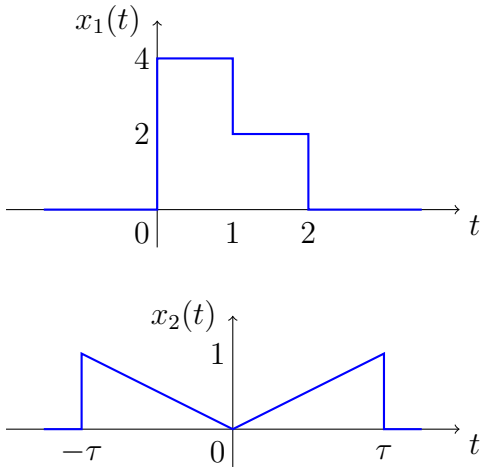
EE1101 Signals and Systems JAN—MAY 2019
Tutorial 7

Note: The Fourier Transform of $y(t) = \text{rect}(\frac{t}{\tau})$ is given as $Y(j\omega) = \tau\,\text{sinc}(\frac{\omega\tau}{2\pi})$, where

$$\text{rect}(\frac{t}{\tau}) = \begin{cases} 1, & |t| \le \frac{\tau}{2} \\ 0, & |t| > \frac{\tau}{2} \end{cases}$$

$$\text{sinc}\,\theta = \frac{\sin \pi\theta}{\pi\theta}$$

1. Find the Fourier Transform of the following signals.



2. Use the Fourier Transform analysis equation to calculate Fourier Transform of the following signals

   (a) $\delta(t+1) + \delta(t-1)$

   (b) $\frac{d}{dt}\{u(t-2) + u(-2-t)\}$

   Sketch and label the magnitude of each Fourier Transform.

3. For each of the following signals $x(t)$, compute the Fourier transform $X(j\omega)$:

   (a) $x(t) = e^{-\frac{|t|}{2}}$

   (b) $x(t) = (\sin 2\pi t) \cdot e^{-t}u(t)$

   For part (a), use the Fourier Transform synthesis equation to verify that the inverse Fourier Tranform of your result yields the given $x(t)$.

4. Consider the signal

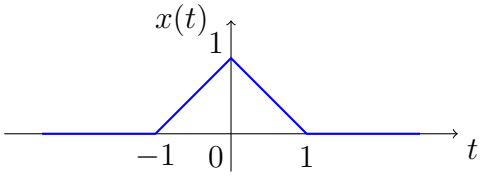
$$x(t) = \begin{cases} 0, & |t| > 1 \\ \dfrac{t+1}{2}, & -1 \le t \le 1 \end{cases}$$

   (a) Determine the Fourier Transform of $x(t)$.

   (b) Sketch the even part of $x(t)$ and find its Fourier Transform. Verify that it is equal to the real part of your answer to part (a).

   (c) What is the Fourier Transform of the odd part of $x(t)$?

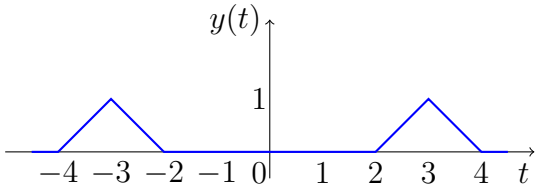
5. (a) Find the Fourier Transform of the signal $x(t)$ shown below.



   (b) For any arbitrary signal $x(t)$, show that if $x(t) \Longleftrightarrow X(j\omega)$, then

$$x(t+T)+x(t-T) \Longleftrightarrow 2X(j\omega)\cos\omega T$$

   (c) Using the above result, find the Fourier Transform of the signal $y(t)$ shown below.



6. (a) Show that if $x(t)$ is an even function of t, then

$$X(j\omega) = 2\int_0^\infty x(t)\cos\omega t dt$$

(b) Show that if $x(t)$ is an odd function of t, then

$$X(j\omega) = -2j \int_0^\infty x(t) \sin \omega t dt$$

7. Use the Fourier Transform synthesis equation to determine the inverse Fourier Transforms of:

(a) $X_1(j\omega) = 2\pi\delta(\omega) + \pi\delta(\omega - 4\pi) + \pi\delta(\omega + 4\pi)$

(b)

$$X_2(j\omega) = \begin{cases} 2 & 0 \leq \omega \leq 2 \\ -2 & -2 \leq \omega < 0 \\ 0 & |\omega| > 2 \end{cases}$$

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