Department of Physics
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PH1020 Physics II Problem set 4

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1. A cylinder of length $2L$ and radius a is centred at the origin, with the z-axis as its symmetry axis. The cylinder is uniformly polarized with polarization $\mathbf{P} = P_0\hat{e}_z$ where P_0 is a constant. (i) Find the bound charge densities ρ_b and σ_b. (ii) Find the electric field at all points on the positive z-axis, and verify that it satisfied the appropriate boundary condition at $z = L$ (iii) Find the electric field at the origin, and sketch its magnitude as a function of ratio a/L.

2. Consider a uniform spherical free charge distribution of radius a and charge density ρ_0. This region is filled with a medium of dielectric constant K_1, and surrounded by a medium of dielectric constant K_2. Find (i) the bound volume charge density everywhere in space, and (ii) the bound surface charge density on the surface of the sphere.

3. A capacitor is formed of two concentric conducting spheres of radii a and b ($a < b$), and the space between is filled with a substance. The dielectric constant of the substance at a distance r from the centre is $\frac{c+r}{r}$, where c is constant. The outer sphere is earthed and the inner sphere is charged. Calculate the capacitance of the system.

4. An insulated spherical conductor in air carries a charge q. The conductor is now surrounded by a concentric spherical shell of dielectric of radii b and c, ($c > b$), whose dielectric constant is a function $k(r)$ of the radial distance r from the centre. Calculate the electrostatic energy.

5. A dielectric sphere of radius R contains a uniform distribution of free charge with charge density ρ_f. Find the potential at the centre of the sphere.

6. A sphere of radius R and dielectric constant k, centered at the origin of coordinates, is placed in a constant field E_o directed along the z-axis. The corresponding electrostatic potential is given by $\emptyset(r,\theta,\varphi) = (-E_o r + b_1 r^{-2})\cos\theta$ outside the sphere, and $\emptyset(r,\theta,\varphi) = (b_2 r)\cos\theta$ inside the sphere. Find (i) the constants b_1 and b_2, in terms of k, E_o and R, (ii) the electric field at all points in space, (iii) the polarization $\boldsymbol{P}$ of the sphere, and the dipole moment of the sphere about the origin, (iv) the volume and surface densities of the bound charge in the sphere.