Continoous r.v's

$|Supp(X)| = |\mathbb{R}|$

$P(X = x) = 0 \Rightarrow$ No PMF

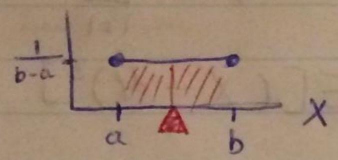
$f(x) := F'(x)$ (PDF) rate of change in a PMF
intgral of $f(x)$ over all support $= 1$

$X \sim Uniform(\{1, 7, 10\})$

Parameter Space
$a \in \mathbb{R}$
$b \in \mathbb{R}$
but $a < b$

$X \sim Uniform(a, b) := \frac{1}{b-a}$ ← F(x)

$Supp[X] = [a, b]$



$X \leq x$
CDF

$F(x) = \int f(x)\,dx + C$

$= \int \frac{1}{b-a}\,dx + C$

$= \frac{x}{b-a} + C$

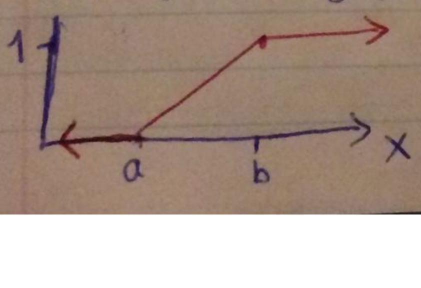
To Find CDF (?)

$F(a) = 0$
$X \leq a = 0$

$F(a) = \frac{a}{b-a} + C = 0$

$\Rightarrow C = -\frac{a}{b-a}$

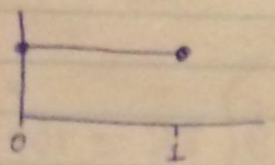
$F(x) = \frac{x}{b-a} - \frac{a}{b-a} = \frac{x-a}{b-a}$

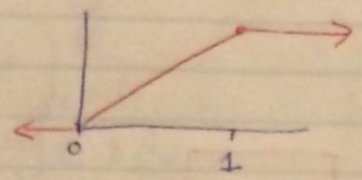


Exp(1) std exponential

Uniform(0,1) std Uniform random #

$:= 1$





$$* \ \mu = E[X] = \int_{x \in \mathrm{supp}[X]} x f(x)\,dx \quad \int_a^b x \frac{1}{b-a}\,dx = \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b = \frac{b^2-a^2}{2(b-a)}$$

$$= \frac{\cancel{(b-a)}(b+a)}{2\cancel{(b-a)}} = \boxed{\frac{b+a}{2}}$$

$$* \ \mathrm{Med}[X] = \{x : F(x) \geq 0.5\} \quad \frac{x-a}{b-a} = \frac{1}{2}$$

$$\Rightarrow x-a = \frac{b-a}{2} \Rightarrow x = \frac{b-a}{2} + a \Rightarrow x = \frac{b-a}{2} + \frac{2a}{2}$$

$$= \boxed{\frac{b+a}{2}}$$

$$\sigma^2 = \mathrm{Var}[X] = E[X^2] - \mu^2 = \int_a^b x^2 \frac{1}{b-a}\,dx - \left(\frac{b+a}{2}\right)^2$$

$$= \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b - \frac{b^2+2ab+a^2}{4}$$

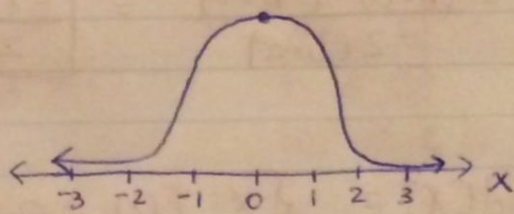
$$\frac{b^3-a^3}{3(b-a)} \quad = \left(\frac{4}{4}\right)\frac{\cancel{(b-a)}(b^2+ab+a^2)}{3\cancel{(b-a)}} - \frac{b^2+2ab+a^2}{4}\left(\frac{3}{3}\right)$$

$$= \frac{4b^2+4ab+4a^2-3b^2-6ab-3a^2}{12} = \frac{b^2-2ab+a^2}{12} = \boxed{\frac{(b-a)^2}{12}}$$

$$\begin{array}{r l} & \quad b^2 + ab + a^2 \\ b-a & \overline{)\, b^3 - a^3} \\ & -(b^3 - ab^2) \\ \hline & \quad ab^2 - a^3 \\ & -(ab^2 - a^2b) \\ \hline & \quad a^2b - a^3 \\ & -(a^2b - a^3) \\ \hline & \quad 0 \end{array}$$

$$\sigma = \sqrt{\frac{(b-a)^2}{12}} = \boxed{\frac{b-a}{\sqrt{12}}}$$

$$Z \sim N(0,1) := \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$



"Standard normal"
"Standard Gaussian"
"Bell curve"

$$\text{Supp}[Z] = \mathbb{R}$$

let $u = \frac{x}{\sqrt{2}} \Rightarrow \frac{du}{dx} = \frac{1}{\sqrt{2}}$

$\Rightarrow dx = \sqrt{2}\, du$

WTS (want to show)

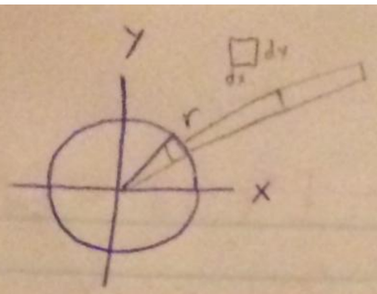
$$\int_{\mathbb{R}} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} = 1 \Rightarrow \int_{\mathbb{R}} e^{-\frac{x^2}{2}} dx = \sqrt{2\pi} \Rightarrow \int_{\mathbb{R}} e^{-u^2} \sqrt{2}\, du = \sqrt{2\pi}$$

show

$$\downarrow$$

$$\underbrace{\int_{\mathbb{R}} e^{-u^2} du = \sqrt{\pi}}_{\text{Gaussin integral}}$$

No exam

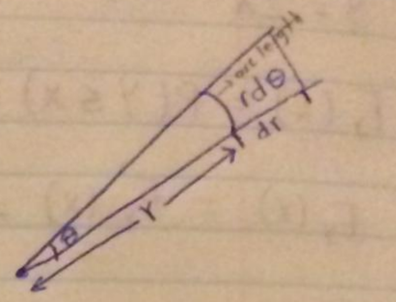


$r^2 = x^2 + y^2$ ← polar world

$x = r\cos\theta$

$y = r\sin\theta$

$$\iint_{\mathbb{R}\,\mathbb{R}} e^{-(x^2+y^2)} \underbrace{dx\,dy}_{dA} = \int_{\mathbb{R}} e^{-x^2}\,dx \int_{\mathbb{R}} e^{-y^2}\,dy \overset{\text{WTS}}{=} \pi$$



radians – 2π in a circle

$360°$

↳ good to divide

Polar

$$\int_0^{2\pi}\int_0^{\infty} e^{-r^2}\, r\,dr = 2\pi \int_0^{\infty} e^{-r^2}\, r\,dr = 2\pi \int_{u_1}^{u_2} e^{-u}\, r\,\frac{du}{2r}$$

let $u = r^2$

$\frac{du}{dr} = 2r \Rightarrow dr = \frac{du}{2r}$

$$= \pi\left[-e^{-u}\right]_{u_1}^{u_2}$$

$$= \pi\left[-e^{-r^2}\right]_0^{\infty}$$

$$= \pi(0 - -1) = \underline{\pi}$$

$dA = r\,dr\,d\theta$

$$dA = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{dy}{dr} & \frac{dx}{d\theta} \end{vmatrix} dr\,d\theta$$

$$\begin{vmatrix} \cos\theta & -r\sin\theta \\ \sin\theta & r\cos\theta \end{vmatrix} dr\,d\theta$$

$= r\cos^2\theta + r\sin^2\theta$

$= r\,dr\,d\theta$

• $X \sim \text{Exp}(\lambda) = \lambda e^{-\lambda x}$

$Y = 2X \sim ?$

$F_Y(x) = P(Y \leq x) = P(2X \leq x) = P(X \leq \frac{x}{2}) = F_X(\frac{x}{2})$

$f_Y(x) = F_Y'(x) = \frac{d}{dx}\left[F_X(\frac{x}{2})\right] = \frac{d}{dx}\left[1 - e^{-\lambda \frac{x}{2}}\right] = \frac{\lambda}{2} e^{-\frac{\lambda}{2}x} = \text{Exp}\left[\frac{\lambda}{2}\right]$

$X \sim \text{Unif}(a, b)$

$Y = cX + d \sim \text{Unif}(ca+d, cb+d)$

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exam again

$X = \sigma Z + \mu$

$E[X] = \sigma E[Z] + \mu$

$\text{Var}[X] = \sigma^2 \text{Var}[Z]$

let $u = \frac{x^2}{2}$

$\frac{du}{dx} = x \quad dx = \frac{du}{x}$

$E[Z] = \int_{\mathbb{R}} x \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} e^{-\frac{x^2}{2}} x \, dx = \frac{1}{\sqrt{2\pi}} \int_{u_1}^{u_2} e^{-u} \cancel{x} \frac{du}{\cancel{x}}$

$= \frac{1}{\sqrt{2\pi}} \left[-e^{-u}\right]_{u_1}^{u_2} = \frac{1}{\sqrt{2\pi}} \left[-e^{-\frac{x^2}{2}}\right]_{-\infty}^{\infty} = \frac{1}{\sqrt{2\pi}} (0 - -0)$

$= \boxed{0}$

$$\mathrm{Var}[Z] = E[Z^2] - \cancel{\mu^2}^{\,0}$$

$$= \int_{\mathbb{R}} x^2 \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \, dx \; := \underline{\underline{1}}$$

$$\Rightarrow \boxed{\sigma = 1}$$

Z already standarized

Pivot of 0

Var of 1

No need to be standarized

$$F_X(x) = P(X \le x) = P(\sigma P + \mu \le x)$$

$$= P\left(Z \le \frac{x-\mu}{\sigma}\right) = F_Z\left(\frac{x-\mu}{\sigma}\right)$$

let $u = \frac{x-\mu}{\sigma}$

$\frac{du}{dx} = \frac{1}{\sigma}$

$$f_X(x) = F_X'(x) = \frac{d}{dx}\left[F_Z\left(\frac{x-\mu}{\sigma}\right)\right] =$$

$$= \frac{du}{dx}\left[\frac{d}{du}\left[F_Z(u)\right]\right] = \frac{1}{\sigma} f_Z(u) = \frac{1}{\sigma}\frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}}$$

$$\boxed{= \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} \; := N(\mu, \sigma^2)}$$

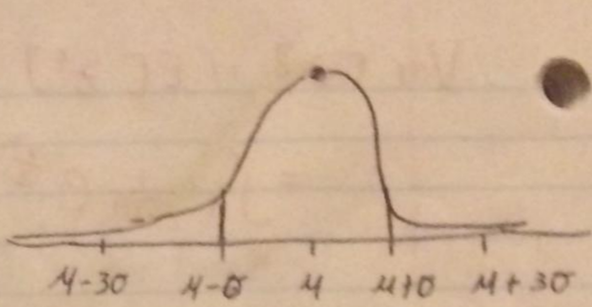
var

$$F_X(x) = \int \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} \, dx + C \Big\} \text{ NO Possible}$$

(Rizch Algorithm)

Best we can do:

$$F_X(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{1}{2\sigma^2}(y-\mu)^2}\, dy$$



$$F(\mu) = \frac{1}{2}$$

Mean

- $P(Z \in [-1, 1])$
  $= P(X \in [\mu-\sigma, \mu+\sigma])$
  $= 0.68$

"68-95-99.7 Rule"

"Empirical rule"

- $P(Z \in [-2, 2])$
  $= P(X \in [\mu-2\sigma, \mu+2\sigma])$
  $= 0.95$

- $P(Z \in [-3, 3])$
  $= P(X \in [\mu-3\sigma, \mu+3\sigma])$
  $= 0.997$

He smiled at her; She smiled back.