Teachers Initials
DV
KJ
SB

Marks	1	2	3	4	5	6	Total

Solutions & Grading Scheme.

Department of Electrical Engineering - IIT Madras

EE1101 - Signals and Systems - Quiz I

8:00 am - 8:50 am February 15, 2018 20 marks

Name:	Roll Number:

Write your answers in the space provided, using correct units, and showing all steps on the question book itself. No marks will be given without steps and clear explanations.

1. Is the signal $x[n] = 3exp\left\{\frac{j3\pi\left(n-\frac{1}{2}\right)}{5}\right\}$ periodic? If so, what is its fundamental period? (3)

$x[n]$ is periodic if $x[n+N] = x[n]$

$\Rightarrow 3\, e^{-j3\pi/10}\, e^{j\frac{3\pi}{5}(n+N)} = 3\, e^{-j3\pi/10}\, e^{j\frac{3\pi}{5}n}$ Periodicity (1)

$\Rightarrow e^{j\frac{3\pi N}{5}} = e^{j2\pi k} \quad \Rightarrow N = \frac{2\pi k}{(3\pi/5)} = \frac{10k}{3}$ Fundamental period - (1)

$\therefore x[n]$ is periodic & its fundamental period is $N=10$.

Is this function an even function? If not, derive its even component.

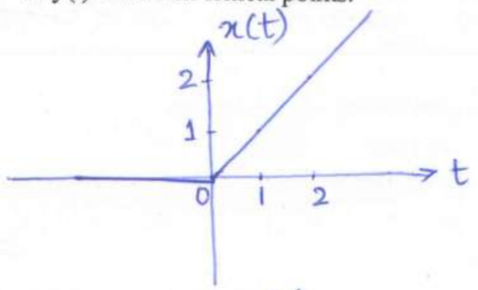
The signal is not even.

The even part is given by $x_e[n] = \frac{x[n]+x[-n]}{2}$

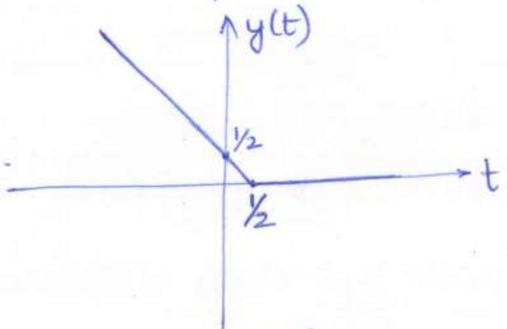
$$x_e[n] = \frac{3\, e^{-j3\pi/10}\, e^{j\frac{3\pi n}{5}} + 3\, e^{-j3\pi/10}\, e^{-j\frac{3\pi n}{5}}}{2}$$

$x_e[n] = 3\, e^{-j3\pi/10} \cos\left(\frac{3\pi}{5}n\right)$ Finding even part - (1)

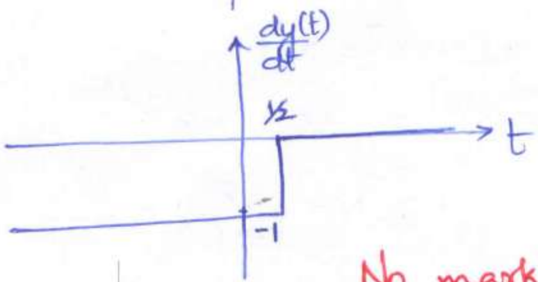
2. If y(t) = 0.5 x(1-2t) and x(t) = t u(t), sketch the following: x(t), y(t) and the derivative of y(t). Mark all critical points. (4)



$x(t) \rightarrow 1$



$y(t) \rightarrow 2$



$\frac{dy(t)}{dt} \rightarrow 1$

No marks if critical points are not labelled.

3. Evaluate the following: (2)

a) $\int_{-\infty}^{\infty} \delta(2t+3)e^{-t}dt$

$$= \int_{-\infty}^{\infty} \frac{1}{2}\delta(t+3/2)\, e^{-t}\, dt$$

$$= \frac{1}{2} e^{3/2}. \quad 1$$

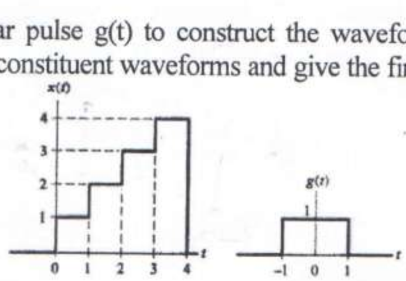
(b) $\sum_{-\infty}^{\infty} \delta[n-3]cos\left(\frac{\pi}{2}[n-5]\right)$

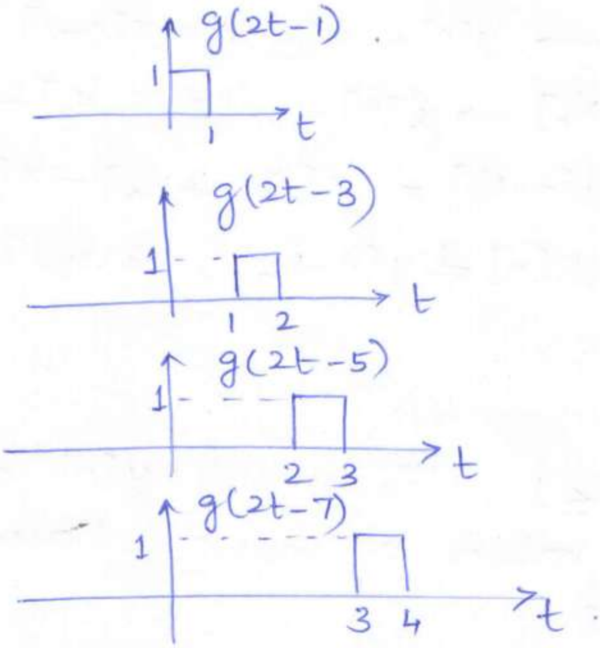
$$= \cos\left(\frac{\pi}{2}(3-5)\right)$$

$$= \cos(-\pi)$$

$$= -1 \quad 1$$

4. Use the rectangular pulse g(t) to construct the waveform x(t) shown in the figures below. Sketch the constituent waveforms and give the final expression for x(t) in terms of g(t). (4)





$$x(t) = g(2t-1) + 2g(2t-3) + 3g(2t-5) + 4g(2t-7)$$

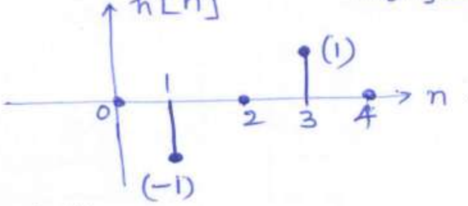
Concept → 2

Final expression → 2

5. The output of a system is y[n] = x[3-n]-x[n-1], where x[n] is the input.

(a) Sketch the impulse response of this system. (3)

$h[n] = \delta[3-n] - \delta[n-1]$



Sketch $h[n] \rightarrow$ 1/2

(b) Is the system time invariant?

$x[n] \longrightarrow y[n] = x[3-n] - x[n-1]$

$x[n-N_0] \longrightarrow y_1'[n] = x[3-n-N_0] - x[n-N_0-1]$

But $y[n-N_0] = x[3-n+N_0] - x[n-N_0-1]$

Since $y_1[n] \neq y[n-N_0]$, the system is not time invariant $\rightarrow$ 1

(c) Is the system causal and stable?

For $n \leq 1$, the output $y[n]$ depends on the future values of input. Hence the system is not causal. $\rightarrow$ 1/2

Consider a bounded input $x[n]$

i.e, $-B < x[n] < B$

Since $x[3-n]$ and $x[n-1]$ involve only time shifting & scaling operations, the resultant signals are also bounded.

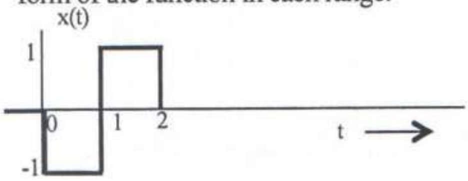
$$-B < x[3-n] < B$$
$$-B < x[n-1] < B.$$

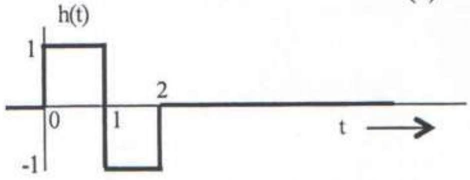
The difference of the two bounded signals will also be bounded.

$$-2B < x[3-n] - x[n-1] < 2B.$$

Hence $y[n]$ is also bounded & the system is stable $\rightarrow$ 1

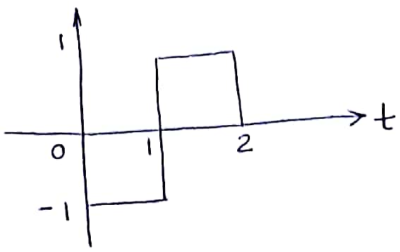
6. Evaluate and sketch the convolution for a system with impulse response h(t) and input x(t), as shown in the figures below. Clearly mark all critical points and indicate the form of the function in each range. (4)





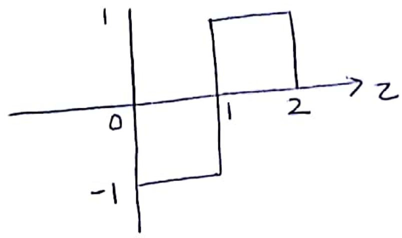
Rough Work

6. $x(t)$

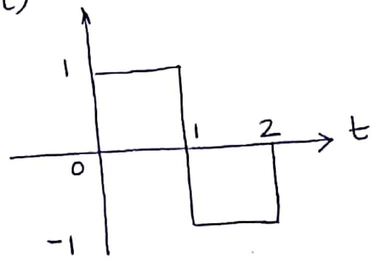


$x(\tau)$

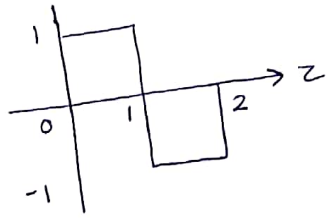
(½ mark)



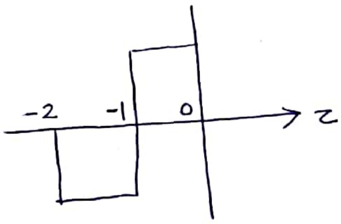
$h(t)$



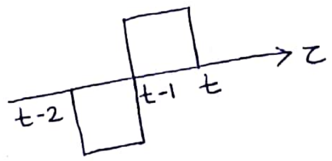
$h(\tau)$



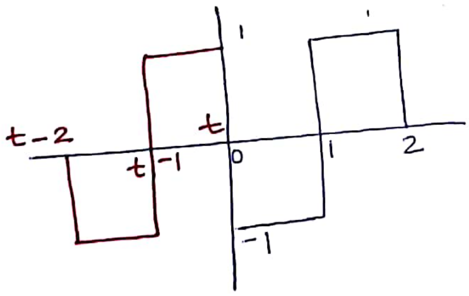
$h(-\tau)$



$h(t-\tau)$

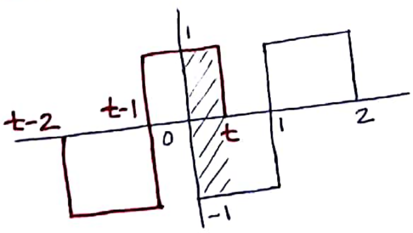


$\underline{t<0}$



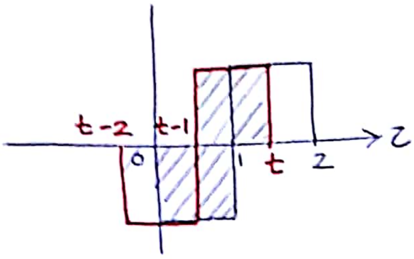
No overlap, $y(t) = 0$

$\underline{0<t<1}$



$$y(t) = \int_0^t (1)(-1)\,d\tau = [-\tau]_0^t = -t$$

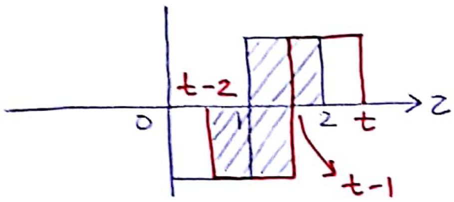
$1 < t < 2$



$$y(t) = \int_0^{t-1} (-1)(-1)\, d\tau + \int_{t-1}^{1} (-1)(1)\, d\tau + \int_1^t (1)(1)\, d\tau$$

$$= t-1 + [-\tau]_{t-1}^{1} + [\tau]_1^t = t-1+t-1-1+t-1 = 3t-4$$

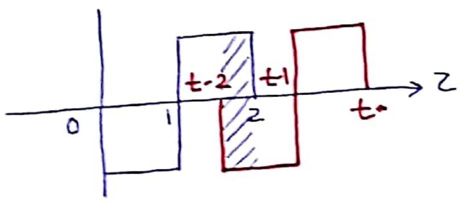
$2 < t < 3$



$$y(t) = \int_{t-2}^{1} d\tau + \int_1^{t-1} (-1)\, d\tau + \int_{t-1}^{2} d\tau = [\tau]_{t-2}^{1} + [-\tau]_1^{t-1} + [\tau]_{t-1}^{2}$$

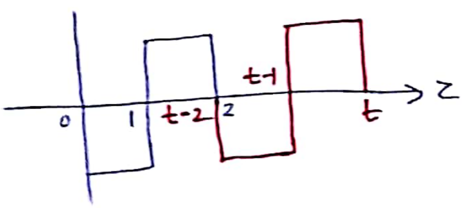
$$= 1-(t-2) + 1-(t-1) + 2-(t-1) = 8-3t$$

$3 < t < 4$



$$y(t) = \int_{t-2}^{2} (-1)(1)\, d\tau = [-\tau]_{t-2}^{2} = t-2-2 = t-4.$$

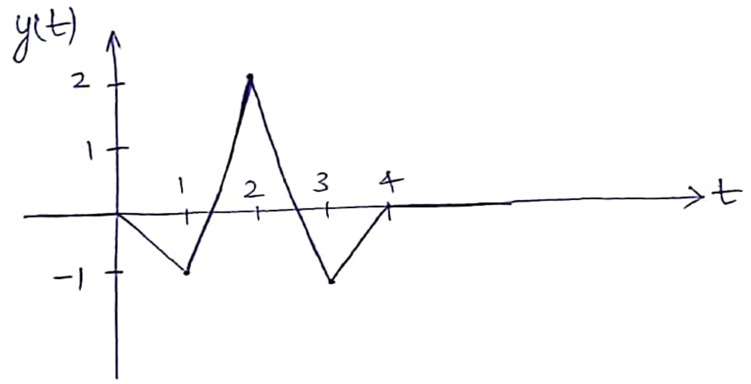
$t > 4$



No overlap

$y(t) = 0$

$$y(t) = \begin{cases} 0, & t<0 \\ -t, & 0<t<1 \quad - \tfrac{1}{2} \\ 3t-4, & 1<t<2 \quad - \tfrac{1}{2} \\ 8-3t, & 2<t<3 \quad - \tfrac{1}{2} \\ t-4, & 3<t<4 \quad - \tfrac{1}{2} \\ 0, & t>4 \end{cases} \Bigg\} \ \tfrac{1}{2}$$



1 mark