Cont r.v's

$|Supp(X)| = |\mathbb{R}|$

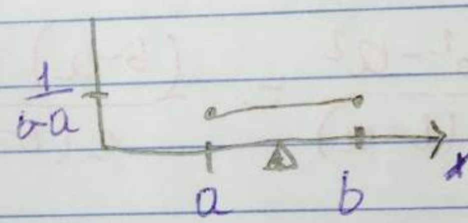
$P(X=x) = 0 \Rightarrow$ No PMF

$f(x) := F'(x)$ (PDF)

$X \sim UNIform(\{1, 7, 10\})$

$X \sim Uniform(a, b) := \frac{1}{b-a}$ $\swarrow f(x)$

$supp(X) = [a, b]$



$b = 1.001 \quad \frac{1}{1001} < 1000$

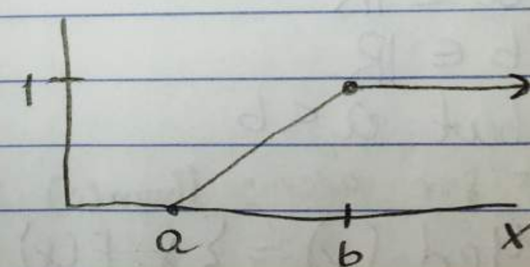
$a = 1$

cdf $\downarrow X \le x$

$\cdot\ F(X) = \int f(x)\,dx + C$

$= \int \frac{1}{b-a}\,dx + C$

$= \frac{X}{b-a} + C$



$F(a) = 0$

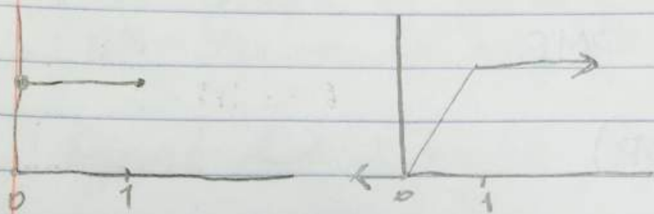
$F(a) = \frac{a}{b-a} + C = 0$

$\Rightarrow C = \frac{-a}{b-a}$

$$F(x) = \frac{x}{b-a} - \frac{a}{b-a} = \frac{x-a}{b-a}$$

Exp(1) std exponential

Unif(0,1) std uniform := 1



$$\mu = E(X) = \int_{x \in supp(x)} x f(x)\, dx \int_a^b y \frac{1}{b-a}\, dx = \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b =$$

$$= \frac{b^2-a^2}{2(b-a)} = \frac{(b-a)(b+a)}{2(b-a)} = \boxed{\frac{b+a}{2}}$$

$a \in \mathbb{R}$
$b \in \mathbb{R}$
but $a < b$

prove for average (bounce) $\frac{a+b}{2}$

$$Med(x) = \{x : F(x) \geq 0.5\} \quad \frac{x-a}{b-a} = \frac{1}{2}$$

$$\Rightarrow x - a = \frac{b-a}{2} \Rightarrow x = \frac{b-a}{2} + a \Rightarrow x = \frac{b-a}{2} + \frac{2a}{2}$$

$$= \boxed{\frac{b+a}{2}}$$

$$\sigma^2 = Var(X) = E(X^3) - \mu^3 = \int_a^b x^2 \frac{1}{b-a}\,dx - \left(\frac{b+a}{2}\right)^2 =$$

$$= \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b - \frac{b^2+2ab+a^2}{4} = \left(\frac{4}{4}\right)\frac{(b-a)(b^2+ab+a^2)}{3(b-a)} -$$

$$\frac{b^3-a^3}{3(b-a)} - \frac{b^2+2ab+a^2}{4}\left(\frac{3}{3}\right)$$

$$\begin{array}{r|l} & b^2+ab+a^2 \\ \hline b-a & b^3-a^3 \\ & -(b^3-ab^2) \\ \hline & ab^2-b^3 \\ & -(ab^2-a^2b) \\ \hline & a^2b-a^3 \\ & -(a^2b-a^3) \\ \hline & 0 \end{array}$$

$$= \frac{4b^2+4ab+4a^2-3b^2-6ab-3a^2}{12}$$

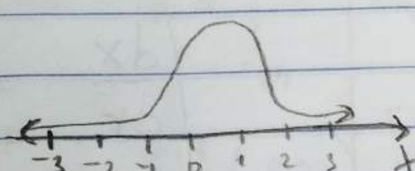
$$= \frac{b^2-2ab+a^2}{12} =$$

$$= \frac{(b-a)^2}{12}$$

$$\sigma = \sqrt{\frac{(b-a)^2}{12}} = \frac{b-a}{\sqrt{12}}$$

$$Z \sim N(0,1) := \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

"standard normal"
"standard Gaussian"
"Bell curve"



$supp(z) = \mathbb{R}$

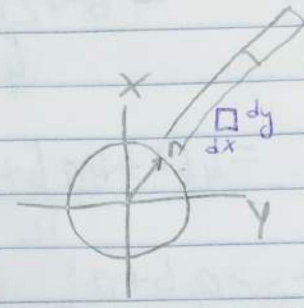
WTS $\int_{\mathbb{R}} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} = \int_{\mathbb{R}} e^{-\frac{x^2}{2}} dx = \sqrt{2\pi} \Rightarrow \int_{\mathbb{R}} e^{-u^2} \sqrt{2}\, du =$

$= \sqrt{2\pi}$

Let $u = \frac{x}{\sqrt{2}} \Rightarrow \frac{du}{dx} = \frac{1}{\sqrt{2}}$

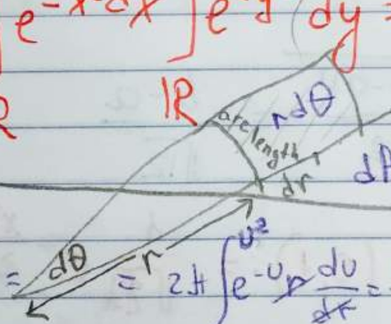
Gaussian Integral

$$\int_{\mathbb{R}} e^{-u^2} du = \sqrt{\pi}$$



$r^2 = x^2 + y^2$
$x = r\cos\theta$
$y = r\sin\theta$

$$\iint_{R\ R} e^{-(x^2+y^2)} \underbrace{dx\,dy}_{dA} = \int_{\mathbb{R}} e^{-x^2} dx \int_{\mathbb{R}} e^{-y^2} dy \overset{wts}{=} \pi$$



$dA = r\,dr\,d\theta$

$$\boxed{\iint e^{-r^2} r\,dr = 2\pi \int^{\infty} e^{-r^2} r\,dr = 2\pi \int_{u_1}^{u_2} e^{-u} \cancel{r} \frac{du}{\cancel{dr}} = \pi\left(-e^{-u}\right)_{u_1}^{u_2} = \pi\left(-e^{-u}\right)_0^{\infty} = \pi(0 - -1) = \pi}$$

$$dA = \begin{vmatrix} \frac{dx}{dr} & \frac{dx}{d\theta} \\ \frac{dx}{dr} & \frac{dx}{d\theta} \end{vmatrix} dr\,d\theta$$

$$\begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} dr\,d\theta = r\cos^2\theta + r\sin^2\theta = $$

$= r\,dr\,d\theta$

- $X \sim Exp(\lambda) = \lambda e^{-\lambda x}$

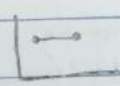
$y = 2x \sim ?$

$F_y(x) = P(y \le x) = P(2x \le x) = P\left(x \le \frac{x}{2}\right) = F_x\left(\frac{x}{2}\right)$

$f_y(x) = f_y'(x) = \frac{d}{dx}\left[F_x\left(\frac{x}{2}\right)\right] = \frac{d}{dx}\left[1 - e^{-\lambda \frac{x}{2}}\right] = \frac{\lambda}{2} e^{-\frac{\lambda}{2}x} =$

$= Exp\left(\frac{\lambda}{2}\right)$

$X \sim U(a,b)$



$Y = cX + d \sim UNIf(ca+d, cb+d)$

$X = \sigma Z + \mu$

$$\boxed{\begin{aligned} E(X) &= \sigma E(Z) + \mu = \mu \\ Var(X) &= \sigma^2 Var[Z] = \sigma^2 \end{aligned}}$$

$E(Z) = \int_{\mathbb{R}} x \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} e^{-\frac{x^2}{2}} x\, dx = \frac{1}{\sqrt{2\pi}} \int_{u_1}^{u_2} e^{-u} * \frac{du}{dx}$

$= \frac{1}{\sqrt{2\pi}} \left[-e^{-u}\right]_{u_1}^{u_2} = \frac{1}{\sqrt{2\pi}} \left[-e^{-\frac{x^2}{2}}\right]_{-\infty}^{\infty} = \frac{1}{\sqrt{2\pi}}(0-0) = \boxed{0}$