Department of Physics
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PH1020 Physics II Problem set 8

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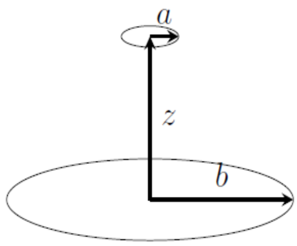
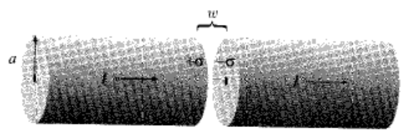
1. An infinitely long straight wire located along the *z*-axis carries a current *I* in the positive z-direction. A square wire loop of side L lies in the *yz*-plane, with its centre at (0, *d*, 0) (where $d > L/2$), and its sides parallel to the *y* and *z* axes. (a) If the magnetic flux Φ_m through the square loop can be written as $\Phi_m = M I$, find the value of the constant M. (b) If the current through the wire has a time-dependence given by I = $I_0 e^{-\alpha t}$ where I_0 and α are positive constants, find the direction and the value of the emf induced in the square loop.

2. A toroidal coil of rectangular cross-section with inner radius *a* and outer radius *b* has height *h* and *n* turns. If a current *I* flows through its windings, find the magnetic flux Φ_m and hence the self-inductance of the toroid.

3. A small circular loop of wire (of radius *a*) lies at a distance *z* above the centre of a larger circular loop (of radius $b \gg a$). The planes of the loop are parallel to each other and perpendicular to the common axis of symmetry (see figure). (a) Suppose a current *I* flows in the larger loop. Determine the magnetic flux through the smaller loop. (Assume that the field across the smaller loop is uniform.) (b) Suppose a current *I* flows in the small loop. Determine the magnetic flux through the big loop. (Assume the small loop as a pointmagnetic dipole.)(c) Find the mutual inductance.



4. A fat wire, radius, *a*, carries a constant current *I*, uniformly distributed over its cross section. A narrow gap in the wire, of width $w \ll a$, forms a parallel-plate capacitor.as shown in Figure. Find the magnetic field in the gap, at a distance $s < a$ from the axis.



5. If an alternating field $\vec{E} = \vec{E}_0 cos\omega t$, where $\vec{E}_0$ is a constant vector, is applied to a conductor, show that the displacement current is negligible compared to the conduction current at any frequency lower than optical frequencies. For a good conductor $\varepsilon_r \approx 1$and conductivity σ=10^7 mhos/m.

6. If constant current charges a large parallel plate capacitor, show that the displacement current will be given by $I_d = C\frac{dV}{dt}$ and is equal to the conduction current. (Hint: $I_d = |\vec{J}_d|A$, where A is the cross-sectional area of the capacitor).

7. We know, $\vec{D} = \epsilon\vec{E}$, $\vec{B} = \mu\vec{H}$, $\vec{J}_f = \sigma\vec{E}$. For a metal under normal circumstances, J_f is much larger than $\frac{\partial\vec{D}}{\partial t}$. (a) Neglect ρ_f and show that for metal, **E** satisfies the equation $\nabla^2\vec{E} = \mu\sigma\frac{\partial\vec{E}}{\partial t}$. (b) Consider a “plane wave” solution of the above equation of the form $\vec{E} = \vec{E}_0 \exp i(kz - \omega t)$, for z>0. Find the allowed values of the wave number k as a function of the frequency ω. (c) Interpret the form of the solution. How does the amplitude of the electric field vary with k, and at what distance does it decay to 1/e of its value at z=0?