

Math 369 / 690.03 Fall 2021 (3 credits)

Course Syllabus

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Course Homepage	https://github.com/kapelner/QC_Math_369_Fall_2021
Slack Homepage	https://QCMath369Fall2021.slack.com/
Contact	Write to @kapelner on slack
Lecture Time / Loc	Tues and Thurs 5 – 6:15PM / on zoom
Instructor Office Hours / Loc	see course homepage
TA / TA Office Hours / Loc	Kennly Weerasinghe / see course homepage

Course Overview

MATH 369 is an introduction to the intermediate concepts of mathematical statistics, statistical inference and theory of experimentation. Main topics are listed below:

- Estimation
 - Data generating processes, parameters, estimators, estimates, samples, population
 - Point parameter estimators and estimates: loss, risk, bias-variance decomposition
 - Likelihood, log-likelihood, score function
 - Un/biased estimators and asymptotically unbiased estimators, consistent estimator
 - Method of Moments (MM), Maximum likelihood estimator (MLE)
 - Cramer-Rao Lower Bound, Fisher information, UMVUEs
 - Asymptotical normality and efficiency of MLE's, uni/multivariate delta methods

- The Empirical CDF estimator
- The non-parametric bootstrap
- Testing
 - Neyman-Pearson paradigm of Hypothesis testing
 - Left-sided/tailed, right-sided/tailed, two-sided/tailed tests
 - One proportion z-test, One sample z-test, one sample t-test, two sample z-test (un/equal variances), two sample t-test (un/equal variances), two-proportion z-test.
 - Size (α), Fisher's p -value and “statistical significance”
 - Statistical power and the power function
 - Effect sizes and “practical / clinical significance”
 - uniformly most powerful tests?, equivalence testing?
 - Wald, score and the generalized likelihood ratio testing procedures
 - Chi-squared tests for goodness of fit and independence
 - Kolmogorov-Smirnov's goodness of fit test and distribution equivalence test
 - Permutation Tests
 - Multiple testing comparison problem: familywise error control via the Bonferroni, Sidak and Simes procedures, false discovery rate control via the Benjamini-Hochberg procedure
 - The non-parametric bootstrap
- Experimentation and Causality
 - Casual inference vs statistical inference
 - Neyman-Rubin model, the counterfactual
 - Observational study, confounding variable, biased causal estimation
 - Experimentation, completely randomized experiment, unbiased causal estimation
- Model Selection
 - Naive model selection bias based on number of parameters (overfitting)
 - Model section criterions: AIC, AICC
 - Akaike weights / scores

This is more of a typical mathematics theory course than the rest of the data science series but it is still not like the other math courses you're used to; it will be more philosophical and very high in conceptual density while the mathematics itself is kept deliberately simple. But we will still attempt to keep our eye on developing ideas and concepts for helping to make decisions in the real world. Thus we may make limited use of computation using the R statistical language.

Prerequisites

MATH 241 (basic probability), 201 (multivariable calculus) and 231 (linear algebra) or equivalents. I expect a 241 class that covers more or less what I cover in 241. See the course homepage for links under “prerequisite review”. The multivariable calculus and linear algebra we will use I will try to review in class.

Course Materials

Textbook: I will be referencing Larry Wasserman’s *All of Statistics: A concise course in statistical inference* which can be purchased on Amazon and Casella and Berger’s *Statistical Inference* which can be purchased on Amazon. There is no excuse not to have these books. They are *required*. However, I will not usually be teaching “from the book” — most of the material in the class comes from the lecture notes. The textbooks are a way to get “another take” on the material and they will only cover about only half of the material done in class. For the other half, you will have to make use of other resources. I also recommend Rice’s *Mathematical Statistics and Data Analysis*, 3rd edition which can be purchased on Amazon as well but I will not reference it during class.

Computer Software: During lectures, there will be demos using R which is a free, open source statistical programming language and console. You can download it from: <http://cran.mirrors.hoobly.com/>. I do not expect you to do *any* programming. If R will be part of the homework, I will give you the code to run and expect you to interpret the results based on concepts explained during the course.

Calculator: You can use a TI-84, 85, 89 or any calculator which you wish. I strongly suggest you use Wolfram Alpha and its smartphone app.

The 690.x Section

You are the students taking this course as part of a masters degree in mathematics. Thus, there will be extra homework problems for you and you will be graded on a separate curve.

Announcements

Course announcements will be made via slack in the #general channel (not on email). I am known to send a few slack messages per week on important issues. **If you are not on slack, you will miss all class announcements!!!** Slack notifies you when there are messages. You may wish to mute all channels except for #general. See this article for how to do that.

The Use of Slack as a Learning Management System

As the course homepage is updated (e.g. a new homework assignment is posted), you will hear about it in slack. You will also find the video recordings of lectures there. (If there are multiple sections of the class, only one section's lectures will be recorded). Each assignment will have its own channel. You can feel free to discuss things with your fellow students there. If you are asking me a question, you must do so in the **#discussions** channel for a general questions or the assignment-specific channel (e.g. **#HW03**) so other students can see the question and benefit from the answer. If you pm me, I will just ask you to move it to the public channel. Do not be afraid to ask questions. There are many people who will have your same question!

Slack is a wildly successful company that recently got bought by Salesforce because businesses *really* use it. Pretend you are working at one of these businesses: **no posting about random stuff; keep things professional!**

Class Meetings

There are 28 scheduled meetings. Of these, 23 will be lectures, 2 will be midterm exams which are in class and 3 will be review periods (the meeting before the exams). The exam schedule is given on page 10. The last class of the semester (Dec 9) will be rescheduled to be a review period that is conveniently before the final. We will discuss later in the semester.

Reschedule of Two Class Meetings

Due to the Jewish holidays, I cannot be here to teach Tuesday, September 21 and Tuesday, September 28. Because my policy is to give an entire lecture period as review before exams, we will reschedule those periods to be night periods before the exams.

Zoom Policies

- Use your real name as the display name. This is important for auditing for the attendance score.
- Your video must always be on and your sound should always be on (except in cases of background noise)
- You can use an appropriate background but it must be a static image.
- No snap camera or similar nonsense. No chatting on zoom.
- No text chatting or verbal chatting with other students (no talking in class!)

Detailed Lecture Information

Individual lectures' topics with time estimates based on the previous semester's lectures are below:

- Lec 1 [20min] Surveys, populations, sampling, sample size n , representativeness, SRS; [30min] definition of parameter, parameter space, inference, statistic, the three goals of statistical inference: point estimation, theory testing and confidence set construction.
- Lec 2 [20min] Sampling with/without replacement, iid assumption, equivalence of populations and data generating processes (DGPs); [55min] estimators, point estimation metrics: biasedness and unbiasedness, loss functions (absolute, squared error, others), risk function, mean squared error (MSE), bias-variance decomposition of MSE, maximum risk for iid Bernoulli model
- Lec 3 [35min] Introduction to hypothesis testing, the intellectual honesty of assuming the null hypothesis H_0 , the theory you wish to prove H_a , right, left and two-sided tests of single parameters, test statistics; [30min] the Binomial exact test of one proportion in the iid Bernoulli DGP, retainment region (RET), rejection rejection, statistical significance; [10min] Type I error, α , Type II error, 2×2 confusion table of testing errors.
- Lec 4 [10min] review of central limit theorem (CLT); [20min] employing the CLT to create the approximate one-proportion z-test; [20min] Fisher's p-value; [25min] Definition of power, assuming an H_a to compute the probability of Type II errors and power
- Lec 5 [15min] The power function for a one-sided z-test; [15min] the iid normal DGP; [15min] exact one-sample z-test with known variance, its power function; [30min] the naive estimator for σ^2 , proof of its biasedness, definition of asymptotic unbiasedness, Bessel's correction and the unbiased sample variance estimator S^2
- Lec 6 [20min] motivation of the t statistic, Student's t distribution, the one-sample t-test; [20min] the concept of two populations, two sample testing for mean differences, H_0 specification for left-sided, right-sided, two sided tests; [15min] the exact two-sample z-test under different variances and shared variance; [20min] the exact two-sample t-test under shared variance, the pooled standard deviation estimator
- Lec 7 [25min] the approximate t-test (Welch-Satterthwaite approximation) under different variances, the Behrens-Fisher distribution; [5min] review of moments, definition of sample moments; [15min] the system of equations yielding the method of moments estimators (MME); [10min] MME for the expectation, MME for the variance; [20min] MME for the two parameters in the iid Binomial DGP, nonsensical MME values

- Lec 8 [10min] MME for the iid Uniform DGP with one unknown endpoint parameter; [10min] definition of likelihood, the equivalence of the likelihood function with the JDF/JMF; [10min] definition of argmax, equivalence of argmax under strictly increasing functions; [10min] definition of maximum likelihood estimators (MLE) and estimates, the log likelihood; [10min] MLE for the iid Bernoulli DGP; [10min] MLEs for the iid Normal DGP; [10min] MLE for iid Uniform DGP with one unknown endpoint parameter; [5min] variances of MME and MLE for that cases
- Lec 9 [5min] definition of relative efficiency and comparison of two estimators; [10min] the nonexistence of a minimum MSE estimator; [5min] definition of uniformly minimum variance unbiased estimators (UMVUEs); [10min] definition of the Cramer-Rao Lower Bound (CRLB), definition of Fisher Information; [55min] proof of the CRLB, definition of the score function, expectation of the score function is zero,
- Lec 10 [10min] proof that the sample average is the UMVUE for the iid Bernoulli DGP; [15min] proof that the sample average is the UMVUE for the iid normal DGP; [10min] definition of asymptotically normal estimators; [15min] definition of consistent estimators, continuous mapping theorem, Slutsky's theorem; [10min] asymptotic normality of asymptotically normal estimators when using the estimator for its standard error; [5min] statement of main theorem for MMEs and MLEs: consistency, asymptotic normality, asymptotic efficiency of the MLE
- Lec 11 [5min] review of Taylor series; [40min] proof that the MLE is asymptotically normal and asymptotically efficient; [5min] definition of the Wald test, the one-proportion z-test as a Wald test; [35min] the two-proportion z-test as a Wald test, the pooled proportion estimator
- Lec 12 [5min] the one-sample t-test as an approximate one-sample z-test (Wald test); [10min] the approximate two-sample t-test as an approximate two-sample z-test (Wald test); [15min] derivation of the MLE, Fisher Information and a Wald test for the iid Gumbel DGP with known scale parameter; [15min] introduction to confidence sets, interval estimators, coverage probability; [20min] definition of the confidence interval (CI), CI construction via hypothesis test inversion
- Lec 13 [10min] comparison of hypothesis testing with CI construction; [15min] approximate CIs for the iid normal DGP under the four assumptions; [10min] CI for one proportion; [15min] CI for the difference of two proportions; [10min] CIs for MLEs; [15min] proof that MSE improvements improve all three statistical inference goals, illustration of all three goals
- Lec 14 [10min] meaninglessness of single inferences; [5min] odds-against reparameterization, odds-against point estimation; [15min] univariate delta method; [10min] CI for odds-against via delta method, CI for log-mean; [10min] risk ratio versus proportion difference; [20min] multivariate delta method; [5min] CI for the risk ratio

- Lec 15 [10min] equivalence of the two-sided z and χ^2 tests, equivalence of the two-sided t and F tests; shape of the χ^2 and F distributions; [30min] χ^2 goodness of fit test for multinomial parameters, its H_0 and H_a , observed and expected counts, the test statistic; [40min] the χ^2 test of independence among two categorical variables, its H_0 and H_a , observed and expected counts, the test statistic
- Lec 16 [35min] concept of assuming DGP models, discussion of what models are, discussion of many model candidates; [15min] model selection via largest likelihood, asymptotic bias of the log-likelihood estimator with substituted MLEs; [15min] the AIC metric, AIC model selection algorithm, penalizing complexity; [5min] the AICC metric
- Lec 17 [5min] Akaike weights; [25min] statistical significance vs. clinical / practical significance of the effect; [5min] review of Type I/II errors; [40min] the multiple hypothesis testing / comparison problem, the 2×2 frequency table of test results, false discoveries,
- Lec 18 [10min] definition of familywise error rate (FWER), weak FWER control; [10min] Bonferroni procedure; [15min] Sidak procedure; [25min] Simes procedure; [15min] definition of false discovery proportion, false discovery rate (FDR), setting for equivalence of FWER and FDR, statement that the Simes procedure provides strong control of the FDR
- Lec 19 [10min] proof that p-values are uniformly distributed under H_0 ; [50min] derivation of the score test as a Wald test, the score test statistic for the logistic distribution with known scale parameter; [15min] definition of likelihood ratio (LR) test statistic, statement of its asymptotic convergence to a χ^2_1 , LR test (LRT) derivation of the LR statistic for the iid Bernoulli DGP
- Lec 20 [25min] Proof of the asymptotic convergence of the LR test statistic; [50min] generalized likelihood ratio test for reduced models nested in a full model, statement of asymptotic convergence to a χ^2 with degrees of freedom equal to the difference of number of parameters, demonstration that it differs from the goodness of fit test, demonstration in the normal iid DGP
- Lec 21 [10min] visualizing the Wald, score and LR tests; [5min] testing entire DGPs; [10min] definition and illustration of the empirical CDF; [20min] one-sample Kolmogorov-Smirnov (KS) test, statement of Kolmogorov distribution and its critical values; [10min] tests of DGP equivalence among two populations; [15min] two-sample KS test
- Lec 22 [45min] two-sample permutation test, partitioning the master population, discussion of possible test statistics, computational construction of its RET via resampling; [30min] the nonparametric bootstrap procedure, typical use cases, approximate CI construction, approximate hypothesis testing
- Lec 23 [20min] causal inference vs statistical inference, confounding variable, an illustration of one scenario with confounding and one scenario without confounding; [15min]

definition of a two-arm treatment vs control experiment, definition of assignment / allocation / manipulation; [10min] the Rubin causal model, counterfactuals, additive treatment effect, selection bias inducing bias in the naive treatment estimator; [15min] response model linear in confounder and noise, noise as an approximate normal realization, the explicit bias term and its explanation; [20min] randomized experiments, bernoulli trial design, completely randomized trial design, statement that causal estimates are unbiased over errors and randomized assignments, statement that bias is small in any random assignment

Homework

There will be 6–9 theory homework assignments. Homeworks will be assigned and placed on the course homepage and will usually be due a week later in class. Homework will be **graded** out of 100 with extra credit getting scores possibly > 100 . I will be doing the grading and will grade an *arbitrary subset of the assignment* which is determined after the homework is handed in.

During this pandemic, homework must be handed in by emailing it to me as a PDF. You must do one of two things:

- **Print out the homework and handwrite your answers in the allotted space for each question. Then scan your homework as a PDF. There are a ton of good photo-PDF apps for both iPhone and droid.**
- **Open the PDF on your device and use a PDF-editing program to electronically handwrite your answers and save the PDF.**

I will NOT accept homework that is not atop the original rendered homework PDF file. Remember to write your name. There are no regrades during this pandemic semester. Homework must be at maximum 5MB.

You are encouraged to seek help from me if you have questions. After class and office hours are good times. *You are highly recommended to work with each other and help each other. You must, however, submit your own solutions, with your own write-up and in your own words. There can be no collaboration on the actual writing. Failure to comply will result in severe penalties.* The university honor code is something I take seriously and I send people to the Dean every semester for violations.

Philosophy of Homework

Homework is the *most* important part of this course.¹ Success in Statistics and Mathematics courses comes from experience in working with and thinking about the concepts. It's kind of like weightlifting; you have to lift weights to build muscles. My job as an instructor is to provide assistance through your zone of proximal development. With me,

¹In one student's observation, I give a "mind-blowing homework" every week.

you can grow more than you can alone. To this effect, homework problems are color coded **green** for easy, **yellow** for harder, **red** for challenging and **purple** for extra credit. You need to know how to do all the greens by yourself. If you've been to class and took notes, they are a joke. Yellows and reds: feel free to work with others. Only do extra credits if you have already finished the assignment. The "[Optional]" problems are for extra practice — highly recommended for exam study.

Time Spent on Homework

This is a three credit course. Thus, the amount of work outside of the 4hr in-class time per week is 6-9 hours. I will aim for 7.5hr of homework per week on average. However, doing the homework well is your sole responsibility since I will make sure that by doing the homework you will study and understand the concepts in the lectures and you won't have all that much to do when the exams roll around.

Late Homework Policy

Late homework will be penalized 10 points per business day (Monday–Friday save holidays) for a maximum of five days. *Do not ask for extensions*; just hand in the homework late. After five days, **you can hand it in whenever you want** until May 18 at noon. As far as I know, this is one of the most lenient and flexible homework policies in college. I realize things come up. Do not abuse this policy; you will fall far, far behind.

L^AT_EX Homework Bonus Points

Part of good mathematics is its beautiful presentation. Thus, **there will be a 1–5 point bonus** added to your theory homework grade for typing up your homework using the L^AT_EX typesetting system based on the elegance of your presentation. The bonus is arbitrarily determined by me.

I recommend using overleaf to write up your homeworks (make sure you upload both the hw#.tex and the preamble.tex file). This has the advantage of (a) not having to install anything on your computer and thus not having to maintain your L^AT_EX installation (b) allowing easy collaboration with others (c) always having a backup of your work since it's always on the cloud. If you insist to have L^AT_EX running on your computer, you can download it for Windows [here](#) and for MAC [here](#). For editing and producing PDF's, I recommend T_EXworks which can be downloaded [here](#). Please use the L^AT_EX code provided on the course homepage for each homework assignment.

If you are handing in homework this way, read the comments in the code; there are two lines to comment out and you should replace my name with yours and write your section. The easiest way to use overleaf is to copy the raw text from hwxx.tex and preamble.tex into two new overleaf tex files with the same name. If you are asked to make drawings, you can take a picture of your handwritten drawing and insert them as figures or leave space using the “\vspace” command and draw them in after printing or attach them stapled.

Since this is extra credit, do not ask me for help in setting up your computer with L^AT_EX in class or in office hours. Also, **never share your L^AT_EX code with other students** — it is cheating and if you are found I will take it seriously.

Homework Extra Credit

There will be many extra credit questions sprinkled throughout the homeworks. They will be worth a variable number of points arbitrarily assigned based on my perceived difficulty of the exercise. Homework scores in the 140's are not unheard of. They are a good boost to your grade; I once had a student go from a B to and A- based on these bonuses.

Examinations

Examinations are solely based on homeworks (which are rooted in the lectures)! If you can do all the green and yellow problems on the homeworks, the exams should not present any challenge. I will *never* give you exam problems on concepts which you have not seen at home on one of the weekly homework assignments. There will be three exams and the schedule is below.

Exam Schedule

- Midterm examination I will be Thursday, October 7 on zoom during class time with the first review session on the class meeting prior
- Midterm examination II will be Thursday, November 11 on zoom during class time with a review on the class meeting prior.
- The final examination will be TBD but on zoom with a review TBA.

Exam Policies and Materials

I allow you to bring any calculator you wish but it cannot be your phone. The only other items allowed are pencil and eraser. I do not recommend using pen but it is allowed. **Food is not allowed** during exams **but beverages are allowed**.

Zoom Exams

Exams will be given on Zoom screen-shared from my computer. In a nutshell, each page of the PDF exam is a “problem scenario” with text and/or illustrations. The questions that follow are enumerated a, b, c, d, etc. and they are all “true or false” questions. You will enter the true question letters into a google form. Each problem scenario page is individually timed. Once the timer expires, you can no longer edit your answers.

I also allow “cheat sheets” on examinations. For both midterms, you are allowed to bring *one* 8.5” × 11” sheet of paper (front and back). **Two sheets single sided are not allowed**. On this paper you can write anything you would like which you believe will help

you on the exam. For the final, you are allowed to bring *three* 8.5" × 11" sheet of paper (front and back). **Six sheets single sided are not allowed.** I will be handing back the cheat sheets so you can reuse your midterm cheat sheets for the final if you wish.

Cheating on Exams

If I catch you cheating, you can either take a zero on the exam, or you can roll the dice before the University Honor Council who may choose to suspend you. **I have extremely elaborate means to detect cheating on my zoom exams. These means have been iterated upon and improved for three semesters. I suggest you don't try me.**

Missing Exams

There are no make-up exams. If you miss the exam, you get a zero. If you are sick, I need documentation of your visit to a hospital or doctor. Expect me to call the doctor or hospital to verify the legitimacy of your note.

Special Services

If you are a student who takes exams at the special services center, I need to see your blue slip or receive email evidence one week before the exam to make proper arrangements with the center.

Class Participation

This portion of your grade is assessed based on your level of interaction during the course lectures e.g. asking and answering questions. Participation on slack also counts towards this total.

Grading and Grading Policy

Your course grade will be calculated based on the percentages as follows:

Homework	20%
Midterm Examination I	20%
Midterm Examination II	20%
Final Examination	35%
Class participation	5%

The semester is split into three periods :

- (a) From the beginning until midterm I. Midterm I covers material during this time..
- (b) From midterm I to midterm II. Midterm II covers material in this period only.

- (c) From midterm II until the final. The final is cumulative and covers all course material.

Each of the periods is assessed evenly. Thus, each period must count the same towards your grade. Since there is 75% of the grade allotted to exams, there is 25% allotted to each period. Thus, the final is upweighted towards the material covered in the third period. In summary, the final will have 5/35 points $\approx 14\%$ for the first period's material, 5/35 points $\approx 14\%$ for the second period's material and 25/35 points $\approx 71\%$ for the last period's material. A good strategy for the final is to just study the material after Midterm II and minimal studying for the previous material.

The Grade Distribution

As this is a small and advanced class, the class curve will be quite generous. Last year, it was approximately 40% A's and 40% B's. If you do your homework and demonstrate understanding on the exams, you should expect to be rewarded with an A or a B. C's are for those who "dropped out" somewhere mid-semester or who cannot demonstrate basic understanding. I have never given an F in this class. Don't give me a reason to change this tradition!

Checking your grade and class standing

You can always check your grades in real-time using <http://qc.gradesly.com>. You will enter in your QC ID number (or CUNYfirst email address). I will provide you with your password by email after the first assignment is completed.

Auditing

Auditors are welcome in both sections. They are encouraged to do all homework assignments. I will even grade them. Note that the university does not allow auditors to take examinations.