lecture 13 10/28/14

Review

discrete r.v. X, $Supp(X) = \{x : f(x) > 0\}$

w/ PMF $f(x) := P(X = x)$, CDF $F(x) := P(X \le x)$

X is within a certain range

on the a lot

How about $P(a \le X \le b) = P(X \in [a,b]) = \sum_{x=a}^{b} f(x)$

point just in supp before b

$P(X \in (a,b)) = \sum_{x=\uparrow} f(x)$ gotta know support! tricky!

least item on support after a

$X \sim binomial(n,p) \implies Supp(X) = \{0, \ldots, n\}$

prior natural #

$P(X \in (a,b)) = \sum_{x=a+1}^{b-1} f(x)$

next natural #

New idea...

$Quantile[X, p] := \{x : F(x) = p\}$ ideally

or "percentile"

25%ile of X is the first x in support s.t. $F(x) \ge 0.25$

~~X ~ Binomial(0.1, 6)~~

$X \sim \text{Binom}(10, 0.4)$

x	$f(x)$	$F(x)$
0	0.0060	0.0060
1	0.0403	0.0464
2	0.1209	0.1673
3	0.2150	0.3823
4	0.2508	0.6331
5	0.2007	0.8338
6	0.1115	0.9452
7	0.0425	0.9877
8	0.0106	0.9983
9	0.0016	0.9999
10	0.0001	1.0000
	sum = 1	N/A

What is the 10%ile of X? 2

What is the 20%ile of X? 3

What is the 90%ile of X? 6

What is the 50%ile of X? 4

$\text{Median}[X] = 4 \quad := \text{Quantile}[X, .5]$

$\text{IQR}[X] := \text{Quantile}[X, .75] - \text{Quantile}[X, .25]$
$= 5 - 3 = 2$

"inter-quartile range"

Note: All distributions have all quantiles defined since $f(x)$ and $F(x)$ must be defined $\forall x \in \text{Supp}[X]$.

$$\text{Mode}[X] := \underset{x \in \text{Supp}[X]}{\text{argmax}} \{ f(x) \}$$ here mode is ... 4

Back to expectation... Roulette... Make \$1 bet on Red

$\Rightarrow$ Even odds $\Rightarrow$ win \$1 on red, lose \$1 on Not Red

$$X \sim \begin{cases} \$1 & \text{w.p. } 10/30 \\ \$-1 & \text{w.p. } 20/30 \end{cases}$$

r.v.'s usually have units in the real world.

$$E[X] := \sum_{x \in \text{Supp}[X]} x f(x) = (1) f(1) + (-1) f(-1)$$
$$= (1)\frac{10}{30} + (-1)\frac{20}{30}$$
$$= -\frac{2}{30} \approx -\$0.0\overline{53}$$

If I play over and over again.. I lose on average \$ 5¢ each time.

"Strategy"

Bet on lucky #7. Payout is 35:1

$$X \sim \begin{cases} \$35 & \text{w.p. } \frac{1}{38} \\ \$-1 & \text{w.p. } \frac{37}{38} \end{cases}$$

$$E[X] = \sum_{x \in \text{Supp}(X)} x f(x) = (35)\frac{1}{38} + (-1)\left(\frac{37}{38}\right) = \frac{35-37}{38} = -\frac{2}{38} \approx -0.053$$

"Dozen Bet" $\{1, \ldots, 12\}$ Payout 2:1

$$X \sim \begin{cases} \$2 & \text{w.p. } \frac{12}{38} \\ \$-1 & \text{w.p. } \frac{26}{38} \end{cases}$$

$$E(X) = (2)\frac{12}{38} + (-1)\frac{26}{38} = \frac{24-26}{38} = -\frac{2}{38} \approx \$-0.053$$

All bets in roulette have the same expectation!

In Europe, bet on red

$$X \sim \begin{cases} \$1 & \text{w.p. } 18/37 \\ -\$1 & \text{w.p. } 19/37 \end{cases} \qquad E(X) = -1/37 \approx -\$0.027$$

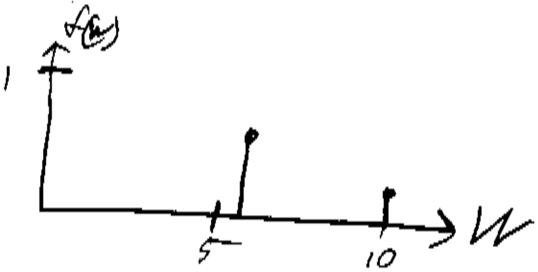
much "fairer"

In the long run, you're doomed! But in the short run, you can win. (See L4)

(HW)

New question. I Ride Uber. If I take the Van Wyck home, it's 6 min. If I take Jewel Ave, it's 10 min. \$0.40/min. P(traffic on Van Wyck) = 30%.

$$W \sim \begin{cases} 6 \text{ min w.p. } 70\% \\ 10 \text{ min w.p. } 30\% \end{cases}$$

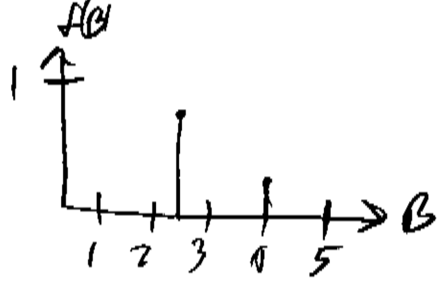


$$E[W] = 6 \cdot .7 + 10 \cdot .3 = \frac{42+30}{10} = 7.2 \text{ min}$$

What should how much I pay for my mileage?

$$B := \$0.40 \ \text{min/mi} \ W$$

$$Supp(B) = \$0.40 \ Supp(W) = \{0.4 \cdot 6, 0.4 \cdot 10\} = \{\$2.40, \$4.00\}$$



M

$$E[B] = \$2.40(0.7) + \$4.00(0.3) = \$2.88$$

$$= 0.4 \cdot 6 \cdot 0.7 + 0.4 \cdot 10 \cdot 0.3$$

b/c $P(B = \$2.40) = P(W = 6 \text{ min})$

$$E[B] = \sum_{b \in Supp[B]} b \, f_B(b) = \sum_{w \in Supp(W)} 0.4 w \, f_W(w) \quad \text{since } B = 0.4 W!$$

$$= 0.4 \, E[W]$$

$\Rightarrow$ In general, $E[aX] = a\,E[X]$ since

$$E[aX] = \sum_{x \in Supp(X)} a\,x\,f(x) = a \sum x\,f(x) = a\,E[X]$$

PMF same!

Uber charges a base fare, so my true bill is

$$B = \$3 + \$0.40/\text{min}\; W$$

$$E[B] = \sum_{w \in Supp(W)} (3 + 0.4w)\,f(w) = \sum 3 f(w) + \sum 0.4w\,f(w)$$

$$= 3 \underbrace{\sum f(w)} + 0.4 \sum w f(w)$$

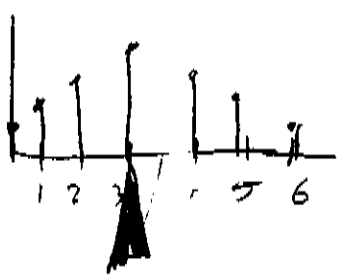
$$= \$3 + 0.4\,E[W]$$

$$= \$3 + \$2.80 = \boxed{\$5.80}$$

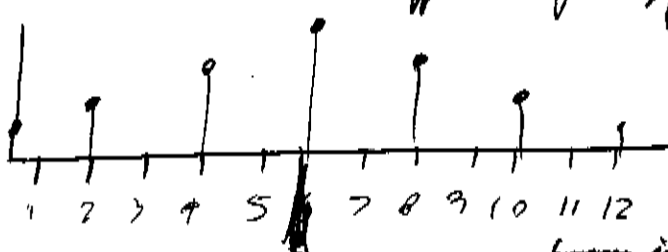
In general, $E[aX + c] = c + a\,E[X]$

STOP

Now say for standard $X \sim Bin(6, \frac{1}{2})$



2X



Support is expanded, PMF is expanded by same factor

X+5

support is shifted, PMF is shifted