

## 2 Gamma ray absorption

**Abstract:** In this lab, you will measure the absorption of gamma rays in matter.

### 2.1 Pre-lab preparation

Just as the electrons in an atom can transition between energy levels by emitting photons of specific energies, the nucleus of an atom can also transition between energy levels. However, the energy levels in the nucleus typically differ by MeV rather than eV, so the radiation absorbed or emitted is highly energetic. Such highly energetic photons are called gamma rays, or just  $\gamma$ 's for short. Nuclear transitions result in gamma rays with specific energies just like the atomic transitions and, just like atomic spectra, the spectrum of gamma ray energies absorbed or emitted by a sample can be used to identify its composition.

A *radioactive* element is simply an element with a nucleus that is in an excited state. Decays of this unstable state emit radiation and leave the nucleus in a lower energy state. There are three kinds of nuclear radiation:  $\alpha$ ,  $\beta$  and  $\gamma$ . Their names are a legacy from when they were first identified in the 1890's. When a nucleus decays by emitting  $\gamma$  radiation, we say it underwent  $\gamma$  decay. While  $\gamma$  decay involves emission of a high energy photon,  $\beta$  decay involves emission of an electron (or anti-electron) from the nucleus. These electrons are produced when a neutron changes into a proton ( $n \rightarrow p + e^- + \bar{\nu}$ ) or *vice versa* ( $p \rightarrow n + e^+ + \nu$ ); the latter is not possible for an isolated proton because  $m_n > m_p$ , but it can occur within a nucleus if the accompanying change in the binding energy of the nucleus makes it energetically allowed. Finally,  $\alpha$  decay involves ejecting part of the nucleus, consisting of two protons and two neutrons (which is essentially the nucleus of a Helium atom).

The particles emitted during  $\alpha$  and  $\beta$  decay are often called  $\alpha$  and  $\beta$  particles. Sometimes they are called  $\alpha$  and  $\beta$  “rays” because that was the original terminology, used before their nature was fully understood. The “ray” terminology still has a use because it carries additional information about the origin of the particle. For example, although they are all electrons, the names cathode ray,  $\beta$  ray,  $\delta$  ray, and photoelectron each tell you something about how the electron was produced. (Never heard of  $\delta$  rays? I encourage you to look up the details if you are curious.)

You'll use Cobalt-60 in this lab. The complete symbol for this isotope is  ${}_{27}^{60}\text{Co}$ , where the lower number (27) is the atomic number,  $Z$ , *i.e.*, the number of protons, and the upper number (60) is the number of nucleons (*i.e.*, the number of protons plus neutrons). The atomic number is redundant with the Co symbol, so it is often just called Co-60. However, explicitly listing the atomic number sometimes helps when looking at decays to remind you of how many protons there are. I'll use the various naming options interchangeably.

Cobalt-60 decays to Nickel-60 through the processes illustrated in Fig. 3. This diagram is structured like an energy level diagram, where the *relative* energy of the states is shown by the numbers along the right side, specified in MeV. The lowest energy Ni-60 state (at the bottom) is the zero point for the energy scale, and the Co-60 nucleus has 2.823 MeV higher energy. There are two intermediate states at energy levels of 2.505 and 1.332 MeV. The arrows (solid, dashed and wavy) between the states represent decay paths, with the corresponding energy change listed next to the arrow along with the type of decay ( $\beta$  or  $\gamma$ ). The dashed arrow is a rare process that we will neglect here, so the decay proceeds as follows:

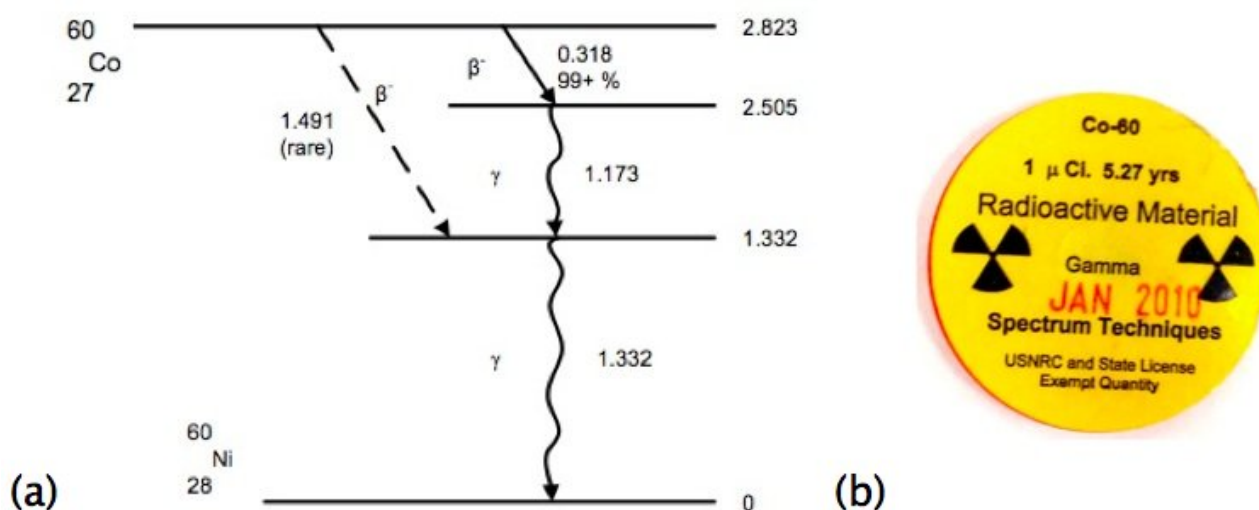


Figure 3: (a) Decay diagram for Co-60. (b) Image of a Co-60 source. The radioactive material is encapsulated at the center of the plastic disk; the yellow label specifies information about the source.

1. The  $^{60}_{27}\text{Co}$  nucleus  $\beta$  decays to a  $^{60}_{28}\text{Ni}$  nucleus. A neutron is converted to a proton so the number of nucleons stays constant at 60, but the number of protons increases by one so it is now a Nickel nucleus. That nucleus is in an excited state, 2.505 MeV above its ground state.
2. The excited nucleus decays to a lower energy, but not yet ground state by emitting a 1.173 MeV  $\gamma$ .
3. A second decay brings the nucleus down to its ground state by emitting a 1.332 MeV  $\gamma$ .

The timescale for the  $\beta$  decay, which starts the cascade, is quite long; the half-life of Co-60 is 5.27 years. The ‘**half-life**’ is the time required for half of the nuclei to decay. The two  $\gamma$  decays happen very quickly afterward, so although they are in fact decays of a Nickel nucleus, we still call this a Co-60 source. If the decays were to happen slowly, we could isolate the material part way through the decay to obtain a different source. That is, in fact, how many radioactive sources are produced; they are metastable products of the decays of other radioactive elements, typically from the Uranium and Thorium decay chains. The Co-60 source that we will use, however, is produced synthetically by bombarding Co-59 with neutrons. It is commonly produced for a variety of applications, including radiation therapy for cancer treatment. The activity of the source is measured in Curie, where 1Ci corresponds to  $3.7 \times 10^{10}$  decays per second. The sources you will use had activities of  $1\mu\text{Ci}$  when they were originally manufactured, but they have since been decaying. You can figure out the current activity of a source by multiplying its original activity by one-half raised to the power of the number of half-lives that have elapsed since its manufacture.

With the appropriate equipment, it is possible to measure the energy of emitted gammas. This is called gamma ray spectroscopy in analogy to atomic spectroscopy. That won’t be today’s project. Rather, we will study how the emitted radiation interacts with material. So, I’ll begin with a short introduction to that.

### 2.1.1 Interaction of radiation with matter

Each of the three types of nuclear radiation,  $\alpha$ ,  $\beta$ , and  $\gamma$ , transfer their energy to matter by electromagnetically interacting with the electrons in the material, but they do so with qualitatively different features. Because  $\beta$  particles are high energy electrons, they scatter off the electrons in a material’s atoms, ioniz-

ing the atoms and losing energy in the process.  $\alpha$  particles also lose energy by ionization, but they don't penetrate as far into a material as  $\beta$ 's because the  $\alpha$ 's are more charged ( $+2e$ ) and (much!) more massive. For a given material (*i.e.*, a given density of electrons), the rate of energy loss by ionization is proportional to the square of a particle's charge and inversely proportional to the square of its velocity. As a result,  $\alpha$  particles lose all their energy in a layer of material that is usually less than a mm thick, while  $\beta$ 's typically traverse a few mm before stopping. (The rate of energy loss is described by the Bethe-Bloch formula, the details of which are beyond the scope of this lab).

Gamma rays, by contrast, have no charge and no mass. They interact with a material in several ways, including: ionization of the atoms via an "atomic photo-electric effect"; Compton scattering off of atomic electrons, which also ionizes the atom but gives the ejected electron high enough energy to cause further ionization of its own; and "pair production", which involves creation of an electron and anti-electron (positron) that have sufficient energy to further ionize. Each of these processes depends differently on energy. The first two processes dominate for the  $\sim 1$  MeV gamma rays from Co-60 that you will use; pair production becomes important only at higher energies.

The rate at which  $\gamma$ 's interact is much smaller than for  $\beta$ 's. Gamma rays can penetrate several cm of material before interacting. When they do interact, they typically deposit all their energy in a small volume at the interaction point. Because of this feature, we can describe their interaction with a given material in terms of an "absorption length",  $\mu$ , such that the number of gamma rays remaining after traversing a material of thickness  $x$  is  $N = N_0 e^{-x/\mu}$ .

The principal characteristic that determines a material's absorption length is the density of atoms along the gamma's path, which is related to the mass density of the material. We can account for that, and obtain a definition of absorption length that is very nearly material independent, by defining a **mass absorption length**,  $\lambda = \rho * \mu$ , which has units of  $\text{g}/\text{cm}^2$ . (Don't confuse this with wavelength! While it may be the same symbol, we're using it for a different purpose.)

We can also define a "density thickness",  $z$ , measured in  $\text{g}/\text{cm}^2$  rather than cm. If you have a chunk of material, such as aluminum, whose thickness  $x$  you have measured, then you can get the **density thickness** from  $z = \rho * x$  using the known density,  $\rho$ , of aluminum. So the number of gammas remaining after passing through a piece of material with density  $\rho$  and thickness  $x$  is

$$N = N_0 e^{-x/\mu} = N_0 e^{-(x*\rho)/(\rho*\mu)} = N_0 e^{-z/\lambda} \quad (9)$$

In this lab, you'll measure the mass absorption length,  $\lambda$ , for the  $\gamma$ 's from Co-60.

### 2.1.2 Apparatus

The apparatus you'll use is called a *Geiger counter*. It consists of a tube of gas with a metallic cathode around the outside and a thin wire anode situated along the center of the tube. A high voltage applied between them produces a large electric field inside the tube. When the gas is ionized by incident radiation, due to the processes described above, the released electrons and ionized atoms will be accelerated by the electric field toward the anode and cathode, respectively. Since the anode wire is very thin, the electric field near it is very high, and the electrons accelerated by it obtain enough energy to ionize additional gas atoms. This leads to an avalanche of ionization, resulting in a large and detectable current pulse. Counting the number of such pulses over a period of time measures the rate of incident radiation. By varying the density thickness,  $z$ , of absorber between the source and the Geiger counter, and measuring the count rate  $N(z)$  you can use Equation (9) to determine  $\lambda$ .

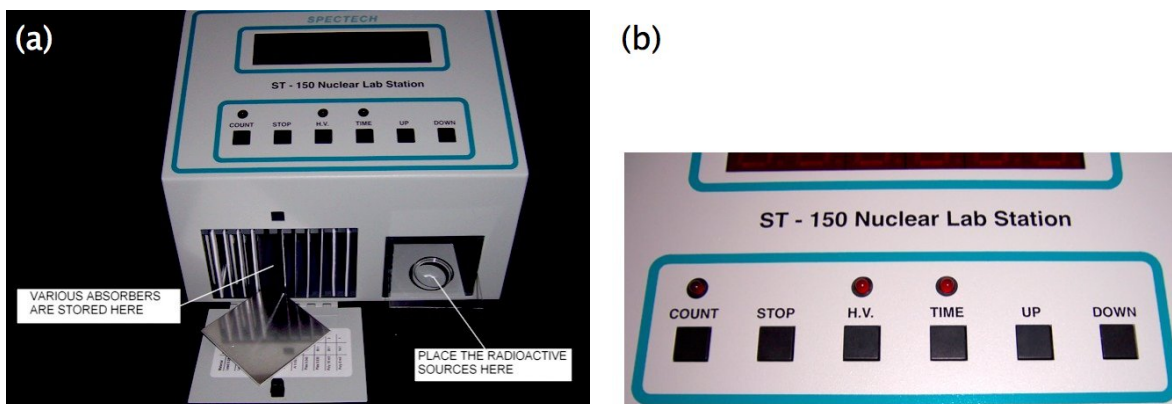


Figure 4: (a) Photo of the Geiger counter apparatus. (b) Close-up of the control buttons.

You will be counting the number of radiation-induced ionization events detected over some period of time. So, what is the uncertainty in the number that you measure? This idea of counting unrelated, randomly occurring “events” is common to many experiments and there is a standard method for determining the uncertainty in a count based on “counting statistics”. Stated simply<sup>3</sup>, the uncertainty in the number of events,  $N$ , that you count is  $\delta N = \sqrt{N}$ . This works for counting any sort of random variable, radiation events or the number of birds that fly overhead per hour.

You might protest that there is no uncertainty in the number that you count. If you saw 15 birds, then there were 15 birds. That is correct, but the reason we are counting these things is to measure the underlying event rate. What the uncertainty represents is the possibility of fluctuations in the number of events that could have happened during the counting time, not how many actually did happen. If you were to repeat the measurement, counting over many different time periods of the same duration, you wouldn’t always get the same number. You would see fluctuations between measurements of the number of events in the different time periods. So,  $\delta N$  represents the range that will contain 68% of those measurements. It accounts for the fact that the time of occurrence of these random events is, well, random.

The Geiger counter that you will use is shown in Fig. 4. On the left is an area where the absorber sheets are stored. On the right is an open area with slots into which you can slide the plastic source holder and the absorber sheets. The Geiger counter is situated above the top of that open area, so absorber sheets placed above the source will be between it and the counter. You can set the length of time for which to count using the *time* button, and then use the *count* button to begin a counting run. It will stop automatically after the selected time period. The number of counts are displayed above the control buttons. Note that when the red LED above the “time” button is lit, the display will show the time rather than the number of counts. To switch back to displaying the number of counts, press the time button again. The “HV” button operates similarly.

### 2.1.3 Safety and care of equipment

While the radioactive sources that we are using have a low activity, it is appropriate to follow standard safety protocols when dealing with them. These, and other precautions, are listed below.

- *Do not damage the sources.* The radioactive material is encapsulated in the plastic disks. Releasing the material, *e.g.*, by physically breaking the plastic, could cause contamination that is difficult to

<sup>3</sup>This method is valid when  $N$  is not too small, say  $N \geq 5$ . For smaller numbers, the more complicated “Poisson statistics” approach is required. Using  $\sqrt{N}$  should be fine for all your measurements here.

contain.

- *Do not eat or drink in the lab, and wash your hands when you leave the lab.* These protocols are required in any experiment. Their motivation is well-illustrated by the details in this case. If your hands became contaminated by radioactive material or lead dust, which is also hazardous, you wouldn't want to let contaminants into your body by eating, either in lab or afterward, before you have washed your hands.
- *Do not lose the sources.* The sources are kept in a locked box at the front of the room. You can check one in and out by contacting me or the TA.
- *Do not touch the active part of the Geiger counter.* There is a thin window up in the top of the open shelf area, which covers the active part of the Geiger Counter. It has to be thin to minimize the likelihood of interactions of  $\alpha$  or  $\beta$  particles in it, rather than inside the gas tube. Thin windows are notoriously easy to damage and difficult to clean. To avoid damaging the window, do not put anything, like your fingers, up into the region above the shelf area.
- *Don't set the voltage for the Geiger counter too high.* The operating voltage for the Geiger counter should be limited to avoid sparking within the tube because sparks cause damage. Our Geiger counters operate best at 500 V; set the HV to that value, but no higher.

## 2.2 Getting started and gaining familiarity

Before beginning to collect data for the measurement of  $\lambda$ , you should do some simple measurements to get a feel for the basic operation and a sense for how best to design your measurement. With the HV set to 500 V and the count time set to 60 seconds, measure the count rate per minute with the source placed in the top slot (*i.e.*, as close to the Geiger counter as possible). Do this twice so that you can compare the measurements and see the  $\sqrt{N}$  fluctuations. Then collect data with the source in two other positions: at the bottom and about halfway up. You should find that the count rate decreases approximately as the inverse square of the distance between the source and the counter.

Next, remove the source. Place it at least 50 cm away from the counter, and measure the count rate. That will give you a measure of the **background counts**, *i.e.*, the rate of counts that come from sources other than your Co-60 source.

For subsequent measurements, you'll want to place the source in a slot toward the bottom to make sure that you have room to stack at least three different absorbers above it. On the other hand, placing the source close to the detector allows you to collect data faster. Picking the third or fourth slot from the top seems like a good balance; letting you combine the mass thickness of at least three different absorbers to sample a large range of  $z$ .

You already have an idea of the count rate with no absorber. To get an idea of what the count rate will be with absorbers in place, you should take a quick, 60 second long, run with the source in whatever position you've decided upon, and with the thickest set of absorbers available between the source and detector. Then you will know approximately the range of count rates and can decide on a good duration for your subsequent runs. You want them to be long enough that the uncertainty in your measurement of the count rate is less than 5 or 10%, but short enough that you can take many runs with different absorber combinations. The duration you choose needn't be exactly calculated. Rather, it is best to pick something that is about right, and round it to the nearest multiple of 60 so it is easy to calculate counts per minute.



## 2.3 Measuring the mass absorption length

With the source in your chosen position, collect a set of runs starting with no absorber and then with increasing thickness of absorber. You should measure the count rate with at least 10 different  $z$  values, and you'll want to include a large range of  $z$ , both very thin absorbers like the aluminum sheets and also very thick ones like a stack of the thickest available. Your apparatus only contains one of each type of absorber, but you can borrow thick lead sheets from a neighbor or the TA; stacking them will allow larger summed  $z$  values. The density thickness,  $z$ , of each absorber sheet is listed on the door that flips down. The uncertainty on these values is small compared to the effect of the counting uncertainty; you can neglect it.

Make sure not to move the source between runs so that the only thing changing is the absorber thickness.

Record your data carefully in a table. It is a good idea to give each separate run a unique number to help with the bookkeeping. Record the type of absorber(s) used and the total  $z$  value. You should record the raw count rate and its uncertainty, then subtract the background counts (measured above) and record the counts per minute as well as its uncertainty in a different column of your table. Since both the raw counts and the background that you subtract have an uncertainty, the uncertainty on the difference between them should be calculated by adding the two uncertainties in quadrature. An example data table is shown in Fig. 5 to illustrate what is expected.

Once you have collected your data, make a plot of the rate, after background subtraction, as a function of  $z$  and see if it fits to an exponential function as expected from Equation (9). Make your plot with both a linear scale and with a log scale on the y-axis. A log scale is useful in this case because an exponential should look like a straight line on a log scale. You will likely find that your data does not fit well, which

Run #	Time	Source Position	Absorber List	$z$ [g/cm <sup>2</sup> ]	$\Delta t$ [sec]	Nraw [cts]	$\delta N_{\text{raw}}$ [cts]	N [cts/min]	$\delta N$ [cts/min]	N-B [cts/min]	$\delta(N-B)$ [cts/min]
1	15:55	No source	None	0	300	125	11.2	25.0	2.2	0.0	2.2
2	16:02	Slot 1, up	None	0	60	3266	57.1	3266.0	57.1	3241.0	57.2
3	16:03	Slot 1, up	None	0	60	3374	58.1	3374.0	58.1	3349.0	58.1
4	16:04	Slot 1, up	None	0	60	3294	57.4	3294.0	57.4	3269.0	57.4
5	16:05	Slot 1, up	None	0	60	3225	56.8	3225.0	56.8	3200.0	56.8
6	16:06	Slot 1, up	None	0	60	3307	57.5	3307.0	57.5	3282.0	57.5
7	16:08	Slot 1, up	None	0	60	3279	57.3	3279.0	57.3	3254.0	57.3
8	16:09	Slot 1, up	None	0	60	3344	57.8	3344.0	57.8	3319.0	57.9
9	16:11	Slot 1, down	None	0	60	3518	59.3	3518.0	59.3	3493.0	59.4
10	16:12	Slot 2, up	None	0	60	1522	39.0	1522.0	39.0	1497.0	39.1
11	16:13	Slot 3, up	None	0	60	779	27.9	779.0	27.9	754.0	28.0
12	16:14	Slot 4, up	None	0	60	548	23.4	548.0	23.4	523.0	23.5
13	16:16	Slot 5, up	None	0	60	342	18.5	342.0	18.5	317.0	18.6
14	16:18	Slot 4, up	None	0	200	1851	43.0	555.3	12.9	530.3	13.1
15	16:22	Slot 4, up	1	0.0096	200	1807	42.5	542.1	12.8	517.1	12.9
16	16:26	Slot 4, up	2	0.0192	200	1814	42.6	544.2	12.8	519.2	13.0
17	16:30	Slot 4, up	3	0.0591	200	1746	41.8	523.8	12.5	498.8	12.7
18	16:34	Slot 4, up	4	0.1020	200	1732	41.6	519.6	12.5	494.6	12.7
19	16:38	Slot 4, up	5	0.1700	200	1638	40.5	491.4	12.1	466.4	12.3
20	16:42	Slot 4, up	6	0.3280	200	1608	40.1	482.4	12.0	457.4	12.2
21	16:46	Slot 4, up	7	0.6450	200	1574	39.7	472.2	11.9	447.2	12.1
22	16:49	Slot 4, up	8	1.1200	300	2417	49.2	483.4	9.8	458.4	10.1
23	16:55	Slot 4, up	9	2.0660	300	2296	47.9	459.2	9.6	434.2	9.8
24	17:00	Slot 4, up	10	3.4480	300	2123	46.1	424.6	9.2	399.6	9.5
25	17:07	Slot 4, up	11	7.3670	400	2458	49.6	368.7	7.4	343.7	7.8
26	17:20	Slot 4, up	11+11	14.7340	600	2582	50.8	258.2	5.1	233.2	5.5
27	17:30	Slot 4, up	11+11+11	22.1010	600	1882	43.4	188.2	4.3	163.2	4.9
28	17:35	Slot 4, up	1	0.0096	200	1862	43.2	558.6	12.9	533.6	13.1

Figure 5: Example of a data table, illustrating how you might collect the raw data and calculate what you will later plot. Your numbers will differ, of course. The black entries are set values, the red ones are raw measurements, and the blue ones are calculated values. This table was made with Microsoft Excel, which is useful for tabulating the data and doing simple arithmetic. (If you are not familiar with Excel, you can see how these calculations are done by downloading this spreadsheet from the Gauchospace website for this course.

would be revealed by having a large value for  $\chi^2/\text{dof}$ . Inspect the plot to see if you can figure out why. The answer is explained on the next page, but you should be able to figure it out for yourself by looking at the full set of decays in Fig. 3(a) and the discussion of the way  $\beta$  and  $\gamma$  rays interact with matter.

Your plot should show that the data fits well to an exponential function for most of the data points, except for those at the lowest values of  $z$ . That occurs because the Co-60 source involves both  $\beta$  and  $\gamma$  decay. The  $\beta$  particles lose energy quickly and can be stopped by a thin layer of absorber. Once they are all absorbed, the remaining  $\gamma$ 's follow Equation (9). So, you can best measure  $\lambda$  by excluding the data points at low  $z$  from your fit. It is a good exercise to try to overlay two graphs to plot the  $\gamma$ -dominated region at high  $z$  with one symbol (e.g., filled circle) of one color, with an exponential fit, and overlay the  $\beta$  dominated region at low  $z$  using a different symbol of another color.<sup>4</sup> That would show the whole story in a single plot.

## 2.4 Data analysis

Since this is the first lab, we'll go through a detailed example of the type of plots you should make to analyze the data and explain a bit about how. For subsequent labs, the two paragraphs above should be sufficient to guide you, and you'll be expected to develop and execute the analysis on your own.

First, you need to load all your data into the plotting software of your choice, which I will refer to as the *plotter*. (See Chapter 1 for suggested software. Do NOT use MS Excel.) Here, I'll show plots using the sample data presented in Fig. 5. We'll want to look separately at the measurements with varying  $z$  and constant source position, but you can select that subset later, so just put all the data in together. The relevant information is the run number, slot number,  $z$ ,  $N - B$ , and  $\delta(N - B)$ . For most plotters, you need to give them variable names without special characters, i.e., no minus signs or spaces. I'll call them **run**, **slot**, **z**, **N**, and **dN**, where the subtraction of background is implicit rather than explicit in the names **N** and **dN**.

After entering the data into the plotter, you can plot the count rate, **N**, as a function of **run** for the measurements made with the source positioned label-up in the top slot (runs 2 through 8 in Fig. 5). You can select that data by defining a *selection criterion* of  $2 \leq \text{run} \leq 8$ . The resulting graph of **N** vs **run** is shown in Fig. 6(a). Nothing changed between these runs, so they should have count rates that are consistent within uncertainty. Fitting them to a constant (specified by "pol0" or 0th order polynomial) shows what we expect, i.e., about 68% of the data points have error bars crossing the best fit line and about 32% are further away than that. If this were not the case, it would indicate a problem, either because we calculated the uncertainty incorrectly or because something was not stable. Another way to validate that the data and check whether the uncertainties are appropriate is by looking at the "fit quality" as reported by  $\chi^2$  over the number of degrees of freedom (ndf). As explained in Chapter 1, this ratio should be close to unity. The value of  $\chi^2/\text{ndf} = 4.4/6$  is reasonable.

Next, we'll add the one data point taken with the source in the same position but flipped upside down. That is run # 9, which we can pick out using a new selection criterion. Using the "Overlay Graphs" feature, we can plot the data from runs 2 through 8 with a fit as before and then overlay a graph of the count rate for run # 9. That is shown in Fig. 6(b). A different color and marker shape was selected for the run # 9 point to highlight its different configuration. It is worth taking the time to choose a style for your plots that makes them clear with proper axis labeling and choice of colors.

---

<sup>4</sup>By varying both color and symbol you ensure that your plot will remain informative despite vagaries potentially beyond your control (e.g., if the viewer is color-blind, if the printer is black-and-white, or if the resolution of the display is low).

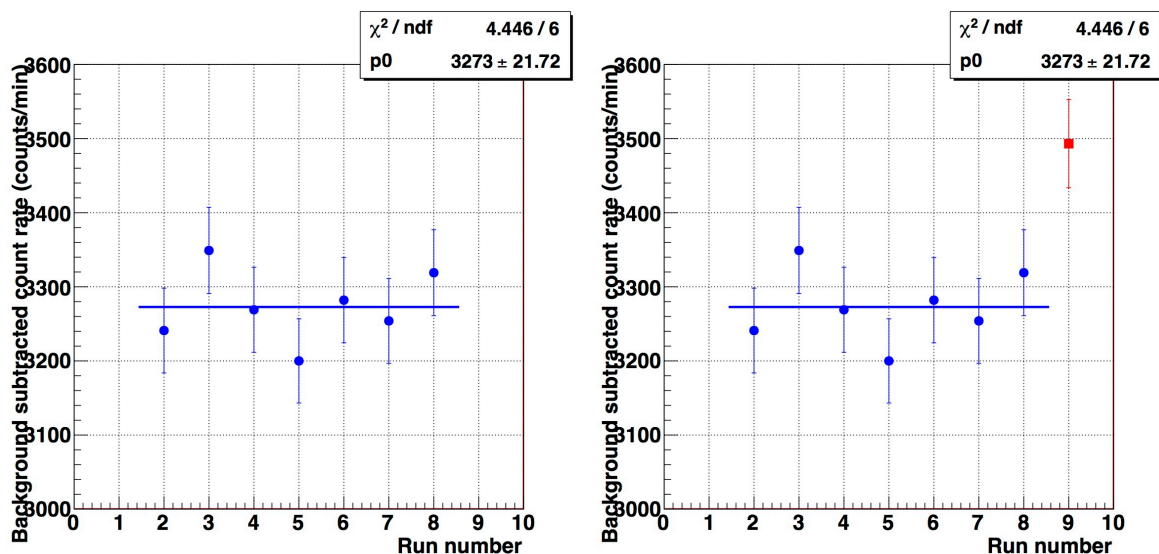


Figure 6: (a) Count rate vs run number taken with the source positioned in the top slot with no absorber. (b) The same graph as in (a) but including run # 9, which is identical except that the source was turned upside down. A significantly different count rate is obtained in that configuration.

The run taken with the label side down has a count rate that is higher than the others by about three times the size of its uncertainty. This indicates that there is a significant difference between these two configurations. After completing the lab, you should have enough information to figure out why. A properly skeptical physicist would point out that the difference between run # 9 and the other runs could simply be due to inadvertently moving the source to the left or right in its holder while flipping it over. Perhaps that affected the counting rate, regardless of whether it was facing label up or label down. How could you check this possibility? The answer is in this footnote<sup>5</sup>.

Next, we should plot the count rate as a function of  $z$  for the data with the source held fixed while the absorber thickness varied (runs 14-27). (You could include run 28, though it is mostly useful as a check that there is no significant difference between the start of the data taking period and the end. However, comparing the count rates for runs 15 and 28, we see that they are consistent, so run 28 has served its purpose.)

To make the desired plot, we select the data for these runs with an appropriate criterion and choose a min and max for the plotting range that will display all the data. Figure 7 shows this plot with a linear scale for the abscissa (y-axis) on the left and with a logarithmic scale on the right. To get the logarithmic scale, click the “Log y” checkbox. But beware that you have to change the minimum value for the y-axis to be something greater than zero for any log plots.

In both plots, the data is fit to an exponential function. The resulting fitted function is the same in both cases, but the logarithmic scale in the right hand plot makes it look linear. (That is one benefit of using a log scale.) Notice, however, that the fit quality is poor. You can tell that because the reported  $\chi^2$  is quite a bit larger than the number of degrees of freedom, 34.8 *versus* 12. You can also see that many of the data points are farther away from the fit line than expected based on their error bars. We need to investigate and understand this to proceed with the analysis.

A hint of what is going on comes from noticing that the points at high  $z$  match the fit line better than

<sup>5</sup>We should have taken several runs where we flipped the source repeatedly to see if the label up and label down runs were consistent with each other despite repositioning and showed a stable discrepancy based only on up/down orientation.



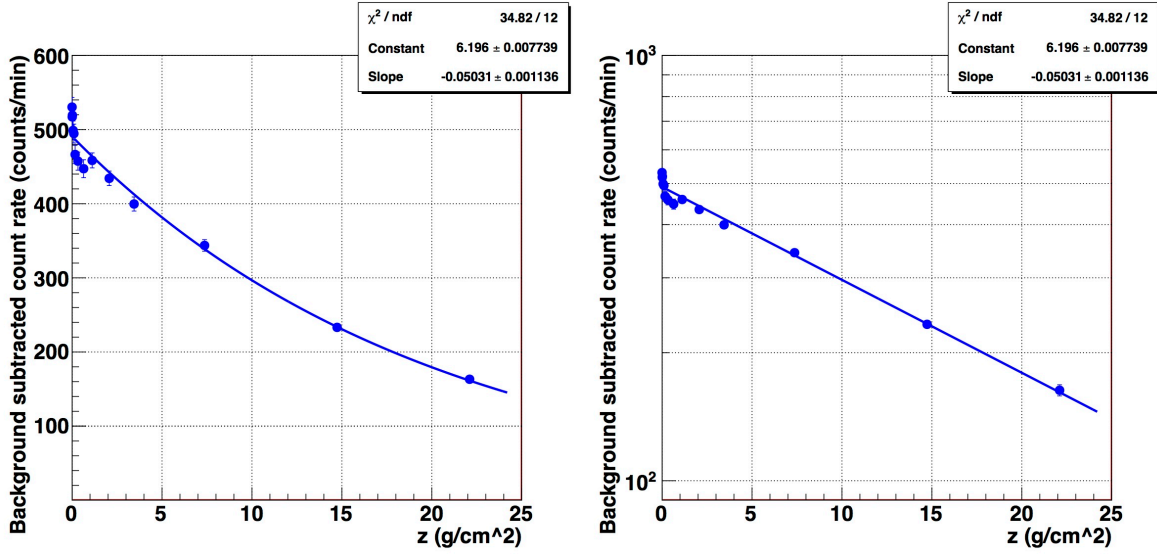


Figure 7: Measured count rate *vs* absorber density thickness,  $z$ , with an exponential fit, plotted with a linear abscissa scale on the left and a logarithmic abscissa scale on the right.

the points at low  $z$ . In fact, the largest discrepancies seem to be from runs taken at very low  $z$ . Perhaps there is something different about the low  $z$  points? Of course, when the fit is done using all the points it will be pulled around by the combined effect of all of them. We can more cleanly investigate how the low *vs* high  $z$  points behave by fitting them separately. In Fig. 8(a) we use the “overlay graphs” feature to fit the high  $z$  data points to an exponential and then overlay the low  $z$  points. I decided to separate low and high at  $z = 0.5 \text{ g/cm}^2$  after making some test plots where I zoom the x-axis scale into the low  $z$  region. Figure 8(b) illustrates this with such a zoomed plot.

These plots clearly show that the data taken with very thin absorbers,  $z$  less than about  $0.2 \text{ g/cm}^2$ , have a different behavior from the data with larger  $z$ . The high  $z$  data points fit well to an exponential,

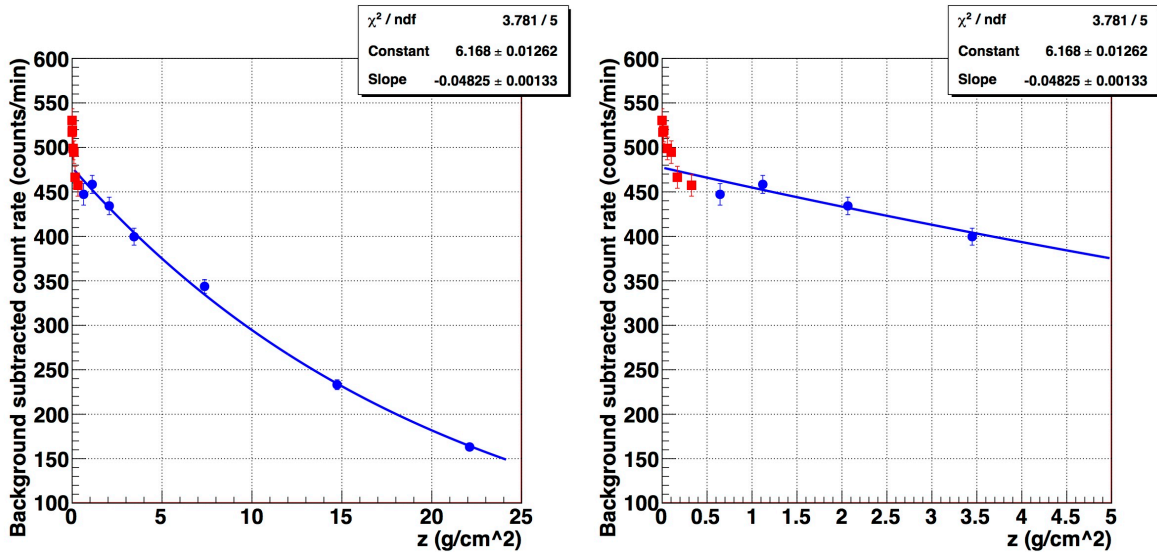


Figure 8: Measured count rate *vs* absorber density thickness,  $z$ , with an exponential fit to only data points with  $z > 0.5 \text{ g/cm}^2$ . The full range of  $z$  is shown on the left, while the right-hand plot shows the low  $z$  region where a discrepancy is observed for  $z$  less than about  $0.2 \text{ g/cm}^2$ .

while the low  $z$  points have a different shape that could be a much steeper exponential. You should be able to figure out why. Since we want to measure  $\lambda$  for gamma ray absorption, it is only the high  $z$  data that is useful for our purpose. The slope of the exponential fit in Fig. 8 is related to  $\lambda$ . But beware that the fitted function is  $y(x) = e^{mx+b}$ , where  $m$  is the reported “Slope” and  $b$  is the reported “Constant”. So the  $\lambda$  we want from Equation (9) is  $\lambda = 1/m$ . That is easy to calculate, but remember to properly propagate the uncertainty in the slope to an uncertainty on  $\lambda$ .

### 2.4.1 Beware biases

Finally, it is worth discussing the points with  $0.2 < z < 0.5$  in Fig. 8(b). They are between the regime of high  $z$ , which we want to use for our measurement, and the regime of low  $z$  that we want to avoid due to bias from some other effect. What should we do with them? It is best to leave them out of the fit and use only points that are well-separated from the problematic region, as I have done in the figure. You might notice, however, that those points are actually *below* the fit while the bias effect at low  $z$  pushes points above the fit. If we were to include those two data points in the fit they would change the slope, so should we include them? In general, no. In particular, you should never decide whether to include or exclude a data point based on its value and what it would do to the final result. Such *a posteriori* “cherry-picking” of the data would bias your results! On the other hand, if our selection of the minimum fit range of  $z = 0.5$  is based on how the data looked around that region, either consciously or unconsciously, then we could have already introduced a bias.

There are two ways to get around this problem. First of all, don’t pick the boundaries too precisely; just choose a round number that is well-separated from the region that you want to exclude. The choice of  $z > 0.5$  is a reasonably round number, and so is  $z > 1.0$ . Secondly, the best thing to do is to collect a new set of data. Having figured out from a first round of data that there is a problematic region to exclude and established a boundary beyond which to exclude it (either 0.5 or 1.0 would be fine in this case), you should collect a whole new set of data to make the final measurement. The fit of this new data will not be affected by fluctuations that might have biased our choices in the first data set. Taking multiple data sets requires more time, but rigor requires effort.

### 2.4.2 A concise summary of results

When you write your conclusions for a measurement that you have made, you want it to be both concise and complete. One way to achieve this is with a single, well-crafted plot. Figure 8(a) is an example of such a plot. It shows both the low  $z$  and high  $z$  data (clearly distinguished by color as well as symbol shape) so it can be used to illustrate the low  $z$  bias. It also shows the fit result, which conveys the outcome of the measurement, and the fit quality ( $\chi^2$ ), which illustrates its validity. You should take the time to properly prepare one or two “summary plots” such as this for each of your measurements. Place graphs in your notes near your conclusions.

## 2.5 Going Beyond

There are several ways that exceptional students would go beyond the basics in this lab. I list a few ideas below, but I encourage you to use your own creativity.

- Check the  $\sqrt{N}$  fluctuations by taking repeated measurements and measuring the RMS of their histogram.

- Measure the uncertainty in the thickness of the absorbers. You can do this by measuring the count rate with several different absorbers that are listed as having the same density thickness; use the spread in count rate and the measured  $\lambda$  to deduce the variation in thickness.
- Do you see twice as many gammas as betas, as predicted in Fig. 3(a)? If not, why not? (Hint: where is the source material?)
- Measure the total activity of the source.
- If the source is in the bottom slot, is there any difference between putting a plastic absorber right above it vs in the top slot? If so, why?
- Carefully investigate whether there is a difference in the count rate when the source is placed label up or label down. Suggest an explanation for this based on what else you observed in the lab. Can you test that explanation somehow?
- Measure the count rate for any object of your choice, like a dollar bill. Is it significantly higher than the background count rate? Why or why not?