Electric Circuit and Networks EE2015

Tutorial 7

September 27 , 2019

1. a)

$$\begin{aligned} x(t) &= 2\sin 30t \\ &= 2\cos(30t - 90^\circ) \\ \therefore x_{phasor} &= 2\underline{/-90^\circ} \end{aligned}$$

b)

$$\begin{aligned} x(t) =& 2\sin(30t + 2) \\ &= 2\sin(30t + 114.59^\circ) \\ &= 2\cos(30t + 24.59^\circ) \\ \therefore x_{phasor} &= 2\underline{/24.59^\circ} \end{aligned}$$

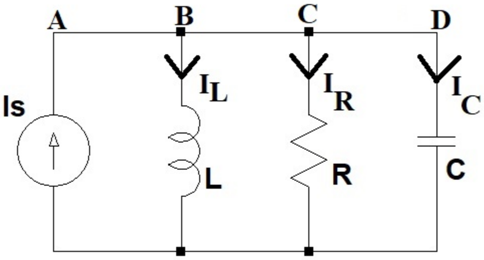
c)

$$\begin{aligned} x(t) &= -2\sin(30t - 150^\circ) + 2\cos(30t - 150^\circ) \\ &= 2\cos(30t - 60^\circ) + 2\cos(30t - 150^\circ) \\ \therefore x_{phasor} &= 2\underline{/-60^\circ} + 2\underline{/-150^\circ} \\ &= 2.828\underline{/-105^\circ} \end{aligned}$$

d)

$$\begin{aligned} x(t) &= 2\sin(-30t + 150^\circ) + 2\sin(30t + 150^\circ) \\ &= 2\cos(30t - 60^\circ) + 2\cos(30t + 60^\circ) \\ \therefore x_{phasor} &= 2\underline{/-60^\circ} + 2\underline{/60^\circ} \\ &= 2\underline{/0^\circ} \end{aligned}$$

2.



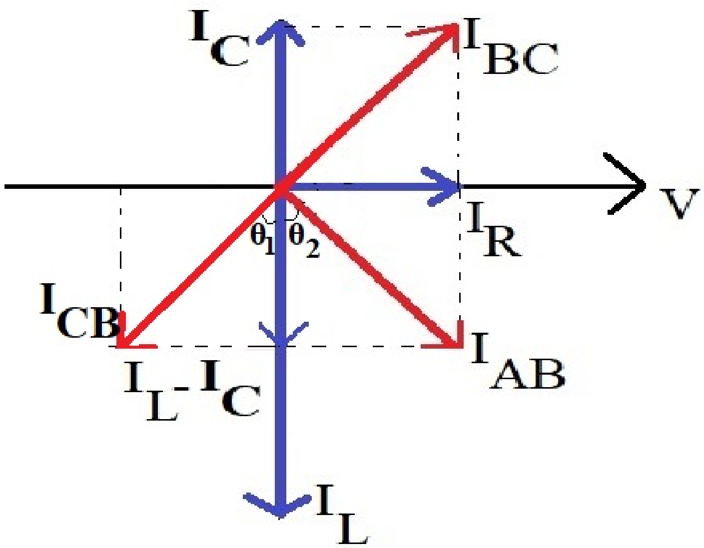


Figure 1: Phasor diagram

$$
\begin{aligned}
{I_{BC}}^2 &= {I_R}^2 + {I_C}^2 \\
5^2 &= {I_R}^2 + 4^2 \\
I_R &= 3\ A
\end{aligned}
$$

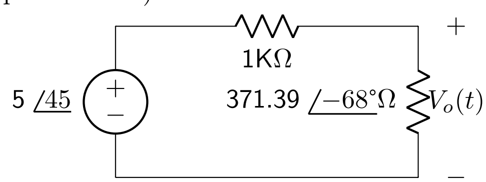
$$
\begin{aligned}
{I_{AB}}^2 \;=\; {I_S}^2 &= {I_R}^2 + (I_L - I_C)^2 \\
5^2 &= 3^2 + (I_L - 4)^2 \\
I_L &= 8\ A\ or\ 0\ A \\
I_L\ cannot\ be\ 0A & \\
I_L &= 8\ A
\end{aligned}
$$

Given the maximum voltage across the parallel combination is 120V

$$
\begin{aligned}
Resistance & \\
R &= \frac{V}{I_R} \;=\; \frac{120}{3} \;=\; 40\Omega \\
Inductive\ Reactance & \\
X_L &= \frac{V}{I_L} \;=\; \frac{120}{8} \;=\; 15\Omega \\
Capacitive\ Reactance & \\
X_C &= \frac{V}{I_C} \;=\; \frac{120}{4} \;=\; 30\Omega
\end{aligned}
$$

$$\theta_1 = tan^{-1}\frac{I_R}{I_C}$$
$$\theta_1 = tan^{-1}\frac{3}{4}$$
$$\theta_1 = 36.869°$$
$$\theta_2 = tan^{-1}\frac{I_R}{(I_L - I_C)}$$
$$\theta_2 = tan^{-1}\frac{3}{4}$$
$$\theta_2 = 36.869°$$
$$Phase\ angle\ between\ I_{AB}\ and\ I_{CB}\ = \theta_1\ +\ \theta_2\ =\ 73.73°$$

3. For the steady state response,the circuit can be redrawn as the figure below (We are solving using the phasor form):

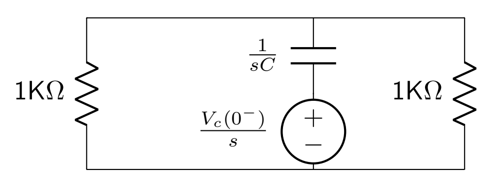


Applying the voltage divider principle,

$$\begin{aligned}
V_o(t) &= V_i(t) \times \frac{Z}{Z+R} \\
&= 5\angle 45° \times \frac{371.39\angle -68°}{371.39\angle -68° + 10^3\angle 0°} \\
&= 1.5617\angle -6.34° \text{ (Steady state response in phasor form)}
\end{aligned}$$

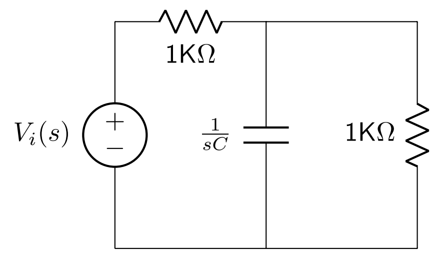
Now, we need to find the natural response, when $V_i(t) = 5\sin(\omega t + \frac{\pi}{4})u(t)$.
$V_o(t)$ = Zero input response + Zero state response

Zero input response: The circuit in S - domain is as follows:



$V_{o_{\text{zero input}}}(s) = \frac{CV_c(0^-)R}{2+sCR}$

Zero state response: The circuit in S - domain is as follows:



$V_{o_{\text{zero state}}}(s) = \frac{5}{\sqrt{2}RC}[\frac{A}{s+\alpha} + \frac{Bs+C}{s^2+\omega^2}]$
Where,

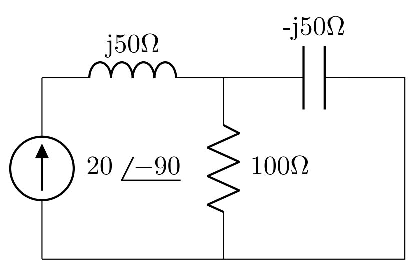
$\alpha = \frac{2}{RC}$, $A = -\frac{\alpha+\omega}{\alpha^2+\omega^2}$

For the natural response to be zero,
$\frac{5}{\sqrt{2}RC}[\frac{A}{s+\alpha}] + \frac{CV_c(0^-)R}{2+sCR} = 0$

On solving, $V_o(0^-) = 1.548V$

4.

$$X_L = j50\Omega$$
$$X_C = -j50\Omega$$



Current flowing through $500mH$ inductor

$$I_L = 20\angle{-90}A$$

Voltage across $500mH$ inductor is

$$V_L = 20\angle{-90} * j50$$
$$= 20\angle{-90} * 50\angle{90}$$
$$= 1000\angle{0}V$$

Current flowing through 100Ω resistor

$$I_R = 20\angle{-90} * \frac{-j50}{100 - j50}$$
$$= 20\angle{-90} * \frac{50\angle{-90}}{111.8\angle{-26.57}}$$

$$= 8.94\underline{/-153.43}A$$

Voltage across 100Ω resistor

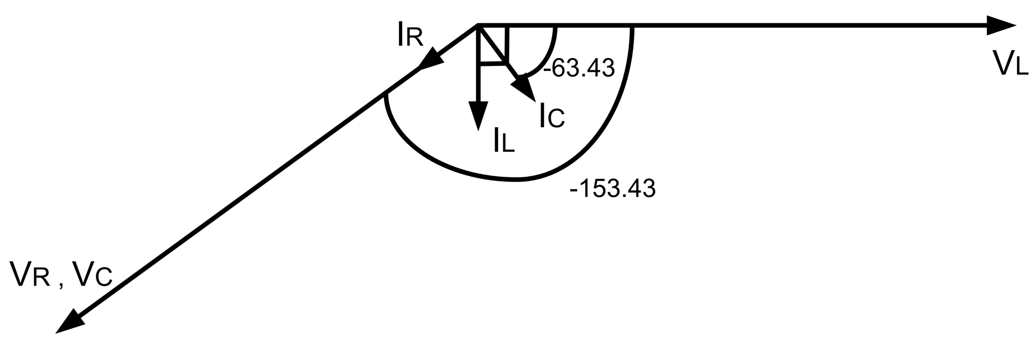
$$V_R = 8.94\underline{/-153.43} * 100\underline{/0}$$
$$= 894\underline{/-153.43}V$$

Current flowing through $200\mu F$ capacitor

$$I_C = 20\underline{/-90} * \frac{100}{100 - j50}$$
$$= 20\underline{/-90} * \frac{100\underline{/0}}{111.8\underline{/-26.57}}$$
$$= 17.88\underline{/-63.43}A$$

Voltage across $200\mu F$ capacitor is

$$V_C = 17.88\underline{/-63.43} * 50\underline{/-90}$$
$$= 894\underline{/-153.43}V$$



5. The question will be reframed. Hence, not incuded here.

6. Let, the box is replaced with its Thevenin equivalent with values V_{th}= V $\underline{/\phi}$ and Z_{th}=a+ib and let, E1 as the reference phasor.
   When the element X is capacitor, X_c= -1000 i

$$So, 100\underline{/0} \quad = \quad V_{th} \ * \frac{X_C}{X_C + Z_{th}} - - - - - (1)$$

When the element X is inductor, X_L= 800 i

$$So, 40\ \underline{/90} \quad = \quad V_{th} \ * \frac{X_L}{X_L + Z_{th}} (since,\ angle\ of\ E_2\ is\ 90\ degrees\ more\ that\ E_1) - - - (2)$$

dividing both equations, we get-

$$0.4\ \underline{/90} \quad = \quad \frac{X_L}{X_C} * \frac{X_C + Z_{th}}{X_L + Z_{th}}$$

Putting the values of X_L, X_C, and replacing Z_{th} by a+ i b and equating the real and imaginary parts, we get-

$$\begin{aligned} a &= 720 \\ b &= 640 \\ so,\ Z_{th} &= 720 + i640 \\ and,\ \frac{X_L}{X_L\ +\ Z_{th}} &= 0.5\ \underline{/26.56} \end{aligned}$$

Equating the magnitude of the equation (2) on both sides, we get-

$$40\ \underline{/90} \quad = \quad (V\ \underline{/\phi}) * (0.5\ \underline{/26.56})$$

Hence, V_{th}=80 $\underline{/63.43}$
Z_{th}=720+i 640 = 963.32 $\underline{/41.63}$
So, I_{sc}=0.083 $\underline{/21.8}$

7. **Solution for first circuit**

Deactivating $10\angle 0°$ source and then writing loop equations

$$20 -\ j2I_1 - 2\,(I_1 - I_2) = 0 \tag{1}$$

$$j2I_2 - 2\,(I_2 - I_1) = 0 \tag{2}$$

$$I_1 = I_2(1 - j)$$

Solving equations (1) and (2)

$$I_2\,(1 + j)\,(1 - j) - I_2 = 10$$

$$I_2 = 10$$

$$I_1 = 10\ \ (1 - j)$$

Deactivating $20\angle 0°$ source and then writing loop equations

$$j2I_1 + 2\,(I_1 - I_2) = 0 \tag{3}$$

$$I_2 = I_1(1 + j)$$

$$-10 + j2I_2 - 2\,(I_2 - I_1) = 0 \tag{4}$$

Solving equations (3) and (4)

$$I_1\,(1 + j)\,(-1 + j) + I_1 = 10$$

$$I_1 = -5$$

$$I_2 = -5\ \ (1 + j)$$

Applying superposition theorem to get I_1 and I_2

$$I_1 = 5 - 10j = 11.18\angle - 63.43°\ A$$

$$I_2 = 5 - 5j = 7.07\angle - 45°\ A$$

Solution for second circuit

Deactivating $5\angle 0$ source and then writing loop equations

$$10\angle 50° -\ \ j45I_1 - 2\,(I_1 - I_2) = 0 \tag{5}$$

$$j100I_2 - 2\left(I_2 - I_1\right) = 0 \tag{6}$$

$$I_1 = I_2(1 - j50)$$

Solving equations (5) and (6)

$$10\angle 50^\circ + 2I_2 = I_1(2 + j45)$$

$$10\angle 50^\circ + 2I_2 = I_2(1 - j50)(2 + j45)$$

$$I_2 = \frac{10\angle 50^\circ}{2250 - j55}$$

$$I_1 = \frac{10\angle 50^\circ}{2250 - j55} \times (1 - j50)$$

Deactivating $10\angle 50$ source and then writing loop equations

$$-\,j45I_1 - 2\left(I_1 - I_2\right) = 0 \tag{7}$$

$$2I_2 = I_1(2 + j45)$$

$$j100I_2 - 2\left(I_2 - I_1\right) - 5 = 0 \tag{8}$$

Solving equations (7) and (8)

$$I_1 = \frac{5}{-2250 + j55}$$

$$I_2 = \frac{2.5}{-2250 + j55} \times (2 + j45)$$

Applying superposition theorem to get I_1 and I_2

$$I_1 = \frac{10\angle 50^\circ}{2250 - j55} \times (1 - j50) - \frac{5}{2250 - j55}$$

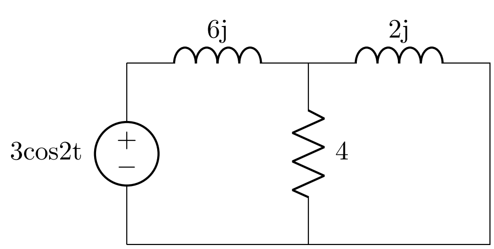
$$I_1 = 220.4\angle - 37.81^\circ \ mA$$

$$I_2 = \frac{2.5}{-2250 + j55} \times (2 + j45) - \frac{10\angle 50^\circ}{-2250 + j55}$$

$$I_2 = 46.5\angle - 87.81^\circ \ mA$$

8. Here the two voltage sources are operated at different frequencies. Therefore, impedance offered by the inductors will not be same for both the frequencies. We need to use superposition theorem by activating only one source at a time such that we know the corresponding impedance offered by the inductors for that particular source at that particular frequency.

   Now, considering only $3cos2t$ acting alone,



   Total impedance offered by all the elements is

$$Z_1 = \frac{4 * j2}{4 + j2} + j6\Omega$$

$$Z_1 = 0.8 + j7.6\Omega = 7.642\angle 84^\circ$$

Therefore, Total current flowing out of the source is

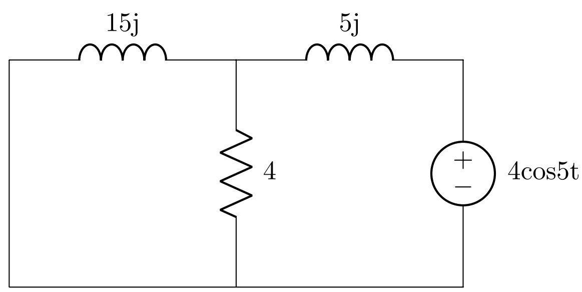
$$I_1 = \frac{3\angle 0^\circ}{7.642\angle 84^\circ}$$

Current flowing through the 4Ω resistor is

$$I_1(4\Omega) = I_1 * \frac{j2}{4+j2}$$

$$I_1(4\Omega) = 0.179\angle -20.97^\circ = 0.179cos(2t - 20.97^\circ)\ A$$

Now, considering only $4cos5t$ acting alone,



Total impedance offered by all the elements is

$$Z_2 = \frac{4 * j15}{4+j15} + j5\Omega$$

$$Z_2 = 3.734 + j6\Omega = 7.067\angle 58.1^\circ$$

Therefore, Total current flowing out of the source is

$$I_2 = \frac{4\angle 0^\circ}{7.067\angle 58.1^\circ}$$

Current flowing through the 4Ω resistor is

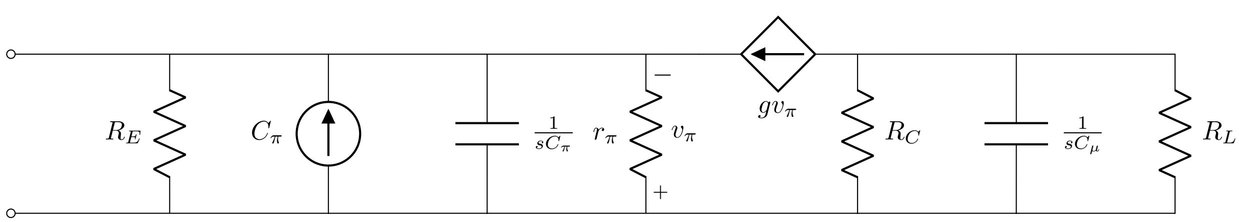
$$I_2(4\Omega) = I_2 * \frac{j15}{4+j15}$$

$$I_2(4\Omega) = 0.547\angle -43^\circ = 0.547cos(5t - 43^\circ)\ A$$

Therefore, total current flowing through 4Ω resistor is

$$0.179cos(2t - 20.97^\circ) + 0.547cos(5t - 43^\circ)\ A$$

9. The s-domain equivalent of the highlighted network is drawn below



Let us assume

$$Z_1 = R_E \parallel \frac{1}{sC_\pi} \parallel r_\pi = \frac{r_\pi R_E}{r_\pi + R_E + sR_E r_\pi C_\pi}$$

and

$$Z_2 = R_C \parallel \frac{1}{sC_\mu} \parallel R_L = \frac{R_C R_L}{R_C + R_L + sR_E R_L C_\mu}$$

Here,

$$V_{oc} = -v_\pi = (C_\pi + gv_\pi) * Z_1$$
$$-v_\pi(1 + gZ_1) = C_\pi Z_1$$
$$v_\pi = -\frac{C_\pi Z_1}{(1 + gZ_1)} = -\frac{C_\pi}{\frac{1}{Z_1} + g} = -\frac{C_\pi}{\frac{1}{R_E} + \frac{1}{r_\pi} + sC_\pi + g}$$

Therefore,

$$V_{oc} = \frac{C_\pi}{\frac{1}{R_E} + \frac{1}{r_\pi} + sC_\pi + g}$$

and

$$I_{sc} = C_\pi + gv_\pi = C_\pi$$
$$\because V_\pi \text{ is zero}$$

$$\therefore R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{\frac{C_\pi}{\frac{1}{R_E} + \frac{1}{r_\pi} + sC_\pi + g}}{C_\pi} = \frac{1}{(\frac{1}{R_E} + \frac{1}{r_\pi} + sC_\pi + g)}$$