Lecture 9 ~~Sept 30, 2014~~ Oct 7

$\rightarrow$ Midterm Sol Posted

10 cards, 4R, 6B

$P(\text{seeing } x \text{ R cards})? = \dfrac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$ # ways (multiplication principle) $X \sim \text{Hypergeom}(3,4,10)$

3 cards

$\text{Support}(X) = \{0,1,2,3\}$

$P(\text{seeing } x \text{ R cards, drawing } 7) = \dfrac{\binom{4}{x}\binom{6}{7-x}}{\binom{10}{7}}$ $X \sim \text{Hypergeom}(7,4,10)$

$\text{Support}(x) = \{1,2,3,4\}$

$P(\text{seeing } x \text{ R, drawing } 9) = \dfrac{\binom{4}{x}\binom{6}{9-x}}{\binom{10}{9}}$ $X \sim \text{Hypergeom}(9,4,10)$

$\text{Support}(x) = \{3,4\}$

$P(\text{seeing } x \text{ R, drawing } 10) = \dfrac{\binom{4}{x}\binom{6}{10-x}}{\binom{10}{10}}$ $\rightarrow 1$ $X \sim \text{Hypergeom}(10,4,10)$

$\text{Support}(x) = \{4\}$ $P(X=4) = \binom{4}{4}\binom{6}{6} = 1$

$X \sim \text{Degenerate}(4) \Rightarrow X \sim \{4 \text{ w.p. } 1$

$X \sim \text{Degenerate}(c) \Rightarrow f(x) = f(c)=1,\ f(x)=0,\ x \neq c$

$F(x) = \begin{cases} 0 & \text{if } x < c \\ 1 & \text{if } x \geq c \end{cases}$

Back to hypergeometric $\frac{\overset{\text{chose successes}}{\binom{K}{x}}\overset{\text{chose failures}}{\binom{N-K}{n-x}}}{\binom{N}{n}}$

$X \sim \text{Hypergeometric}(n, K, N) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$

n: # samples

K: # of total possible successes

N: population size

$N \in \mathbb{N}$

$K \in \{0, 1, \ldots, N\}$ why?

$n \in \{0, 1, \ldots, N\}$ why?

Supp$[X]$ depends...

$\min(\text{Supp}[X])$? $\max(\text{Supp}[X])$?

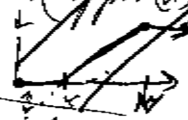
$K = 4$ successes

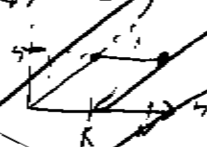
$n = 3$

if $n \leq K \Rightarrow \min(\text{Supp}[X]) = 0$ ← $n-K<0$ and $\max(\text{Supp}[X]) = n$

if $n > K \Rightarrow \min(\text{Supp}[X]) = n-K$ and $\max(\text{Supp}[X]) = K$

$n-K>0$





$\text{Supp}(X) = \{\max\{0, n+K-N\}, \ldots, \min\{n, K\}\}$

Imagine if K, the # of total successes, was a fraction of N, $K = pN$

$X \sim \text{Hypergeometric}(n, pN, N)$ different parameterization

$$f(x) = \frac{\binom{pN}{x}\binom{N(1-p)}{n-x}}{\binom{N}{n}}$$

$N \in \mathbb{N}$

$n \in \{1, \ldots, N\}$

$p \in \{0, \frac{1}{N}, \frac{2}{N}, \ldots, 1\} \neq [0,1]$

What if $N \to \infty$? This means we aren't really doing without replacement anymore...

$$\lim_{N\to\infty} \frac{(pN)!}{(pN-x)!\,x!}\cdot\frac{(N(1-p))!}{(N(1-p)-(n-x))!\,(n-x)!}\cdot\frac{n!\,(N-n)!}{N!}$$

$$= \underbrace{\frac{n!}{(n-x)!\,x!}}_{\binom{n}{x}} \lim_{N\to\infty} \frac{(pN)!}{(pN-x)!}\;\frac{(N(1-p))!}{(N(1-p)-(n-x))!}\;\frac{(N-n)!}{N!}$$

$$\underbrace{(pN)(pN-1)(pN-2)\cdots(pN-x+1)}_{x\text{ terms}}$$

$$\underbrace{N(1-p)\,(N(1-p)-1)\,(N(1-p)-2)\cdots(N(1-p)-(n-x)+1)}_{n-x\text{ terms}}$$

$$\frac{1}{\underbrace{N\cdot(N-1)(N-2)\cdot\,\cdots\,\cdot(N-n+1)}_{n\text{ terms}}}$$

As $N\to\infty$, other terms are just constants, so it looks like...

$$\underbrace{\left(\frac{pN-c_1}{N-c_2}\right)\left(\frac{pN-c_3}{N-c_4}\right)\cdots\left(\frac{pN-c_?}{pN-c_?}\right)}_{x}\cdot\left(\frac{N(1-p)-c}{N-c}\right)\cdots\left(\frac{N(1-p)-c}{N-c}\right)\ n-x$$

$$\cancel{=\left(\frac{pN-c_1}{N-c_2}\right)^{x}\left(\frac{N(1-p)-c_3}{N-c_4}\right)^{n-x}}$$ with differen constants

taking the limit for each ...

$\lim \frac{pN-c_1}{N-c_2} = p$ by l'Hopital's Rule...

$\lim \frac{N(1-p)-c_{102}}{N-c_{101}} = 1-p$ by ''

because $\lim_{x\to a} f(x)g(x) = \lim_{x\to a} f(x)\cdot\lim_{x\to a} g(x)$ for $\forall f,g$ w/ the limit existing..

$\Rightarrow p^x(1-p)^{n-x}$

$\Rightarrow \lim_{N\to\infty} f(x) = \binom{n}{x}p^x(1-p)^{n-x}$ New r.v.