

Department of Physics
Indian Institute of Technology, Madras

PH1020 Physics II

Problem set 2

1. Two equal and opposite charges ($+q$ and $-q$) separated by a distance d constitute a dipole. Assume the charges to be situated on the z -axis so that its coordinates are $(0, 0, +d/2)$ and $(0, 0, -d/2)$.
 - (i) Calculate the electrostatic potential at an arbitrary point $P(x, y, z)$ and show that at large distance from the origin the potential is given by $V = (p \cos \theta) / 4\pi\epsilon_0 r^2$, where $p = qd$ dipole moment and θ is the polar angle.
 - (ii) Calculate the corresponding electric field in Cartesian coordinates. Plot the electric field and equipotential surfaces.
2. A sphere of radius R_1 has uniform charge density ρ within its volume, except for a small spherical hollow region of radius R_2 located at a distance a from the center (center to center).
 - (i) Find the electric field at the center of the hollow region. (ii) Find the potential at the same point.
3. The total charge within a sphere of radius r is given by $q \frac{r^2}{a^2} e^{-r/a}$, where a is real constant of appropriate dimensions. Determine: (i) the electric potential (ii) the charge density.
4. Find the electric potential on the axis of a charged ring of radius a lying in the xoy plane when the charge density on the ring varies as $\rho = \alpha(1 + \sin\phi)\delta(r - a)\delta(z)$, where α is real constant of suitable dimensions.
5. Consider a situation with cylindrical symmetry, where the potential is independent of the coordinates ϕ and z i.e, $\Phi = \Phi(\rho)$. Show that the most general solution to Laplace's equation in this case takes the form $\Phi = A \log \rho + B$ for suitable constants A and B . Determine A and B using the following boundary conditions: $\Phi(\rho)|_{\rho=a} = V_0$ and $\Phi(\rho)|_{\rho=b} = 0$ for $a > b > 0$. Take the region of interest to be $b \leq \rho \leq a$. Compute the electric field for this electrostatic potential. Plot lines of equipotential as well as electrical lines of force in the plane $z = 0$.
6. Consider the situation where two (infinite) metal plates are parallel to each other and separated by a distance d . One plate is grounded and the other is kept at V_0 . Choose the direction of the separation to be the y -axis. Solve Laplace's equation in the region between the two plates subject to the given boundary conditions. Compute the electric field for this electrostatic potential. Plot lines of equipotential as well as electrical lines of force.