1. In the boxes below provided clearly indicate your answers to each of the questions. (Each box carries 1 **mark).**

a) A cylinder of length $2L$ and radius L centred at the origin with its axis oriented along the z-axis has a magnetization $\vec{M} = k\,\rho^2\,\hat{z}$, where k is positive constant of appropriate dimension. Assume there are no free currents.

i. The bound surface current density on the curved surface of the cylinder is

$$\vec{K}_b = kL^2\,\hat{e}_\phi$$

ii. The bound volume current density in the cylinder is

$$\vec{J}_b = -2k\rho\,\hat{e}_\phi$$

iii. Magnetic dipole moment of the cylinder is

$$\vec{m} = \pi k L^5\,\hat{e}_z$$

iv. Magnetic field $\vec{B}\,(r)$ for $r >> 2L$ is

$$\vec{B}(r) = \frac{\mu_0 k L^5}{4r^3}\left[2\cos\theta\,\hat{e}_r + \sin\theta\,\hat{e}_\theta\right]$$

or $\frac{\mu_0 k L^5}{4r^3}\left[3(\hat{e}_r\cdot\hat{e}_z)\hat{e}_r - \hat{e}_z\right]$

b) A thin insulating rod of length L lying on the x-y plane has line charge density $\lambda = k\rho^2$ glued to it. The rod has one end fixed at origin and rotates with angular velocity $\omega_0\,\hat{z}$.

i. The current due to infinitesimal line element of the rod is

$$\frac{\omega_0 k\rho^2}{2\pi}\,d\rho\,\hat{e}_\phi$$

ii. The magnetic field at the origin is

$$\frac{\mu_0\omega_0 k L^2}{8\pi}\,\hat{e}_z$$

c) Direction of vector potential for a current carrying infinite solenoid with its axis along z-axis is

$\hat{e}_\phi$ or $-\hat{e}_\phi$

$\frac{1}{2}$ mark penalty for not indicating direction.

Q.1 (a) (i) $\vec{K}_b = \vec{M} \times \hat{n} = kL^2 \hat{e}_3 \times \hat{e}_\rho = \boxed{kL^2 \hat{e}_\phi}$

(ii) $\vec{J}_b = \vec{\nabla} \times \vec{M} = -e_\phi \dfrac{\partial M_3}{\partial \rho} = \boxed{-2k\rho \hat{e}_\phi}$

(iii) $\vec{m} = \int \vec{M}\, d\tau = \int_{-L}^{L}\int_0^{2\pi}\int_0^{L} k\rho^2\, \rho\, d\rho\, d\phi\, dz\ \hat{e}_3$

$= 2\pi . k . 2L . \dfrac{\rho^4}{4}\Big|_0^L \hat{e}_3 = \boxed{\pi k L^5 \hat{e}_3}$

(iv) $\vec{B} = \dfrac{\mu_0}{4\pi r^3}\left[3(\vec{m}.\hat{r})\hat{r} - \vec{m}\right]$

$= \dfrac{\mu_0 m}{4\pi r^3}\left[3\cos\theta\, \hat{e}_r - (\cos\theta\, \hat{e}_r - \sin\theta\, \hat{e}_\theta)\right]$

$= \boxed{\dfrac{\mu_0 k L^5}{4 r^3}\left(2\cos\theta\, \hat{e}_r + \sin\theta\, \hat{e}_\theta\right)}$

(b) (i) $dI = \dfrac{dq}{T} = k\rho^2 d\rho . \dfrac{\omega_0}{2\pi} = \boxed{\dfrac{\omega_0 k \rho^2 d\rho}{2\pi} \hat{e}_\phi}$

(ii) $\vec{K} = \dfrac{dI}{dl_\perp} = \dfrac{\omega_0 k \rho^2}{2\pi} \hat{e}_\phi$

$\vec{B} = \dfrac{\mu_0}{4\pi}\int \dfrac{\vec{K} \times \hat{r}}{r^2}\, da = \dfrac{\mu_0}{4\pi}\dfrac{\omega_0 k}{2\pi}\int_0^L\int_0^{2\pi} \dfrac{(\rho^2 e_\phi) \times (-\hat{e}_\rho)}{\rho^2}\, \rho\, d\rho\, d\Phi$

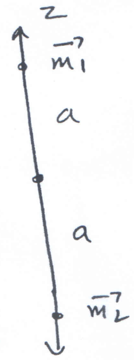
$= \boxed{\dfrac{\mu_0 \omega_0 k L^2}{8\pi} \hat{e}_3}$

2. In the box provided, indicate whether the following statements are **True** or **False**.

[4 × 1 = 4 marks]

(a) The equation $\vec{\nabla}.\vec{B} = 0$ implies that magnetic charges do not exist.	TRUE
(b) When the magnetic field ($\vec{B}$) crosses a boundary carrying a surface current, the component of magnetic field normal to the surface is discontinuous.	FALSE
(c) In a linear homogenous medium $\vec{J}_b = \chi_m \vec{J}_{free}$.	TRUE
(d) The magnitude of a magnetic field within a long solenoid filled with a diamagnetic materials is greater than that inside an identical solenoid filled with a paramagnetic material.	FALSE

3. Two magnetic dipoles $\vec{m}_1 = m_0\,\hat{z}$ and $\vec{m}_2 = -\,m_0\,\hat{z}$ are placed at (0, 0, a) and (0, 0, -a), respectively (Cartesian coordinates). Evaluate the vector potential and the magnetic field (the leading order) on the z-axis for a point z, where $z >> a$. (3 Marks)



$$\vec{A}(r) = \frac{\mu_0}{4\pi}\,\frac{\vec{m}\times\hat{e}_r}{r^2}$$

$$\vec{m} = \pm\, m_0\,\hat{e}_z \;;\; \hat{e}_r = \hat{z} \;,\; \vec{A} = 0$$

$\vec{A} = 0$ ON z- axis → 1

$$\vec{B}_{dip} = \frac{\mu_0}{4\pi}\left[\frac{3(\vec{m}\cdot\hat{e}_r)\,\hat{e}_r - \vec{m}}{r^3}\right] \rightarrow \frac{1}{2}$$

From m_1 $\quad \vec{B}_{dip} = \frac{\mu_0}{4\pi}\left[\frac{2m_0}{(z-a)^3}\right]\hat{e}_z \rightarrow \frac{1}{2}$

m_2 $\quad \vec{B}_{dip} = \frac{\mu_0}{4\pi}\left(\frac{-2m_0}{(z+a)^3}\right)\hat{e}_z$

Net field $\quad \vec{B}_{dip} = \frac{\mu_0 m_0}{2\pi}\left[\frac{1}{(z-a)^3} - \frac{1}{(z+a)^3}\right]\hat{e}_z \rightarrow \frac{1}{2}$

$$= \frac{\mu_0 m_0}{2\pi z^3}\left[\frac{1}{(1-\frac{a}{z})^3} - \frac{1}{(1+\frac{a}{z})^3}\right]\hat{e}_z .$$

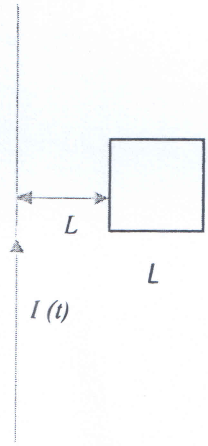
$$\vec{B}_{Net} = \frac{\mu_0 m_0}{2\pi z^3}\left[1 + \frac{3a}{z} - 1 + \frac{3a}{z}\right] = \frac{3\mu_0 m_0 a}{\pi z^4} \rightarrow \frac{1}{2}$$

4. A square loop of side L and resistance R lies at a distance L from an infinite straight wire that carries time varying current $I\,(t)$ as follows (see fig.)

$$I(t) = \begin{cases} (1-\alpha t)I_0 & \text{for } 0 \le t \le {}^1/_\alpha \\ 0 & \text{for } t > {}^1/_\alpha \end{cases}$$

a) In what direction does the induced current flow in the loop?

b) Calculate the total charge that passes through any given point in the loop during the time this current flows? (3 Marks)



$$\Phi = \int \vec{B}\cdot d\vec{a} = \frac{\mu_0 I}{2\pi}\int_{-L/2}^{L/2}\int_{L}^{2L}\frac{d\rho}{\rho}\,dz$$

$$= \frac{\mu_0 I L \ln(2)}{2\pi} \longrightarrow \frac{1}{2}$$

$$\mathcal{E} = I_{loop}R = \frac{dQ}{dt}R = -\frac{d\phi}{dt} = -\frac{\mu_0 L \ln(2)}{2\pi}\frac{dI}{dt} \rightarrow 1$$

$$dQ = \frac{-\mu_0 L \ln(2)\, dI}{2\pi R}$$

$$\therefore\ Q = \frac{I_0 \mu_0 L \ln(2)}{2\pi R} \qquad 1/2$$

Current in clockwise — 1

5. Assume that the magnetic field of earth arises from a point dipole at its centre and is directed towards the North Pole. Determine the moment of this dipole, given that field at the North Pole is 6×10^{-5} Tesla. Consider the earth as sphere with radius 5×10^{6} meter and $\frac{\mu_0}{4\pi} = 1 \times 10^{-7}$ Newton/Ampere². (3 Marks)

$$\vec{B}_{dip} = \frac{\mu_0}{4\pi r^3}\left[3(\vec{m}\cdot\hat{e}_r)\hat{e}_r - \vec{m}\right]$$

$$= \frac{\mu_0 m}{4\pi r^3}\left(2\cos\theta\,\hat{e}_r + \sin\theta\,\hat{e}_\phi\right) \rightarrow \frac{1}{2}$$

$$\theta = 0 \qquad = \frac{2\mu_0 m}{4\pi r^3}\,\hat{e}_r \qquad - \frac{1}{2}$$

$$\frac{\mu_0}{4\pi}\,\frac{2m}{r^3} = 6 \times 10^{-5}$$

$$\frac{10^{-7} \times 2m}{(5 \times 10^{6})^3} = 6 \times 10^{-5} \qquad 1$$

$$\vec{m} = 3 \times 10^{-5} \times 5^3 \times 10^{18} \times 10^{7}$$

$$= \underline{3.75 \times 10^{22}} \text{ Amp. mt}^2 \qquad 1$$

$\hat{e}_z$ or $\hat{e}_r$