(c) $\vec{\theta}_b := s^2_{b_1} - s^2_{b_2}$

(d) $\vec{\theta}_b := \dfrac{\bar{x}_{b,1}}{\bar{x}_{b,2}}$

(e) $\vec{\theta}_b := \dfrac{s^2_{b_1}}{s^2_{b_2}}$

K-S statistics

(f) $\vec{\theta}_b := d_{b_1} - d_{b_2}$

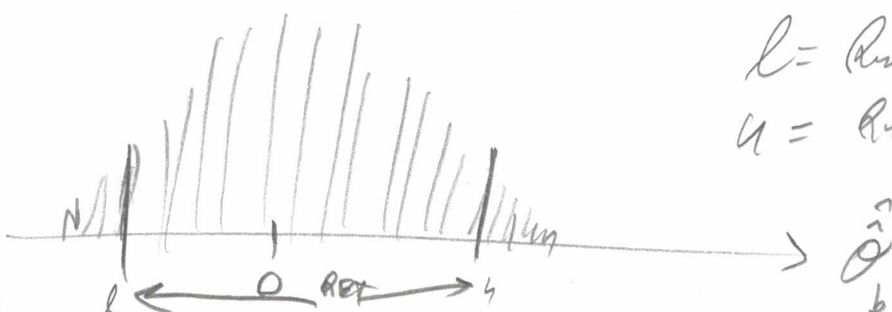
⋮ many more!!!

If $n_1 = n_2 = 100$, then are $\binom{201}{100} = 10^{58}$ possible subsets.

Now, using the computer, we create B many, many subsets b and calculate whichever $\vec{\theta}_b$ we choose to estimate the statistics. This approximates the null distr.

$l = \text{Quantile}\left[\vec{\theta}_b, \frac{\alpha}{2}\right]$

$u = \text{Quantile}\left[\vec{\theta}_b, 1-\frac{\alpha}{2}\right]$



Based on α,

Now we order the $\vec{\theta}_b$'s and the $\frac{\alpha}{2}$ and $1-\frac{\alpha}{2}$ percentiles form the endpoints of the RET. Now we compute $\hat{\theta}$ in the <u>ACTUAL</u> sample and see

$\hat{\theta} \in RET \Rightarrow$ RETAIN H_0.

This is an exact test if we would simulate all $\binom{n}{n_1} = \binom{n}{n_2}$ samples, otherwise it is approximate. With small sample sizes it is exact. However, if B is very large, in the millions, it's pretty much exact.

This test is non-parametric as we didn't make any distribution assumptions. The K-S test is also non-parametric.

"Fisher's Permutation Test"

Each $\hat{\theta}$ definition is a new test!

Why have many tests for the same Ha?

Because each test has different power based on the underlying DGP's.

Non-parametric tests tend to have higher power than parametric tests when the parametric tests are highly approximate (see demo).

How to get pval? Expand RET_α until $\hat{\hat{\theta}} \in RET$.

This is equivalent to $\min\{2P(\hat{\theta}_b > \hat{\hat{\theta}}), 2P(\hat{\theta}_b < \hat{\hat{\theta}})\}$

Ways to get CI's? Yes, beyond scope of course.

Very powerful test: Efron's "Bootstrap" (1979)

Let $X_1, \ldots, X_n \overset{iid}{\sim} DGP(\theta_1, \ldots, \theta_K)$

Let $\phi = g(\theta_1, \ldots, \theta_K)$ which could be almost anything

e.g. $\phi = Med[X]$, $\phi = Q[X, 25\%]$, etc.

$\Rightarrow \hat{\phi} = Med[X_1, \ldots, X_n]$, $\hat{\phi} = Q[X_1, \ldots, X_n, 25\%]$

Generally speaking, we do not know how to get the distr. of $\hat{\phi}$ so we can get inference.

The bootstrap gives you $\hat{\phi}_{boot_n}$ an asymptotically valid distr of $\hat{\phi}$.

Method: for many B resamplings, draw $\{x_{b,1}, \ldots x_{b,n}\}$ with replacement from $\{x_1, \ldots, x_n\}$, the original dataset. Then compute $\hat{\hat{\phi}}_b$ for that resampling. Doing this many times,

this gives you $\approx \frac{2}{3}$ of the original data pts.

$\{\hat{\hat{\phi}}_1, \ldots, \hat{\hat{\phi}}_B\}$ can be thought of as samples from $\hat{\phi}_{boot_n}$ which is approximately $\hat{\phi}$ i.e. $\hat{\phi}_{boot_n} \xrightarrow{d} \hat{\phi}$

"Pull yourself up by the bootstrap" means save yourself miraculously which is exactly what this is!

How to get CI's?

$$\hat{\hat{CI}}_{\phi, 1-\alpha} = \left[Q\left[\{\hat{\hat{\phi}}_1, \ldots, \hat{\hat{\phi}}_B\}, \frac{\alpha}{2}\right], Q\left[\{\hat{\hat{\phi}}_1, \ldots, \hat{\hat{\phi}}_B\}, 1 - \frac{\alpha}{2}\right] \right]$$

How to do testing? $H_a: \phi \neq \phi_0$

If $\phi_0 \in \hat{\hat{CI}}_{\phi, 1-\alpha} \Rightarrow$ Retain H_0

One-sided tests... make 1-sided CI's (not covered).