EE1101 Signals and Systems JAN—MAY 2018
Tutorial 10
April 23, 2018

1. Sketch the pole-zero plot corresponding to the following causal system functions:
   (a) $\dfrac{s-2}{s^2+8s+15}$
   (b) $\dfrac{s+1}{(s+2)^2(s+3)}$
   (c) $\dfrac{2s^2+s+1}{s(s+2)}$
   (d) $\dfrac{2s+1}{(s+2)(s^2+1)^2}$

   Which of the above system functions correspond to BIBO stable systems?

2. Determine the BIBO stability and causality for the following Laplace transforms:
   (a) $\dfrac{2s+5}{(s+2)(s+3)}; -3 < Re(s) < -2$
   (b) $\dfrac{2s-5}{(s-2)(s-3)}; 2 < Re(s) < 3$
   (c) $\dfrac{2s+3}{(s+1)(s+2)}; Re(s) > -1$
   (d) $\dfrac{2s+3}{(s+1)(s+2)}; Re(s) < -2$

3. A causal LTI system is described by the system function $H(s) = \dfrac{s+3}{(s+2)^3}$.
   (a) Find the impulse response of the system.
   (b) For the input signal $x(t) = 10u(t)$, calculate the final value of the output $y(t)$ of the above system without explicitly evaluating $y(t)$.

4. Consider a continuous time LTI system for which the input $x(t)$ and output $y(t)$ are related by the differential equation

$$\frac{\mathrm{d}^2 y(t)}{\mathrm{d}t^2} - \frac{\mathrm{d}y(t)}{\mathrm{d}t} - 2y(t) = x(t).$$

   Let $X(s)$ and $Y(s)$ denote Laplace transforms of $x(t)$ and $y(t)$, respectively, and let $H(s)$ denote the Laplace transform of $h(t)$, the system impulse response.
   (a) Determine $H(s)$ as a ratio of two polynomials in s. Sketch the pole-zero pattern of $H(s)$.
   (b) Determine $h(t)$ for each of the following cases:
      (a) The system is stable.
      (b) The system is causal.
      (c) The system is neither stable nor causal.

5. The system function of a causal LTI system is

$$H(s) = \frac{s+1}{s^2+2s+2}.$$

   Determine the response $y(t)$ when the input is

$$x(t) = e^{-|t|}, \qquad -\infty < t < \infty.$$

6. Suppose we are given the following information about a causal and stable LTI system with impulse response $h(t)$ and a rational system function $H(s)$:
   (a) $H(1) = 0.2$.
   (b) When the input is $u(t)$, the output is absolutely integrable.
   (c) When the input is $tu(t)$, the output is not absolutely integrable.
   (d) The signal $\dfrac{d^2h(t)}{dt^2} + 2\dfrac{dh(t)}{dt} + 2h(t)$ is of finite duration.
   (e) $H(s)$ has exactly one zero at infinity.

   Determine $H(s)$ and its region of convergence.

7. The autocorrelation function of a signal $x(t)$ is defined as

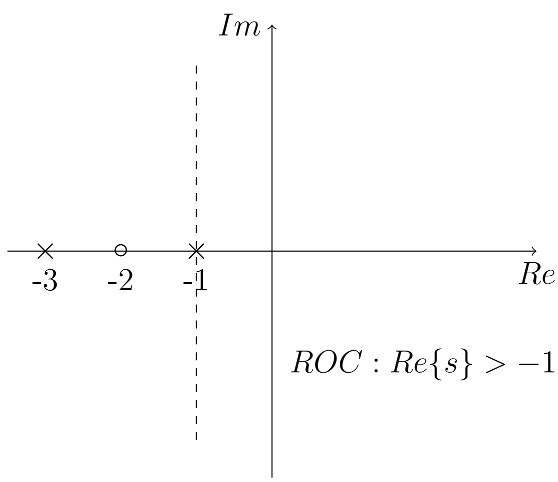
$$\phi_{xx}(\tau) = \int_{-\infty}^{\infty} x(t)x(t+\tau)dt$$

   (a) Determine, in terms of $x(t)$, the impulse response $h(t)$ of an LTI system for which, when the input is $x(t)$, the output is $\phi_{xx}(t)$.

$$x(t) \longrightarrow \boxed{\quad h(t) \quad} \longrightarrow \phi_{xx}(t)$$

   (b) From your answer in part (a), determine $\Phi_{xx}(s)$, the Laplace transform of $\phi_{xx}(\tau)$ in terms of $X(s)$. Also express $\Phi_{xx}(j\omega)$, the Fourier transform of $\phi_{xx}(\tau)$ in terms of $X(j\omega)$.

   (c) If $x(t)$ has the pole zero pattern and ROC as shown in figure, sketch the pole-zero pattern and indicate the ROC for $\phi_{xx}(\tau)$.



8. For each of the following signals $x(t)$ given below, calculate the unilateral Laplace transform using direct integration.

   (a) $x(t) = u(t-2)$

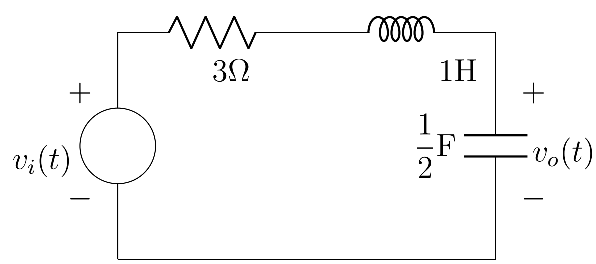
   (b) $x(t) = u(t+2)$

   (c) $x(t) = e^{3t}u(t)$

   (d) $x(t) = te^{t}u(t)$

   (e) $x(t) = \sin t \cdot u(t)$

9. (a) Determine the differential equation relating $v_i(t)$ and $v_o(t)$ for the RLC circuit shown below.



$$v_o(0^-) = 1$$
$$\frac{dv_o(t)}{dt}\Big|_{t=0^-} = 2$$

   (b) Suppose that $v_i(t) = e^{-3t}u(t)$. Using the unilateral Laplace transform, determine $v_o(t)$ for $t > 0$.

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