

Gibbs Sampling
Illustration

If we cannot sample from $p(\theta_j | \theta_{-j}, \bar{x})$, can we approximate it?

Or at least move around the space effectively? Yes.

Consider \textcircled{I} $q(\theta_{j,t-1}, \phi)$, the "proposal distr", which
proposes $\theta_{j,t}^*$ e.g. $q(\theta_{j,t-1}, \phi) = N(\theta_{j,t-1}, 1)$.

\textcircled{II} Then calculate:

$$r := \frac{p(\theta_{1,t}, \dots, \theta_{j,t}^*, \theta_{j+1,t-1}, \dots, \theta_{K,t-1})}{f_\ell(\theta_{j,t}^*; \theta_{j,t-1}, \phi)} \leftarrow \text{"prob of transitioning from } \theta_{j,t-1} \rightarrow \theta_{j,t}^* \text{"}$$

$$\uparrow$$

$$\frac{p(\theta_{1,t-1}, \dots, \theta_{j,t-1}, \theta_{j+1,t}, \dots, \theta_{K,t})}{f_\ell(\theta_{j,t-1}; \theta_{j,t}^*, \phi)} \leftarrow \text{"prob of reversing from } \theta_{j,t}^* \rightarrow \theta_{j,t-1} \text{"}$$

"Metropolis Ratio"

If $f_\ell(\theta_{j,t}^*; \theta_{j,t-1}, \phi) = f_\ell(\theta_{j-1,t}; \theta_{j,t}^*, \phi)$, r simplifies
and is called the Hastings Ratio.

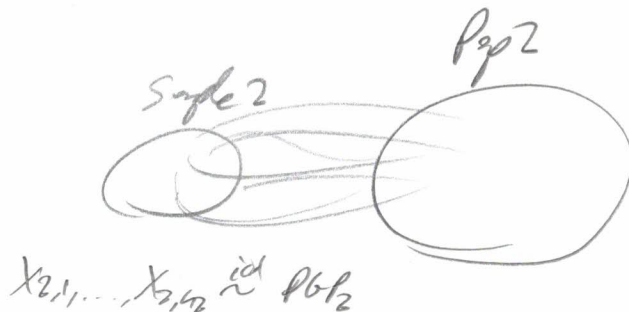
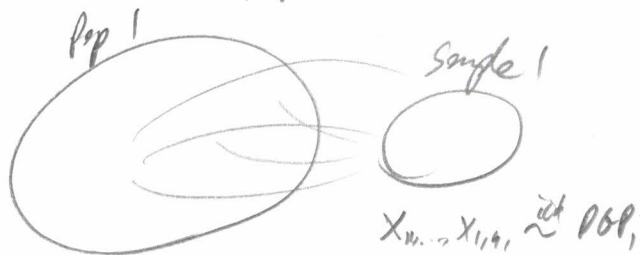
\textcircled{III} Sample u from $U(0,1)$. If $u \leq r$, then accept $\theta_{j,t}^*$
otherwise return $\theta_{j,t-1}$ and draw again.

Done with Bayes... for now...

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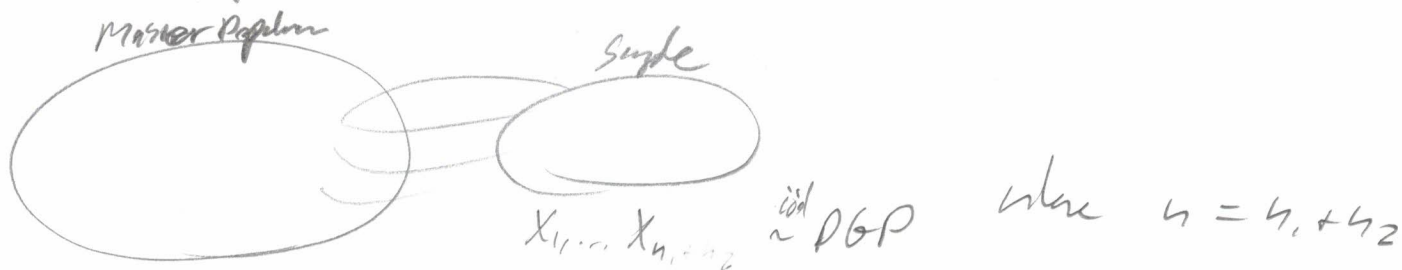
Back to the frequentist paradigm...

Two populations



$$H_a: DGP_1 \neq DGP_2, \quad H_0: DGP_1 = DGP_2 = DGP$$

We previously talked about the 2-sample K-S test. Here's a new test. Under H_0 , there's no difference between the two pop's. Remember the $\hat{\theta}$ for the one sample pop test?



Thus, there should be no difference if we divide the sample into two subsets: one size n_1 , the other size n_2 . Let the subset be called "b". Let $I_{b,1}$ be index of subset 1 and $I_{b,2}$ be index of subset 2. and then compare any ^{test} statistic between the two groups. eg.

$$(a) \quad \hat{\theta}_b := \bar{X}_{b,1} - \bar{X}_{b,2} = \frac{1}{n_1} \sum_{i \in I_{b,1}} x_i - \frac{1}{n_2} \sum_{i \in I_{b,2}} x_i$$

$$(b) \quad \hat{\theta}_b := \text{Med}_{b,1} - \text{Med}_{b,2} = \text{Med}[\{x_i: i \in I_{b,1}\}] - \text{Med}[\{x_i: i \in I_{b,2}\}]$$