$X_{1,...,}X_{n}$  seque of rivs  $X_{n} \xrightarrow{d} M$  Pf:  $Y \sim \log_{2}(n)$  50  $X_{n} \xrightarrow{d} Y$ . Let  $T_{n} = X_{n} + ... + X_{n}$  $\Rightarrow n_{Y}(t) = e^{t n}$ 

But still... why is the normal distribution special?

Let  $X_1,\dots,X_n \stackrel{iid}{\sim}$  with mean  $\mu$  and variance  $\sigma^2$  $\mathbb{E}\left[\overline{X}_{h}\right] = M$ ,  $\sqrt{a} \cdot \left[\overline{X}\right] = \frac{\overline{\sigma}^{2}}{h}$  $Z_{n} := \frac{X_{n} - A_{n}}{6} \qquad E[Z_{n}] := E\left[\frac{X_{n}}{S_{n}}\right] - E\left[\frac{A_{n}}{S_{n}}\right] := \frac{A_{n}}{S_{n}} = 0$ 

tandardizing a rv  $\sqrt{4v} \left[ \overline{Z_n} \right] = \sqrt{4r} \left[ \frac{\overline{X}}{\frac{C}{\sqrt{n}}} \right] = \frac{1}{\frac{C}{\sqrt{n}}} \sqrt{4r} \left[ \overline{X} \right] = \frac{C}{\frac{C}{\sqrt{n}}} = 1$ i.e. subtract the mean and divid by the standard deviation yields a rv with mean zero and Var & SD = 1  $\sum_{n} = \frac{\sqrt{\frac{x_{1} + ... + x_{n}}{n}} - x_{1}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}} = \frac{\sqrt{\frac{x_{1} + ... + x_{n}}{n}} - x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}}} = \frac{x_{1} + ... + x_{n}}{\sqrt{\frac{x_{1} + ... + x_{n}}{n}}}$ 

 $= \frac{1}{\sqrt{n}} \left( \left( X_{1} - M_{1} \right) + \left( X_{2} - M_{1} \right) + \dots + \left( X_{n} - M_{n} \right) \right)$   $= \frac{1}{\sqrt{n}} \left( \frac{X_{1} - M_{1}}{\sigma} + \frac{X_{2} - M_{1}}{\sigma} + \dots + \frac{X_{n} - M_{n}}{\sigma} \right) \stackrel{\text{def}}{=} \frac{Z_{1} + \dots + Z_{n}}{\sqrt{n}} \stackrel{\text{def}}{=} \frac{Z_{1} + \dots +$  $= e^{h \ln \left( n_{2} \left( \frac{t}{\sigma_{n}} \right) \right)} = e^{h \ln \left( n_{2} \left( \frac{t}{\sigma_{n}} \right) \right)}$ 

 $h_{Z_{i}}(t) = h_{Z_{i},x_{i},z_{i}}(t) = h_{Z_{i}}(t) \cdot \dots \cdot n_{Z_{i}}(t) = h_{Z_{i}}(t)^{h}$ Now we take the limit of the MGF for Z\_n as  $n \to \infty$  in the hope of finding something cool  $\lim_{h \to \infty} h_{n}(t) = \lim_{h \to \infty} \lim_{h \to \infty} \frac{\ln \left( \ln_{2}(t) \right)}{\ln_{n}(t)} = \lim_{h \to \infty} \frac{\ln_{n} \left( \ln_{2}(t) \right)}{\ln_{n}(t)}$ (Hapisal's Rule + M2(t) let u= In > 4 > 0

so the result is the MGF for the rv N(0, 1) hence.....  $\rightarrow N(0,1)$ This is called the "Central Limit Theorem" (CLT) which is probably the most famous thm in all of probability. This is unbelievable since we made no assumptions about the X's except that they were iid and they had mean  $\mu$  and variance  $\sigma^2$ . They could've been any rv's!! But their standardized average no matter what becomes more and more normal. Implications of the CLT. Recall, limits don't exist so, it's never exactly normally distribution, only "approximately distributed" so:

 $Z_h = \frac{X_h - h}{\frac{\sigma}{\sqrt{h}}} \sim N(0, 1)$ 

Imagine you're standing on a line and with 50% prob you take a step forward and with 50% prob you take a step backward (indep) Let's say you take 100 such random steps. (1) Where do you finish on average? (2) What is the approx probability you are more than 10 steps away from where you started? X, ..., X,000 2 3-1 up 50% 62 = (1-52(5) + (1-52.5 ,5+,5=| >0=| E[X] = n = 0(2) P(|T| > 10) = P(T > 10) + P(T < -10)CLT: TniN(nn, (650)2)

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 $P(T>10) = P\left(\frac{T\cdot 0}{10} > \frac{10\cdot 1}{10}\right)$ 

 $= \rho(\geq > 1) = 16^{\prime}$ 

P(T<-10) = P/T-0 <

Here is the distribution of light bulb failure time, rv X: 6 = 500 day

You have 50 lightbulbs. What is the probability the average burn out is more than 1300 hr?