

$$\left(\frac{\partial^2 \psi}{\partial t^2} - \frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial y^2} - \frac{\partial^2 \psi}{\partial z^2} \right) \psi^* + m \psi \psi^* = 0$$

$$\left(\frac{\partial^2 \psi^*}{\partial t^2} - \frac{\partial^2 \psi^*}{\partial x^2} - \frac{\partial^2 \psi^*}{\partial y^2} - \frac{\partial^2 \psi^*}{\partial z^2} \right) \psi + m \psi \psi^* = 0$$

focus on time derivatives:

$$\therefore \psi^* \frac{\partial^2 \psi}{\partial t^2} - \psi \frac{\partial^2 \psi^*}{\partial t^2} + \underbrace{\frac{\partial \psi^*}{\partial t} \frac{\partial \psi}{\partial t} - \frac{\partial \psi^*}{\partial t} \frac{\partial \psi}{\partial t}}_{\text{add and subtract to complete derivative}}$$

$$= \frac{\partial}{\partial t} \left(\psi^* \frac{\partial \psi}{\partial t} - \frac{\partial \psi^*}{\partial t} \psi \right)$$

similar game for spatial derivatives

$$- (\nabla^2 \psi \psi^* - \nabla^2 \psi^* \psi + \tilde{\nabla} \psi^* \tilde{\nabla} \psi - \tilde{\nabla} \psi^* \tilde{\nabla} \psi)$$

$$= - \tilde{\nabla} \cdot (\psi^* \tilde{\nabla} \psi - (\tilde{\nabla} \psi^*) \psi)$$

∴ putting all together

$$\frac{\partial}{\partial t} \left(\psi^* \frac{\partial}{\partial t} \psi - \frac{\partial \psi^*}{\partial t} \psi \right)$$

$$- \vec{\nabla} \cdot \left(\psi^* \vec{\nabla} \psi - \vec{\nabla} \psi^* \psi \right) = 0$$

So multiply LHS / RHS by i

$$\text{and } \frac{\partial}{\partial t} \rho + \vec{\nabla} \cdot \vec{j} = 0 \text{ as}$$

required