EE1101: Signals and Systems
Quiz - 2
April 5, 2018

Roll. No.:____________________

Name:____________________

Instructor (Circle it): DV / KJ / SB / SU

Answer all FOUR questions clearly and concisely.

Marks: 40
Time: 50 minutes

1. Consider a sinusoidal signal, $x(t) = A\sin(\pi t)$ $\{A=1\}$

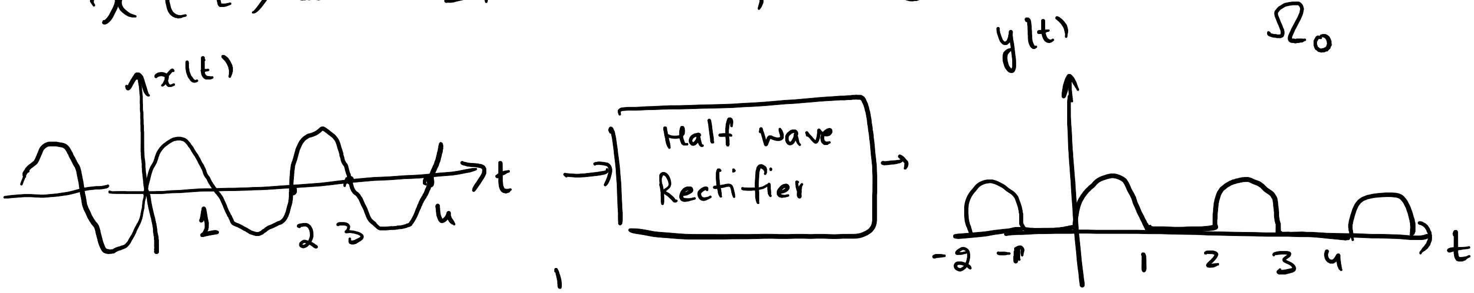
(a) What are the exponential Fourier series coefficients of the output of half-wave rectifier, i.e. $y(t)$ where

$$y(t) = \begin{cases} x(t) & x(t) \geq 0 \\ 0 & x(t) < 0 \end{cases}$$

(Hint: Find a_k for $k = 0, 1, -1$ and then for k even and k odd. Use $\cos(1 \pm k)\pi = -\cos(k\pi)$)

(b) Which F.S. coefficient of $y(t)$ has its value as $\frac{-j}{4}$? What is the frequency, $\hat{\Omega}$ of the corresponding complex exponential? [10]

$x(t) = \sin(\pi t)$, $\Omega_0 = \pi \Rightarrow T = \frac{2\pi}{\Omega_0} = 2$



$a_0 = \frac{1}{T}\int_0^T y(t)\cancel{e^{-j\Omega_0 t}}\,dt = \frac{1}{2}\int_0^1 \sin\pi t\,dt \quad \frac{1}{2}\left[\frac{-1}{\pi}\cos\pi t\right]_0^1$

$= \frac{-1}{2\pi}\left[(-1)-1\right] = \frac{1}{\pi}$ 1/2

$a_1 = \frac{1}{2}\int_0^1 \sin\pi t\left[\cos\pi t + j\sin\pi t\right]dt = \frac{1}{4}\cancel{\int_0^1 \sin 2\pi t\,dt}^{\,0} + \frac{j}{2}\int_0^1 \sin^2\pi t\,dt$

$= 0 + \frac{j}{2}\left[\int_0^1 \frac{1-\cos 2\pi t}{2}\,dt = 0 + \frac{j}{2}\int_0^1 \frac{1}{2} - 0 = j/4\right.$

$a_1 = \boxed{j/4} \rightarrow 1$

More Work Space for Q1

$$a_{-1} = \frac{1}{2}\int_0^1 \sin\pi t\,[\cos\pi t - j\sin\pi t] = -(j/4) = -j/4 \rightarrow 1/2$$

$$a_k = \frac{1}{2}\int_0^1 \underbrace{\sin\pi t\,[\cos k\pi t}_{\mathrm{I}} + j \sin k\pi t]\,dt \quad \mathrm{II}$$

I $$= \frac{1}{4}\int_0^1 \{\sin(1+k)\pi t + \sin(1-k)\pi t\}\,dt = \frac{-1}{4\pi}\left[\frac{\cos(1+k)\pi t}{1+k} + \frac{\cos(1-k)\pi t}{1-k}\right]_0^1$$

$$= \frac{1}{2}\left(\frac{1+\cos k\pi}{\pi(1-k^2)}\right) \quad \begin{cases} = 0 & \text{for } |k|>1 \text{ and } k \text{ odd} \\ = \frac{1}{\pi(1-k^2)}, & k \text{ even} \end{cases} \quad 1$$

II $$\frac{1}{2}\int_0^1 (\sin\pi t)(\sin k\pi t)\,dt = \frac{1}{4}\int_0^1 [\cos(1-k)\pi t - \cos(1+k)\pi t]\,dt$$

$$= \frac{1}{4\pi}\left[\frac{\sin(1-k)\pi t}{1-k} + \frac{\sin(1+k)\pi t}{1+k}\right]_0^1$$

$$= 0 \quad \text{for all } k \quad 1$$

Hence $a_0 = \frac{1}{\pi}$, $a_1 = \frac{j}{4}$, $a_{-1} = -j/4$

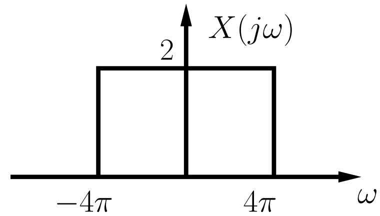
For $|k|>1$ $$\begin{cases} a_k = 0 & \text{if } k \text{ is odd} \\ \quad = \frac{1}{\pi(1-k^2)} & \text{if } k \text{ is even} \end{cases}$$

$a_{-1} = -j/4 \Rightarrow$ frequ $k \cdot 2\pi/T = 1 \cdot \frac{2\pi}{2} = \pi$ radians

$\uparrow k=-1$ $\hat{\Omega} = \pi$

$\rightarrow 1$

2. Let $x(t)$ (**which is real and even**) have its Fourier transform, $X(j\omega)$, as shown in the figure below. Using Fourier transform properties:
   (a) Plot the Fourier transform of $y(t) = x(t)\cos(4\pi t)$, i.e. plot $Y(j\omega)$. Please label properly indicating all important values.
   (b) Plot the Fourier transform of $z(t) = x(2t)$.
   (c) What is the relation between $y(t)$ and $z(t)$? [10]



$$y(t) = x(t)\cdot\cos 4\pi t \longleftrightarrow Y(\omega) = \frac{1}{2\pi}\left[X(\omega) * F.T\{\cos 4\pi t\}\right]$$

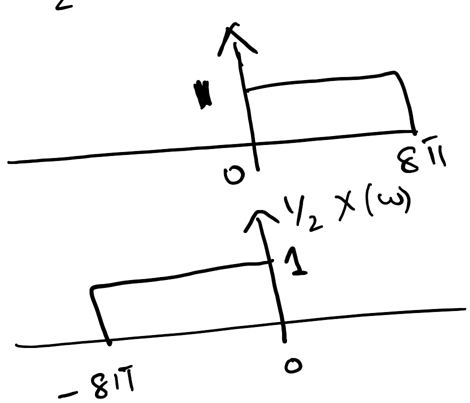
1/2

$$= \frac{1}{2\pi}\left[X(\omega) * \left[\pi\,\delta(\omega - 4\pi) + \pi\,\delta(\omega + 4\pi)\right]\right]$$

1/2

$$= \frac{\cancel{\pi}}{2\cancel{\pi}}\left[X(\omega - 4\pi) + X(\omega + 4\pi)\right]$$

$\frac{1}{2}X(\omega - 4\pi)$



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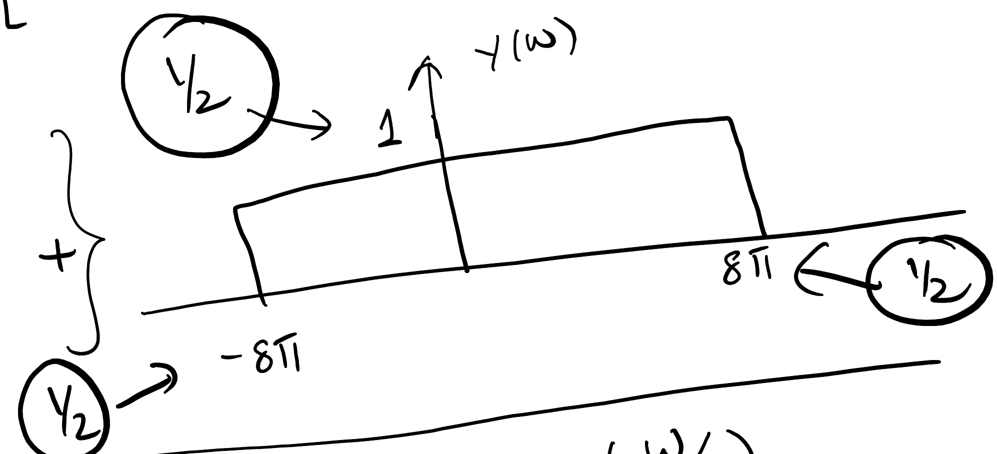
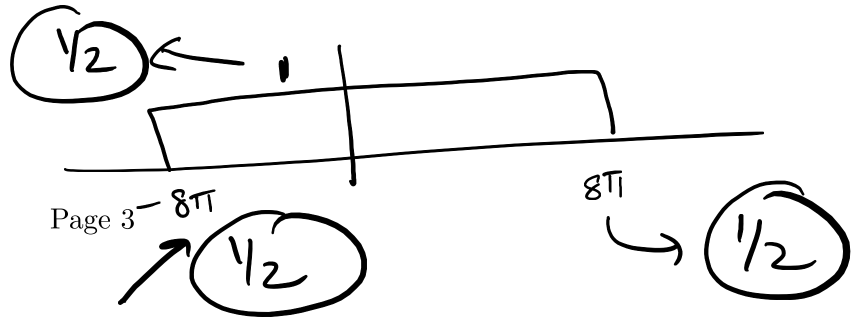


(b) $z(t) = x(2t) \longleftrightarrow Z(\omega) = \frac{1}{|2|}X\left(\frac{\omega}{2}\right)$



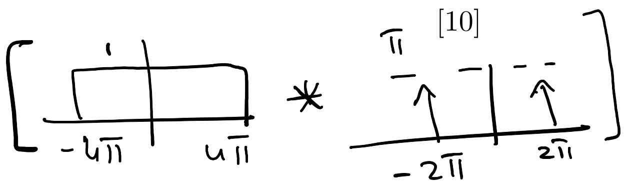
(c) $z(t) = y(t)$ — T

3. Evaluate the energy of the signal, $x(t)$, given below

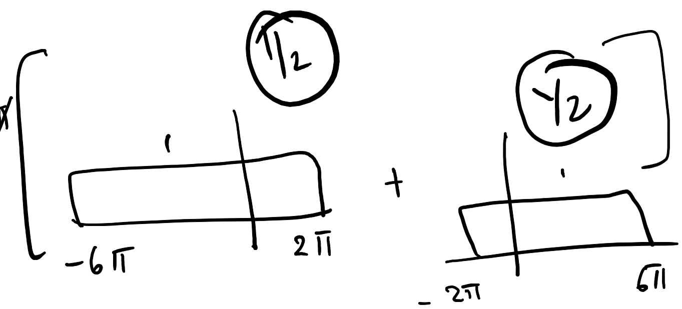
$$x(t) = 8\frac{\sin(4\pi t)}{\pi t}\cos(2\pi t)$$

[10]

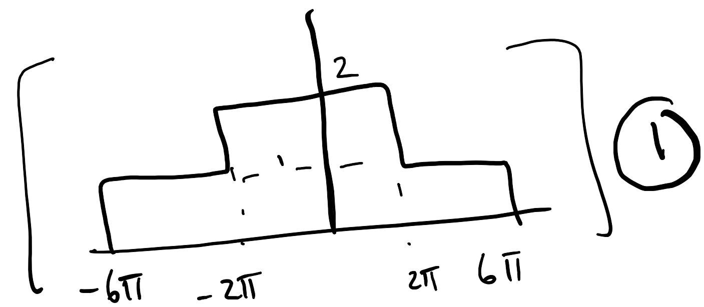
$x(t) = 8\,\frac{\sin 4\pi t}{\pi t}\cos(2\pi t) \longleftrightarrow 8\cdot\frac{1}{2\pi}\left[\;\ast\;\right]$



$8\cdot\frac{1}{2\cancel{\pi}}\cdot\cancel{\pi}\left[\;+\;\right]$



$= 4\,[\;]$



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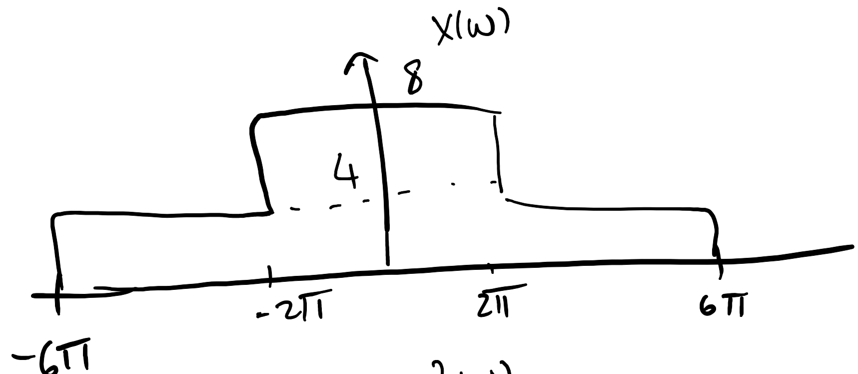

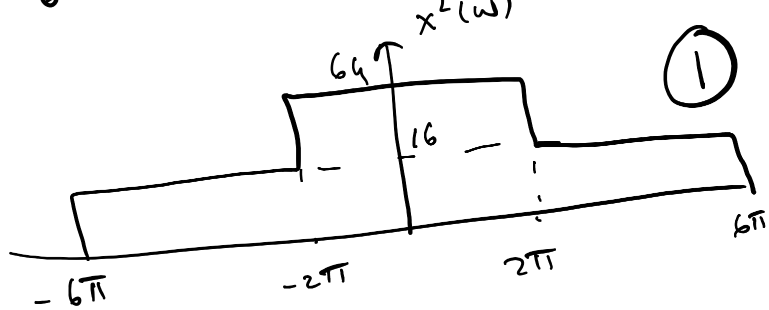

1

Energy $= \int_{-\infty}^{\infty}|x(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty}|X(\omega)|^2\,d\omega = \frac{1}{2\pi}\left[4\pi\times 16 + 4\pi\times 64 + 4\pi\times 16\right]$

$= 2\,[96] = \boxed{192}$ — 1

4. An LTI system has an impulse response $h(t) = e^{-4t}u(t)$. The input to this system is $x(t) = \sin(4\pi t) + \cos(6\pi t + \frac{\pi}{4})$. What is the period of $x(t)$? Find the exponential Fourier series representation of the corresponding output. [10]

$$\boxed{H(\Omega) = \frac{1}{4+j\Omega}} \; - 1$$

Idea: $$e^{j\Omega_0 t} \longrightarrow \boxed{h(t)} \longrightarrow e^{j\Omega_0 t} H(\Omega_0) = e^{j\Omega_0 t} \frac{1}{4+j\Omega_0}$$

$\sin(4\pi t)$, $\cos(6\pi t + \pi/4)$ $\Big\}$ Common-Period $= 1 = T$ — 1

$\Rightarrow T = \frac{1}{2}$, $T = \frac{1}{3}$

Fundamental frequency $= \Omega_0 = 2\pi$

Therefore F.S. of output $y(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\Omega_0 t}$ where $\Omega_0 = 2\pi$

$= \dots a \dots + a_{-1}e^{-j2\pi t} + a_0 + a_1 e^{j2\pi t} + a_2 e^{j4\pi t} + a_3 e^{j6\pi t}$

$\sin 4\pi t$

$$= \frac{e^{j4\pi t} - e^{-j4\pi t}}{2j} \longrightarrow \boxed{h(t)} \longrightarrow \underbrace{\frac{e^{j4\pi t}}{2j} \cdot \frac{1}{4+j4\pi}}_{a_2} - \underbrace{\frac{e^{-j4\pi t}}{2j} \cdot \frac{1}{4-j4\pi}}_{a_{-2}}$$

1/2 1/2

$\cos\left(6\pi t + \frac{\pi}{4}\right)$

$$= e^{j6\pi t} \cdot \frac{e^{j\pi/4}}{2} + e^{-j6\pi t} \cdot \frac{e^{-j\pi/4}}{2} \longrightarrow \boxed{h(t)} \longrightarrow e^{j6\pi t} \cdot \underbrace{\frac{e^{j\pi/4}}{2} \cdot \frac{1}{4+j6\pi}}_{a_3} + e^{-j6\pi t} \cdot \underbrace{\frac{e^{-j\pi/4}}{2} \cdot \frac{1}{4-j6\pi}}_{a_{-3}}$$

1/2 1/2

F.S. coeff of $y(t)$ are $\begin{cases} a_0 = 0 \\ a_1 = 0 \\ a_2 = \frac{1}{2j(4+j4\pi)}, \\ a_3 = \end{cases}$ $a_{-1} = 0$, $a_{-2} =$, $a_{-3} =$

Getting the correct coefficient number 1/2 $\longrightarrow a_2 =$

1/2 $\longrightarrow a_3 =$

all the rest are zero

More Work Space for Answering Questions