Department of Physics
Indian Institute of Technology Madras

Quiz I

Date: Febraury 20, 2018 — Time: 08:00 – 08:50 AM

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Instructor:	DR. SHANTANU MUKHERJEE	Batch:	11

Instructions

1. Please write your name, roll number, instructor name, and batch number.
2. This question paper cum answer sheet booklet contains 7 single-sided pages.
3. The answers have to be written in the boxes provided. Answers written elsewhere in the booklet will not be evaluated.
4. Write the answers, including sketches, with a blue or black pen. Note that answers written with pencils or pens of other colors will not be evaluated.
5. You can use the empty reverse sides for rough work. No extra sheets will be provided.
6. You are not allowed to use a calculator or any other electronic device during the quiz.

For use by examiners

(Do not write in this space)

Page 2	Page 3	Page 4	Page 5	Page 6	Total
5	4	2	NA	2½	13.5

✦ **Multiple choice questions** (write the correct option in the box provided) **[1 x 5 = 5]**

1. A spherical conductor of radius a carrying a net charge q is surrounded by a linear dielectric of relative permittivity ϵ_r. The boundary conditions on D at the surface ($r = a$) are

   [A] $D^{\perp}_{above} - D^{\perp}_{below} = q$, $\quad D^{\parallel}_{above} - D^{\parallel}_{below} = 0$

   [B] $D^{\perp}_{above} - D^{\perp}_{below} = \frac{q}{4\pi a^2}$, $\quad D^{\parallel}_{above} - D^{\parallel}_{below} = 0$

   [C] $D^{\perp}_{above} - D^{\perp}_{below} = 0$, $\quad D^{\parallel}_{above} - D^{\parallel}_{below} = \frac{q}{4\pi a^2}$

   [D] $D^{\perp}_{above} - D^{\perp}_{below} = 0$, $\quad D^{\parallel}_{above} - D^{\parallel}_{below} = 0$

   B

2. A particle of mass m with charge q and initial velocity $\mathbf{v} = v_0\hat{e}_y$ is subject to electric field $\mathbf{E} = E_0\hat{e}_z$ and magnetic field $\mathbf{B} = B_0\hat{e}_x$, where E_0 and B_0 are positive constants. Which of the following is true?

   [A] The particle will not undergo deflection only if B = 0.
   [B] For E=0, the work done by the B-field transforms into rotational energy of the particle.
   [C] The trajectory of the particle will be a helix.
   [D] For $\mathbf{v} = \frac{E_0}{B_0}\hat{e}_y$, the trajectory of the particle will be a straightline along $\hat{e}_y$.

   D

3. Electrostatic potential for a charge configuration is given by $V(x, y) = \frac{a}{\sqrt{\epsilon_0}}(6x + 8y)$ Volts, where a is an appropriate constant and the medium is free-space. The energy density in J/m^3 is

   [A] $50a^2$ [B] $100a^2$ [C] $25a^2$ [D] $75a^2$

   A

4. Consider three particles of unit charge placed at (x, y, z) : (0,0,1), (0,0,0), and (0,0,-1). The z component of the electric field at the point (0,1,1) is

   [A] $\frac{1}{4\pi\epsilon_0}\left[1 + \frac{1}{2\sqrt{2}}\right]$ [B] $\frac{1}{4\pi\epsilon_0}\left[1 + \frac{1}{5\sqrt{5}}\right]$ [C] $\frac{1}{4\pi\epsilon_0}\left[1 + \frac{1}{2\sqrt{2}} + \frac{1}{5\sqrt{5}}\right]$ [D] $\frac{1}{4\pi\epsilon_0}\left[\frac{1}{2\sqrt{2}} + \frac{2}{5\sqrt{5}}\right]$

   D

5. Consider an infinitely long hollow-cylindrical conductor of radius a carrying current I parallel to its axis ($\hat{e}_z$). The magnetic field $\mathbf{B}_{in}$ in the region $r < a$ and $\mathbf{B}_{out}$ in the region $r > a$ are

   [A] $\mathbf{B}_{in} = \mathbf{B}_{out} = 0$

   [B] $\mathbf{B}_{in} = \frac{\mu_0 I}{2\pi a}\hat{e}_\phi$ and $\mathbf{B}_{out} = \frac{\mu_0 I}{2\pi r}\hat{e}_\phi$

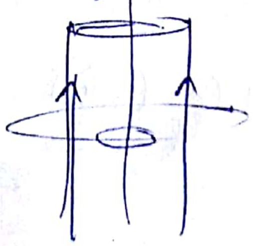
   [C] $\mathbf{B}_{in} = 0$ and $\mathbf{B}_{out} = \frac{\mu_0 I}{2\pi r}\hat{e}_\phi$

   [D] $\mathbf{B}_{in} = \frac{\mu_0 I}{2\pi a}\hat{e}_\phi$ and $\mathbf{B}_{out} = \frac{\mu_0 I}{2\pi a}\hat{e}_\phi$

   C

5

a.



~~μ_0~~

$B \cdot 2\pi S =$

$B \cdot 2\pi S = \mu_0 I$

✦ **Fill in the blanks** (1 mark each, write the answer in the box provided)

6. Consider a pyramid carrying a charge q at the center of its square base. What is the flux through any one face of the pyramid other than the base?

1 $\boxed{\dfrac{q}{8\epsilon_0}}$

7. Two infinitely long parallel wires at $x = a$ and $x = -a$ carry current I along y−direction. What is $\dfrac{\partial B_z}{\partial x}$ at the origin?

0 $\boxed{-\dfrac{\mu_0 I}{\pi a^2}\hat{z}}$ ✗

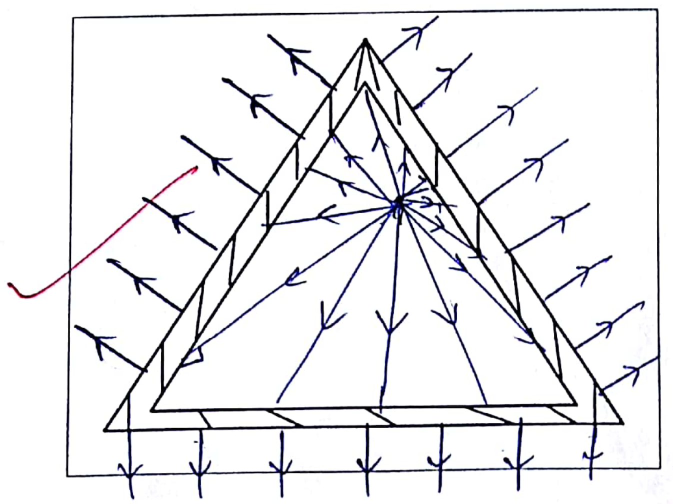
8. Consider the charge configuration (x, y, z): q at (0,0,0), $-\frac{q}{2}$ at (1,0,0), and $-\frac{q}{2}$ at (0,1,0). The dipolar potential at (1,1,0) is given by

1 $\boxed{-\dfrac{1}{4\pi\epsilon_0}\left(\dfrac{q}{2\sqrt{2}}\right)}$

9. The volume-current density in a straight wire of radius a is $\mathbf{J} = ks\hat{e}_z$, where k is an appropriate constant and s is the distance from the axis. What is the force per unit length on the wire due to a magnetic field $\mathbf{B} = B_0\hat{e}_x$, where B_0 is an appropriate constant?

1 $\boxed{\dfrac{2B_0\pi}{l}\cdot\dfrac{a^3}{3}\hat{y}}$

10. A long conducting tube with triangular cross-section encloses a point charge q as shown below. Mark the E-field lines everywhere inside the rectangular box.



1

Note
Please assume all $\bar{E}$ lines contact surface perpendicularly.

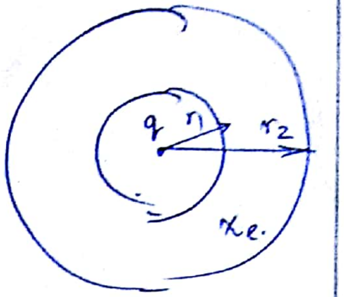
✦ **Questions with detailed answers** (write the calculations and answers within the boxes provided)

11. A point charge q is embedded at the center of a spherical shell of linear dielectric material (with susceptibility χ_e, inner and outer radii r_1 and r_2, respectively). Find the electric field everywhere, polarization, and the bound-charge densities ρ_b and σ_b. [1+1+1 marks]

~~Electric field:~~

i) ~~For $r < r_1$ using Gauss'~~ Law

$\cancel{E_1 \cdot 4\pi r^2 = \frac{q}{\epsilon_0} \Rightarrow E_1 = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}}$.



~~Before that~~ . . .

$\epsilon_r = 1 + \chi_e$

$\Rightarrow \epsilon = \epsilon_0(1+\chi_e)$

For i) $r < r_1$

$D \cdot 4\pi r^2 = 0 \Rightarrow \boxed{D_1 = 0}$. ?

For ii) $r_1 < r < r_2$

$D \cdot 4\pi r^2 = q \Rightarrow \boxed{D_2 = \frac{q}{4\pi r^2}\hat{r}}$.

For iii) $r > r_2$

$D \cdot 4\pi r^2 = q \Rightarrow \boxed{D_3 = \frac{q}{4\pi r^2}\hat{r}}$

1/2

Then $\boxed{E_1 \Big|_{r<r_1} = \frac{D}{\epsilon_0} = 0}$

$\boxed{E_2 \Big|_{r_1<r<r_2} = \frac{D}{\epsilon} = \frac{q}{4\pi r^2 \epsilon_0 (1+\chi_e)}\hat{r}}$

$\boxed{E_3 \Big|_{r>r_2} = \frac{D}{\epsilon_0} = \frac{q}{4\pi\epsilon_0 r^2}\hat{r}}$.

$\boxed{P_1 \Big|_{r<r_1} = \epsilon_0 \chi_e E = 0}$

$\boxed{P_2 = \epsilon_0 \chi_e E_2 = \frac{\chi_e}{1+\chi_e}\frac{q}{4\pi r^2}\hat{r}}$

1

σ_b at r_1

$\Rightarrow -P_2 \cdot \hat{r}$

$= -\frac{q}{4\pi r^2}\frac{\chi_e}{1+\chi_e}$ at r_1

σ_b at r_2

$= P_2 \cdot \hat{r}$

$\boxed{\sigma_b = \frac{q}{4\pi r^2}\frac{\chi_e}{1+\chi_e}}$ at r_2

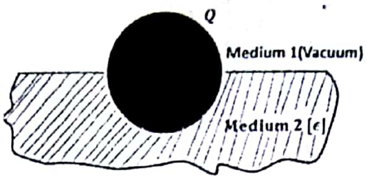
0.5

$\cancel{P_3 = \epsilon_0 \chi_e E_3 = \frac{q}{4\pi r^2}} \Rightarrow \boxed{P_3 = 0}$

ρ_b = only for $r_1 < r < r_2$ = $-\bar{\nabla}\cdot\bar{P}$ =

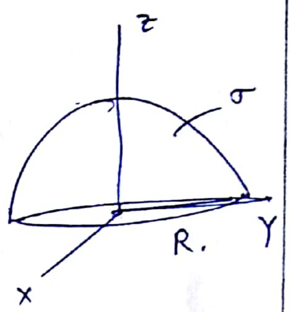
$\rho_b = \boxed{\frac{\chi_e}{1+\chi_e}\cdot\frac{q}{r^2}\hat{r}}$

12. A conducting sphere of radius R carries charge Q. One half of the sphere is immersed into a linear dielectric with permittivity ϵ, as shown below. The space above this dielectric is vacuum. (a) Find the electric field in both the media. (b) Find the electric displacement



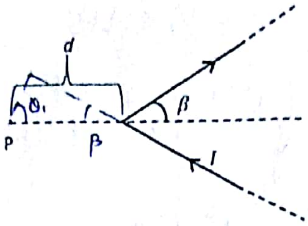
in both the media. (c) Determine the total energy stored in the system. [*Hint:* $\mathbf{E}$ at the interface between the two media is tangential.] $\boxed{\text{1+1+1 marks}}$

(a) For vacuum: $\bar{E}$ due to upper hemisphere only.

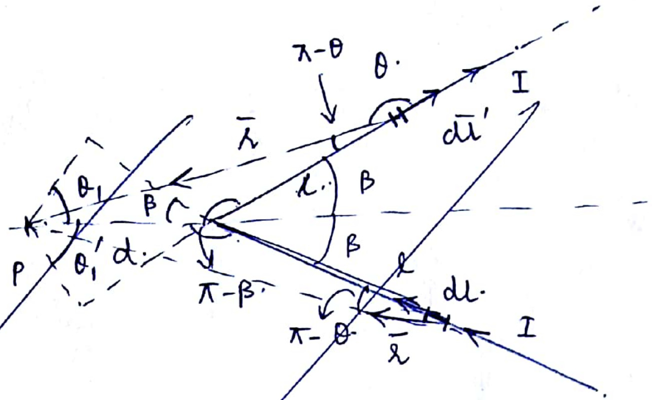


NA

13. Consider an infinitely long wire carrying current I. If the wire is bent by an angle 2β as shown in the figure, find the magnetic field at a point P, lying on the angle bisector, at a distance d from the bending point.



4 marks



Due to upper part by Biot-Savart's Law.

$$|B| = \frac{\mu_0 I}{4\pi}\int \frac{dl \sin\theta}{r^2} \; ; \quad \cos(\pi-\beta) = \frac{d^2+l^2-r^2}{2dl}$$

$$\Rightarrow r^2 = d^2+l^2+2dl\cos\beta$$

$$|B| = \frac{\mu_0 I}{4\pi}\int \frac{dl \sin\theta}{d^2+l^2+2dl\cos\beta}$$

By sine rule; $\frac{\sin(\pi-\theta)}{d} = \frac{\sin(\pi-\beta)}{r}$

or $\sin\theta = \frac{d\sin\beta}{r}$

$$\Rightarrow |B| = \frac{\mu_0 I}{4\pi}\int \frac{dl\left(\frac{d}{r}\right)\sin\beta}{d^2+l^2+2dl\cos\beta}$$

$$\Rightarrow |B| = \frac{\mu_0 I d \sin\beta}{4\pi}\int_0^{\infty}\frac{dl}{(d^2+l^2+2dl\cos\beta)^{3/2}}$$

For the other segment we have.

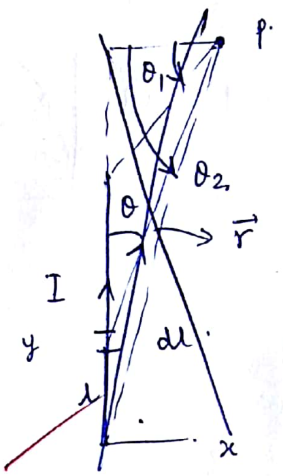
$$|\bar{B}| = \frac{\mu_0 I}{4\pi}\int_{\infty}^{0}\frac{dl\,\sin(\pi-\theta)}{(d^2+l^2+2dl\cos\beta)}$$

$$= -\frac{\mu_0 I}{4\pi}\int_0^{\infty}\frac{dl\,\sin\theta}{(d^2+l^2+2dl\cos\beta)}$$

$$= -\frac{\mu_0 I d\sin\beta}{4\pi}\int_0^{\infty}\frac{dl}{(d^2+l^2+2dl\cos\beta)^{3/2}}$$

Soln: By biot-savart's Law:

$$|B| = \frac{\mu_0 I}{4\pi}\int\frac{d\bar{l}\times\hat{r}}{r^2}$$

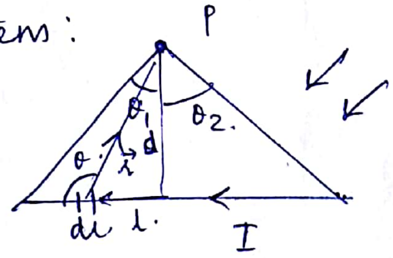


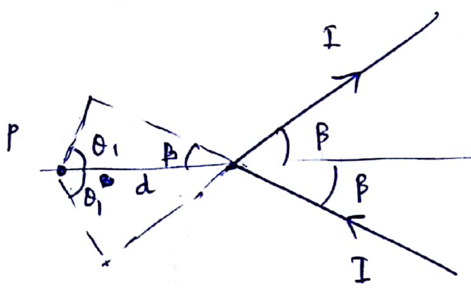
By solving for shown system we get

$$|B| = \frac{\mu_0 I}{4\pi d}[\sin\theta_2 - \sin\theta_1]$$

$1\frac{1}{2}$

Using this on our system:





Please read overleaf

$2\frac{1}{2}$