MATH 214 Fall 2014 Homework #1

Professor Adam Kapelner

Due 5PM in my office, Tues Sept 9, 2014

Instructions and Philosophy

Once again, the path to success in this class is to do many problems. Unlike other courses, exclusively doing reading(s) will not help. Coming to lecture is akin to watching workout videos; thinking about and solving problems on your own is the actual "working out".

Reading is still *required*. For this homework set, please read Chapter 1 of Hogg, Tanis and Zimmerman (H, T & Z) pages 1–6.

The problems below are color coded: green problems are considered *easy*; yellow problems are considered *intermediate*, red problems are considered *difficult*; and purple problems are for *extra credit*. The *easy* problems are intended to be "giveaways" if you went to class. Do as much as you can of the others; I expect you to at least attempt the *difficult* problems.

This homework is worth 100 points but the point distribution will not be determined until after the due date. Late homework will be penalized 10 points per day up to a maximum of three days. After three days, it will receive a zero because I will post the solutions.

15 points are given as a bonus if the homework is typed using LaTeX. Links to instaling LaTeX and program for compiling LaTeX is found on the syllabus. If you are handing in homework this way, read the comments in the code; there are two lines to comment out and place to put your name.

If you are not doing the homework in LaTeX, there are two options for hand-in formats: (1) you handwrite on this paper itself and (2) you handwrite on a separate paper. Thus, the compiled PDF of this document will be available on the course homepage in two forms: (1) with vertical spaces for your handwritten answers and (2) without spaces. You must show your work! At least enough of it so that I know you derived the result and not only copied the answer from someone else.

I STRONGLY recommend using strategy number 1. It is not only easy for me to grade your homework, but you always have the questions and answers in one place and it makes for easier referencing.

Set Theory Problems below are related to set theory. The sets we talk about in class are composed of outcomes in a universe that are events. Some of the problems below will be about abstract sets that are divorced from the sets used in probability.

Problem 1 These are questions on abstract set theory. Assume capital letters are arbitrary sets and Ω is the universe for all the following questions. Answer as succinctly as possible. Some of this will be review.

(a) $A \cup A =$ $\quad A \cap A =$ $\quad A \cup \varnothing =$ $\quad A \cap \varnothing =$ $\quad A \cup \Omega =$ $\quad A \cap \Omega =$
$A \cup A^C =$ $\quad A \cap A^C =$ $\quad \varnothing^C =$ $\quad \Omega^C =$ $\quad A \backslash A =$ $\quad A \backslash \Omega =$ $\quad A \backslash \varnothing =$

(b) Are the following true (T) or false (F) for arbitrary sets $A,\ B,\ C$?
$A \subseteq \Omega$ $\quad A \subset \Omega$ $\quad \varnothing \subseteq A \subseteq \Omega$ $\quad A \subseteq A \cup B$ $\quad A \subseteq A \cap B$

(c) Are the following true (T) or false (F)? The symbol "$\Rightarrow$" denotes logical implication *i.e.* if the conditions on the l.h.s are met, the statement on the r.h.s is always true. Commas should be interpreted to mean "and."
$A \subseteq B \Rightarrow A \cap B = A$ $\quad A \subseteq B \Rightarrow A \cup B = A$ $\quad A \subseteq B,\ B \subseteq C \Rightarrow A \subseteq C$
$A \subseteq B,\ B \subseteq C \Rightarrow A \subset C$ $\quad A \subseteq B,\ A \subseteq C \Rightarrow A \subset B \cap C$ $\quad A \subset A \cup B$

(d) Express $A \cap B$ only in terms of set subtraction (by using the symbol "\").

(e) Explain why $A \cup B = B \cup A$ *in English*.

(f) Draw three pictures illustrating the distributive law $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ one for each of three configurations of $A, B,\ C$ that you decide.

Problem 2 A "full deck of cards" has 52 cards where each card has two characteristics: (1) one of four suits $\spadesuit$, $\heartsuit$, $\clubsuit$ and $\diamondsuit$ and (2) one of 13 ranks: A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K and each card is unique. The game Euchre (see `http://en.wikipedia.org/wiki/Euchre` for more information), 24 playing cards are used consisting of only aces, kings, queens, jacks, tens, and nines.

(a) Let Ω_E denote the sample space of a Euchre deck and Ω denote the sample space of a full deck. Is $\Omega_E \subset \Omega$ true?

(b) Construct Ω_E, the event space of a Euchre deck by using set notation and operations on Ω, the event space of a full deck of cards. Use the "..." notation used in class to specify your sets explicitly and use rank and suit such as 4$\clubsuit$ to denote the ω's $\in \Omega$.

(c) Let B be the set of black cards, F the set of face cards and $\spadesuit$ the set of spades. Describe the set on the r.h.s of:

$$\{A\spadesuit, 9\heartsuit, K\diamondsuit\} \cup \varnothing \;\subseteq\; \Big((B \cup F)^C \cup \spadesuit\Big)^C \setminus (\{10\spadesuit, 10\clubsuit, 10\heartsuit\} \cap \Omega)$$

(d) Is the statement in the previous question true?

(e) Do this problem after completing the last questions since it has to do with counting. How many ways are there to order 5 Euchre cards?

Problem 3 We will review the notation $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$ as well as their subsets as introduced in Lecture 2.

(a) Draw a number line for x and shade in the area that represents the set $[1,3] \cup [4,9]$. If the set includes a number on the endpoint, draw a solid circle "$\bullet$" and if does not include the number, draw an open circle "$\circ$."

(b) Draw a number line for $Z \subset \mathbb{R}$ where $Z := \{x \in \mathbb{R} : |x| \geq 2\}$. This Z notation we'll be using in a couple months when we get to the normal distribution.

(c) Draw on the number line the set $[0,1] \cap \left[0, \frac{1}{2}\right] \cap \left[0, \frac{1}{4}\right]$.

(d) Find the set $A := \bigcup_{i=1}^{\infty} \left[0, \frac{1}{2^i}\right]$.

(e) Find the set $B := \bigcap_{i=1}^{\infty} \left[0, \frac{1}{2^i}\right]$.

(f) Find the set $\mathbb{N} \backslash \mathbb{Z}$.

(g) Describe the set $\mathbb{R} \backslash \mathbb{Q}$ as best as you can in English and give an example of an element of this set.

Counting Problems below are related to counting. We will review the methods learned in class and expand our horizons.

Problem 4 In this problem, we imagine rolling different sized-dice. Assume the outcomes (each face of each die) are equally likely for that die (see middle of page 9 in H, T & Z for a definition).

Let R be a standard 6-sided die, let S be an 8-sided die, let T be a 12-sided die, and let U be a 20-sided die. What is the sample size of Ω for the experiment where we...

(a) roll R 3 times?

(b) roll R then S then T then U?

(c) roll R 53 times, then roll S 32 times, then roll T 47 times, then roll U 87 times.

(d) Roll R and then roll S only if R rolled greater than or equal to 4. Would each $\omega \in \Omega$ here be equally likely?

Problem 5 Examine the following words and tell me how many *permutations* there are of the letters. We do not care about keeping track of the individual common letters. For example, in the word *dad*, there are two $d's$ and we want to treat the permutation $d_1 d_2 a$ the *same* as $d_2 d_1 a$.

(a) town

(b) mississippi

(c) supercalifragilisticexpialidocious

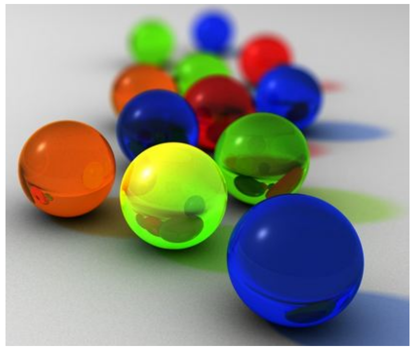
Problem 6 Below is a standard chessboard.

(a) How many ways are there to place the white king on a white square?

(b) How many ways are there to set up the pieces in the back ranks of both white and black *i.e.* arrange the two rooks, two knights, two bishops, king and queen on the first row of 8 squares. Note that this game is called "Fischer Random Chess" after the famous grandmaster Bobby Fischer who proposed the idea to make standard chess more exciting.

(c) How many ways are there to arrange the pieces on the board? We don't care about pieces of a type being unique (*i.e.* all white pawns are the same, all black rooks are the same, etc)

Problem 7 We have 4 blue marbles, 4 green marbles, 2 orange marbles, and 2 red marbles. For the following questions, if you are using "choose notation", please write your choose notation, then write the formula using factorials, then write the actual number after you compute it.



(a) Viewing all the marbles as *unique*, how many ways are there to order the marbles?

(b) Viewing all marbles of the same color as *interchangeable*, how many ways is there to order the marbles?

(c) If I pick 4 marbles at random from the collection, how many ways are there to get two-of-a-kind *i.e.* two marbles of one color and two marbles of a different color.