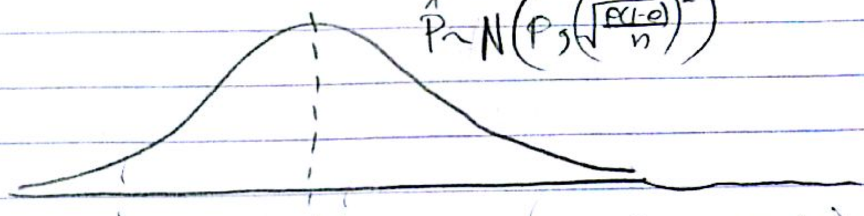
## ② Test Theories about p

Goal: Test a Theory. ("Test an hypotesis")

Kapelner's Theory: "Males/females are born in equal proportion"

$p := \mathbb{P}(\text{Male}) = 0.5$

$\hat{P} \sim N\left(p, \left(\sqrt{\frac{p(1-p)}{n}}\right)^2\right)$



Verification: ⓐ → Assume p

ⓑ Take sample: → $n = 20$ babies → See Results

$p = 0.5$

$= N\left(0.5, \left(\sqrt{\frac{0.5(0.5)}{n}}\right)^2\right)$

$\hat{p} = ✓$

$\hat{p} = ✓$

$\hat{p} \approx ✓$

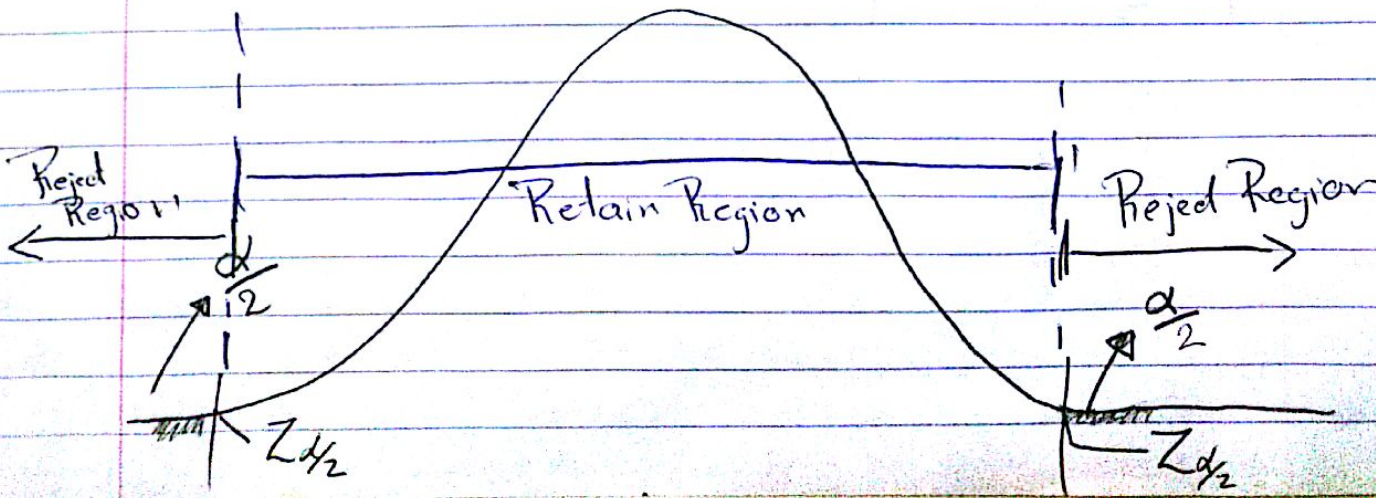
$\hat{p} = ✗$

**Dec 03**

P(p

$\boxed{\text{At a certain poin we will say that we do/don't believe the theory.}}$

Pick $\alpha = P(\hat{p}$ because results are too weird)



$\hat{p} \in$ Retaining Region $\Rightarrow$ Retain Theory

$\hat{p} \notin$ R " " $\Rightarrow$ Reject Theory

$$1-\alpha = P\left(|Z| \le Z_{\alpha/2}\right)$$

$$= P\left(\left|\frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}}\right| \le Z\right)$$

$$= P\left(-Z \le \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}} \le Z\right)$$

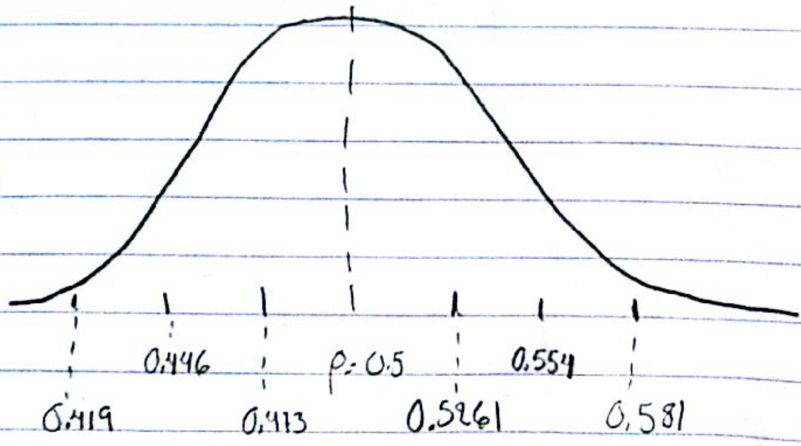
$$= P\left(-Z\sqrt{\frac{p(1-p)}{n}} \le \hat{p}-p \le Z\sqrt{\frac{p(1-p)}{n}}\right)$$

$$= P\left(p - Z \le \hat{p} \le p - Z\right)$$

$$= P\left(\hat{p} \in \left[p \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{p(1-p)}{n}}\right]\right)$$

Survey of $n = 345$ babies.

$P = P(\text{Male}) = 0.5$




$$\left[0.5 \pm 2 \cdot 0.269\right]$$

$$= \left[0.446,\ 0.554\right]$$

$\hat{p} \in$ Retaining Theory $\Rightarrow$ Retain theory

There is no evidence to suggest that there is an uneven ratio ~~number~~ of males to females.

Flip 100 coins (fair)

See: 1: 51 Heads $\Rightarrow$ Fair
See: 2: 98 Heads $\Rightarrow$ ~~Fair~~ Unfair
See: 3: 61 Heads $\Rightarrow$ ?

Theory Of NULL Hypotesis denoted "Ho", is in a subset of the parameter space. The alternative hypotesis that everything else is not "Ho", is called "Ha"

$H_0: p = 0.5$ $\alpha = 5\%$

$H_a: p \neq 0.5$ Retain Region $= \left[0.5 \pm \sqrt{\frac{0.5(0.5)}{100}}\right] = [0.4, 0.6]$

$\hat{p} \notin$ Retaining Region $\Rightarrow \underbrace{\text{Reject } H_0}$

$\boxed{\text{The Coin Is Not Fair}}$

---

Eminems Experiment:

- $H_0: p = 0.2$

  Population Parameter: of MM's produced in a certain factory from a certain shipment, at around this time.

  Not a Perfect sample of all MM's in the world

- $H_a: p \neq 0.2$ My Portion 8 total
  1 blue

Total MM's = 377
Blue MM's = 180

$n = 377$ mm's

$\alpha = 1\%$

$\hat{p} \sim N\left(0.2, \sqrt{\frac{0.2(0.8)}{377}}\right)$

$\Rightarrow Z = 2.56$

$\hat{p} \sim N(0.2, 0.0206)$

$\hat{p} = \frac{108}{377} = 0.286 \in$ Rej. Region

$\Rightarrow$ Rejects $H_0$

__Retaining Region__ $= [0.2 \pm 2.56]$

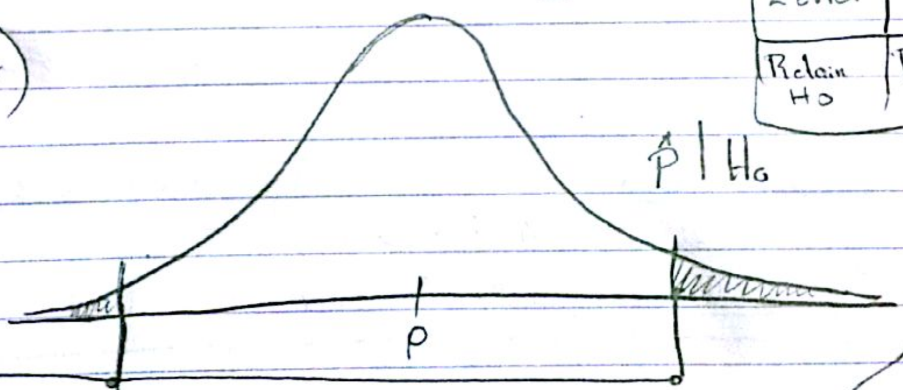
$= [0.147, 0.253]$

Picking different $\alpha$ makes things come up differently.

__The truth__ is unknowable

| | $H_0$ | $H_a$ |
|---|---|---|
| $H_0$ true | ✓ | Type 1 error |
| $H_0$ false | Type 2 error | ✓ |
| | Retain $H_0$ | Reject $H_a$ |

P(Type 1 error)




$\alpha$ is arbitrary

$\alpha = P(\text{Type 1 error})$

$\alpha \uparrow = P(\text{Type II error}) \downarrow$

$1 - P(\text{Type II error})$

$\alpha \downarrow = P(\text{Type II error}) \uparrow$

Wack'em-All-Game Scenario

$n \uparrow \Rightarrow P(\text{Type II error}) \downarrow$

Random Variable spits out data; "random data".

Clinical trial for cancer cure

$H_0$: drug does not work

$H_a$: drug does work

Type I error: Say that drug does work but in reality it doesn't work

Type II error Say that drug doesn't work but in reality it does work.

→ Both are bad and costly.

Fire Alarm in Kielly Hall (again)

$H_0$: no fire

$H_a$: fire

Type I error Alarm sounds but no fire (false alarm).

Type II error No alarm but there is a fire.

* → Make $\alpha$ big so there is more "Type I" errors than "Type II" errors, to avoid people dying.

When using random data to make decisions you ~~can~~ could be wrong!

$H_0 = p = 0.5$ $\quad\quad \alpha = 5\% \Rightarrow z = 2$

$H_a: p \neq 0.5$ $\quad\quad \cancel{\beta}:$

$n: 4{,}247{,}000$

$$\text{Retain Region} = \left[0.5 \pm \underbrace{2\sqrt{\frac{0.5(0.5)}{4{,}247{,}000}}}_{0.00242}\right]$$

$$= [0.49516,\ 0.50484]$$

$$\hat{p} = \frac{2173{,}000}{4{,}247{,}000} = 0.51165 \rightarrow \text{Male/Female}$$

$\hat{p} \notin$ Reject $\Rightarrow$ Reject $H_0 \Rightarrow$ $\boxed{\text{Male/Female birth ratio is un-even!!}}$

$\cdot$ $\boxed{\text{More Males than Females}}$

as $n \uparrow \Rightarrow P(\text{Type II error}) \downarrow$

$$\text{Power} = 1 - P(\text{Type II error})$$

probability of failure in effect.

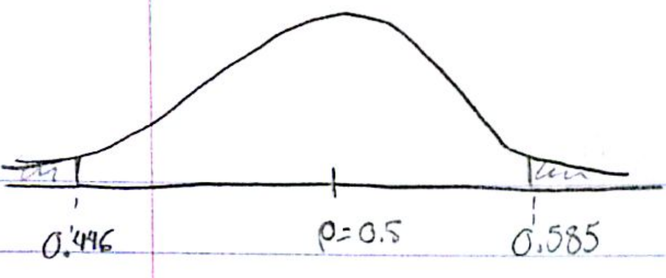
---

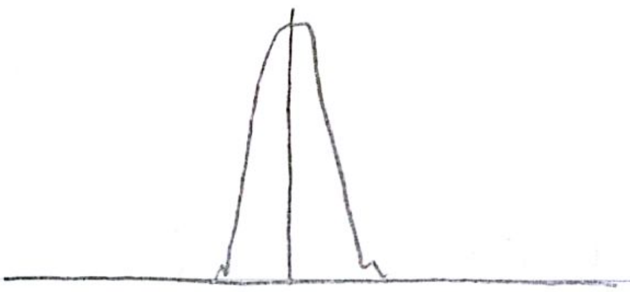
We either Retain or Reject:

| Retain | Reject |
|---|---|
| $H_0$ | $H_a \Rightarrow$ Accept $H_a$ |

$H_0 = 0.5$

$H_a \neq 0.5$



<u>Retain</u> means not rejecting