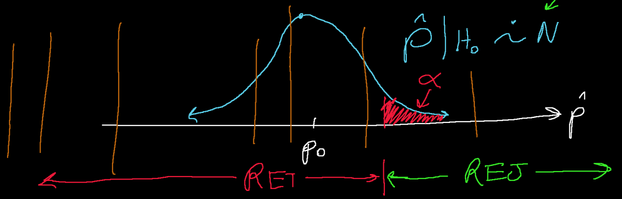
One-sided one-proportion z-test. Let's consider hypotheses:

$H_a: p > p_0 \overset{e.g.}{=} 0.5 \Rightarrow H_0: p \le p_0$

by CLT

$\hat{p} \mid H_0 \sim N$



$RET = \left[0,\ p_0 + z_\alpha \sqrt{\frac{p_0(1-p_0)}{n}}\right]$

for $\alpha = 2.5\%$, $z_\alpha = 2$

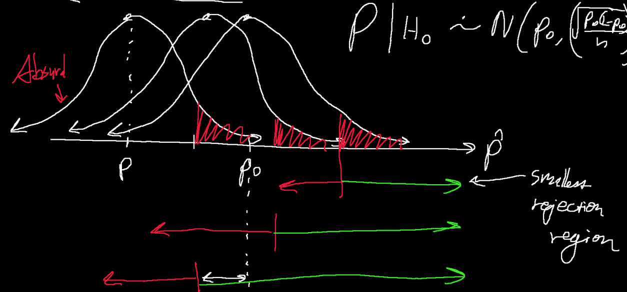
$\hat{p} \overset{?}{\in} RET$ $\overset{Yes}{\Rightarrow}$ Retain H_0

$\overset{No}{\Rightarrow}$ Reject H_0

$H_a: p > p_0 \Rightarrow H_0: p \le p_0$

$(\sqrt{2})^2 = 2$

$\hat{P} \mid H_0 \sim N\left(p_0, \left(\sqrt{\frac{p_0(1-p_0)}{n}}\right)^2\right)$

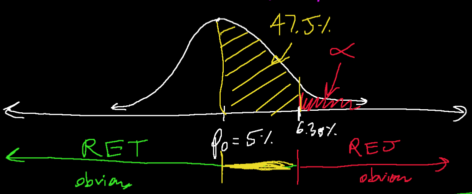


To give the *most* benefit of the doubt to the null, you provide the sampling distribution with p so that rejections are the most difficult. This is p = p_0.

If more than 5% of *all* customers do not give 5★ to an uber driver, then that driver is fired.

This is a statement about an unknowable parameter p. Uber doesn't like firing people. So Uber wants to give the most benefit of the doubt to its drivers. The hypothesis testing framework can be used here in a non-traditional way.

$H_0: p \le p_0 = 5\% \Rightarrow H_a: p > p_0 = 5\%, \quad \alpha = 2.5\%$



$RET = \left[0,\ 0.05 + 2\sqrt{\frac{0.05(1-0.05)}{n}}\right] = [0,\ 0.0638]$

Uber makes this decision after n = 1000 rides. Thus, if p-hat > 0.0638, Uber fires the driver.

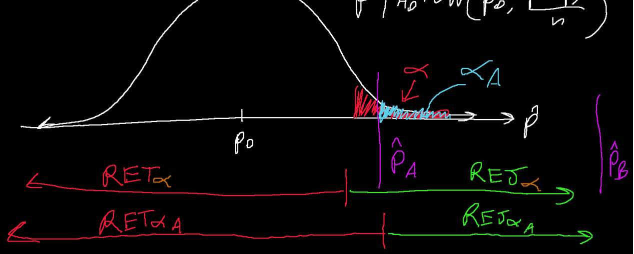
Why is 0.05 ≠ 0.0638? If the rule is "more than 5% of riders don't give 5★ then the driver is fired" then why can't we fire a driver if p-hat > 0.05??

MATH241 ↑ FINAL

MATH 369 ↓ NOT ON FINAL

Consider $H_0: p \le p_0$ vs. $H_a: p > p_0$, α declared

$\hat{P} \mid H_0 \sim N\left(p_0, \frac{p_0(1-p_0)}{n}\right)$



Which is a "stronger rejection" (the null hypothesis is more "nullified")? p-hat-A or p-hat-B? Yudava said p-hat-B is the "stronger rejection" because it's further away from the mass of the sampling distribution - it's more "extreme".

RA Fisher developed a way to "measure" the strength of the estimate and it's called "Fisher's p value" or "p-val".

$p_{val} := \max_{\alpha} \left\{ \hat{p} \in RET_\alpha \right\} = P(\hat{P} > \hat{p} \mid H_0)$ in the right-sided one-prop z-test

The probability of observing your estimate or "more extreme" given the null hypothesis is true.

$\Rightarrow p_{val} < \alpha \Rightarrow$ Reject H_0, $p_{val} \ge \alpha \Rightarrow$ Retain H_0