Math 369 / 690 Fall 2021
Midterm Examination Two

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Code of Academic Integrity

Since the college is an academic community, its fundamental purpose is the pursuit of knowledge. Essential to the success of this educational mission is a commitment to the principles of academic integrity. Every member of the college community is responsible for upholding the highest standards of honesty at all times. Students, as members of the community, are also responsible for adhering to the principles and spirit of the following Code of Academic Integrity.

Activities that have the effect or intention of interfering with education, pursuit of knowledge, or fair evaluation of a student's performance are prohibited. Examples of such activities include but are not limited to the following definitions:

Cheating Using or attempting to use unauthorized assistance, material, or study aids in examinations or other academic work or preventing, or attempting to prevent, another from using authorized assistance, material, or study aids. Example: using an unauthorized cheat sheet in a quiz or exam, altering a graded exam and resubmitting it for a better grade, etc.

By taking this exam, you acknowledge and agree to uphold this Code of Academic Integrity.

Instructions

This exam is 70 minutes (variable time per question) and closed-book. You are allowed **one** page (front and back) of a "cheat sheet", blank scrap paper and a graphing calculator. Please read the questions carefully. I recommend answering all questions that are easy first and then circling back to work on the harder ones. Gray text means the text is repeated verbatim from a previous problem. No food is allowed, only drinks.

Problem 1 [10min] (and 10min will have elapsed) These are conceptual questions about statistical inference and sampling.

- [16 pt / 16 pts] Record the letter(s) of all the following that are **true** in general. At least one will be true. We are interested in understanding times in milliseconds for router packet switching under heavy traffic situations on the Internet at two different sites: New York and Chicago which we will denote population 1 and 2 respectively. $n = 15$ data points are recorded for New York and $n = 15$ data points are recorded for Chicago. We can assume the DGP for samples from population 1 is iid and we can assume the DGP for samples from population 1 is iid and that samples between populations are independent. We also assume that both populations have their first two moments defined. Here are sample statistics: $\bar{x}_1 = 6.01$, $\bar{x}_2 = 10.50$, $s_1 = 14.54$ and $s_2 = 31.62$. We seek to prove a difference between the populations.

  We first would like to prove that the population means are different. Denote the means of both populations by θ_1 and θ_2. We will employ $\hat{\theta}_1 = \bar{X}_1$ and $\hat{\theta}_2 = \bar{X}_2$.

  (a) The DGP that produced sample 1 has the same DGP that produced sample 2 but with different
  (b) $\hat{\theta}_1$ and $\hat{\theta}_2$ are unbiased estimators
  (c) $\hat{\theta}_1$ and $\hat{\theta}_2$ are the method of moments estimators for θ_1 and θ_2 respectively
  (d) $\hat{\theta}_1$ and $\hat{\theta}_2$ are the maximum likelihood estimators for θ_1 and θ_2 respectively
  (e) $\hat{\theta}_1$ and $\hat{\theta}_2$ are asymptotically normal due to the central limit theorem
  (f) $\hat{\theta}_1$ and $\hat{\theta}_2$ are asymptotically normal due to the MLE monster theorem
  (g) It is a reasonable assumption that the variances of the DGP's for the two populations are equal
  (h) We have enough information to justify that the sampling distribution of $\hat{\theta}_1 - \hat{\theta}_2$ is exactly normally distributed
  (i) We have enough information to justify that the sampling distribution of $\hat{\theta}_1 - \hat{\theta}_2$ is approximately normally distributed
  (j) We have enough information to justify that the sampling distribution of $\hat{\theta}_1 - \hat{\theta}_2$ is exactly T distributed
  (k) We have enough information to justify that the sampling distribution of $\hat{\theta}_1 - \hat{\theta}_2$ is approximately T distributed
  (l) We have enough information to justify that the sampling distribution of $(\hat{\theta}_1 - \hat{\theta}_2)^2$ is exactly χ^2 distributed
  (m) We have enough information to justify that the sampling distribution of $(\hat{\theta}_1 - \hat{\theta}_2)^2$ is approximately χ^2 distributed
  (n) We have enough information to justify that the sampling distribution of $(\hat{\theta}_1 - \hat{\theta}_2)^2$ is exactly F distributed
  (o) We have enough information to justify that eth sampling distribution of $(\hat{\theta}_1 - \hat{\theta}_2)^2$ is approximately F distributed
  (p) $H_0 : \theta_1 \neq \theta_2$

Your answer will consist of a lowercase string (e.g. `aebgd`) where the order of the letters does not matter.

Problem 2 [7min] (and 17min will have elapsed) These are conceptual questions about statistical inference and sampling.

- [13 pt / 29 pts] Record the letter(s) of all the following that are **true** in general. At least one will be true. We are interested in understanding times in milliseconds for router packet switching under heavy traffic situations on the Internet at two different sites: New York and Chicago which we will denote population 1 and 2 respectively. $n = 15$ data points are recorded for New York and $n = 15$ data points are recorded for Chicago. We can assume the DGP for samples from population 1 is iid and we can assume the DGP for samples from population 1 is iid and that samples between populations are independent. We also assume that both populations have their first two moments defined. Here are sample statistics: $\bar{x}_1 = 6.01$, $\bar{x}_2 = 10.50$, $s_1 = 14.54$ and $s_2 = 31.62$. We seek to prove a difference between the populations.

  We first would like to prove that the population means are different. Denote the means of both populations by θ_1 and θ_2. We will employ $\hat{\theta}_1 = \bar{X}_1$ and $\hat{\theta}_2 = \bar{X}_2$ to test $H_a : \theta_1 \neq \theta_2$. We will first assume both DGP's are normal and that $\sigma = 24.61$ Note that $F_Z(.975) = 1.96$ and $F_{T_{14}}(.975) = 2.15$.

  (a) We have enough information to justify that the sampling distribution of $\hat{\theta}_1 - \hat{\theta}_2$ is exactly normally distributed
  (b) We have enough information to justify that the sampling distribution of $\hat{\theta}_1 - \hat{\theta}_2$ is approximately normally distributed
  (c) We have enough information to justify that the sampling distribution of $\hat{\theta}_1 - \hat{\theta}_2$ is exactly T distributed
  (d) We have enough information to justify that the sampling distribution of $\hat{\theta}_1 - \hat{\theta}_2$ is approximately T distributed
  (e) We have enough information to justify that the sampling distribution of $\hat{\theta}_1 - \hat{\theta}_2$ is exactly Fisher-Behrens distributed
  (f) A justified standardized test statistic is -0.4996635 rounded to the nearest 7 digits
  (g) A justified standardized test statistic is -0.4996494 rounded to the nearest 7 digits
  (h) A justified standardized test statistic is -0.7066109 rounded to the nearest 7 digits
  (i) A justified standardized test statistic is -0.1824462 rounded to the nearest 7 digits
  (j) The null is rejected at $\alpha = 5\%$
  (k) The null is retained at $\alpha = 5\%$
  (l) Fisher's p-value $< \alpha = 5\%$
  (m) Fisher's p-value $\geq \alpha = 5\%$

Your answer will consist of a lowercase string (e.g. `aebgd`) where the order of the letters does not matter.

Problem 3 [5min] (and 22min will have elapsed) These are conceptual questions about statistical inference and sampling.

- [13 pt / 42 pts] Record the letter(s) of all the following that are **true** in general. At least one will be true. We are interested in understanding times in milliseconds for router packet switching under heavy traffic situations on the Internet at two different sites: New York and Chicago which we will denote population 1 and 2 respectively. $n = 15$ data points are recorded for New York and $n = 15$ data points are recorded for Chicago. We can assume the DGP for samples from population 1 is iid and we can assume the DGP for samples from population 1 is iid and that samples between populations are independent. We also assume that both populations have their first two moments defined. Here are sample statistics: $\bar{x}_1 = 6.01$, $\bar{x}_2 = 10.50$, $s_1 = 14.54$ and $s_2 = 31.62$. We seek to prove a difference between the populations.

  We first would like to prove that the population means are different. Denote the means of both populations by θ_1 and θ_2. We will employ $\hat{\theta}_1 = \bar{X}_1$ and $\hat{\theta}_2 = \bar{X}_2$ to test $H_a : \theta_1 \neq \theta_2$. We will first assume both DGP's are normal and that $\sigma_1 \neq \sigma_2$ and both values are unknown. Note that $F_Z(.975) = 1.96$ and $F_{T_{14}}(.975) = 2.15$.

  (a) We have enough information to justify that the sampling distribution of $\hat{\theta}_1 - \hat{\theta}_2$ is exactly normally distributed
  (b) We have enough information to justify that the sampling distribution of $\hat{\theta}_1 - \hat{\theta}_2$ is approximately normally distributed
  (c) We have enough information to justify that the sampling distribution of $\hat{\theta}_1 - \hat{\theta}_2$ is exactly T distributed
  (d) We have enough information to justify that the sampling distribution of $\hat{\theta}_1 - \hat{\theta}_2$ is approximately T distributed
  (e) We have enough information to justify that the sampling distribution of $\hat{\theta}_1 - \hat{\theta}_2$ is exactly Fisher-Behrens distributed
  (f) A justified standardized test statistic is -0.4996635 rounded to the nearest 7 digits
  (g) A justified standardized test statistic is -0.4996494 rounded to the nearest 7 digits
  (h) A justified standardized test statistic is -0.7066109 rounded to the nearest 7 digits
  (i) A justified standardized test statistic is -0.1824462 rounded to the nearest 7 digits
  (j) The null is rejected at $\alpha = 5\%$
  (k) The null is retained at $\alpha = 5\%$
  (l) Fisher's p-value $< \alpha = 5\%$
  (m) Fisher's p-value $\geq \alpha = 5\%$

Your answer will consist of a lowercase string (e.g. `aebgd`) where the order of the letters does not matter.

Problem 4 [5min] (and 27min will have elapsed) These are conceptual questions about statistical inference and sampling.

- [12 pt / 54 pts] Record the letter(s) of all the following that are **true** in general. At least one will be true. We are interested in understanding times in milliseconds for router packet switching under heavy traffic situations on the Internet at two different sites: New York and Chicago which we will denote population 1 and 2 respectively. $n = 15$ data points are recorded for New York and $n = 15$ data points are recorded for Chicago. We can assume the DGP for samples from population 1 is iid and we can assume the DGP for samples from population 1 is iid and that samples between populations are independent. We also assume that both populations have their first two moments defined. Here are sample statistics: $\bar{x}_1 = 6.01$, $\bar{x}_2 = 10.50$, $s_1 = 14.54$ and $s_2 = 31.62$. We seek to prove a difference between the populations.

  We first would like to prove that the population means are different. Denote the means of both populations by θ_1 and θ_2. We will employ $\hat{\theta}_1 = \bar{X}_1$ and $\hat{\theta}_2 = \bar{X}_2$ to test $H_a : \theta_1 \neq \theta_2$. ~~We will first assume both DGP's are normal.~~ We have reason to suspect these distributions are very non-normal and extreme-tailed. Note that $F_Z(.975) = 1.96$ and $F_{T_{14}}(.975) = 2.15$.

  (a) We have enough information to justify that the sampling distribution of $\hat{\theta}_1 - \hat{\theta}_2$ is exactly normally distributed
  (b) We have enough information to justify that the sampling distribution of $\hat{\theta}_1 - \hat{\theta}_2$ is approximately normally distributed
  (c) We have enough information to justify that the sampling distribution of $\hat{\theta}_1 - \hat{\theta}_2$ is exactly T distributed
  (d) We have enough information to justify that the sampling distribution of $\hat{\theta}_1 - \hat{\theta}_2$ is exactly Fisher-Behrens distributed
  (e) A justified standardized test statistic is -0.4996635 rounded to the nearest 7 digits
  (f) A justified standardized test statistic is -0.4996494 rounded to the nearest 7 digits
  (g) A justified standardized test statistic is -0.7066109 rounded to the nearest 7 digits
  (h) A justified standardized test statistic is -0.1824462 rounded to the nearest 7 digits
  (i) The null is rejected at $\alpha = 5\%$
  (j) The null is retained at $\alpha = 5\%$
  (k) Fisher's p-value $< \alpha = 5\%$
  (l) Fisher's p-value $\geq \alpha = 5\%$

Your answer will consist of a lowercase string (e.g. `aebgd`) where the order of the letters does not matter.

Problem 5 [6min] (and 33min will have elapsed) These are conceptual questions about statistical inference and sampling.

- [11 pt / 65 pts] Record the letter(s) of all the following that are **true** in general. At least one will be true. We have reason to suspect these distributions are very non-normal and extreme-tailed. One such model is the Lomax distribution which is a two-parameter survival model. To keep things tractable for the purposes of this class, we will set one parameter to be equal to one. Assume $X_1, \ldots, X_n \overset{iid}{\sim} \text{Lomax}(1, \theta) := f(x; \theta) = \theta(1+x)^{-(\theta+1)}$ and $\mathbb{S}\text{upp}[X] = (0, \infty)$. Note: we are redefining the definition of θ now — it is now a parameter within this assumed DGP! The mean of this rv is $\mu_1 := \mathbb{E}[X] = 1/(\theta - 1)$. Here are sample statistics: $\bar{x}_1 = 6.01$, $\bar{x}_2 = 10.50$, $s_1 = 14.54$ and $s_2 = 31.62$.

  (a) $\theta = (1 + \mu_1)/\mu_1$

  (b) $\hat{\theta}^{\text{MM}} = (1 + \hat{\mu}_1)/\hat{\mu}_1$

  (c) $\hat{\theta}^{\text{MM}} = (1 + \bar{X})/\bar{X}$

  (d) $\hat{\theta}^{\text{MM}} = (1 + \hat{\sigma})/\hat{\sigma}$

  (e) $\hat{\theta}^{\text{MM}}$ is definitely unbiased without further proof

  (f) $\hat{\theta}^{\text{MM}}$ is definitely asymptotically normal without further proof

  (g) $\hat{\theta}^{\text{MM}}$ is definitely asymptotically efficient (i.e. its variance approaches the CRLB of variance) without further proof

  (h) Since there is only one parameter of the DGP, $\hat{\theta}^{\text{MM}} = \hat{\theta}^{\text{MLE}}$ definitely without further proof

  (i) $\hat{\hat{\theta}}^{\text{MM}} = (1 + \bar{x})/\bar{x}$

  (j) $\hat{\hat{\theta}}^{\text{MM}} = (1 + s)/s$

  (k) $\hat{\hat{\theta}}_2^{\text{MM}} = 1.095238$ rounded to the nearest 6 digits

Your answer will consist of a lowercase string (e.g. `aebgd`) where the order of the letters does not matter.

Problem 6 [10min] (and 43min will have elapsed) These are conceptual questions about statistical inference and sampling.

- [17 pt / 82 pts] Record the letter(s) of all the following that are **true** in general. At least one will be true. We have reason to suspect these distributions are very non-normal and extreme-tailed. One such model is the Lomax distribution which is a two-parameter survival model. To keep things tractable for the purposes of this class, we will set one parameter to be equal to one. Assume $X_1, \ldots, X_n \overset{iid}{\sim} \text{Lomax}(1, \theta) := f(x; \theta) = \theta(1 + x)^{-(\theta+1)}$ and $\mathbb{S}\text{upp}[X] = (0, \infty)$. Note: we are redefining the definition of θ now — it is now a parameter within this assumed DGP! The mean of this rv is $\mu_1 := \mathbb{E}[X] = 1/(\theta - 1)$. Here are sample statistics: $\bar{x}_1 = 6.01$, $\bar{x}_2 = 10.50$, $s_1 = 14.54$ and $s_2 = 31.62$.

(a) $\mathcal{L}(\theta; x) = f(x; \theta)$

(b) $\ell(\theta; x) = f(x; \theta)$

(c) $\mathcal{L}(\theta; x_1, \ldots, x_n) = \prod_{i=1}^n f(x_i; \theta)$

(d) $\ell(\theta; x_1, \ldots, x_n) = \prod_{i=1}^n \ln(f(x_i; \theta))$

(e) $\hat{\theta}^{\text{MLE}} = \bar{X}$

(f) $\hat{\theta}^{\text{MLE}} = (1 + \bar{X})/\bar{X}$

(g) $\hat{\theta}^{\text{MLE}} = n \left(\sum_{i=1}^n \ln(1 + X_i)\right)^{-1}$

(h) $I(\theta) = \int_{\mathbb{R}} \ell'(\theta; X)^2 \theta(1 + x)^{-(\theta+1)} dx$

(i) $I(\theta) = \int_{\mathbb{R}} x\ell'(\theta; X)^2 dx$

(j) $I(\theta) = -\mathbb{E}[\ell''(\theta; X)]$

(k) $I(\theta) = 1/(\theta - 1)$

(l) $I(\theta) = 1/(\theta - 1)^2$

(m) $I(\theta) = 1/\theta^2$

(n) $I(\theta) \propto 1$

(o) $\hat{\theta}^{\text{MLE}}$ is definitely unbiased without further proof

(p) $\hat{\theta}^{\text{MLE}}$ is definitely asymptotically normal without further proof

(q) $\hat{\theta}^{\text{MLE}}$ is definitely asymptotically efficient (i.e. its variance approaches the CRLB of variance) without further proof

Your answer will consist of a lowercase string (e.g. `aebgd`) where the order of the letters does not matter.

Problem 7 [5min] (and 48min will have elapsed) These are conceptual questions about statistical inference and sampling.

- [10 pt / 92 pts] Record the letter(s) of all the following that are **true** in general. At least one will be true. We have reason to suspect these distributions are very non-normal and extreme-tailed. One such model is the Lomax distribution which is a two-parameter survival model. To keep things tractable for the purposes of this class, we will set one parameter to be equal to one. Assume $X_1, \ldots, X_n \overset{iid}{\sim} \text{Lomax}(1, \theta) := f(x; \theta) = \theta(1+x)^{-(\theta+1)}$ and $\mathbb{S}\text{upp}[X] = (0, \infty)$. Note: we are redefining the definition of θ now — it is now a parameter within this assumed DGP! The mean of this rv is $\mu_1 := \mathbb{E}[X] = 1/(\theta - 1)$. Here are sample statistics: $\bar{x}_1 = 6.01$, $\bar{x}_2 = 10.50$, $s_1 = 14.54$ and $s_2 = 31.62$. In this setting, $\hat{\theta}^{\text{MLE}} = n\left(\sum_{i=1}^n \ln(1 + X_i)\right)^{-1}$ and $I(\theta) = 1/\theta^2$.

  (a) For an estimator $\hat{\theta}$ whose expectation is θ, the standard deviation of $\hat{\theta}$ must be at least $\theta/\sqrt{n}$

  (b) The variance of $\hat{\theta}^{\text{MLE}}$ is exactly θ^2/n

  (c) The variance of $\hat{\theta}^{\text{MLE}}$ is approximately θ^2/n

  (d) $\hat{\theta}^{\text{MLE}} \xrightarrow{p} \theta$

  (e) $I(\hat{\theta}^{\text{MLE}}) \xrightarrow{p} I(\theta)$

  (f) $I(\hat{\theta}^{\text{MLE}})^{-1} \xrightarrow{p} I(\theta)^{-1}$

  (g) $I(\hat{\theta}^{\text{MLE}}) \xrightarrow{d} \mathcal{N}(0,\, 1)$

  (h) $\hat{\theta}^{\text{MLE}} \xrightarrow{d} \mathcal{N}(0,\, 1)$

  (i) $\dfrac{\hat{\theta}^{\text{MLE}}}{\text{SE}\left[\hat{\theta}^{\text{MLE}}\right]} \xrightarrow{d} \mathcal{N}(0,\, 1)$

  (j) If $\dfrac{\hat{\theta}^{\text{MLE}}}{\text{SE}\left[\hat{\theta}^{\text{MLE}}\right]} \xrightarrow{d} \mathcal{N}(0,\, 1)$ then $\dfrac{\left(\hat{\theta}^{\text{MLE}}\right)^2}{\mathbb{V}\text{ar}\left[\hat{\theta}^{\text{MLE}}\right]} \xrightarrow{d} \chi^2_1$

Your answer will consist of a lowercase string (e.g. `aebgd`) where the order of the letters does not matter.

Problem 8 [7min] (and 55min will have elapsed) These are conceptual questions about statistical inference and sampling.

- [10 pt / 102 pts] Record the letter(s) of all the following that are **true** in general. At least one will be true. We have reason to suspect these distributions are very non-normal and extreme-tailed. One such model is the Lomax distribution which is a two-parameter survival model. To keep things tractable for the purposes of this class, we will set one parameter to be equal to one. Assume $X_1, \ldots, X_n \overset{iid}{\sim} \text{Lomax}(1, \theta) := f(x; \theta) = \theta(1 + x)^{-(\theta+1)}$ and $\mathbb{S}\text{upp}[X] = (0, \infty)$. Note: we are redefining the definition of θ now — it is now a parameter within this assumed DGP! The mean of this rv is $\mu_1 := \mathbb{E}[X] = 1/(\theta - 1)$. Here are sample statistics: $\bar{x}_1 = 6.01$, $\bar{x}_2 = 10.50$, $s_1 = 14.54$ and $s_2 = 31.62$. In this setting, $\hat{\theta}^{\text{MLE}} = n\left(\sum_{i=1}^n \ln(1 + X_i)\right)^{-1}$ and $I(\theta) = 1/\theta^2$. We wish to prove that the θ's in the two populations are different. Given the raw data, we can compute the maximum likelihood estimates: $\hat{\hat{\theta}}_1^{\text{MLE}} = 0.996$ and $\hat{\hat{\theta}}_1^{\text{MLE}} = 0.998$ rounded to the nearest 3 digits. Note that $F_Z(.975) = 1.96$ and $F_{T_{14}}(.975) = 2.15$.

  (a) The standardized test statistic can be expressed as $\dfrac{\hat{\hat{\theta}}_1^{\text{MLE}} - \hat{\hat{\theta}}_2^{\text{MLE}}}{\theta/\sqrt{n}}$

  (b) The standardized test statistic can be expressed as $\dfrac{\hat{\hat{\theta}}_1^{\text{MLE}} - \hat{\hat{\theta}}_2^{\text{MLE}}}{\theta\sqrt{2/n}}$

  (c) A justified standardized test statistic is -0.4996635 rounded to the nearest 7 digits

  (d) A justified standardized test statistic is -0.01509039 rounded to the nearest 7 digits

  (e) A justified standardized test statistic is -0.005493704 rounded to the nearest 7 digits

  (f) A justified standardized test statistic is -0.0002225672 rounded to the nearest 7 digits

  (g) If the null hypothesis in the test of no mean difference was retained, then the confidence interval for the mean different *must* contain zero (for any valid α).

  (h) $\hat{\hat{CI}}_{\theta_1, 95\%}$ cannot be computed given the data at hand

  (i) $\hat{\hat{CI}}_{\theta_1 - \theta_2, 95\%}$ cannot be computed given the data at hand

  (j) $\hat{\hat{CI}}_{\theta_1, \alpha}$ can only be computed for some values of α but not other values of α

Your answer will consist of a lowercase string (e.g. `aebgd`) where the order of the letters does not matter.

Problem 9 [5min] (and 60min will have elapsed) These are conceptual questions about statistical inference and sampling.

- [11 pt / 113 pts] Record the letter(s) of all the following that are **true** in general. At least one will be true. We have reason to suspect these distributions are very non-normal and extreme-tailed. One such model is the Lomax distribution which is a two-parameter survival model. To keep things tractable for the purposes of this class, we will set one parameter to be equal to one. Assume $X_1, \ldots, X_n \overset{iid}{\sim} \text{Lomax}(1, \theta) := f(x; \theta) = \theta(1+x)^{-(\theta+1)}$ and $\mathbb{S}\text{upp}[X] = (0, \infty)$. Note: we are redefining the definition of θ now — it is now a parameter within this assumed DGP! The mean of this rv is $\mu_1 := \mathbb{E}[X] = 1/(\theta - 1)$. Here are sample statistics: $\bar{x}_1 = 6.01$, $\bar{x}_2 = 10.50$, $s_1 = 14.54$ and $s_2 = 31.62$. In this setting, $\hat{\theta}^{\text{MLE}} = n\left(\sum_{i=1}^n \ln(1 + X_i)\right)^{-1}$ and $I(\theta) = 1/\theta^2$. We wish to prove that the θ's in the two populations are different. Given the raw data, we can compute the maximum likelihood estimates: $\hat{\hat{\theta}}_1^{\text{MLE}} = 0.996$ and $\hat{\hat{\theta}}_1^{\text{MLE}} = 0.998$ rounded to the nearest 3 digits. Note that $F_Z(.975) = 1.96$ and $F_{T_{14}}(.975) = 2.15$.

  (a) The construction of a confidence interval estimate requires knowledge of the parameter(s)

  (b) $\hat{\hat{CI}}_{\theta_1 - \theta_2, 95\%} \approx [0.4919545, 1.500046]$

  (c) $\hat{\hat{CI}}_{\theta_1 - \theta_2, 95\%} \approx [-0.7155441, 0.7115441]$

  (d) $\hat{\hat{CI}}_{\theta_1 - \theta_2, 95\%} \approx [-0.2617679, 0.2577679]$

  (e) The construction of $\hat{\hat{CI}}_{\theta_1 - \theta_2, 95\%}$ in this problem relied on the central limit theorem

  (f) The construction of $\hat{\hat{CI}}_{\theta_1 - \theta_2, 95\%}$ in this problem relied on the monster MLE theorem

  (g) The construction of $\hat{\hat{CI}}_{\theta_1 - \theta_2, 95\%}$ in this problem relied on the asymptotic normality of $\hat{\theta}_1^{\text{MLE}} - \hat{\theta}_2^{\text{MLE}}$

  (h) The construction of $\hat{\hat{CI}}_{\theta_1 - \theta_2, 95\%}$ in this problem relied on the delta method

  (i) The construction of $\hat{\hat{CI}}_{\theta_1 - \theta_2, 95\%}$ in this problem relied on Slutsky's theorem

  (j) $\mathbb{P}\left(\theta_1 - \theta_2 \in \hat{\hat{CI}}_{\theta_1 - \theta_2, 95\%}\right) \approx 95\%$

  (k) $\mathbb{P}\left(\theta_1 - \theta_2 \in \hat{CI}_{\theta_1 - \theta_2, 95\%}\right) \approx 95\%$

Your answer will consist of a lowercase string (e.g. `aebgd`) where the order of the letters does not matter.

Problem 10 [10min] (and 70min will have elapsed) These are conceptual questions about statistical inference and sampling.

- [7 pt / 120 pts] Record the letter(s) of all the following that are **true** in general. At least one will be true. We have reason to suspect these distributions are very non-normal and extreme-tailed. One such model is the Lomax distribution which is a two-parameter survival model. To keep things tractable for the purposes of this class, we will set one parameter to be equal to one. Assume $X_1, \ldots, X_n \overset{iid}{\sim} \text{Lomax}(1, \theta) := f(x; \theta) = \theta(1+x)^{-(\theta+1)}$ and $\mathbb{S}\text{upp}[X] = (0, \infty)$. Note: we are redefining the definition of θ now — it is now a parameter within this assumed DGP! The mean of this rv is $\mu_1 := \mathbb{E}[X] = 1/(\theta - 1)$. Here are sample statistics: $\bar{x}_1 = 6.01$, $\bar{x}_2 = 10.50$, $s_1 = 14.54$ and $s_2 = 31.62$. In this setting, $\hat{\theta}^{\text{MLE}} = n\left(\sum_{i=1}^n \ln(1 + X_i)\right)^{-1}$ and $I(\theta) = 1/\theta^2$. ~~We wish to prove that the θ's in the two populations are different.~~ Given the raw data, we can compute the maximum likelihood estimates: $\hat{\hat{\theta}}_1^{\text{MLE}} = 0.996$ and $\hat{\hat{\theta}}_1^{\text{MLE}} = 0.998$ rounded to the nearest 3 digits. Note that $F_Z(.975) = 1.96$ and $F_{T_{14}}(.975) = 2.15$. We are now interested in proving that $\theta_1^2 > \theta_0^2$ where $\theta_0 = 1$ which is a critical value in the Lomax distribution.

  (a) The univariate delta method applies in our setting and where $g(\theta) = \theta^2$

  (b) $\dfrac{\left(\hat{\theta}_1^{\text{MLE}}\right)^2 - \theta_0^2}{2\theta_0^2/\sqrt{n}} \overset{d}{\to} \mathcal{N}(0,\, 1)$

  (c) $\dfrac{\left(\hat{\theta}_1^{\text{MLE}}\right)^2 - \theta_0^2}{2\left(\hat{\theta}_1^{\text{MLE}}\right)^2/\sqrt{n}} \overset{d}{\to} \mathcal{N}(0,\, 1)$

  (d) A justified standardized test statistic is -0.01546095 rounded to the nearest 7 digits

  (e) A justified standardized test statistic is -0.01558538 rounded to the nearest 7 digits

  (f) $\hat{CI}_{\theta_1, 95\%} \approx \left[\hat{\theta}_1^{\text{MLE}} \pm 1.96 \times 2\dfrac{\theta_0^2}{\sqrt{n}}\right]$

  (g) $\hat{CI}_{\theta_1, 95\%} \approx \left[\hat{\theta}_1^{\text{MLE}} \pm 1.96 \times 2\dfrac{\left(\hat{\theta}_1^{\text{MLE}}\right)^2}{\sqrt{n}}\right]$

Your answer will consist of a lowercase string (e.g. `aebgd`) where the order of the letters does not matter.