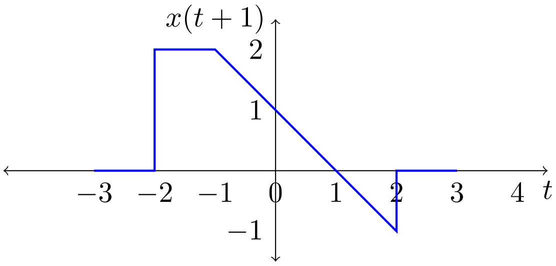
EE1101: Signals and Systems JAN—MAY 2019

Tutorial Quiz 1 Solutions

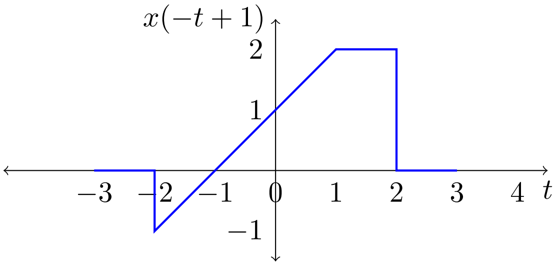
Set A

Solution 1

$x(1-t) = x(-t+1)$ can be obtained by shifting $x(t)$ left by one unit and time reversing the signal (or time reversing first and then shifting right by one unit).



1 mark for graph and 1 mark for labeling.



1 mark for graph and 1 mark for labeling.

Solution 2

1.5 mark (digital marking) for each bit correctly solved with justification.

a

Let $y_1(t)$ is the output of a system to input $x_1(t)$ and $y_2(t)$ is the output to input $x_2(t)$. Let $y(t)$ be the output of the system to the input $ax_1(t) + bx_2(t)$.

$$\begin{aligned} y(t) &= ax_1\left(\frac{t}{2}\right) + bx_2\left(\frac{t}{2}\right) \\ &= ay_1(t) + by_2(t) \end{aligned}$$

Hence the system is **linear**.

Let $y(t)$ is the output of a system to input $x(t)$. Let $y_1(t)$ be the output of the system to the input $x(t-t_0)$.

$$y_1(t) = x\left(\frac{t}{2} - t_0\right)$$

But,

$$\begin{aligned} y(t-t_0) &= x\left(\frac{t-t_0}{2}\right) \\ &= x\left(\frac{t}{2} - \frac{t_0}{2}\right) \\ \therefore y(t-t_0) &\neq y_1(t) \end{aligned}$$

Hence the system is **time variant**.

b

Let $y_1[n]$ be the response of $x_1[n]$, $y_2[n]$ be the response of $x_2[n]$ and $y[n]$ be the response of the combined input $x[n] = ax_1[n] + bx_2[n]$.

$$y_1[n] = x_1[n]x_1[n+1]$$

$$y_2[n] = x_2[n]x_2[n+1]$$

$$\begin{aligned} y[n] = ax_1[n] \times ax_1[n+1] + ax_1[n] \times bx_2[n+1] + \\ ax_1[n+1] \times bx_2[n] + bx_2[n] \times bx_2[n+1] \\ = a^2x_1[n]x_1[n+1] + abx_1[n]x_2[n+1] + \\ abx_1[n+1]x_2[n] + b^2x_2[n]x_2[n+1] \\ \neq ay_1[n] + by_2[n] \end{aligned}$$

The given system is **non-linear**.

Let $y[n]$ is the output of a system to input $x[n]$. Let $y_1[n]$ be the output of the system to the input $x[n-n_0]$.

$$y_1[n] = x[n-n_0]x[n+1-n_0]$$

And,

$$y[n-n_0] = x[n-n_0]x[n+1-n_0]$$

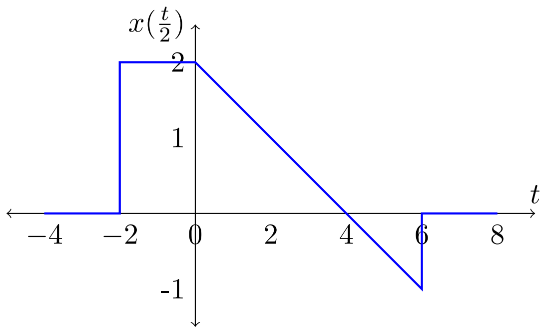
$$\therefore y[n-n_0] = y_1[n]$$

Hence the system is **time invariant**.

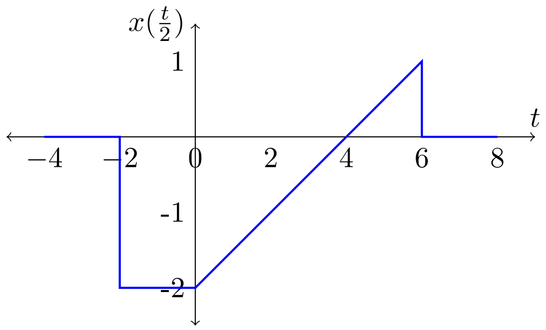
Set B

Solution 1

$-x(\frac{t}{2})$ can be obtained by time compressing $x(t)$ and flipping it about the X-axis (or flipping first and then compressing).



1 mark for graph and 1 mark for labeling.



1 mark for graph and 1 mark for labeling.

Solution 2

1.5 mark (digital marking) for each bit correctly solved with justification.

a

Let $y(t)$ is the output of a system to input $x(t)$. Let $y_1(t)$ be the output of the system to the input $x(t-t_0)$.

$$y_1(t) = x\big(2t - 4 - t_0\big)$$

And,

$$y(t-t_0) = x(2(t-t_0) - 4)$$

$$\therefore y(t-t_0) \neq y_1(t)$$

Hence the system is **time variant**.

For $t > 4$, $y(t)$ is dependent on future values of $x(t)$. Hence, this system is **not causal**.

b

Let $y[n]$ is the output of a system to input $x[n]$. Let $y_1[n]$ be the output of the system to the input $x[n-n_0]$.

$$y_1[n] = x^2[n-n_0]$$

And,

$$y[n-n_0] = x^2[n-n_0]$$

$$\therefore y[n-n_0] = y_1[n]$$

Hence the system is **time invariant**.

$y[n]$ is not dependent on future values of $x[n]$. Hence, this system is **causal**.