$X \sim Binom(8, \frac{1}{2})$, $E[X] = 4$ we never proved this, we just got it from looking at the PMF plot

$$E[X] = \sum_{x=0}^{8} x\, p(x) = \cancel{0 \cdot p(0)} + 1\, p(1) + \ldots + 8\, p(8)$$

= .031 + 2 * .109 + 3 * .219 + 4 * .273 + 5 * .219
6 * 109 + 7 * .031 + 8 * 0.004 = 4

$X \sim Binom(9873, 0.316)$, $E[X] = \sum_{x=0}^{9873} x\, p(x) =$ ~~too~~ much!, we need a general formula

$X \sim Binom(n, p)$ $E[X] = np$ intuitive!

$$\frac{x}{x!} = \frac{x}{x(x-1)!}$$

$$E[X] = \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x} = \sum_{x=1}^{n} \cancel{x} \frac{n!}{\underbrace{\cancel{x!}}_{(x-1)!}(n-x)!} p^x (1-p)^{n-x}$$

If x = 0, that term in the sum is 0

$$= \sum_{x=1}^{n} \frac{\overbrace{n!}^{n(n-1)!}}{(x-1)!\underbrace{(n-x)!}_{((n-1)-(x-1))!}} p^{\overbrace{x}^{1+x-1}} (1-p)^{\overbrace{n-x}^{(n-1)-(x-1)}} = np \sum_{x=1}^{n} \underbrace{\frac{(n-1)!}{(x-1)!((n-1)-(x-1))!}}_{\binom{n-1}{x-1}} p^{x-1} (1-p)^{(n-1)-(x-1)}$$

$$= np \overbrace{\sum_{y=0}^{m} \underbrace{\binom{m}{y} p^y (1-p)^{m-y}}_{\text{PMF for } Bin(m,p)}}^{1} = \boxed{np}$$

let $y = x - 1 \Rightarrow x = y + 1$, $m = n - 1 \Rightarrow n = m + 1$

$X \sim Hyper(n, K, N)$, $E[X] = \sum_{\underbrace{x \in Supp[X]}_{4 \text{ cases}}} x \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}} \overset{HARD}{=} \ldots = n\frac{K}{N}$

there is an easier way to solve this

$X \sim U(\{1, 10, 100\}) = \frac{1}{3}$, $Supp[X] = \{1, 10, 100\}$, $E[X] = 1\frac{1}{3} + 10\frac{1}{3} + 100\frac{1}{3}$

$X \sim U(A) = \frac{1}{|A|}$, $E[X] = \frac{1}{|A|}\sum_{x \in A} x$

$X \sim Geom(p) := (1-p)^{x-1} p = p(x)$

$f(x) = x^2$
$g(y) = y^2$
$\not\Downarrow$
$f = g$

$$\Rightarrow \mu = \sum_{x=1}^{\infty} x (1-p)^{x-1} p = \sum_{y=0}^{\infty} (y+1)(1-p)^y p$$

$y = x - 1 \Leftrightarrow x = y + 1$

$\sum (a+b)c = \sum ab + \sum ac$

$$= \sum_{y=0}^{\infty} y (1-p)^y p + \overbrace{\sum_{y=0}^{\infty} (1-p)^y p}^{1}$$

If $y = 0$, term is 0

$$= \sum_{y=1}^{\infty} y (1-p)^{y-1} (1-p) p + 1 = (1-p) \overbrace{\sum_{y=1}^{\infty} y (1-p)^{y-1}}^{\mu} + 1 \Rightarrow (1-p)\mu + 1 = \mu$$

$$\Rightarrow \cancel{\mu} - p\mu + 1 = \cancel{\mu} \Rightarrow -p\mu + 1 = 0 \Rightarrow 1 = p\mu \Rightarrow \boxed{\mu = \frac{1}{p}}$$

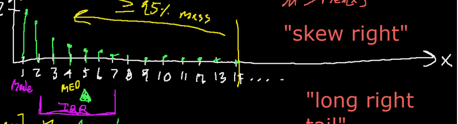
Is E[X] the only useful "summary" of p(x)? No. Let's talk about other univariate ways of summarizing the rv X.

$X \sim Geom(0.2) = 0.8^{x-1} 0.2$

| x | p(x) | F(x) |
|---|---|---|
| 1 | 0.200 | 0.200 |
| 2 | 0.160 | 0.360 |
| 3 | 0.120 | 0.488 |
| 4 | 0.102 | 0.590 |
| 5 | 0.082 | 0.672 |
| 6 | 0.066 | 0.738 |
| 7 | 0.052 | 0.790 |
| 8 | 0.042 | 0.832 |
| 9 | 0.034 | 0.866 |
| 10 | 0.027 | 0.893 |
| 11 | 0.021 | 0.914 |
| 12 | 0.017 | 0.931 |
| 13 | 0.014 | 0.945 |
| 14 | 0.011 | 0.956 |
| 15 | 0.009 | 0.965 |
| 16 | 0.007 | 0.972 |
| 17 | 0.006 | 0.978 |
| 18 | 0.005 | 0.983 |
| 19 | 0.004 | 0.987 |
| 20 | 0.003 | 0.990 |
| 21 | 0.002 | 0.992 |
| 22 | 0.001 | 0.994 |
| 23 | 0.001 | 0.995 |
| 24 | 0.001 | 0.996 |
| 25 | 0.001 | 0.997 |
| 26 | 0.001 | 0.998 |
| 27 | 0.001 | 0.999 |
| 28 | 0.000 | 0.999 |
| ⋮ | | |

Med[X] = 4
IQR = 7 − 2 = 5
Quantile[X, .95] = 14

Effective Support is the set {x : p(x) ≥ 0.001}
so here, the effective support is {1, 2, ..., 27}




$\mu > Med[X]$
"skew right"

"long right tail"

$\mu = \frac{1}{p} = \frac{1}{0.2} = 5$

You cannot read μ from this table.

Q[X, q] := Quantile[X, q] := $\min_{x \in Supp[X]}$ {F(x) ≥ q}

Pctile[X, q] := Quantile[X, q / 100]

Med[X] = Median[X] = Pctile[X, 50] = Q[X, 0.5]
which is another way to summarize the "central tendency" of a rv model other than E[X].

1 = Mode[X] = $\underset{x \in Supp[X]}{\text{argmax}}$ {p(x)} another metric of central tendency but least popular of the three we discussed.
x s.t. p(x) is max

If |Mode[X]| = 1 we call the rv "unimodal"
If |Mode[X]| = 2 we call the rv "bimodal"

IQR[X] = Q[X, 0.75] - Q[X, 0.25]
interquartile range

Range[X] = max{Supp[X]} - min{Supp[X]}

Custom build rv models.
For example, roulette in America. Bet $1 on black. Payout is 1:1.
The rv model for your winnings is:

$$X \sim \begin{cases} \$1 & \text{wp } \frac{18}{38} \\ -\$1 & \text{wp } \frac{20}{38} \end{cases} \qquad \mu = \$1 \cdot \frac{18}{38} + -\$1\frac{20}{38} = -\$\frac{2}{38} = -\$0.053$$

$$X \sim \begin{cases} \$35 & \text{wp } \frac{1}{38} \\ -\$1 & \text{wp } \frac{37}{38} \end{cases} \qquad \mu = \$35 \cdot \frac{1}{38} + -\$1\frac{37}{38} = -\$\frac{2}{38} = -\$0.053$$

For example, roulette in America. Bet $1 on 1-12.
Payout is 2:1. The rv model for your winnings is:

$$X \sim \begin{cases} \$2 & \text{wp } \frac{12}{38} \\ -\$1 & \text{wp } \frac{26}{38} \end{cases} \qquad \mu = 2\frac{12}{38} + -1\frac{26}{38} = -\frac{2}{38} = -\$0.053$$

All bets in a roulette table payout so that the expectation is -2/38 and those 2 come from the 0 and 00 slots (the house edge).