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Math 241 Prof : Adam Kapelner

8/27 Set : A collection of distisct objects without order.

collection : none , one , many
distisct : no diplicates
object : anything , numbers , elephants . } meaning of words.

$F \{ \text{Jam, mary, Susan, Dama} \}$

$M = \{ \text{Bob, Joe, Max, Dana} \}$

Equaling

$A = B \Rightarrow$ Same exact object

$F \neq M \Rightarrow$ not are in equal

Subject

Denoted with " $\subset$ " or " $\subseteq$ "

$A \subset B$ means proper ~~subject~~ subset all elements A are in B , but $A \neq B$.

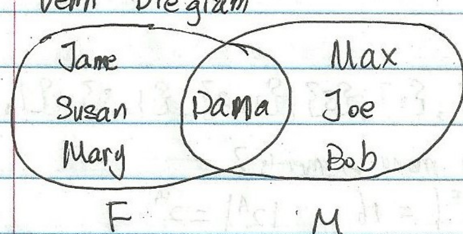
$\{\text{Jam}\} \subset F$ T $\{\text{Jam}\} \notin F$ Jam $\notin F$

Set Inclusion

Jane $\in F$, " $\in$ " means "in" or "element of"

Max $\notin F$, " $\notin$ " means "not in"

Venn Diegram



Set Union : combine elements.

$\{\text{Jane}\} \cup \{\text{Susan, mary, Dam}\} = F$

$\{Jane\} \cup \{Jane\} = \{Jane\}$ "non-excluse or"

$A \subseteq B$ is $A \subset B$ or $A = B$

$\mathbb{N} := \{1, 2, 3 \dots\}$ Naturel #'s

$\mathbb{N}_0 := \mathbb{N} \cup \{0\}$

$\mathbb{Z} := \{\cdots -2, -1, 0, 1, 2 \cdots\}$ Integer #'s

$\mathbb{Q} := \{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{N}\}$ Rational #'s

Set buider ~~notion~~ notation.

Set Intersection. → "and"

finds common elements

$F \cap M = \{Dana\}$

$F \cap \{Bob, max\} = \{\emptyset\}$, $\emptyset := \{\}$ ⇒ empty / null set.

$A \cap B = \emptyset$ ⇒ A and B are disjoint (nothing in common)

$\emptyset \subset F$ T

$\emptyset \notin F$

Set Subtration

$F \setminus M = \{Jane, Susan, mary\}$

F less all elements in M.

$M \setminus F = \{max, Joe, Bob\}$.

$A \setminus B = A \iff A \cap B = \emptyset$ [nothing in common].

Power sets

Set Functions

$\mathcal{P}(A) := 2^A = \{B : B \subseteq A\}$

$A = \{1, 2, 3\}$, $2^A = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, A\}$

Set size / Cardinling. ⇒ How many elements?

$|A| = 3$ $|2^A| = 8$ $|2^F| = 16$ $|2^A| = 2^{|A|}$

$|F| = 4$, $|M| = 7$, $|F \cup M| = 7 \neq |F| + |M| = |F| + |M| - |F \cap M|$

Special Ω

9/1 Universe single space

space of discourse

$\Omega = F \cup M$

$F \subseteq \Omega$

$M \subseteq \Omega$

$\forall A, A \in \Omega$

Joe $\in \Omega$

$F \cup \Omega = \Omega$

$F \cap \Omega = F$

$A_1, \ldots, A_n$ are sets and $\bigcup_{i=1}^{n} A_i \Rightarrow \{A_1, \ldots A_n\} = A_1 \cup A_2 \cup \cdots A_n$

are called collectivelly exhastive

$A_1 \ldots A_n$, $\underline{A_i \cap A_j = \{A_1 \ldots A_n\}}$ $(\nexists \forall i \neq j)$

are called "mutally exclusive", $\boxed{\emptyset \text{ is disjoin}}$

"mutally exclusive" $\neq$ "collective exhastive

$|\Omega| = |A_1| + |A_2| \cdots = \sum_{i=1}^{n} |A_i|$ (finite Ω)

Set complement

$A^c = \Omega \setminus A$

$F^c = \{Bob, Joe, Max\} \neq M$

$A \cup A^c = \Omega$ $\quad \emptyset^c = \Omega$

$A \cap A^c = \emptyset$ $\quad \Omega^c = \emptyset$

$\{A, A^c\}$ mutally exclusive and collective exhasive

$(A \cup B)^c = A^c \cap B^c$ DeMorgan's Laws

$(A \cap B)^c = A^c \cup B^c$

$|\mathbb{N}| = \aleph_0$ countble ∞

$|\mathbb{Z}| = \aleph_0$ countble ∞

$\frac{p}{q} = \frac{p_1 \cdots p_k}{p q_1 \cdots q_r}$

share no factore

$\frac{p}{q} = \sqrt{2}$

$\frac{p^2}{q^2} = 2$

$\frac{p^2}{2} = q^2 \in \mathbb{N} \Rightarrow \frac{p_1 p_1 \cdots p_k p_{k+1} \cancel{2}}{\cancel{2}}$

$2 \in \{p_1 \cdots p_k\} \quad = \frac{q_1 q_1 \cdots q_r q_r}{2}$

$\mathbb{R} = \mathbb{Q} \cup \{\text{all} \quad \text{in } \mathbb{Q}\}$

$|\mathbb{R}| = c > \aleph_0$ uncountble ∞

$[0,1] = \{x : x \geq 0 \;\&\; x \leq 1\}$

$\subset \mathbb{R}$

$(0,1) = \{x : x > 0 \;\&\; x < 1\}$

$[a,b] = \{x : x \geq a \;\&\; x \leq b\}$

st $a < b$

$(a,b) = \{x : x > a \;\&\; x < b)$ $a < b$

Order tuple

$(a,b) \neq \{a,b\}$

1st element 2nd element

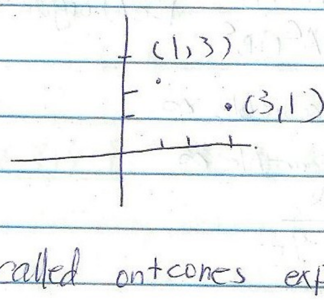
$A \times B = \{(a,b), a \in A, b \in B\}$

Cartesian product of A and B

$|A+B| = |A|\,|B|$

$\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$

Cartesian plane



Elements of sets are called outcomes experiments (thing that happen $w \in \Omega$

$\Omega = \{H, T\}$ $\qquad \{H\}, \{T\}$

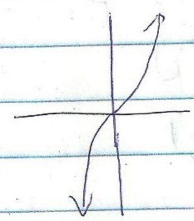
$2^\Omega = \{\emptyset, \{H\}, \{T\}, \{H, T\}\}$ $\qquad 2^A = \{B : B \subseteq A\}$

elements of 2^Ω are called "events"
(subset of Ω)

$P(\{H\}) = \frac{|\{H\}|}{|\Omega|} = \frac{1}{2}$ $\qquad P(\{H, T\}) = \frac{|\{H, T\}|}{|\Omega|} = \frac{2}{3} = 1$

$P(A) = \frac{|A|}{|\Omega|}$ $\qquad P(\emptyset) = \frac{|\emptyset|}{|\Omega|} = \frac{0}{2} = 0$

$P(F) = \frac{|F|}{|\Omega|} = \frac{4}{7}$



$f(x) = x^3$

$f : \mathbb{R} \to \mathbb{R}$ (Range)

$p : \underbrace{2^\Omega}_{\text{all events}} \to [0, 1]$ (Domain)

$\Omega^2 = \Omega \times \Omega$ $\qquad P(\{(H, H)\})$

(H, H)	(H, T)
(T, H)	(T, T)

$P(H\,H) = \frac{|(H, H)|}{|\Omega^2|} = \frac{1}{4}$

A: at lease one Head $\qquad P(A) = \frac{|A|}{|\Omega^2|} = \frac{3}{4}$

$|\{HH, HT, TH\}|$

B: at lease one tail

$P(B) = \frac{3}{4}$ $\qquad |\{HT, TH, TT\}|$

C: at lease one H and T

$P(A \cap B) = \frac{1}{2}$

Ω^4 $\qquad |\Omega^4| = |\Omega|^4$

HHHH	HHTT	TTHH	THTH
HHHT	HHTH	HTTH	HTHT
HTHH	THHH	TTTH	THHT
TTHT	THTT	THHH	TTTT

$P(HHHH) = P(HTHT)$

$P(HHHH) < P(\{2H \,\&\, 2T\})$

$= \frac{1}{16}$ $\qquad = \frac{6}{16}$

Ω = coin flip, x Roll die

H_1	H_2 ✓	H_3	H_4 ✓	H_5	H_6 ✓
T_1	T_2	T_3	T_4	T_5	T_6

$P(\text{even} \& H) = \frac{3}{12}$

$\Omega = \{J, m, s\}$

How many way to seat them in 3 distinct chairs?

	Seat 1	Seat 2	Seat 3
Way 1	J	m	s
		s	m
	m	J	s
		s	J
	s	J	m
		m	J

$\}$ 6.

Way 2. $\underset{\#1}{3} \cdot \underset{\#2}{2} \cdot \underset{\#3}{1} = 3!$

How many ways to seat n people in n distisct chairs?

$\underset{\#1}{n} \quad \underset{\#2}{n-1} \quad \underset{\#3}{n-2} \quad \_\_ \ldots \quad \underset{\#n}{1} = n! = \prod_{i=1}^{n} i$

"n - factorial"

$P((J, m, s)) = \frac{1}{6} = |\Omega|$

$|\Omega^3| = 3 \cdot 3 \cdot 3 = 27 \neq 6$

Sampling with replacement.

Sampling without replacement (no dups allowed)

permutation: sampling with replacement when order matter)

How many perm's 10 people 3 chairs?

$_{10}P_3 = \underset{\#1}{10} \cdot \underset{\#2}{9} \cdot \underset{\#3}{8} = \frac{10!}{7!} = \frac{10!}{(10-3)!}$ $\quad {}_nP_k = \frac{n!}{(n-k)!}$

$n \in \mathbb{N}$

$_3P_3 = \frac{3!}{(3-3)!} = \frac{6!}{0!}$ $\quad 0! = 1$ $\quad k \in \{0 \ldots n\}$