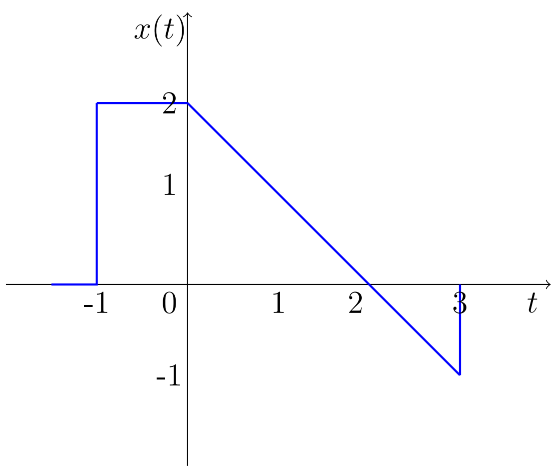
EE1101 Signals and Systems JAN—MAY 2019
Tutorial 1

1. Given a continuous-time signal specified by

$$x(t) = \begin{cases} 1-|t|, & -1 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}$$

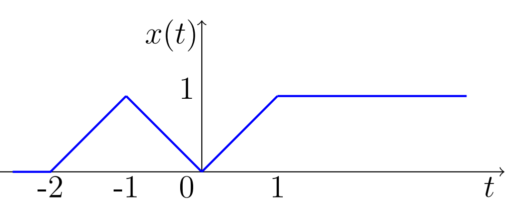
Plot the discrete-time sequence that results from sampling of $x(t)$ for the following sampling intervals: (a) 0.25 s, (b) 0.5 s, and (c) 1 s. The discrete time signal is given by x[n] = x(nT_s)

2. For the continuous time signal $x(t)$ shown below, sketch and label carefully each of the following signals: (a) $x(t-1)$, (b) $x(2-t)$, (c) $x(2t+1)$, (d) $x(4-\frac{t}{2})$, (e) $[x(t)+x(-t)]u(t)$ and (f) $x(t)[\delta(t+\frac{3}{2})-\delta(t-\frac{3}{2})]$.

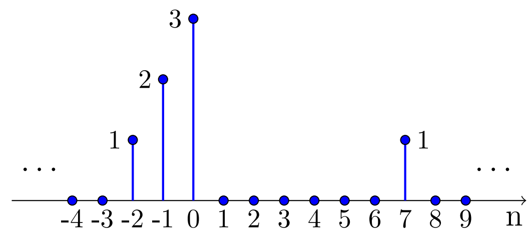


3. Determine and sketch the even and odd parts of the signals depicted in figures below. Label your sketches carefully.

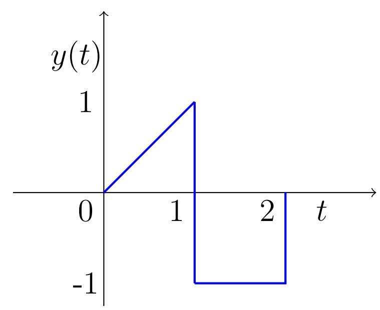
(a)



(b)



4. Consider the signal $y(t) = (-1/2)x(-2t-3/2)$ shown below.
   (a) Determine and carefully sketch the original signal $x(t)$.
   (b) Determine and carefully sketch $y_o(t)$, the odd portion of $y(t)$.

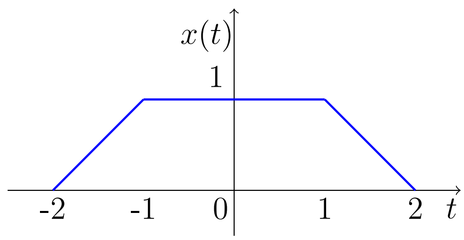


5. A sinusoid $e^{\sigma t}\cos\omega t$ can be expressed as a sum of exponentials e^{st} and e^{s^*t} with complex frequencies $s = \sigma + j\omega$ and $s^* = \sigma - j\omega$. Locate in the complex plane the frequencies of the following sinusoids:

   (a) $\sin 2t$, (b) $e^{-5t}\cos 3t$, (c) $e^{2t}\cos 3t$, (d) e^{-2t}, (e) e^{2t}, and (f) 5.

6. (a) Show that the power of the signal $x(t) = \sum_{k=m}^{n} D_k e^{j\omega_k t}$ is $P_x = \sum_{k=m}^{n} |D_k|^2$ assuming all frequencies to be distinct i.e. $\omega_i \neq \omega_j$ for $i \neq j$.
   (b) Determine the power of the signals (a) $x(t) = 10\cos 5t \cos 10t$ and (b) $x(t) = 10\cos(100t+\frac{\pi}{3})+5\sin(100t+\frac{\pi}{6})$. Use the result from part (a). (Hint: Write cos and sin as sum of complex exponentials).

7. Consider the signal $x(t)$ shown in figure.



   (a) Determine and carefully sketch $y(t) = 3x\big(-\frac{1}{2}(t+1)\big)$.

   (b) Determine the energy and power of $y(t)$.

   (c) Determine and carefully sketch the even portion of $y(t)$, $y_e(t)$.

   (d) Let $a = 2$ and $b = 3$, sketch $y(at+b)$, $y(at) + b$, $ay(t+b)$ and $ay(t) + b$.

8. Let the energy of signal $x(t)$ be denoted by E_x. Show that

   (a) The signals $-x(t)$, $x(-t)$ and $x(t - T)$ have the same energy.

   (b) The energy of $x(at)$ and $x(at - b)$ is $\frac{E_x}{|a|}$. $a \neq 0$.

9. The unit pulse function $\Pi : R \longrightarrow R$ is defined as

$$\Pi(t) = \begin{cases} 1, & -\frac{1}{2} \leq t \leq \frac{1}{2} \\ 0, & \text{otherwise} \end{cases}$$

   Sketch the following signals and evaluate the energy of each one of them

   (a) $\Pi(2t)$.

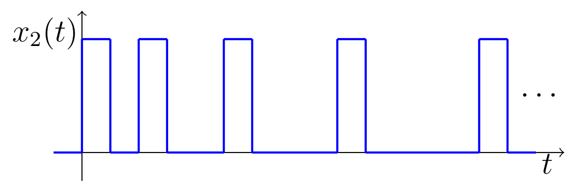
   (b) $6\Pi(0.5t)$.

   (c) $\Pi(t - 4)$.

   (d) $\Pi(\frac{t+1}{2}) + \Pi(t - 1)$.

10. (a) A binary signal $x_1(t) = 0$ for $t < 0$. For positive time, $x_1(t)$ toggles between one and zero in every one second. Determine the energy and power of $x_1(t)$.

   (b) A binary signal $x_2(t) = 0$ for $t < 0$. For positive time, $x_2(t)$ toggles between one and zero as follows: one for 1 second, zero for 1 second, one for 1 second, zero for 2 seconds, one for 1 second, zero for 3 seconds, and so forth. That is, the "on" time is always one second, but the "off" time successively increases by one second between each toggle. A portion of $x_2(t)$ is shown below. Determine the energy and power of $x_2(t)$.



11. Determine which of the following signals are periodic. If a signal is periodic, what is the fundamental period and average power?

   (a) $\cos(\pi t)$.

   (b) $A\sin(10t)$.

   (c) $\sin(\sqrt{3}\pi t)$.

   (d) e^{jt}.

   (e) $A\sin(4\pi t + \pi)$.

   (f) $\sum_{n=-\infty}^{\infty} \Pi(t - 2n)$.

12. Carefully sketch the following signals. Mark all the critical points.

   (a)

$$h(t) = \begin{cases} e^{-tu(t)}, & -1 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

   (b) $p[n] = (\frac{1}{2})^n u[n-1]$

   (c) $g(t) = \dfrac{d}{dt}\big(u(t-2)r(t)\big)$

   (d) $f(t) = sgn(e^{-2t}\sin\pi t)$, $sgn(t) = u(t) - u(-t)$