EE1101 Signals and Systems Jan—May 2019
Tutorial 5 Solutions

1) (a) The signal $x(t)$ is periodic with period $T_0 = 1$ and the fundamental frequency $f_0 = \frac{1}{T_0} = 1$ Hz, and $\omega_0 = \frac{2\pi}{T_0} = 2\pi$.

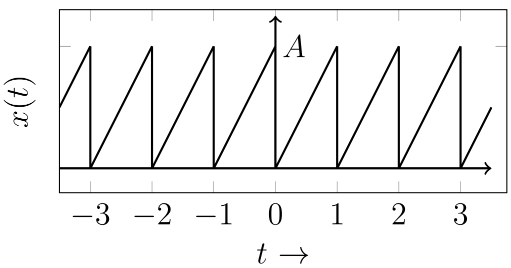


Figure 1

The Fourier series representation of the signal is,

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

where a_k's are the Fourier coefficients given by

$$\begin{aligned}
a_k &= \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t} dt \\
&= A \int_0^1 t e^{-jk2\pi t} dt \\
&= \frac{Aj}{2\pi k} && \text{for } k \neq 0 \\
a_0 &= \frac{1}{T_0} \int_{T_0} x(t) dt = \frac{A}{2}
\end{aligned}$$

The magnitude of Fourier coefficient $|a_k| = \frac{A}{2\pi k}, k \neq 0$ and $|a_0| = \frac{A}{2}$. The phase $\arg(a_k) = sign(k)j$ i.e.,

$$\arg(a_k) = \begin{cases} \pi/2 & \text{if } k > 0 \\ -\pi/2 & \text{if } k < 0 \\ 0 & \text{if } k = 0 \end{cases}$$

The magnitude and phase spectrum are presented in Fig. 2. For these plots we assume $A = 1$.

And the Fourier series representation of the signal is

$$\begin{aligned} x(t) &= \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \\ &= a_0 + \sum_{k\neq 0} a_k e^{jk\omega_0 t} \\ &= \frac{A}{2} + \sum_{k\neq 0} \frac{A}{2\pi k} e^{j(k\omega_0 t + \frac{\pi}{2})} \end{aligned}$$

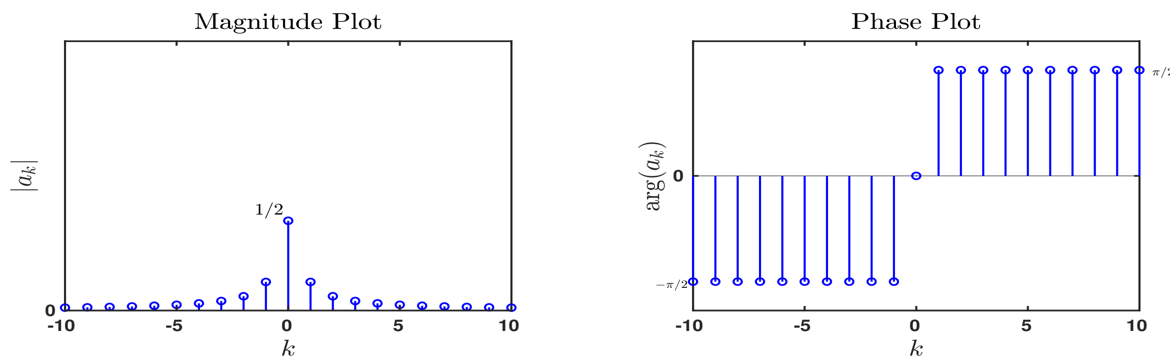


Figure 2: Q1.a) Magnitude and phase spectra of $x(t)$ for $A = 1$.

(b) The period of function $y(t)$ is $T_0 = 1$ and the fundamental frequency $\omega_0 = \frac{2\pi}{T_0} = 2\pi$. Let b_k indicate the Fourier coefficients of the signal $y(t)$. The Fourier series representation of the signal is,

$$y(t) = \sum_{k=-\infty}^{\infty} b_k e^{jk\omega_0 t}$$

where b_k's are given by

$$b_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t} dt$$

Consider the period $-1/2 \leq t \leq 1/2$, $y(t) = -At + \frac{3A}{2}$. Computing the Fourier coefficients,

$$\begin{aligned} b_k &= \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t} dt \\ &= \int_{-.5}^{.5} (-At + \frac{3A}{2}) e^{-jk\omega_0 t} dt \\ &= -A \int_{-.5}^{.5} t e^{-j2\pi k t} dt + \frac{3A}{2} \int_{-.5}^{.5} e^{-j2\pi k t} dt \end{aligned}$$

The second term integrates to zero. Using integration by parts on first term ($\int udv = uv - \int vdu$),

$$\int_{-.5}^{.5} te^{-j2\pi kt}dt = \frac{j}{2\pi k}cos(\pi k)$$
$$= \frac{j}{2\pi k}e^{-j\pi k}$$

Therefore,

$$b_k = \frac{-jA}{2\pi k}e^{-j\pi k} \quad k \neq 0$$

$$\begin{aligned} b_0 &= \frac{1}{T_0}\int_{T_0} x(t)dt \\ &= \int_{-.5}^{.5}(-At + \frac{3A}{2})dt \\ &= -\frac{-At^2}{2} + \frac{3A}{2}t|_{-.5}^{.5} \\ &= \frac{3A}{2} \end{aligned}$$

The magnitude of Fourier coefficient $|b_k| = \frac{A}{2\pi k}, k \neq 0$ and $|b_0| = \frac{3A}{2}$. The phase $\arg(b_k)$ for $k \geq 0$ is given by

$$\arg(b_k) = \begin{cases} \pi/2 & \text{if } k > 0 \text{ is odd} \\ -\pi/2 & \text{if } k > 0 \text{ is even} \\ 0 & \text{if } k = 0 \end{cases}$$

Since the signal is real, the phase spectrum should be antisymmetric. Thus, the phase for $k < 0$ is

$$\arg(b_k) = \begin{cases} \pi/2 & \text{if } k < 0 \text{ is even} \\ -\pi/2 & \text{if } k < 0 \text{ is odd} \\ 0 & \text{if } k = 0 \end{cases}$$

The magnitude and phase spectra are presented in Fig. 3. For these plots we assume $A = 1$.

(c) The given function is $y(t) = A + x(-t + 0.5)$.
Fourier coefficients of

$$\begin{aligned} x(t) &\longrightarrow a_k \\ x(t+0.5) &\longrightarrow e^{jk\pi}a_k \\ x(-t+0.5) &\longrightarrow e^{-jk\pi}a_{-k} \end{aligned}$$

Therefore, Fourier coefficients of $y(t)$

$$b_k = \frac{-Aj}{2\pi k}e^{-jk\pi}$$
$$b_0 = \frac{3A}{2}$$

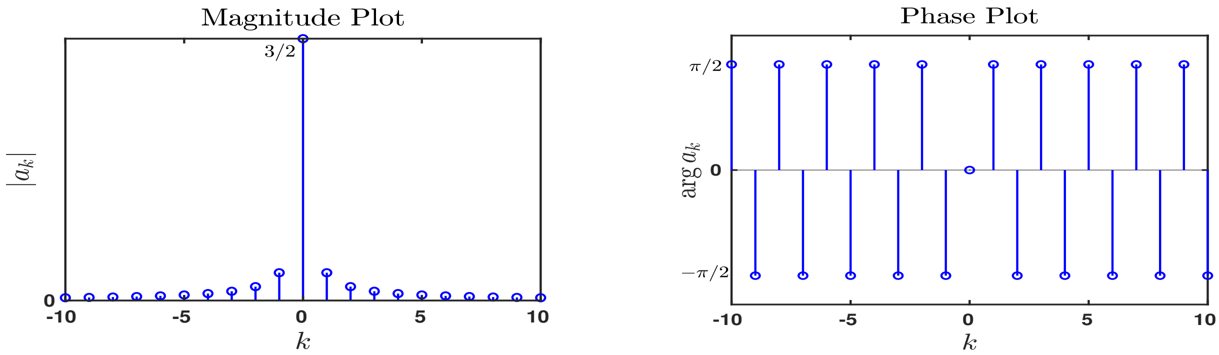


Figure 3: Q1.b) Magnitude and phase spectra of $y(t)$ for $A = 1$.

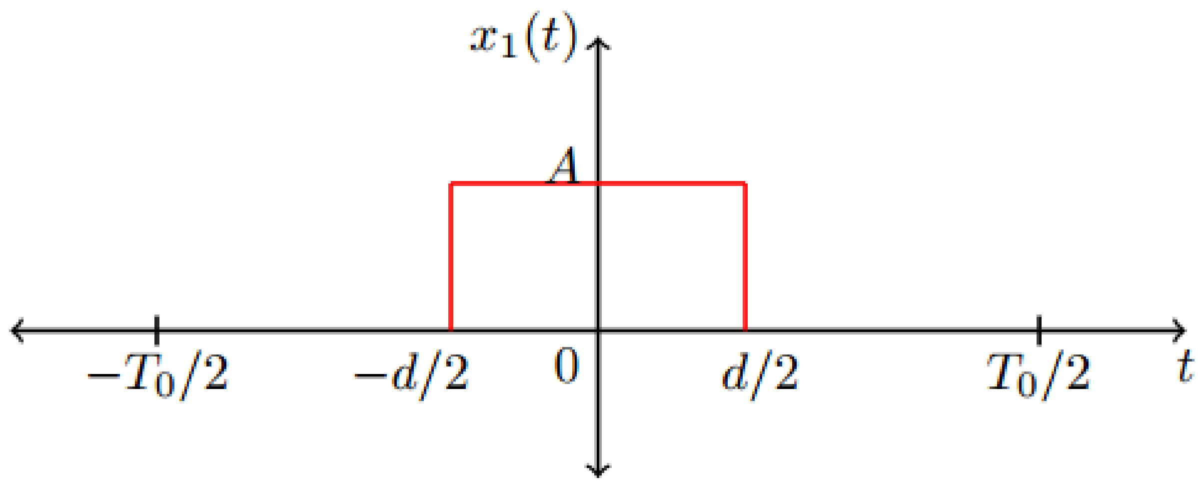


Figure 4: Q2. a)

2) (a) The plot of signal $x_1(t)$ is shown below,

$$x_1(t) = \begin{cases} A/2 & |t| < \frac{d}{2} \\ 0 & \frac{d}{2} < |t| < \frac{T_0}{2} \end{cases}$$

The signal $x(t)$ is periodic with period T_0. The Fourier series representation of the signal is,

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

where a_k's are the Fourier coefficients given by

$$a_k = \frac{1}{T_0} \int_{\frac{-T_0}{2}}^{\frac{T_0}{2}} x(t) e^{-jk\omega_0 t} dt$$

$$\begin{aligned}
a_k &= \frac{1}{T_0} \int_{\frac{-d}{2}}^{\frac{d}{2}} A e^{-jk\omega_0 t} dt \\
a_k &= \frac{-A}{jk\omega_0 T_0} \left(e^{-jk\omega_0 \frac{d}{2}} - e^{jk\omega_0 \frac{d}{2}} \right) \\
a_k &= \frac{2A}{k\omega_0 T_0} \left(\sin \left(k\omega_0 \frac{d}{2} \right) \right), \quad \text{for } k \neq 0.
\end{aligned}$$

$$a_0 = \frac{1}{T_0} \int_{-\frac{d}{2}}^{\frac{d}{2}} A dt = \frac{Ad}{T_0}$$

The magnitude of Fourier coefficient $|a_k| = |\frac{A}{k\pi} \left(\sin \left(k\omega_0 \frac{d}{2} \right) \right)|, k \neq 0$ and $|a_0| = \frac{Ad}{2}$. Phase spectrum of $x_1(t)$ is either zero or π for all n because there is no imaginary part in a_n. The magnitude and phase spectra are shown in Fig. 5. For these plots, we assume $A = 1, d = 1, T_0 = 2, \omega_0 = \pi$.

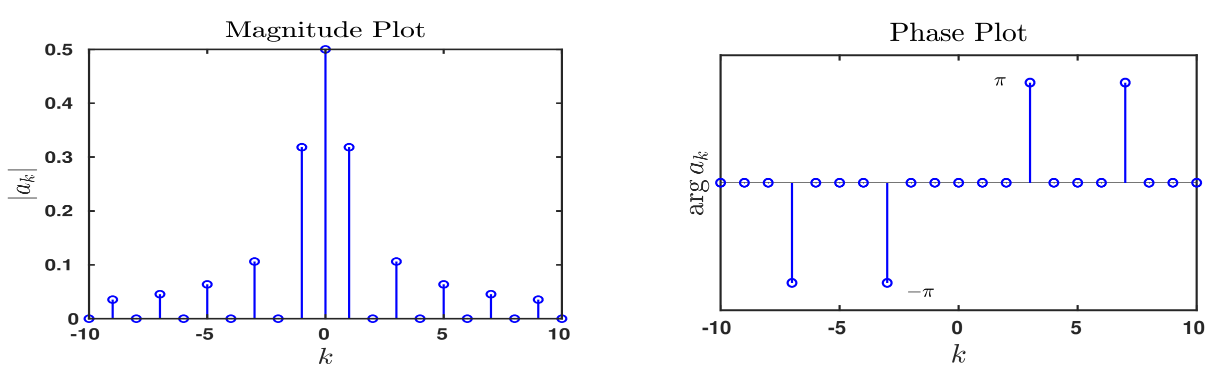


Figure 5: Q2.a) Magnitude and phase spectra of $x_1(t)$ for $A = 1, d = 1, T_0 = 2, \omega_0 = \pi$.

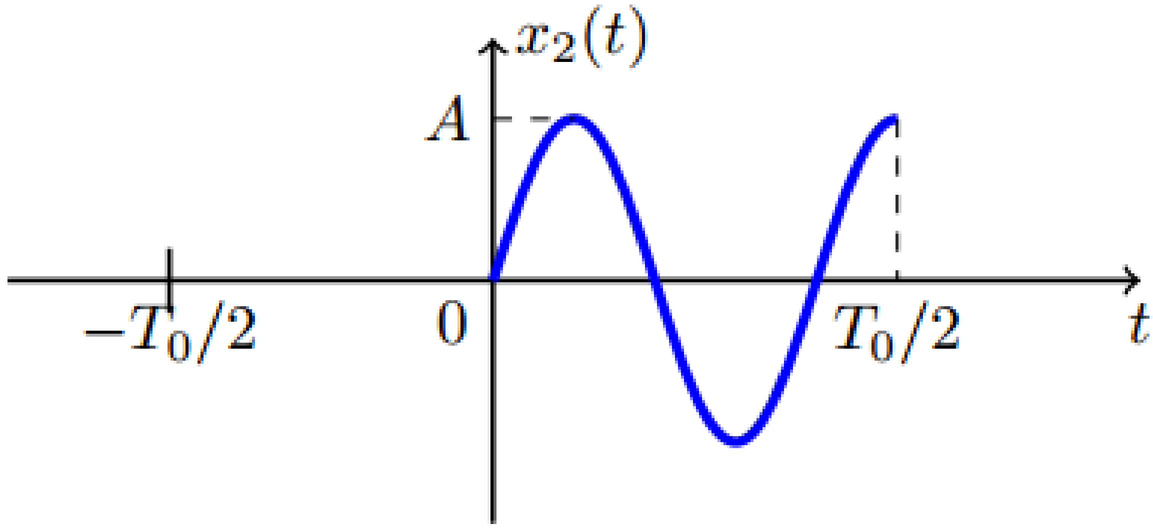


Figure 6: Q2. b)

(b)

$$x_2(t) = \begin{cases} A\sin(\frac{2\pi t}{T_0}) & 0 \leq t < \frac{T_0}{2} \\ 0 & \frac{-T_0}{2} \leq t < 0 \end{cases}$$

The signal $x(t)$ is periodic with period T_0. The Fourier series representation of the signal is,

$$x(t) = \sum_{k=-\infty}^{\infty} a_n e^{jk\omega_0 t}$$

where a_n's are the Fourier coefficients given by

$$\begin{aligned} a_0 &= \frac{A}{T_0} \int\limits_0^{\frac{T_0}{2}} \sin(\omega_0 t) d(t) \\ a_0 &= \frac{-A}{\omega_0 T_0} \ (\cos(\pi) - 1) = \frac{A}{\pi} \end{aligned}$$

$$
\begin{aligned}
a_n &= \frac{A}{T_0} \int_0^{\frac{T_0}{2}} \sin(\omega_0 t) e^{-jn\omega_0 t} dt \\
&= \frac{A}{2jT_0} \int_0^{\frac{T_0}{2}} \left(e^{j\frac{2\pi t}{T_0}} - e^{-j\frac{2\pi t}{T_0}} \right) e^{-jn\omega_0 t} dt \\
&= \frac{-A}{2T_0} \left(\frac{e^{j(1-n)\pi} - 1}{\left(\frac{(1-n)2\pi}{T_0}\right)} - \frac{e^{-j(1+n)\pi} - 1}{-\left(\frac{(1+n)2\pi}{T_0}\right)} \right) \\
&= \frac{-A}{4\pi} \left(\frac{e^{j(1-n)\pi} - 1}{(1-n)} + \frac{e^{-j(1+n)\pi} - 1}{(1+n)} \right)
\end{aligned}
$$

$$
\begin{aligned}
a_n &= \frac{A}{2\pi} \left(\frac{2}{1-n^2} \right), \quad n = \text{even} \\
a_n &= 0, \quad n = \text{odd}, \; n \neq 1, \; n \neq -1
\end{aligned}
$$

$$
\begin{aligned}
a_1 &= \frac{A}{T_0} \int_0^{\frac{T_0}{2}} \sin(\omega_0 t) e^{-j\omega_0 t} dt = \frac{-Aj}{4} \\
a_{-1} &= \frac{A}{T_0} \int_0^{\frac{T_0}{2}} \sin(\omega_0 t) e^{j\omega_0 t} dt = \frac{Aj}{4}
\end{aligned}
$$

Phase spectrum of $x_2(t)$ for $n = 1$ is $-\pi/2$, for $n = -1$ is $\pi/2$. The magnitude and phase spectra are shown in Fig. 7. For these plots, we assume $A = 1$.

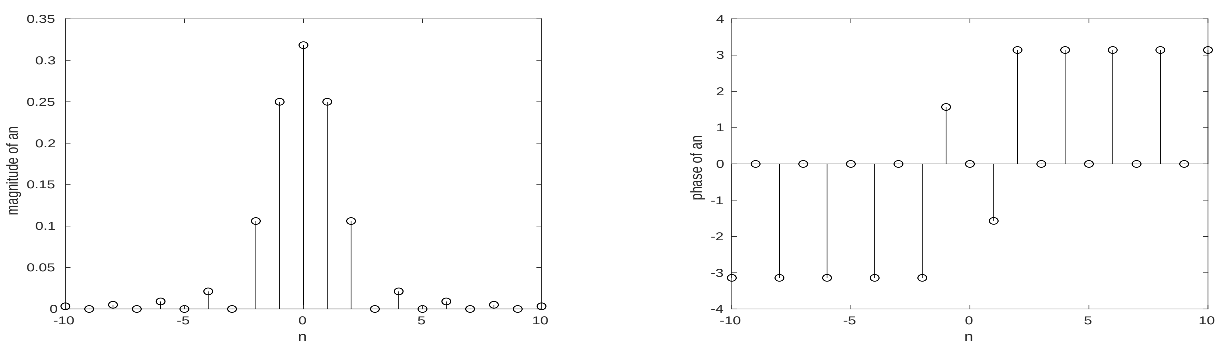


Figure 7: Q2.b) Magnitude and phase spectra of $x_2(t)$ for $A = 1, d = 1, T_0 = 2, \omega_0 = \pi$.

3) We have

$$
x(t) = \sum_{n=-\infty}^{+\infty} a_n e^{jnt}
$$

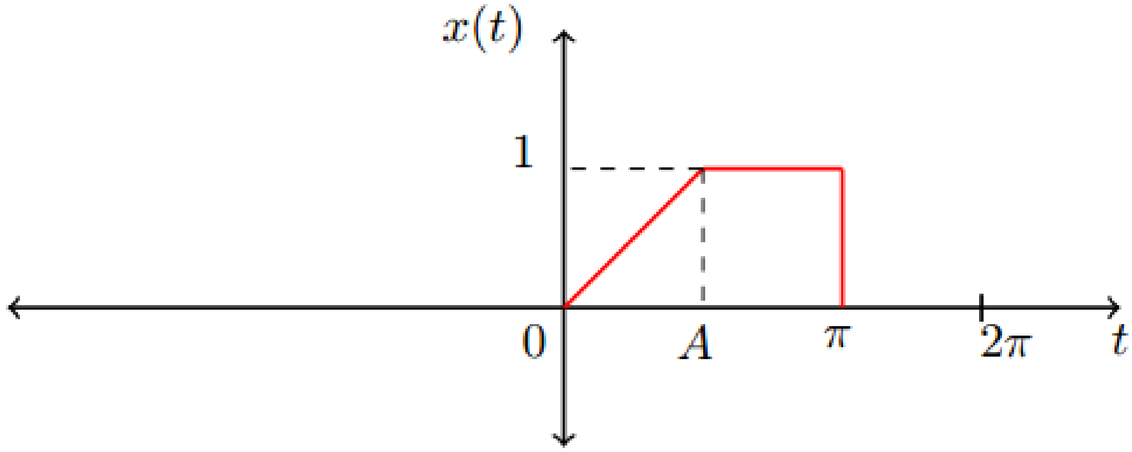


Figure 8: Q3

where

$$a_o = \frac{1}{2\pi}\int_0^{2\pi} x(t)dt$$

$$= \frac{1}{2\pi}\left(\int_0^A \frac{t}{A}dt + \int_A^\pi dt\right)$$

Therefore,

$$a_o = \frac{1}{2\pi}\left(\pi - \frac{A}{2}\right)$$

and

$$a_n = \frac{1}{2\pi}\int_0^{2\pi} x(t)e^{-jnt}dt$$

$$= \frac{1}{2\pi}\left(\int_0^A \frac{t}{A}e^{-jnt}dt + \int_A^\pi e^{-jnt}dt\right)$$

Integrating by parts, we have

$$\int_0^A \frac{t}{A}e^{-jnt}dt = j\frac{e^{-jnA}}{n} - \frac{1-e^{-jnA}}{An^2}$$

and

$$\int_A^\pi e^{-jnt}dt = -\frac{j}{n}\left(e^{-jnA} - (-1)^n\right)$$

Therefore,

$$a_n = \frac{1}{2\pi}\left(j\frac{(-1)^n}{n} - \frac{1-e^{-jnA}}{An^2}\right).$$

4) $a_k = jk,\ |k| < 3$

$$\begin{aligned}
x(t) &= \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \\
x(t) &= j\Big(- 2e^{j(-2)\frac{2\pi}{4}t} - 1e^{j(-1)\frac{2\pi}{4}t} + 1e^{j\frac{2\pi}{4}t} \\
&\quad + 2e^{j(2)\frac{2\pi}{4}t}\Big) \\
x(t) &= (-1)\left(4\sin(\pi t) + 2\sin(\frac{\pi}{2}t)\right).
\end{aligned}$$

5) Given $x(t)$ is a periodic signal with fundamental period T and Fourier series coefficients a_k

(a) Let b_k be the Fourier series coefficient of $x(t-t_0)$ $b_k = \frac{1}{T}\int_T x(t-t_0)e^{-jk\omega_0 t}dt$. The Fourier series representation of the signal is,

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

Substitute $t = \tau - t_0$, then

$$\begin{aligned}
x(\tau - t_0) &= \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 \tau - t_0} \\
&= \sum_{k=-\infty}^{\infty} a_k e^{-jk\omega_0 t_0} e^{jk\omega_0 \tau}
\end{aligned}$$

Comparing with the synthesis equation, $b_k = a_k e^{-jk\omega_0 t_0}$

(b)

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$\Rightarrow$

$$x(-t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0(-t)} = \sum_{k=-\infty}^{\infty} a_k e^{j(-k)\omega_0 t}$$

Replace k by $-m$ which implies

$$x(-t) = \sum_{m=-\infty}^{\infty} a_{-m} e^{jm\omega_0 t}$$

Thus the Fourier series coefficients of $x(-t)$ is a_{-k}

(c)

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

$\Rightarrow$

$$x^*(t) = \sum_{k=-\infty}^{\infty} (a_k e^{jk\omega_0 t})^*$$

$$= \sum_{k=-\infty}^{\infty} a_k^* e^{-jk\omega_0 t}$$

substituting k by $-m$ implies

$$x^*(t) = \sum_{m=-\infty}^{\infty} a_{-m}^* e^{jm\omega_0 t}$$

Thus the Fourier series coefficients of $x^*(t)$ is a_{-k}^*

(d) $x(t-t_0) + x(t+t_0)$

By time shifting property,

$$x(t-t_0) \stackrel{\mathcal{F}}{\leftrightarrow} a_k e^{-jk\omega_0 t_0}$$

where $\omega_0 = 2\pi/T$. Therefore, Fourier coefficients of $x(t-t_0) + x(t+t_0)$ are

$$a_k e^{-jk\omega_0 t_0} + a_k e^{jk\omega_0 t_0} = 2a_k \cos(k\omega_0 t_0)$$

(e) Even $\{x(t)\} = \dfrac{x(t) + x(-t)}{2}$

$$x(t) \stackrel{\mathcal{F}}{\leftrightarrow} a_k$$
$$x(-t) \stackrel{\mathcal{F}}{\leftrightarrow} a_{-k}$$

Therefore, Fourier coefficients of Even $\{x(t)\}$ are $\dfrac{a_k + a_{-k}}{2}$

(f) Real $\{x(t)\} = \dfrac{x(t) + x^*(t)}{2}$

$$x(t) \stackrel{\mathcal{F}}{\leftrightarrow} a_k$$
$$x^*(t) \stackrel{\mathcal{F}}{\leftrightarrow} a_{-k}^*$$

Therefore, Fourier coefficients of Real $\{x(t)\}$ are $\dfrac{a_k + a_{-k}^*}{2}$

6) $\cos(t)$ and $\cos(3t)$ are periodic with period $T = 2\pi$. Using the definition of periodic convolu-

tion,

$$\begin{aligned}
\cos(t) * \cos(3t) &= \int_T \cos(t-\tau)\cos(3\tau)d\tau \\
&= \int_T \big(\cos(t)\cos(\tau)\cos(3\tau) \\
&\quad + \sin(t)\sin(\tau)\cos(3\tau)\big)d\tau \\
&= \cos(t)\int_T \cos(\tau)\cos(3\tau)d\tau \\
&\quad + \sin(t)\int_T \sin(\tau)\cos(3\tau)d\tau \\
&= \cos(t)\int_T \cos(2\tau)+\cos(4\tau)d\tau \\
&\quad + \sin(t)\int_T \sin(4\tau)-\sin(2\tau)d\tau \\
&= 0.
\end{aligned}$$

Thus the periodic convolution yields 0, whose FS coefficients will then be 0.

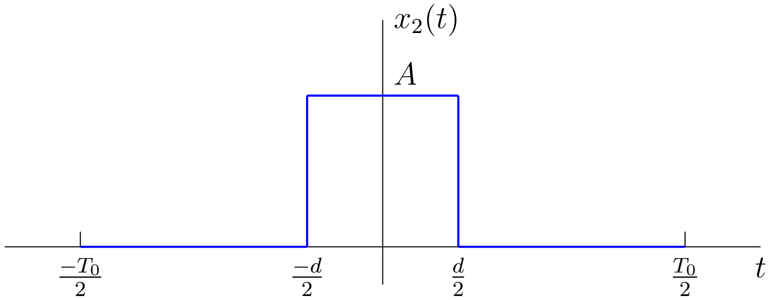
Using properties of F.S

Let the Fourier Series coefficients of $\cos(t)$ be a_k and that of $\cos(3t)$ be b_k. Then, by the periodic convolution property of Fourier series, the FS coefficients of $\cos t * \cos(3t)$ will be Ta_kb_k where T is the period.

$$\cos(t) = \frac{e^{jt}+e^{-jt}}{2} \quad \text{and} \quad \cos(3t) = \frac{e^{j3t}+e^{-j3t}}{2}.$$

Thus we see that only a_1, a_{-1}, b_3 and b_{-3} are non-zero. Therefore the FS coefficients of $\cos t * \cos(3t)$ will all be 0. This verifies the previous result.

7) (a) The signal $x_1(t)$ is periodic with period T_0 and $\omega_0 = \frac{2\pi}{T_0}$. Evaluating the Trigonometric



Fourier series coefficients,

$$\begin{aligned}
a_0 &= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)dt \\
&= \frac{1}{T_0}\int_{-d/2}^{d/2} Adt \\
&= A \times d
\end{aligned}$$

$$
\begin{aligned}
a_k &= \frac{2}{T_0}\int\limits_{-T_0/2}^{T_0/2} x(t)cos(\omega_0 kt)dt \\
&= \frac{2}{T_0}\int\limits_{-d/2}^{d/2} Acos(\omega_0 kt)dt \\
&= \frac{2A}{T_0}\frac{T_0}{2\pi k}(2\sin(2\omega_0 d)) \\
&= \frac{2A}{\pi k}\sin(\frac{\pi kd}{T_0})
\end{aligned}
$$

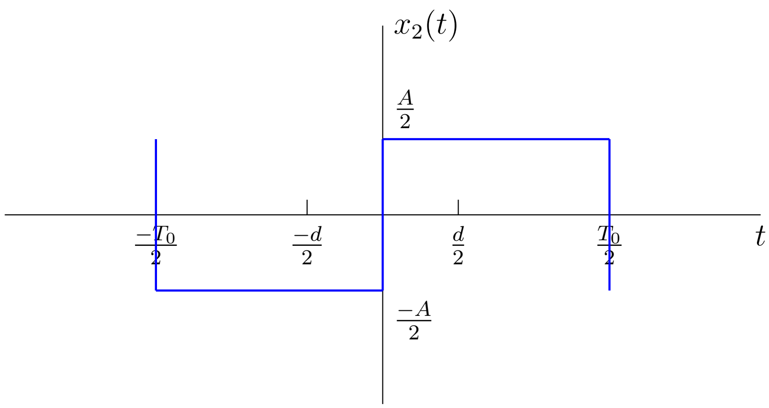
$$
\begin{aligned}
b_k &= \frac{2}{T_0}\int\limits_{-T_0/2}^{T_0/2} x(t)sin(\omega_0 kt)dt \\
&= \frac{2}{T_0}\int\limits_{-d/2}^{d/2} Asin(\omega_0 kt)dt \\
&= \frac{2A}{T_0}\frac{T_0}{2\pi k}(-\cos(2\omega_0 t))|_{-d//2}^{d/2} \\
&= 0
\end{aligned}
$$

Thus the Fourier series representation of the signal is,

$$
x(t) = a_0 + \sum_{k=1}^{\infty} a_k cos(\omega_0 kt)
$$

Note that the sigal is even, therefore the Fourier series representation of the signal has only cosine components.

(b) The signal $x_2(t)$ is periodic with period $T_0 = 2d$ and $\omega_0 = \frac{\pi}{d}$. Evaluating the Trigono-



metric Fourier series coefficients,

$$a_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)dt$$

$$= \frac{1}{T_0}\int_0^d Adt - \int_d^{2d} Adt = 0$$

$$a_k = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} x(t)cos(\omega_0 kt)dt$$

$$= \frac{2}{T_0}\int_0^d Acos(\omega_0 kt)dt + \int_d^{2d} -Acos(\omega_0 kt)dt$$

$$a_k = 0 \quad \text{for all k}$$

$$b_k = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} x(t)sin(\omega_0 kt)dt$$

$$= \frac{2}{T_0}\int_0^d Asin(\omega_0 kt)dt + \int_d^{2d} Asin(\omega_0 kt)dt$$

$$= \frac{2A}{T_0}\frac{T_0}{2\pi k}\left[-cos(\frac{\pi t}{d})|_0^d + cos(\frac{\pi t}{d})|_d^2 d\right]$$

$$= \frac{A}{\pi k}[-2cos(\pi k) + 1]$$

$$b_k = \frac{A}{\pi k}[2(-1)^{n+1} + 1]$$

Thus the Fourier series representation of the signal is,

$$x(t) = \sum_{k=1}^{\infty} b_k \sin(\omega_0 kt)$$

Note that the signal is odd, therefore the Fourier series representation of the signal has only sine terms (sine is odd).

8) (a) Given signal is a pulse train as shown below

$$a_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \sum_{p=-\infty}^{\infty} \delta(t - pT)e^{-jk\omega_0 t}dt$$

$$= \frac{1}{T}\sum_{p=-\infty}^{\infty}\int_{-\frac{T}{2}}^{\frac{T}{2}} \delta(t - pT)e^{-jk\omega_0 t}dt$$

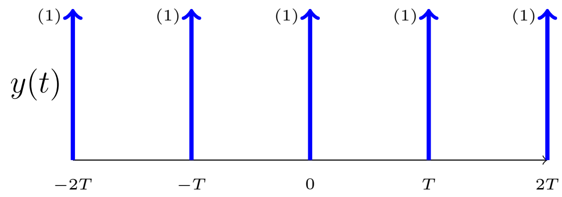


Figure 9: Q8.(a)

For the given range, $p = 0$

$$a_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \delta(t)e^{-jk\omega_0 t}dt = \frac{1}{T}$$

(b) The plots for signals $x(t)$, $\frac{dx}{dt}$ and $\frac{d^2x}{dt^2}$ are shown in Fig. 10.

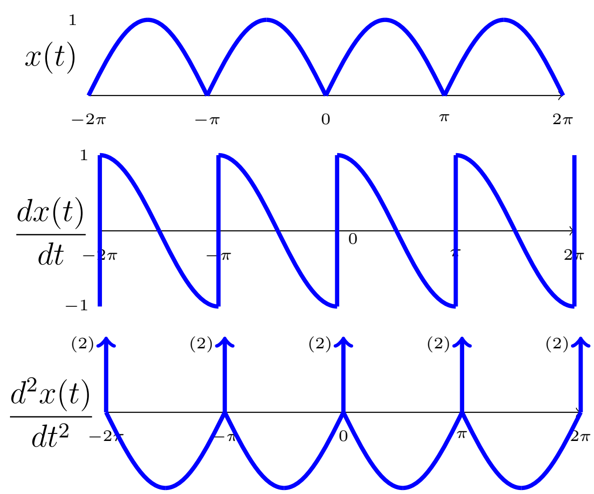


Figure 10: Q8.(b)

(c) The plot for $x(t) + \frac{d^2x(t)}{dt^2}$ is shown in Fig. 11.

$$\begin{aligned} x(t) + \frac{d^2x(t)}{dt^2} &= \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \\ &= \sum_{k=-\infty}^{\infty} a_k e^{jk(2)t} \end{aligned}$$

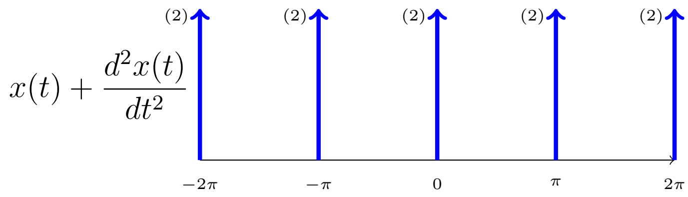


Figure 11: Q8.(c)

where

$$
\begin{aligned}
a_k &= \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sum_{p=-\infty}^{\infty} 2\delta(t-p\pi)e^{-jk2t}dt \\
&= \frac{1}{\pi}\sum_{p=-\infty}^{\infty}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 2\delta(t-p\pi)e^{-jk2t}dt
\end{aligned}
$$

Using part (a) and linearity, $a_k = \frac{2}{\pi}$

(d)

$$
x(t) \overset{\mathcal{F}}{\leftrightarrow} c_k \qquad \text{and} \qquad \frac{d^2x(t)}{dt^2} \overset{\mathcal{F}}{\leftrightarrow} (jk\omega_0)^2\ c_k
$$

$$
x(t) + \frac{d^2x(t)}{dt^2} \overset{\mathcal{F}}{\leftrightarrow} \left(1 + (jk\omega_0)^2\right)c_k
$$

$$
\begin{aligned}
a_k &= \left(1 + (jk\omega_0)^2\right)c_k \\
c_k &= \frac{a_k}{\left(1 + (jk\omega_0)^2\right)} = \frac{2}{\pi(1-4k^2)}
\end{aligned}
$$

9) It is given that $a_k = 0$ for $k > 2$. This implies that $a_{-k} = a_k^* = 0$ for $k > 2$.
Also it is given that $a_0 = 0$. Therefore the only non-zero Fourier coefficients are $a_1 a_{-1}, a_2, a_{-2}$.
Since x(t) is a real signal, $a_k = a_{-k}^*$. Thus, $a_{-1} = a_1^*$ and $a_{-2} = a_2^*$
It is also given that a_1 is positive real number. Therefore $a_{-1} = a_1$. Thus we have,

$$
\begin{aligned}
x(t) &= a_1\left(e^{j\frac{2\pi}{T}t} + e^{-j\frac{2\pi}{T}t}\right) + a_2 e^{j\frac{4\pi}{T}t} + a_2^* e^{-j\frac{4\pi}{T}t} \\
&= 2a_1 \cos\frac{2\pi}{T}t + a_2 e^{j\frac{4\pi}{T}t} + a_2^* e^{-j\frac{4\pi}{T}t}
\end{aligned}
$$

It is given that $T = 6$. This gives

$$
= 2a_1 \cos\frac{\pi}{3}t + a_2 e^{j\frac{2\pi}{3}t} + a_2^* e^{-j\frac{2\pi}{3}t}.
$$

Now using the condition $x(t) = -x(t-3)$,

$$
\begin{aligned}
x(t-3) =& 2a_1 \cos\frac{\pi}{3}(t-3) + a_2 e^{j\frac{2\pi}{3}(t-3)} + a_2^* e^{-j\frac{2\pi}{3}(t-3)} \\
=& -2a_1 \cos\frac{\pi}{3}t + a_2 e^{j\frac{2\pi}{3}t} + a_2^* e^{-j\frac{2\pi}{3}t}.
\end{aligned}
$$

Since $e^{j\frac{2\pi}{3}t}$ and $e^{-j\frac{2\pi}{3}t}$ are both periodic with period 3 above equality follows. Thus, $x(t) = -x(t-3)$ implies

$$\begin{aligned} 2a_1 \cos\frac{\pi}{3}t + a_2 e^{j\frac{2\pi}{3}t} + a_2^* e^{-j\frac{2\pi}{3}t} &= -\left(-2a_1 \cos\frac{\pi}{3}t + a_2 e^{j\frac{2\pi}{3}t} + a_2^* e^{-j\frac{2\pi}{3}t}\right) \\ 2\left(a_2 e^{j\frac{2\pi}{3}t} + a_2^* e^{-j\frac{2\pi}{3}t}\right) &= 0. \end{aligned}$$

Therefore we have,

$$x(t) = 2a_1 \cos\frac{\pi}{3}t.$$

Finally, it is given that

$$\begin{aligned} &\frac{1}{6}\int_{-3}^{3} \mid x(t) \mid^2 dt = \frac{1}{2} \\ &\implies \frac{4}{6}\int_{-3}^{3} a_1^2 \cos^2\left(\frac{\pi}{3}t\right) dt = \frac{1}{2} \\ &\implies a_1 = \frac{1}{2} \end{aligned}$$

Therefore, $x(t) = \cos\frac{\pi}{3}t$ and the constants $A = 1$, $B = \frac{\pi}{3}$ and $C = 0$.