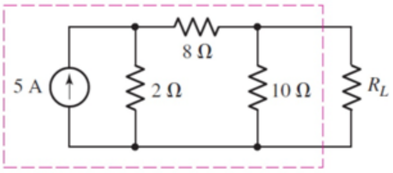
EC2015 Electric Circuits and Networks - Tutorial 6

September 20, 2019

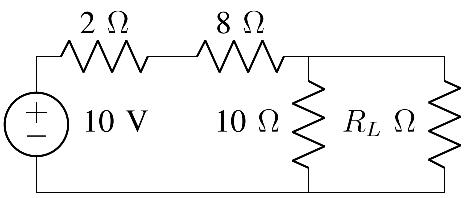
***Topics covered*— Superposition theorem, Source transformation theorem, Thevenin theorem, Norton theorem.**

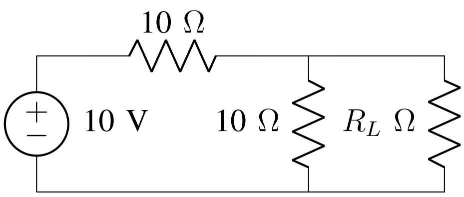
5. Using repeated source transformations, determine the Thevenin and Norton equivalent (i) for the highlighted portion of 1st the network given below and (ii) across the terminals a and b of the 2nd network.

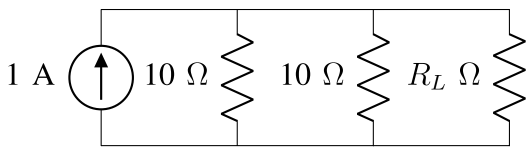
i).



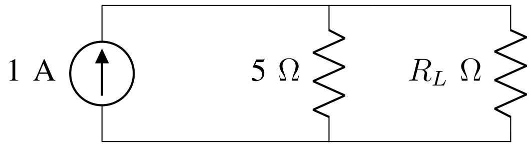
By applying repeated source transformation



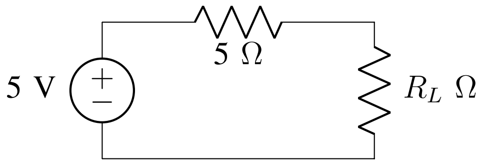




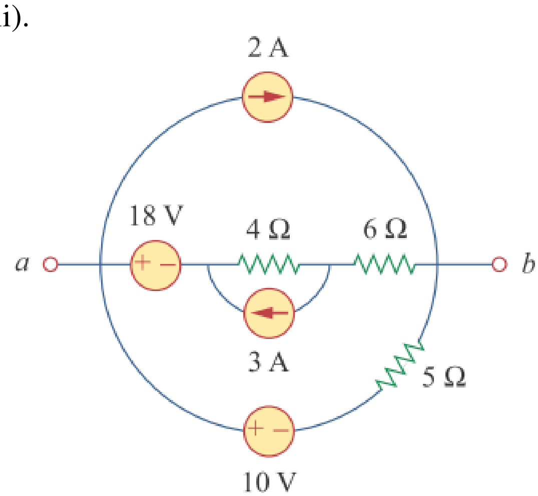
The Norton equivalent circuit is



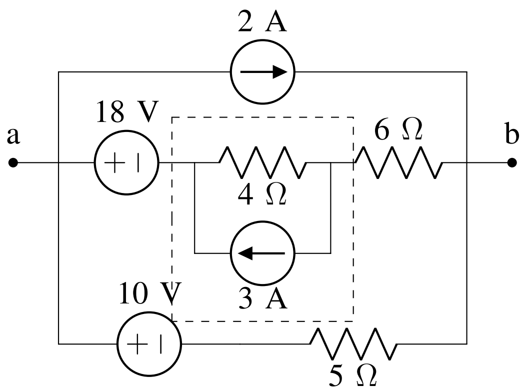
The Thevenin equivalent circuit is

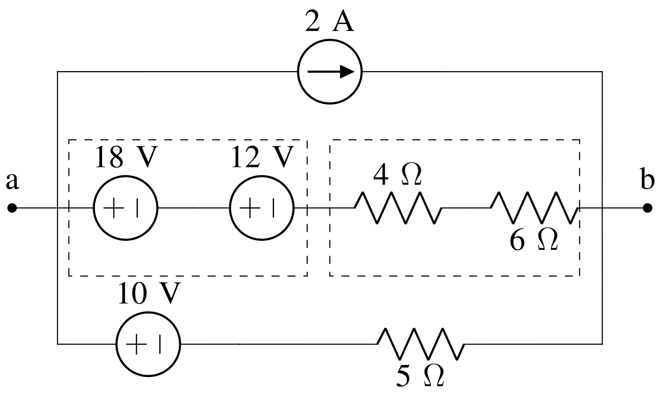


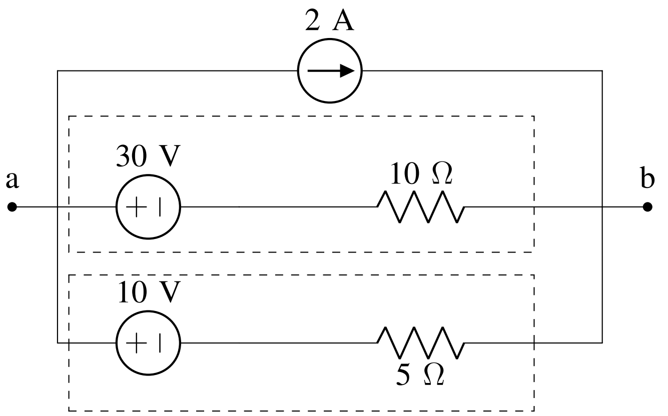
ii).

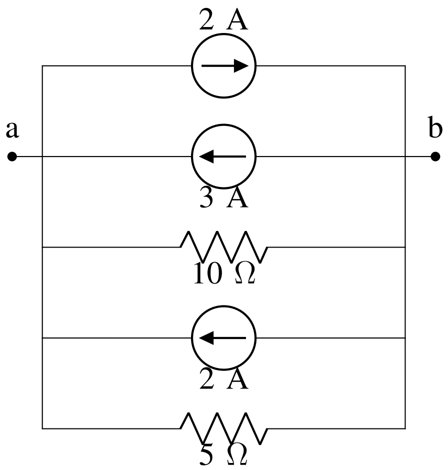


The given circuit can be redrawn as shown below

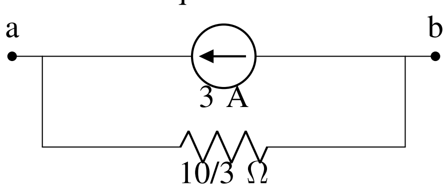




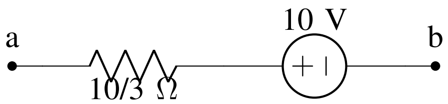




The Norton equivalent network is shown below

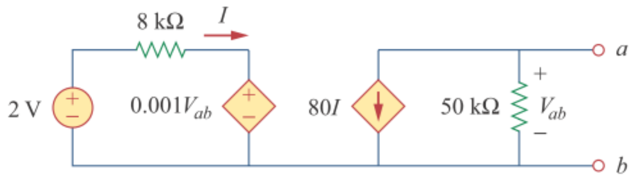


The Thevenin equivalent network is shown below



6. Find all possible (Thevenin and Norton) equivalent circuits for the following networks across the marked terminals:

a.



Calculation of opencircuit voltage V_{oc}:

$$V_{oc} = V_{ab} = -80I * 50k$$

By writing KVL to the first loop

$$-2 + 8k * I + 0.001V_{ab} = 0$$

$$-2 + 8k * I + 0.001 * 80I * 50k = 0$$

$$-2 + 4k * I = 0$$

$$I = 0.5mA$$

$$V_{oc} = V_{ab} = -80 * 0.5m * 50k = -2000V$$

Calculation of short circuit current I_{sc}:
By shorting the terminals a and b

$$I_{sc} = -80I$$

By writing KVL to the first loop

$$-2 + 8k * I = 0$$

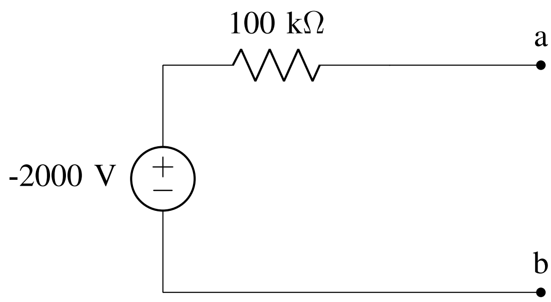
$$I = 2/8k = 0.25mA$$

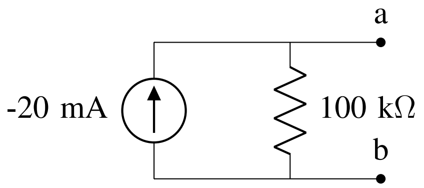
$$I_{sc} = -80 * 0.25mA = -20mA$$

Calculation of R_{Th} or R_N:

$$R_{Th} = R_N = \frac{V_{oc}}{I_{sc}} = \frac{-2000}{-20 \times 10^{-3}} = 100k\Omega$$

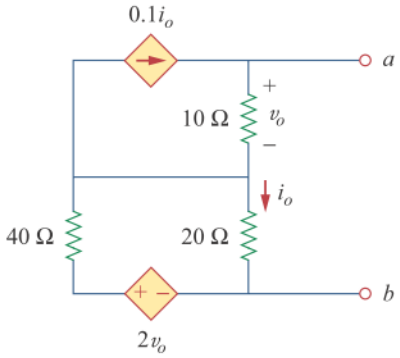
Thevenin equivalent circuit for the given circuit:



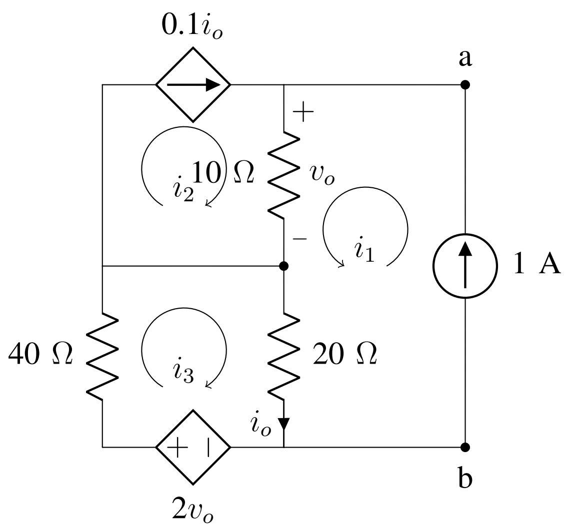


Norton equivalent circuit for the given circuit is shown above.

b.



As there is no independent sources $V_{oc} = 0$ and $I_{sc} = 0$. So the equivalent circuit having single resistor between the terminals a and b. The value of resistance is calculated by connecting a dependent source between a and b.



From this circuit

$$i_1 = 1$$

$$i_2 = 0.1 i_o = 0.1 * (i_1 + i_3) = 0.1 + 0.1 i_3$$

$$v_o = 10(i_1 + i_2) = 10(1 + 0.1 + 0.1 i_3) = 11 + i_3$$

Applying KVL to loop 3:

$$-2v_o + 40 i_3 + 20(i_1 + i_3) = 0$$

$$-2(11 + i_3) + 40 i_3 + 20 i_3 + 20 = 0$$

$$i_3(-2 + 40 + 20) = 2$$

$$i_3 = \frac{2}{58} A$$

Volage across source:

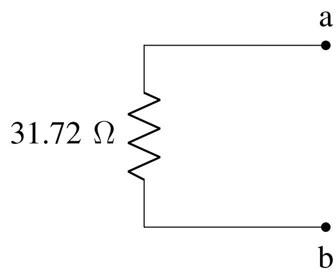
$$v_s = 10(i_1 + i_2) + 20(i_1 + i_3) = 31 + 21 i_3$$

$$v_s = 31 + 21 * \frac{2}{58} = 31.72V$$

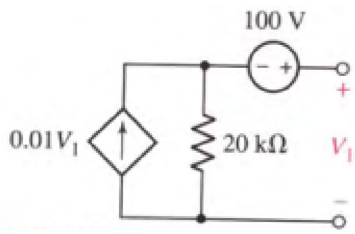
Therefore,

$$R_{Th} = R_N = \frac{v_s}{i_s} = \frac{31.72}{1} = 31.72\Omega$$

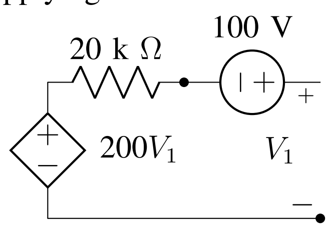
Thevenin and Norton equivalent circuit for the given circuit:



c.



By applying source transformation



Calculation of opencircuit voltage V_{oc}:
By writing KVL to the first loop

$$-200V_1 - 100 + V_1 = 0$$

$$-199V_1 = 100$$

$$V_1 = -0.502V$$

$$V_{oc} = V_1 = -0.502V$$

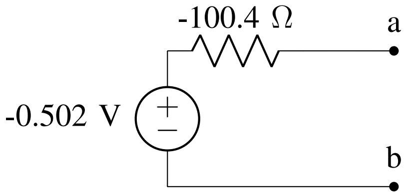
Calculation of short circuit current I_{sc}:
By shorting the terminals a and b

$$I_{sc} = -\frac{100}{20k} = 5mA$$

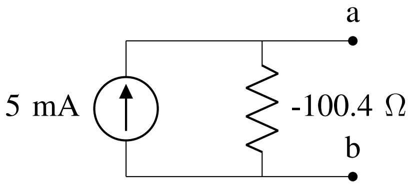
Calculation of R_{Th} or R_N:

$$R_{Th} = R_N = \frac{V_{oc}}{I_{sc}} = \frac{-0.502}{5m} = -100.4\Omega$$

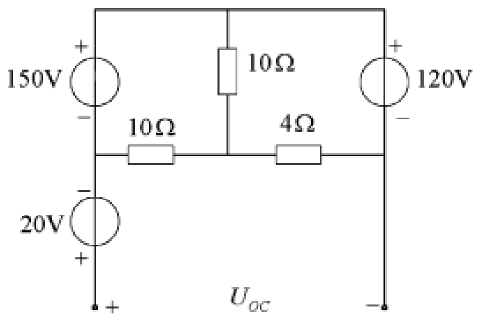
Thevenin equivalent circuit for the given circuit:



Norton equivalent circuitfor the given circuit:



d.



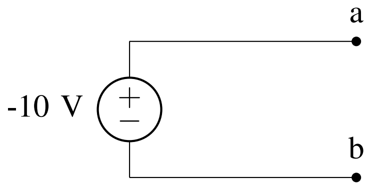
Calculation of opencircuit voltage V_{oc}: By writing KVL outer loop

$$20 - 150 + 120 - U_{oc} = 0$$

$$V_{oc} = -10V$$

Calculation of short circuit current I_{sc}:
By shorting the terminals a and b. The outer loop doesn't satisfies KVL. So, calculation of I_{sc} is not possible.

Thevenin equivalent circuit for the given circuit:

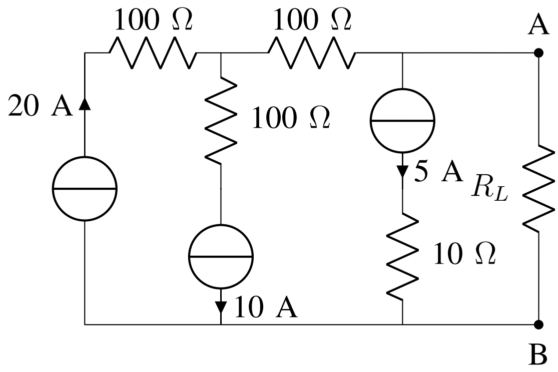


Norton equivalent circuit for the given circuit:
Not possible

Calculation of R_{Th} or R_N: When all source are nullified

$$R_{Th} = 0$$

e.



Here the symbols are current sources in European style of representation.
Calculation of opencircuit voltage V_{oc}: By opening R_L. The network doesn't satisfies KCL. So, calculation of V_{oc} is not possible.
Calculation of short circuit current I_{sc}:
By shorting the terminals R_L.
By writing KCL

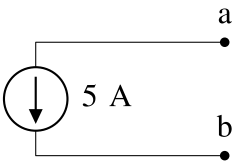
$$-20 + 5 + 10 + I_{sc} = 0$$

$$I_{sc} = -5A$$

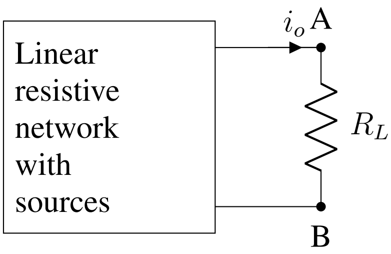
Calculation of R_{Th} or R_N: When all source are nullified

$$R_N = \infty = opencircuit$$

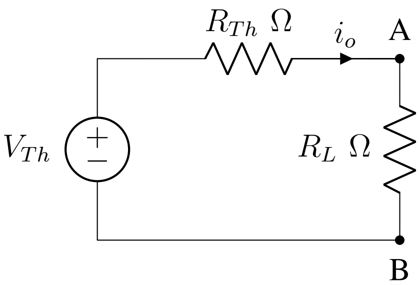
Thevenin equivalent circuit for the given circuit:
Not possible.
Norton equivalent circuit for the given circuit is a 5 A current source connected from a to b as shown in follwing figure.



7. The following box consist of linear resistive circuit with sources. It gives a current $i_o = 2A$ when $R_L = 3\Omega$ and current $i_o = 1A$ when $R_L = 8\Omega$. Find the value of i_o when $R_L = 5\Omega$.



The black box can be replaced with its Thevenin equivalent. Then, the given network can be re-drawn as From the figure



$$i_o = \frac{V_{Th}}{R_{Th} + R_L}$$

By substituting given conditions

$$2 = \frac{V_{Th}}{R_{Th} + 3}$$

$$1 = \frac{V_{Th}}{R_{Th} + 8}$$

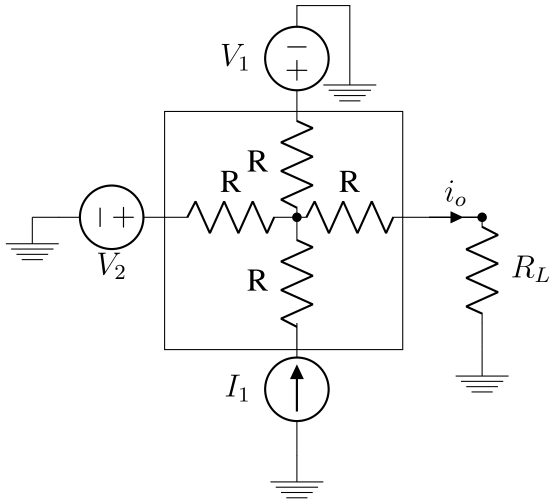
By solving,

$$V_{Th} = 10V \ and \ R_{Th} = 2\Omega$$

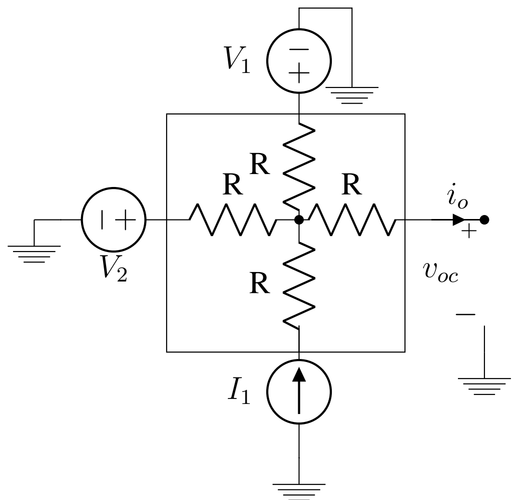
When $R_L = 5\Omega$

$$i_o = \frac{10}{2+5} = \frac{10}{7}A$$

8. Find the generalized equivalent circuit for the following network seen from load resistance R_L and solve the following problems.
(a) Find the value of i_o when $R_L = 5\Omega$, $R = 10\Omega$, $I_1 = 5A$,$V_1 = 10V$ and $V_2 = 5V$.
(b) Find the value of i_o when $R_L = 5\Omega$, $R = 10\Omega$, $I_1 = 5A$,$V_1 = 0V$ and $V_2 = 5V$.



Calculation of open circuit voltage:
The circuit after opening R_L is shown below



Let Let us consider voltage voltage at common node as v
Then,

$$v_{oc} = v$$

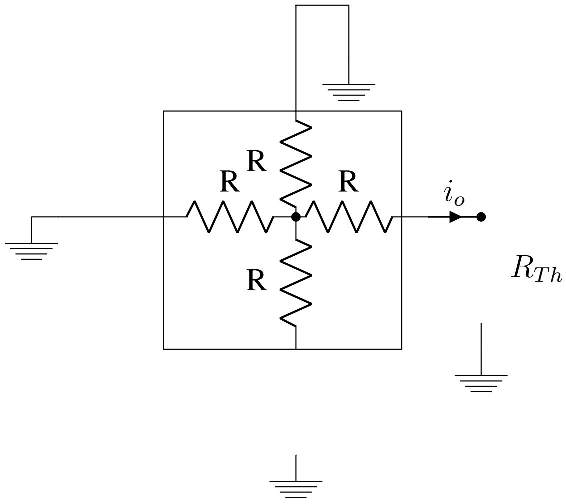
By applying KCL at common node

$$\frac{v - V_1}{R} + \frac{v - V_2}{R} - I_1 = 0$$

$$v(\frac{1}{R}+\frac{1}{R})=\frac{V_1}{R}+\frac{V_2}{R}+I_1$$

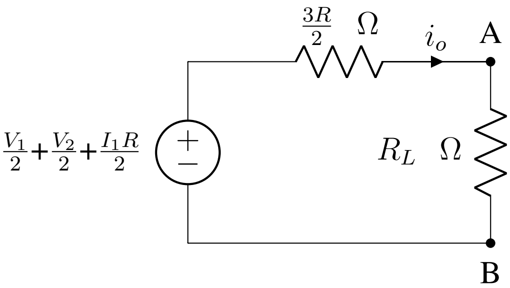
$$v=\frac{V_1}{2}+\frac{V_2}{2}+\frac{I_1R}{2}$$

Calculation of equivalent resistance:
The circuit after nullifying all sources is shown below From the circuit



$$R_{Th}=R+R//R=\frac{3R}{2}$$

Thevenin equivalent circuit for given circuit is:



From this

$$i_o=\frac{\frac{V_1}{2}+\frac{V_2}{2}+\frac{I_1R}{2}}{\frac{3R}{2}+R_L}$$

(a) when $R_L=5\Omega$, $R=10\Omega$,$I_1=5A$,$V_1=10V$ and $V_2=5V$.

$$i_o=\frac{\frac{10}{2}+\frac{5}{2}+\frac{5*10}{2}}{\frac{3*10}{2}+5}=\frac{5+2.5+25}{15+5}=\frac{32.5}{20}$$

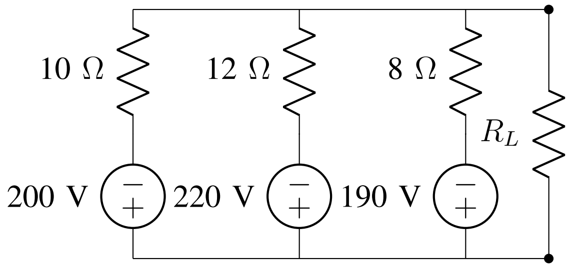
$$i_o=1.625A$$

(b) when $R_L=5\Omega$, $R=10\Omega$, $I_1=5A$,$V_1=0V$ and $V_2=5V$.

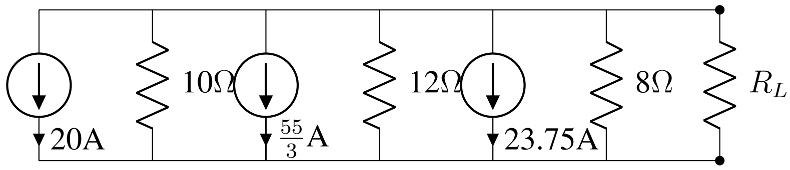
$$i_o=\frac{\frac{0}{2}+\frac{5}{2}+\frac{5*10}{2}}{\frac{3*10}{2}+5}=\frac{0+2.5+25}{15+5}=\frac{27.5}{20}$$

$$i_o=1.375A$$

9. In an industry three DC generators are connected in parallel to supply electrical power to the load as shown below. Draw the equivalent source seen by load R_L.



The given circuit can be redrawn as From this



$$I_{eq}=20+\frac{55}{3}+23.75=62.08A$$

and

$$\frac{1}{R_{eq}}=\frac{1}{10}+\frac{1}{12}+\frac{1}{8}=\frac{37}{120}$$

$$R_{eq}=\frac{120}{37}$$

$$V_{eq}=I_{eq}*R_{eq}=62.08*\frac{120}{37}$$

$$V_{eq}=201.34$$

The equivalent circuits are: