

**EE1101 Signals and Systems JAN—MAY 2019**  
**Tutorial 8**

1. (a) If  $x(t) \longleftrightarrow X(j\omega)$ , then show that

$$X(0) = \int_{-\infty}^{\infty} x(t) dt$$

- (b) Given  $\text{sinc } t = \frac{\sin \pi t}{\pi t}$ , show that

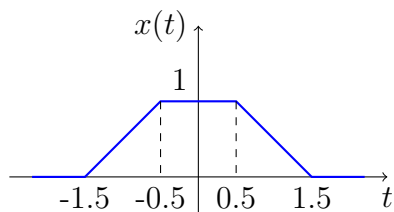
$$\int_{-\infty}^{\infty} \text{sinc } t \, dt = \int_{-\infty}^{\infty} \text{sinc}^2 t \, dt = 1$$

2. Given two signals  $f_1(t) = 2\text{rect}(t/4)$  and  $f_2(t) = \text{rect}(t/2)$

- (a) Sketch  $g(t) = f_1(t) \star f_2(t)$   
 (b) Evaluate  $G(j\omega)$ , the fourier transform of  $g(t)$ .  
 (c) Sketch the magnitude and phase of  $G(j\omega)$

$$\text{Note : } \text{rect}\left(\frac{t}{\tau}\right) = \begin{cases} 1 & , |t| \leq \frac{\tau}{2} \\ 0 & , |t| > \frac{\tau}{2} \end{cases}$$

3. Using properties, find the Fourier transform  $X(j\omega)$  of the signal  $x(t)$  shown below.



4. Determine the signal  $x(t)$  corresponding to the following Fourier transforms:

(a)  $X(j\omega) = \frac{2 \sin[3(\omega - 2\pi)]}{(\omega - 2\pi)}$

(b)  $X(j\omega) = \cos\left(4\omega + \frac{\pi}{3}\right)$

5. Let  $X(j\omega)$  be the Fourier transform of the signal  $x(t)$  shown below. Do the following computations without explicitly evaluating  $X(j\omega)$ .

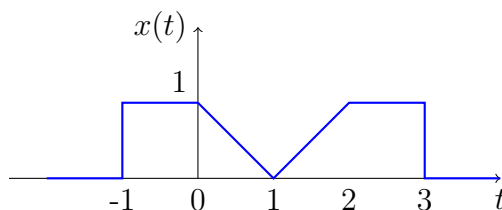
- (a) Find  $X(0)$

(b) Evaluate  $\int_{-\infty}^{\infty} X(j\omega) d\omega$

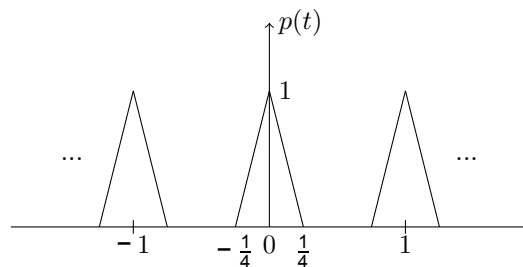
(c) Evaluate  $\int_{-\infty}^{\infty} X(j\omega) e^{j\omega} d\omega$

(d) Evaluate  $\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$

- (e) Sketch the inverse Fourier transform of  $\text{Re}\{X(j\omega)\}$ .



6. Let  $p(t)$  be periodic triangular pulse train shown below.

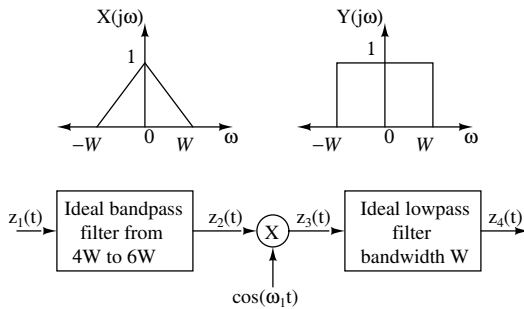


- (a) Calculate  $P_n$ , the exponential Fourier series coefficients of  $p(t)$  and sketch  $P_n$  vs.  $n$ . Calculate  $P(j\omega)$ , the Fourier transform of  $p(t)$ . Sketch  $P(j\omega)$  vs.  $\omega$ .

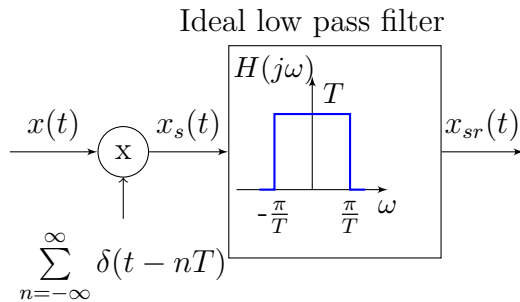
- (b) Let  $x(t)$  be an aperiodic signal having Fourier transform  $X(j\omega)$ . Define  $y(t) = p(t) \cdot x(t)$ . Find an expression for  $Y(j\omega)$ , the Fourier transform of  $y(t)$ .

- (c) Let  $x(t) = \text{sinc}(t)$ . Sketch  $Y(j\omega)$ .

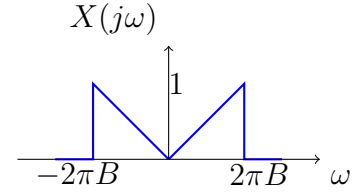
7.  $x(t)$  and  $y(t)$  have Fourier transforms as shown below. Sketch the Fourier transform of the various signals  $z_i(t)$  for  $i = 1, 2, 3, 4$  in the system shown below given that  $z_1(t) = x(t) \cos(\omega_1 t) + y(t) \cos(\omega_2 t)$ . Determine  $z_4(t)$  in terms of  $x(t)$  and  $y(t)$ ? Assume that  $\omega_1 = \omega_2 - 2W = 5W$ .



8. In the sampling process,  $x(t)$ , the signal to be sampled is multiplied by the impulse train, yielding the sampled signal  $x_s(t)$ . In order to perform reconstruction, we pass  $x_s(t)$  through the ideal low pass filter, yielding the reconstructed signal  $x_{sr}(t)$ .



- (a) Write  $X_s(j\omega)$ , the Fourier transform of  $x_s(t)$  in terms of  $X(j\omega)$ , the Fourier transform of  $x(t)$ . Your answer should be an explicit, closed form expression, not one involving a convolution integral.
- (b) Let  $x(t)$  have the Fourier transform shown.



Sketch  $X_s(j\omega)$  for the cases (i)  $T = \frac{1}{4B}$  (ii)  $T = \frac{1}{2B}$  (iii)  $T = \frac{1}{B}$ .

- (c) For the three cases considered in part (b), sketch  $X_{sr}(j\omega)$ , the Fourier transform of  $x_{sr}(t)$ .
- (d) For an arbitrary signal  $x(t)$  that occupies a bandwidth of  $2\pi B$  rad/s, what is the largest  $T$  such that  $X_{sr}(j\omega) = X(j\omega)$ , i.e.  $x_{sr}(t) = x(t)$ ?
9. (a) Determine the Nyquist rate corresponding to each of the following signals:

(i)  $x_1(t) = \frac{\sin(4000\pi t)}{\pi t}$

(ii)  $x_2(t) = \left( \frac{\sin(4000\pi t)}{\pi t} \right)^2$

- (b) The signal  $y(t)$  is generated by convolving a band-limited signal  $x_1(t)$  with another band-limited signal  $x_2(t)$ , that is,

$$y(t) = x_1(t) * x_2(t)$$

where

$$X_1(j\omega) = 0 \quad \text{for } |\omega| > 1000\pi$$

$$X_2(j\omega) = 0 \quad \text{for } |\omega| > 2000\pi$$

Suppose impulse train sampling is performed on  $y(t)$ , specify the range of values for the sampling period  $T$  which ensures that  $y(t)$  is recoverable from the samples.