Marks	1	2	3	4	5	Total

Please tick yours instructors name
Instructor: Pradeep Sarvepalli /Deepa Venkitesh/Shanti Bhattacharya/Venkatesh Ramaiyan

Department of Electrical Engineering - IIT Madras
EE1101 - Signals and Systems - Quiz I
February 14, 2019

8:00 am - 8:50 am

20 marks

Name:

Roll Number:

Write your answers in the space provided, using correct units, and showing all steps on the question book itself. No extra answer sheets will be given. No marks will be given without steps and clear explanations.

1. Calculate the fundamental period of $x[n] = e^{j\left(\frac{2\pi}{3}\right)n} + e^{j\left(\frac{\pi}{5}\right)n}$. (2)

$x_1[n] = e^{j\left(\frac{2\pi}{3}\right)n}$

$x_2[n] = e^{j\left(\frac{\pi}{5}\right)n}$

Fundamental period of $x_1[n] = \dfrac{2\pi}{\frac{2\pi}{3}} = \underline{\underline{3}}$ ------- $(1/2)$

Fundamental period of $x_2[n] = \dfrac{2\pi}{\frac{\pi}{5}} = \underline{\underline{10}}$ ------- $(1/2)$

$\therefore$ Fundamental period of $x[n] = \text{LCM}(3, 10)$

$= \underline{\underline{30}}$. ------- (1)

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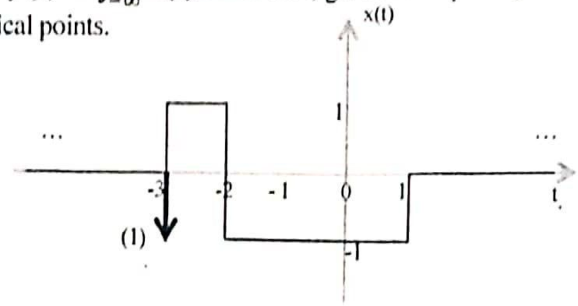
2. Sketch $f(t) = \int_{-\infty}^{t} x(t)dt$ for all t, given the input signal x(t) as shown below. Mark all critical points.



(5)

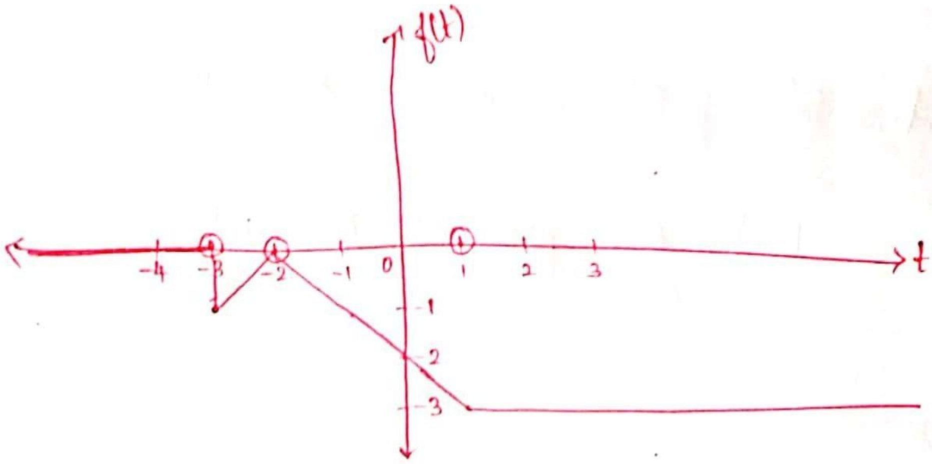
$x(t) = -r(t+3) + u(t+3) - 2u(t+2) + u(t-1)$

$f(t) = -u(t+3) + r(t+3) - 2r(t+2) + r(t-1)$

Case 1 :- If sketch is correct → 5 marks

Case 2 :- If sketch is wrong

based on number of mistakes, marks will be deducted



3. The output of a system y(t) for an input x(t) is given by 5 + cos(2t) x(t). Determine whether the system is linear, causal and stable. (3)

Linearity check : ('1' mark for answer with correct explanation, else '0')

method 1 : check additivity

$x_1(t) \longrightarrow y_1(t) = 5 + \cos(2t)\, x_1(t)$

$x_2(t) \longrightarrow y_2(t) = 5 + \cos(2t)\, x_2(t)$

$x_3(t) = x_1(t) + x_2(t) \rightarrow y_3(t) = 5 + \cos 2t\,(x_1(t) + x_2(t))$

$y_1(t) + y_2(t) = 10 + \cos(2t)\, x_1(t) + \cos(2t)\, x_2(t)$

$\therefore \quad y_3(t) \neq y_1(t) + y_2(t)$

$\therefore$ $\boxed{\text{Non linear}}$

method 2 : check homogenity

$x_1(t) = \alpha\, x(t) \longrightarrow y_1(t) = 5 + \alpha \cos(2t)\, x(t)$

$\alpha\, y(t) = 5\alpha + \alpha \cos(2t)\, x(t)$

$\alpha\, y(t) \neq y_1(t)$

$\boxed{\text{Non linear}}$

method 3 : check superposition (both additivity & homogenity)

method 4 : if $x(t) = 0$

$y(t) = 5$

Zero i/p give non zero o/p.

Non linear

Checking Causality ('1' mark for answer with correct explanation else '0')

$$y(t) = 5 + \cos(2t)\, x(t).$$

$y(t)$ depends on only present value of $x(t)$

$\therefore$ S/m is $\boxed{\text{causal}}$

Checking Stability ('1' mark for answer with correct explanation, else '0')

let $-A \le x(t) \le A$ for Bounded i/p.

$\cos(2t)$ is also bounded

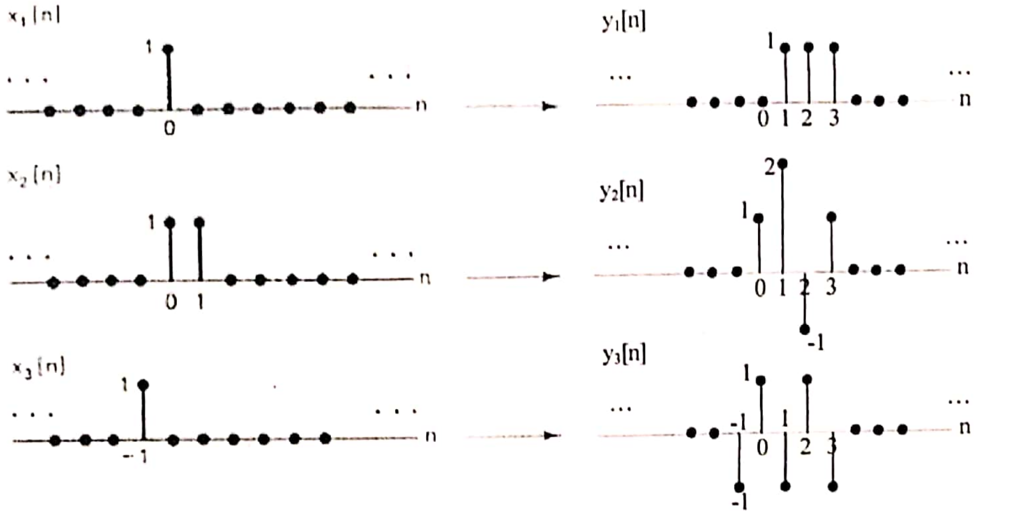
$$-1 \le \cos(2t) \le 1$$

$\therefore$ $y(t) = 5 + \cos(2t)\, x(t)$ is bounded between $5-A$ & $5+A$.

$$5-A \le y(t) \le 5+A$$

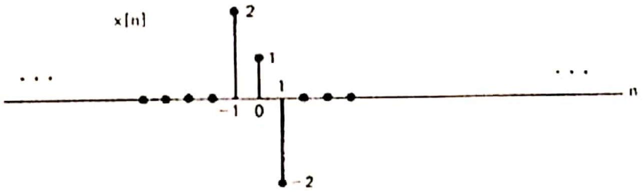
$\therefore$ BIBO $\boxed{\text{stable}}$

4. A discrete-time linear system has the responses $y_1[n]$, $y_2[n]$, and $y_3[n]$ to the inputs $x_1[n]$, $x_2[n]$, and $x_3[n]$, respectively, as illustrated below. (5)



a) Is the system time invariant? Justify.

b) If the input to this system is x[n] (given below), give the expressions for x[n] and y[n]. Sketch y[n]. Explain your answer!



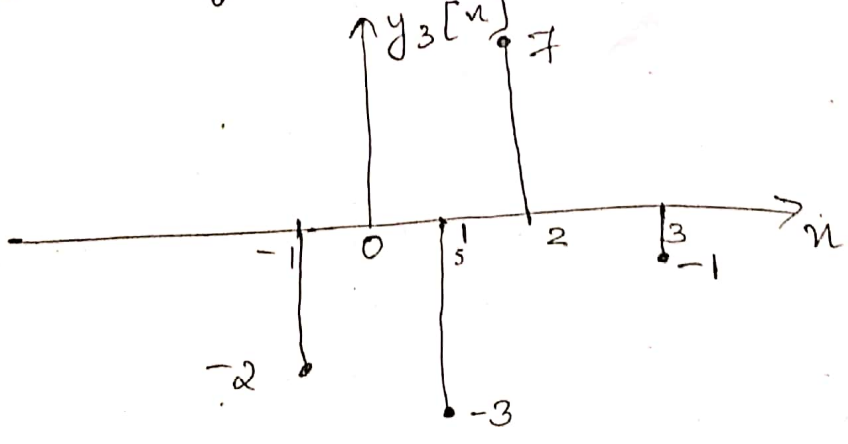
a) system is time variant

$x_3[n] = x_1[n+1]$ — 1 mark

$y_3[n] \neq y_1[n+1]$

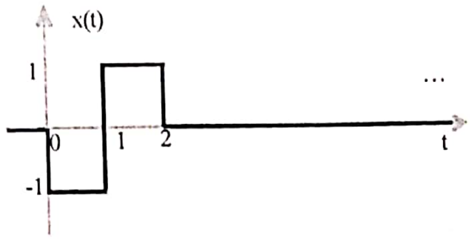
b). $x[n] = 3x_1[n] - 2x_2[n] + 2x_3[n]$ — 1

$y[n] = 3y_1[n] - 2y_2[n] + 2y_3[n]$ — 1

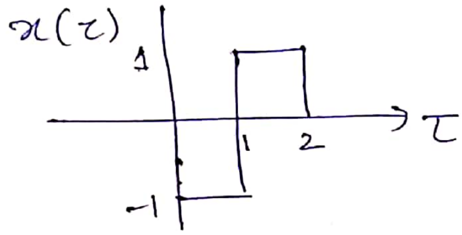


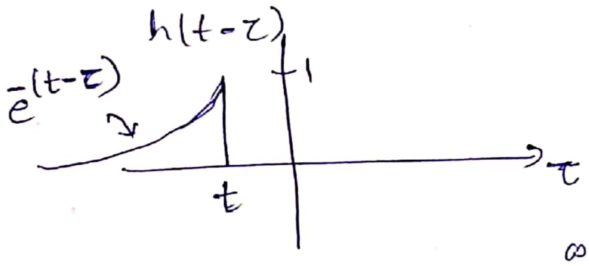
— 2

5. Evaluate and sketch the convolution for an LTI system with impulse response $h(t) = e^{-t}u(t)$ and input x(t), as shown below. Clearly mark all critical points and indicate the form of the function in each range. (5)



Solution:

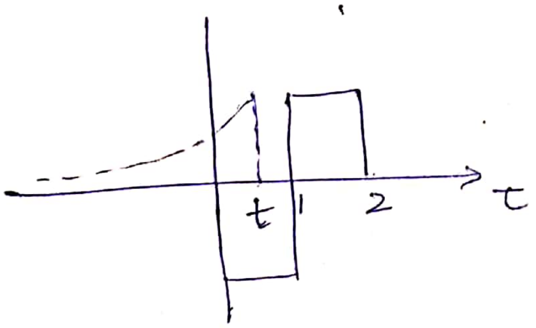




1 Mark

case a) $t < 0$. $y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \int 0\, d\tau = 0.$

case b) $0 \le t \le 1$



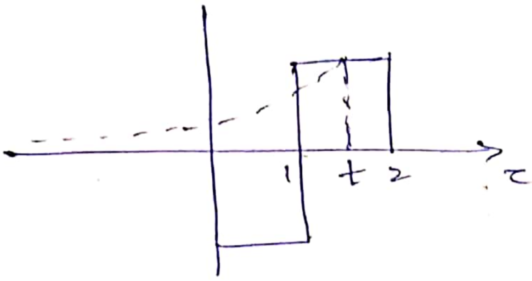
$x(\tau)\, h(t-\tau) \neq 0 \qquad 0 < \tau < t.$

$$y(t) = \int_{0}^{t} (-1) e^{-(t-\tau)}\, d\tau$$

$$= -1 \cdot \left. \frac{e^{-(t-\tau)}}{-(-1)} \right|_{0}^{t} = -1 + e^{-t}.$$

1 Mark

case c: $1 \leq t \leq 2$



$x(\tau)\, h(t-\tau) \neq 0 \qquad 0<\tau<t$

$$y(t) = \int_0^1 (-1)\, e^{-(t-\tau)}\, d\tau + \int_1^t (1)\, e^{-(t-\tau)}\, d\tau$$

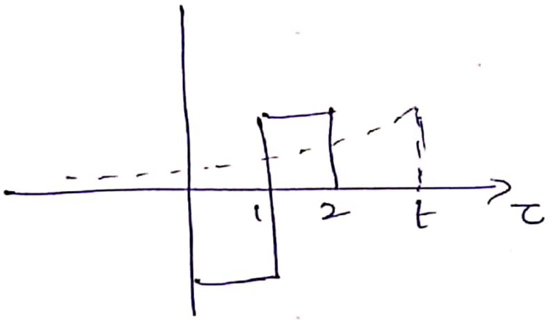
$$= -e^{-t}\; e^{\tau}\Big|_0^1 + e^{-t}\; e^{\tau}\Big|_1^t$$

$$= -e^{-t}\left[e-1\right] + e^{-t}\left[e^{t}-e\right]$$

$$= 1+e^{-t}-2e^{-t+1}$$

1 Mark

case d: $t > 2$



$x(\tau)\, h(t-\tau) \neq 0 \qquad 0<\tau<2$

$$\therefore\ y(t) = \int_0^1 (-1)\, e^{-(t-\tau)}\, d\tau + \int_1^2 (1)\, e^{-(t-\tau)}\, d\tau$$

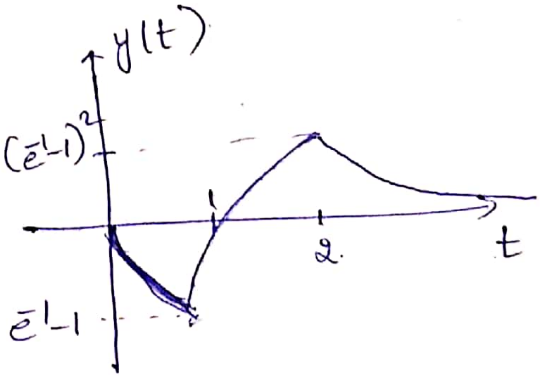
$$= -e^{-t}\left[e^{\tau}\Big|_0^1 \right] + e^{-t}\; e^{\tau}\Big|_1^2$$

1 Mark

$$= -e^{-t}\left[e-1\right] + e^{-t}\left[e^{2}-e\right]$$

$$= e^{-t} - 2e^{-t+1} + e^{-t+2}$$

$$\therefore\ y(t) = \begin{cases} 0 & t<0 \\ -1+e^{-t} & 0\leq t\leq 1 \\ 1+e^{-t}-2e^{1-t} & 1\leq t\leq 2 \\ e^{-t}-2e^{1-t}+e^{2-t} & t>2 \end{cases}$$



1/2 — Graph.

1/2 — Critical Points