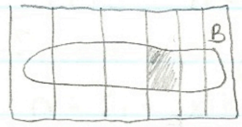
Math 241 Lecture 8

$$P(A_\ell|B)=\frac{P(B|A_\ell)P(A_\ell)}{\sum_{i=1}^{n} P(B|A_i)P(A_i)}$$

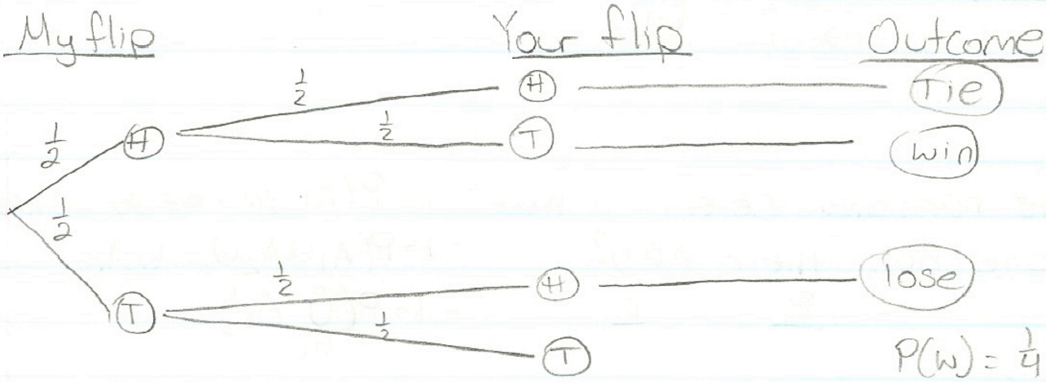


$A_1,\ldots,A_n$

$A_1 \ldots A_n$ non-exclusive, collectively exhaustive

Condition on V

$$P(A_\ell|B,V)=\frac{P(B|A_\ell,V)P(A_\ell|V)}{\sum_{i=1}^{n} P(B|A_i,V)P(A_i|V)}$$



$P(W)=\frac{1}{4}$

$P(tie)=\frac{1}{2}$

$P(W)=\frac{1}{4}+\frac{1}{2}P(W)$

$\frac{1}{2}P(W)=\frac{1}{4}$

$P(W)=\frac{1}{2}$

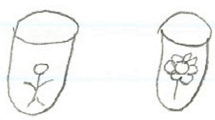
$P(A|B)\overset{?}{=}P(A)$

$P(\text{win}|\text{tie})\overset{?}{=}P(\text{win})$

10 Balls 

Indistinct

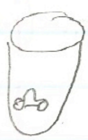
2 urns (distinct)



How many ways to put 10 balls in 2 urns such that no urn is empty?

$\langle 2,8\rangle$ $\langle 8,2\rangle$, $\langle 1,9\rangle$ $\langle 9,1\rangle$, $\langle 3,7\rangle$, $\langle 4,6\rangle$, $\langle 5,5\rangle$

$\langle 6,4\rangle$, $\langle 7,3\rangle$



3 urns $\langle 1,1,8\rangle$

$\langle 1,2,7\rangle$

n balls

K urns

2

$\binom{n-1}{k-1}$ ← # places between balls; ← # of dividers

$$\underset{\text{\# in urn 1}}{x_1} + \underset{\text{\# in urn 2}}{x_2} + \dots + \underset{\text{\# in urn K}}{x_K} = \underset{\text{\# of balls}}{n} \quad s.t. \; x_1, \dots x_K \in \mathbb{N}$$

How many solutions? $\binom{n-1}{K-1}$

$x_1 + x_2 + \dots + x_K = n \quad s.t. \; x_1, \dots, x_K \in \mathbb{N}_0$

How many solutions? $\Rightarrow \binom{n+K-1}{K-1}$

$(x_1' - 1) + (x_2' - 1) + \dots + (x_K' - 1) = n$

$x_1' + x_2' + \dots + x_K' = \underbrace{n+K}_{n'}$

$x_1' = x_1 + 1 \Rightarrow x_1 = x_1' - 1$

$x_2' = x_2 + 1$

$\vdots$

$x_K = x_K + 1$

$x_1', \dots x_K' \in \mathbb{N}$

How many ways to pair n balls into K urns?

① $\binom{n+K-1}{K-1}$ 10 balls, 2 urns $\binom{10+2-1}{2-1} = \binom{11}{1}$

12 flowers
10 red
2 blue

$\frac{12!}{10!\,2!} = \binom{12}{2}$

n people randomly select a hat $= 1 - P(\text{at least one person has their hat})$

$P(\text{no one has their hat})$?

$= 1 - P(A_1 \cup A_2 \cup \dots \cup A_n)$

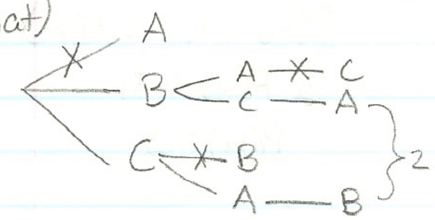
$= 1 - P(\bigcup_{i=1}^{n} A_i)$

$= 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \dots = e^{-1} \approx \boxed{.368}$

$P\binom{3 \text{ don't}}{\text{have hat}}$

$\frac{|\cdot|}{|\Omega|}$

$\frac{2}{3!}$



A_1: event that person 1 has their hat

A_2: "   2   "

A_3: "   3   "

$\vdots$

A_n " n "

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_i P(A_i) - \sum_{i \neq j} P(A_i \cap A_j) + \sum_{i \neq j \neq K} P(A_i \cap A_j \cap A_K) - \dots + \dots (-1^n) P\left(\bigcap_{i=1}^{n} A_i\right)$$

$\frac{n(n-1)!}{n!} = \frac{n!}{n!} = 1$ $\binom{n}{2}\frac{(n-2)!}{n!} = \frac{n!}{n-2!\,2!}\,\frac{(n-2)!}{n!} = \frac{1}{2!}$

$= 1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \dots (-1)^n \frac{1}{n!}$

$$P(A_1) = \frac{1 \; \underline{n-1} \; \underline{n-2} \; \underline{n-3} \ldots}{n!} = \frac{(n-1)!}{n!}$$

$$P(A_2) = \frac{\underline{n-1} \; \underline{1} \; \underline{n-2} \ldots}{n!} = \frac{(n-1)!}{n!}$$

$$P(A_1 \cap A_2) = \frac{\underline{1} \; \underline{1} \; \underline{n-2} \; \underline{n-3} \; \underline{n-4}}{n!} = \frac{(n-2)!}{n!}$$

$$\sum_{i \neq j \neq k} P(A_i \cap A_j \cap A_k) \rightarrow \binom{n}{3} \frac{(n-3)!}{n!} = \frac{\cancel{n!}}{\cancel{(n-3)!}\,3!} \frac{\cancel{(n-3)!}}{\cancel{n!}} = \frac{1}{3!}$$

$$(-1)^n P\left(\bigcap_{i=1}^{n} A_i\right) \rightarrow P(\;) = \frac{\underline{1}\,\underline{1}\,\underline{1}\,\underline{1}\,\underline{1}}{n!} = \frac{1}{n!}$$

$$f(x) = \sum_{i=0}^{\infty} \frac{f^{(i)}(c)}{(i)!} (x-c)^i \quad \forall c \in \mathbb{R}$$

$$e^x = \frac{1}{0!} + \frac{1}{1!}x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \ldots$$

centered at 0

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \ldots$$

$$e^{-1} = 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \ldots\ldots$$

<u>Midterm 2 Material</u>

Random Variable (R.V.) theory

$$\Omega = \{H, T\}$$

$$1\$ \leftarrow \boxed{H \mid T} \rightarrow 0\$$$

$$P: 2^{\Omega} \rightarrow [0,1]$$

$$X: \Omega \rightarrow \mathbb{R}$$

$$X(H) = \$1$$

$$X(T) = \$0$$

r.v. X (capital X)

$$X \sim \begin{cases} \$1 & \text{w.p. } \frac{1}{2} \\ \$0 & \text{w.p. } \frac{1}{2} \end{cases}$$

$\sim$ distributed as

w.p. with probability

Review Midterm 1

$$P(A\mid \underbrace{B,C,D,E}_{Q}) = \frac{P(A,Q)}{P(Q)} = \frac{P(Q\mid A)\,P(A)}{P(Q)} = \frac{P(B,C,D,E\mid A)\,P(A)}{P(B,C,D,E)}$$