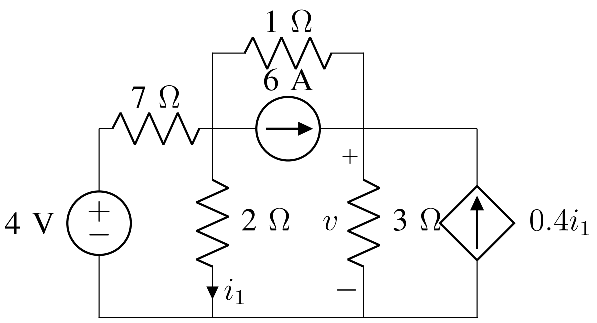
EC2015 Electric Circuits and Networks - Tutorial 6

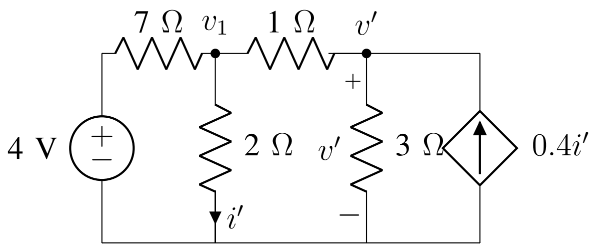
September 20, 2019

***Topics covered*— Superposition theorem, Source transformation theorem, Thevenin theorem, Norton theorem.**

1. Employ superposition to determine the individual contribution from each independent source to the voltage v and i_1 as labeled in the circuit. Compute the power absorbed by the 2 Ohm resistor.



When voltage source alone acting, the circuit will modified as shown below



$$\frac{v_1 - 4}{7} + \frac{v_1}{2} + \frac{v_1 - v'}{1} = 0$$

$$\frac{v' - v_1}{1} + \frac{v'}{3} - 0.4i' = 0$$

here

$$i' = \frac{v_1}{2}$$

After substituting i_1 in above equations can written as,

$$\begin{bmatrix} \frac{1}{7} + \frac{1}{2} + 1 & -1 \\ -1 - 0.2 & \frac{1}{3} + 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \frac{4}{7} \\ 0 \end{bmatrix}$$

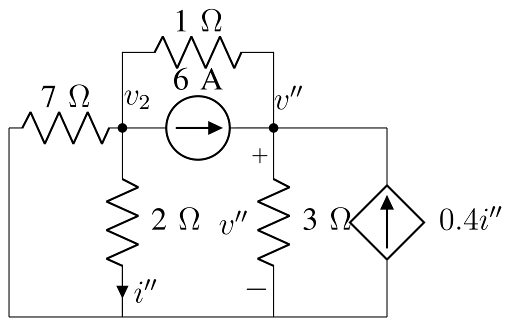
By solving

$$v_1 = 0.769\ V,\ v' = 0.692\ V$$

and

$$i' = \frac{v_1}{2} = 0.385\ A$$

When current source alone acting, the circuit will modified as shown below



$$\frac{v_2}{7} + \frac{v_2}{2} + \frac{v_2 - v''}{1} + 6 = 0$$

$$\frac{v'' - v_2}{1} + \frac{v''}{3} - 0.4i'' - 6 = 0$$

here

$$i'' = \frac{v_2}{2}$$

After substituting i'' in above equations,

$$\begin{bmatrix} \frac{1}{7} + \frac{1}{2} + 1 & -1 \\ -1 - 0.2 & \frac{1}{3} + 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} -6 \\ 6 \end{bmatrix}$$

By solving these equations

$$v_2 = -2.02\ V,\ v'' = 2.68\ V$$

and

$$i'' = \frac{v_2}{2} = -1.01\ A$$

Thefore

$$v = v' + v'' = 0.69 + 2.68 = 3.37\ V$$

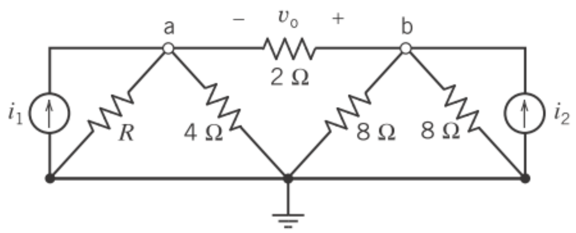
$$i = i' + i'' = 0.385 - 1.01 = -0.625\ A$$

Power dissipated in 2Ω is

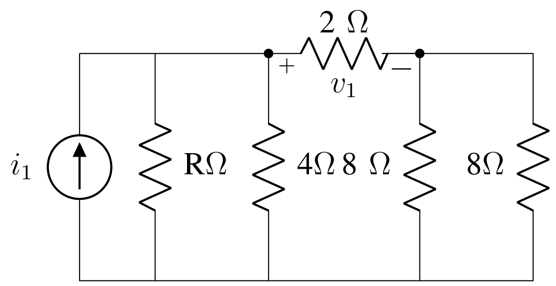
$$P_{2\Omega} = i^2 * 2 = 0.782\ W$$

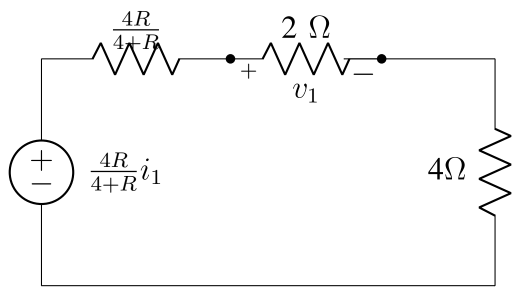
2. For the following circuit, the current source i_2 is used to adjust the relationship between the input and output. Determine values of the current i_2 and the resistance R, that cause the output to be related to the input by the equation

$$v_o = -0.5i_1 + 4$$



When i_1 alone acting, the circuit can be redrawn as below The circuit can be redrawn as





By applying voltage division rule

$$v_1 = \frac{2}{2+4+\frac{4R}{4+R}}\frac{4R}{4+R}i_1 = \frac{8R}{24+10R}$$

When i_2 alone acting, the circuit can be redrawn as below

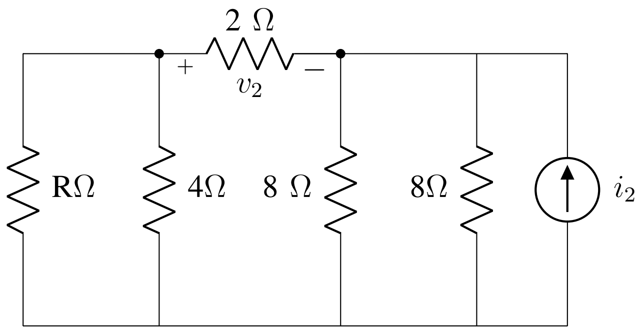
By applying voltage division rule

$$v_2 = \frac{2}{2+4+\frac{4R}{4+R}}i_2 = \frac{32+8R}{24+10R}i_2$$

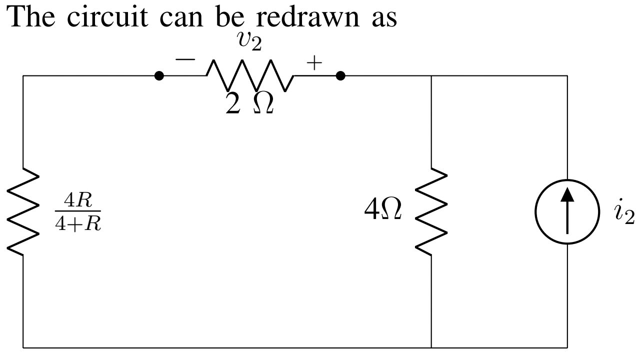
Now

$$v_o = -v_1 + v_2$$

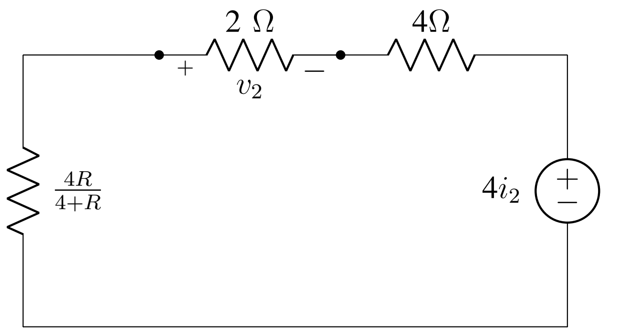
$$-0.5i_1 + 4 = -\frac{8R}{24+10R}i_1 + \frac{32+8R}{24+10R}i_2$$



The circuit can be redrawn as



The circuit can be redrawn as



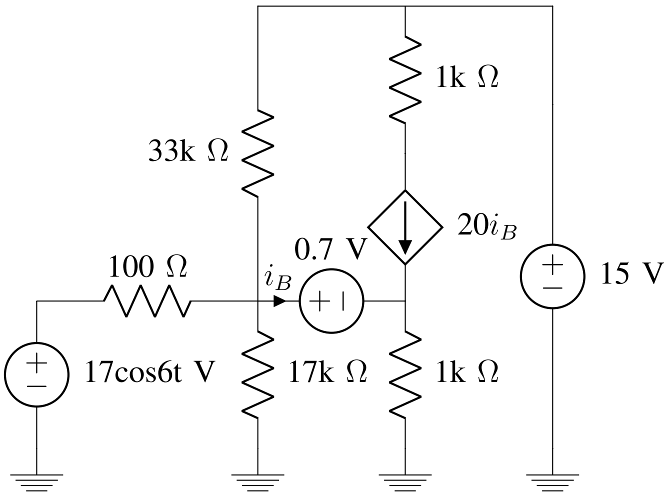
By equating

$$\frac{8R}{24+10R} = 0.5, \ \frac{32+8R}{24+10R}i_2 = 4$$

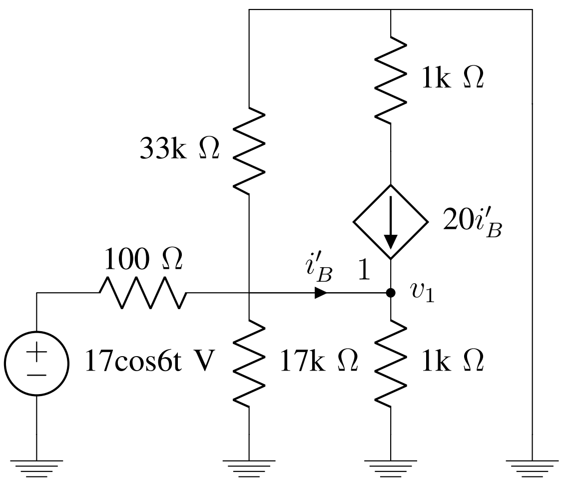
By solving,

$$R = 4\Omega \ and \ i_2 = 4A$$

3. The following circuit is a commonly used as a model for bipolar junction transistor amplifier. Find the value of base current i_B using superposition theorem.



When the 17cos6t source alone acting, the circuit is redrawn as



By writing KCL at node 1:

$$i'_B + 20i'_B = \frac{v_1}{1k}$$

and

$$v_1(\frac{1}{100} + \frac{1}{33k} + \frac{1}{17k} + \frac{1}{1k}) - 20i'_B - \frac{17cos6t}{100} = 0$$

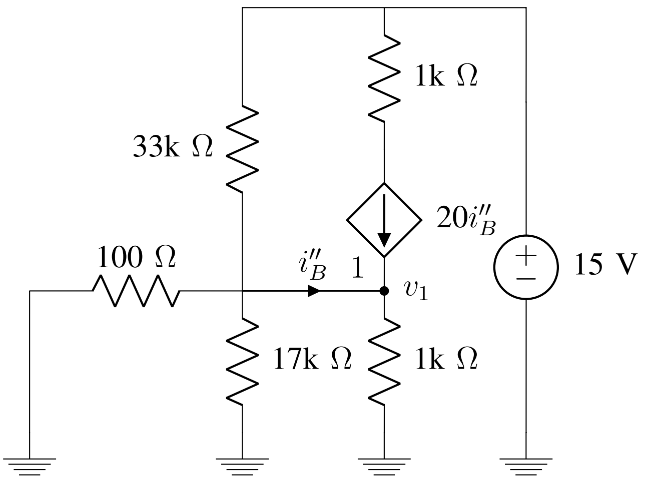
by substituting i'_B in this equation

$$v_1(\frac{1}{100} + \frac{1}{33k} + \frac{1}{17k} + \frac{1}{1k} - \frac{20}{21k}) = \frac{17cos6t}{100}$$

$$v_1 = 16.77cos6t$$

$$i'_B = \frac{v_1}{21k} = 0.7986\ cos6t\ mA$$

When the 15 V source alone acting, the circuit is redrawn as



By writing KCL at node 1:

$$i''_B + 20i''_B = \frac{v_1}{1k}$$

and

$$v_1(\frac{1}{100} + \frac{1}{33k} + \frac{1}{17k} + \frac{1}{1k}) - 20i''_B - \frac{15}{33k} = 0$$

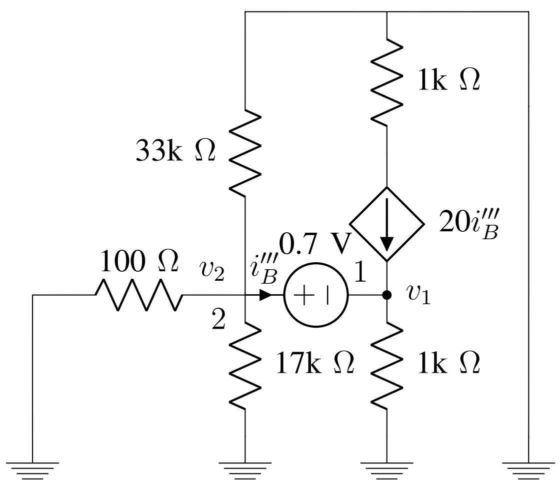
by substituting i''_B in this equation

$$v_1(\frac{1}{100} + \frac{1}{33k} + \frac{1}{17k} + \frac{1}{1k} - \frac{20}{21k}) = \frac{15}{33k}$$

$$v_1 = 44.84mV$$

$$i''_B = \frac{v_1}{21k} = 2.13\ \mu A$$

When the 0.7 V source alone acting, the circuit is redrawn as



By writing KCL at node 1:

$$i_B''' + 20i_B''' = \frac{v_1}{1k}$$

and

$$v_2 - v_1 = 0.7$$

The super node equation is:

$$v_2(\frac{1}{100} + \frac{1}{33k} + \frac{1}{17k}) + v_1(\frac{1}{1k}) - 20i_B''' = 0$$

by substituting i_B''' in this equation

$$v_2(\frac{1}{100} + \frac{1}{33k} + \frac{1}{17k}) + v_1(\frac{1}{1k} - \frac{20}{21k}) = 0$$

By solving

$$v_1 = -0.6967\ V\ and\ v_2 = 0.003\ V$$

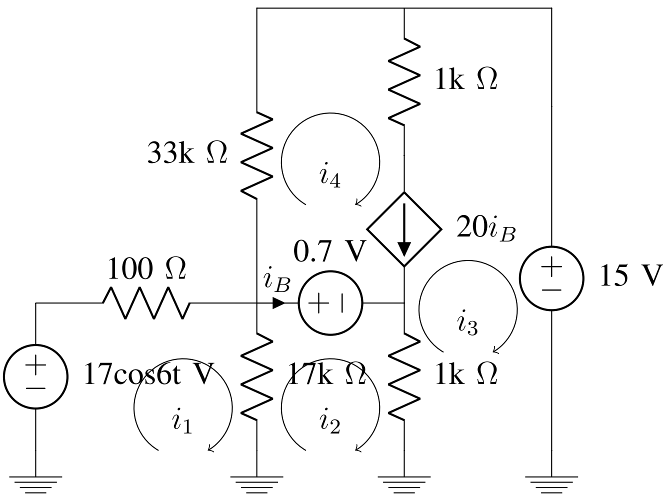
$$i_B''' = \frac{v_1}{21k} = -33.17\ \mu A$$

Now

$$i_B = i_B' + i_B'' + i_B''' = 798.6\ cos6t\ -33.17 + 2.13\mu A$$

$$i_B = 798.6\ cos6t\ -31.04\mu A$$

Alternative method:



Here

$$i_2 - i_4 = i_B$$

$$i_4 - i_3 = 20i_B$$

By applying Kvl to loop 1:

$$-17cos6t + 100i_1 + 17k(i_1 - i_2) = 0$$

By applying Kvl to loop 1:

$$17k(i_2 - i_1) + 0.7 + 1k(i_2 - i_3) = 0$$

By writing super mesh equation:

$$17k(i_2 - i_1) + 33ki_4 + 15 = 0$$

We can write these equation in matrix form as

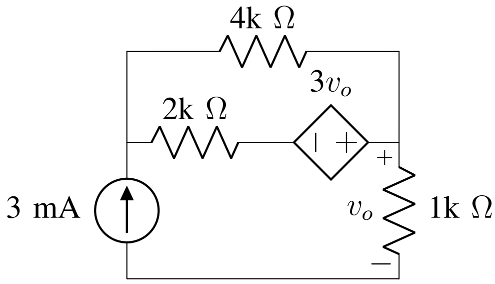
$$\begin{bmatrix} 17.1k & -17k & 0 & 0 & 0 \\ -17k & 18k & -1k & 0 & 0 \\ -17k & 17k & 0 & 33k & 0 \\ 0 & 1 & 0 & -1 & -1 \\ 0 & 0 & -1 & 1 & -20 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \\ i_B \end{bmatrix} = \begin{bmatrix} 17cos6t \\ -0.7 \\ -15 \\ 0 \\ 0 \end{bmatrix}$$

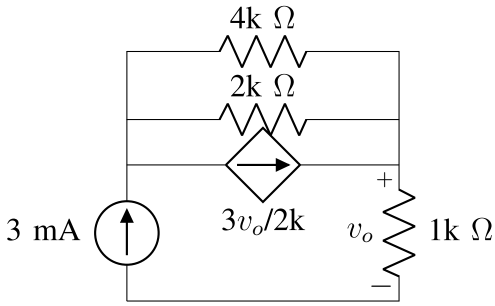
By solving above matrices using Gauss elimination method,

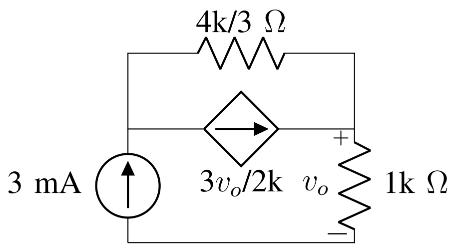
$$i_B = 798.6\ cos6t\ -31.04\mu A$$

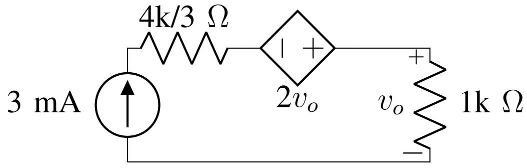
4. For the given circuits here, compute the voltage v_0 (in the 1st circuit) and i_x (in the 2nd circuit) using source transformation technique.

a. By applying source transformation technique on dependent source, the circuit modifies as shown below





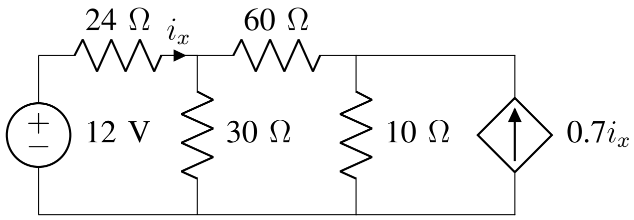




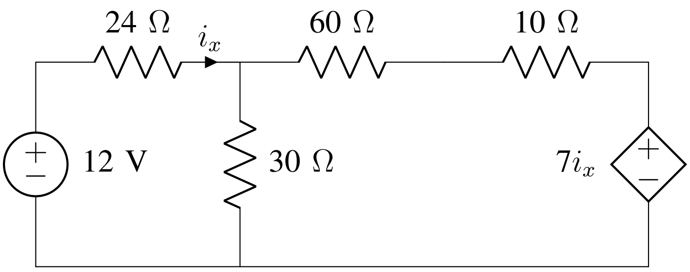
As all elements are in series, the current flowing through 1k Ω resister is 3 mA. Therefore,

$$v_o = 3m * 1k = 3\ V$$

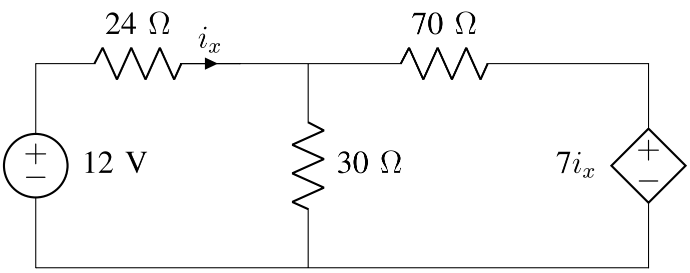
b.



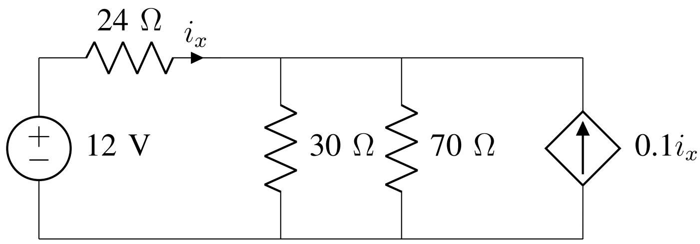
By applying source transformation technique on dependent source, the circuit modifies as shown below. As we need to find the value of i_x. So, the branch which is having i_x is not disturbed throughout the process.



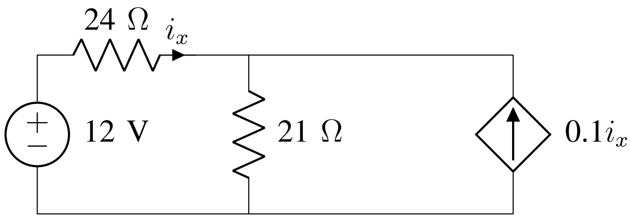
The 60Ω and 10Ω are in series, so these two can replace with 70Ω as shown below



By applying source transformation technique on dependent source, the circuit modifies,



The 70Ω and 30Ω are in parallel, so these two can replace with 21Ω as shown below



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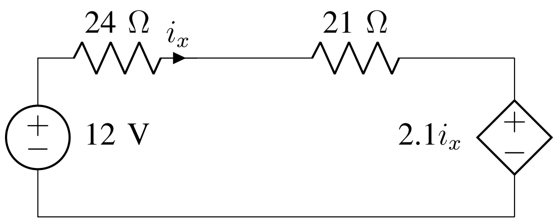

By writing KVL to the loop:

$$-12 + 24i_x + 21i_x + 2.1i_x = 0$$

$$47.1i_x = 12$$

$$i_x = 0.254A$$