EE1101 Signals and Systems JAN—MAY 2019
Tutorial 3

1. Convolve the two signals $x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3]$ and $h[n] = 2\delta[n+1] + 2\delta[n-1]$ using the graphical method.

2. Convolve the signals
   (a) $\alpha^n u[n]$ and $\beta^n u[n]$ $(\alpha \neq \beta)$
   (b) $u[n]$ and $a^n u[-n-1]$, given that $|a| > 1$.

3. Show that
   (a) $x[n] \star y[n] = y[n] \star x[n]$
   (b) $x[n] \star (y[n] + z[n]) = x[n] * y[n] + x[n] \star z[n]$
   (c) $x[n] \star \delta[n-a] = x[n-a]$

4. Let $x[n] = \delta[n] + 2\delta[n-1] - \delta[n-3]$ and $h[n] = 2\delta[n+1] + 2\delta[n-1]$. Using the properties derived in (3) compute and plot each of the following convolutions.
   (a) $y_1[n] = x[n] \star h[n]$
   (b) $y_2[n] = x[n+2] \star h[n]$

5. Let
$$x[n] = \begin{cases} 1, & 0 \leq n \leq 9 \\ 0, & \text{elsewhere} \end{cases}$$
and
$$h[n] = \begin{cases} 1, & 0 \leq n \leq N \\ 0, & \text{elsewhere} \end{cases}$$
where $N \leq 9$ is a positive integer. Determine the value of N, given that $y[n] = x[n] \star h[n]$, $y[4] = 5$ and $y[14] = 0$.

6. Let $y(t) = x(t) \star h(t)$. $x(t)$ is non-negative for $t \in (2,3)$ and zero elsewhere, and is symmetric about $t = 5/2$. $h(t) = 1$ for $t \in (3,4)$ and zero elsewhere.
   (a) During what times will the values $y(t)$ be non-zero?
   (b) At what time(s) will $y(t)$ achieve its maximum value.

7. Consider a system with input $x(t)$ and output $y(t)$ related by:
$$y(t) = \int_{-\infty}^{t+1} \sin(t-\tau)x(\tau)\, d\tau.$$
   (a) Is the system time-invariant? Prove.
   (b) What is the system impulse response?
   (c) Is the system causal?

8. Perform the following convolutions where $\star$ indicates convolution.
   (a) For $u(t)$ a unit step function, find $u(t) \star u(t)$.
   (b) Find $x(t) \star h(t)$, where $h(t) = (-e^{-t} + 2e^{-2t})u(t)$ and $x(t) = 10e^{-3t}u(t)$.
   (c) Find the output $y(t)$ of an LTI system with impulse response $h(t) = 2e^{-2t}u(t)$ when excited with an input $x(t)$ given by
      (i) $x(t) = \begin{cases} 1, & 2 \leq t \leq 4 \\ 0, & \text{otherwise} \end{cases}$
      (i) $x(t) = \cos(4\pi t)$.
   (d) Using the result of part (a), evaluate and sketch $y(t) = [u(t) \star u(t-2)]\, u(4-t)$.
   (e) Determine graphically $h(t) = f(t) \star g(t)$, where
      (i) $f(t) = u(-t)$ and $g(t) = 2(u(t) - u(t-1))$.
      (ii) $f(t) = r(t) - r(t-2)$ and $g(t) = u(t-3) - u(t-6)$ [Note: $r(t) = tu(t)$].

9. Let the output of a discrete time LTI system, with impulse response $h[n]$, be given by, $y[n] = x[n] \star h[n]$, where the input $x[n] = 0$ outside the range $0 \leq n \leq N-1$.

Let the column vector $\mathbf{y}$ represent the output $y[n]$ from 0 to $N-1$, and the column vector $\mathbf{x}$, the values of $x[n]$ from 0 to $N-1$. If $\mathbf{y} = \mathbf{H}\mathbf{x}$, find the matrix $\mathbf{H}$.

10. Given that $f(t) \star g(t) = y(t)$, where $\star$ denotes convolution,

    (a) Find $f(t-T_1) \star g(t-T_2)$, for some finite-valued real numbers T_1 and T_2.

    (b) Use the result of (a) and the fact that $u(t) \star u(t) = r(t)$, to find $(u(t+1) - u(t-2)) \star (u(t-3) - u(t-4))$. Verify the result graphically.

11. Given $y(t) = f(t) \star g(t)$, derive a general formula to compute $f(ct) \star g(ct), c \neq 0$. Hence, if $f(t) = u(t+1) - u(t-2)$ and $g(t) = r(t)(u(t) - u(t-1))$, find $f(2t) \star g(2t)$.

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