Exercise 1 Consider the two following problems:

- Sorting problem: Given a set $S = \{s_1, \ldots, s_n\}$ of n non-negative integer values, order them in non-decreasing order.
- Median problem: Given a set $R = \{r_1, \ldots, r_m\}$ of m non-negative integer values, find its median.

a) Reduce the median problem to the sorting problem. What is the time complexity of this transformation?

Exercise 2 Consider the following problem:

- Graph coloring: Given a graph $G = (V, E)$, and a positive integer k, color the nodes with at most k colors such that no two adjacent nodes have the same color.

a) Sketch an algorithm that, given a graph that is arbitrarly coloured with k colors, and a function $color(u)$ that returns the color of node u, it outputs the number of pairs of nodes that violate the constraint above. Discuss its time complexity.

b) Assume that you have an algorithm $\mathcal{A}$ to solve the Graph Coloring problem as stated above. Sketch an algorithm to solve the optimization version of this problem (to minimize the number of colors) that uses $\mathcal{A}$. Give the number of calls to $\mathcal{A}$.

Exercise 3 Consider the two following problems:

- 3-Sum: Given a set of n distinct integers, find whether it contains three elements that sum to 0.
- 3-Collinear: Given a set of m distinct points in the plane, find whether it contains three collinear points.

a) A set of points is collinear if the slopes between any pair of points of that set match. Explore this property to solve the 3-Collinear problem. Discuss its time complexity.

b) Show that the 3-Sum problem can be reduced to the 3-Collinear problem. Hint: Consider the property above.

c) A widely known conjecture claims that it is not possible to solve the 3-Sum problem in less than $O(n^2)$ amount of time. If this conjecture is true, what can you state about the time complexity of solving the 3-Collinear problem?

Programming Exercise

- Read the problem A - *Party Icebreaker* in EA2023_PL at mooshak. Relate it with the 3-Sum problem and solve it. Hint: An $O(n^3)$-time algorithm may not be enough to solve all test cases.