

1. A spherical conductor A of radius  $R$  contains two spherical cavities with radii  $a$ ,  $b$  respectively as shown in the figure. The total charge on the conductor itself is zero. At the center of each cavity a point charges  $+q_a$  and  $+q_b$  are placed. (i) Find the surface charges  $\sigma_a$ ,  $\sigma_b$  and  $\sigma_R$ . (ii) What is the field outside the conductor? (iii) What is the field within each cavity? (iv) What is the force on  $+q_a$  and  $+q_b$ ? (v) What is the force on a third charge  $+q_c$  placed at a large distance? (vi) Which of these answers would change if  $q_c$  were brought near the conductor?

2. Two parallel infinite conducting plates at  $x = 0$  and  $x = L$  have potentials  $\Phi_0$  and 0 respectively. Using Poisson's equation with the appropriate boundary conditions, find (i) the electric field between the plates and (ii) the surface charge densities on the plates, when the free volume charge density between the plates is equal to a constant  $k$  ( $k \neq 0$ ).

3. Find the capacitance per unit length of two coaxial metallic cylindrical tubes of radii  $a$  and  $b$ , where  $b > a$ .

4. A point dipole of moment  $\vec{p} = p_0(\hat{e}_x + 2\hat{e}_y + 3\hat{e}_z)$  is placed in an electrostatic potential  $\Phi$  given by

$$\Phi(x, y, z) = \Phi_0 \left[ 1 + \frac{x^2 + y^2 + z^2}{a^2} + \frac{(x^4 + y^4 + z^4)}{a^4} \right]$$

Where  $\Phi_0$  and  $a$  are appropriate constants. Find the force and the couple acting on the dipole when it is located at the point  $(a, a, a)$ . Find also the torque of the force about the origin.

5. A sphere of radius  $R$  carries a charge density  $\rho(r) = kr$  (where  $k$  is a constant). Find the energy of the configuration.
6. Find the monopole, dipole and quadrupole moments of the following charge distributions about the origin. (i) a line charge of constant line charge density  $\lambda_0$  and of length  $L$  lying in the first quadrant of the  $xoy$  plane with one end at the origin making an angle  $\alpha$  with the positive  $x$ -axis. (ii) A spherical shell of radius  $R$  with surface charge density  $\sigma_c = \sigma_0 \cos\theta$ , where  $\sigma_0$  is a constant with its center at the origin.

7. Show that the interaction energy of two dipoles separated by a displacement  $r$  is

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{1}{r^3} [\vec{p}_1 \cdot \vec{p}_2 - 3(\vec{p}_1 \cdot \hat{r})(\vec{p}_2 \cdot \hat{r})]$$

