$\{J, m, S\}$

9/3 $_3P_3 = \frac{3!}{(3-3)!} = \frac{3!}{0} = 3!$

$\boxed{{}_nP_K = \frac{n!}{(n-K)!}}$

~~Bob and~~ Bob <-> Jane $\quad P(\text{all couple sit together}) = \frac{6 \cdot 4 \cdot 2}{6!} = \frac{1}{15}$.

Max <-> Mary $\quad \underline{6}\ \underline{1}\ \underline{4}\ \underline{1}\ \underline{2}\ \underline{1}$

Joe <-> Susan $\quad = \underline{3}\ \underline{2}\ \underline{1} \cdot 2^3$ <- put 2 couple as 1 unit.

$P(\text{Alternarting gender seating}) = \frac{6 \cdot 3 \cdot 4}{6!} = \frac{1}{10}\ \underline{6}\ \underline{3}\ \underline{2}\ \underline{2}\ \underline{1}\ \underline{1}$

or $(\underline{3}\ \underline{3}\ \underline{2}\ \underline{2}\ \underline{1}\ \underline{1})2$

$P(\text{Mary Max together}) = \frac{|V|}{6!} = \frac{1}{3}(\underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1})2$

or $\underline{5} \cdot \underline{4!} \cdot \underline{2}$

100 balls. 3 positions without replacement $\quad \underline{100} \cdot \underline{99} \cdot \underline{98}$

100 balls, 3 positions with replacement $\quad \underline{100} \cdot \underline{100} \cdot \underline{100}$

10,000 balls, 3 positions without replacement $\quad 10000 \cdot 9999 \cdot 9998$

10,000 balls, 3 positions with replacement $\quad 10000 \cdot 10000 \cdot 10000$

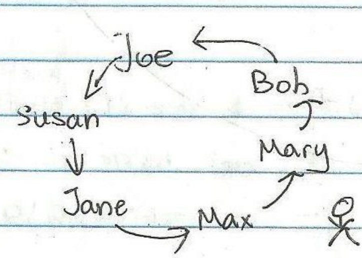
If n is large, K is fixed

Sampling with or without replacement ~~to~~ is the same

$$1 = \lim_{n \to \infty} \frac{{}_nP_K}{n^K} = \frac{\overbrace{n(n-1)(n-2)\cdots(n-K+1)}^{K\ \text{time}}}{\underbrace{n \cdot n \cdot n \cdots n}_{K\ \text{time}}}$$

$n-K+1 \Rightarrow$ #s b/t 7 and 10 is 4.

$$= \lim \underbrace{\frac{n}{n}}_{1} \lim \underbrace{\frac{n-1}{n}}_{\underbrace{1-\frac{1}{n}}_{1}} \lim \underbrace{\frac{n-2}{n}}_{\underbrace{1-\frac{2}{n}}_{1}} \cdots \lim \underbrace{\frac{n-K+1}{n}}_{\underbrace{1-\frac{K+1}{n}}_{1}}$$



$\frac{6!}{6}$ <- principle of diving out invarince. (divide what you don't care about Same order)

8 flowers, 5 Blue, 3 red. $\frac{8!}{3!5!}$ → care about all flowers.
red → 3!, 5! ← Blue.

Put in 8 flowerpots.

5 flowers, 3 Blue, 2 red.

$$BBBRR \left\{ \begin{array}{l} B_1\, B_2\, B_3\, RR \begin{cases} R_1\, R_2 \\ R_2\, R_1 \end{cases} \\ B_1\, B_3\, B_2\, RR \\ B_2\, B_1\, B_3\, RR \\ B_2\, B_3\, B_1\, RR \\ B_3\, B_1\, B_2\, RR \\ B_3\, B_2\, B_1\, RR \end{array} \right\} \times 12$$

$$\frac{5!}{\underset{B}{3!}\,\underset{R}{2!}} = 10.$$

BBBRR	BRBRB
BBRBR	RBBBR
BBRRB	RBRBB
BRBBR	RRBBB
BRRBB	RBBBR.

$$P(2H, 2T) = \frac{\frac{4!}{2!2!}}{|\Omega^k|} = \frac{\ }{16}.$$

$$P(50H, 50T) = \frac{\frac{100!}{50!50!}}{|\Omega|^{100}} = \frac{\ }{2^{100}} = 11\%.$$

$$P(500H, 500T) = \frac{\frac{1000!}{(500!)^2}}{2^{1000}}$$

$$\ln(p) = \ln(1000!) - 2\ln(500!) - 1000\ln(2)$$

$$= \frac{1}{2}\ln(2\pi) + 1000.5\ln(1000) - 1000 - \ln(2\pi) - 500.5\ln(500) + 500 - 1000\ln(2)$$

$$= -3.6797$$

~~Stirl~~ Stirling Approx

$$n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

$$\ln(n!) = \frac{1}{2}\ln(2\pi) + (n+\tfrac{1}{2})\ln(n) - n$$

$$P = e^{\ln P} \approx e^{-3.6797} = 2.52\%.$$

$_4P_2 = \frac{4!}{2!} = 12$

$m = \{J, B, m, D\}$ $\frac{_4P_2}{2!} = \frac{\frac{4!}{2!}}{2!} = 6$.

$\{\{J,B\}, \{J,m\}, \{J,D\}, \{B,m\} \{B,D\}, \{m,D\}\}$. care about order.

"# combinations of 4 people taken 2 at a time".

"# combinations of n objects taken k at a time".

$\binom{n}{k} {}_nC_k = \frac{_nP_k}{k!} = \frac{\frac{n!}{(n-k)!}}{k!} = \frac{n!}{(n-k)!\,k!}$ "n - choose - k"

Given A s.t $|A| = n$

$\binom{n}{k} := |\{B : B \subseteq A, |B| = k\}|$.

① $\binom{n}{1} = n$

② $\binom{n}{n} = 1$

③ $\binom{n}{n-1} = n$

④ $\binom{n}{n-k} = \frac{n!}{(n-(n-k))!\,(n-k)!} = \frac{n!}{k!(n-k)!} = \binom{n}{k}$

⑤ $\binom{n}{0} = 1$

$\binom{n}{k} = \frac{n!}{(n-k)k!} \cdot \frac{n-k+1}{n-k+1} = \frac{n! \;\; n-k+1}{(n-k+1)!\,k(k-1)!} = \frac{n!}{(n-k+1)!\,(k-1)!} \cdot \frac{n-k+1}{k}$

$= \binom{n}{k-1} \cdot \frac{n-k+1}{k}$

$\binom{n}{k} = \frac{n!}{(n-k)k!} = \frac{(n-1)!\; n}{\underbrace{(n-k)!}_{((n-1)-(k-1))!}\,(k-1)!\; k} = \binom{n-1}{k-1} \cdot \frac{n}{k}$

$\binom{n}{k} = \frac{n!}{(n-k)!\,k!} = \frac{(n-1)!}{(n-1-k)!\,k!\,(n-k)} = \binom{n-1}{k} \cdot \frac{n}{n-k}$

$\Omega = \{\underbrace{A, 2, 3, \ldots, 10, J, Q, K}_{\text{Ranks}}\} \times \{\underbrace{C, D, S, H}_{\text{Suit}}\}$

~~Hand~~ Hand: Random 5 cards of the 52 . How many hand?

order does not matter $\binom{52}{5}$

$P(\text{Royal Flush}) = \frac{1 \cup 1}{\binom{52}{5}}$