Probability and statistics (20 lectures on former, 3 on latter).

Set theory (1870's): sets are fundamental units of all mathematical entities. Math is built atop sets. Sets: unordered collections of unique elements e.g.

F := {Jane, Mary, Susan, Dana}n

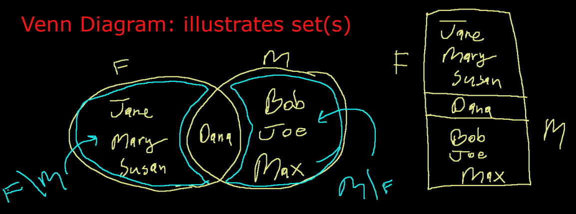
elements

separator

descriptive letter (females)

M := {Bob, Joe, Max, Dana}

Venn Diagram: illustrates set(s)



Sets can have infinite elements e.g.

$\mathbb{N}$:= {1, 2, 3, ...} natural numbers $\mathbb{N}_0 := \{0, 1, 2, 3, \ldots\}$

$\mathbb{Z}$:= {..., -2, -1 ,0, 1, 2, ...} integers

Operators on sets

Jane ∈ F "set inclusion operator" "element of operator"
It's either true or false. As convention, we only write statements that are true. E.g. we never write 1 = 2.

Bob ∉ F "not element of"

{Jane, Mary} ⊆ F "subset operator" (also true or false).
Every element on the left hand side (lhs) is an element of the right hand side (rhs).

G := {Jane, Dana, Mary, Susan}

G = F (set equality) defined as G ⊆ F & F ⊆ G

{Jane, Mary} ≠ F (set inequality)

{Jane, Mary} ⊂ F "proper subset operator" (also true or false)
Subset but not equal.

FYI: =, ⊆, ∉, etc are "predicate functions ∉(A, B) RETURN {TRUE, FALSE}

{Jane} ⊂ F, Jane ∈ F

Jane ⊄ F

Functions that return other sets. For example: union

F ∪ M = {Jane, Mary, Susan, Dana} ∪ {Bob, Joe, Max, Dana}
= {Jane, Mary, Susan, Dana, Bob, Joe, Max}

Union combines all elements together and drops duplicates. It's not really addition, but kinda close. Union is "non-exclusive or" (XOR

$\mathbb{N}_0 = \mathbb{N} \cup \{0\}$ singleton set: a set with one element

Intersection is the function which returns a set of elements in both sets (AND).

F ∩ M = {Dana}

F ∩ {Bob, Joe} = {}, $\emptyset := \{\}$ the "empty set"

If A ∩ B = ∅, then "A and B are 'mutually exclusive'".

Can two infinitely large sets be "mutually exlcusive"?

E = {2, 4, 6, ...}, O = {1, 3, 5, ...}, E ∩ O = ∅

∅ ⊂ F, this is called "vacuously true"

∅ ∉ F

Set subtraction

F \ M all elements in F that are not in M

M \ F all elements in M that are not in F

F\M ≠ M\F

Can you find a situation where A \ B = B \ A?

A and B being mutually exclusive e.g.

A = {2, 4}, B = {1, 3},

A \ B = {2, 4} = A

B \ A = {1, 3} = B

=> A \ B ≠ B \ A

A = {2, 4}, B = {2, 4},

A \ B = ∅

B \ A = ∅

A \ B = B \ A

∅ \ ∅ = ∅

A ⊄ B, A \ B = ∅

"Set Builder" Notation. E.g. all elements 2n such that n is an integer.

E := {2n : n ∈ $\mathbb{Z}$} = {..., -4, -2, 0, 2, 4, ...}

condition of set membership

elements