

Cylindrical coordinate system, (r, z) plane. $S = S(r, z, t)$.

$$\begin{aligned}
\Delta S &= \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial S}{\partial r} \right) + \frac{\partial^2 S}{\partial z^2} \approx \\
&\approx \frac{1}{r} \frac{\partial}{\partial r} \left(r_i \frac{S_{i,j,k} - S_{i-1,j,k}}{g} \right) + \frac{\partial^2 S}{\partial z^2} \approx \\
&\approx \frac{r_{i+1/2} \frac{S_{i+1,j,k} - S_{i,j,k}}{g} - r_{i-1/2} \frac{S_{i,j,k} - S_{i-1,j,k}}{g}}{r_i g} + \frac{\partial^2 S}{\partial z^2} \approx \\
&\approx \frac{r_{i+1/2} S_{i+1,j,k} - (r_{i+1/2} + r_{i-1/2}) S_{i,j,k} + r_{i-1/2} S_{i-1,j,k}}{r_i g^2} + \frac{S_{i,j+1,k} - 2S_{i,j,k} + S_{i,j-1,k}}{h^2}.
\end{aligned} \tag{1}$$

Here g and h are step sizes correspondingly in r and z axes. Note that $r_{i+1/2} = r_i + \frac{g}{2}$ and $r_{i-1/2} = r_i - \frac{g}{2}$.