EE1101 Signals and Systems JAN—MAY 2018
Tutorial 8
April 2, 2018

1. (a) If $x(t) \longleftrightarrow X(j\omega)$, then show that

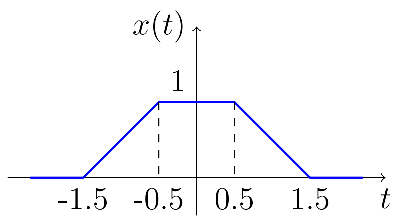
$$X(0) = \int_{-\infty}^{\infty} x(t)dt$$

   (b) Given $\operatorname{sinc} t = \dfrac{\sin \pi t}{\pi t}$, show that $\int\limits_{-\infty}^{\infty} \operatorname{sinc} t \; dt = \int\limits_{-\infty}^{\infty} \operatorname{sinc}^2 t \; dt = 1$

2. Given two signals $f_1(t) = 2\,\mathrm{rect}(t/4)$ and $f_2(t) = \mathrm{rect}(t/2)$
   (a) Sketch $g(t) = f_1(t) \star f_2(t)$
   (b) Evaluate $G(j\omega)$, the fourier transform of $g(t)$.
   (c) Sketch the magnitude and phase of $G(j\omega)$

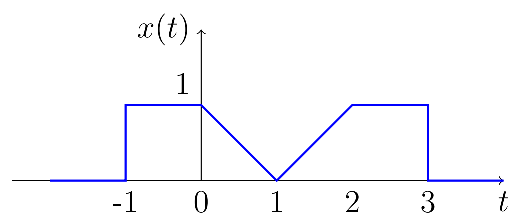
$$\textit{Note}: \ \mathrm{rect}\left(\frac{t}{\tau}\right) = \begin{cases} 1 & , |t| \leq \frac{\tau}{2} \\ 0 & , |t| > \frac{\tau}{2} \end{cases}$$

3. Using properties, find the Fourier transform $X(j\omega)$ of the signal $x(t)$ shown below.

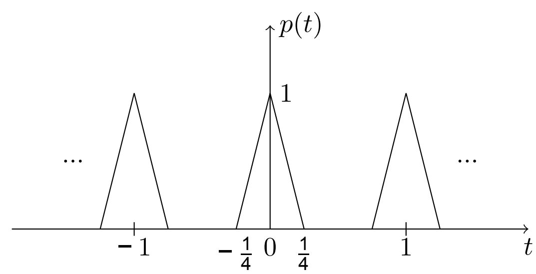


4. Determine the signal $x(t)$ corresponding to the following Fourier transforms:
   (a) $X(j\omega) = \dfrac{2\sin[3(\omega - 2\pi)]}{(\omega - 2\pi)}$
   (b) $X(j\omega) = \cos\left(4\omega + \dfrac{\pi}{3}\right)$

5. Let $X(j\omega)$ be the Fourier transform of the signal $x(t)$ shown below. Do the following computations without explicitly evaluating $X(j\omega)$.
   (a) Find $X(0)$
   (b) Evaluate $\int\limits_{-\infty}^{\infty} X(j\omega)d\omega$
   (c) Evaluate $\int\limits_{-\infty}^{\infty} X(j\omega)e^{j\omega}d\omega$
   (d) Evaluate $\int\limits_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$
   (e) Sketch the inverse Fourier transform of $Re\{X(j\omega)\}$.

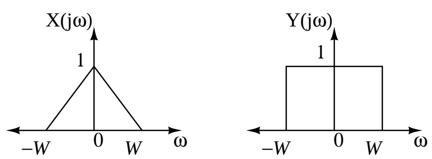


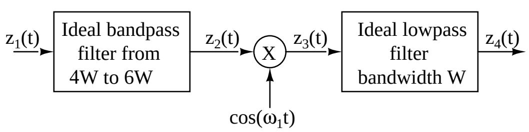
6. Let $p(t)$ be periodic triangular pulse train shown below.



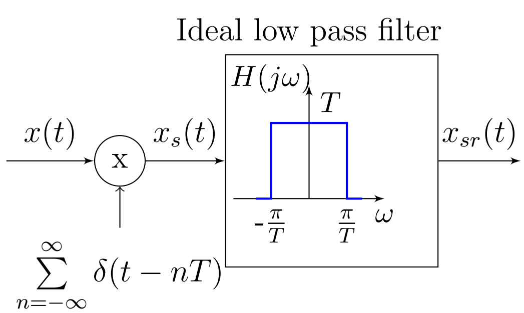
   (a) Calculate P_n, the exponential Fourier series coefficients of $p(t)$ and sketch P_n vs. n. Calculate $P(j\omega)$, the Fourier transform of $p(t)$. Sketch $P(j\omega)$ vs. ω.
   (b) Let $x(t)$ be an aperiodic signal having Fourier transform $X(j\omega)$. Define $y(t) = p(t) \cdot x(t)$. Find an expression for $Y(j\omega)$, the Fourier transform of $y(t)$.
   (c) Let $x(t) = \mathrm{sinc}(t)$. Sketch $Y(j\omega)$.

7. $x(t)$ and $y(t)$ have Fourier transforms as shown below. Sketch the Fourier transform of the various signals $z_i(t)$ for $i = 1$, 2, 3, 4 in the system shown below given that $z_1(t) = x(t)\cos(\omega_1 t) + y(t)\cos(\omega_2 t)$. Determine $z_4(t)$ in terms of $x(t)$ and $y(t)$? Assume that $\omega_1 = \omega_2 - 2W = 5W$.



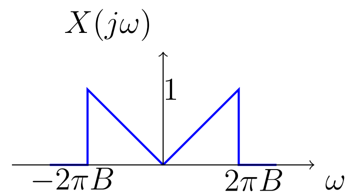


8. In the sampling process, $x(t)$, the signal to be sampled is multiplied by the impulse train, yielding the sampled signal $x_s(t)$. In order to perform reconstruction, we pass $x_s(t)$ through the ideal low pass filter, yielding the reconstructed signal $x_{sr}(t)$.



(a) Write $X_s(j\omega)$, the Fourier transform of $x_s(t)$ in terms of $X(j\omega)$, the Fourier transform of $x(t)$. Your answer should be an explicit, closed form expression, not one involving a convolution integral.

(b) Let $x(t)$ have the Fourier transform shown.



Sketch $X_s(j\omega)$ for the cases (i) $T = \dfrac{1}{4B}$ (ii) $T = \dfrac{1}{2B}$ (iii) $T = \dfrac{1}{B}$.

(c) For the three cases considered in part (b), sketch $X_{sr}(j\omega)$, the Fourier transform of $x_{sr}(t)$.

(d) For an arbitrary signal $x(t)$ that occupies a bandwidth of $2\pi B$ rad/s, what is the largest T such that $X_{sr}(j\omega) = X(j\omega)$, i.e. $x_{sr}(t) = x(t)$?

9. (a) Determine the Nyquist rate corresponding to each of the following signals:

(i) $x_1(t) = \dfrac{\sin(4000\pi t)}{\pi t}$

(ii) $x_2(t) = \left(\dfrac{\sin(4000\pi t)}{\pi t}\right)^2$

(b) The signal $y(t)$ is generated by convolving a band-limited signal $x_1(t)$ with another band-limited signal $x_2(t)$, that is,

$$y(t) = x_1(t) * x_2(t)$$

where

$$X_1(j\omega) = 0 \qquad \text{for } |\omega| > 1000\pi$$

$$X_2(j\omega) = 0 \qquad \text{for } |\omega| > 2000\pi$$

Suppose impulse train sampling is performed on $y(t)$, specify the range of values for the sampling period T which ensures that $y(t)$ is recoverable from the samples.