2021 1K

$P(X|\theta) = Bin(n,\theta) = \binom{n}{x}\theta^x(1-\theta)^{n-x}$

$P(\theta) = U(0,1) = Beta(1,1)$

$\Rightarrow P(\theta|X) = Beta(x+1,\ n-x+1)$

$$P(X|\theta) = \frac{n!}{x!(n-x)!}\,\theta^x(1-\theta)^n(1-\theta)^{-x}$$

$$\propto \frac{1}{x!(n-x)!}\left(\frac{\theta}{1-\theta}\right)^x$$

HW 6 2(a)

$$P(\theta,\sigma^2|X) \propto P(X|\theta,\sigma^2)\,\underbrace{P(\theta,\sigma^2)}_{\propto \frac{1}{\sigma^2}}$$

$$\propto \prod_{i=1}^{n}\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2\sigma^2}(X_i-\theta)^2}\ \frac{1}{\sigma^2}$$

$$= (2\pi)^{-n/2}(\sigma^2)^{-n/2-1}e^{-\frac{1}{2\sigma^2}\sum(X_i-\theta)^2}$$

$$\propto (\sigma^2)^{-n/2-1}e^{-\frac{1}{2\sigma^2}\left((n-1)s^2+n(\bar{x}-\theta)^2\right)}$$

HW 6 2(b)

$$= e^{-\frac{1}{2\sigma^2}n(\bar{x}-\theta)^2}(\sigma^2)^{-n/2-1}e^{-\frac{1}{2\sigma^2}(n-1)s^2}$$

$$P(\theta,\sigma^2|X) = \underbrace{e^{-\frac{1}{2\sigma^2/n}(\bar{x}-\theta)^2}}_{\propto P(\theta|X,\sigma^2)}\ \underbrace{(\sigma^2)^{-n/2-1}e^{-\frac{(n-1)s^2/2}{\sigma^2}}}_{\propto P(\sigma^2|X)}$$

$$\underbrace{N\left(\bar{x},\frac{\sigma^2}{n}\right) \qquad InvGamma\left(\frac{n}{2},\frac{(n-1)s^2}{2}\right)}_{NormInvGamma\left(\bar{x},\frac{\sigma^2}{n},\frac{n}{2},\frac{(n-1)s^2}{2}\right)}$$

let $\sigma^2=\sigma_s^2$

HW 6 2(c)

1. Sample σ_s^2 from
2. Use σ_s^2 to sample from
3. Repeat steps 1-2 to get ...

$$\left\{\begin{bmatrix}\sigma_1^2\\ \theta_1\end{bmatrix},\begin{bmatrix}\sigma_2^2\\ \theta_2\end{bmatrix},\ \dots,\ \begin{bmatrix}\sigma_S^2\\ \theta_S\end{bmatrix}\right\}$$

e.g. $S=10{,}000$

HW 6 2(d)

$$E[\theta|x] \approx \frac{1}{S}\sum_{s=1}^{S}\theta_s$$

order them

HW6 2(e)

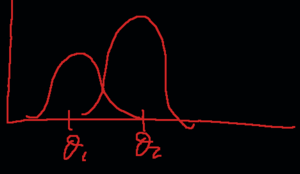
$$CR_{\theta,95\%} = \left[\theta_{(2.5\%\cdot S)},\ \theta_{(97.5\%\cdot S)}\right] \qquad \left\langle \underbrace{\theta_{(1)}}_{min},\theta_{(2)},\dots,\underbrace{\theta_{(S)}}_{max}\right\rangle$$

2018 4

$$X_1,\dots,X_{300} \sim \begin{cases} \xrightarrow{p} N(\theta_1,\sigma_1^2) \\ \xrightarrow{1-p} N(\theta_2,\sigma_2^2)\end{cases}$$

$n_1+n_2=n$

$\underset{p}{58\%}\cdot\underset{n}{300} = \#$ from $N(\theta_1,\sigma_1^2)$ expected $= n_1$



$$CR_{p,95\%} = [63\%, 77\%]$$

$$\Downarrow$$

$$CR_{n_1,95\%} = [189, 231]$$

$$\begin{pmatrix} p\\ \theta_1\\ \sigma_1^2\\ \theta_2\\ \sigma_2^2\end{pmatrix}$$

$I_k \sim Bern(p)$

$$\begin{cases} I_k = 1 \Rightarrow X\sim N(\theta_1,\sigma_1^2)\\ I_k = 0 \Rightarrow X\sim N(\theta_2,\sigma_2^2)\end{cases}$$