Math 340 / 640
Fall 2025 (4 credits)
Course Syllabus

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Course Homepage	`https://github.com/kapelner/QC_Math_340_Fall_2025`
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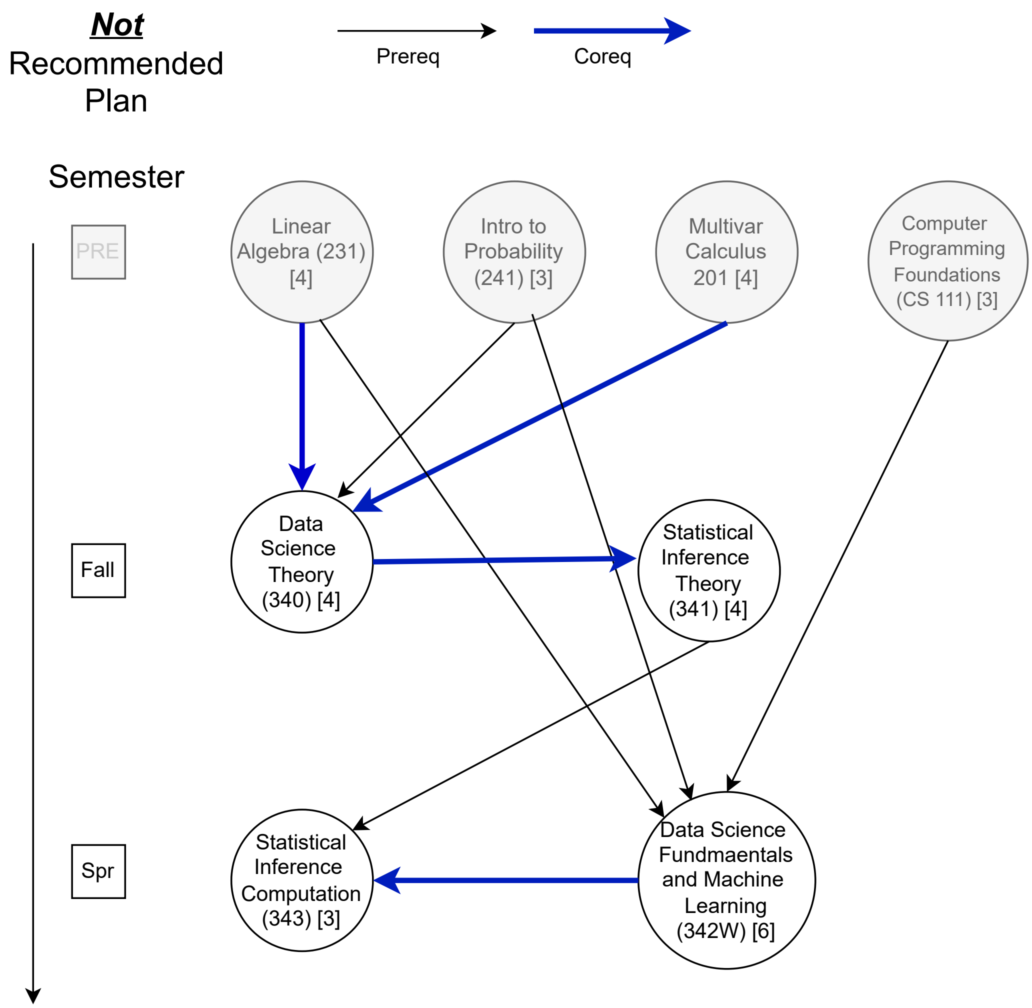
Instructor Office Hours / Loc	see course homepage
TA(s) / Office Hours Time / Loc	Elliot Gangaram / see course homepage

Course Overview

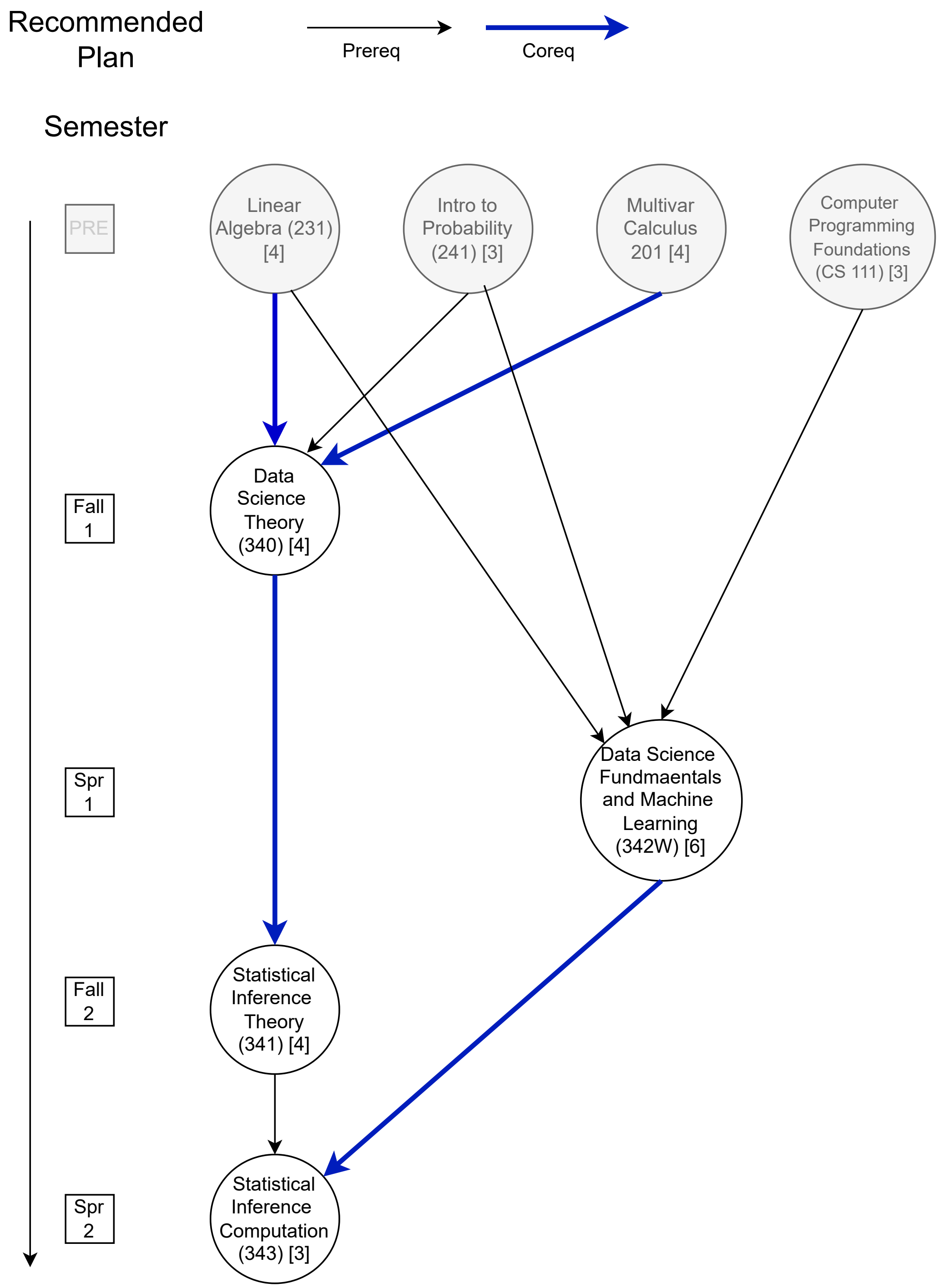
MATH 340 / 640. Probability Theory for Data Science and Statistics. 4 hr.; 4 cr. Prereq.: 241. Coreq.: MATH 201 and 231. Convolutions, multivariate transformation of variables, the poisson process, beta, logistic, laplace, Weibull distributions, characteristic functions, central limit theorem, Cochran's Theorem, the multivariate normal, chi-squared, T, F, Cauchy distributions, vector random variables, covariance matrices, multinomial, multivariate normal, order statistics, law of large numbers, famous inequalities, convergence in distribution and probability, laws of large numbers, Slutsky's Theorem, optimization including stochastic gradient descent. Probability computation demos using modern software. Possible special topics. Not open to students who are taking or who have received credit for MATH 340. Students cannot receive credit for both: MATH 340, and 640. Fall, Spring.

This class is part of the four 34x series which constitute the data science core. There is an order that the four classes need to be taken. Below are three plans, the first is over two semesters and hence it is not *not recommended* as it will be a *very* heavy workload. The second is over four semesters and it is the recommended plan as I believe it will allow students to absorb the material more effectively.



Examining the above, we note that MATH 340 can be taken as a standalone course as a higher-level math elective. It can be thought of as advanced probability following MATH 241. This course is also recommended for students who are considering an actuarial career as this course provides great practice for Exam P.

There is also a third way (not pictured) which is Spring year 1: MATH 342W, Fall year 2: Math 340 and Math 341, Spring year 2: Math 343 which is a middle path with a very difficult Fall semester.


Recommended Plan
Prereq
Coreq
Semester
PRE
Fall 1
Spr 1
Fall 2
Spr 2
Linear Algebra (231) [4]
Intro to Probability (241) [3]
Multivar Calculus 201 [4]
Computer Programming Foundations (CS 111) [3]
Data Science Theory (340) [4]
Data Science Fundmaentals and Machine Learning (342W) [6]
Statistical Inference Theory (341) [4]
Statistical Inference Computation (343) [3]

Tentative Day-by-Day Schedule

Lectures and their topics with rough time estimates per topic are below:

D1 - Lec 1 [10min] Review of discrete random variables (rvs), support; [10min] the Bernoulli rv, parameter space, degenerate rv, indicator functions; [10min] old-style probability mass functions (PMFs), new-style PMFs using the indicator function; [10min] cumulative distribution functions (CDFs), survival functions, [10min] vector rvs, joint mass functions (JMFs), independence, identical distributedness, iid; [25min] sum of two Bernoulli rvs, convolution operator, tree diagram for two rvs, conditional probability, marginal probability; [10min] Plotting the PMF of the convolution; [20min] discrete convolution formulas (general, independent rvs, iid rvs, old and new style), convolution support

D2 - Lec 2 [5min] definition of data as realization from rv's [5min] Uniform discrete rv [5min] defining combinatorial terms with indicator functions; [5min] visualizing convolutions as passing PMFs "through each other" [35min] Binomial rv with PMF derivation, Pascal's identity [40min] Sequences of rv's, derivation of the geometric rv as a "waiting time" or "survival" distribution, geometric rv PMF, geometric series [30min] convolution of geometric rvs, the negative binomial rv

D3 - Lec 3 [30min] Strategy to compute $\mathbb{P}(X > Y)$ for two rvs X, Y, the sum reindexing trick [15min] CDF / survival function of geometric rv [30min] derivation of the exponential rv as a limit of geometric rv's, function limits [15min] definition of continuous rvs, PDF's, PDF rules, JDF's [10min] Uniform rv [30min] derivation of convolution formulas for continuous rvs;

D4 - Lec 4 [35min] Convolution of two iid standard uniform rv's using both derivation methods [30min] derivation of the Erlang rv as a sum of iid exponential rv's [20min] review of complex numbers, Euler's identity [15min] definition of forward and inverse Fourier integral transform [20min] definition of characteristic functions (chf's)

D5 - Lec 5 [10min] demo of Fourier analysis with singing head and chest voices and the same note on the electric piano; [40min] definition of chf's and properties 0, 1, ..., 8 [20min] chf's for Bernoulli, Binomial, Geometric, Exponential [25min] Proof of the very weak Law of Large Numbers (LLN) [20min] Setup of the central limit theorem (CLT), standardization of rv's

D6 - Lec 6 [45min] Proof of the central limit theorem (CLT): derivation of the limiting chf, inversion of the limitiing ch.f [30min] properties of the standard normal, the general normal rv, the normal CDF (Φ function), approximate distributions due to asymptotic convergence [30min] applications of the CLT [5min] intro to Multinomial rv

D7 - Lec 7 [45min] derivation of JMF of multinomial rv, support, parameter space, humpty-dumpty rule [20min] proof that marginal distribution of multinomial is binomial [30min] derivation of covariance, covariance properties, [25min] proof of Cauchy-Schwartz inequality and the covariance inequality

D8 Midterm I Review

D9 Midterm I

D10 - Lec 8 [5min] expectation of vector rv's, special case of multinomial, expectation of matrix rv's [15min] definition of variance-covariance matrix, special case of independence and iid [25min] proof of covariance of components of the multinomial rv [20min] proof of multinomial being the conditional distribution within the multinomial [20min] Markov's inequality [20min] Markov's corrolaries including Chebyshev's inequality

D11 - Lec 9 [30min] derivation of formulas for PMF's of invertible transformations of discrete rv's [10min] example of noninvertible transformation of discrete rv's [40min] derivation of formulas for PDF's of invertible transformations of continuous rv's [30min] logistic rv as a transformation of the exponential, logistic function

D12 - Lec 10 [10min] CDF of the logistic rv [20min] definition of convergence in probability to a constant [30min] proof that convergence in probability to a constant is equivalent to convergence in distribution to a degenerate rv [30min] convergence mapping theorem with proof [20min] statements of Slutsky's theorem A and B

D13 - Lec 11 [20min] proof of Slutsky's thm A for one rv converging to a constant [30min] proof that sample variance estimator converges to variance for all rv's with finite variance [10min] proof that Wilson's estimate for Binomial parameter converges to the mean [20min] proof that T-statistic converges to a standard normal for any iid rv sequence [10min] derivation of the ParetoI distribution as a transformation from the exponential rv [10min] derivation of the Weibull distribution as a transformation from the exponential rv [10min] derivation of the Gumbel distribution as a transformation from the exponential rv

D14 - Lec 12 [10min] derivation of the Laplace rv as a difference of standard exponentials [20min] definition of an error distribution [10min] Laplace and normal rv's as error distributions [10min] derivation of the log normal distribution from the normal rv [20min] revealing and defining the gamma function from the humpty-dumpty property of the Erlang rv [10min] the gamma rv as an extended form of the Erlang rv [10min] lower, upper, regularized gamma functions [10min] common integrals in the gamma family [10min] extending the negative binomial rv using the gamma function [10min] derivation of the chi-squared rv

D15 - Lec 13 [10min] proving the special value of the gamma function at 1/2 [10min] showing the chi-squared rv is really a gamma rv [20min] convolution of gammas / chi-squared rv's using chf's [10min] derivation of chi rv using transformation of rv's [30min] preliminaries for Cochran's theorem and statement of Cochran's theorem [30min] proof that sample variance for iid normal rv is chi-squared-distributed and independent of the sample average via Cochran's theorem

D16 - Lec 14 [20min] formula for jdf of vector rv tranformation [30min] step-by-step procedure to find jdf's for arbitrary transformations [30min] derivation of the quotient transformation PDF [20min] derivation of Student's T rv using the quotient transformation [30min] derinvation of betaprime rv

D17 - Lec 15 [20min] derivation of Snecedor-Fisher F rv [20min] derivation of Cauchy rv from quotient formula [20min] derivation of Cauchy rv from transformation of the uniform rv [30min]

Cauchy chf derivation using an integral trick [20min] proof that sample average of iid Cauchy rv's is Cauchy and not degenerate

D18 Midterm II Review

D19 Midterm II

D20 - Lec 16 [10min] definition of order statistics [15min] derivation of CDF and PDF of the maximum of an iid sequence of continuous rv's [15min] derivation of CDF and PDF of the minimum of an iid sequence of continuous rv's [30min] derivation of CDF and PDF of an arbitrary order statistic of an iid sequence of continuous rv's [20min] derivation of CDF and PDF of an arbitrary order statistic of an iid sequence of standard uniform rv's [20min] definition of kernels and the example of the normal kernel

D21 - Lec 17 [20min] kernels of common example rv's [20min] using kernels to solve the convolution of gamma rv's [10min] definition of the beta function [20min] beta-gamma function equivalence, rewriting PDF of the betaprime and F PDF using the beta function [10min] definition of the beta rv, its CDF and demonstrating equivalnce to order statistics of standard uniform rv's [10min] incomplete and regularized beta functions [20min] proof that proportion of a gamma rv divided by the sum of gamma rv's is beta

D22 - Lec 18 [30min] Proof that CDF transformation of continuous rv's are standard uniform rv's [40min] algorithms for sampling realizations from rv's for different scenarios of discrete and continuous rv's [40min] two examples of conditional, mixture, compound and marginal distributions

D23 - Lec 19 [20min] Betabinomial rv derivation as a compound distribution [20min] BetaNegativebinomial rv derivation as a compound distribution [10min] kernel representations of the normal rv [20min] Normal rv with known variance compounded with a normal rv is normal [20min] derivation of the Poisson rv [20min] Poisson rv compunded with a gamma yields an extended negative binomial

D24 - Lec 20 [40min] Definition of joint chf's and its properties [10min] derivation of the joint characteristic function of the multinomial rv [10min] proof of convolutions of sums of multinomial rvs are multinomial using joint chf's [10min] calculation of the covariance of a multinomial rv using joint chf's [30min] definition and properties of a standard multivariate normal (MVN) rv and general MVN rv [10min] proof of expectation of a vector rv times a matrix identity

D25 - Lec 21 [20 min] proof of variance of a vector rv times a matrix identity [10min] proof that mahalanobis distance is chi-squared-distributed [20min] parameter space of the MVN rv [10min] proof that subvectors of a MVN are MVN [20min] definition and properties of truncated distribution [30min] properties of the Weibull distribution defined by the three sets of Weibull modulus values, memorylessness for the exponential rv

D26 - Lec 22 [40min] 80-20 rule for the ParetoI rv and its physical meaning, fractal property of ParetoI rv truncations [20min] definition, proof and examples of the law of iterated expectation [20min] definition, proof and examples of the law of total variance / variance decomposition [10min] differing views of the philosophy of probability [20min] sampling a multinomial rv iteratively using binomials and conditional distributions

D27 - Lec 23 [30min] Definition of the mode of rv's and how to calculate them [40min] Newton-Raphson method for finding maxima/minima with an example from the beta rv [20min] introduction to the Newton-Raphson method in multiple dimensions using the Hessian matrix [20min] introduction to gradient descent and stochastic gradient descent

D28 Final Review

This is more of a typical mathematics theory course than the rest of the data science series. But we will still attempt to keep our eye on developing ideas and concepts for helping to make decisions in the real world. Thus we may make limited use of computation using the `R` statistical language.

Prerequisites

MATH 241 (basic probability), 201 (multivariable calculus) and 231 (linear algebra) or equivalents. I expect a 241 class that covers more or less what I cover in 241. See the course homepage for links under "prerequisite review". The multivariable calculus and linear algebra we will use will be light and I will try to review those concepts in class as we need them.

Course Materials

Textbook: Introduction to Probability Theory by Hoel, Port & Stone. This book is out of print but you can buy it used on Amazon for less than $20 (as far as textbooks go). There is no excuse not to have this book. It is *required*. However, I will not ususally be teaching "from the book" — most of the material in the class comes from the lecture notes. The textbook is a way to get "another take" on the material. The textbook covers about only half of the material done in class (yes, sometimes we will be following the textbook page by page). For the other half, you will have to make use of other resources.

Computer Software: We will also be using `R` which is a free, open source statistical programming language and console. You can download it from: `http://cran.mirrors.hoobly.com/`. I do not expect you to do *any* programming. I will be giving you `R` code to run and expect you to interpret the results based on concepts explained during the course.

Calculator: You can use a TI-84, 85, 89 or any calculator which you wish. I strongly suggest you use Wolfram Alpha and its smartphone app.

The 640 Section

You are the students taking this course as part of a masters degree in mathematics. Thus, there will be extra homework problems for you and you will be graded on a separate curve.

Discord Coupled with Github as a Learning Management System

Each assignment and exam will have its own channel. You can feel free to discuss things with your fellow students there. If you are asking me a question, you must do so in the `#discussions` channel for a general questions or the assignment-specific channel (e.g. `#hw03`) so other students can see the question and benefit from the answer. Do not open "issues" on github! If you pm me for help with a class assignment, I will not answer and just ask you to move it to the appropriate public channel. Do not be afraid to ask questions. There are many people who will have your same question!

Discord is to be used professionally so no posting about random stuff!

Announcements

Course announcements will be made via discord in the `#general` channel (not on email). I am known to send a few discord messages per week on important issues.

I can't stress the following enough: if you are not on discord, you will miss all class announcements!!! Discord notifies you when there are messages.

Class Meetings

There are 28 scheduled meetings. Of these, 23 will be lectures, 2 will be midterm exams which are in class and 3 will be review periods during the meeting before the exams (see lecture schedule section above). The exam schedule is given on page 10. The last class of the semester *may* be rescheduled to be a review period that is conveniently before the final. We will discuss later in the semester.

Reschedule of One Class Meeting

Due to the Jewish holidays, I cannot be here to teach on one day (see course homepage).

Homework

There will be 6–9 theory homework assignments. Homeworks will be assigned and placed on the course homepage and will usually be due a week later. Homework will be **graded** out of 100 with extra credit getting scores possibly > 100. I (or the TA if one exists) will be doing the grading and will either grade an *arbitrary subset of the assignment* which is determined after the homework is handed in or *will grade based on completeness.*

You will send by email your finished homework as a PDF to the address on the course homepage. To generate this PDF file that you push to your repository, **you must do one of two things:**

- **Print out the homework and handwrite your answers in the alotted space for each question. Then scan your homework as a PDF. There are a ton of good photo $\Rightarrow$ PDF apps for both iPhone and android.**

- **Open the PDF on your device and use a PDF editing program to electronically handwrite your answers and save the PDF.**

I will NOT accept homework that is not atop the original rendered homework PDF file. Remember to write your name. There are no regrades during this pandemic semester. Homework must be at maximum 5MB.

You are highly recommended to work with each other and help each other. You must, however, submit your own solutions, *with your own write-up* and in *your own words*. There can be no collaboration on the actual *writing*. Failure to comply will result in severe penalties. The university honor code is something I take seriously and I send people to the Dean every semester for violations.

Philosophy of Homework

Homework is the *most* important part of this course.[1] Success in Statistics and Mathematics courses comes from experience in working with and thinking about the concepts. It's kind of like weightlifting; you have to lift weights to build muscles. My job as an instructor is to provide assistance through your zone of proximal development. With me, you can grow more than you can alone. To this effect, homework problems are color coded green for easy, yellow for harder, red for challenging and purple for extra credit. You need to know how to do all the greens by yourself. If you've been to class and took notes, they are a joke. Yellows and reds: feel free to work with others. Only do extra credits if you have already finished the assignment. The "[Optional]" problems are for extra practice — highly recommended for exam study.

Time Spent on Homework

This is a four credit course. Thus, the amount of work outside of the 4 hours in-class time per week is 8–10 hours per the QC bulletin. I will aim for 9 hours of homework per week on average. This will be time well-spent. I guarantee that by doing the homework well, you will study and understand the concepts in the lectures and you won't have all that much to do when the exams roll around.

Late Assignment Policy

Late homework will be penalized 10 points per business day (Monday–Friday save holidays) for a maximum of five days. *Do not ask for extensions*; just hand in the homework late. After five days, **you can hand it in whenever you want** until *the last scheduled class meeting according to the official academic calendar.* As far as I know, this is one of the most lenient and flexible homework policies in college. I realize things come up. Do not abuse this policy; you will fall far, far behind.

LaTeX Homework Bonus Points

Part of good mathematics is its beautiful presentation. Thus, there will be a 1–5 point bonus added to your theory homework grade for typing up your homework using the LaTeX typesetting

[1] In one student's observation, I give a "mind-blowing homework" every week.

system based on the elegance of your presentation. The bonus is arbitrarily determined by me.

I recommend using overleaf to write up your homeworks (make sure you upload both the hw#.tex and the preamble.tex file). This has the advantage of (a) not having to install anything on your computer and thus not having to maintain your LaTeX installation (b) allowing easy collaboration with others (c) alway having a backup of your work since it's always on the cloud. If you insist to have LaTeX running on your computer, you can download it for Windows here and for MAC here. For editing and producing PDF's, I recommend TeXworks which can be downloaded here. Please use the LaTeX code provided on the course homepage for each homework assignment.

If you are handing in homework this way, read the comments in the code; there are two lines to comment out and you should replace my name with yours and write your section. The easiest way to use overleaf is to copy the raw text from hwxx.tex and preamble.tex into two new overleaf tex files with the same name. If you are asked to make drawings, you can take a picture of your handwritten drawing and insert them as figures or leave space using the "\vspace" command and draw them in after printing or attach them stapled.

Since this is extra credit, do not ask me for help in setting up your computer with LaTeX in class or in office hours. Also, **never share your LaTeX code with other students** — it is cheating and if you are found I will take it seriously.

Homework Extra Credit

There will be many extra credit questions sprinkled throughout the homeworks. They will be worth a variable number of points arbitrarily assigned based on my perceived difficulty of the exercise. Very high homework scores are not unheard of. They are a good boost to your grade; I once had a student go from a B to and A- based on these bonuses.

Examinations

Examinations are solely based on homeworks (which are rooted in the lectures)! If you can do all the green and yellow problems on the homeworks, the exams should not present any challenge. I will *never* give you exam problems on concepts which you have not seen at home on one of the weekly homework assignments. There will be three exams and the schedule is below.

Exam Schedule

- Midterm examination I will be on [see course homepage] with the first review session on the class meeting prior
- Midterm examination II will be on [see course homepage] with a review on the class meeting prior.
- The final examination will be on [see course homepage] with a review on the final class meeting.

Exam Policies and Materials

I allow you to bring any calculator you wish but it cannot be your phone. The only other items allowed are pencil and eraser. I do not recommend using pen but it is allowed. Food is not allowed during exams but beverages are allowed. I also allow "cheat sheets" on examinations. For midterm I, you are allowed to bring *one* 8.5" $\times$ 11" sheet of paper (front and back). Two sheets single sided are not allowed. Midterm II, you are allowed to bring *one* 8.5" $\times$ 11" cheat sheet. For the final, you are allowed to bring *three* 8.5" $\times$ 11" cheat sheets. Six sheets single sided are not allowed. On these sheet(s) of paper you can write anything you would like which you believe will help you on the exam. I will be handing back the cheat sheets so you can reuse your midterm cheat sheets for the final if you wish.

Cheating on Exams

If I catch you cheating, you can either take a zero on the exam, or you can roll the dice before the University Honor Council who may choose to suspend you.

Missing Exams

There are no make-up exams. If you miss the exam, you get a zero. If you are sick, I need documentation of your visit to a hospital or doctor. Expect me to call the doctor or hospital to verify the legitimacy of your note.

Accommodations for Students with Disabilities

Candidates with disabilities needing academic accommodation should: 1) register with and provide documentation to the Special Services Office, Frese Hall, Room 111; 2) bring a letter indicating the need for accommodation and what type. This should be done during the first week of class. For more information about services available to Queens College candidates, contact: Special Service Office; Frese Hall, Room 111; 718-997-5870 (Monday – Thursday 8:00 a.m. to 5:00 p.m. & Friday 8:00 a.m. to 4 p.m.).

Class Participation

This portion of your grade is assessed based on your level of interaction during the course lectures e.g. asking / answering questions. Participation on discord counts towards this total.

Grading and Grading Policy

Your course grade will be calculated based on the percentages as follows:

Homework	20%
Midterm Examination I	20%
Midterm Examination II	20%
Final Examination	35%
Class participation	5%

The semester is split into three periods :

(a) From the beginning until midterm I. Midterm I covers material during this time.

(b) From midterm I to midterm II. Midterm II covers material in this period only.

(c) From midterm II until the final. The final is cumulative and covers all course material.

Each of the periods is assessed evenly. Thus, each period must count the same towards your grade. Since there is 75% of the grade allotted to exams, there is 25% allotted to each period. Thus, the final is upweighted towards the material covered in the third period. In summary, the final will have $5/35$ points $\approx 14\%$ for the first period's material, $5/35$ points $\approx 14\%$ for the second period's material and $25/35$ points $\approx 71\%$ for the last period's material. A good strategy for the final is to just study the material after Midterm II and minimal studying for the previous material.

The Grade Distribution

As this is a small and advanced class; thus, the class curve will be quite generous. Last year, it was approximately 40% A's and 40% B's. If you do your homework and demonstrate understanding on the exams, you should expect to be rewarded with an A or a B. C's are for those who can only demonstrate basic understanding. F's are rare, but occur.

Checking your grade and class standing

You can always check your grades in real-time using `https://qc.gradesly.com`. You will enter in your QC ID number (or CUNYfirst email address). I will provide you with your password by email the first week of class.

Auditing

Auditors are welcome. They are encouraged to do all course assignments. I will even grade them. Note that the university does not allow auditors to take examinations.