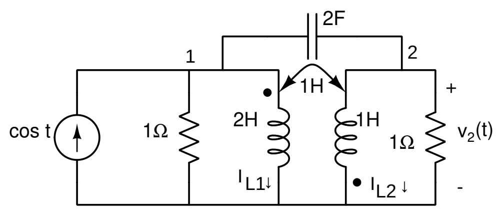
Tutorial 8

October 11, 2019



Q.1

1. Using nodal analysis we get,

$$V_1(1+j2) - V_2(2j) + I_{L1} = 1\underline{/0^\circ} \quad (1)$$
$$V_2(1+j2) - V_1(2j) + I_{L2} = 0 \quad (2)$$

The voltage across $2H$ inductor is given by:

$$V_1 = j2I_{L1} - jI_{L2} \quad (3)$$

The voltage across $1H$ inductor is given by:

$$V_2 = jI_{L2} - jI_{L1} \quad (4)$$

Using (3), (4), we get:

$$I_{L1} = -j(V_1 + V_2) \quad (5)$$
$$I_{l2} = -j(V_1 + 2V_2) \quad (6)$$

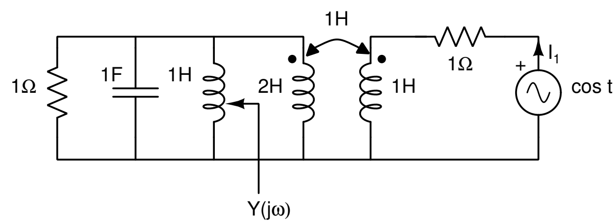
Plugging (5) , (6) in (1) and (2), we get:

$$\begin{bmatrix} 1+j & -3j \\ -3j & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1\underline{/0^\circ} \\ 0 \end{bmatrix}$$

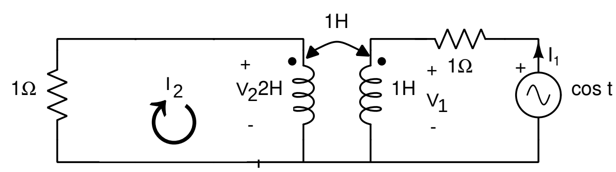
$$V_2 = 0.2985\underline{/84.29^\circ}$$
$$\therefore v_2(t) = 0.2985\cos(t + 84.29^\circ)$$

2.

$$Y(j\omega) = 1 + j\omega + \frac{1}{j\omega}$$
$$= 1 + j(\omega - \frac{1}{\omega})$$



Q.2



In the question, $\omega = 1$, which is resonant frequency. Therefore, the admittance is purely resistive; $Y = 1S$. The circuit can be re-drawn as shown above.

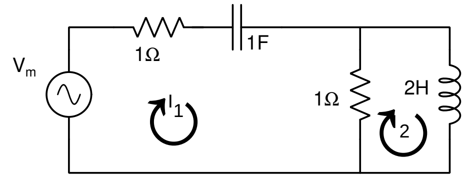
Using mesh analysis on the left side, we get:

$$
\begin{aligned}
-1I_2 - V_2 &= 0 \\
\because V_2 &= j2I_2 + jI_1 \\
\therefore -I_2 &= j2I_2 + jI_1 \\
\text{Also,} & \\
V_1 &= j(I_1 + I_2)
\end{aligned}
$$

Using the above 2 equations, we get:

$$Y_1 = \frac{I_1}{V_1} = 1.582\underline{/-71.565^\circ} S$$

The admittance as seen by the voltage source is $\frac{1}{Z_1+1} = 0.745\underline{/-26.565^\circ} S$



Q.3

3. Assuming $\omega = 1$ and applying mesh analysis we get:

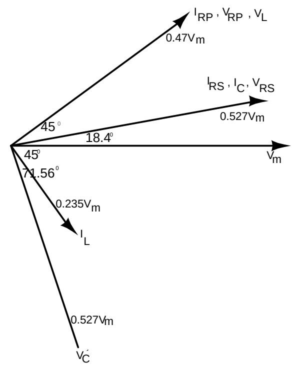
$$\begin{bmatrix} 2-j & -1 \\ -1 & 1+2j \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_m \\ 0 \end{bmatrix}$$

On solving,

$$
\begin{aligned}
I_1 &= 0.528V_m\underline{/18.43^\circ} \\
I_2 &= 0.235V_m\underline{/-45^\circ}
\end{aligned}
$$

$$
\begin{aligned}
I_{RS}(\text{resistor in series}) &= I_C = I_1 = 0.527V_m\underline{/18.43^\circ} \\
I_{RP}(\text{resistor in parallel}) &= I_1 - I_2 = 0.47V_m\underline{/45^\circ} \\
I_L &= I_2 = 0.235V_m\underline{/-45^\circ}
\end{aligned}
$$

$$
\begin{aligned}
V_{RS} &= 0.527V_m\underline{/18.43^\circ} \\
V_C &= 0.527V_m\underline{/-71.565^\circ} \\
V_{RP} &= V_L = I_{RP} = 0.47V_m\underline{/45^\circ}
\end{aligned}
$$

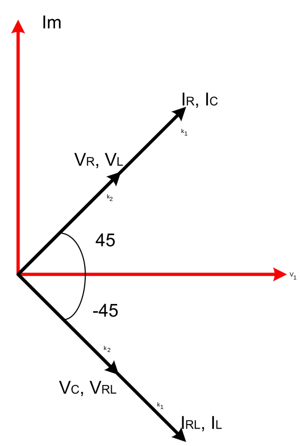


Phasor diagram for Q.3 with $V_m\underline{/0^\circ}$ as the reference.

4. For $|I_L| = |I_C|$,

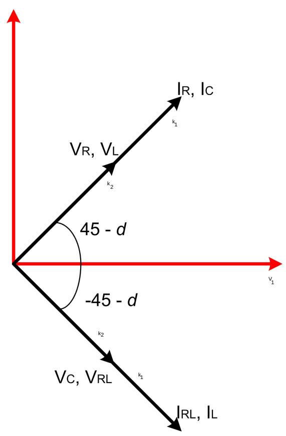
$$
|R + \frac{1}{j\omega C}| = |R_L + j\omega L|
$$
$$
\implies \omega = \frac{1}{\sqrt{LC}}
$$

In the phasor diagrams, $k_1 = \frac{V_1}{\sqrt{\frac{2L}{C}}}$ and $k_2 = \frac{V_1}{\sqrt{2}}$

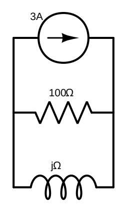


Phasor diagram for Q.4 at ω_1

When, ω is increased, in the inductor current lags further and in the capacitor, current leads less.



Phasor diagram for Q.4, when ω_1 is increased to ω_2



Due to $3\angle 0^\circ$ current source

5. Due to $3\angle 0^\circ$ current source:

$$I_{100\Omega} = 3\angle 0^\circ \times \frac{j}{100+j}$$
$$V_{100\Omega} = -I_{100\Omega} \times 100$$

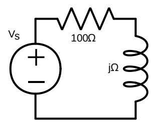
Due to $12\angle -30^\circ$ voltage source:

$$V_{100\Omega} = 12\angle -30^\circ \times \frac{100}{100+j}$$

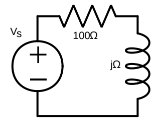
Due to $6\angle 0^\circ$ voltage source:

$$V_{100\Omega} = 6\angle 0^\circ \times \frac{100}{100+j}$$

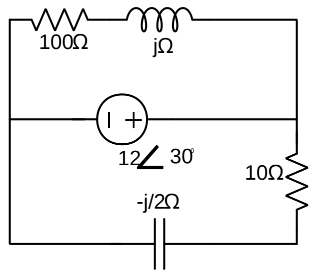
Due to $12\angle 30^\circ$ voltage source:



Due to $12\angle -30^\circ$ voltage source. $V_s = 12\angle -30^\circ$



Due to $6\underline{/0^\circ}$ voltage source. $V_s = 6\underline{/0^\circ}$



Due to $12\underline{/30^\circ}$ voltage source

$$V_{100\Omega} = -12\underline{/30^\circ} \times \frac{100}{100+j}$$
$$V_{10\Omega} = 12\underline{/30^\circ} \times \frac{10}{10-j/2}$$

$$V_{100\Omega} = 16.155\underline{/-68.2^\circ}$$
$$P_{100\Omega} = \frac{1}{2} \times \frac{16.155^2}{100} = 1.3W$$

$$V_{10\Omega} = 12\underline{/30.286^\circ}$$
$$P_{10\Omega} = \frac{1}{2} \times \frac{12^2}{10} = 7.2W$$