To approximate the sampling distribution of phi-hat, you take a sample with replacement from $x_1, \ldots, x_n$ and call it $x_{b,1}, \ldots, x_{b,n}$. Then calculate phi-hat-hat_b using the w function. Then repeat this many times (e.g. B = 1,000,000) to produce

$\{\hat{\hat{\phi}}_1, \ldots, \hat{\hat{\phi}}_B\}$ and then create an empirical distribution which we

call the "bootstrap distribution". Thm: the bootstrap distribution is asymptotically convergent to the true sampling distribution. Hence, the bootstrap distribution approximates the sampling distribution. This thm works under highly general conditions (we're skipping the technical details).

The word "bootstrap" in English is famous for the expression "pull oneself up by the bootstrap" which means succeeding miraculously like defying gravity.

$$CI_{\phi,1-\alpha} \approx \left[Q\left[\{\hat{\hat{\phi}}_1, \ldots, \hat{\hat{\phi}}_B\}, \frac{\alpha}{2}\right], Q\left[\{\hat{\hat{\phi}}_1, \ldots, \hat{\hat{\phi}}_B\}, 1-\frac{\alpha}{2}\right]\right]$$

To test the plausibility of the null hypothesis $H_0: \phi = \phi_0$ we use the equivalence of CI and testing so we just check if

$$\phi_0 \overset{?}{\in} CI_{\phi,1-\alpha}$$

The actual statistic on the dataset phi-hat-hat does not play a role here.

<u>end of Math 369</u>

We did not cover many fundamental topics:

* Sufficiency / sufficient statistics. If thetahat = $w(X_1, \ldots, X_n)$ it seems like you're doing dimension reduction from dimension n to dimension 1. If you can show that thetahat contains "all the information" about theta, then thetahat is called a "sufficient statistic" and then you can discard the raw data because it won't ever tell you anything more about theta.

* Identifiability. Some DGP's have parameters that aren't possible to estimate even with infinite data (unidentifiable). Everything in this class we did was identifiable.

* Neyman-Pearson Lemma. We've seen many tests for the same null. You can prove some tests are the most powerful tests for some situations.

* Paired comparisons. Data is realized in pairs.

* Correlation vs causation, causal estimation, randomized experiments.