MA2020 Differential Equations (July - November 2019)
Assignment Sheet- 1 (Covering Quiz-I Syllabus)

1. Solve:

   (a) $x\frac{dy}{dx} + y \ = \ x^3y^6$

   (b) $x\frac{dy}{dx} + y \ = \ y^2 log(x)$

   (c) $(x^2y^3 + xy)\frac{dy}{dx} \ = \ 1$

   (d) $\frac{dy}{dx} + (2x\tan^{-1} y - x^3)(1 + y^2) = 0$

   (e) $(1 + e^{\frac{x}{y}})dx + e^{\frac{x}{y}}(1 - \frac{x}{y})dy \ = \ 0$

   (f) $(xdx + ydy)(x^2 + y^2) = ydx - xdy$

   (g) $\frac{dy}{dx} = 3y + 2e^{3x}$, $y(0) = 2$

   (h) $\frac{dy}{dx} = y\tan(x) + \sec(x)$, $y(0) = -1$

2. Define the Wronskian $w(y_1, y_2)$ of any two differentiable functions y_1 and y_2 defined in an interval $(a, b) \subset R$. Show that $w(y_1, y_2) = 0$ if y_1 and y_2 are linearly dependent.

3. If y_1 and y_2 are any two solutions of a second order linear homogeneous ordinary differential equation which is defined in an interval $(a, b) \subset R$, then $w(y_1, y_2)$ is either identically zero or non-zero at any point of the interval (a, b).

4. If y_1 and y_2 are two linearly independent solutions of a second order linear homogeneous ordinary differential equation then prove that $y = c_1y_1 + c_2y_2$, where c_1 and c_2 are constants, is a general solution.

5. Find the general solution of the following second order equations using the given known solution y_1.

   (a) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} - 4y \ = \ 0$ where $y_1(x) = x^2$.

   (b) $(x - 1)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y \ = \ 0$ where $y_1(x) = x$.

   (c) $x\frac{d^2y}{dx^2} - (2x + 1)\frac{dy}{dx} + (x + 1)y \ = \ 0$ where $y_1(x) = e^x$.

6. Find the general solution of each of the following equations ($D^n \equiv \frac{d^n}{dx^n}$)

   (a) $(D^3 - 4D^2 + 5D - 2)y = 0$

   (b) $(D^2 - 5D - 6)y \ = \ 3\sin 2x$

   (c) $(D^2 - 4D + 4)y = \cos 2x$

   (d) $(D^2 - 3D + 2)y \ = \ (4x + 5)e^{3x}$

   (e) $(D^2 - 1)y \ = \ 3e^{2x}\cos 2x$

   (f) $(D^2 - 2D - 3)y \ = \ 3e^{-x}\cos x$

7. Solve the following using the method of variation of parameters ($D^n \equiv \frac{d^n}{dx^n}$)

   (a) $(D^2 + 1)y \ = \ cosecx$

   (b) $(D^2 - D - 6)y \ = \ e^{-x}$

(c) $(D^2 + a^2)y \;=\; \tan ax$

(d) $x^2y'' - 2xy' + 2y \;=\; x^3 \cos x$

8. Locate and classify the singular points of the following differential equaitons

    (a) $x^2(x+2)y^{''} + xy^{'} - (2x-1)y = 0$

    (b) $(x-1)^2(x+3)y^{''} + (2x+1)y^{'} - y = 0$

    (c) $(2x+1)x^2y^{''} - (x+2)y^{'} + 2e^xy = 0$

9. Solve, using the power series method

    (a) $(1-x^2)y^{''} - 2xy^{'} + 2y = 0$

    (b) $(1+x^2)y^{''} + 2xy^{'} - 2y = 0$

    (c) $y^{''} + \frac{1}{x}y^{'} - \frac{1}{x^2}y = 0, y(1) = 1, y^{'}(1) = 0$

    (d) $y^{''} + \frac{x}{1-x^2}y^{'} - \frac{1}{1-x^2}y = 0, y(0) = 1, y^{'}(0) = 1$