

DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH1020 Physics II

Problem Set 9

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1. An ideal parallel plate capacitor of capacitance C has circular plates located at $z = 0$ and $z = d$ respectively. The medium between the plates is a linear, homogeneous, isotropic dielectric of dielectric constant κ . The capacitor is connected to a resistance R in series, and a voltage V is applied to the circuit. The charge q on the capacitor plates increases with time according to $q = CV(1 - e^{-t/RC})$. Find the magnitude of the magnetic field H inside the dielectric.
2. An infinitely long straight non-magnetic conductor with a circular cross section of radius a carries a steady current I . The current is distributed uniformly over the cross-section of the wire. The conductivity of the wire is σ . Find the rate at which energy flows into unit length of the conductor.
3. The electric field of a plane EM wave in vacuum is given by

$$\vec{E} = E_0 \hat{e}_x \cos kz \cos \omega t$$

Find the corresponding magnetic field \vec{H} , given that $\vec{H} = 0$ at $t = 0$. Find the mean energy flux density of the wave.

4. Take two linearly polarized plane electromagnetic waves propagating in the z -direction, with their planes of polarization along the x and y directions respectively. The electric fields of the two waves have equal amplitudes, given by $|E_0|$. The frequency of each wave is ω , and its wave number is k .
 - (a) Write down the electric and magnetic fields of the two waves.
 - (b) Find the value of $\frac{\partial w}{\partial t} + \vec{\nabla} \cdot \vec{S}$ for the plane waves, where w is the energy density and \vec{S} is the Poynting vector.
 - (c) Construct superpositions of the two waves to represent a wave that is (i) left circularly polarized, (ii) right circularly polarized. What are the electric and magnetic fields of these waves?
5. A plane wave travelling in a medium of refractive index of n_1 falls at normal incidence on a medium of refractive index n_2 .

(a) Show that ratio of reflected intensity to incident intensity is $\left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2$ and the ratio of transmitted intensity to incident intensity is $\frac{4n_1 n_2}{(n_1 + n_2)^2}$

- (b) Now instead of normal incident, the plane wave falls at an oblique incident. Show that
 - i Incident wave, reflected wave and transmitted wave lies in the same plane, called **plane of incidence**.
 - ii Angle of reflection is equal to angle of incident- **Law of reflection**.
 - iii For the transmitted angle θ_T , **law of refractions (Snell's Law)** is $\frac{\sin \theta_T}{\sin \theta_I} = \frac{n_1}{n_2}$

6. As in example 5b, if the polarization of incident wave is in the plane of incidence, derive the Fresnel equations for the reflected and transmitted amplitudes of a plane EM wave when it passes from one medium to another, and also find incident angle for which reflected wave extinguished (Brewster's angle)

7. As in example 5b, if the polarization of incident wave is perpendicular to the plane of incidence, derive the Fresnel equations for the reflected and transmitted amplitudes of a plane EM wave when it passes from one medium to another and show that there is no Brewster's angle.