I take Uber

Van Wqak. : 7 min (no traffic)

Jewel Ave: 12 min (traffic)

P(traffic) = 0.3

Law of large #'s $\bar{X} \to M$

$$W \sim \begin{cases} 7 \text{ min} & \text{wp } 0.7 \\ 12 \text{ min} & \text{wp } 0.3 \end{cases}$$

$$E[W] = 7 \times 0.7 + 12 \cdot 0.3 = 8.5 \text{ min}$$

average time in the car 8.5 if I take many, many Ubers. Meaning of 8.5

average - Sample

Mean - Expectation

Expectation - Long Run avera

Uber charges $0.40 Min

$$B = \underbrace{\$0.40/m \times W}_{g(w)}$$

$$E[B] = E[g(w)] = \sum_{\text{we supp}(w)} g(w) \overbrace{p(w)}^{P(W=w)}$$

$$E[B] = \underbrace{g(7)}_{0.4 \times 7} p(7) + \underbrace{g(12)}_{0.4 \times 12} p(12)$$

$$= \$2.80 \times 0.7 + \$4.80 \times 0.3 = \$3.40$$

If you take many many Uber the in the long run the avarage that you spend in the bill will be $3.40

$Y := g(x)$ $\qquad$ X is discrete $\Rightarrow \operatorname{supp}[X] = \{x_1, x_2, \cdots\}$

$$E[Y] := \int_{\Omega} Y(\omega)\, dP(\omega)$$

$$\Omega = \int_{\Omega} g(X(\omega))\, dP(\omega) = \int_{\{\omega : X(\omega) = x_1\}} g(X(\omega))\, dP(\omega) + \int_{\{\omega_i\, X(\omega) = x_2\}} g(X(\omega))\, dP(\omega) + \cdots$$

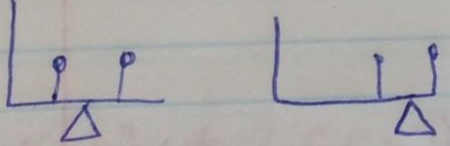
$$= g(X(\omega)) \int_{\{\omega_1\}} dP(\omega) + g(X) \int dP(\omega) + \cdots$$

$$= \underbrace{\sum_{x \in \operatorname{supp}[X]} g(x)\, p(x)} = \underbrace{E[g(x)]}$$

$$E[B] = \sum_{b \in \operatorname{supp}[B]} b\, p(b) \quad \leftarrow P(B = b)$$

* $Y = y(X) = aX, \quad a \in \mathbb{R}$

$$E[aX] = \sum_{x \in \operatorname{supp}[X]} a x\, p(x) = 1 \sum_{x \in \operatorname{supp}(X)} x\, p(x) = a\, E[X]$$



* There is a $3 base fee

$$T = \underbrace{\$3 + B}_{h(B)} = \$3 + \underbrace{\$0.40/m}_{g(W)} \cdot W$$

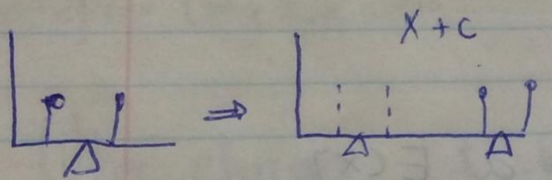
$$E[T] = g(7)\,p(7) + g(12)\,p(12)$$

$$= (3 + 0.4 \times 7)\cdot \underset{0.7}{p(7)} + (3 + 0.4 \times 12)\,\underset{0.3}{p(12)} = \$6.40$$

over a long term period of time, that will be the bill

- $Y = X + c\,,\quad c \in \mathbb{R}$

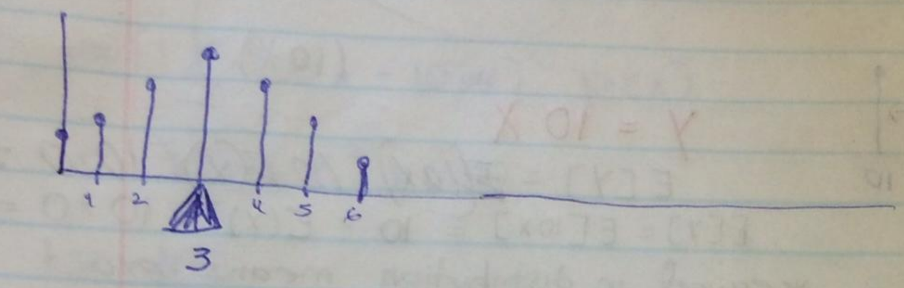
$$E[\underbrace{X+c}_{g(x)}] = \sum g(x)p(x) = \sum (x+c)p(x) = \underbrace{\sum xp(x)}_{\boxed{E[X]+c}} + \underbrace{\sum cp(x)}_{c\overbrace{\sum p(x)}^{=1}}$$

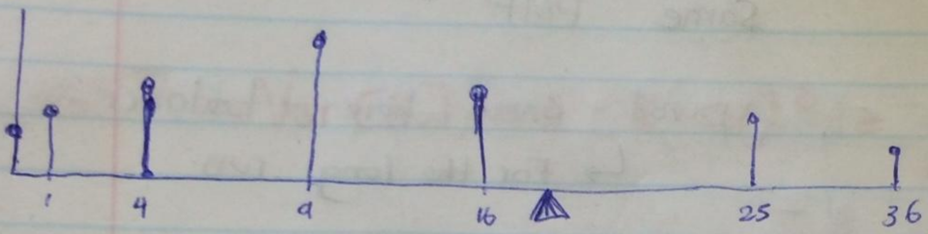


Linear transformations

$$\boxed{\begin{aligned} &E[aX + c] \\ &= aE[X] + c \end{aligned}}$$

$X \sim \text{Bin}\left(6, \frac{1}{2}\right) \qquad Y = X^2 = g(x)$





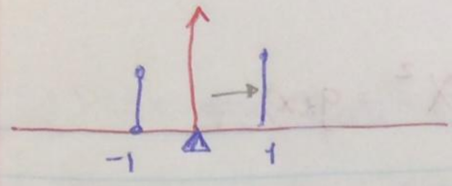
$$\text{Supp}[g(x)] = \{ g(x) : x \in \text{Supp}[X] \}$$

$$E[X^2] = \sum_{x \in \text{Supp}[X]} x^2 \, p(x)$$

$$= \sum_{x=0}^{6} x^2 \binom{6}{x} \frac{1}{2^6} = \frac{1}{2^6}\left(\binom{6}{1} + 4\binom{6}{2} + 9\binom{6}{3} + 16\binom{6}{4} + 25\binom{6}{5} + 36 \right)$$

$$= 17.5$$

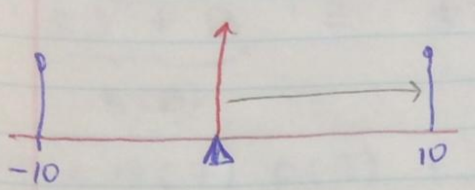
$$E[X^2] \neq (E[X])^2$$



$X \sim$ Rademacher
$E[X] = 0$

?

$(10X)$



$Y = 10X$

$E[Y] = E(10X) = 10\,E(X) = 10 \cdot 0 = \boxed{0}$

$E[Y] = E[10X] = 10 \times E(X) = 10 \times 0 = 0$

*equal in distribution means exact same PMF

$E[X] = E[Y]$

$\not\Rightarrow X \overset{d}{=} Y$

pivot - know where it balances.
↳ For the long run

*Why are they different?

→ Different distance from the pivot

*Spit out → support

Dispersion Far from the pivot
Distance
error Far from where I want to be
cost
Loss
Variance
Diviance

$e(X, \mu) = X - \mu$
$e(X, \mu) = |X - \mu|$
$e(X, \mu) := (X - \mu)^2$ right loss function
quadratic loss,
square error loss

on avarege what's out square error loss.

$$E(X-\mu)^2 = \sum_{x \in supp[X]} (x-\mu)^2 \, p(x)$$

$$= \sum_{x \in \{-1,1\}} x^2 \frac{1}{2} = 1$$

How far I'm away on avarage

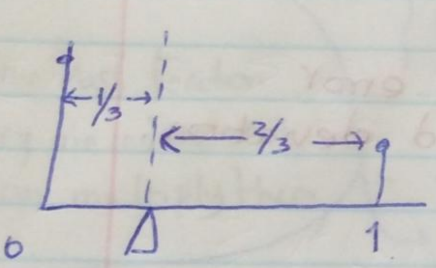
$$= \sum (10x - 10\overset{0}{\cancel{\mu}})^2 \, p(x)$$

$$= 100 \sum x^2 \, p(x)$$

$$= 100$$

$$\sigma^2 := Var[X] := E[X-\mu]^2 = \sum_{x \in supp(x)} (x-\mu)^2 p(x)$$

$X \sim Bern\left(\frac{1}{3}\right)$ Variance of X

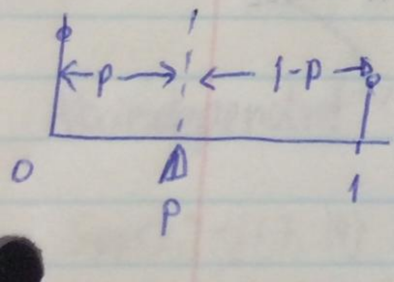


$$\sigma^2 = \left(0-\frac{1}{3}\right)^2 \frac{2}{3} + \left(1-\frac{1}{3}\right)^2 \frac{1}{3}$$

$$= \frac{1}{9} \cdot \frac{2}{3} + \frac{4}{9} \cdot \frac{1}{3}$$

$$= .259$$

* $X \sim Bern(p)$



$$\sigma^2 = (0-p)^2 \underbrace{(1-p)}_{p(0)} + (1-p)^2 \underbrace{p}_{p(1)}$$

$$= p^2(1-p) + (1-p)^2 p$$

$$= p(1-p)(p + 1 - p)$$

Varianceofany Bernuly $= \underline{p(1-p)}$

• Bet on lucky # 7

$$X \sim \begin{cases} \$35 & \text{wp } 1/38 \\ \$-1 & \text{wp } 37/38 \end{cases}$$

$E[X] = -\$0.053$

$$\underset{\text{"}}{\overset{E[X-\mu]^2}{}} \quad Var[X] = (\$35 - -\$0.053)^2 \frac{1}{38} + (-\$1 - -\$0.053)\frac{37}{38}$$

$= 33.21 \ \$^2$ — Distance away

$SE[X] := \sqrt{Var[X]} = \$5.76$

$\boxed{\sigma := SE[X] := \sqrt{var[X]}}$ Standard error
Standard deviation.

• $T = \underbrace{X_1 + X_2}_{g(X_1, X_2)}$

$E[T] = \sum_{t \in supp(T)} t\, p(t)$

X_1	X_2	T	$P(X_1, X_2)$
X_{11}	X_{21}	$X_{11} + X_{21}$	$P(X_{11})P(X_{21} \mid X_{11})$
	X_{22}	$X_{11} + X_{22}$	.
	⋮		
X_{12}			
X_{13}			
⋮			

$$E[g(X_1,X_2)] = \sum_{\substack{\langle X_1, X_2\rangle \in \\ Supp[X_1] \times \\ Supp[X_2]}} g(X_1,X_2)\, p(X_1,X_2)$$

$$= \sum_{X \in Supp[X_1]} \sum_{X_2 \in Supp[X_2]} g(X_1,X_2)\, p(X_1,X_2)$$

$$E[X_1+X_2] = \sum_{X_1} \sum_{X_2} (X_1+X_2)\, p(X_1,X_2)$$

Under no independence will be stuck here

$$= \sum_{X_1}\sum_{X_2} X_1\, p(X_1,X_2) + \sum_{X_1}\sum_{X_2} X_2\, p(X_1,X_2)$$

If X_1, X_2 independent $\Rightarrow p(X_1,X_2) = p(X_1)p(X_2)$

Join Mass Function
If they are independent
I can multiply them

$$= \sum_{X_1}\sum_{X_2} X_1\, p(X_1)\, p(X_2) + \sum_{X_1}\sum_{X_2} X_2\, p(X_1)\, p(X_2)$$

Independence

$$= \underbrace{\sum_{X_1} X_1\, p(X_1)}_{E[X_1]} \underbrace{\sum_{X_2} p(X_2)}_{1} + \underbrace{\sum_{X_1} p(X_1)}_{1} \underbrace{\sum_{X_2} X_2\, p(X_2)}_{E[X_2]}$$

No independent $\sum_{X_1} X_1 \sum_{X_2} p(X_1,X_2) + \sum_{X_2} X_2 \sum_{X_1} p(X_1,X_2)$

$Supp[X_1] = \{1, 7, 19\}$
$Supp[X_2] = \{5, 23, 88\}$

	1	7	19
5	1/15	1/3	2/15
23	1/30	1/10	1/30
88	1/30	1/5	1/15