EE1101 Signals and Systems JAN—MAY 2019
Tutorial 2

1. (a) Let $x_1(t)$ and $x_2(t)$ be periodic signals with periods T_1 and T_2. Derive the conditions under which the sum $x(t) = x_1(t) + x_2(t)$ is periodic. What is the fundamental period of $x(t)$?

   (b) Determine the fundamental period of the following signals:

   (a) $x(t) = \cos(10\pi t + 1) - 2\sin(4\pi t - 1)$

   (b) $x[n] = 1 + e^{j\frac{4\pi}{7}n} - e^{j\frac{2\pi}{3}n}$

2. Using the generalized function definition of impulse, show that : $\delta(at) = \frac{1}{|a|}\delta(t)$.

3. Evaluate the following integrals:

   (a) $\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\, d\tau$

   (b) $\int_{-\infty}^{\infty} \delta(\tau)x(t-\tau)\, d\tau$

   (c) $\int_{-\infty}^{\infty} \delta(t)e^{-j\omega t}\, dt$

   (d) $\int_{-\infty}^{\infty} \delta(2t-3)\sin \pi t\, dt$

   (e) $\int_{-\infty}^{\infty} \delta(t+3)e^{-t}\, dt$

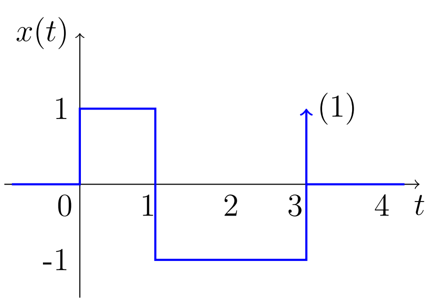
   (f) $\int_{-\infty}^{\infty} (t^3+4)\delta(1-t)\, dt$

   (g) $\int_{-\infty}^{\infty} x(2-t)\delta(3-t)\, dt$

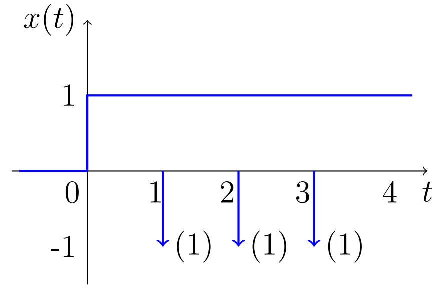
   (h) $\int_{-\infty}^{\infty} e^{(x-1)}\cos\left[\frac{\pi}{2}(x-5)\right]\delta(x-3)\, dx$

4. Find and sketch $\int_{-\infty}^{t} x(t)\, dt$ for the signal $x(t)$ illustrated in the following figures.

(a)



(b)



5. Determine whether the following systems are (a) linear, (b) time-invariant, (c) causal, (d) stable and (e) invertible.

   (a) $y(t) = \frac{dx(t)}{dt}$ where $\frac{d}{dt}$ represents the left differentiator.

   (b) $y(t) = \int_{-\infty}^{3t} x(\tau)d\tau$

   (c) $y(t) = x(t/2)$

   (d)
$$y(t) = \begin{cases} x(t) - x(t-100) & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

   (e) $\frac{dy(t)}{dt} + 3ty(t) = t^2\frac{dx(t)}{dt}$

   (f) $y(t) = \sum_{n=-\infty}^{\infty} x(t)\delta(t-nT)$

   (g) $y(t) = x(2t-4)$

6. Consider a discrete-time system with input $x[n]$ and output $y[n]$. The input-ouput relationship for the systems is

$$y[n] = x[n]x[n-2].$$

   (a) Is the system memoryless?

   (b) Determine the output of the system when the input is $A\delta[n]$, where A is any real or complex number.

   (c) Is the system invertible?

7. For each of the following input-output relationships, determine whether the corresponding system is linear, time-invariant or both.

   (a) $y(t) = t^2 x(t-1)$

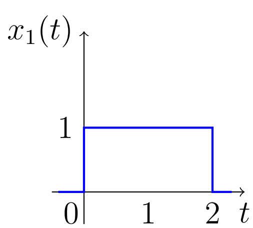
   (b) $y[n] = x^2[n-2]$

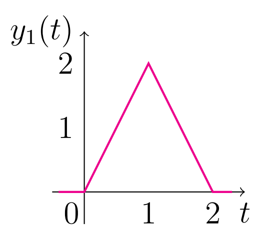
   (c) $y[n] = x[n+1] - x[n-1]$

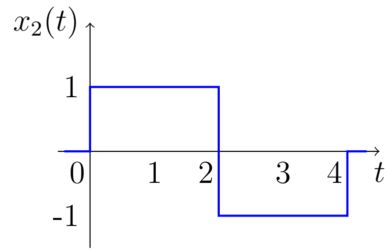
   (d) $y[n] = Odd\{x[n]\}$

8. Let $\mathbf{H}$ represent a continuous time Linear Time-invariant (LTI) system. Then show that $\mathbf{H}\{e^{st}\} = \lambda e^{st}$ where s is a complex variable and λ is a complex constant.

9. Consider a continuous time LTI system whose response to the signal $x_1(t)$ in figure below is the signal $y_1(t)$ illustrated below. Determine and sketch carefully the response of the system to the input $x_2(t)$ shown below.







10. In frequency modulation (FM), the modulated signal $y(t)$ is related to the modulating signal $m(t)$ by

$$y(t) = A\cos\left(\omega_c t + \omega_\Delta \int_{-\infty}^{t} m(\tau)d\tau\right)$$

where ω_Δ is the frequency-deviation constant. This is called FM because the instantaneous frequency is proportional to the modulating signal:

$$\begin{aligned}\omega(t) &= \frac{d}{dt}\left[\omega_c t + \omega_\Delta \int_{-\infty}^{t} m(\tau)d\tau\right] \\ &= \omega_c + \omega_\Delta m(t).\end{aligned}$$

   (a) Sketch $y(t)$ for $\omega_c = 8\pi$, $\omega_\Delta = 2\pi$ and $m(t) = u(t+2) - u(t-1)$.

   (b) Is the modulation system, with input $m(t)$ and output $y(t)$, linear? Time invariant? Memoryless? Causal?

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