\circ = T_2 \sin 50^\circ$

$\sin 50^\circ > \sin 30^\circ$ 故 $T_1 > T_2$

左邊繩子張力較大

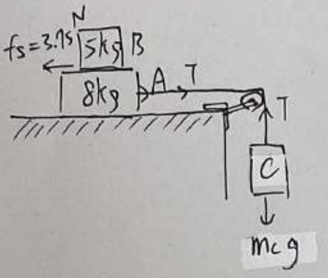
(b) 因左繩張力大，設左 rope 張力 = 5000 N

$T_2 = \dfrac{2500}{\sin 50^\circ} \approx 3263.7\ N$

$mg = 5000 \cdot \cos 30^\circ + 3263.7 \cdot \cos 50^\circ \approx 6428\ N$

7.

AB之間的靜摩擦力 : $5 \times 9.8 \times 0.750 = 36.75\ N$



~~設C為x kg~~

~~A的加速度 :~~ $\cancel{\frac{9.8x}{13}\ m/s^2}$

$\cancel{\frac{9.8x}{8+x} = \frac{3.75}{5}}$

B相對於A的 a_{max} : $\frac{36.75}{5} = 7.35\ m/s^2$

$T = 13.00 \times 7.35 = 95.55\ N$

$m_c g - T = m_c a_{max}$ $\quad T = m_c(g - a_{max})$

$m_c(g - a_{max}) = (m_A + m_B)\, a_{max}$ $\quad m_c = \frac{13 \times 7.35}{9.8 - 7.35} = 39\,(kg)$

8.

(a)

$ma = mg - cv^2$ $\quad m\frac{dv}{dt} = mg - cv^2$

(b)

達到 terminal velocity $\Rightarrow a = 0$

$0 = mg - cv_t^2$ $\quad V_t = \sqrt{\frac{mg}{c}}$

(c)

$m\frac{dv}{dt} = mg - cv^2$ $\quad \frac{dv}{dt} = g - \frac{c}{m}v^2$

$\frac{dv}{g - \frac{c}{m}v^2} = dt$ 設 $k = \sqrt{\frac{c}{mg}}$ 則 $\frac{dv}{g(1-k^2v^2)} = dt$ $\quad \int \frac{dv}{1-k^2v^2} = \int g\,dt$

$\frac{1}{2k}\ln\left(\frac{1+kv}{1-kv}\right) = gt$ 得 $\frac{1+kv}{1-kv} = e^{2kgt}$ $\quad kv = \tanh(kgt)$ $\quad V(t) = \frac{1}{k}\tanh(kgt)$

回代 $k = \sqrt{\frac{c}{mg}}$ $\Rightarrow$ $v(t) = \sqrt{\frac{mg}{c}}\tanh\left(\sqrt{\frac{gc}{m}}\,t\right)$

(d)

$m = 60kg$

$c = 0.430$

$g = 9.8\ m/s^2$

$V_t = \sqrt{\frac{mg}{c}} = \sqrt{\frac{60 \times 9.8}{0.430}} \approx 36.98\ m/s$

$0.9V_t \approx 0.9 \times 36.98 \approx 33.28\ m/s$

$v(t) = V_t \tanh\left(\sqrt{\frac{gc}{m}}\,t\right)$

代入 $0.9V_t$ $\quad 0.9 = \tanh\left(\sqrt{\frac{gc}{m}}\,t\right)$

$\sqrt{\frac{gc}{m}}\,t = \tanh^{-1}(0.9)$

$\tanh^{-1}(0.9) = \frac{1}{2}\ln\left(\frac{1+0.9}{1-0.9}\right) = \frac{1}{2}\ln(19)$

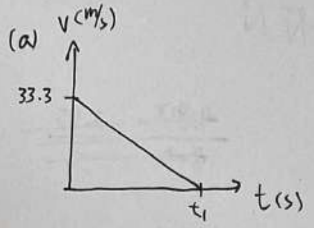
$\approx \frac{1}{2} \cdot 2.944 \approx 1.472$

$\sqrt{\frac{gc}{m}} = \sqrt{\frac{9.8 \times 0.430}{60.0}} \approx \sqrt{0.0702} \approx 0.265$

$t = \frac{1.472}{0.265} \approx 5.55\ s$

9.

(a)



$$\frac{33.3 \times t_1}{2} = 180 \qquad t_1 \approx 10.81\ s$$

$$a = \frac{33.3}{10.81} \approx 3.08\ m/s^2$$

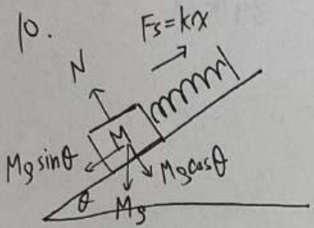
(b)

$$1400 \times 3.08 = 4312\ (N)$$

(c)

$$f_s = N\mu_s \qquad 4312 = 1400 \cdot 9.8 \cdot \mu_s \qquad \mu_s \approx 0.314$$

10.



(a) $f_{s.max} = \mu N$

$$N - Mg\cos\theta = 0 \qquad N = Mg\cos\theta$$

$$f_{s.max} = \mu Mg\cos\theta \qquad kx_{max} + \mu Mg\cos\theta = Mg\sin\theta$$

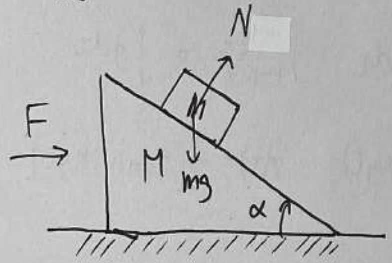
$$kx_{max} = Mg\sin\theta - \mu Mg\cos\theta \qquad x_{max} = \frac{Mg\sin\theta - \mu Mg\cos\theta}{k}$$

(b)

f_s 方向向下

$$kx_{max} = Mg\sin\theta + \mu Mg\cos\theta \qquad x_{max} = \frac{Mg\sin\theta + \mu Mg\cos\theta}{k}$$

11.



(a) m與M水平方向加速度相同 設為a

m在垂直分量的 $a=0$

$$\Sigma F_y = N\cos\alpha - mg = 0 \qquad N = \frac{mg}{\cos\alpha}$$

$$\Sigma F_x = N\sin\alpha = ma \qquad N代入 \Rightarrow \frac{mg}{\cos\alpha}\sin\alpha = ma$$

$$a = g\tan\alpha$$

考慮整個系統 $(m+M)$ $\quad F = (M+m)(g\tan\alpha)$

(b) I：物向上滑

f_s 朝下

$$\begin{cases} N\cos\alpha - mg - f_s\sin\alpha = 0 & \text{①} \\ N\sin\alpha + f_s\cos\alpha = ma & \text{②} \end{cases} \qquad f_s \le \mu_s N$$

$$N = \frac{mg + f_s\sin\alpha}{\cos\alpha}$$ 代入② 使用 $F=(M+m)a$

$$\Rightarrow F_{min} = (M+m)g\tan(\alpha+\phi_s) \qquad \tan\phi_s = \mu_s$$

II：物向下滑

f_s 朝上

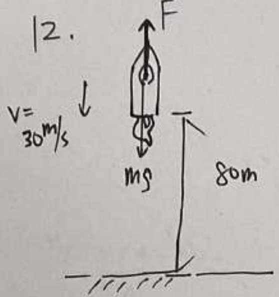
$$\begin{cases} N\cos\alpha - mg + f_s\sin\alpha = 0 & \text{①} \\ N\sin\alpha - f_s\cos\alpha = ma & \text{②} \end{cases}$$

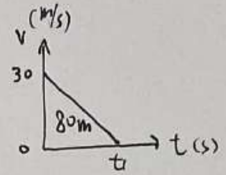
$$f_s < \mu_s N \qquad N = \frac{mg - f_s\sin\alpha}{\cos\alpha}$$

$$F_{min} = (M+m)g\tan(\alpha - \phi_s)$$

$$(M+m)g\tan(\alpha-\phi_s) \le F \le (M+m)g\tan(\alpha+\phi_s)$$

12.





$20 \times \frac{45}{8}$

$15t_1 = 80$

$t_1 = \frac{80}{15} = \frac{16}{3}$

$a = 30 \div \frac{16}{3} = \frac{30}{15} \times \frac{3}{16_8} = \frac{45}{8}\ m/s^2$ 方向向下

$F - mg = ma \qquad F - 20 \cdot 9.8 = 20 \cdot \frac{-45}{8} \qquad F = 196 - 112.5 = 83.5\ (N)$

13.

設 x 自然長度座標（由上到下）

$y(x)$ 拉伸後，原本處在 x 點，變到的位置

一段長度 dx 的彈簧，位置為 x 上方受拉力 $T(x)$，下方拉力 $T(x+dx)$

$$T(x+dx) - T(x) = \rho g\, dx \quad \Rightarrow \quad \frac{dT}{dx} = \rho g \quad \text{積分得 } T(x) = \rho g x + C$$

由於最下端 $x=L$ 沒有再受拉力，所以 $T(L)=0 \qquad 0 = \rho g L + C \qquad C = -\rho g L$

拉力分佈為 $T(x) = \rho g(x-L)$

* 虎克定律關係應變與張力 $T(x) = k\left(\frac{dy}{dx} - 1\right) \Rightarrow \frac{dy}{dx} = 1 + \frac{T(x)}{k} = 1 + \frac{\rho g(L-x)}{k}$

彈簧伸長後長度：$y(L) = \int_0^L \frac{dy}{dx}dx = \int_0^L \left(1 + \frac{\rho g(L-x)}{k}\right)dx$

$$y(L) = L + \frac{\rho g}{k}\int_0^L (L-x)dx = L + \frac{\rho g}{k}\left[Lx - \frac{x^2}{2}\right]_0^L = L + \frac{\rho g}{k}\left(L^2 - \frac{L^2}{2}\right) = L + \frac{\rho g L^2}{2k}$$

$$L_{new} = L + \frac{\rho g L^2}{2k}$$

14.

$W = \Delta K = Fd$

$\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}m(v_f^2 - v_i^2)$ $\quad v_f$ 為末速 $\quad v_i$ 為初速

$40 \times d = \frac{1}{2} \times 0.420(6^2 - 2^2) \qquad d = \frac{6.72}{40} = 0.168m$

15.

$W = \int_0^{14} F(x)\,dx$

將 $F(x) = 18 - 0.530x$ 代入

$\Rightarrow W = \int_0^{14}(18 - 0.530x)dx$

$= \int_0^{14} 18\,dx - \int_0^{14} 0.530x\,dx$

$= 18x\Big|_0^{14} - 0.530\frac{x^2}{2}\,dx$

$= 18 \cdot 14 - 0.530 \cdot \frac{14^2}{2} = 200.06\ (J)$

$W = \frac{1}{2}mv^2 = 200.06\ (J)$

$\frac{1}{2} \cdot 6 \cdot v^2 = 200.06$

$v^2 = 66.69$

$v = \sqrt{66.69} \approx 8.17\ m/s$

16.

$t_{run} = \frac{5}{10} = 0.5\,hr = 1800\,s$

$t_{walk} = \frac{5}{3} \approx 1.67\,hr \approx 6000\,s$

$E_{run} = P_r \times t_{run} = 700 \times 1800 = 1260000\,J$

$E_{walk} = P_w \times t_{walk} = 290 \times 6000 = 1740000\,J$

① 選擇 walk , 1740000J

② 走路相比跑步花費時間長很多,因此 burn up more energy

$\frac{-2x^3-2x+2x^4}{x^4}$

$\frac{-2x^2-2+2x^3}{x^3}$

$\frac{-2x \cdot x^2-(1-x^2) \cdot 2x}{x^4}$

$\frac{2x}{x^4}$

$\frac{1-x^2}{x^2}$

$2\left(\frac{u-1}{u+1}\right)\left(\frac{(u+1)-(u-1)}{(u+1)^2}\right)$

$\frac{4(u-1)}{(u+1)^3}$

$\frac{1-1}{x^2}$ x^2