Subjective type questions [30]

1. The response of an LTI system to the input $x_1(t) = 1,\ t \in [0,1]$, (and zero otherwise) is given by $y_1(t) = (1-e^{-t})u(t) - (1-e^{-(t-1)})u(t-1)$. Determine the output $y_2(t)$ of the system, when the input is as given below [4].

$$x_2(t) = \begin{cases} t,\ t \in [0,1], \\ 2-t,\ t \in [1,2], \\ 0 \text{ otherwise} \end{cases}.$$

2. State whether the following statements are true or false, with justification or counterexample [10].

   (i) If the region of convergence (RoC) of the transfer function of an LTI system is a right half plane, then the system is causal.

   (ii) If the RoC of $X(s)$ is a right half plane, then $x(t)$ is a right-sided signal.

   (iii) If the RoC of a transfer function of an LTI system includes the origin, then the system is BIBO-stable.

   (iv) If the frequency response $H(j\omega)$ of an LTI system is well-defined and finite for all $\omega \in \mathbb{R}$, then the system is BIBO stable.

   (v) If $x(t)$ is an even signal, then the ROC of $X(s)$ is symmetric about the $j\omega$ axis.

3. Find the Laplace transform, or the inverse Laplace transform of the following (a and b are positive, real): [4]

   (i) $x_1(t) = te^{-\frac{t^2}{2}},\ t \in \mathbb{R}$.

   (ii) $X_3(s) = \ln[(s+a)^2 + b^2]$; $x_3(t)$ is a causal signal.

4. Find the transfer function relating $Y(s)$ to $X(s)$ corresponding to the following block diagram [2]:

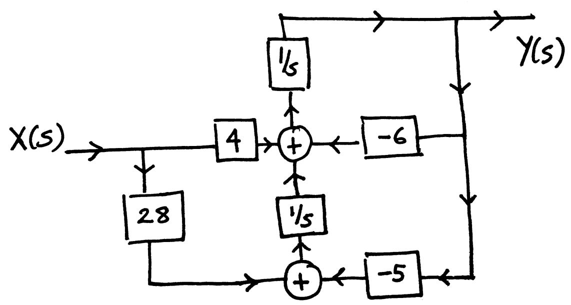

Figure 1: Block diagram

5. Consider a real bandpass signal, which has non-zero frequency components only in the range $|\omega| \in (\frac{\omega_0}{2}, \omega_0]$, where $\omega_0 > 0$. We are interested in perfectly recovering this signal, from its samples.

   (i) What is the minimum sampling rate required, if we have access only to an ideal low-pass filter? Specify the cut-off frequencies of the filter. [2]

   (ii) What is the minimum sampling rate, if we have access to an ideal bandpass filter? Specify the cut-off frequencies of the filter. [3]

6. Consider an RLC circuit, where the three components are connected in series. The inductor value L and the capacitor value C are given constants, but the resistance R can be changed. At time $t = 0^-$, suppose that the capacitor voltage and inductor current are both non-zero. For what values of R does the voltage across the capacitor behave monotonically for $t > 0$? [5]