DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH1020 Physics II

Tutorial 7 (19.3.2018)

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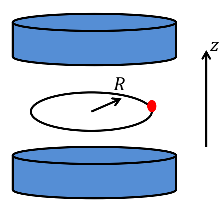
1. A charge q is moving in a circular orbit of radius R about the center of a cylindrically symmetrical magnet, as shown. Assume that the orbit



of the charge lies in the median plane between the poles of the magnet and hence the only component of magnetic field that it sees is in the z direction. The field is now allowed to increase with time. Determine the condition under which the particle will accelerate without any change in the radius of the orbit.

Solution:

Using Faraday's law,

$$2\pi RE = \left|\frac{d\Phi}{dt}\right|$$

where, due to cylindrical symmetry,

$$\Phi = \int_0^R B(r)\ rdrd\phi = 2\pi \int_0^R B(r)\ rdr$$

Thus

$$E = \frac{1}{R}\frac{d}{dt}\left[\int_0^R B(r)\ rdr\right]$$

On the other hand, the equation of motion has the form

$$\frac{dp}{dt} = qE = \frac{q}{R}\frac{d}{dt}\left[\int_0^R B(r)\ rdr\right] \tag{1}$$

Further the centripetal force relation gives

$$\frac{mv^2}{R} = qvB(R) \quad or \quad p = qRB(R)$$

Since R is a constant, we have

$$\frac{dp}{dt} = qR\frac{dB(R)}{dt} \tag{2}$$

Comparing Eqns. (1) and (2), we get

$$\frac{dB(R)}{dt} = \frac{1}{R^2}.\frac{d}{dt}\left[\int_0^R B(r)\ rdr\right]$$

The average magnetic field for $r \leq R$,

$$\bar{B}(R) = \frac{1}{\pi R^2}\int_0^R B(r)\ rdrd\phi = \frac{2}{R^2}\int_0^R B(r)\ rdr$$

Thus the required condition is,

$$2\frac{dB(R)}{dt} = \frac{d}{dt}\bar{B}(R)$$

The magnetic field on the electron's orbit should be equal to half the average magnetic field piercing the orbit. Consequently, the magnetic field should decrease from the centre to the orbit, maintaining the above condition.

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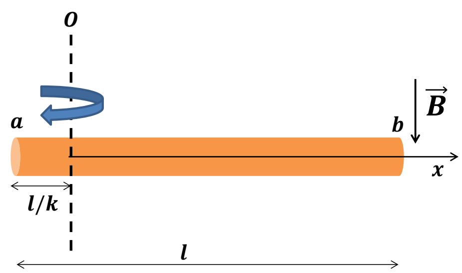
2. A horizontal metallic rod of length l rotates at a frequency ν about a vertical axis which is at a distance l/k from one of its ends. Find the potential difference between the ends of the rod if it rotates in a uniform vertical magnetic field $\mathbf{B}$. Assume that $k = 3$, $l = 12$ m, $\nu = 6$ s^{-1} and B $= 10^{-2}$ Tesla.



Solution: During the rotation, free electrons in the segments Ob and Oa will be pushed towards O by the Lorentz force. Hence the $e.m.f$ is generated in both segments. At an instant when the rod is along x axis, the total $e.m.f$ is given by

$$\mathscr{E} = \int_0^b vBdx - \int_0^a vBdx$$

Since the translational velocity $v = 2\pi\nu x$, we have

$$\mathscr{E} = \int_0^{l(1-1/k)} 2\pi\nu xBdx - \int_0^{l/k} 2\pi\nu xBdx = \pi\nu Bl^2\left(\frac{k-2}{k}\right)$$

For the given parameters, the potential difference between the two ends of the rod turns out to be 9.04 V.

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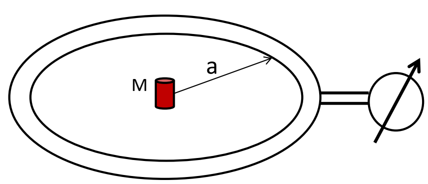
3. A tiny magnet M is placed at the center of a thin coil of radius a containing N turns. The coil is connected to a ballistic galvanometer. The resistance of the circuit is R. When the magnet was rapidly removed



away from the coil a charge q passed through the galvanometer. Find the magnetic moment of the magnet. You may assume the magnetic field in the plane of the coil to be nearly uniform when a current is passed through it.

Solution:

The current I in the coil upon removal of the magnet is given by

$$RI = -\frac{d\Phi}{dt} - L\frac{dI}{dt}$$

Since $Idt = dq$

$$Rdq = -d\Phi - LdI$$

Integrating we get

$$Rq = -\Delta\Phi - L\Delta I$$

The initial and final current are zero($\Delta I = 0$). Further, the final flux is zero. Hence

$$q = \frac{\Phi}{R}$$

Φ is the initial flux through the coil due to the tiny magnet. Let us view the tiny magnet, with magnetic moment μ_m, to be a tiny loop carrying current I and enclosing an area S such that $\mu_m = IS$.

The flux Φ due to the magnet through the circular area of the coil will be same as the flux within the tiny loop if I was passed through the coil of N turns. Assuming uniform field within the circular area of the coil,

$$\Phi = \frac{\mu_0 NI}{2a}S = \frac{\mu_0\mu_m N}{2a}$$

Recalling $q = \frac{\Phi}{R}$

$$q = \frac{\mu_0\mu_m N}{2aR}$$

Thus

$$\mu_m = \frac{2aRq}{\mu_0 N}$$

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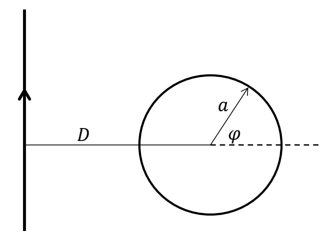
4. Consider a long straight wire placed in the same plane of a circular loop of radius a and at a distance D from its center. Find the mutual inductance.

$$[\int \frac{dx}{a + b\cos x} = \frac{2}{\sqrt{a^2 - b^2}}\tan^{-1}\left[\frac{\sqrt{a^2 - b^2}\tan(x/2)}{a+b}\right]$$

$$\int \frac{rdr}{\sqrt{D^2 - r^2}} = -\sqrt{D^2 - r^2}]$$



Solution:

The field inside the circular loop $\mathbf{B}$(r,ϕ), where r is the radial distance from its center, due to current in the long wire is

$$B_\phi = \frac{\mu_0 I}{2\pi s} = \frac{\mu_0 I}{2\pi\left[D + r\cos\phi\right]}$$

The magnetic flux through the circular loop due to the current in the long wire is

$$\Phi = \int_0^a \int_0^{2\pi} B_\phi \; r dr d\phi = \frac{\mu_0 I}{2\pi} \int_0^a \int_0^{2\pi} \frac{r dr d\phi}{2\pi\left[D + r\cos\phi\right]}$$

$$= \frac{\mu_0 I}{\pi} \int_0^a \frac{2}{\sqrt{D^2 - r^2}} \tan^{-1}\left[\frac{\sqrt{D^2 - r^2}\tan(\phi/2)}{D + r}\right]_0^\pi r dr$$

$$= \mu_0 I \int_0^a \frac{r}{\sqrt{D^2 - r^2}} dr = -\mu_0 I \sqrt{D^2 - r^2} \;\;\Big|_0^a = -\mu_0 I \left[\sqrt{D^2 - a^2} - D\right]$$