**EE1101 Signals and Systems JAN—MAY 2019**
**Tutorial 8, Extra Questions**
March 25, 2019

1. The Fourier transform of a signal $x(t)$ is given by

$$X(j\omega) = 2\cos(\omega)\frac{\sin^2(2\omega)}{(2\omega)^2}.$$

What is the value of $x(0)$?

2. Find the fourier transform of $\text{sgn}(t)$ (pronounced as signum (t)) given by

$$\text{sgn}(t) = \begin{cases} 1 & t > 0 \\ -1 & t < 0 \end{cases}$$

Hint: Use $\lim_{a \longrightarrow \infty} e^{-at}u(t)$ for t $> 0$.

3. Given that $x(t)$ has the Fourier transform $X(j\omega)$, express the Fourier transforms of the signals listed below in terms of $X(j\omega)$.

   a) $x_1(t) = x(1-t) + x(-1-t)$
   b) $x_2(t) = x(3t-6)$
   c) $x_3(t) = \frac{d^2}{dt^2}x(t-1)$

4. **(Filtering)** Consider an auditorium with an echo which can be modelled as an impulse response:

$$h(t) = \sum_{k=0}^{\infty} e^{-kT}\delta(t-kT)$$

where $e^{-kT}$ represents the attentuation of the $k$th echo. Find $G(j\omega)$ that cancels the effects of the echos; that is for input $X(j\omega)$, $H(j\omega)G(j\omega) = 1$. Find $g(t)$ from $G(j\omega)$ and show that $g(t) * h(t) = \delta(t)$.

5. Use properties of the Fourier transform to show by induction that the Fourier transform of

$$x(t) = \frac{t^{n-1}}{(n-1)!}e^{-at}u(t), \quad a > 0$$

is

$$X(j\omega) = \frac{1}{(a+j\omega)^n}$$

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