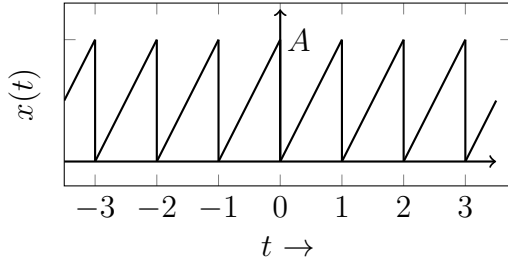
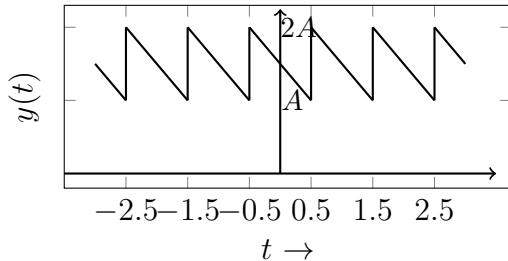


EE1101 Signals and Systems JAN—MAY 2018
Tutorial 5
 March 5, 2018

1. A periodic signal $x(t)$ is given below.



- (a) Determine its Fourier Series coefficients in the exponential form. Sketch its magnitude and phase spectrum.
- (b) Using the results in part (a) above and without doing elaborate integrations, determine the coefficients of the Fourier series of the periodic signal $y(t)$ shown below. Sketch the magnitude and phase spectrum.



2. Determine the coefficients of the Fourier series of the periodic signals $x_1(t)$ and $x_2(t)$ with period T_0 and defined in the interval $[-\frac{T_0}{2}, \frac{T_0}{2})$ as follows.

$$x_1(t) = \begin{cases} A & |t| < d/2 \\ 0 & \text{otherwise} \end{cases}$$

$$x_2(t) = \begin{cases} A \sin(2\pi t/T_0) & 0 \leq t < T_0/2 \\ 0 & -T_0/2 \leq t < 0 \end{cases}$$

Sketch the magnitude and phase spectrum in each case for $A = 1$, $d = 1$ and $T_0 = 2$.

3. A 2π periodic signal $x(t)$ is specified over one period as

$$x(t) = \begin{cases} \frac{t}{A} & 0 \leq t < A \\ 1 & A \leq t < \pi \\ 0 & \pi \leq t < 2\pi \end{cases}$$

Represent the function as an exponential Fourier series.

4. The (exponential) Fourier series coefficients of a periodic signal $x(t)$ is given by

$$d_k = \begin{cases} jk, & |k| < 3 \\ 0, & \text{otherwise} \end{cases}$$

The fundamental period of the signal is $T_0 = 4$. Determine the signal $x(t)$.

5. $x(t)$ is a periodic signal with fundamental period T and Fourier series coefficients a_k . Derive the Fourier series coefficients of the following signals in terms of a_k .

- (a) $x(t - t_0) + x(t + t_0)$
 (b) $\text{Even}\{x(t)\}$
 (c) $\text{Real}\{x(t)\}$

6. Use the (periodic) convolution property of Fourier series to determine the Fourier series coefficients of the signal

$$x(t) = \cos t \star \cos 3t.$$

Verify your result by actually performing a time-domain periodic convolution.

7. $x(t) = |\sin t|$.

- (a) Plot the signals $\frac{dx}{dt}$, $\frac{d^2x}{dt^2}$.
 (b) Plot and find the Fourier series coefficients of the signal $x(t) + \frac{d^2x}{dt^2}$.

- (c) Use the differentiation property to find the Fourier series coefficients of the signal $x(t)$.
8. Suppose we are given the following information about signal $x(t)$:
- i) $x(t)$ is a real signal
 - ii) $x(t)$ is periodic with period $T=6$ and has Fourier coefficients a_k
 - iii) $a_k=0$ for $k = 0$ and $k > 2$
 - iv) $x(t) = -x(t - 3)$
 - v) $\frac{1}{6} \int_{-3}^3 |x(t)|^2 dt = \frac{1}{2}$
 - vi) a_1 is a positive real number
- Show that $x(t) = A \cos(Bt + C)$ and determine the value of constants A, B and C .

— END —