

Exercise 1 Show by induction:

- a) $1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$
- b) For any $n \geq 7$, we have that $n! > 3^n$

Exercise 2 Show that the following recursive algorithm computes correctly the factorial of a number. Assume that $n \geq 0$.

Function $F(n)$

```
if  $n = 0$  then
     $s = 1$ 
else
     $s = n \cdot F(n - 1)$ 
return  $s$ 
```

Exercise 3 Show that the following insertion sort algorithm is able to sort a list A of n numbers in nondecreasing order.

Function $IS(n, A)$

```
if  $n \geq 2$  then
     $IS(n - 1, A)$ 
     $i = n - 1$ 
    while  $i \geq 1$  and  $A[n] < A[i]$  do
         $i = i - 1$ 
     $i = i + 1$ 
     $p = A[n]$ 
     $j = n - 1$ 
    while  $j \geq i$  do
         $A[j + 1] = A[j]$ 
         $j = j - 1$ 
     $A[i] = p$ 
```

Exercise 4 Read the problem *A new chess game* in Mooshak. Consider a recursive approach to solve it.