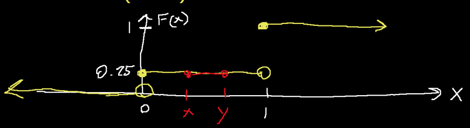
F(x) := P(X ≤ x) this is called the "cumulative distribution function" (CDF) and it is 1:1: with p(x), the PMF.

X ~ Bern(0.75). Here is an illustration of F(x)



$Supp[X] \subset \mathbb{R}$

$P(X \le \infty) = 1$

$P(X \le -\infty) = 1 - \underbrace{P(X > -\infty)}_{1} = 0$

Properties of the CDF

(1) $\lim_{x \to \infty} F(x) = 1$

(2) $\lim_{x \to -\infty} F(x) = 0$

(3) x ≤ y ⇔ F(x) ≤ F(y)

i.e. F is monotonically increasing

$x \le y \Rightarrow y = x + \delta,\ \delta \ge 0$

$F(x) \le F(x+\delta) = P(X \le x+\delta) = P(X \le x) + P(X \in (x, x+\delta)) = F(x) + \bigcirc = F(y)$

(4) F(x) ∈ [0, 1] since it's a probability

(5) # of discontinuities in F is |Supp[X]|

$X \sim Bern(p) := \begin{cases} 1 & wp\ p \\ 0 & wp\ 1-p \end{cases} = \underbrace{p(x) = p^x(1-p)^{1-x}}_{PMF}$

$X_1 \sim Bern(p) := p^x(1-p)^{1-x} = p_{X_1}(x)$

$X_2 \sim Bern(p) := p^x(1-p)^{1-x} = p_{X_2}(x)$

Is $x_1 = x_2$ always? No. But is $X_1 = X_2$?

We call this "equals in distribution" and denote it $X_1 \overset{d}{=} X_2$ which means

$p_{X_1}(x) = p_{X_2}(x)$ or $F_{X_1}(x) = F_{X_2}(x)$

Consider 10 cards, 4 red (R), 6 blue (B)

Let's draw cards without replacement from these 10.

P(2 R out of 3 cards) = $\dfrac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}}$

P(x R out of 3 cards) = $\dfrac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$

P(x R out of n cards) = $\dfrac{\binom{4}{x}\binom{6}{n-x}}{\binom{10}{n}}$

Consider 10 cards, K red (R) and 10-K blue (B)

P(x R out of n cards) = $\dfrac{\binom{K}{x}\binom{10-K}{n-x}}{\binom{10}{n}}$

Consider N cards, K red (R) and N-K blue (B)

P(x R out of n cards) = $\dfrac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}} = p(x)$

X ~ Hyper(n, K, N)

This is a new rv called the "hypergeometric" rv model. It has three parameters: n, K, N. The value x is the realization value (it's the number of cards that you care about in your sample of n cards).

There are 20 people in our classroom today. 6 female, 14 males. If we sample 8 students at random, what is the probability 4 will be female?

X ~ Hyper(n = 8, K = 6, N = 20) := $\dfrac{\binom{6}{x}\binom{14}{8-x}}{\binom{20}{8}} = p(x)$

P(X = 4) = $\dfrac{\binom{6}{4}\binom{14}{4}}{\binom{20}{8}}$

What is the parameter space of the hypergeometric rv?

$N = 0 \Rightarrow K = 0, n = 0 \Rightarrow x \in \{0\} \Rightarrow Hyper(0,0,0) = Deg(0)$

$N = 1 \Rightarrow K = 1, n = 1 \Rightarrow x \in \{1\} \Rightarrow Hyper(1,1,1) = Deg(1)$

$ \Rightarrow K = 0, n = 1 \Rightarrow x \in \{0\} \Rightarrow Hyper(1,0,1) = Deg(0)$

$n \in \{0, 1\}$

we also know that Hyper(0, K, N) = Deg(0)

$N = 2 \Rightarrow K = 2 \Rightarrow Hyper(n, 2, 2) = Deg(n)$

$n \in \{0, 1, 2\}$

$ \Rightarrow K = 1 \Rightarrow Hyper(2, 1, 2) = Deg(1)$

$ \Rightarrow K = 0 \Rightarrow Hyper(n, 0, 2) = Deg(0)$

$Hyper(1,1,2) = \dfrac{\binom{1}{x}\binom{2-1}{1-x}}{\binom{2}{1}} = p(x) = Bern\left(\tfrac{1}{2}\right)$

$p(0) = \dfrac{\binom{1}{0}\binom{1}{1-0}}{\binom{2}{1}} = \dfrac{1 \cdot 1}{2} = \dfrac{1}{2},\quad p(1) = \dfrac{\binom{1}{1}\binom{1}{1-1}}{\binom{2}{1}} = \dfrac{1}{2}$

Param Space:

$N \in \mathbb{N} \setminus \{1\},\ K \in \{1, \ldots, N-1\},\ n \in \{1, \ldots, N-1\}$

$X \sim Hyper(1, K, N) := \dfrac{\binom{K}{x}\binom{N-K}{1-x}}{\binom{N}{1}} = p(x) = \begin{cases} 1 & wp\ \frac{K}{N} \\ 0 & wp\ \frac{N-K}{N} \end{cases} = Bern\left(\frac{K}{N}\right)$

$p(0) = \dfrac{\binom{K}{0}\binom{N-K}{1-0}}{\binom{N}{1}} = \dfrac{1(N-K)}{N} = \dfrac{N-K}{N},\quad p(1) = \dfrac{\binom{K}{1}\binom{N-K}{1-1}}{\binom{N}{1}} = \dfrac{K(1)}{N} = \dfrac{K}{N}$

We want to now elucidate the Supp[X]. It is not easy. So let's consider a few cases and hopefully find some sort of pattern.

X ~ Hyper(2, 4, 10), Supp[X] = {0, 1, 2}

X ~ Hyper(5, 4, 10), Supp[X] = {0, 1, 2, 3, 4}

X ~ Hyper(8, 4, 10), Supp[X] = {2, 3, 4}

X ~ Hyper(5, 7, 10), Supp[X] = {2, 3, 4, 5}

There are four cases.