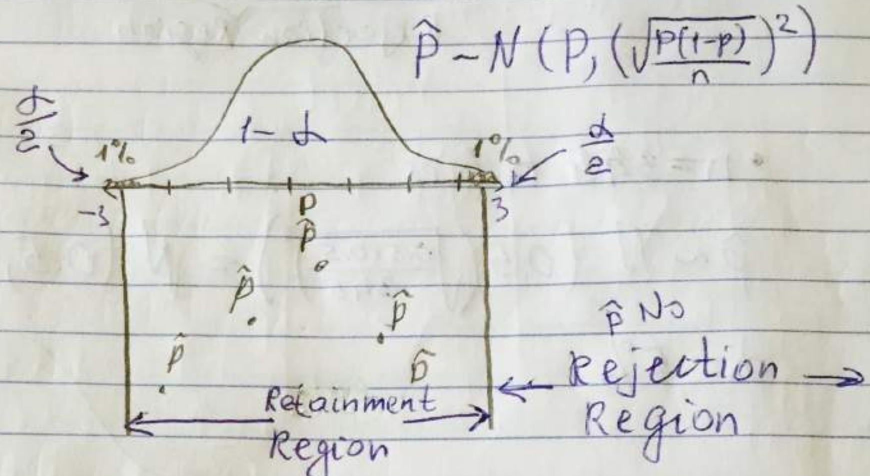
Theory: M/F gender ratio is

$p := P(\text{male}) = 0.5$



$\alpha = P(\hat{p} \text{ being "too" } \quad)$

$\hat{p} \in$ Retainment Region $\Rightarrow$ Reject theory

$\hat{p} \notin$ Retainment Region $\Rightarrow$ Reject theory

$\alpha = 5\%$

$$1.\ \alpha = p\left(|Z| \le z_{\frac{\alpha}{2}}\right)$$
$$= p(|Z| \le Z)$$
$$= p\left(\left|\frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}}\right| \le Z\right)$$
$$= p(-Z \le \quad \le Z)$$

$$= P\left(-z\sqrt{\frac{p(1-p)}{n}} \le \hat{p} - p \le z\sqrt{\frac{p(1-p)}{n}}\right)$$

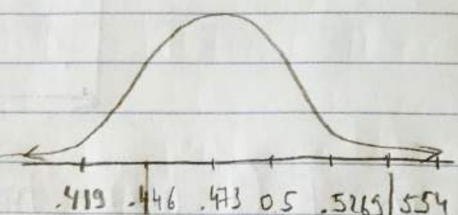
$$= P\left(p - z\sqrt{\frac{p(1-p)}{n}} \le \hat{p} \le p + z\sqrt{\frac{p(1-p)}{n}}\right)$$

$$= P\left(\hat{p} \in \underbrace{\left[p \pm z_{\frac{\alpha}{2}}\sqrt{\frac{p(1-p)}{n}}\,\right]}_{\text{Rejection Region}}\right)$$

• $n = 345$ babies

$$\hat{p} \sim N\left(0.5, \left(\sqrt{\frac{0.5 \cdot 0.5}{345}}\right)^2\right) = N(0.5, .0269^2)$$

$\alpha = 5\%$



RetRegn = $[0.5 \pm 2 \cdot .0269]$ $= [.446, .554]$

There is no evidence to suggest that gender proportion is not equal

• Is a coin fair? $(p = 0.5)$ → "Null Hypothesis" statement about a theoretical paramete range denoted "H_0"

Flip coin 100 times

Scen: #1: 51 H ⇒ fair

Scen: #2: 18 H ⇒ unfair

Scen: #3: 61 H ⇒

$H_0: p = 0.5$

ypothesis → H_A: param space $\setminus H_0: p \neq 0.5$ "H_0^c"

$\alpha = 5\%$

$$\hat{p} \sim N\left(0.5, \left(\sqrt{\frac{0.5 \cdot 0.5}{100}}\right)^2\right) = N\left(0.5, .05^2\right)$$

Retain Region $= (0.5 \pm 2 \cdot .05] = [0.4, 0.6]$

$\hat{p} = 0.61 \notin$ Retain Region $\rightarrow$ Reject H_0 (accept H_a)

- $H_0: p = 0.2$

$H_a: p \neq 0.2$

$\alpha = 1\%$

$\Rightarrow Z_{0.5} = 2.56$

$n = 380$

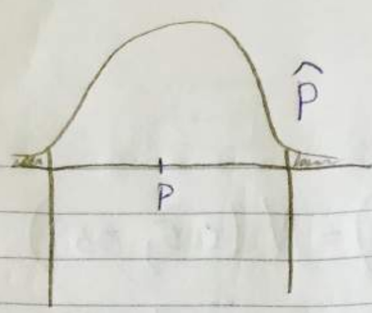
$$\hat{p} \sim N\left(0.2, \underbrace{\left(\sqrt{\frac{0.2 \cdot 0.8}{380}}\right)^2}_{.0205^2}\right)$$

Ret Region $= [0.2 \pm \overbrace{2.56 \cdot .0205}^{.0525}] =$

$= [.1475, .2525]$

$\hat{p} = \frac{69}{380} = .1816$

$\hat{p} \in$ Ret Region $\Rightarrow$ Retain H_0



our decision

	Retain H_0	Reject H_0
H_0 should be retained	✓	Type I error
H_0 false (should be rejected)	Type II error	✓

$\alpha = P(\text{Type I error})$

$\alpha \downarrow \Rightarrow P(\text{type II error}) \uparrow$

$\alpha \uparrow \Rightarrow P(\text{type II error}) \downarrow$

$n \uparrow \Rightarrow P(\text{type II error}) \downarrow$, $P(\text{type I error})$ no change

Costs

type I, type II errors

- Clinical trial

H_0: drug does not work

H_a: drug does work

- Type I error: drug doesn't work but you say it does.
- Type II error: drug does work but you say it doesn't

Big Building

- Fire Alarm System

H_0: No fire

H_a: fire

Type I error: No fire, but alarm goes off

Type II error: Fire, but no alarm

* $\hat{p}$ is a random variable

* $\hat{p}$ is a random realization (theta)

* with random data, you can make errors

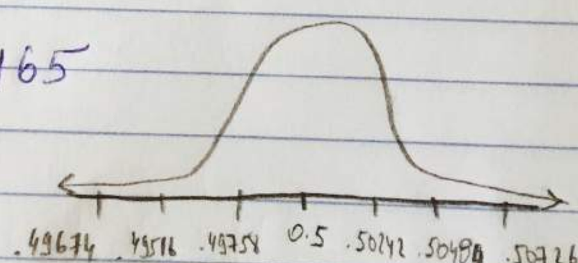
$H_0: p = 0.5$

$H_a: p \neq 0.5$

$\alpha = 5\%$

$n = 4{,}247{,}000$ babies in 2008

$\hat{p} = \dfrac{2{,}173{,}000}{4{,}247{,}000} = .51165$



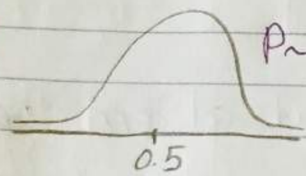
RetRegion = [.49516, .50484]

$\hat{p} \notin$ Retain Region $\Rightarrow$ Reject H_0

more "power" → prob of

"Retain" = "Accept"

$p \sim N(0.5, .0269)$



Ret Region $[.446, .554]$ $\hat{p} = 0.48 \in \Rightarrow$ Retain H_0

$H_0: p = 0.501$

$H_a: p \neq 0.501$