EE1101 Signals and Systems JAN—MAY 2019
Tutorial 4

1. Determine whether the LTI systems with following impulse responses is causal and/or stable. Justify your answers.
   (a) $h[n] = (-\frac{1}{2})^n u[n] + (1.01)^n u[1-n]$
   (b) $h[n] = n(\frac{1}{3})^n u[n-1]$
   (c) $h(t) = e^{2t}u(-1-t)$
   (d) $h(t) = e^{-6|t|}$
   (e) $h(t) = (2e^{-t} - e^{(t-100)/100})u(t)$

2. Evaluate the step response for the LTI systems represented by the following impulse responses:
   a) $h[n] = (-1/2)^n u[n]$
   b) $h[n] = nu[n]$
   c) $h(t) = e^{-|t|}$
   d) $h(t) = (1/4)(u(t) - u(t-4))$
   e) $h(t) = u(t)$

3. Determine whether each of the following statements concerning LTI systems is true or false. Justify your answers.
   (a) If $h(t)$ is the impulse response of an LTI system, and h(t) is periodic and nonzero, the system is unstable.
   (b) The inverse of a causal LTI system is always causal.
   (c) If $|h[n]| < K$ for each n, where K is a given number, then the LTI system with $h[n]$ as its impulse response is stable.
   (d) If a discrete-time LTI system has an impulse response $h[n]$ that is bounded and of finite duration, the system is stable.
   (e) If an LTI system is causal, it is stable.
   (f) The cascade of a noncausal LTI system with a causal one is necessarily noncausal.
   (g) A continuous-time LTI system is stable if and only if its step response $s(t)$ is absoultely integrable, that is,
   $$\int_{-\infty}^{+\infty} |s(t)| dt < \infty$$
   (h) A discrete-time LTI system is causal if and only if its step response $s[n]$ is zero for $n < 0$.

4. Consider two systems A and B. It is given that system A is LTI and system B is an inverse of system A.
   (a) Prove that system B is linear.
   (b) Prove that system B is time-invariant.

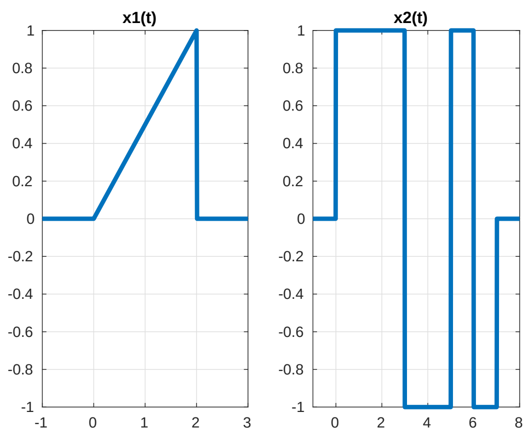
5. Consider a discrete-time LTI system with unit sample response
   $$h[n] = (n+1)\alpha^n u[n]$$
   where $|\alpha| < 1$. Show that the step response of this system is
   $$s[n] = \left[\frac{1}{(\alpha-1)^2} - \frac{\alpha}{(\alpha-1)^2}\alpha^n + \frac{\alpha}{(\alpha-1)}(n+1)\alpha^n\right] u[n] \quad (1)$$
   Given that
   $$\sum_{k=0}^{N}(k+1)\alpha^k = \frac{d}{d\alpha}\sum_{k=0}^{N+1}\alpha^k$$

6. The cross-correlation function between two continuous-time real signals $x(t)$ and $y(t)$ is
   $$\phi_{xy}(t) = \int_{-\infty}^{+\infty} x(t+\tau)y(\tau)d\tau$$

The autocorrelation function of a signal $x(t)$ is obtained by setting $y(t)$=$x(t)$ in the above equation:

$$\phi_{xx}(t) = \int_{-\infty}^{+\infty} x(t+\tau)x(\tau)d\tau$$

(a) Compute the autocorrelation function of each of the two signals $x_1(t)$ and $x_2(t)$ depicted in the figure below.



(b) Compute the cross correlation function of $x_1(t)$ and $x_2(t)$.

(c) Prove that the cross-correlation function

$$\phi_{xy}(t) = x(t) * y(-t)$$

7. Multipath propagation model can be generalized as $y[n] = x[n] + ax[n-k]$. Find impulse response of the causal inverse system.

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