DEPARTMENT OF PHYSICS
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

PH1020 Physics II

Tutorial 1 (22.1.2018)

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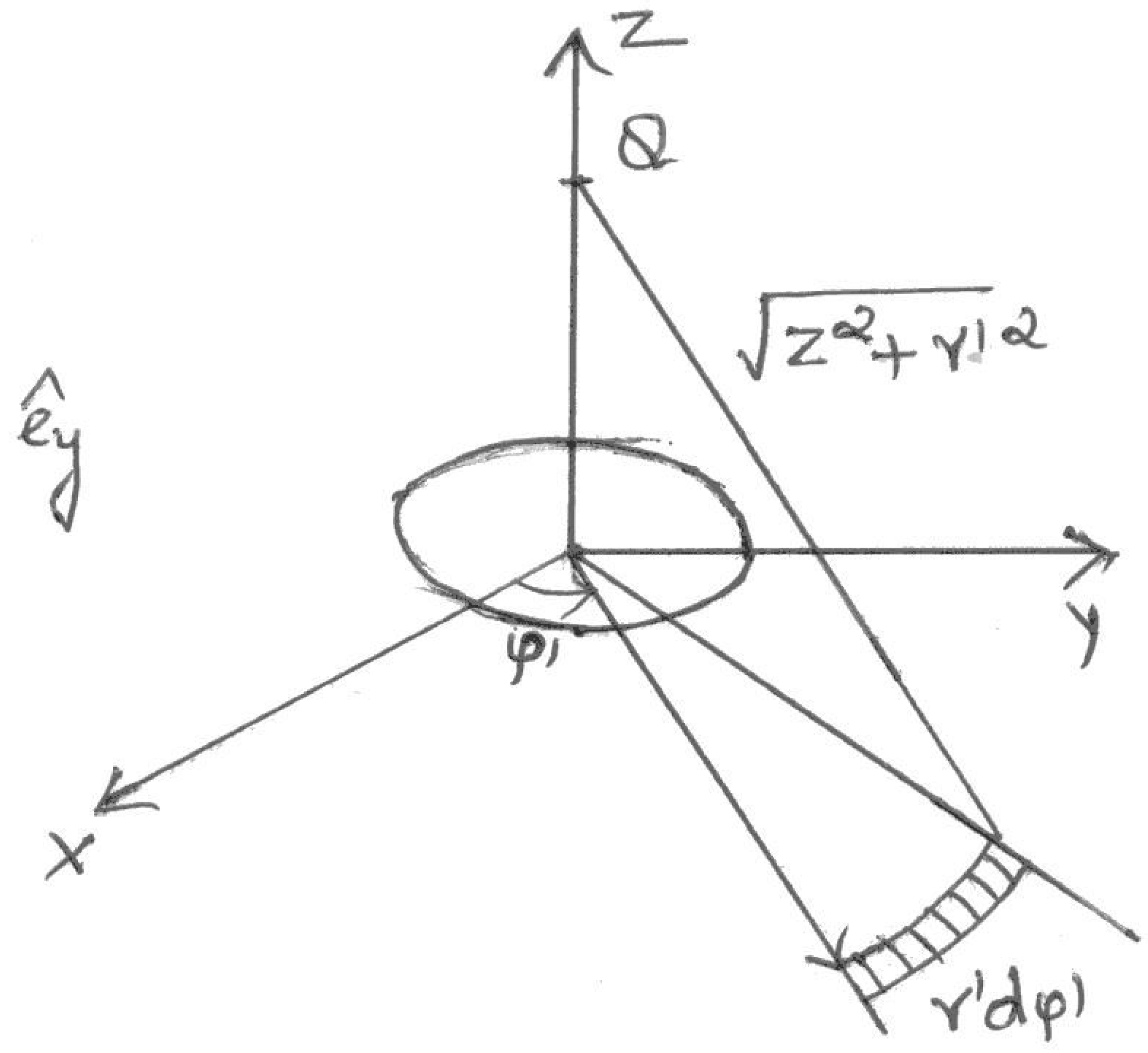
1. Consider an infinite uniformly-charged plate occuping the xoy plane, carrying a surface charge density σ, with a circular hole of radius a centered at the origin. Find the force on a charge Q lying on the z−axis. If the charge is negative, (i.e., $Q < 0$), discuss the force on it when released at a distance x close to the origin (i.e., $z << a$).

<u>Ans.</u>



The field pt : $\vec{r} = z\hat{e}_z$

The source pts : $(r', \varphi') = r'\cos\varphi'\hat{e}_x + r'\sin\varphi'\hat{e}_y$

$= \vec{r'}$

$\vec{r} - \vec{r'} = z\hat{e}_z - r'\cos\varphi'\hat{e}_x - r'\sin\varphi'\hat{e}_y$

Element of charge $dq' = \sigma r'dr'd\varphi'$

The force on $Q = \dfrac{Q}{4\pi\varepsilon_0}\displaystyle\int_a^\infty\int_0^{2\pi} \frac{\sigma(z\hat{e}_z - r'\cos\varphi'\hat{e}_x - r'\sin\varphi'\hat{e}_y)}{(z^2+r'^2)^{3/2}} r'dr'd\varphi'$

Integrations over $\cos\varphi'$ and $\sin\varphi'$ vanish

The force on Q

$= \vec{F}_Q = \dfrac{Q}{4\pi\varepsilon_0} 2\pi \displaystyle\int_a^\infty \frac{\sigma z\,\hat{e}_z}{(z^2+r'^2)^{3/2}} r'dr'$

$$= \frac{Q}{4\pi\varepsilon_0} 2\pi\sigma z \frac{1}{2} \int_a^\infty \frac{d(r'^2 + z^2)}{(z^2 + r'^2)^{3/2}}$$

$$= \frac{-Q}{2\varepsilon_0} \frac{\sigma z}{\sqrt{z^2 + r'^2}} \hat{e}_z \Bigg|_{r'=a}^{\infty} = \frac{Q\sigma z}{2\varepsilon_0\sqrt{z^2 + a^2}} \hat{e}_z$$

(Note : At large distances, $\vec{F}_Q = \frac{Q\sigma}{2\varepsilon_0} \hat{e}_z$, which is exactly same as corresponding to the situation in which the hole is absent ! So, at large distances, Q does not see the hole !)

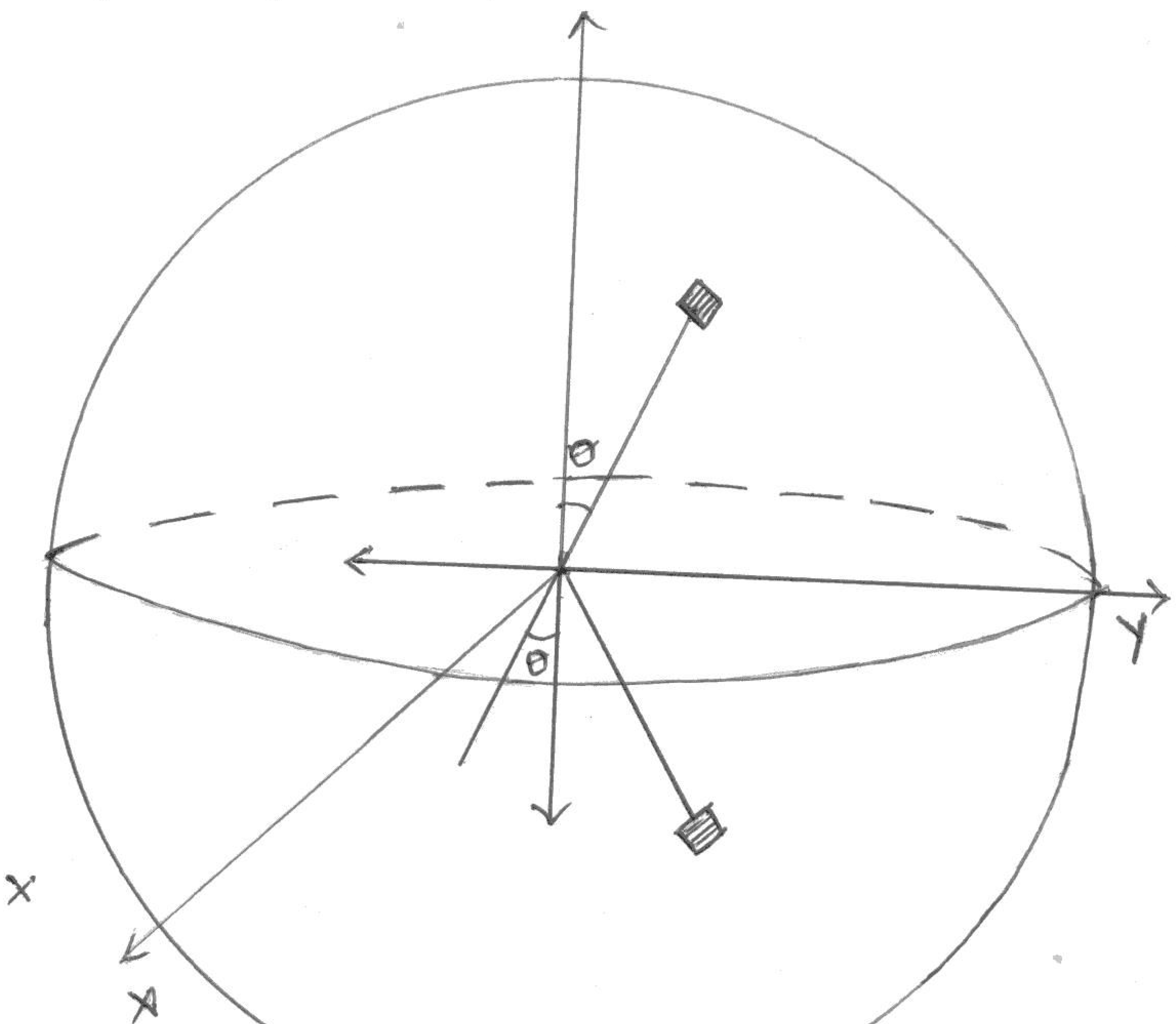
For $z \ll a$ and $Q < 0$

$$\vec{F}_Q = \frac{Q\sigma z \, \hat{e}_z}{2\varepsilon_0 a \left(1 + \frac{z^2}{a^2}\right)^{1/2}} = \frac{Q\sigma z}{2\varepsilon_0 a} \left(1 + \frac{z^2}{a^2}\right)^{-1/2} \hat{e}_z$$

$$\simeq \frac{Q\sigma z}{2\varepsilon_0 a} \hat{e}_z$$

The motion of charge is simple harmonic.

2. Determine the electric field at the center of a sphere of radius R that carries a charge on its surface with charge density $\sigma = \vec{k} \cdot \vec{r}$, where $\vec{k}$ is a constant vector.



Ans :

$$\sigma = \vec{k} \cdot \vec{r} = kR\cos\theta'$$

$$\vec{dE} = \frac{1}{4\pi\varepsilon_0 R^2} (kR\cos\theta') (R^2 \sin\theta' d\theta' d\varphi') (-\hat{e}_r)$$

$$\vec{dE} = \frac{kR\cos\theta'}{4\pi\varepsilon_0 R^2} (R^2 \sin\theta' d\theta' d\varphi') \times (-\cos\theta' \hat{e}_z + \sin\theta' \hat{e}_{XOY})$$

Where $\hat{e}_{XOY}$ is the appropriate unit vector in the XOY plane

x and y components of the electric field vanish

$$\vec{dE}\Big|_{Net} = 2 \times \frac{kR\cos\theta'}{4\pi\varepsilon_0 R^2} (R^2 \sin\theta' d\theta' d\varphi') (-\cos\theta') \hat{e}_z$$

$$\vec{E} = \int \vec{dE}\Big|_{Net} = \int_{\varphi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \frac{(-2kR\cos^2\theta')(R^2 \sin\theta' d\theta' d\varphi')}{4\pi\varepsilon_0 R^2} \hat{e}_z$$

$$= \frac{2\pi}{4\pi\varepsilon_0} (-2kR) \int_0^{\pi/2} \cos^2\theta' \sin\theta' d\theta' \hat{e}_z$$

$$= -\frac{kR}{\varepsilon_0}\hat{e}_z \int_{\theta'=0}^{\pi/2} \cos^2\theta' \, d(-\cos\theta') = \frac{kR}{\varepsilon_0}\hat{e}_z \left(\frac{\cos^3\theta'}{3}\right)_{\theta'=0}^{\pi/2}$$

$$= \frac{-kR}{3\varepsilon_0}\hat{e}_z$$

3. A fixed charge $+q$ at a point O is surrounded by a continuous distribution of charge whose density $\rho(< 0)$ is a function only of the distance r from O. The total negative charge *exceeds* q in magnitude. A point charge $+q'$, with a mass m, is free to move on a line passing through O. (a) Obtain the condition for $+q'$ to be in equilibrium at a distance r_0 from O. (b) If q' is released at a point very close to r_0, find the force acting on it.

Ans

(a) $$4\pi r^2 E_r = \left\{ q + 4\pi \int_0^r s^2 \rho(s)\, ds \right\} \frac{1}{\varepsilon_0}$$

For q' to be in equilibrium at $r = r_0$, $E_{r_0} = 0$

$$\Rightarrow \quad q + 4\pi \int_0^r s^2 \rho(s)\, ds = 0$$ which is the required condition

(b) How let us apply the Gauss theorem to a sphere of radius

$$r = r_0 + \delta$$

$$4\pi (r_0+\delta)^2 E_r = \left\{ q + 4\pi \int_0^{r_0+\delta} s^2 \rho(s)\, ds \right\} \frac{1}{\varepsilon_0}$$

$$= \left\{ q + 4\pi \int_0^{r_0} s^2 \rho(s)\, ds + 4\pi \int_{r_0}^{r_0+\delta} s^2 \rho(s)\, ds \right\} \frac{1}{\varepsilon_0}.$$

$$= \left\{ 0 + 4\pi \int_{r_0}^{r_0+\delta} s^2 \rho(s)\, ds \right\} \frac{1}{\varepsilon_0}$$

$$= \left\{ (4\pi r_0^2 \delta)\, \rho(r_0) \right\} \frac{1}{\varepsilon_0}$$

i.e $$E_r = \frac{r_0^2}{(r_0+\delta)^2} \frac{\delta \rho(r_0)}{\varepsilon_0} \simeq \frac{\delta \rho(r_0)}{\varepsilon_0}$$

This means the equation of motion for the charge is

$$m\ddot{\delta} = q' \delta \rho(r_0) / \varepsilon_0$$

Since, $f < 0$ the motion of the charge q' is simple harmonic.

Such oscillations are known as radial oscillations.