Extra problems based on Tutorial 1

(1) Let $x[n]$ be a signal with $x[n] = 0$ for $n < -2$
1.4 and $n > 4$. For each signal given below, determine the values of n for which it is guaranteed to be zero.

(a) $x[n-3]$ (b) $x[-n]$ (c) $x[-n-2]$

(2) Determine whether or not each of the following
1.6 signals is periodic.

(a) $x_1(t) = 2e^{j(t+\pi/4)}u(t)$ (b) $x_2[n] = u[n] + u[-n]$

(c) $x_3[n] = \sum_{k=-\infty}^{\infty} \{\delta[n-4k] - \delta[n-1-4k]\}$

(3) For each of the following signals given below, determine
1.7 all the values of the independent variable at which the even part of the signal is guaranteed to be zero.

(a) $x_1[n] = \left(\frac{1}{2}\right)^n u[n-3]$ (b) $x_2(t) = \sin(t/2)$

(c) $x_3(t) = e^{-5t}u(t+2)$

(4) Consider the signal $x(t) = \delta(t+2) - \delta(t-2)$.
1.13 Calculate the value of E_∞ for the signal

$$y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$$

(5) (a) Show that if $x[n]$ is an odd signal, then
1.34

$$\sum_{n=-\infty}^{\infty} x[n] = 0$$

(b) Show that if $x_1[n]$ is an odd signal and $x_2[n]$ is an even signal, then $x_1[n]\,x_2[n]$ is an odd signal

(c) Let $x[n]$ be an arbitrary signal, with even and odd parts denoted by

$$x_e[n] = \mathcal{E}v\{x[n]\} \quad \text{and}$$

$$x_o[n] = \mathcal{O}d\{x[n]\}$$

Show that

$$\sum_{n=-\infty}^{\infty} x^2[n] = \sum_{n=-\infty}^{\infty} x_e^2[n] + \sum_{n=-\infty}^{\infty} x_o^2[n].$$

(6) An important concept in many communication
1.37 applications is the 'correlation' between two signals. Let $x(t)$ and $y(t)$ be two signals, then the correlation function is defined as

$$\phi_{xy}(t) = \int_{-\infty}^{\infty} x(t+\tau)\, y(\tau)\, d\tau$$

The function $\phi_{xx}(t)$ is usually referred as the autocorrelation function of the signal $x(t)$, while $\phi_{xy}(t)$ is often called a cross-correlation function.

(a) What is the relationship between $\phi_{xy}(t)$ and $\phi_{yx}(t)$?

(b). Compute the odd part of $\phi_{xx}(t)$.

(c) Suppose $y(t) = x(t+T)$. Express $\phi_{xy}(t)$ and $\phi_{yy}(t)$ in terms of $\phi_{xx}(t)$.

Extra Questions based on Tutorial-2

① Check if the following systems are (a) linear (b) time invariant (c) memoryless (d) causal and (e) stable.

(i) $y(t) = x(t-2) + x(2-t)$

(ii) $y(t) = (\cos 3t)\, x(t)$

(iii) $y[n] = n\, x[n]$

(iv) $y[n] = x[-n]$

(v) $y[n] = \mathcal{E}v\left[x[n-1]\right]$

② Determine whether the following systems are invertible. If so, find the inverse system.

(i) $y(t) = \cos(x(t))$

(ii) $y[n] = n\, x[n]$

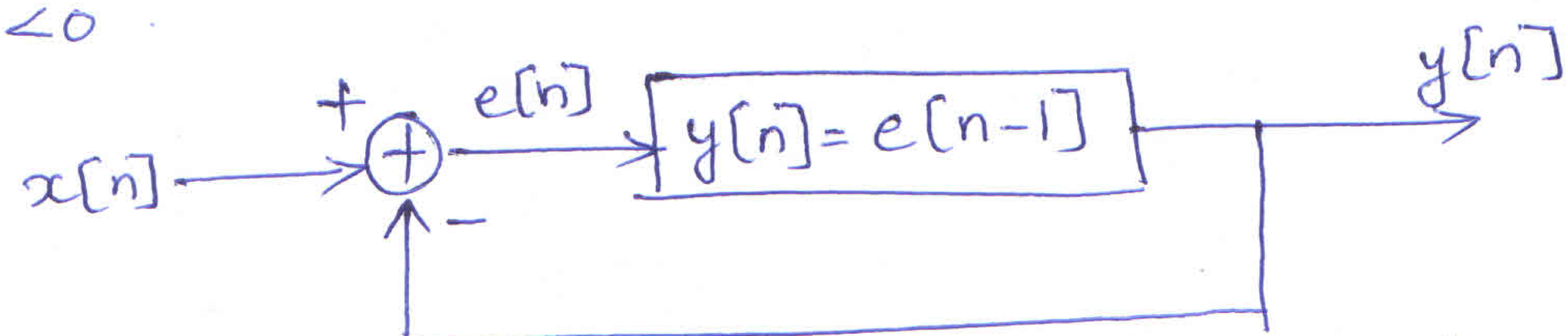
(iii) $y[n] = x[1-n]$

(iv) $y[n] = x[2n]$

(v) $y(t) = x(2t)$

③ For a time invariant system, show that if the input $x(t)$ is periodic, then the output $y(t)$ is also periodic.

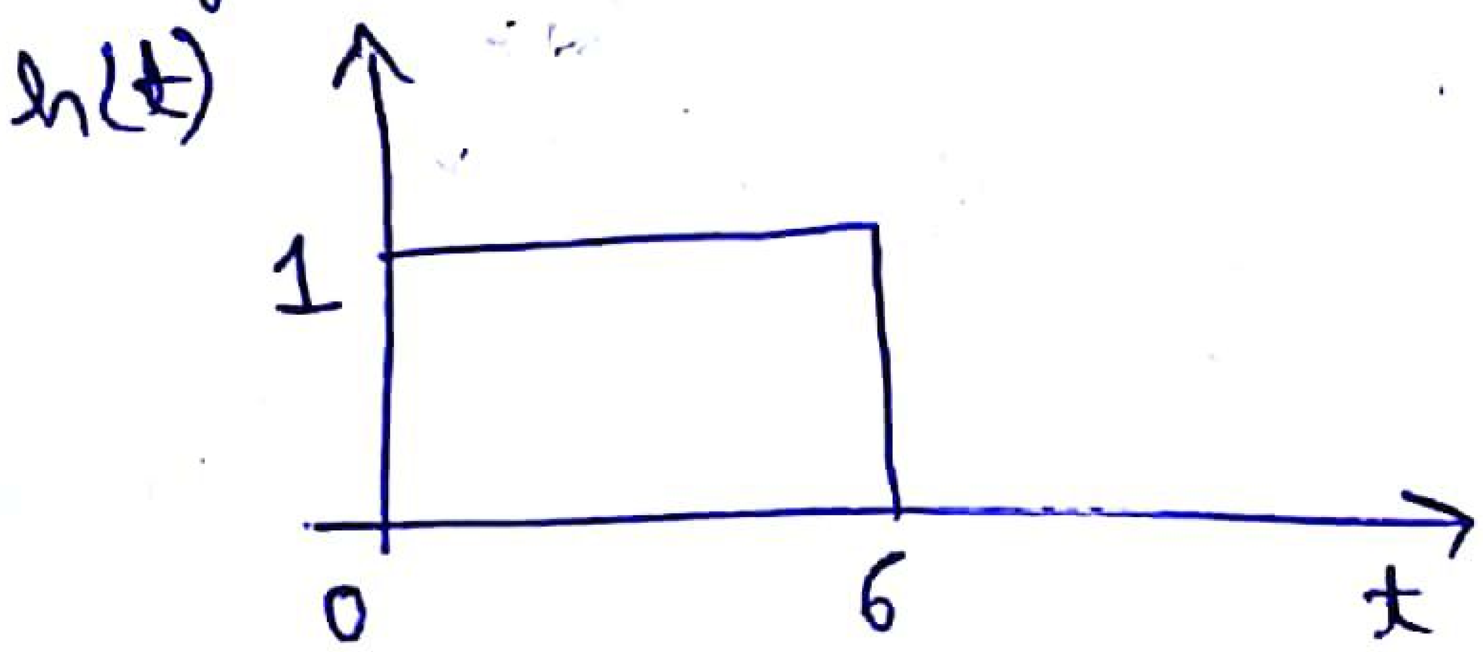
④ For the feedback system shown below, sketch the output $y[n]$ for (i) $x[n] = \delta[n]$ and (ii) $x[n] = u[n]$. Assume $y[n] = 0$ for $n < 0$.

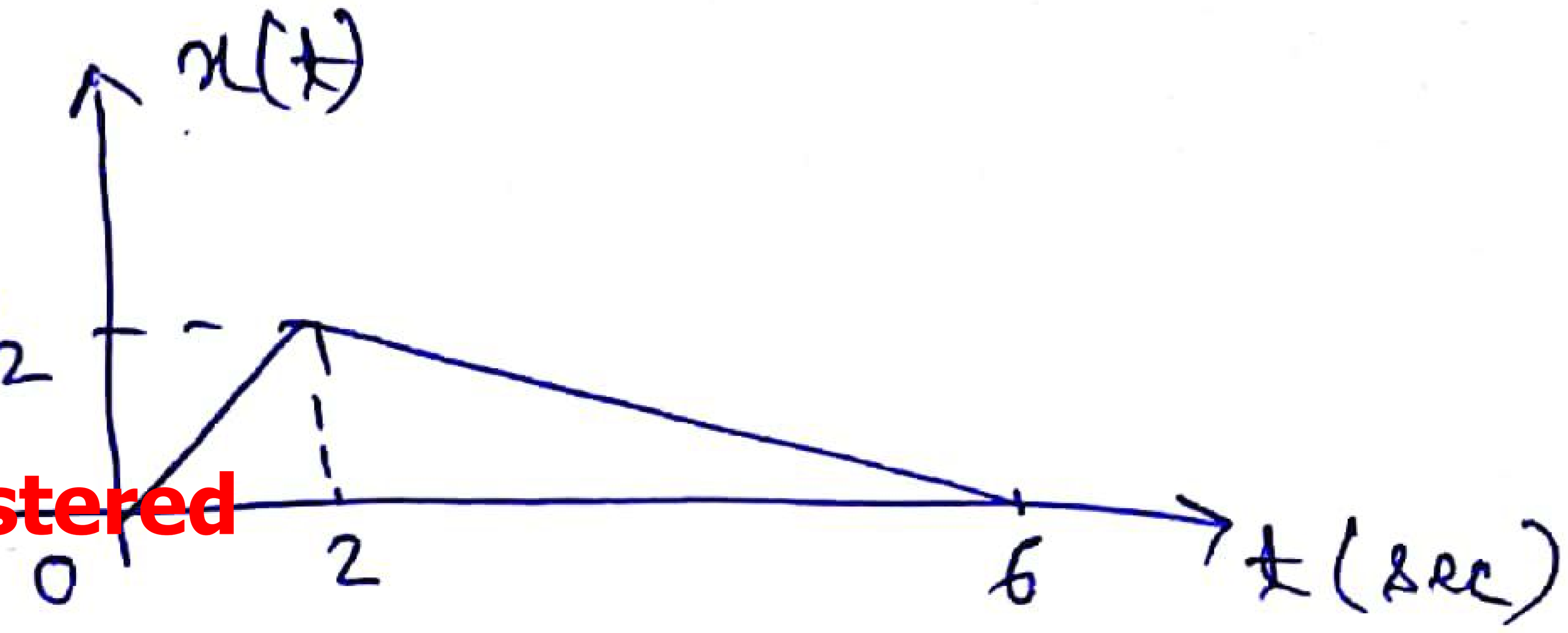


⑤ Give an example of a system which satisfies the condition of additivity but not the condition of homogeneity.

EXTRA QUESTIONS

Q1) The impulse response and the excitation function of a LTI and causal system are shown in Fig (a) and (b) respectively. The output of the system at $t = 2$ sec is equal to –

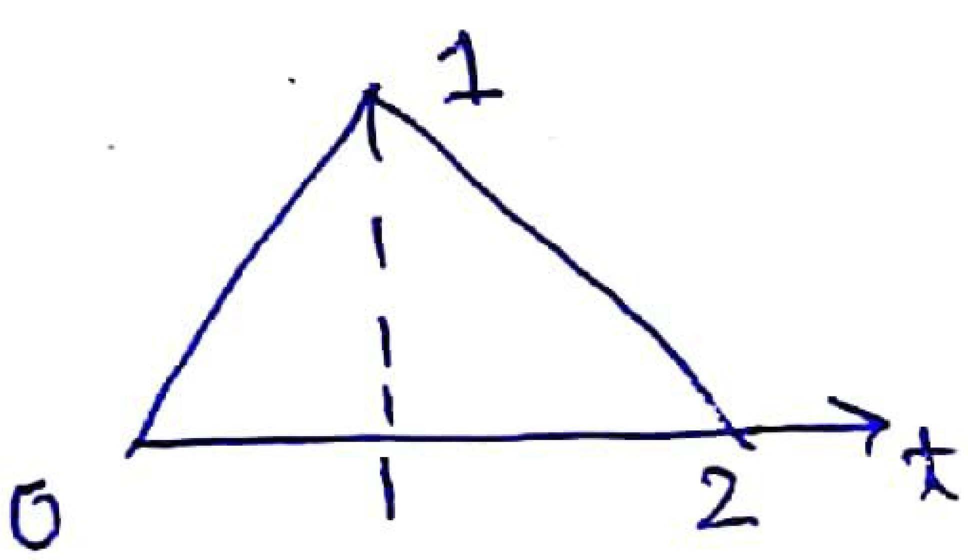




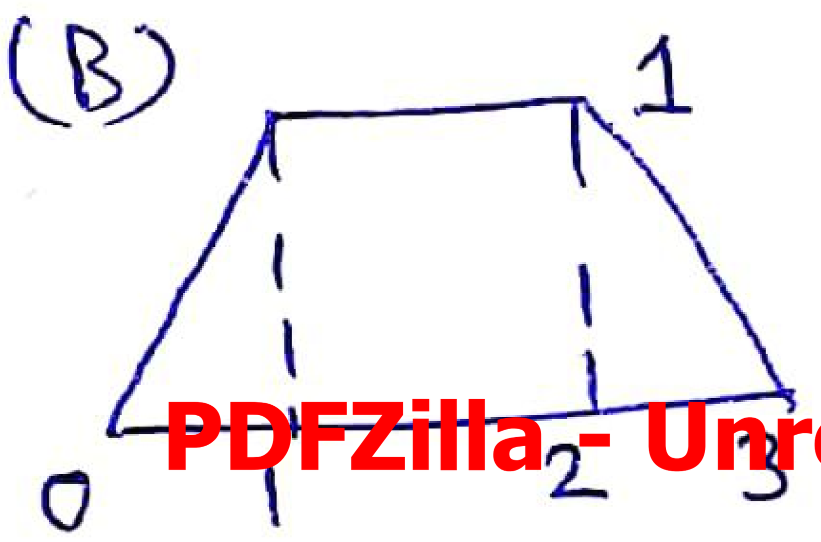
(A) 0 (B) 1/2 (C) 3/2 (D) 1.

Q2) Let $u(t)$ be the unit step function. Which of the following then corresponds to the convolution of $[u(t) - u(t-1)]$ with $[u(t) - u(t-2)]$?

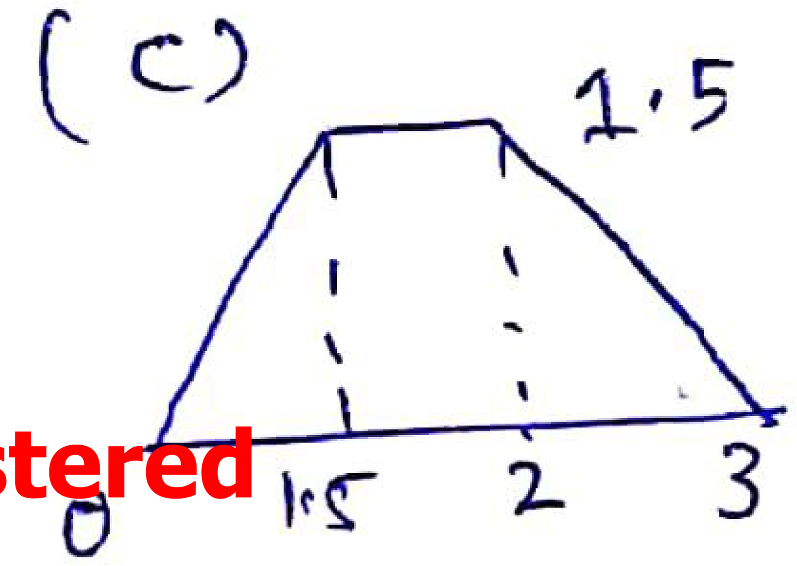
(A)



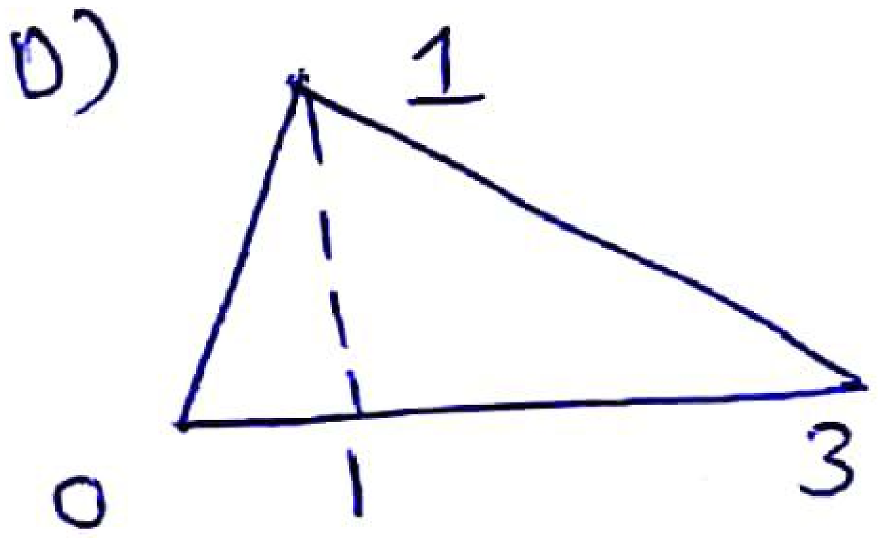
(B)



(C)



(D)



Q3) The impulse response of a continuous time system is given by $h(t) = \delta(t-1) + \delta(t-3)$. The value of the step response at $t = 2$ is :

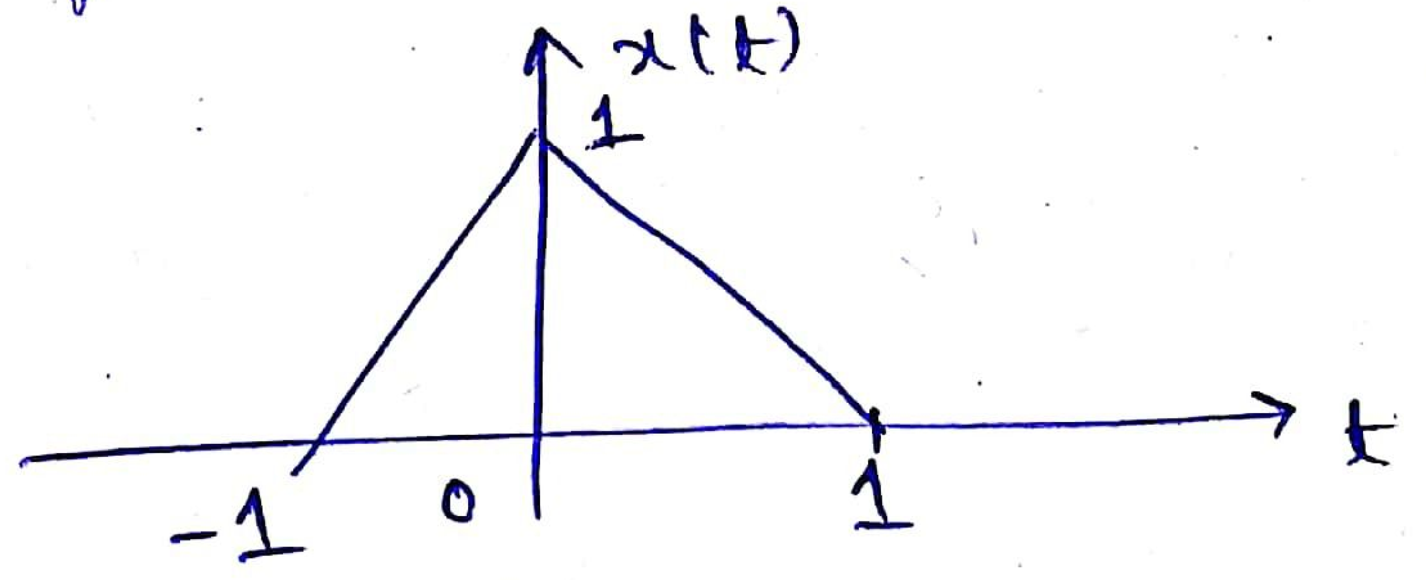
(A) 0 (B) 1 (C) 2 (D) 3.

EXTRA QUESTIONS

Q1) An input signal $x(t) = \begin{cases} 0 ; & t<0 \\ e^{-t} ; & 0 \le t \le 1 \\ 0 ; & t > 1 \end{cases}$ is applied to an unstable LTI system having impulse response $h(t) = e^{t/2} u(t)$. Find the output $y(t)$?

Q2) Suppose that the signal $x(t) = u(t+0.5) - u(t-0.5)$ and the signal $h(t) = e^{j\omega_0 t}$. Determine a value of ω_0 which ensures that $y(0) = 0$, where $y(t) = x(t) \circledast h(t)$?

Q3) An input signal $x(t)$ shown in figure is applied to the system with impulse response $h(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$. Find the output, for the values of (i) $T = 1$, (ii) $T = 2$.



Q4) The impulse response and the excitation function of a causal LTI system are given fig (a) & (b). Find the output of the system at $t = 2$ sec.

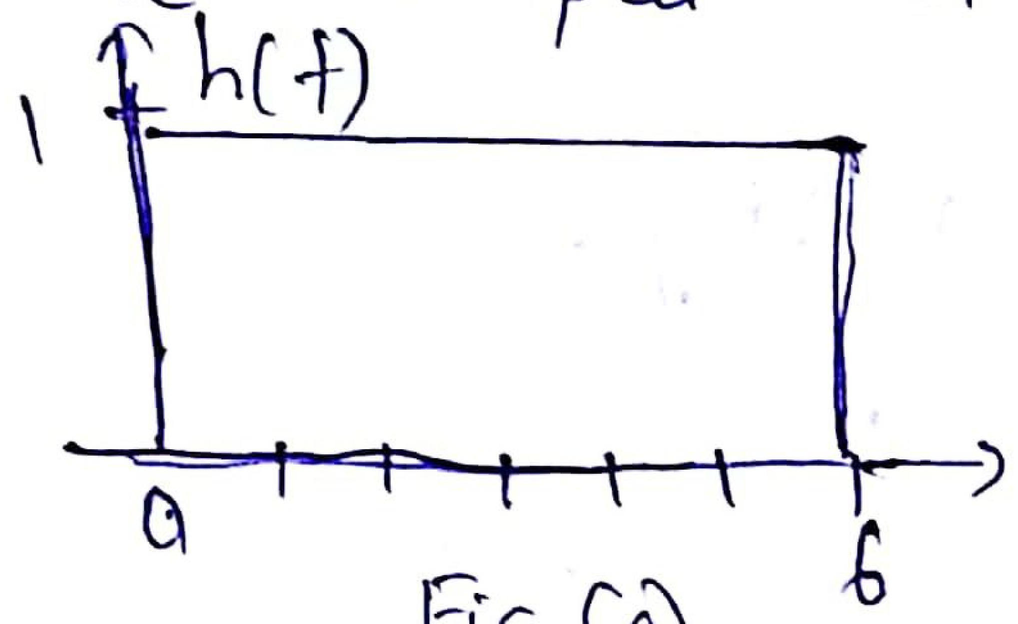


Fig (a)

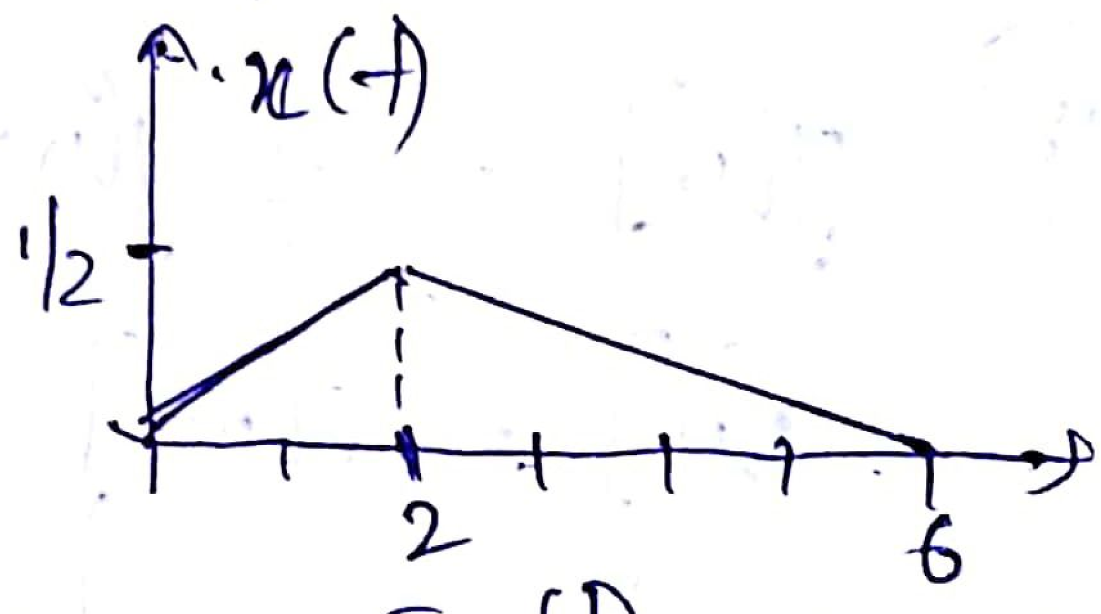


Fig (b)

Q5) The impulse response of a continuous time system is given by
$h(t) = \delta(t-1) + \delta(t-3)$.
Find the value of step response at $t = 2$ sec?

Q6) If $y(t) = x(-t) * \delta(t - t_0)$, where $*$ denotes convolution, express $y(t)$ only in terms of $x(t)$.

Q 3. Let $x[n]$ be a real, odd periodic s/l with period

(TB 3.10) $N=7$ & FS coeff. a_k. Given

$$a_{15} = j, \quad a_{16} = 2j, \quad a_{17} = 3j.$$

Find a_0, a_{-1}, a_{-2} & a_{-3}.

Q 4. For the CT periodic s/l
(TB 3.3)
$$x(t) = 2 + \cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{5\pi}{3}t\right),$$

Find the fundamental freq. ω_0 & the FS coeff. a_k

such that $$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}.$$

Q1. Let $x_1(t)$ be a continuous-time periodic s/l with fundamental freq. ω_1 & Fourier coeff. a_k. Given
(TB 3.5)

$$x_2(t) = x_1(1-t) + x_1(t-1).$$

How is the fundamental freq ω_2 of $x_2(t)$ related to ω_1?
Also, find the relation b/w FS coeff. b_k of $x_2(t)$ & a_k.

Q2. Consider 3 CT periodic s/ls whose FS representations are
(TB 3.6)

$$x_1(t) = \sum_{k=0}^{100} \left(\frac{1}{2}\right)^k e^{jk\frac{2\pi}{50}t}$$

$$x_2(t) = \sum_{k=-100}^{100} \cos(k\pi) e^{jk\frac{2\pi}{50}t}$$

$$x_3(t) = \sum_{k=-100}^{100} j\sin\left(\frac{k\pi}{2}\right) e^{jk\frac{2\pi}{50}t}.$$

Use FS properties to answer the following–

i) Which of these s/ls are real-valued?

ii) Which of these s/ls are even.

Tutorial - 6

Extra questions

1. Let $x[n]$ be a periodic signal with period N and Fourier coefficients a_k.

   (a) Express the Fourier coefficients b_k of $|x[n]|^2$ in terms of a_k.

   (b) If the coefficients a_k are real, is it guaranteed that the coefficients b_k are also real?

2. The concept of *eigenfunctions*, which is and important tool in the study of LTI systems, is also relevant for linear, but time-variant systems. Consider a system with input $x(t)$ and output $y(t)$. We say that a signal $\phi(t)$ is an *eigenfunction* of the system if $x(t) = \phi(t) \implies y(t) = \lambda\phi(t)$, where the complex constant λ is called the *eigenvalue* associated with $\phi(t)$.

   (a) Suppose we can represent the input $x(t)$ to the system as a linear combination of eigenfunctions $\phi_k(t)$ with the corresponding eigenvalues λ_k, that is

   $$x(t) = \sum_{k=\infty}^{\infty} c_k \phi_k(t)$$

   Express the output $y(t)$ of the system in terms of $\{c_k\}$, $\{\phi_k(t)\}$ and $\{\lambda_k\}$.

   (b) Consider the system characterized by the differential equation

   $$y(t) = t^2 \frac{d^2x(t)}{dt^2} + t\frac{dx(t)}{dt}$$

   Is the system linear? Is it time invariant?

   (c) Show that the functions $\phi_k(t) = t^k$ are eigenfunctions of the system defined in part (b). Find the corresponding eigenvalues λ_k for each $\phi_k(t)$.

   (d) Determine the output of the system if $x(t) = 10t^{-10} + 3t + \frac{1}{2}t^4 + \pi$.

3. Two functions $u(t)$ and $v(t)$ are said to be *orthogonal over an interval (a,b)* if

   $$\int_a^b u(t)v^*(t)dt = 0$$

   If, in addition,

   $$\int_a^b |u(t)|^2 dt = 0 = \int_a^b |v(t)|^2 dt$$

   the functions are said to be *normalized* and hence *orthonormal.* A set of functions $\{\phi_k(t)\}$ are said to be *orthogonal (orthonormal)* set if each pair of functions in the set are *orthogonal (orthonormal).*

(a) Consider the pair of signals defined below. Determine if they are orthogonal over the interval $(0, 4)$.

$$\text{u(t)}=\begin{cases} 1, & 0 \leq t < 1 \\ -1, & 1 \leq t < 3 \\ 1, & 3 \leq t \leq 4 \end{cases} \qquad \text{v(t)}=\begin{cases} 1, & 0 \leq t < 2 \\ -1, & 2 \leq t \leq 4 \end{cases}$$

(b) Are the functions $\sin n\omega_0 t$ and $\sin m\omega_0 t$ orthogonal over the interval $(0, T)$, where $T = 2\pi/\omega_0$? Are they also orthonormal?

(c) Show that the functions $\phi_k(t) = e^{jk\omega_0 t}$ are orthogonal over any interval of length $T = 2\pi/\omega_0$. Are they orthonormal?

(d) Let $x(t)$ be an arbitrary signal, and $x_e(t)$ and $x_o(t)$ be its even and odd parts respectively. Show that $x_e(t)$ and $x_o(t)$ are orthogonal over the interval $(-T, T)$ for any T.

Solution

1.
$$x[n] \to a_k \implies x^*[n] \to a^*_{-k}$$
$$|x[n]|^2 = x[n]x^*[n] \to c_k$$

where coefficients c_k are given by

$$c_k = \sum_{l=0}^{N-1} a^*_{-l} a_{k-l}$$

If all a_k's are real, c_k's are also real.