EE1101: Signals and Systems JAN — MAY 2018

Tutorial 7 Solutions

Solution 1

a) Fourier transform of the given function,

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$X(j\omega) = \int_{0}^{1} 4e^{-j\omega t}dt + \int_{1}^{2} 2e^{-j\omega t}dt$$

$$= \frac{4}{j\omega}\left(-e^{-j\omega}+1\right) + \frac{2}{j\omega}\left(-e^{-j2\omega}+e^{-j\omega}\right)$$

$$X(j\omega) = \frac{4}{j\omega} - \frac{2}{j\omega}e^{-j\omega} - \frac{2}{j\omega}e^{-j2\omega}$$

It can also be written in terms of $sinc(x)$ function as

$$X(j\omega) = \frac{4}{j\omega}\left(-e^{-j\omega}+1\right) + \frac{2}{j\omega}\left(-e^{-j2\omega}+e^{-j\omega}\right)$$

$$= 4e^{-j0.5\omega}\frac{sin(0.5\omega)}{0.5\omega} + 2e^{-j1.5\omega}\frac{sin(0.5\omega)}{0.5\omega}$$

Therefore,

$$X(j\omega) = 4sinc(\frac{\omega}{2})e^{-j0.5\omega} + 2sinc(\frac{\omega}{2})e^{-j1.5\omega}$$

b) Given function is

$$x(t) = \begin{cases} \frac{-t}{\tau} & t < 0 \\ \frac{t}{\tau} & t \geq 0 \end{cases}$$

Fourier Transform of given function,

$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$

$X(j\omega) = \int_{-\tau}^{0} \frac{-t}{\tau}e^{-j\omega t}dt + \int_{0}^{\tau} \frac{t}{\tau}e^{-j\omega t}dt$

$= \left[\frac{t}{\tau}\frac{e^{-j\omega t}}{j\omega} - \int \frac{e^{-j\omega t}}{\tau j\omega}dt\right]_{-\tau}^{0} + \left[\frac{-t}{\tau}\frac{e^{-j\omega t}}{j\omega} + \int \frac{e^{-j\omega t}}{\tau j\omega}dt\right]_{0}^{\tau}$

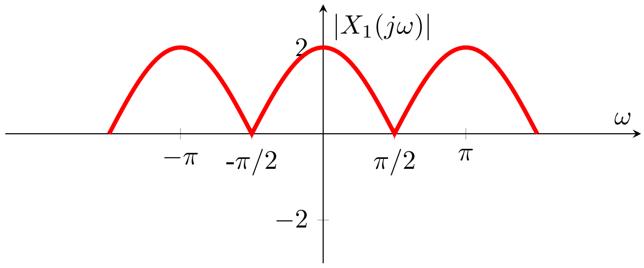
$= \frac{1}{j\omega}\left[e^{j\omega\tau} - e^{-j\omega\tau}\right] + \frac{1}{\tau\omega^2}\left[-2 + e^{j\omega\tau} + e^{-j\omega\tau}\right]$

$X(j\omega) = \frac{2}{\omega}sin(\omega\tau) - \frac{2}{\tau\omega^2}(1 - cos(\omega\tau))$

Solution 2

a) Let $x_1(t) = \delta(t+1) + \delta(t-1)$, then the Fourier transform is given by,

$$\begin{aligned} X_1(j\omega) &= \int_{-\infty}^{\infty} [\delta(t+1) + \delta(t-1)]\, e^{-j\omega t}dt \\ &= e^{j\omega} + e^{-j\omega} = 2\cos\omega \end{aligned}$$

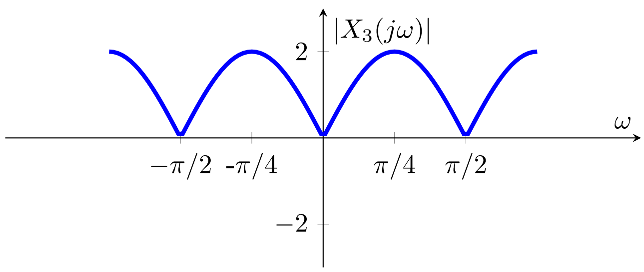


b) Consider the signal $x_2(t) = u(-2-t)+u(t-2)$, Clearly

$$\begin{aligned} x_3(t) = \frac{dx_2(t)}{dt} &= \frac{d}{dt}[u(t-2) + u(-2-t)] \\ &= \frac{d}{dt}[u(t-2) + u(-(t+2))] \\ &= \delta(t-2) - \delta(-(t+2)) \\ &= \delta(t-2) - \delta(t+2) \end{aligned}$$

Therefore,

$$\begin{aligned} X_3(j\omega) &= \int_{-\infty}^{\infty} [\delta(t-2) - \delta(t+2)]\, e^{-j\omega t}dt \\ &= e^{-j2\omega} - e^{j2\omega} = -2j\sin(2\omega) \end{aligned}$$



Solution 3

a)

$$x(t) = e^{-\frac{|t|}{2}} = \begin{cases} e^{\frac{t}{2}} & t < 0 \\ e^{-\frac{t}{2}} & t \geq 0 \end{cases}$$

Fourier transform,

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$
$$= \int_{-\infty}^{0} e^{\frac{t}{2}}e^{-j\omega t}dt + \int_{0}^{\infty} e^{-\frac{t}{2}}e^{-j\omega t}dt$$
$$= \frac{\left[e^{(0.5-j\omega)t}\right]_{-\infty}^{0}}{(0.5-j\omega)} - \frac{\left[e^{-(0.5+j\omega)t}\right]_{0}^{\infty}}{(0.5+j\omega)}$$
$$= \frac{1}{(0.5-j\omega)} + \frac{1}{(0.5+j\omega)}$$
$$X(j\omega) = \frac{1}{0.25+\omega^2}$$

b)

$$x(t) = sin(2\pi t)e^{-t}u(t) = \frac{(e^{j2\pi t} - e^{-j2\pi t})}{2j}e^{-t}u(t)$$

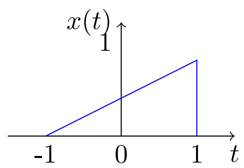
Fourier transform of $x(t)$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$
$$= \int_{0}^{\infty} \frac{(e^{j2\pi t} - e^{-j2\pi t})}{2j}e^{-t}e^{-j\omega t}dt$$
$$= \int_{0}^{\infty} \frac{e^{t(j2\pi - j\omega - 1)}}{2j}dt - \int_{0}^{\infty} \frac{e^{-t(j2\pi + j\omega + 1)}}{2j}dt$$

$$X(j\omega) = \frac{1}{2j}\left[\frac{1}{(1+j(\omega-2\pi))} - \frac{1}{(1+j(\omega+2\pi))}\right]$$
$$X(j\omega) = \frac{2\pi}{(1+j\omega)^2+4\pi^2}$$

Solution 4

a) The x(t) plot is as shown in figure



Using the Fourier transform equation, we have

$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$
$X(j\omega) = \int_{-1}^{1} \frac{(t+1)}{2}e^{-j\omega t}dt = \int_{-1}^{1}\left[\frac{te^{-jwt}}{2} + \frac{e^{-jwt}}{2}\right]dt$

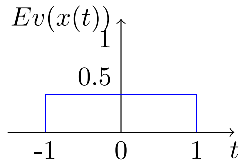
And integrate the first term by parts

$$= \frac{1}{2}\left(\left[\frac{te^{-j\omega t}}{-j\omega} + \frac{e^{-j\omega t}}{\omega^2}\right]_{-1}^{1} + \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-1}^{1}\right)$$
$$= \frac{e^{-j\omega}}{-j\omega} - \frac{sin(\omega)2j}{2\omega^2}$$
$$X(j\omega) = \frac{e^{-j\omega}}{-j\omega} + \frac{sin(\omega)}{j\omega^2}$$

b) Real part of $X(j\omega) = Real\left(\frac{e^{-j\omega}}{-j\omega}\right) = (1/\omega)Re\left[\,j\,(cos\omega - jsin\omega)\right] = \frac{sin\omega}{\omega}$

& Even part of $x(t)$ is given as $\Rightarrow \frac{x(t)+x(-t)}{2} = \frac{(t+1)/2+(-t+1)/2}{2} = 0.5$. In the domain t=[-1,1]

The Ev(x(t)) plot is as shown in figure

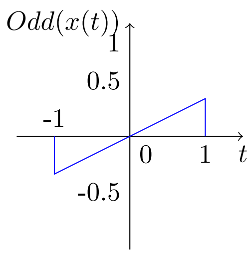


$X(j\omega) = \int_{-1}^{1} 0.5.e^{-j\omega t}d\omega = 0.5\frac{e^-j\omega t|_{-1}^{1}}{-j\omega} = \frac{sin\omega}{\omega}$

And thus they are equal

c) Odd part of $x(t)$ is given as $\Rightarrow \frac{x(t)-x(-t)}{2} = \frac{(t+1)/2-(-t+1)/2}{2} = t/2$. In the domain t=[-1,1]

The Odd(x(t)) plot is as shown in figure



$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$

$X(j\omega) = \int_{-1}^{1} \frac{t}{2}e^{-j\omega t}dt = \frac{-jsin\omega}{\omega^2} + \frac{jcos\omega}{\omega}$

Property:The fourier transform of the odd part of x(t) is the same as j times imaginary part of the answer to part (a). i.e

Let Y= $Im\left[\frac{e^{-j\omega}}{-j\omega} + \frac{sin(\omega)}{j\omega^2}\right] = \frac{-sin\omega}{\omega^2} + \frac{cos\omega}{\omega}$

now $Y * j = \frac{-jsin\omega}{\omega^2} + \frac{jcos\omega}{\omega}$

Solution 5

a)

$$\begin{aligned} X(j\omega) &= \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \\ &= \int_{-1}^{1} e^{-j\omega t}dt = \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-1}^{1} = \frac{2\sin\omega}{\omega} \end{aligned}$$

b) Let $Let, y(t) = x(t+T) + x(t-T) = x_1(t) + x_2(t)$
F.T,

$$X_1(j\omega) = \int_{-\infty}^{\infty} x_1(t)e^{-j\omega t}dt = \int_{-\infty}^{\infty} x(t+T)e^{-j\omega t}dt$$

Take $t + T = z \implies dt = dz$

$$X_1(j\omega) = \int_{-\infty}^{\infty} x(z)e^{-j\omega(z-T)}dz = X(j\omega)e^{j\omega T}$$

Similarly, $X_2(j\omega) = X(j\omega)e^{-j\omega T}$,
Therefore,

$$\begin{aligned} Y(j\omega) &= X_1(j\omega) + X_2(j\omega) \\ &= 2X(j\omega)\left[\frac{e^{j\omega T} + e^{-j\omega T}}{2}\right] \\ &= 2X(j\omega)\cos(\omega T) \end{aligned}$$

c)

$$y(t) = x(t+3) + x(t-3)$$

Therefore, using result from Qn.5(b),

$$Y(j\omega) = 2X(j\omega)\cos(3\omega) = \frac{4\sin(\omega)\cos(3\omega)}{\omega}$$

Solution 6

a) If $x(t)$ is even in t, $x(t) = x(-t)$,

$$\begin{aligned} X(j\omega) &= \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \\ &= \int_{-\infty}^{0} x(t)e^{-j\omega t}dt + \int_{0}^{\infty} x(t)e^{-j\omega t}dt \\ &\quad \text{In first part replace } t \to -t \implies dt = -dt \\ &= -\int_{\infty}^{0} x(-t)e^{j\omega t}dt + \int_{0}^{\infty} x(t)e^{-j\omega t}dt \\ &= \int_{0}^{\infty} x(-t)e^{j\omega t}dt + \int_{0}^{\infty} x(t)e^{-j\omega t}dt \\ &= \int_{0}^{\infty} (x(-t)e^{j\omega t} + x(t)e^{-j\omega t})dt \\ &= \int_{0}^{\infty} (x(t)\left[e^{j\omega t} + e^{-j\omega t}\right])dt \\ &\quad \because x(t) = x(-t) \\ &= 2\int_{0}^{\infty} x(t)\cos(\omega t)dt \end{aligned}$$

b) If $x(t)$ is odd in t, $x(t) = -x(-t)$,

$$\begin{aligned} X(j\omega) &= \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \\ &= \int_{-\infty}^{0} x(t)e^{-j\omega t}dt + \int_{0}^{\infty} x(t)e^{-j\omega t}dt \\ &\quad \text{In first part replace } t \to -t \implies dt = -dt \\ &= -\int_{\infty}^{0} x(-t)e^{j\omega t}dt + \int_{0}^{\infty} x(t)e^{-j\omega t}dt \\ &= \int_{0}^{\infty} x(-t)e^{j\omega t}dt + \int_{0}^{\infty} x(t)e^{-j\omega t}dt \\ &= \int_{0}^{\infty} (x(-t)e^{j\omega t} + x(t)e^{-j\omega t})dt \\ &= \int_{0}^{\infty} (x(t)\left[-e^{j\omega t} + e^{-j\omega t}\right])dt \\ &\quad \because x(t) = -x(-t) \\ &= -2j\int_{0}^{\infty} x(t)\sin(\omega t)dt \end{aligned}$$

Solution 7

a) The inverse fourier transform is

$x_1(t) = (1/2\pi)\int_{-\infty}^{\infty}[2\pi\delta(\omega) + \pi\delta(\omega-4\pi) + \pi\delta(\omega+4\pi)]e^{j\omega t}d\omega$

$= (1/2\pi)[2\pi e^{j0t} + \pi e^{j4\pi t} + \pi e^{-j4\pi t}]$

$= 1 + (1/2)e^{j4\pi t} + (1/2)e^{-j4\pi t} = 1 + cos(4\pi t)$

b) The inverse fourier transform is

$x_2(t) = (1/2\pi)\int_{-\infty}^{\infty} X_2(j\omega)e^{j\omega t}d\omega$

$= (1/2\pi)\int_{0}^{2} 2e^{j\omega t}d\omega + (1/2\pi)\int_{-2}^{0}(-2)e^{j\omega t}d\omega$

$= (e^{j2t} - 1)/(\pi jt) - (1 - e^{-j2t})/(\pi jt)$

$-(4jsin^2 t)/(\pi t)$