EE1101 Signals and Systems JAN—MAY 2018
Tutorial 3

February 12, 2018

1. Find the fundamental period of the signal $x(t) = \sin\left(\frac{3\pi}{5}t\right)$. Let $x[n]$ be obtained from $x(t)$ by sampling at $t = nT_s$ where (a) $T_s = 1$ *sec*, (b) $T_s = 5$ *sec*, and (c) $T_s = 1/\pi$ *sec*. Determine whether $x[n]$ is periodic for each case. If so, find its fundamental period.

2. Let $y_1[n] = x[2n]$ and
$$\begin{aligned} y_2[n] &= x[n/2], \quad n \text{ even} \\ &= 0, \qquad\quad\; n \text{ odd} \end{aligned}$$
If x[n] is periodic, are $y_1[n]$ and $y_2[n]$ periodic? If so, find their fundamental period.

3. Let $x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3]$ and $h[n] = 2\delta[n+1] + 2\delta[n-1]$. Compute and plot each of the following convolutions.
   (a) $y_1[n] = x[n] \star h[n]$
   (b) $y_2[n] = x[n+2] \star h[n]$
   (c) $y_3[n] = x[n] \star h[n+2]$

4. Let the output of a discrete time LTI system, with impulse response $h[n]$, be given by, $y[n] = x[n] \star h[n]$, where the input $x[n] = 0$ outside the range $0 \le n \le N-1$. Let the column vector $\mathbf{y}$ represent the output $y[n]$ from 0 to $N-1$, and the column vector $\mathbf{x}$, the values of $x[n]$ from 0 to $N-1$. If $\mathbf{y} = \mathbf{H}\mathbf{x}$, find the matrix $\mathbf{H}$.

5. Convolve the signals $u[n]$ and $a^n u[-n-1]$, given that $|a| > 1$.

6. Let
$$x[n] = \begin{cases} 1, & 0 \le n \le 9 \\ 0, & \text{elsewhere} \end{cases}$$
and
$$h[n] = \begin{cases} 1, & 0 \le n \le N \\ 0, & \text{elsewhere} \end{cases}$$
where $N \le 9$ is an integer. Determine the value of N, given that $y[n] = x[n] \star h[n]$, $y[4] = 5$ and $y[14] = 0$.

7. Let $y(t) = x(t) \star h(t)$. $x(t)$ is non-negative for $t \in (2, 3)$ and zero elsewhere, and is symmetric about $t = 5/2$. $h(t) = 1$ for $t \in (3, 4)$ and zero elsewhere.
   (a) During what times will the values $y(t)$ be non-zero?
   (b) At what time(s) will $y(t)$ achieve its maximum value.

8. Perform the following convolutions where $\star$ indicates convolution.
   (a) For $u(t)$ a unit step function, find $r(t) = u(t) \star u(t)$.
   (b) Find $x(t) \star h(t)$, where $h(t) = (-e^{-t} + 2e^{-2t})u(t)$ and $x(t) = 10e^{-3t}u(t)$.
   (c) Find the output $y(t)$ of an LTI system with impulse response $h(t) = 2e^{-2t}u(t)$ when excited with an input $x(t)$ given by
   $$x(t) = \begin{cases} 1, & 2 \le t \le 4 \\ 0, & \text{otherwise} \end{cases}$$
   (d) Sketch $y(t) = [u(t) \star u(t-2)]\, u(4-t)$.
   (e) Determine graphically $h(t) = f(t) \star g(t)$, where
      (i) $f(t) = u(-t)$ and $g(t) = 2(u(t) - u(t-1))$.
      (ii) $f(t) = r(t) - r(t-2)$ and $g(t) = u(t-3) - u(t-6)$ [Note: $r(t) = tu(t)$].

9. Consider a system with input $x(t)$ and output $y(t)$ related by:
$$y(t) = \int_{-\infty}^{t+1} \sin(t-\tau)x(\tau)\, d\tau.$$

(a) Is the system time-invariant? Prove.

(b) What is the system impulse response?

(c) Is the system causal?

10. Let $x(t) = 1, 0 \leq t < 1$ and zero elsewhere. And, let $h(t) = x\left(\frac{t}{\alpha}\right)$, with $0 < \alpha \leq 1$.

    (a) Plot $y(t) = x(t) \star h(t)$, where $\star$ denotes convolution operation.

    (b) Plot the first derivative of $y(t)$.

    (c) What should be the value of α such that the first derivative of $y(t)$ contains exactly three discontinuities?

11. Consider a time-invariant system with input $x(t)$ and output $y(t)$. Show that if $x(t)$ is periodic with period T, $y(t)$ is also periodic.

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