Math 281

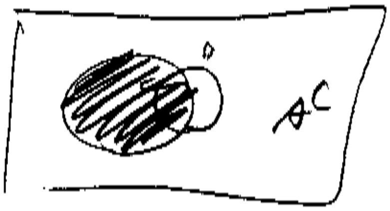
Lecture 7

$P(B) = P(B, A) + P(B, A^c) \Rightarrow .06 = .036 + P(B, A^c)$

$\Rightarrow P(B, A^c) = .024$

$P(B \mid A^c) = \frac{P(B, A^c)}{P(A^c)} = \frac{.024}{.8} = \boxed{.03}$ ...

$A^c$




Risk of smoking $\frac{P(B|A)}{P(B|A^c)} = \frac{.18}{.03} = \boxed{6\times}$

prob of not smoking given you have l.c.? Should be high

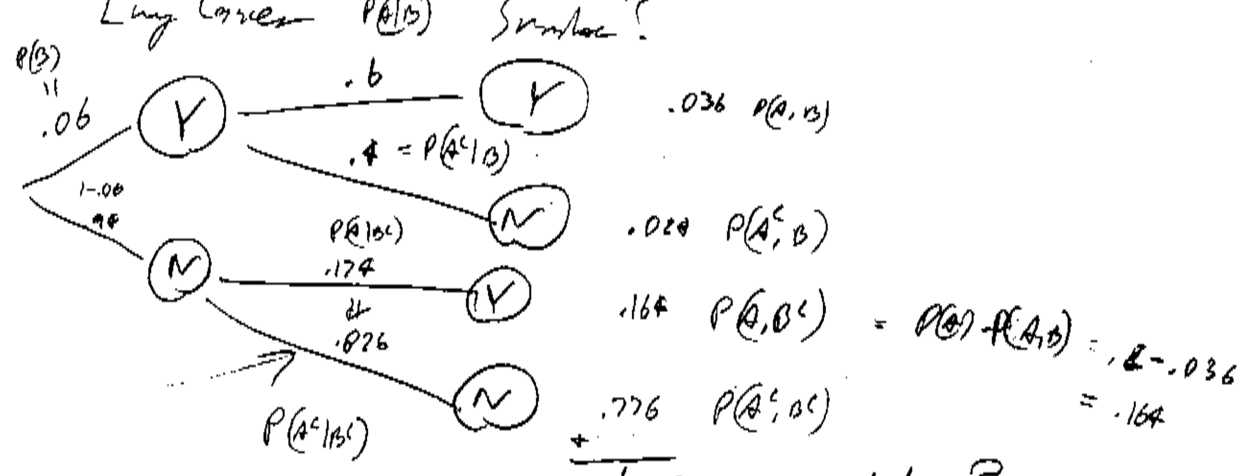
$$P(A^c|B) = 1 - P(A|B) = 1 - .6 = .4$$

complement rule
in $\Omega' = B$

prob of smoking given no l.c.? Should be low

$P(A|B^c)$ We can do it the other way... Bayes...

Lung Cancer $P(A|B)$ Smoke?




Why? $\Omega$ splits into disjoint sets

$$P(\Omega) = P(A,B) + P(A^c,B) + P(A,B^c) + P(A^c,B^c)$$

$$P(B|A) = \frac{P(A,B)}{P(A)} = \frac{P(A,B)}{P(A,B) + P(A,B^c)} = \frac{P(A,B)}{P(A|B)P(B) + P(A|B^c)P(B^c)}$$

can be proven
disjoint

Bayes Rule 3x

$$\Rightarrow P(B|A)\left(P(A|B)P(B) + P(A|B^c)P(B^c)\right) = P(A,B)$$

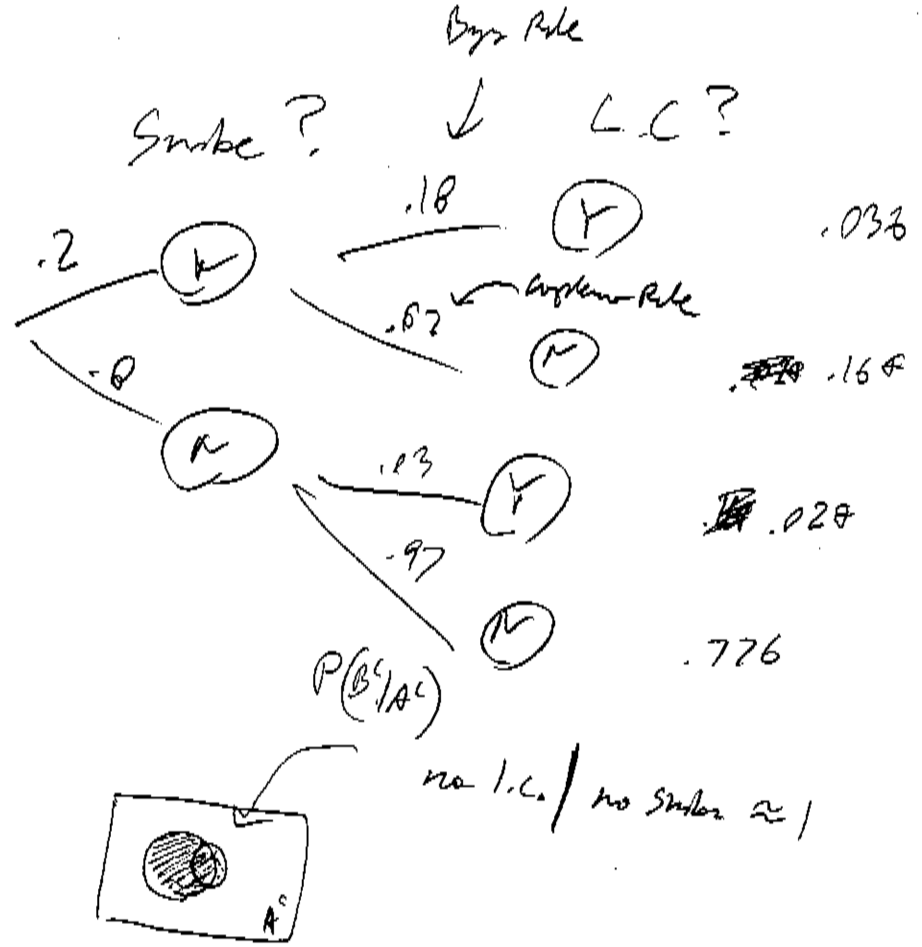
$$\Rightarrow P(B|A)P(A|B)P(B) + P(B|A)P(A|B^c)P(B^c) = P(A,B)$$

$$\Rightarrow P(A|B^c) = \frac{P(A,B) - P(B|A)P(A|B)P(B)}{P(B|A)\,P(B^c)} = \frac{.036 - .18 \cdot .6 \cdot .06}{.18 \cdot .94} = .174$$

$\approx P(A) = .2$

$P(A^c|B^c)$ Prob not smoke given no l.c.

"Invert" the tree




there are 8 cond'l probs! All necessary!

~~Last prob. Regression...~~ ~~DO LATER~~

~~two people flip coins. First person to flip H first wins. T's reflip.~~
~~What $p = P(1^{st}$ person wins$)$?~~

~~$p = \frac{1}{4} + \frac{1}{2}(p) + \frac{1}{4}(1-p)$~~

~~$\Rightarrow p = \frac{1}{4} + \frac{1}{2}p$~~

~~$\Rightarrow \frac{1}{2}p = \frac{1}{4}$~~

~~$\Rightarrow \boxed{p = \frac{1}{2}}$ when you expect...~~

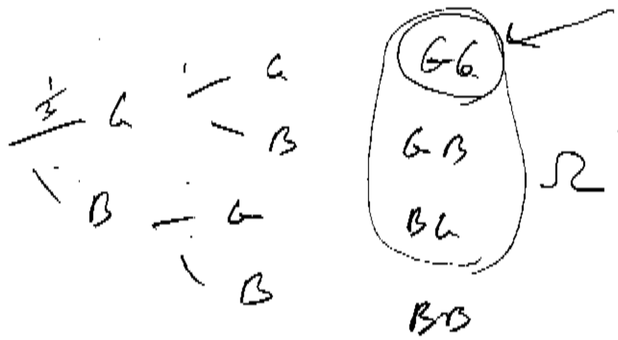
More probs w/ cond. prob. "Children Paradox"

If I know you have two kids and 1 is a G, what is P(other one is a G)?

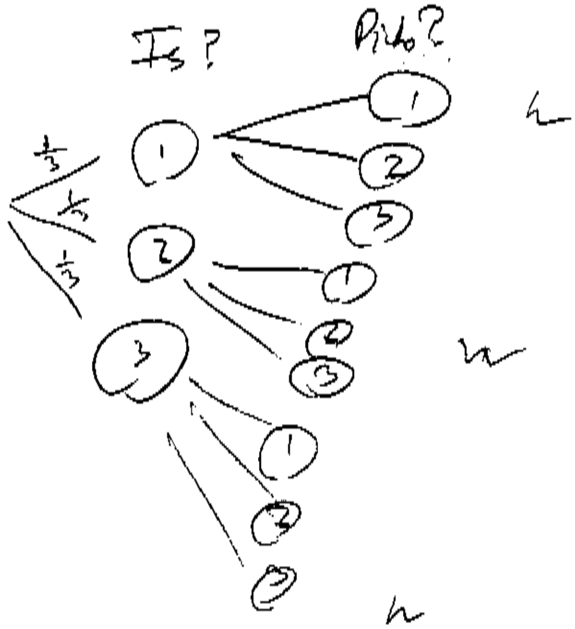
$$P(GG \mid \text{one girl})? = P(GG \mid GG \text{ or } GB \text{ or } BG) = \frac{P(GG \,\&\, (GG \text{ or } GB \text{ or } BG))}{P(GG \text{ or } GB \text{ or } BG)}$$

$$= \frac{P(GG)}{P(\ldots)} = \frac{1}{3}$$

$$\neq \frac{1}{2}$$

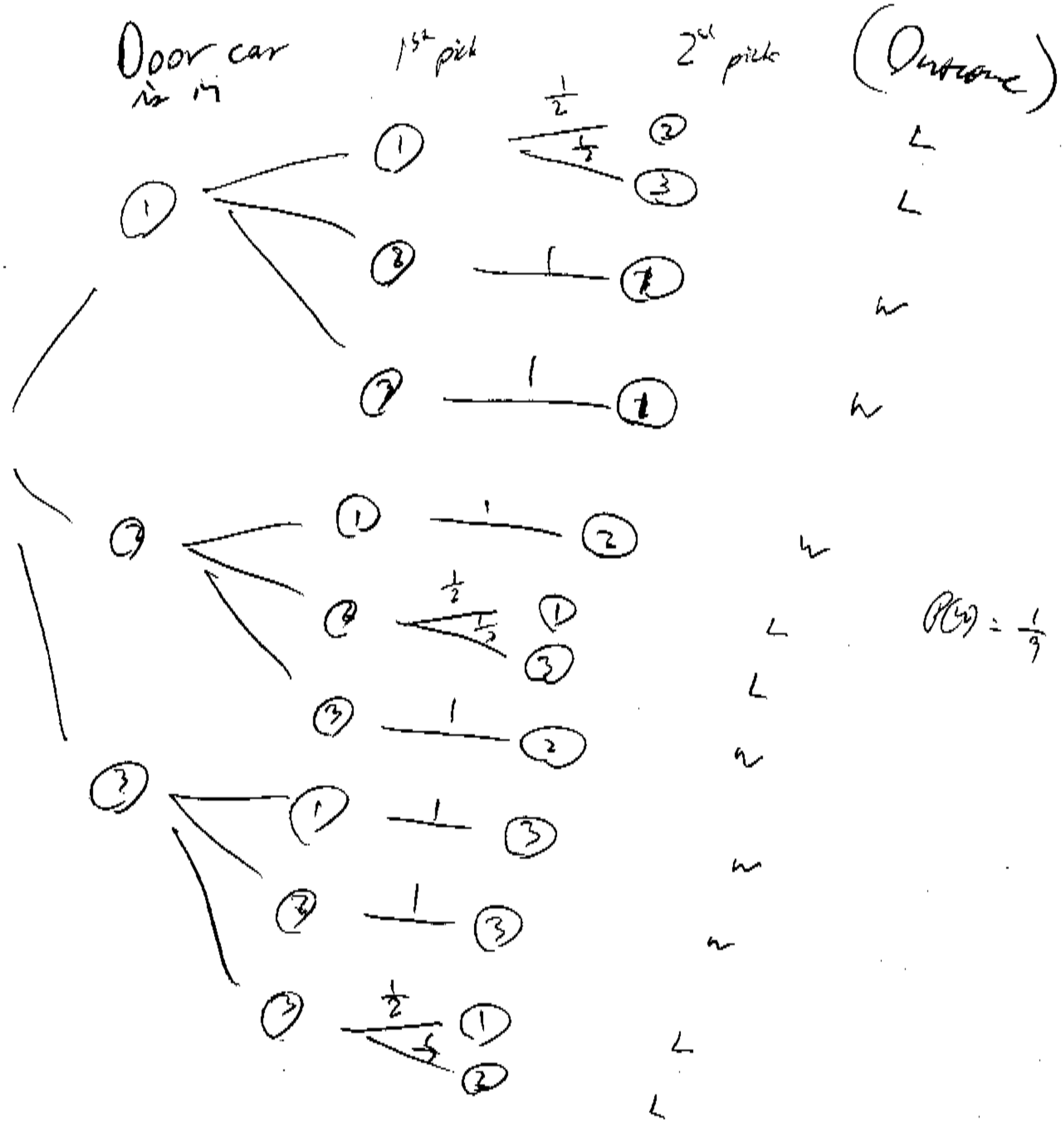



Monty Hall Game




Switch

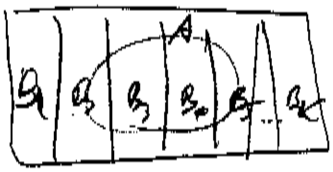
$$P(W) = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{1}{3}$$




$P(W) = \frac{1}{9} \cdot 6 = \frac{2}{3}$ !

$B_i \cap B_j = \emptyset \ \forall i \neq j$

p35-37




$A = (A \cap B_1) \cup (A \cap B_2) \cup (A \cap B_3) \cup \dots \cup (A \cap B_k)$

(good exercise to prove).

$P(A) = P(A \cap B_1) + \dots + P(A \cap B_k)$ since disjoint (also good to prove)

$= \sum_{i=1}^{k} P(A, B_k)$

$\underbrace{P(A) = \sum_{i=1}^{k} P(A \mid B_k) P(B_k)}_{\text{Law of total prob.}}$ Bayes Rule

Bayes Thm...

$$P(B_\ell \mid A) = \frac{P(B_\ell)\, P(A \mid B_\ell)}{\sum_{i=1}^{k} P(B_k)\, P(A \mid B_k)}$$

posterior prob of $B_\ell$ given data A

Always works!