- No Electronic gadgets (including Calculator) are allowed during the examination

- Please write your Name, Roll number and your class teacher's name on the answer script

1. (a) Let $a_n \in \mathbb{R}$, $n \in \mathbb{N}$ be non negative real numbers. Suppose $\sum_{n=1}^{\infty} a_n$ is convergent then prove that $\sum_{n=1}^{\infty} \sqrt{a_n a_{n+1}}$ is also convergent. **(2)**

   (b) Does the series $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n^2+1}}$ converge absolutely, converge conditionally, or diverge? Justify your answer. **(3)**

2. (a) For what values of $p$ does the series $\sum_{n=1}^{\infty} \frac{n^p}{2+n^3}$ converge? **(2)**

   (b) Determine whether the series $\sum_{n=1}^{\infty} 2^{(-1)^n - n}$ converges or not. Justify your answer. **(3)**

3. (a) Determine whether the series $\sum_{n=1}^{\infty} \frac{3^n n!}{n^n}$ converges or not. Justify your answer. **(2)**

   (b) Show that the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^\alpha}$ converges for $\alpha > 1$ and diverges to $\infty$ for $\alpha \leq 1$. **(3)**

4. Prove that the limit of the sequence

$$\sqrt{1}, \sqrt{1+\sqrt{1}}, \sqrt{1+\sqrt{1+\sqrt{1}}}, \cdots$$

exists and find the limit. **(5)**