Math 241 Lecture 6 Sept 16, 2014

MISSISSIPPI

$$\frac{11!}{4!\,2!\,2!}$$

R, S if R ≥ 4

$\{1, 2, 3, (4,1), \ldots\}$ $|\Omega| = 27$

equally likely $P(\omega_i) = \frac{1}{|\Omega|}\ \forall i$

$P(1) = \frac{1}{6} \neq \frac{1}{27}$

Back row ... 2C, 2K, 2B, 1Kg, 1Q

$$\left(\frac{8!}{2!\,2!\,2!}\right)^2$$

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how many ways can we place 32 pieces on 64 sq

$$\frac{{}_{64}P_{32}}{(8!\,2!\,2!\,2!)^2}$$

also ~~Brandon~~ Brandon

how many ways to pick 32 sq's

$$\binom{64}{32} \frac{32!}{(8!(2!)^3)^2}$$

for each, how many arrangements?

4B, 4G, 2O, 2R

$$\frac{12!}{4!\,4!\,2!\,2!}$$

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2 pair (B, G), (B, O), (B, R), (G, O), (G, R), (O, R)

$$\binom{4}{2}\binom{4}{2} \quad \binom{4}{2}\binom{2}{2} \quad \binom{4}{2}\binom{2}{2} \quad \binom{4}{2}\binom{2}{2} \quad \binom{4}{2}\binom{2}{2} \quad \binom{2}{2}\binom{2}{2}$$

$$6 \cdot 6 + 6 + 6 + 6 + 6 + 1 = \boxed{61}$$

Previously, we discussed balls & urns

7 balls 4 urns

$\bigcirc\bigcirc|\bigcirc|\bigcirc|\bigcirc\bigcirc\bigcirc$ $\quad \binom{7-1}{4-1}$ generally $\binom{n-1}{r-1}$

with the restriction ≥ 1 balls in each urn $\Rightarrow r \leq n$

What about if urns are indistinguishable $\rightarrow$ E.C.

$7 = 1 + 2 + 4 + 0 \quad \langle 1,2,4,0 \rangle$

what about $\{1,2,4,0\}$ without order? $\frac{\binom{n-1}{r-1}}{r!}$

What about not caring if urns are empty? $\rightarrow$ E.C.

3 balls 2 urns

$|\bigcirc|\bigcirc|\bigcirc$

3 balls 3 urns $\binom{3+3-1}{2} = \binom{5}{2} = \frac{5 \cdot 4}{2} = 10$

11000	01010
10100	01001
10010	00110
10001	00101
01100	00011

5 places pick 2 for dividers

$\Rightarrow \binom{n+r-1}{r-1}$

Next Class

Independence

If A ind B, is A^c ind of B?

$P(A|B) = P(A)$

$P(A^c|B) = P(A^c)$?

$1 - P(A|B) = P(A^c)$

$1 - P(A) = P(A^c)$ ✓

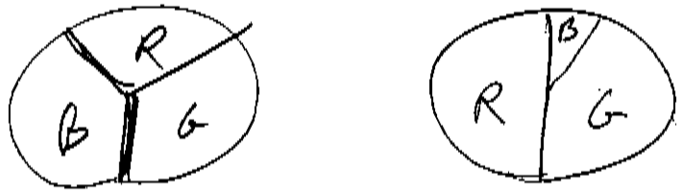
Dependent Coin? H,H or T,T

$P(H_1) = P(H_2) = \frac{1}{2}$

$P(H_1 H_2) = \frac{1}{2} \neq P(H_1)P(H_2) = \frac{1}{4}$ dependent...

Independence $P(A,B) = P(A)P(B) \iff P(A|B) = P(A)$ or $P(B|A) = P(B)$

Sometimes independence doesn't mean "independent" in English e.g.

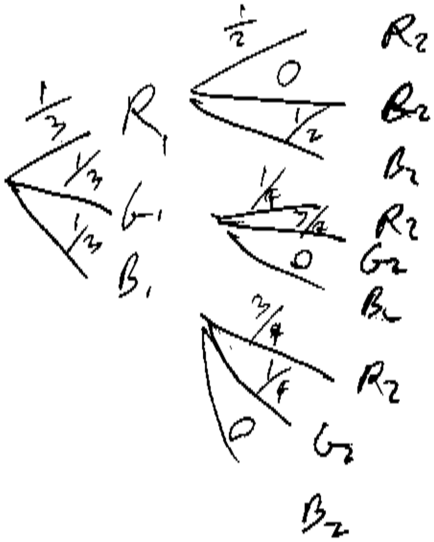


spinner 1 spinner 2

$P(R_1) = \frac{1}{3}$ $P(R_2) = \frac{1}{2}$

If "independent" (in English) $\implies P(R_1, R_2) = \frac{1}{6}$

But what if they were computer controlled like the coin...



Clearly "dependent" in English.
But "dependent" probabilistically?

"Law of total prob"

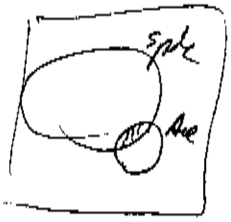
$$\frac{1}{6} = P(R_1, R_2) \overset{?}{=} P(R_1)\,P(R_2) = \left(\frac{1}{3}\right)\left(\underbrace{\underbrace{\frac{1}{3}\cdot\frac{1}{2}}_{\frac{1}{6}} + \underbrace{\frac{1}{3}\cdot\frac{1}{4}}_{\frac{1}{12}} + \underbrace{\frac{1}{3}\cdot\frac{3}{4}}_{\frac{1}{4}}}_{\frac{1}{2}}\right) = \frac{1}{6}$$

independence really means informationally irrelevant..

pairwise ind, mutually independence → NOT COVERED (P.31)

$P(A\cup B) = P(A)+P(B)-P(A\cap B)$ Bode's Inequality $P(A\cup B) \le P(A)+P(B)$

$P(A, \text{spade}) = \frac{4}{52}+\frac{13}{52}-\frac{1}{52}=\frac{16}{52}$



$P(A\mid \text{spade}) = \frac{1}{4}$ Still "informationally irrelevant" but a tad more subtle...

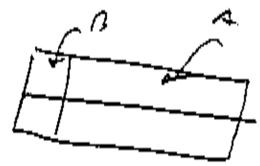
What does "given" mean? It means we no longer operate in Ω. We only operate in $\Omega' \subseteq \Omega$. Here $\Omega' = \{\text{spades}\}$

thus...

and $\frac{|A'|}{|\Omega'|} = \frac{|A|}{|\Omega|}$ due to independence

$\Rightarrow \frac{1}{13} = \frac{4}{52}$ ✓

A: stock ↑
B: rain
$P(A) = 0.5$
$P(B) = 0.1$



$P(A\mid B) = 0.5 = P(A)$

different picture though!

"Conditional Probability" p20-27 (Now, p1-33 covered)

We track $n=1{,}000$ people, 200 smokers, 60 lung cancer, 36 lc & sm.

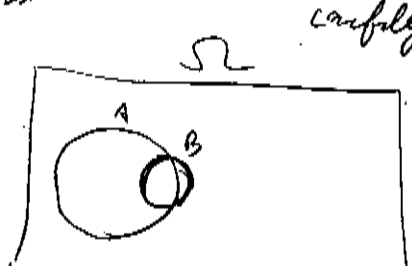
If $n \approx \infty$, frequencies are probs! Assume so. A: smoke
B: Lung Cancer

$P(A) = \frac{200}{1000} = 0.2$, $P(B) = \frac{60}{1000} = 0.06$, $P(A,B) = \frac{36}{1000} = 0.036$ de Morgan's Laws

$P(A^c) = 0.8$, $P(B^c) = 0.94$, $P((A,B)^c) = 0.974 = P(A^c \cup B^c)$

$P(\Omega) = \frac{1000}{1000} = 1$

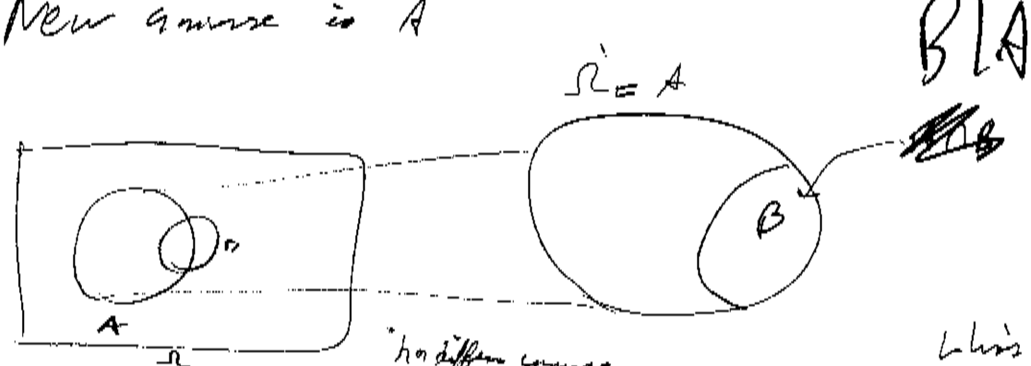
carefully draw sample space...

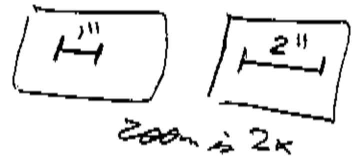


What is $P(\text{lung cancer} \mid \text{smoking}) = P(B \mid A)$

New universe is A $\qquad B \mid A$ $\qquad \frac{\text{orig size}}{\text{new size}} = \frac{1}{\frac{1}{5}} = 2$

$\Omega' = A$





zoom is 2x

What is my zoom factor?

"in different universe"

how much smaller is Ω' than Ω

$$P(B \mid A) = P(A,B)\ \underbrace{\frac{P(\Omega)}{P(\Omega')}}_{\frac{1}{P(A)}} = \frac{P(A,B)}{P(A)}$$

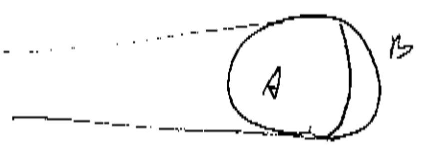
"Bayes Rule" $\quad P(A \mid B) = \frac{P(A,B)}{P(B)}$

$$= \frac{0.036}{0.2} = 0.18 \approx \frac{1}{5} \Rightarrow \text{don't smoke (bad odds!)}$$

Universe shrank and then expanded... but you have to keep track of zoom factor.

$0.2 \longrightarrow 1$.

Flip question: someone walks in with lung cancer. What's the prob they smoke?



$$P(A \mid B) = \frac{P(A,B)}{P(B)} = \frac{.036}{.06} = .6$$

good chance he smoked, but not 99%.

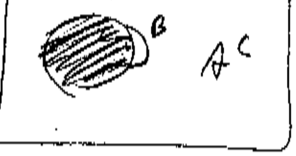
$$P(B \mid A) = \frac{P(A \mid B)\,P(B)}{P(A)}$$

$$P(A \mid B) = \frac{P(B \mid A)\,P(A)}{P(B)} = \frac{.18 \cdot .2}{.06} = .6 \checkmark$$

Flavor #2 of Bayes Rule

Prob of getting l.c. | no smoking $\qquad P(B \cap A^c) = P(B) - P(A,B) = .06 - .036 = .024$

$P(B \mid A^c)$



$$= \frac{P(B, A^c)}{P(A^c)} = \frac{.024}{.8} = .03 \quad \text{still happens..}$$

hard to draw

Risk of getting l.c. in Smoking $= \frac{P(B \mid A)}{P(B \mid A^c)} = \frac{.18}{.03} = \boxed{6\times}$