EE1101 Signals and Systems JAN—MAY 2019
Tutorial 5
March 4, 2019

1. A periodic signals $x(t)$ is given below.

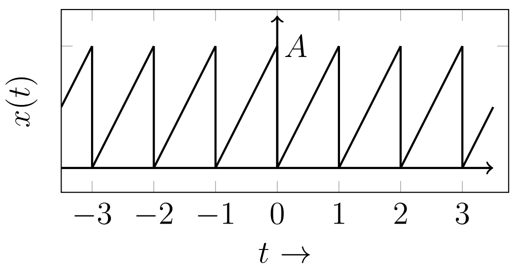

Figure 1

   (a) Determine the Fourier Series coefficients of $x(t)$ in the exponential form. Sketch the magnitude and phase spectrum.

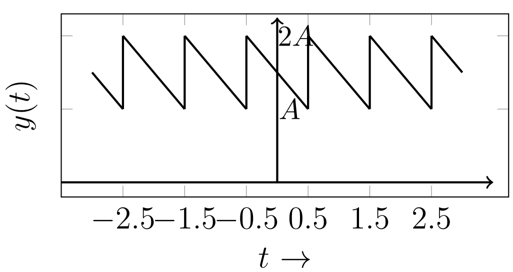

Figure 2

   (b) Determine the Fourier Series coefficients of the periodic signal $y(t)$ given in Figure 2 in the exponential form. Sketch the magnitude and phase spectrum.

   (c) Using the results in part (a) and without doing elaborate integrations, determine the coefficients of the Fourier series of $y(t)$.

2. Determine the Fourier series coefficients for the following periodic signals of period T_0 and defined in the interval $\big[-\frac{T_0}{2}, \frac{T_0}{2}\big)$. Sketch the magnitude and phase spectrum in each case assuming $A = 1, d = .5, T_0 = 1$.

   (a) $x_1(t) = \begin{cases} A & |t| < d/2,\ d < T_0 \\ 0 & \text{otherwise} \end{cases}$

   (b) $x_2(t) = \begin{cases} A\sin(\frac{2\pi t}{T_0}) & 0 \leq t < \frac{T_0}{2} \\ 0 & \frac{-T_0}{2} \leq t < 0 \end{cases}$

3. A 2π periodic signal $x(t)$ is specified over one period as

$$x(t) = \begin{cases} \dfrac{t}{A} & 0 \leq t < A \\ 1 & A \leq t < \pi \\ 0 & \pi \leq t < 2\pi \end{cases}$$

   Represent the function as an exponential Fourier series.

4. The (exponential) Fourier series coefficients of a periodic signal $x(t)$ is given by

$$a_k = \begin{cases} jk, & |k| < 3 \\ 0, & \text{otherwise} \end{cases}$$

   The fundamental period of the signal is $T_0 = 4$. Determine the signal $x(t)$.

5. $x(t)$ is a periodic signal with fundamental period T and Fourier series coefficients a_k. Derive the Fourier series coefficients of the following signals in terms of a_k (Hint: Use analysis/synthesis equation).

   (a) $x(t - t_0)$
   (b) $x(-t)$
   (c) $x^*(t)$, where $()^*$ denotes complex conjugation
   (d) $x(t - t_0) + x(t + t_0)$
   (e) $\text{Even}\{x(t)\}$
   (f) $\text{Real}\{x(t)\}$

6. The Periodic convolution of two signals $x(t)$ and $y(t)$ with period T and Fourier coefficients a_k and b_k resp. is defined as,

$$x(t) * y(t) = \int_T x(\tau)y(t-\tau)d\tau$$

Find the exponential Fourier series coefficients of the signal

$$\cos t \star \cos 3t.$$

where $\star$ denotes periodic convolution with period $T = 2\pi$. Verify your result by using the periodic convolution property of Fourier series ($Ta_k b_k$, See Table 3.1 of Oppenheim).

7. The Trigonometric Fourier series of a periodic signal $x(t)$ with period T and frequency $\omega_0 = \frac{2\pi}{T}$ is,

$$x(t) = a_0 + \sum_{k=1}^{\infty} [a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t)]$$

where,

$$a_0 = \frac{1}{T}\int_T x(t)\, dt$$
$$a_k = \frac{2}{T}\int_T x(t)\cos\left(\frac{2\pi kt}{T}\right)\, dt$$
$$b_k = \frac{2}{T}\int_T x(t)\sin\left(\frac{2\pi kt}{T}\right)\, dt$$

Find the Trigonometric Fourier series coefficients for the periodic signal

(a) $x_1(t)$ in 2(a)

(b) $x_2(t) = x_1(t - d/2) - \frac{A}{2}$ considering $T_0 = 2d$

8. (a) Find Fourier series coefficients of the periodic signal

$$y(t) = \sum_{l=-\infty}^{\infty} \delta(t - lT)$$

where $T > 0$

(b) Let $x(t) = |\sin t|$. Plot the signals $\frac{dx}{dt}, \frac{d^2x}{dt^2}$.

(c) Using part (a), find and plot the Fourier series coefficients of the signal $x(t) + \frac{d^2x}{dt^2}$.

(d) Use the differentiation property to find the Fourier series coefficients of the signal $x(t)$.

9. Suppose we are given the following information about signal $x(t)$:

i) a_k=0 for $k = 0$ and $k > 2$

ii) $x(t)$ is a real signal

iii) a_1 is a positive real number

iv) $x(t)$ is periodic with period T=6 and has Fourier coefficients a_k

v) $x(t) = -x(t-3)$

vi) $\frac{1}{6}\int_{-3}^{3} |x(t)|^2 dt = \frac{1}{2}$

Show that $x(t) = A\cos(Bt + C)$ and determine the value of constants A, B and C.

— END —