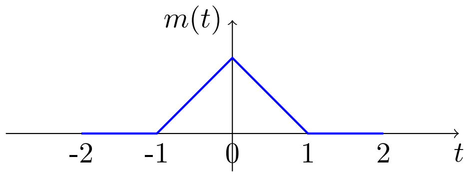
EE1101: Signals and Systems JAN—MAY 2018

Tutorial 1 Solutions

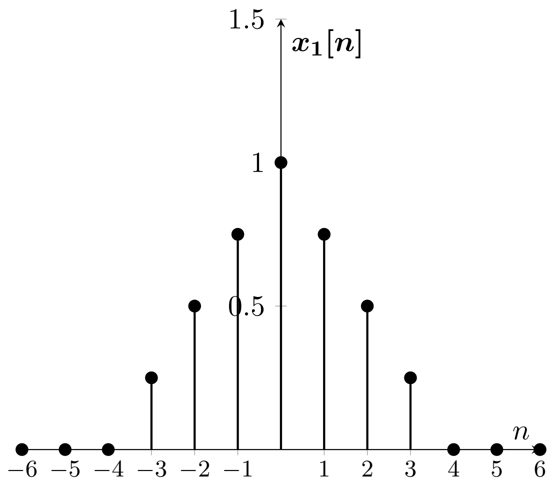
Solution 1

$$x(t) = \begin{cases} 1 - |t| & -1 \le t \le 1 \\ 0 & otherwise \end{cases}$$

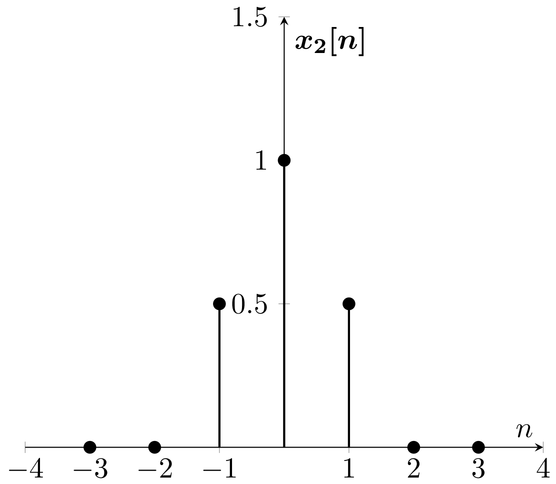
$x(t)$ is a triangular function from -1 to +1. The plot is shown below.



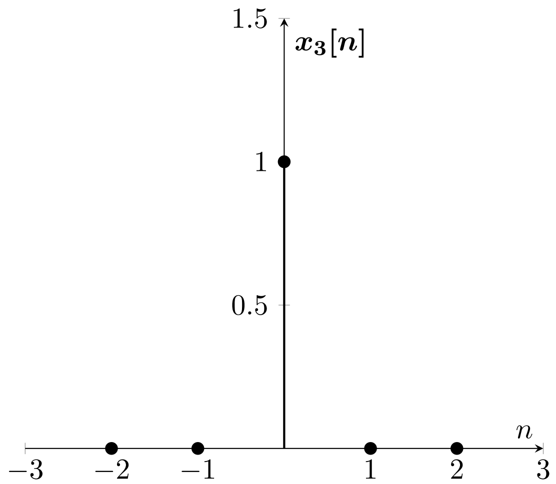
- when sampled at 0.25s, $x(t)$ becomes a discrete sequence taking values at $t = 0.25n$ where $n = 0, \pm1, \pm2, \pm3$ ... The plot is shown below.



- when sampled at 0.5s, $x(t)$ becomes a discrete sequence taking values at $t = 0.5n$ where $n = 0, \pm1, \pm2, \pm3$ ... The plot is shown below.

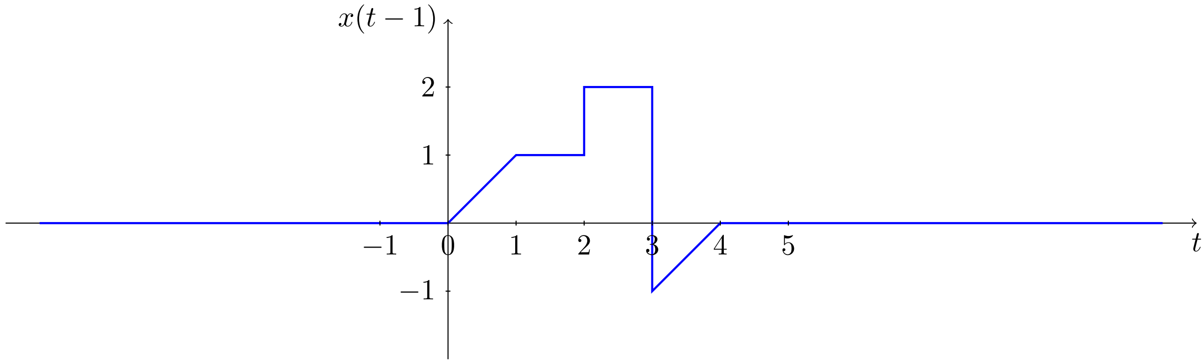


- when sampled at 1s, $x(t)$ becomes a discrete sequence taking values at $t = n$ where $n = 0, \pm1, \pm2, \pm3$... The plot is shown below.

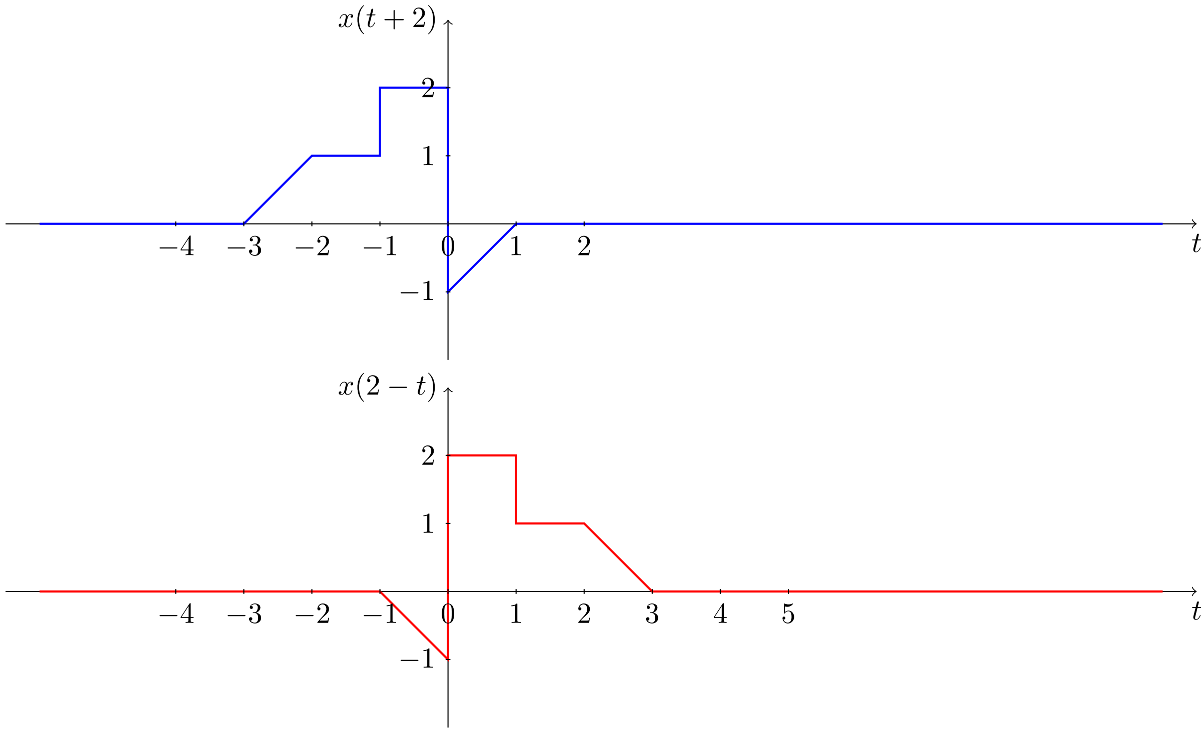


Solution 2

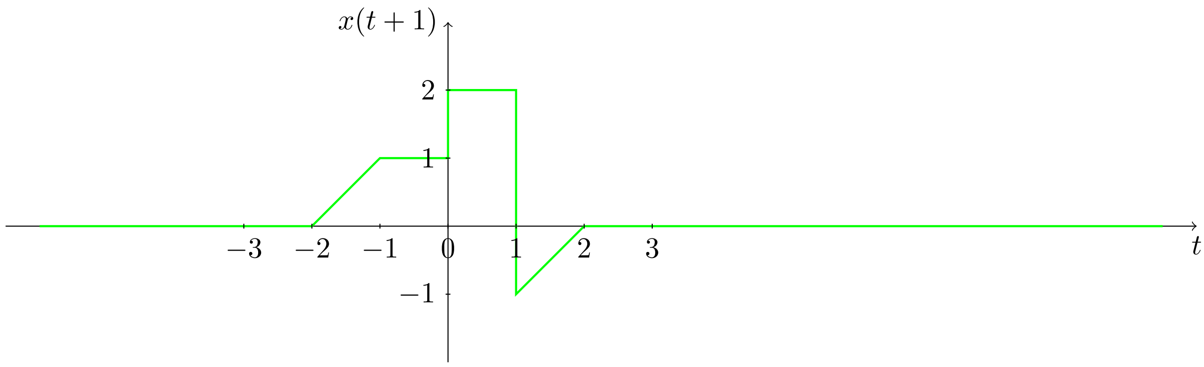
(a) $x(t-1)$ can be obtained by shifting $x(t)$ right by 1 unit as shown below.

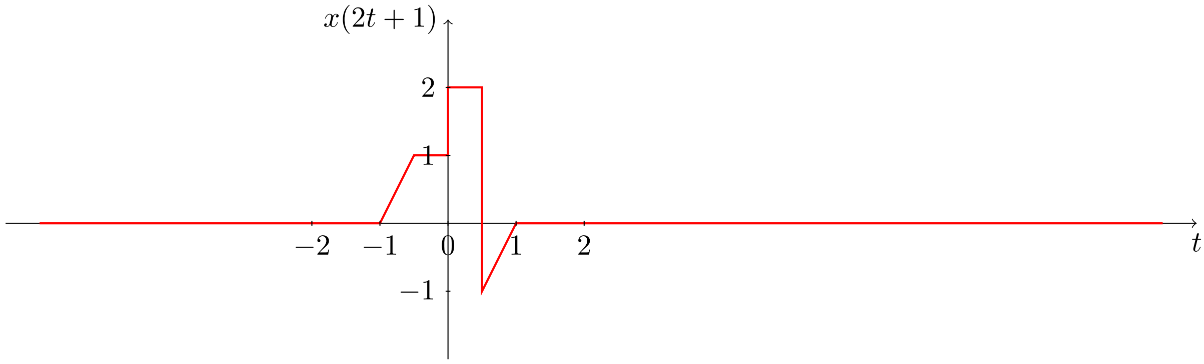


(b) $x(2-t) = x(-t+2)$ can be obtained by shifting $x(t)$ left by 2 units and then reversing the time axis as shown below.

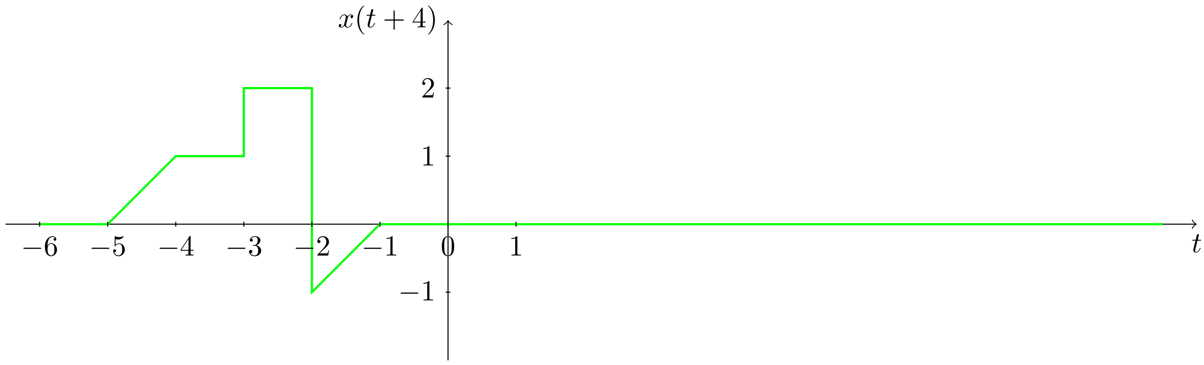


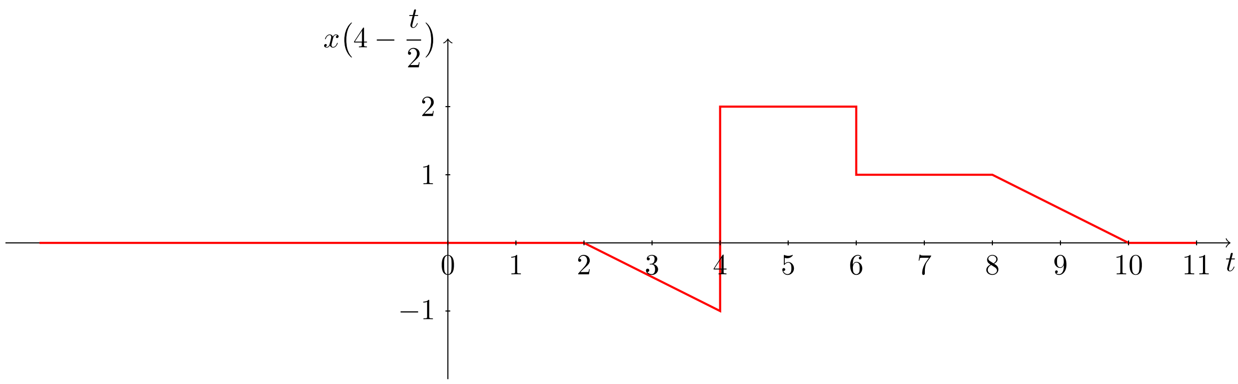
(c) $x(2t+1)$ can be obtained by shifting $x(t)$ left by 1 unit and then scaling the time axis by a factor of 2 as shown below.



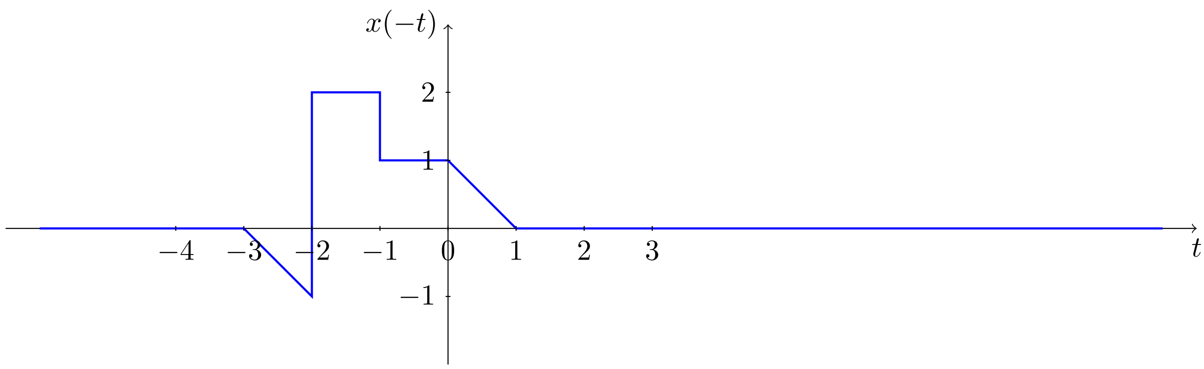


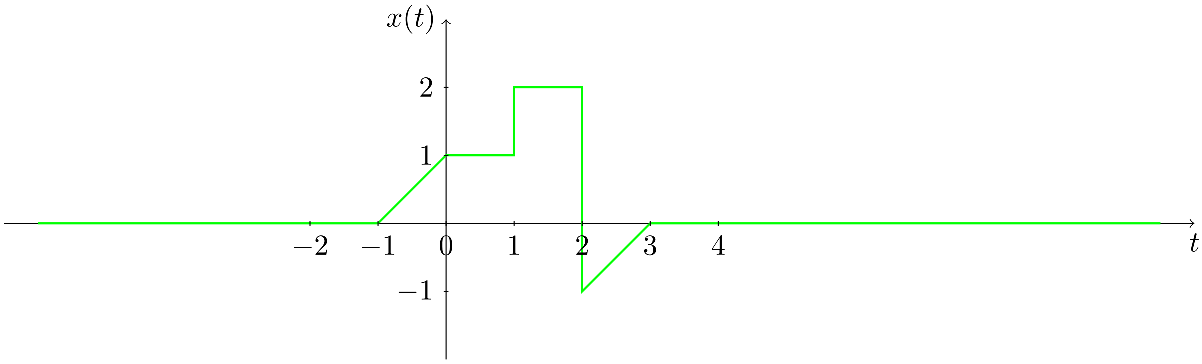
(d) $x\left(4 - \frac{t}{2}\right)$ can be plotted in a similar way as shown below.

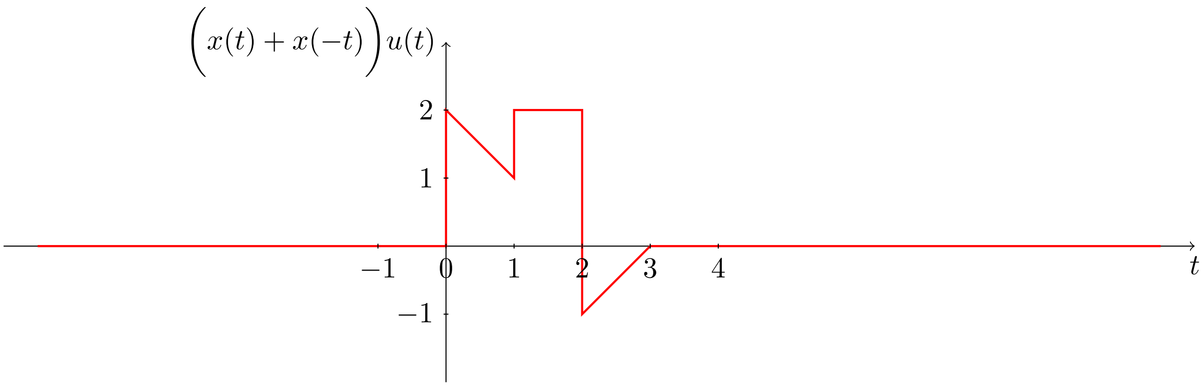




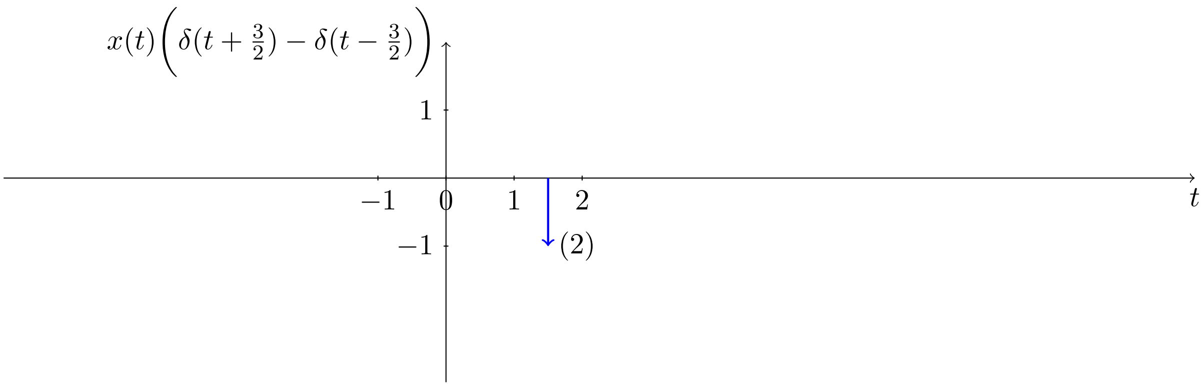
(e) The signals $x(t)$ and $x(-t)$ are added and the result is multiplied with $u(t)$, which makes the resultant signal causal.







(f) The signal $x(t)\Big(\delta(t+\frac{3}{2})-\delta(t-\frac{3}{2})\Big)$ consists of impulse samples of the signal $x(t)$ at $t=\dfrac{3}{2}$ and $t=-\dfrac{3}{2}$.



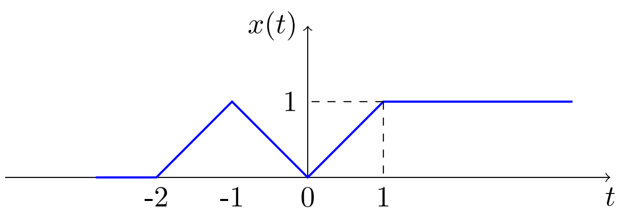
Solution 3

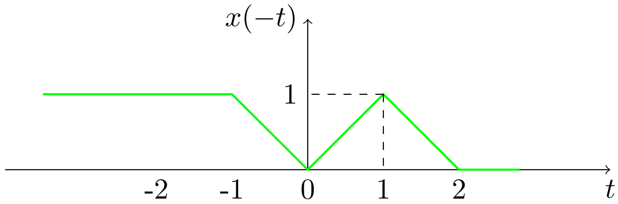
(a) The even part of a signal $x(t)$ can be calculated as

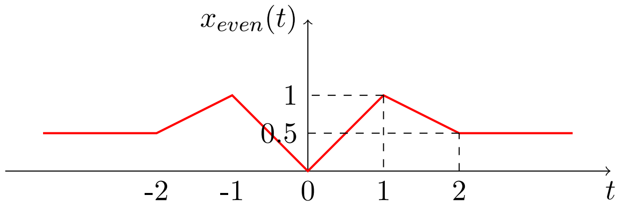
$$x_{even}(t)=\frac{x(t)+x(-t)}{2}$$

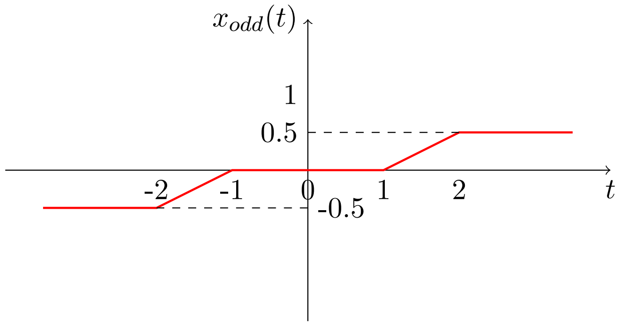
The odd part of a signal $x(t)$ can be calculated as

$$x_{odd}(t)=\frac{x(t)-x(-t)}{2}$$







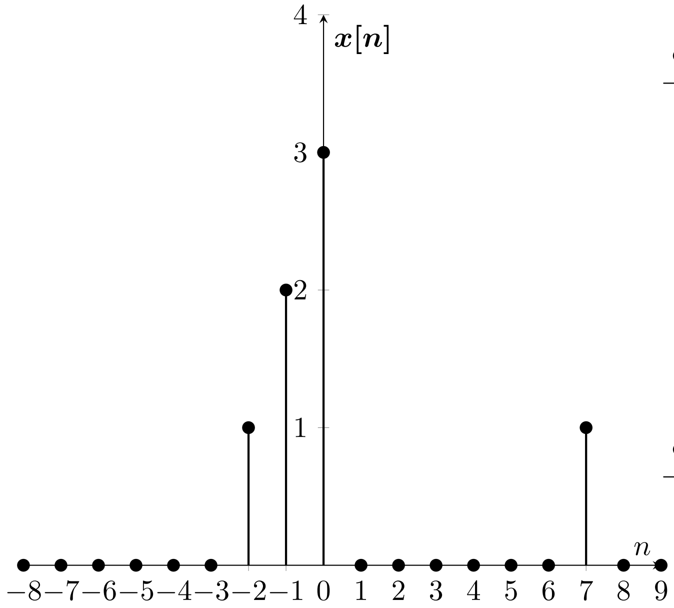


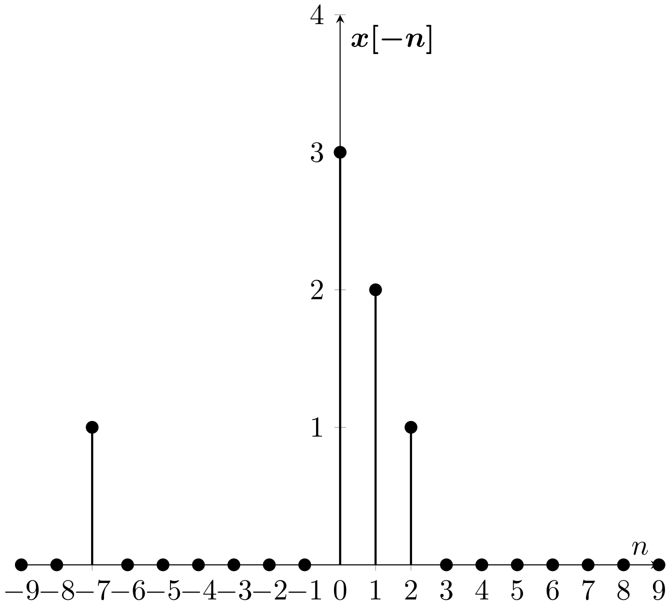
(b) The even part of a signal $x[n]$ can be calculated as

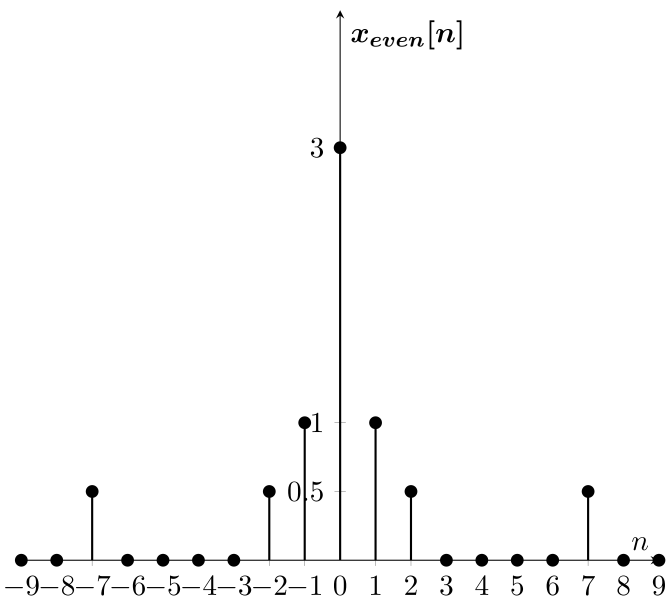
$$x_{even}[n] = \frac{x[n] + x[-n]}{2}$$

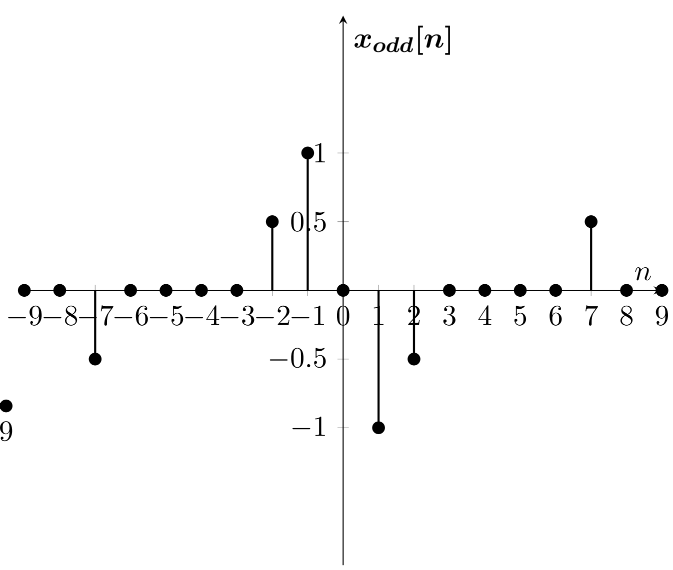
The odd part of a signal $x[n]$ can be calculated as

$$x_{odd}[n] = \frac{x[n] - x[-n]}{2}$$





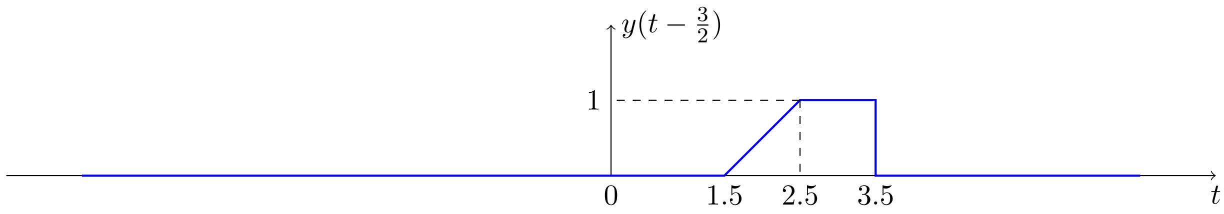


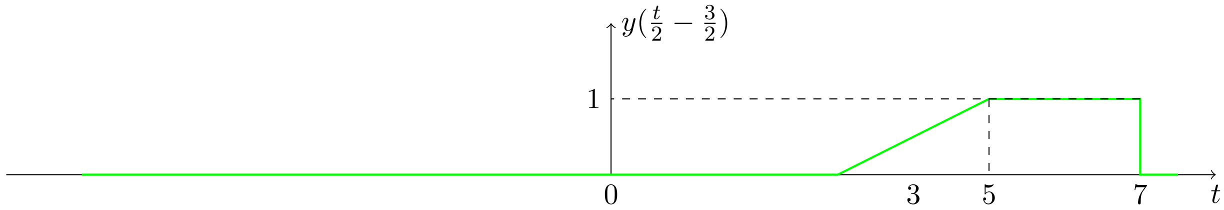


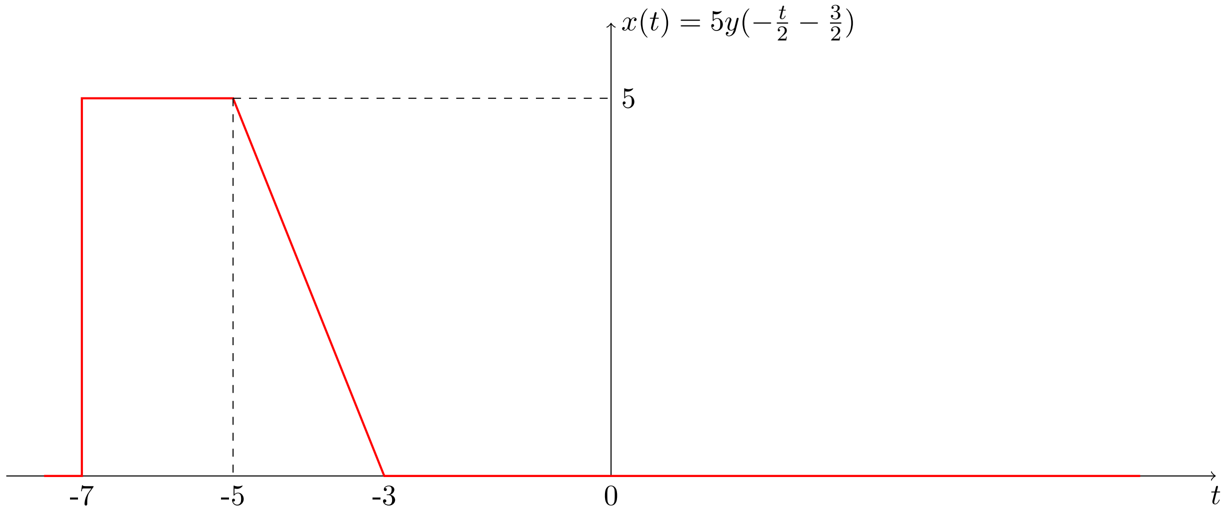
Solution 4

The signal $x(t)$ can be obtained from $y(t)$ through the change of variables as shown below

$$y(t) = \frac{1}{5}x(-2t-3)$$
$$\text{Let} \quad t' = -2t-3$$
$$y\Bigg(-\frac{(t'+3)}{2}\Bigg) = \frac{1}{5}x(t')$$
$$x(t) = 5y\Bigg(-\frac{t}{2}-\frac{3}{2}\Bigg)$$



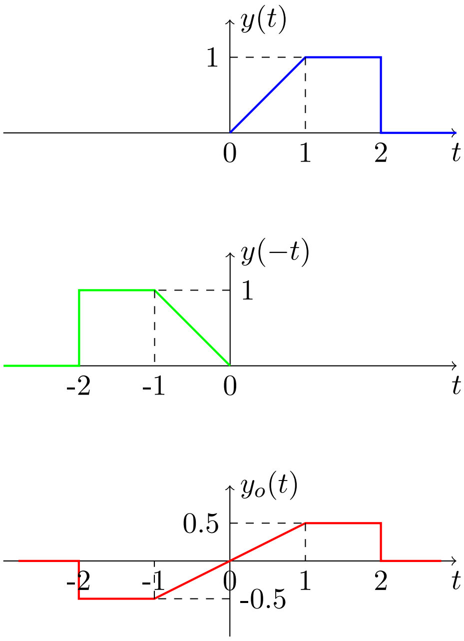


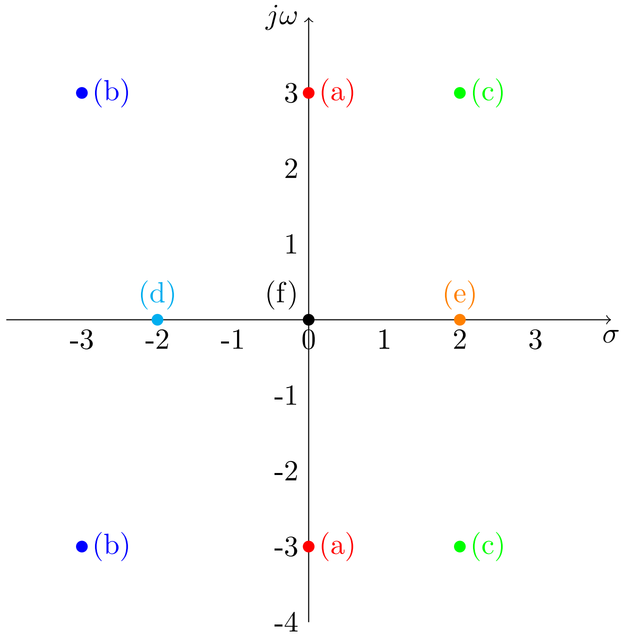


For the odd portion of $y(t)$,

$$y_o(t) = \frac{y(t)-y(-t)}{2}$$

$$y_o = \begin{cases} 0 & t < -2 \\ -0.5 & -2 \le t < -1 \\ \frac{t}{2} & -1 \le t < 1 \\ 0.5 & 1 \le t \le 2 \\ 0 & t > 2 \end{cases}$$





Solution 5

For a signal expanded as $x(t) = \sum_k A_k e^{s_k t}$ where A_k is in general complex, the complex frequency components present are the s_k's and each $s_k = \sigma_k + j\omega_k$

(a) $cos(3t) = \frac{1}{2}(e^{3jt} + e^{-3jt})$
the complex frequencies thus are $j3$ and $-j3$.

(b) $e^{-3t}cos(3t) = \frac{1}{2}e^{-3t}(e^{3jt} + e^{-3jt}) = \frac{1}{2}(e^{-3+3jt} + e^{-3-3jt})$
the complex frequencies thus are $-3+j3$ and $-3-j3$.

(c) $e^{2t}cos(3t) = \frac{1}{2}e^{2t}(e^{3jt} + e^{-3jt}) = \frac{1}{2}(e^{2+3jt} + e^{2-3jt})$
the complex frequencies thus are $2+j3$ and $2-j3$.

(d) e^{-2t}
clearly the complex frequency is -2.

(e) e^{2t}
the complex frequency is 2.

(f) 5
It is a DC signal, hence the frequency is 0.

The complex frequencies are plotted in the complex plane below.

Solution 6

The power of signal $x(t)$ can be calculated as

$$P_x = \lim_{T\to\infty} \frac{1}{T}\int_{t=-T/2}^{T/2} |x(t)|^2 dt$$

For a periodic signal with period T this may be just computed over a time period as:

$$P_x = \frac{1}{T}\int_{t=0}^{T} |x(t)|^2 dt$$

and the RMS value is the square-root of P_x

(a) For the signal $x(t) = \sum_{k=m}^{n} D_k e^{j\omega_k t}$ (periodic with period T), the power can be calculated as

$$\begin{aligned}
P_x &= \frac{1}{T}\int_{t=0}^{T} x(t)x^*(t)dt \qquad (\because |x(t)|^2 = x(t)x^*(t)) \\
&= \frac{1}{T}\int_{t=0}^{T} \sum_{k=m}^{n} D_k e^{j\omega_k t} \sum_{k=m}^{n} D_k^* e^{-j\omega_k t} dt \\
&= \frac{1}{T}\int_{t=0}^{T} \sum_{k=m}^{n} D_k D_k^* e^{j(\omega_k-\omega_k)t} dt \\
&\qquad + \sum_{k=m}^{n} \sum_{l=m;k\neq l}^{n} D_k D_l^* e^{j(\omega_k-\omega_l)t} dt \\
&= \frac{1}{T} * T \sum_{k=m}^{n} D_k D_k^* \\
&\qquad + \sum_{k=m}^{n} \sum_{l=m;k\neq l}^{n} D_k D_l^* \int_{t=0}^{T} e^{j(\omega_k-\omega_l)t} dt \\
&= \sum_{k=m}^{n} |D_k|^2 \qquad (\because \int_{t=0}^{T} e^{j(\omega_k-\omega_l)t} dt = 0)
\end{aligned}$$

The frequencies are distinct and hence the sinusoids are orthogonal and when you integrate $e^{j(\omega_k-\omega_l)t}$ over the period T gives you zero.

(b) (a) $x(t) = 10cos(5t)cos(10t)$

$$\begin{aligned}x(t) &= 2.5(e^{j5t}+e^{-j5t})(e^{j10t}+e^{-j10t})\\ &= 2.5(e^{j15t}+e^{-j5t}+e^{j5t}+e^{-j15t})\end{aligned}$$

From the above expression, power can be calculated as:

$$P_x = 2.5^2 \times 4 = 25$$

RMS value is :

$$RMS = \sqrt{P_x} = 5$$

(b) $x(t) = 10\cos(100t+\frac{\pi}{3})+5\sin(100t+\frac{\pi}{6})$

$$\begin{aligned}x(t) &= 5(e^{j(100t+\frac{\pi}{3})}+e^{-j(100t+\frac{\pi}{3})})\\ &\quad - 2.5j(e^{j(100t+\frac{\pi}{6})}-e^{-j(100t+\frac{\pi}{6})})\\ &= 5(e^{j(100t+\frac{\pi}{3})}+e^{-j(100t+\frac{\pi}{3})})\\ &\quad + 2.5(e^{j(100t+\frac{\pi}{6}-\frac{\pi}{2})}-e^{-j(100t+\frac{\pi}{6}+\frac{\pi}{2})})\\ &= (5e^{j\frac{\pi}{3}}+2.5e^{-j\frac{\pi}{3}})e^{j100t}\\ &\quad + (5e^{-j\frac{\pi}{3}}+2.5e^{j\frac{\pi}{3}})e^{-j100t}\end{aligned}$$

Power:

$$\begin{aligned}P_x &= 2|5e^{j\frac{\pi}{3}}+2.5e^{-j\frac{\pi}{3}})|^2\\ &= 2\times\frac{75}{4}\\ &= 37.5\end{aligned}$$

RMS value is :

$$RMS = \sqrt{P_x} = 6.1237$$

Solution 7

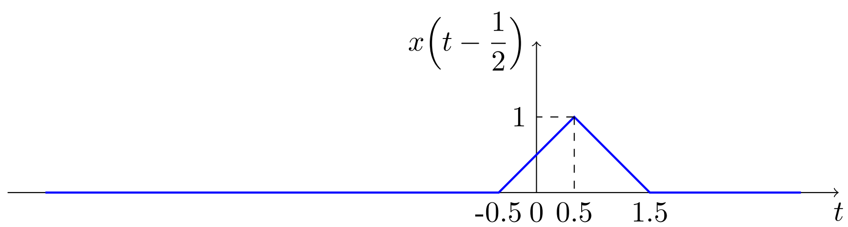
Any signal $x(at+b)$ can be obtained from $x(t)$ in 2 ways

(i) First shifting, then scaling
Find $x(t+b)$ first then replace t with at to obtain $x(at+b)$

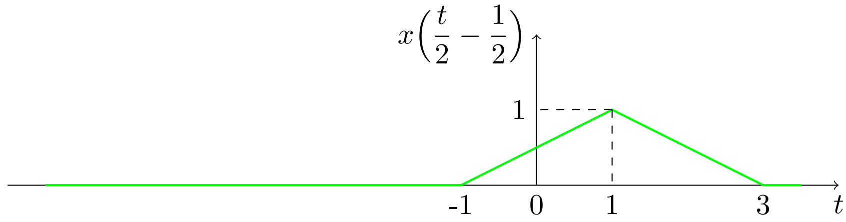
(ii) First scaling, then shifting
$x(at+b) = x(a(t+b/a))$ Find $x(at)$ first then shift left/right depending on the sign of b/a to obtain $x(at+b)$

We shall follow the first method.

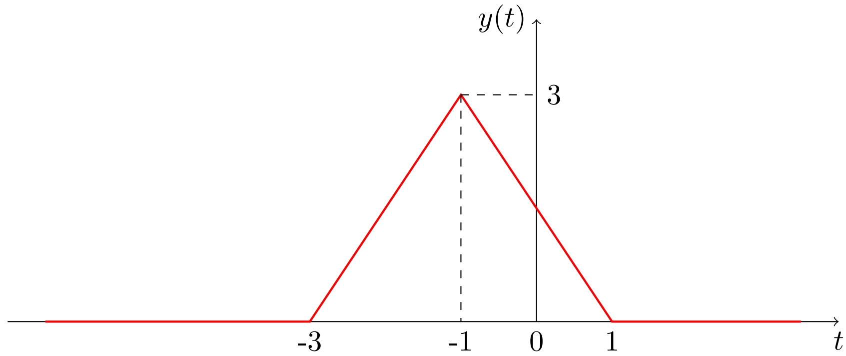
(a) $y(t) = 3x\Big(-\frac{1}{2}(t+1)\Big)$
$y(t) = 3x\Big(-\frac{1}{2}t-\frac{1}{2}\Big)$
First we will find $x\Big(t-\frac{1}{2}\Big)$
Here $a=-1/2, b=1/2$. Since b is positive, we should delay the signal or shift it towards right



Now replace t with $-\frac{1}{2}t$ in the above plot and redraw Since $a = -1/2$, it is a combination of scaling and reflection. We will do scaling with 1/2 first and then take the reflection about y-axis. Since $a = -1/2 < 1$, we should expand the signal.



Now take the reflection about y-axis to get $x\left(-\frac{t}{2} - \frac{1}{2}\right)$. Finally multiply the amplitude by 3 units to obtain $y(t)$



(b) The energy of a signal $x(t)$ can be calculated as

$$E_x = \int_{t=-\infty}^{\infty} |x(t)|^2 dt$$

The signal $y(t)$ can be expressed as $y(t) = y_1(t) + y_2(t)$ where

$$y_1(t) = \frac{3}{2}(t+3), -3 < t < -1$$
$$y_2(t) = -\frac{3}{2}(t-1), -1 < t < 1$$

Hence, Energy of $y(t)$

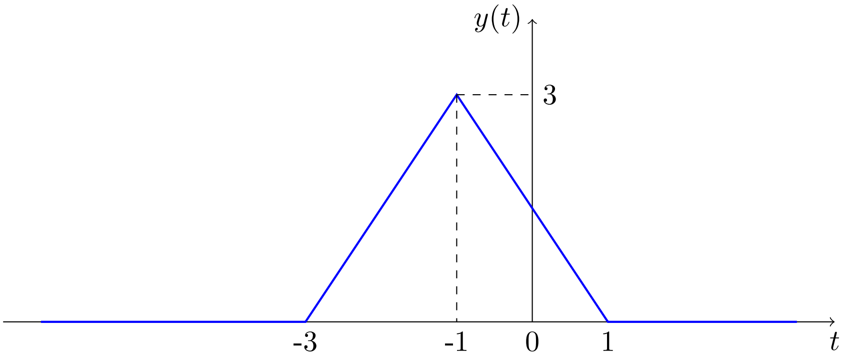
$$\begin{aligned}
E_y &= \int_{-3}^{-1} |y_1(t)|^2 dt + \int_{-1}^{1} |y_2(t)|^2 dt \\
&= \int_{-3}^{-1} \left|\frac{3}{2}(t+3)\right|^2 dt + \int_{-1}^{1} \left|-\frac{3}{2}(t-1)\right|^2 dt \\
&= \frac{9}{4}\left|\frac{(t+3)^3}{3}\right|_{-3}^{-1} + \frac{9}{4}\left|\frac{(t-1)^3}{3}\right|_{-1}^{1} \\
&= \frac{9}{4} \times \frac{8}{3} + \frac{9}{4} \times \frac{8}{3} \\
&= 12
\end{aligned}$$

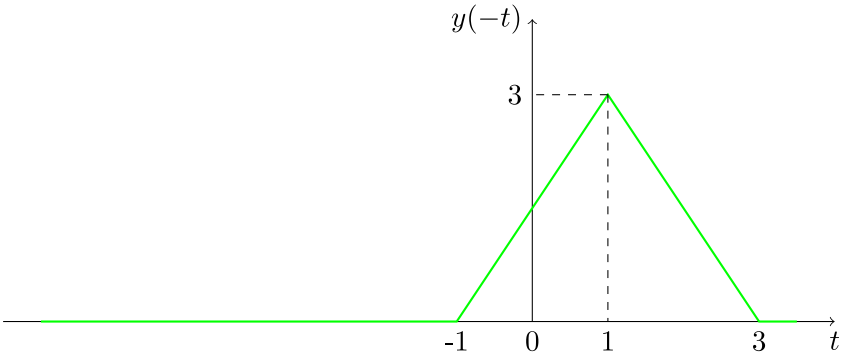
The power of a signal $x(t)$ can be calculated as

$$P_x = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

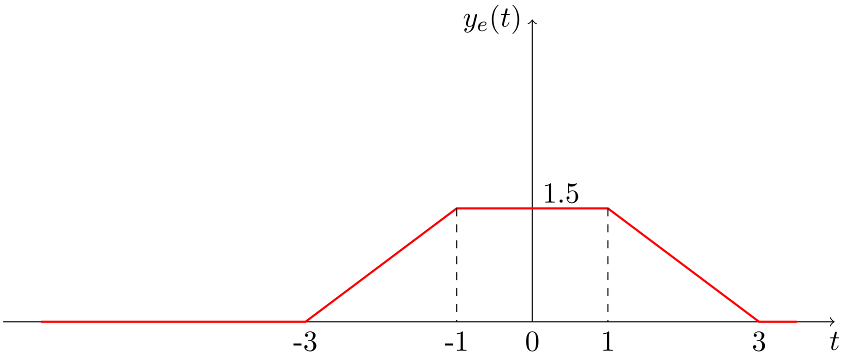
Since the given signal is of finite duration and has finite energy, it is an energy signal. Hence the power is 0.

(c) Even portion of any signal $y(t)$ is found by $y_e(t) = \dfrac{y(t) + y(-t)}{2}$
$y(-t)$ is the reflection of $y(t)$ about y-axis. We will plot $y(-t)$ now.



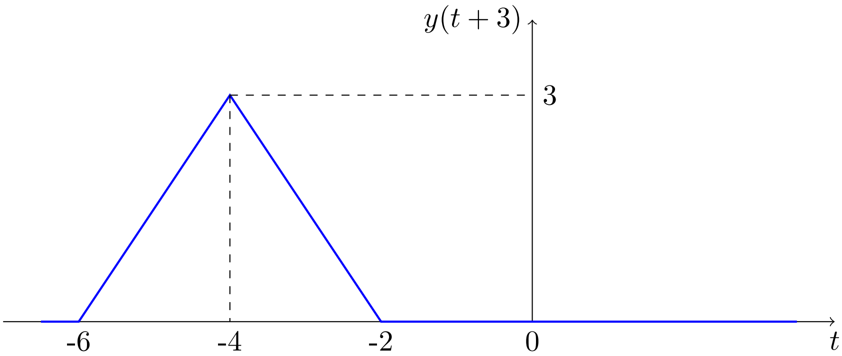


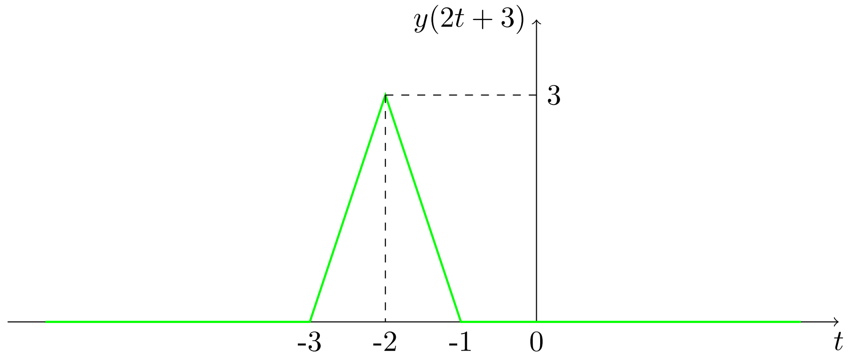
From -1 to +1 lines $-\dfrac{3}{2}(t-1)$ and $\dfrac{3}{2}(t+1)$ add up to give 3, which when divided by 2 gives 1.5



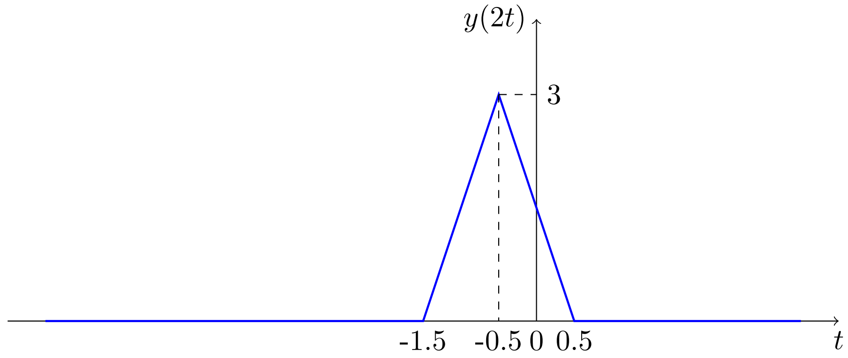
(d)

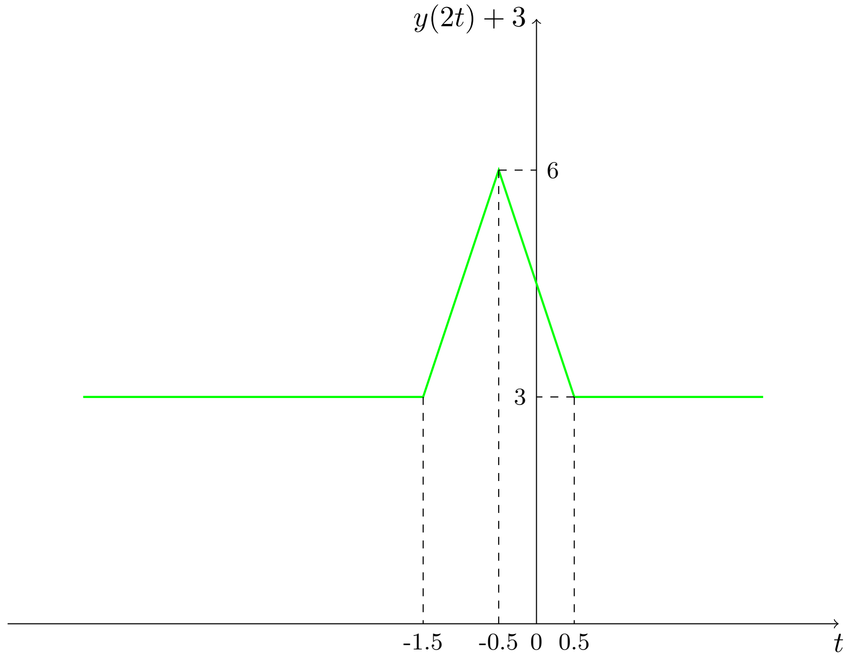
$$a = 2, b = 3$$
$$y(at+b) = y(2t+3)$$



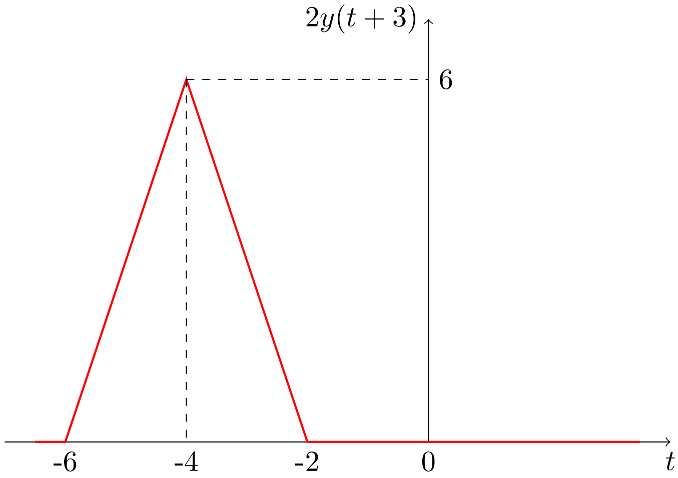


$y(2t) + 3$ is adding a dc value of 3 to $y(t)$

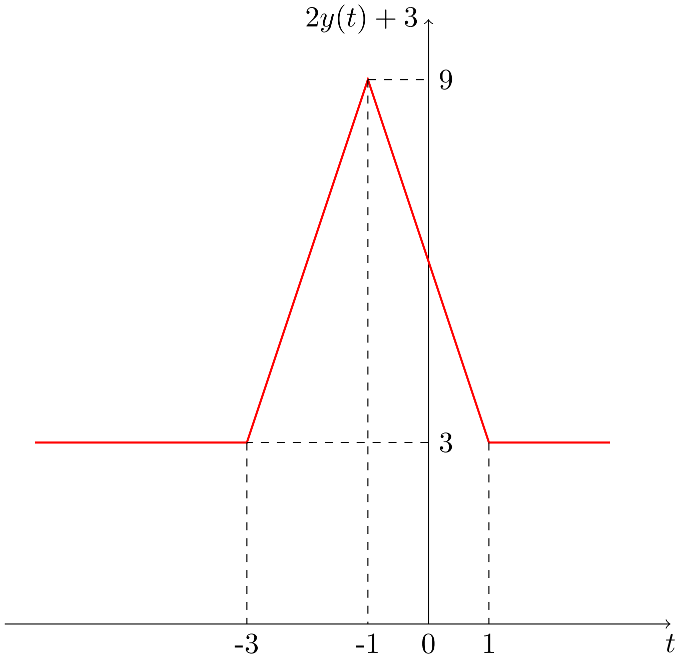




$2y(t+3)$



$2y(t)+3$ is adding a dc of 3 to $y(t)$



Solution 8

The energy of signal $x(t)$ can be calculated as

$$E_x = \int_{t=-\infty}^{\infty} |x(t)|^2 dt$$

(a) (i) The energy of signal $-x(t)$ can be calculated as

$$\begin{aligned} E_1 &= \int_{t=-\infty}^{\infty} \big|\big(-x(t)\big)\big|^2 dt \\ &= \int_{t=-\infty}^{\infty} |x(t)|^2 dt \\ &= E_x \end{aligned}$$

(ii) The energy of signal $x(-t)$ can be calculated as

$$E_2 = \int_{t=-\infty}^{\infty} |x(-t)|^2 dt$$

$$\text{Let } \tau = -t \quad \Rightarrow d\tau = -dt$$

$$\Rightarrow E_2 = -\int_{\tau=\infty}^{-\infty} |x(\tau)|^2 d\tau$$

$$= \int_{\tau=-\infty}^{\infty} |x(\tau)|^2 d\tau$$

$$= E_x$$

(iii) The energy of signal $x(t-T)$ can be calculated as

$$E_3 = \int_{t=-\infty}^{\infty} |x(t-T)|^2 dt$$

$$\text{Let } \tau = t - T \quad \Rightarrow d\tau = dt$$

$$\Rightarrow E_3 = \int_{\tau=-\infty-T}^{\infty+T} |x(\tau)|^2 d\tau$$

$$= \int_{\tau=-\infty}^{\infty} |x(\tau)|^2 d\tau$$

$$= E_x$$

(b) (i) The energy of signal $x(at)$ can be calculated as

$$E_1 = \int_{t=-\infty}^{\infty} |x(at)|^2 dt$$

Case 1: $a > 0 \Rightarrow a = |a|$

Let $\tau = at = |a|t \Rightarrow d\tau = |a|\ dt$

$$\Rightarrow E_1 = \frac{1}{|a|}\int_{\tau=-\infty}^{\infty} |x(\tau)|^2 d\tau$$

$$= \frac{E_x}{|a|}$$

Case 2: $a < 0 \Rightarrow a = -|a|$

Let $\tau = at = -|a|t \Rightarrow d\tau = -|a|\ dt$

$$\Rightarrow E_1 = -\frac{1}{|a|}\int_{\tau=\infty}^{-\infty} |x(\tau)|^2 d\tau$$

$$= \frac{1}{|a|}\int_{\tau=-\infty}^{\infty} |x(\tau)|^2 d\tau$$

$$= \frac{E_x}{|a|}$$

(ii) The energy of signal $x(at-b)$ can be calculated as

$$E_2 = \int_{t=-\infty}^{\infty} |x(at-b)|^2 dt$$

Case 1: $a > 0 \Rightarrow a = |a|$

Let $\tau = at - b = |a|t - b$

$$\Rightarrow d\tau = |a|\ dt$$

$$\Rightarrow E_2 = \frac{1}{|a|}\int_{\tau=-\infty}^{\infty} |x(\tau)|^2 d\tau$$

$$= \frac{E_x}{|a|}$$

Case 2: $a < 0 \Rightarrow a = -|a|$

Let $\tau = at - b = -|a|t - b$

$$\Rightarrow d\tau = -|a|\ dt$$

$$\Rightarrow E_2 = -\frac{1}{|a|}\int_{\tau=\infty}^{-\infty} |x(\tau)|^2 d\tau$$

$$= \frac{1}{|a|}\int_{\tau=-\infty}^{\infty} |x(\tau)|^2 d\tau$$

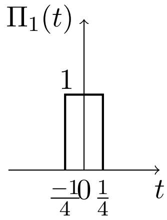
$$= \frac{E_x}{|a|}$$

Solution 9

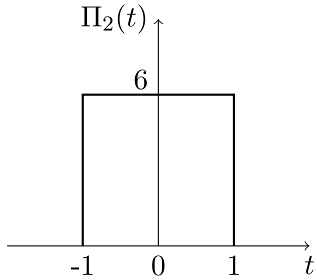
The energy of signal $\Pi(t)$ can be calculated as

$$E_\Pi = \int_{t=-\infty}^{\infty} |\Pi(t)|^2 dt$$

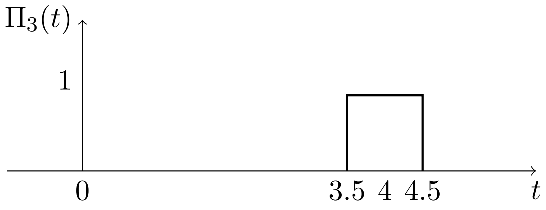
(a) $\Pi_1(t) = \Pi(2t)$
$E_{\Pi_1} = 0.5$



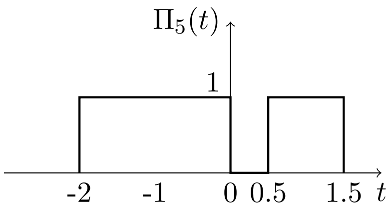
(b) $\Pi_2(t) = 6\Pi(0.5t)$
$E_{\Pi_2} = 72$



(c) $\Pi_3(t) = \Pi(t-4)$
$E_{\Pi_3} = 1$

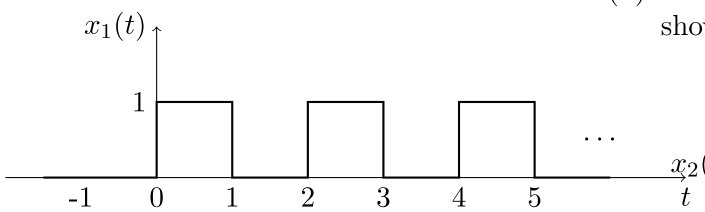


(d) $\Pi_5(t) = \Pi\left(\frac{t+1}{2}\right) + \Pi(t-1)$
$E_{\Pi_5} = 3$



Solution 10

(a) The binary signal $x_1(t)$ can be plotted as shown below:



Energy of the signal

The energy of signal $x_1(t)$ can be evaluated as:

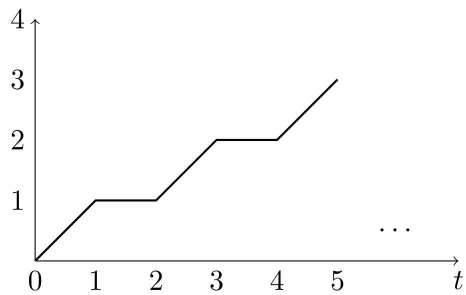
$$
\begin{aligned}
E_1 &= \int_{t=-\infty}^{\infty} |x_1(t)|^2 dt \\
&= \int_{t=0}^{\infty} |x_1(t)|^2 dt \qquad (\because x_1(t) = 0, t < 0) \\
&= \int_{t=0}^{1} 1 \cdot dt + \int_{t=1}^{2} 0 \cdot dt + \int_{t=2}^{3} 1 \cdot dt + \int_{t=3}^{4} 0 \cdot dt + \cdots \\
&= 1 + 0 + 1 + 0 + \cdots \\
&= \infty.
\end{aligned}
$$

Power of the signal

The power of the signal $x_1(t)$ can be evaluated as:

$$
\begin{aligned}
P_1 &= \lim_{T\to\infty} \frac{\int_{t=-T}^{T} |x_1(t)|^2 dt}{2T} \\
&= \lim_{T\to\infty} \frac{1}{2} \frac{\int_{t=0}^{T} |x_1(t)|^2 dt}{T}
\end{aligned}
$$

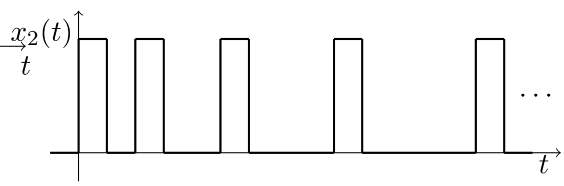
The figure below shows how the numerator function evolves as a function of time.



Consider N pulses. The time elapsed would then be $T = 2N$.

$$
\begin{aligned}
\therefore P_1 &= \lim_{N\to\infty} \frac{1}{2} \frac{N}{2N} \\
&= \frac{1}{4}
\end{aligned}
$$

(b) The binary signal $x_2(t)$ can be plotted as shown below:



Energy of the signal

The energy of signal $x_2(t)$ can be calculated as follows:

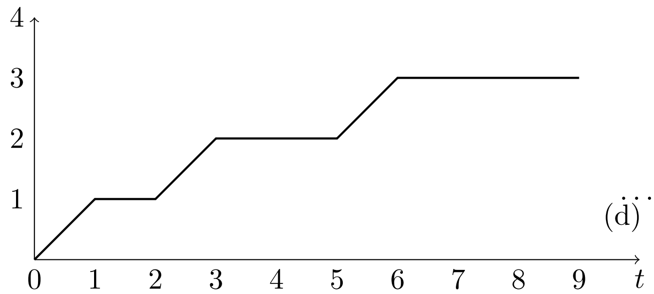
$$
\begin{aligned}
E_2 &= \int_{t=0}^{\infty} |x_2(t)|^2 dt \\
&= \int_{t=0}^{1} 1 \cdot dt + \int_{t=1}^{2} 0 \cdot dt \\
&\quad + \int_{t=2}^{3} 1 \cdot dt + \int_{t=3}^{5} 0 \cdot dt + \cdots \\
&= 1 + 0 + 1 + 0 + \cdots \infty \\
&= \infty
\end{aligned}
$$

Power of the signal

The power of signal $x_2(t)$ can be calculated as follows:

$$P_2 = \lim_{T\to\infty} \frac{\int_{t=-T}^{T} |x_2(t)|^2 dt}{2T}$$
$$= \lim_{T\to\infty} \frac{1}{2} \frac{\int_{t=0}^{T} |x_1(t)|^2 dt}{T}$$

The figure below shows how the numerator function evolves as a function of time.



Consider N pulses. The time elapsed would then be $T = N + (1 + 2 + 3 + \cdots + N)$, i.e. $T = N + \frac{N(N+1)}{2}$

$$\therefore P_2 = \lim_{N\to\infty} \frac{1}{2} \frac{N}{N + \frac{N(N+1)}{2}}$$
$$= \lim_{N\to\infty} \frac{1}{2} \frac{N}{\frac{N(N+3)}{2}}$$
$$= \lim_{N\to\infty} \frac{1}{N+3}$$
$$= 0$$

Solution 11

A continuous time signal $x(t)$ is periodic if and only if $x(t) = x(t+T)$.
Power of the periodic signal can be evaluated as:

$$P = \frac{1}{T} \int_T |x(t)|^2 dt$$

(a) All continuous time sinusoidal and complex exponential are periodic.

$$x(t) = cos(\pi t)$$
$$\omega = \frac{2\pi}{T} = \pi$$
$$T = 2$$

Corresponding power is P = 0.5.

(b)

$$x(t) = A\sin(10\pi t)$$
$$\omega = \frac{2\pi}{T} = 10\pi$$
$$T = 0.2$$

Corresponding power is P $= \frac{A^2}{2}$.

(c)

$$x(t) = \sin(\sqrt{3}\pi t)$$
$$\omega = \frac{2\pi}{T} = \sqrt{3}\pi$$
$$T = \frac{2}{\sqrt{3}}$$

Corresponding power is P = 0.5.

(d)

$$x(t) = e^{jt}$$
$$\omega = 1$$
$$T = 2\pi$$

Corresponding power is P = 1.

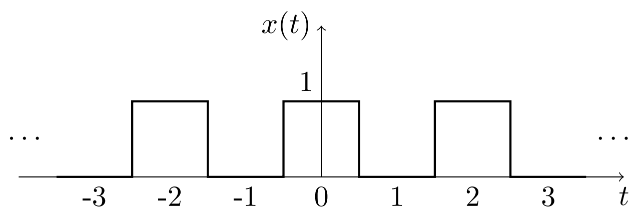
(e)

$$x(t) = A\sin(4\pi t + \pi)$$
$$\omega = \frac{2\pi}{T} = 4\pi$$
$$T = 0.5$$

Corresponding power is P $= \frac{A^2}{2}$.

(f)

$$x(t) = \sum_{n=-\infty}^{\infty} \Pi(t - 2n)$$
$$= \cdots + \Pi(t+4) + \Pi(t+2) + \Pi(t) + \Pi(t-2) + \Pi(t-4) + \cdots$$



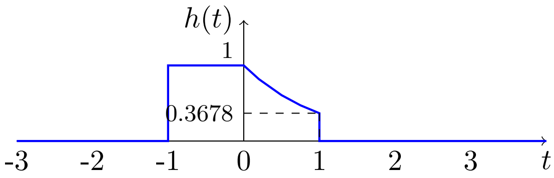
From the figure, it can be observed that T = 2.
Corresponding power is P = 0.5.

Solution 12

(a)

$$h(t) = \exp(-tu(t))$$

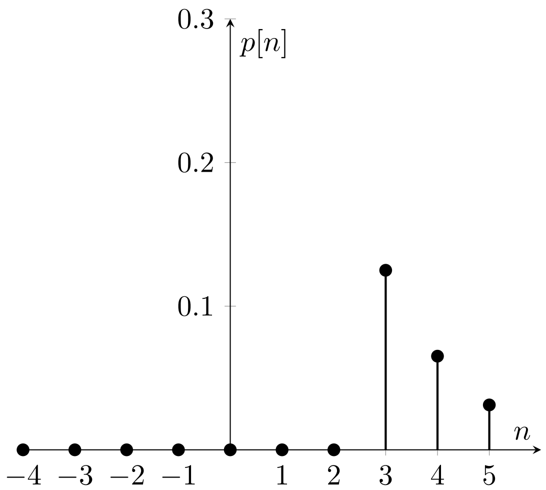
The power of exponential is zero till $t \leq 0$. So, h(t) is 1 till $t \leq 0$. For $t > 0$, h(t) is an exponentially decaying signal. The plot of h(t) is shown below.



(b)

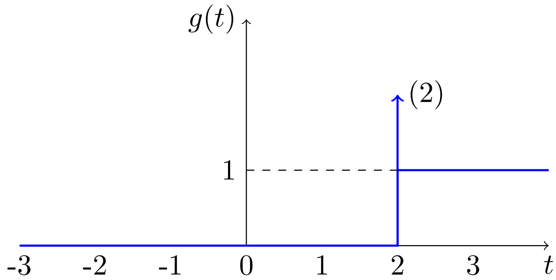
$$p[n] = \frac{1}{2}^n u[n-3]$$

$p[n]$ is a discrete time sequence which is zero for $n \leq 2$ and is an exponentially decaying sequence for $n \geq 3$. The plot is shown below.



(c)

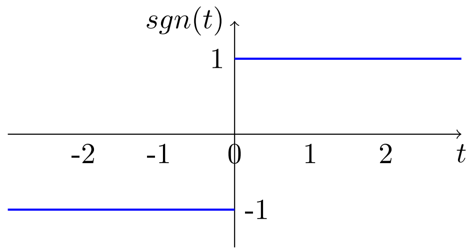
$$\begin{aligned}
g(t) &= \frac{d}{dt}\big(u(t-2)r(t)\big) \\
&= r(t)\frac{d}{dt}\big(u(t-2)\big) + u(t-2)\frac{dr(t)}{dt} \\
&= r(t)\delta(t-2) + u(t-2)u(t) \\
&= r(2)\delta(t-2) + u(t-2) \\
&= 2\delta(t-2) + u(t-2)
\end{aligned}$$



(d)

$$f(t) = sgn(e^{-2t}\sin 2\pi t)$$

The function $sgn(t)$ can be plotted as shown below:



$e^{-2t}\sin 2\pi t$ oscillates with a period of $T = 1$. Hence the signal $f(t)$ can be plotted as shown below: