Assignment-1

6. $1/5$
7. (a) Converges to 1. (b) Diverges. (c) Converges. (d) Diverges.
8. Converges.
9. Converges to π.
10. The area is equal to 1.
11. (a) $3(1+2^{1/3})$ (b) Integral does not converge.
15. Diverges to ∞.
16. Converges.

Assignment-2

1 (a) $[-1,1)$ (b) $[-1,1]$ (c) $(-1,1]$
2. $(-1/2,1/2]$
3. (a) $x - x^2/2 + x^3/3 - \cdots$ for $-1 < x < 1$. (b) $\sum c_n x^n$, where $c_n = \sum_{k=1}^{n} (-1)^{k-1}/n$.
5. $\ln 2$.
7. $f(x) = e^{-1/x^2}$ for $x \neq 0$ and $f(0) = 0$.
8. Find a series for $\sin^{-1} x$ and then show that the given series converges.
9. Fourier series is $\dfrac{1}{2} + \dfrac{2}{\pi}\displaystyle\sum_{n=0}^{\infty} \frac{\sin(2n+1)x}{2n+1}$.
10. Fourier series is $\dfrac{\pi}{2} - \dfrac{4}{\pi}\displaystyle\sum_{n=0}^{\infty} \frac{\cos(2n+1)x}{(2n+1)^2}$.
11. Fourier series is $2\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nx$.
12. Fourier series is $\dfrac{4}{\pi}\displaystyle\sum_{n=0}^{\infty} \frac{\sin(2n+1)x}{2n+1}$.
13. Find sine and cosine series half-range expansions.
14. Fourier series is $\dfrac{1}{2} + \dfrac{4}{\pi^2}\displaystyle\sum_{n=0}^{\infty} \frac{\cos(2n+1)\pi x}{(2n+1)^2}$.

Assignment-3

2. $A^n = \begin{bmatrix} 1 & n & n(n-1) \\ 0 & 1 & 2n \\ 0 & 0 & 1 \end{bmatrix}$.
4. $\begin{bmatrix} I & -A \\ 0 & I \end{bmatrix}$.
10. $(1,0,1/2)$ and $(0,1,3/4)$.
11. The third vector is $\left(-\frac{2}{3}, \frac{2}{3}, -\frac{2}{3}\right)$.
12. $\pm(1/\sqrt{6}, -2/\sqrt{6}, 1/\sqrt{6})^T$.
13 (a) rank is 3. (b) rank is 2.
14. Linearly dependent.
15. Rank of the matrix is 3. Its first row is $[1,1,0]$.
16. $(1,0,1,0,1)$ and $(0,1,0,-1,0)$.

Assignment-4

1. $x_1 = 3 - 2\alpha + 2\beta,\ x_2 = 1 + \alpha - \beta,\ x_3 = \alpha,\ x_4 = 2 + 2\beta,\ x_5 = \beta$.
3 (a) Inconsistent. (b) $x_1 = -\frac{10}{9} + 2\alpha,\ x_2 = \frac{23}{27} + \frac{\alpha}{3},\ x_3 = \frac{121}{27} + \frac{7\alpha}{3},\ x_4 = \alpha$.
4. $k = -3$ no solution, $k = 2$ infinitely many solutions, $k \neq -3,\ k \neq 2$ unique solution.
5. (d) $\lambda = 3, -5, 8$. Eigenvectors: $e_2,\ (-4,1,4)^t,\ (15,-6,50)$.
9 (a) and (d) diagonalizable; (b) and (c) not diagonalizable.
10. Except (b) others are diagonalizable.