stopping.

	Once	Many	PMF	PDF	CDF
Discrete	Geom	NegBin	✓		✓
Cont	Exp	Erlang		✓	✓

$p(X > x_0 + x \mid X > x_0) = \ldots = P(X > x)$

$= \dfrac{p(X > x_0 + x)}{P(X > x_0)} = \dfrac{e^{-\lambda(x_0+x)}}{e^{-\lambda x_0}} = e^{-\lambda x}$

$e^{-\lambda(x_0+x)} \rightarrow e^{-\lambda x_0} e^{-\lambda x}$

11/17
Lec 18

Continuous r.v.'s

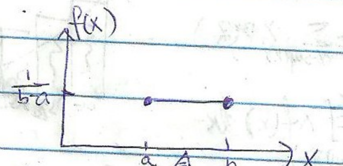
$|supp(x)| = |\mathbb{R}|$

$P(X=x) = 0 \Rightarrow$ No PMF

$f(x) = F'(x)$ (PDF)

$X \sim Unif(\{1, 7, 10\})$

$X \sim Unif(a, b) = \frac{1}{b-a}$

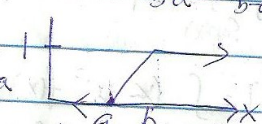


$Supp(x) = [a, b]$

Para space: $a \in \mathbb{R}$, $b \in \mathbb{R}$, but $a < b$

$F(x) = \int f(x)dx + c = \int \frac{1}{b-a} dx + c = \frac{x}{b-a} + c = \frac{x}{b-a} - \frac{a}{b-a} = \frac{x-a}{b-a}$

$F(a) = 0 = \frac{a}{b-a} + c \Rightarrow c = -\frac{a}{b-a}$



$med(x) = \{x : F(x) = 0.5\} \Rightarrow \frac{x-a}{b-a} = \frac{1}{2}$ ~~x-a~~

$\Rightarrow x-a = \frac{b-a}{2} \Rightarrow x = \frac{b-a}{2} + a = \frac{b-a}{2} + \frac{2a}{2} = \frac{b+a}{2}$

$E[X] = \int_{supp(x)} x f(x) dx = \int_a^b x \frac{1}{b-a} dx = \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b = \frac{b^2-a^2}{2(b-a)} = \frac{(b-a)(b+a)}{2(b-a)} = \frac{b+a}{2}$

$Var(X) = E(X^2) - \mu^2 = \int_a^b x^2 \frac{1}{b-a} dx - \left(\frac{b+a}{2}\right)^2 = \frac{1}{b-a}\left(\frac{x^3}{3}\right)_a^b - \frac{b^2+2ab+a^2}{4}$

$= \frac{b^3-a^3}{3(b-a)}$

$\Rightarrow \frac{(b-a)(b^2+ab+a^2)}{3(b-a)}$

$\left(\frac{4}{4}\right)$

$\Rightarrow \frac{b^2+2ab+a^2}{4}$ $\left(\frac{3}{3}\right)$

$\Rightarrow \frac{4b^2+4ab+4a^2-3b^2-6ab-3a^2}{12}$

$\Rightarrow \frac{b^2-2ab+a^2}{12} = \frac{(b-a)^2}{12} = \sigma^2$

$$\begin{array}{r|l} & b^2+ab+a^2 \\ \hline b-a & b^3 - a^3 \\ & -(b^3-ab^2) \\ \hline & -a^3+ab^2 \\ & -(ab^2-a^2b) \\ \hline & a^2b-a^3 \\ & -(a^2b-a^3) \\ \hline & 0 \end{array}$$

$Z \sim N(0,1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

"Standard normal" "standard Geosists" "Standard ball curve"

let $u = \frac{x}{\sqrt{2}} \Rightarrow du = \frac{dx}{\sqrt{2}}$ $dx = \sqrt{2}\,du$

$Supp(Z) = \mathbb{R}$

$\int_{Supp(x)} f(x) = 1 \quad 1 = \int_{\mathbb{R}} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \Rightarrow \int_{\mathbb{R}} e^{-\frac{x^2}{2}} = \sqrt{2\pi} \Rightarrow \int e^{-u^2} \sqrt{2}\, du = \sqrt{2\pi}$

Geosistes Integral $\Rightarrow \int e^{-u^2} du = \sqrt{\pi}$

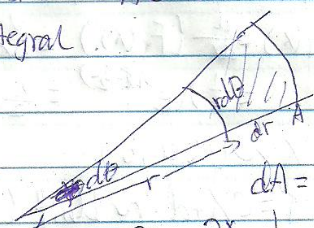
$x^2+y^2=r^2$

$x = r\cos\theta$

$y = r\sin\theta$

$\iint_{\mathbb{R}\,\mathbb{R}} e^{-(x^2+y^2)} \underbrace{dx\,dy}_{dA}$

$\Rightarrow \int_{\mathbb{R}} e^{-x^2} dx \int_{\mathbb{R}} e^{-y^2} dy = \pi$



$dA = r\,dr\,d\theta.$

$= \int\int_{0}^{\infty} e^{-r^2} r\,dr\,d\theta$

$dA = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} dr\,d\theta$

$\Rightarrow \int_0^{2\pi}\int_0^{\infty} e^{-r^2} r\,dr \Rightarrow 2\pi \int_0^{\infty} e^{-r^2} r\,dr$

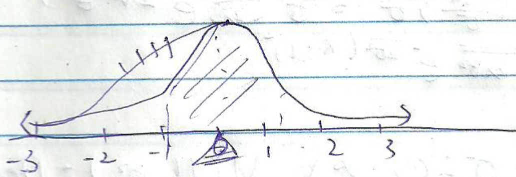
$= \begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} dr\,d\theta$

$r(\cos^2\theta + \sin^2\theta)$

$= r\cos^2\theta + r\sin^2\theta = r\,dr\,d\theta.$

$= 2\pi \int_{u_1}^{u_2} e^{-u}\, r \frac{du}{2r}$

$= \pi[-e^{-u}]_{u_1}^{u_2} \Rightarrow \pi[-e^{-\infty} - -e^{0}] = \pi[0 - -1] = \pi.$



$F_Z(x) = \int \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} + C$ Not possible (Risch Alge)

$F_Z(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy$ $F_Z(0) = \frac{1}{2}$

$E(x) = \int_{Supp(x)} x f(x) dx = \int_{\mathbb{R}} x \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} x \cdot e^{-\frac{x^2}{2}} dx$

let $u = \frac{x^2}{2}$ $\frac{du}{dx} = x \Rightarrow dx = \frac{du}{x}$

$= \frac{1}{\sqrt{2\pi}} \int_{u_1}^{u_2} e^{-u} x \frac{du}{x} = \frac{1}{\sqrt{2\pi}} [-e^{-u}]_{u_1}^{u_2} \Rightarrow \frac{1}{\sqrt{2\pi}} [-e^{-\frac{x^2}{2}}]_{-\infty}^{\infty}$

$\frac{1}{\sqrt{2\pi}} [-e^{-\frac{\infty^2}{2}} - -e^{-\frac{(-\infty)^2}{2}}] = \frac{1}{\sqrt{2\pi}} (0 - -0) = 0.$

$P(Z \in [-1,1]) = 0.68$ "68-95-99.7 rule"

$P(Z \in [-2,2]) = 0.95$ "empirical rule"

$P(Z \in [-3,3]) = 0.997$

$X \sim Exp(\lambda) = \lambda e^{-\lambda x}$ $Y = 2X \sim ?$

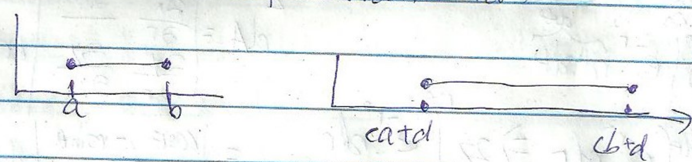
$F_Y(x) = P(Y \le x) = P(2X \le x) = P(X \le \frac{x}{2}) = F_X(\frac{x}{2})$

$f_Y(x) = \frac{d}{dx}(F_Y(x)) = \frac{d}{dx}(F_X(\frac{x}{2})) = \frac{1}{2} f_X(\frac{x}{2})$

$= \frac{1}{2}\lambda e^{-\lambda(\frac{x}{2})} = \frac{\lambda}{2} e^{-\frac{\lambda}{2}x} = Exp(\frac{\lambda}{2})$

$X \sim U(a,b)$

$Y = cX + d \sim Unif(ca+d, cb+d)$



$X = \sigma Z + \mu$, $E(X) = \sigma E(Z) + \mu = \mu$, $Var(X) = \sigma^2 Var(Z) = \sigma^2$

$F_X(X) = P(X \le x) = P(\sigma Z + \mu \le x) = P(Z \le \frac{x-\mu}{\sigma}) = F_Z(\frac{x-\mu}{\sigma})$

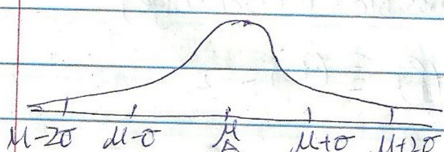
$f_X(x) = \frac{d}{dx}[F_X(x)] = \frac{d}{dx}[F_Z(\frac{x-\mu}{\sigma})]$

$= f_Z(\frac{x-\mu}{\sigma})\frac{1}{\sigma} = \frac{1}{\sigma}\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$

$X \sim N(\mu, \sigma)$ $= \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$

Gen nor R.V.S.

Para space: $\mu \in \mathbb{R}$, $\sigma^2 \in (0, \infty)$ $N(\mu, \sigma^2) = Deg(\mu)$



$P(X \in (\mu-\sigma, \mu+\sigma))$

$= P(\mu - \sigma \le X \le \mu + \sigma)$

$= P(-\sigma \le X - \mu \le \sigma)$

$= P(-1 \le \frac{X-\mu}{\sigma} \le 1)$

$= P(Z \in [-1, 1])$