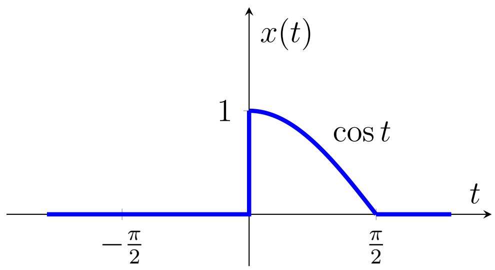
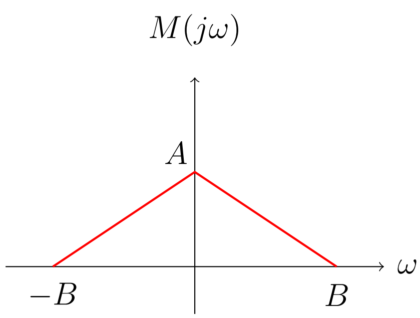
EE1101 Signals and Systems JAN—MAY 2019
Tutorial 7, Extra Questions
March 18, 2019

1. Find the Fourier transform of the signal $x(t)$ shown below.



2. a) Consider a signal $m(t)$ with its Fourier transform as shown in the figure below. The signal $m(t)$ is multiplied by a sinusoid $\cos(\omega t)$ to obtain the signal $x(t) = m(t)\cos(\omega t)$. Plot the Fourier transform of $x(t)$. Assume $\omega \gg B$.



   b) The signal $x(t)$ is again multiplied by the same sinusoid $\cos(\omega t)$ to get a new signal $y(t)$. Plot the Fourier transform of $y(t)$.

   c) The signal $y(t)$ is passed through an ideal low pass filter with cut off frequency ω_l. What should the range of ω_l be so that we recover $m(t)$ at the output of filter.

3. a) The impulse response of a linear time-invariant continuous time system is given by $h(t) = e^{-2t}u(t)$, where u(t) denotes the unit step function. Calculate the frequency response $H(\omega)$ of this system in terms of angular frequency ω.

   b) Find the output of this system, to the sinusoidal input $x(t) = 2cos(2t)$ for all time t.

4. Given that $x(t)$ has the Fourier transform $X(jw)$, Calculate the inverse fourier transform of following functions in terms of $x(t)$.

   a) $X(j(w - w_0)) + X(j(w + w_0))$

   b) Even$[X(jw)]$

5. A signal $x(t)$ can be expressed as the sum of even and odd components as $x(t) = x_e(t) + x_o(t)$.

   a) If $x(t) \iff X(j\omega)$, show that for real $x(t)$, $x_e(t) \iff Re[X(j\omega)]$ and $x_o(t) \iff jIm[X(j\omega)]$.

   b) Verify these results for $x(t) = e^{-at}u(t)$.

6. Prove that in general, the following relationships hold:

$$\frac{dx(t)}{dt} * y(t) = x(t) * \frac{dy(t)}{dt} = \frac{d(x(t) * y(t))}{dt}$$

Here $*$ represents convolution, and $x(t), y(t)$ are differentiable.

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