... conclude that two circularly polarized waves of equal amplitude but with a phase difference of 2τ generate a linearly polarized wave of tilt angle τ.

For a linear polarization with $\tau = \frac{\pi}{3}$, the phase difference between E_L and E_R should be $\frac{2\pi}{3}$ with E_R leading E_L.

4.6 WAVE PROPAGATION IN CONDUCTING MEDIUM

In the previous section, we investigated the plane wave propagation in a source-free medium. We also assumed that the medium is an ideal dielectric, i.e. the conductivity of the medium was zero. In this section we study the wave propagation in a medium which does not have free charges but which has finite conductivity and consequently has conduction current **J**. In the presence of the finite conductivity, the Maxwell's equation for time harmonic fields can be written as

$$\nabla \cdot \mathbf{D} = 0 \quad \Rightarrow \quad \nabla \cdot \mathbf{E} = 0 \tag{4.92}$$

$$\nabla \cdot \mathbf{B} = 0 \quad \Rightarrow \quad \nabla \cdot \mathbf{H} = 0 \tag{4.93}$$

$$\nabla \times \mathbf{E} = -\mu \frac{\partial \mathbf{H}}{\partial t} = -j\omega\mu_o\mu_r \mathbf{H} \tag{4.94}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + j\omega\epsilon\mathbf{E} = \mathbf{J} + j\omega\epsilon_0\epsilon_r\mathbf{E} \tag{4.95}$$

where, μ_r and ϵ_r are relative permeability and relative permittivity of the medium respectively. If the medium has conductivity σ, the conduction current density J is given by the Ohm's law as

$$\mathbf{J} = \sigma\mathbf{E} \tag{4.96}$$

Substituting for $\mathbf{J}$ in Eqn (4.95), we get

$$\nabla \times \mathbf{H} = \sigma \mathbf{E} + j\omega\epsilon_0\epsilon_r \mathbf{E} = (\sigma + j\omega\epsilon_0\epsilon_r)\mathbf{E} \tag{4.97}$$

Equation (4.97) can be re-written as

$$\nabla \times \mathbf{H} = j\omega\epsilon_0 \left\{\epsilon_r - j\frac{\sigma}{\omega\epsilon_0}\right\} \mathbf{E} \equiv j\omega\epsilon_0\epsilon_{rc}\mathbf{E} \tag{4.98}$$

where we define the relative permittivity of the conducting medium as

$$\epsilon_{rc} \equiv \epsilon_r - j\frac{\sigma}{\omega\epsilon_0} \tag{4.99}$$

It is then clear that the relative permittivity (also called the dielectric constant) of a conducting medium is always complex and it is a function of frequency. The behavior of the medium now becomes frequency dependent.

Let us look at Eqn (4.97) once again. We note that, the two terms $\sigma\mathbf{E}$ and $j\omega\epsilon_0\epsilon_r\mathbf{E}$ correspond to the conduction and displacement current densities respectively. The conduction current is a characteristic of a conductor whereas, the displacement current is a characteristic of a dielectric. For a medium which has conduction as well displacement current one would then wonder whether to call the medium a conductor or a dielectric! The answer to this question lies in the relative contributions of the two currents. For a given electric field if the conduction current is larger compared to the displacement current, we can treat the medium like a conductor, whereas if the conduction current is negligible compared to the displacement current, we can treat the medium like a dielectric. That is,

$$\text{If} \quad \frac{\text{Cond. current density}}{\text{Disp. current density}} = \frac{\sigma}{\omega\epsilon_0\epsilon_r} \gg 1 \text{ — good conductor} \tag{4.100}$$

$$\text{If} \quad \frac{\text{Cond. current density}}{\text{Disp. current density}} = \frac{\sigma}{\omega\epsilon_0\epsilon_r} \ll 1 \text{ — good dielectric} \tag{4.101}$$

If the ratio $\sigma/\omega\epsilon_0\epsilon_r$ is ~ 1 then the medium can neither be called a good conductor nor a good dielectric.

Now, since the ratio $\sigma/\omega\epsilon_0\epsilon_r$ is a function of frequency, a medium (with conductivity σ and dielectric constant ϵ_r) can behave like a dielectric at one frequency and like a conductor at another frequency. Noting that the ratio is inversely proportional to ω, we can say in general, that towards the lower end of the electromagnetic spectrum a medium behaves more like a conductor (except

One can find the change over frequency f_T at which the medium behavior changes from conductor to dielectric or vice-versa by making the conduction and the displacement currents equal. At $f = f_T$ we have

$$\omega\epsilon_0\epsilon_r = \sigma \tag{4.102}$$

$$\Rightarrow \qquad f_T = \frac{\sigma}{2\pi\epsilon_0\epsilon_r} \tag{4.103}$$

For copper, taking $\epsilon_r \approx 1$ and $\sigma = 5.6 \times 10^7$ ℧/m, f_T is $\approx 10^{18}$ Hz. For sea water on the other hand, taking $\epsilon_r \approx 80$, and $\sigma = 10^{-3}$ ℧/m, f_T is ≈ 225 kHz.

The concept of complex dielectric constant is quite useful in analysing electromagnetics problems. For a non-ideal dielectric medium, one can first analyse the problem assuming the dielecric to be ideal with dielectric constant ϵ_r. The results for non-ideal dielectric medium can then be obtained by replacing the dielectric constant ϵ_r by ϵ_{rc}.

To analyse the plane wave propagation in a dielectric medium with finite conductivity, we use the concept of the complex dielectric constant. For a conducting medium, the wave Eqns (4.31) and (4.32) respectively, become

$$\nabla^2\mathbf{E} = -\omega^2\mu_0\mu_r\epsilon_0\epsilon_{rc}\mathbf{E} = \sqrt{j\omega\mu_0\mu_r(\sigma + j\omega\epsilon_0\epsilon_r)}\ \mathbf{E} \tag{4.104}$$

$$\nabla^2\mathbf{H} = -\omega^2\mu_0\mu_r\epsilon_0\epsilon_{rc}\mathbf{H} = \sqrt{j\omega\mu_0\mu_r(\sigma + j\omega\epsilon_0\epsilon_r)}\ \mathbf{H} \tag{4.105}$$

Here, we have explicitly written $\mu = \mu_0\mu_r$ and $\epsilon = \epsilon_0\epsilon_{rc}$. μ_0 and ϵ_0 are free space permeability and permittivity respectively. Since, here, we are primarily interested in non-magnetic media, we can take $\mu_r = 1$ in our further discussion of the wave propagation.

The Eqn (4.51) of a plane wave travelling in z direction now becomes

$$\frac{\partial^2 E_x}{\partial z^2} = -\omega^2\mu_0\epsilon_0\epsilon_{rc}E_x \equiv \gamma^2 E_x \tag{4.106}$$

The propagation constant of the wave therefore is

$$\gamma = \sqrt{-\omega^2\mu_0\epsilon_0\epsilon_{rc}} = +j\omega\sqrt{\mu_0\epsilon_0}\sqrt{\epsilon_{rc}} \tag{4.107}$$

Substituting for ϵ_{rc} from Eqn (4.99), the propagation constant of the wave is

$$\gamma = j\omega\sqrt{\mu_0\epsilon_0}\left\{\epsilon_r - j\frac{\sigma}{\omega\epsilon_0}\right\}^{1/2} \tag{4.108}$$

The propagation constant for a conducting medium, therefore, is complex and can be written as

$$\gamma = \alpha + j\beta \tag{4.109}$$

where

$$\alpha = Re(\gamma) = \omega\sqrt{\frac{\mu_0\epsilon_0\epsilon_r}{2}}\left[\sqrt{1 + \frac{\sigma^2}{\omega^2\epsilon_0^2\epsilon_r^2}} - 1\right]^{1/2} \tag{4.110}$$

$$\Rightarrow \quad \sigma = \omega\epsilon_o\epsilon_r \tan\delta$$
$$= 2\pi \times 10^8 \times \frac{1}{36\pi} \times 10^{-9} \times 18 \times 10^{-3}$$
$$= 10^{-4}\ \mho/\text{m}.$$

Since the loss-tangent is very small, the dielectric is a low-loss dielectric, and we can approximately get the attenuation constant of the wave as (see Eqn (4.118))

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu_o}{\epsilon_o\epsilon_r}} = \frac{\sigma}{2}\sqrt{\frac{\mu_o}{\epsilon_o}}.\frac{1}{\sqrt{\epsilon_r}} = \frac{10^{-4}}{2} \times 120\pi \times \frac{1}{\sqrt{18}} = 4.44 \times 10^{-3}\text{nepers/m}$$

The distance over which the wave amplitude reduces to 1/e of its original value is $1/\alpha = 225.08$ m

4.6.2 Good Conductor

As discussed in the previous sections, for a good conductor we have $\sigma/\omega\epsilon >> 1$ and we can apply some approximations to the amplitude and phase constants. The propagation constant in general can be written as

$$\gamma = \alpha + j\beta = \sqrt{j\omega\mu_0(\sigma + j\omega\epsilon_0\epsilon_r)} \quad (4.123)$$

$$\approx \sqrt{j\omega\mu_0\sigma} \qquad \text{since } \sigma >> \omega\epsilon_0\epsilon_r \quad (4.124)$$

since, $\sqrt{j} = \sqrt{e^{j\pi/2}} = e^{j\pi/4} = \cos\frac{\pi}{4} + j\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}} = \frac{1+j}{\sqrt{2}}$, we get

$$\gamma = \alpha + j\beta = \sqrt{\omega\mu_0\sigma}\left(\frac{1+j}{\sqrt{2}}\right) \quad (4.125)$$

giving

$$\alpha = \beta = \sqrt{\frac{\omega\mu_0\sigma}{2}} \quad (4.126)$$

It is interesting to note that for a good conductor, α and β are approximately equal. As the amplitude of a wave varies as $e^{-\alpha z}$, the wave amplitude reduces to $1/e$ of its value over a distance of $1/\alpha$. That is, over a distance of $1/\alpha = 1/\beta = \lambda/2\pi \approx \lambda/6$, the wave amplitude reduces to $1/e$ of its initial value. A conductor, therefore, acts like a very lossy medium. The field decays very rapidly from the point of its origin along the direction of the wave propagation.

At this point it is worthwhile to ask a question, "How is a field excited inside a conductor?" For transportation of electromagnetic energy, again we have to depend upon the wave propagation but the wave propagation is very lossy in a conductor. It is, therefore, apparent that we can not generate energy inside a conductor. We can generate energy inside a dielectric, and transport the energy to a conductor. The question then reduces to what happens to an electromagnetic wave when it reaches a conducting medium? Without worrying about the reflections,

$$= \left[\frac{1}{2}\frac{\sigma^2}{\omega^2\epsilon^2} - \frac{1}{8}\left(\frac{\sigma^2}{\omega^2\epsilon^2}\right)^2 + \cdots \right]^{\frac{1}{2}} \quad (4.114)$$

$$= \sqrt{\frac{\sigma^2}{2\omega^2\epsilon^2}} \left[1 - \frac{1}{2}\left(\frac{\sigma^2}{4\omega^2\epsilon^2}\right) + \cdots \right] \quad (4.115)$$

and the attenuation constant α can be written as

$$\alpha = \omega\sqrt{\frac{\mu_0\epsilon}{2}\frac{\sigma^2}{2\omega^2\epsilon^2}} \left[1 - \frac{\sigma^2}{8\omega^2\epsilon^2} + \cdots \right] \quad (4.116)$$

$$\approx \frac{\sigma}{2}\sqrt{\frac{\mu_0}{\epsilon}} \left[1 - \frac{\sigma^2}{8\omega^2\epsilon^2} \right] \quad (4.117)$$

$$\approx \frac{\sigma}{2}\sqrt{\frac{\mu_0}{\epsilon}} \quad (4.118)$$

Similarly the phase constant β can be approximated as

$$\beta \approx \omega\sqrt{\mu_0\epsilon} \left[1 + \frac{1}{8}\frac{\sigma^2}{\omega^2\epsilon^2} \right] \quad (4.119)$$

$$\approx \omega\sqrt{\mu_0\epsilon} \quad (4.120)$$

It can be seen from Eqn (4.118) that the attenuation constant α is directly proportional to the conductivity of the medium and is practically independent of frequency. The phase constant however is almost same as that of the loss-less medium (see Eqn (4.120)).

The intrinsic impedance of a low-loss medium is

$$\eta_d = \sqrt{\frac{j\omega\mu_0}{\sigma + j\omega\epsilon}} \quad (4.121)$$

$$\approx \sqrt{\frac{\mu_0}{\epsilon_o\epsilon_r}} = \eta_0\sqrt{\frac{1}{\epsilon_r}} \quad (4.122)$$

The phase of η_d is almost zero and hence the electric and magnetic fields are almost in phase with each other.

EXAMPLE 4.9 A dielectric material has relative permittivity 18 and loss tangent 10^{-3} at 100 MHz. Find the conductivity of the medium. Also find the distance over which the wave amplitude reduces to $1/e$ of its original amplitude.

Solution:

The loss tangent is

$$\tan\delta = \frac{\sigma}{\omega\epsilon_o\epsilon_r}$$

$$\beta = Im(\gamma) = \omega\sqrt{\frac{\mu_0\epsilon_0\epsilon_r}{2}}\left[\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon_0^2\epsilon_r^2}}+1\right]^{1/2} \tag{4.111}$$

Now, following the discussion on transmission lines, we note that, α is the attenuation constant and β is the phase constant. α decides the change in amplitude of a wave as it propagates in the medium and β decides the wavelength of the wave in the medium (wavelength $\lambda = 2\pi/\beta$). It is then interesting to note from Eqns (4.110) and (4.111) that in a dielectric medium with non-zero conductivity the attenuation constant and the wavelength of a wave are functions of frequency. It is also interesting that with increasing σ the attenuation of the wave increases and the wavelength of the wave decreases (β increases). From our basic understanding of electrical devices, we expect the attenuation (or loss) to decrease with increase in conductivity. The dielectric medium with finite conductivity however has exactly opposite behavior, i.e. increase in conductivity increases the attenuation. One would then wonder how there are opposite trends for the circuit behavior and the wave behavior! The surprise can be resolved by noting that in electrical circuits we primarily concentrate on the conductive current and treat the medium as a conductor whereas in the above analysis we have treated the medium as a dielectric with finite conductivity. A dielectric medium has attenuation if the conductivity is not zero and a conductor has attenuation if the conductivity is not infinite (this will become clear in later sections).

Since, here, we are primarily investigating the wave propagation in a dielectric medium, there is a wave attenuation if the conductivity of the medium is non-zero and for $\sigma = 0$ the attenuation is zero. A quantity called 'loss tangent' (denoted by $\tan\delta$) which is the tangent of the phase of the complex dielectric constant is normally used as a measure of the medium attenuation.

The loss tangent is

$$\tan\delta = \frac{\sigma}{\omega\epsilon_0\epsilon_r} \tag{4.112}$$

Smaller the loss tangent lesser is the attenuation and better is the dielectric.

4.6.1 Low-loss Dielectrics

A low-loss dielectric is a medium for which the loss tangent is very small, i.e. $\sigma/\omega\epsilon_0\epsilon_r << 1$. For this medium the expressions for the attenuation and the phase constants in Eqns (4.110) and (4.111) can be approximated. Making binomial expansion for the term inside the brackets of (4.110) we get

$$\left[\sqrt{1+\frac{\sigma^2}{\omega^2\epsilon_0^2\epsilon_r^2}}-1\right]^{\frac{1}{2}} = \left[1+\frac{1}{2}\frac{\sigma^2}{\omega^2\epsilon^2}-\frac{1}{8}\left(\frac{\sigma^2}{\omega^2\epsilon^2}\right)^2+\cdots-1\right]^{\frac{1}{2}} \tag{4.113}$$

etc at the conductor surface, let us assume that after all transient adjustments have taken place, the field just inside the conductor surface has some value E_0. Due to high attenuation constant α, the field decreases rapidly as the wave propagates deeper in the conductor. The wave amplitude reduces to $1/e$ of its value at the surface, over a distance of $1/\alpha$ and within a distance of few times $1/\alpha$ the field reduces practically to zero. Figure 4.19 shows the amplitude of a wave as a function of depth inside a conductor. The field is, therefore, effectively confined to a layer which is $\sim 1/\alpha$ deep below the surface of the conductor. The thickness of the layer ($\sim 1/\alpha$) decreases as ω and σ increase. At a frequency of tens of MHz, and for conductivity of $\sim 10^7$ ℧/m (good conductor), the thickness lies in the range of µm. The field confinement then is just in the skin of the conductor. This effect is therefore called the 'skin effect', and the thickness of the layer is called the 'skin depth' or depth of penetration. The skin depth is given as

$$\delta = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu_0\sigma}} = \sqrt{\frac{1}{\pi f \mu_0 \sigma}} \tag{4.127}$$

The skin effect can be wisely exploited for shielding an electromagnetic wave. If a region with electromagnetic radiation is covered with a metal sheet of thickness much larger than the skin depth the fields outside the metal cover would be of negligibly small value.

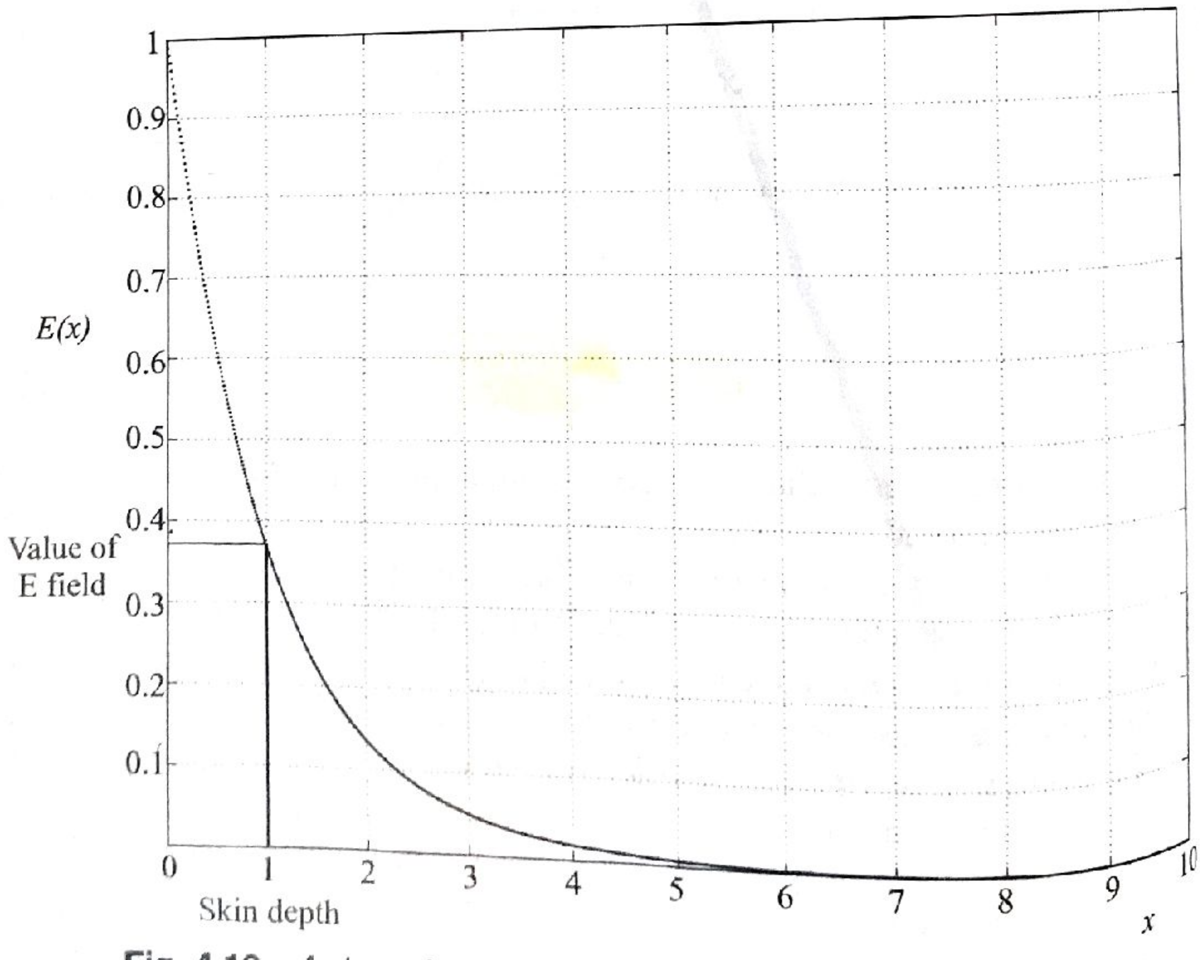

Fig. 4.19 *A decaying electrical field inside a good conductor.*

The skin effect which helps in isolating electromagnetic environments acts as a hinderance for the radio wave communication through the conducting walls. We all have noticed the poor reception of radio signals inside a train. The skin effect is the primary cause of that.

Table 4.1 shows the skin depth values for different conductors at a frequency of 1 MHz. The skin-depth is inversely proportional to the square-root of the frequency, and the square-root of the conductivity.

Table 4.1 Skin depth at 1 MHz

Material	Conductivity	Skin depth
Silver	6.17×10^7 ℧/m	63.87 μm
Copper	5.88×10^7 ℧/m	76.33 μm
Aluminium	3.65×10^7 ℧/m	83.17 μm
Doped Silicon	10^3 ℧/m	6.39 mm
Earth	5×10^{-3} ℧/m	7.12 m

The intrinsic impedance of a good conductor is

$$\eta_c \approx \sqrt{\frac{j\omega\mu_0}{\sigma}} = \sqrt{\frac{\omega\mu_0}{\sigma}}\angle 45^\circ \tag{4.128}$$

The magnetic field, therefore, lags the electric field by $\approx 45^o$ inside a good conductor.

EXAMPLE 4.10 A material has dielectric constant 25 and conductivity 2×10^6 ℧/m. What is the frequency above which the material cannot behave like a good conductor? If a plane wave of 10 MHz is incident on the material, effectively upto what depth can the wave penitrate the material, and what will be the wavelength of the wave inside the material?

Solution:

For any material to behave like a good conductor, we must have $\sigma >> \omega\epsilon_0\epsilon_r$. As a rule of thumb, we may say that when $\sigma > 10\omega\epsilon_0\epsilon_r$, the material is a good conductor. Therefore, the frequency above which the material does not behave like a good conductor, is

$$\omega = \frac{\sigma}{10\epsilon_0\epsilon_r} = \frac{2 \times 10^6}{10 \times \frac{1}{36\pi} \times 10^{-9} \times 25}$$

$$\Rightarrow \qquad f = 1.44 \times 10^{14} \quad \text{Hz.}$$

Effective depth of wave penitration is the skin depth,

$$\delta = \frac{1}{\sqrt{\pi f \mu_0 \sigma}} = \frac{1}{\sqrt{\pi \times 10^7 \times 4\pi \times 10^{-7} \times 2 \times 10^6}} = 112.54\ \mu\text{m}$$

To find the wavelength we first calculate the phase constant $\beta = \sqrt{\pi f \mu_0 \sigma} =$ 8885.5 rad/m.
The wavelength $\lambda = \frac{2\pi}{\beta} = 0.707$ mm.

4.7 PHASE VELOCITY OF A WAVE

As we have seen above, the electric field of a plane wave travelling in $+z$ direction is written as

$$\mathbf{E}(z,t) = \mathbf{E}_0 e^{-\gamma z} \cdot e^{j\omega t} \tag{4.129}$$

For a general medium, γ is complex ($\gamma \equiv \alpha + j\beta$), and the electric field can be written as

$$\mathbf{E}(z,t) = E_0 e^{-\alpha z} \cdot e^{-j\beta z} \cdot e^{j\omega t} \tag{4.130}$$

$$= E_0 e^{-\alpha z} \cdot e^{j(\omega t - \beta z)} \tag{4.131}$$

As has been discussed in detail for the transmission lines, the phase ϕ of **E** is a composite function of space (z) and time (t). The constant phase point moves in $+z$ direction as a function of time. One can then ask a question, "with what speed does the constant phase point move in the space"

Let us consider an observer standing at some location in space and seeing the wave passing by him. Since $z = constant$ for the observer, he observes the phase of the wave to be ωt, i.e. the phase increases linearly as a function of time. Now, consider an observer who is holding on to a particular phase point. Obviously, he has to move with the speed with which the phase point is moving so that he never leaves the point. Then for this observer, the phase appears stationary (constant) as a function of time. The velocity of the wave is same as the velocity of the observer for whom the phase is constant. This velocity is called the 'phase velocity' of the wave. For a wave travelling in $+z$ direction, the phase of the wave is

$$\phi = \omega t - \beta z = constant \tag{4.132}$$

Differentiating Eqn (4.132) with respect to time we get

$$\omega - \beta \frac{\partial z}{\partial t} = 0 \tag{4.133}$$

$$\Rightarrow \qquad \text{phase velocity} : v_p = \frac{\partial z}{\partial t} = \frac{\omega}{\beta} \tag{4.134}$$

Since β in general, is a function of the dielectric constant, ϵ_r of the medium, and the conductivity σ, the phase velocity also changes from medium to medium. In fact, the phase velocity is one of the parameters which characterizes the medium. Let us find the phase velocities of a wave in different media.

1. Free Space: For free space (or vacuum) we have $\epsilon = \epsilon_0 = \frac{1}{36\pi} \times 10^{-9}$ F/m $\mu = \mu_0 = 4\pi \times 10^{-7}$ H/m. We therefore have

$$\beta = \omega\sqrt{\mu_0\epsilon_0} = \frac{\omega}{3} \times 10^{-8} \text{ rad/m} \tag{4.135}$$

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 3 \times 10^8 \text{ m/s} \tag{4.136}$$

This is the velocity of light in the free-space and is normally denoted by letter 'c'. One would appreciate the strength of the Maxwell's equations now which correctly predicted the wave phenomenon and provided correct estimate of the velocity of light in vacuum.

2. Pure Dielectric: For a pure dielectric we have $\sigma = 0$, $\epsilon = \epsilon_0\epsilon_r$, $\mu = \mu_0$

$$\Rightarrow \qquad \beta = \omega\sqrt{\mu_0\epsilon_0\epsilon_r} = \omega\sqrt{\mu_0\epsilon_0}\sqrt{\epsilon_r} \tag{4.137}$$

$$\Rightarrow \qquad v_p = \frac{\omega}{\omega\sqrt{\mu_0\epsilon_0}\sqrt{\epsilon_r}} = \frac{c}{\sqrt{\epsilon_r}} \tag{4.138}$$

Equation (4.138) gives the velocity of light in a medium with dielectric constant ϵ_r. From Eqn (4.138) we note that the velocity of an electromagnetic wave in a dielectric medium is always less than that in the vacuum and is independent of the frequency of the wave.

We may recall from our high school physics that the refractive index n of a medium is defined as

$$n = \frac{\text{velocity of light in vaccum}}{\text{velocity of light in the medium}} \tag{4.139}$$

Therefore, from Eqns (4.138) and (4.139) we get

$$n = \sqrt{\epsilon_r} \tag{4.140}$$

The refractive index of a dielectric material is the square root of its dielectric constant.

3. Dielectric Medium with Loss: For a lossy dielectric, $\sigma \neq 0$, and the phase constant is given as (from Eqn (4.119))

$$\beta = \omega\sqrt{\mu_0\epsilon_0\epsilon_r}\left\{1 + \frac{1}{8}\frac{\sigma^2}{\omega^2\epsilon^2}\right\} \tag{4.141}$$

hence the phase velocity is

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu_0\epsilon_0\epsilon_r}}\left\{1 + \frac{1}{8}\frac{\sigma^2}{\omega^2\epsilon^2}\right\}^{-1} = \frac{c}{\sqrt{\epsilon_r}}\left\{1 + \frac{1}{8}\frac{\sigma^2}{\omega^2\epsilon_0^2\epsilon_r^2}\right\}^{-1} \tag{4.142}$$

From Eqn (4.142) we note that for a lossy dielectric medium, the phase velocity is a function of ω, i.e. the waves of different frequencies travel with different velocities. This phenomenon is called the 'dispersion', and the medium is said to be a 'dispersive medium'. One can then say that a loss-less dielectric is non-dispersive, whereas a lossy medium is dispersive and the dispersion increases with the loss in the medium. For a low-loss medium however, the dispersion is generally small.

4. Good Conductor: For a good conductor $\sigma >> \omega\epsilon$ and the phase constant is (Eqn (4.126))

$$\beta = \sqrt{\frac{\omega\mu_0\sigma}{2}} \tag{4.143}$$

$$\Rightarrow \qquad v_p = \frac{\omega}{\sqrt{\frac{\omega\mu_0\sigma}{2}}} = \sqrt{\frac{2\omega}{\mu_0\sigma}} \tag{4.144}$$

Multiplying numerator and denominator within the square root sign of Eqn (4.144) by $\omega\epsilon_0$ and re-arranging we get

$$v_p = \frac{1}{\sqrt{\mu_0\epsilon_0}}\sqrt{\frac{2\omega\epsilon_0}{\sigma}} = c\sqrt{\frac{2\omega\epsilon_0}{\sigma}} \tag{4.145}$$

For a good conductor $\omega\epsilon_0/\sigma << 1$ and therefore $v_p << c$. The electromagnetic wave therefore slows down considerably in a conductor. As can be seen from the example, the phase velocity of an electromagnetic wave inside copper is few hundred meter/s which is of the order of the velocity of sound in copper.

Compared to a lossy dielectric, the conductor is much dispersive since v_p varies as $\sqrt{\omega}$. It should be noted however, that the dispersion decreases with σ and for an ideal conductor ($\sigma = \infty$) the dispersion is zero. Figures 4.20 and 4.21 show velocity and dispersion as a function of frequency and the conductivity of the medium.

EXAMPLE 4.11 Just outside a train compartment the field strength of a radio station is 0.1 V/m. What will be the approximate field strength inside the compartment? Assume the compartment to be a closed box of metal having conductivity 5×10^6 ℧/m. The thickness of the compartment wall is 5 mm. Frequency of radio station is 600 kHz.

Solution:

The attenuation constant for the compartment material (assuming the compartment material non-magnetic)

$$\alpha = \sqrt{\pi f \mu_o \sigma} = \sqrt{\pi \times 600 \times 10^3 \times 4\pi \times 10^{-7} \times 5 \times 10^6} = 3441.44 \text{ napers/m.}$$

The wave amplitude after passing through the compartment wall will be

$$E = E_o e^{-\alpha z} = 0.1e^{-17.2} = 3.366 \times 10^{-9} \text{ V/m}$$

The wave, therefore, is attenuated by a factor $\sim 3 \times 10^7$.

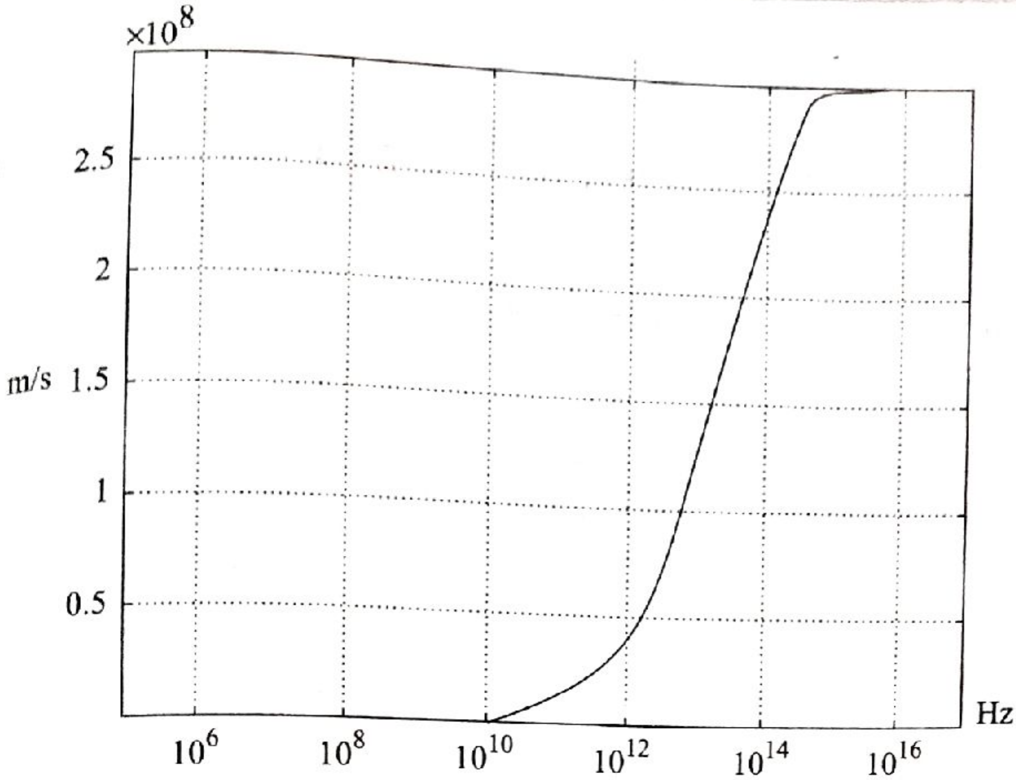

Fig. 4.20 *Variation of the phase velocity as a function of frequency.*

4.8 POWER FLOW AND POYNTING VECTOR

As seen above, the time varying electric and magnetic fields have to form an electromagnetic wave which propagates in the space. Naturally the wave carries some energy with it. It is then worthwhile to investigate, the quantity energy or power (i.e. energy per unit time) is carried by an electromagnetic wave? Note that in general, the electromagnetic wave need neither be a plane wave nor be travelling in an unbound medium. We, therefore, would like to develop a general frame work for power flow from arbitrary time varying fields. Since the electromagnetic phenomenon is completely governed by the Maxwell's equations, we again fall back upon the Maxwell's equations to find the power flow due to time varying fields.

Let us take the two Maxwell's equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu \frac{\partial \mathbf{H}}{\partial t} \tag{4.146}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J} + \epsilon \frac{\partial \mathbf{E}}{\partial t} \tag{4.147}$$

We assume here that μ, ϵ are not varying as a function of time. From the vector identity we have

$$\nabla \cdot (\mathbf{A} \times \mathbf{C}) = \mathbf{C} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot \nabla \times \mathbf{C} \tag{4.148}$$

where $\mathbf{A}$ and $\mathbf{C}$ are any two arbitrary vectors.
Taking $\mathbf{A} = \mathbf{E}$ and $\mathbf{C} = \mathbf{H}$ we have the vector identity

$$\nabla \cdot (\mathbf{E} \times \mathbf{H}) = \mathbf{H} \cdot (\nabla \times \mathbf{E}) - \mathbf{E} \cdot (\nabla \times \mathbf{H}) \tag{4.149}$$