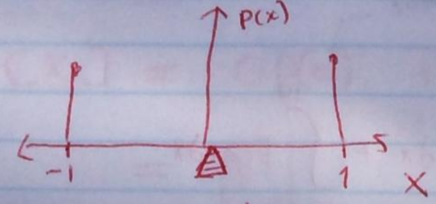
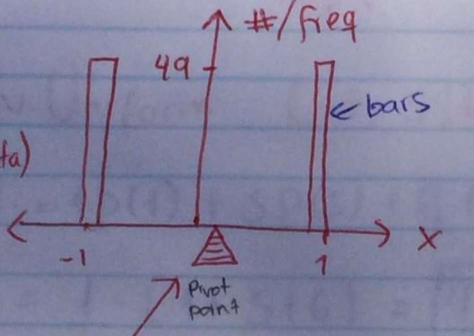
$X \sim$ Rademache

PMF



model

Histogram
(for plotting data)



What happens in the experimental

$$\bar{X} := \frac{1}{n}\sum_{i=1}^{n} X_i$$

$n \to \infty$ $\bar{X}$?

The pivot of the PMF is called the "mean" or "expectation" or "expected value".

$E[X]$ or μ mu

operator

r.v

like $\frac{d}{dx}[x^2]$

$X_1, X_2, \dots X_n \overset{iid}{\sim}$ see $x_1, x_2, \dots, x_n$

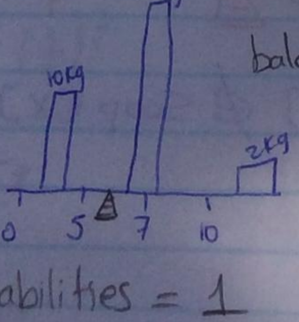
$$\bar{X}_n \xrightarrow{n \to \infty} E[X]$$

degenerate

$$P(A) = \lim_{n \to \infty} \frac{1}{n} \sum \mathbb{1}_{w \in A}$$

No of a exam

balance it



$$\text{Weigh Avg Pivot} = \frac{W_1 X_1 + W_2 X_2 + W_3 X_3}{\sum W_i}$$

Sum of all probabilities = 1

Descrete r.v $\quad Supp[X] = \{X_1, X_2, \dots\}$

$$E[X] := \int_{w \in \Omega} X(w)\, dP(w)$$

$$\int_0^{10} f(x)\,dx = \int_0^{2} f(x)\,dx + \int_2^{7.3} f(x)\,dx + \int_{7.3}^{10} f(x)\,dx$$

$$\int_{\{w: X(w)=x_1\}} X(w)\,dP(w) + \int_{\{w: X(w)=X_2\}} X(w)\,d(w)$$

$$= X_1 \int dP(w) + X_2 \int dP(w) + \dots$$

$$= X_1\, P(X_1) + X_2\, P(X_2)$$

$$= \sum_{x \in supp(X)} x\; p(x_i)$$ — Probability weithed average.

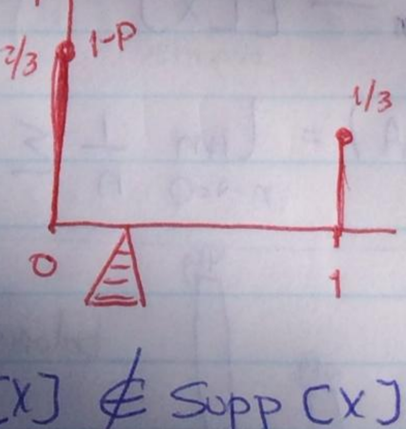
$$E[X] = (-1)\left(\tfrac{1}{2}\right) + (1)\left(\tfrac{1}{2}\right) = 0$$ — Ramedache

$X \sim$ Bern $\left(\frac{1}{3}\right)$

$$E[X] = (0)\,p(0) + (1)\,p(1)$$
expectation

$$= \frac{1}{3}$$

average of data in the long run is close to 1/3 (pivot).



$E[X] \notin Supp[X]$

- $X \sim \text{Bern}(p)$

$$E[X] = 0\,p(\underset{\text{support}}{0}) + 1\,p(\underset{\text{support}}{1})$$

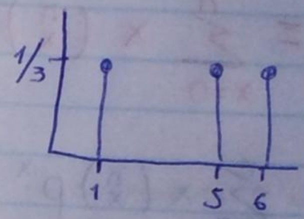
General

$$= \boxed{p}$$

- $X \sim \text{Uniform}(1, 5, 6)$

$$E[X] = 1p(1) + 5p(5) + 6p(6)$$

$$= \frac{1}{3}(1+5+6) = \boxed{4}$$



General

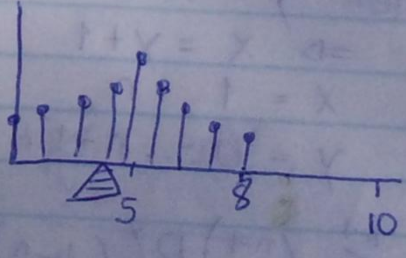
- $X \sim \text{Uniform}(A)$

$$E[X] = \frac{1}{|A|} \sum_{x \in A} x$$

- $X \sim \text{Binomial}(8, \frac{1}{2})$

Supp = $\{0, \ldots, 8\}$

$$E[X] = 0p(0) + 1p(1) + 2p(2) + 3p(3) + 4p(4) + 5p(5) + 6p(6) + 7p(7) + 8p(8)$$



$$p(0) = \binom{8}{0}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^8$$

$$P(1) = \underbrace{\binom{8}{1}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^7}_{\frac{1}{2^8}}$$

$$E(X) = \frac{1}{2^8}\left(1\binom{8}{1} + 2\binom{8}{2} + 3\binom{8}{3} + 4\binom{8}{4} + 5\binom{8}{5} + 6\binom{8}{6} + 7\binom{8}{7} + 8\right)$$

$$= \frac{1}{2^8}(1024) = \boxed{4}$$

General

$X \sim \text{Binomial}(n, p) \rightarrow \text{supp} = \{x, \ldots, n\}$

$\sum_{A \in 2^{\Omega}} P(A)$

$$E[X] = \sum_{x \in \text{Supp}(x)} x \, p(x)$$

$$= \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=1}^{n} x \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=1}^{n} x \frac{n!}{x!\,(n-x)!} p^x (1-p)^{n-x}$$

$$= np \sum_{x=1}^{n} \underbrace{\overset{\text{neg} \sim \text{Bino}}{\frac{(n-1)!}{(x-1)!\,(n-x)!}}}_{\binom{n-1}{x-1}} p^{x-1} (1-p)^{n-x}$$

n 100 avg .3 p

avg 30 succe

n 200 avg .45 p

90

$n \times p$

$\frac{x}{x!} = \frac{1}{(x-1)!}$

let $y = x-1$

$\Rightarrow x = y+1$

$x = 1 \ldots n$

$y = 0 \ldots n-1$

$$= np \sum_{y=0}^{n-1} \binom{n-1}{y} p^y (1-p)^{n-1-y}$$

let $m := n-1$

$$= np \underbrace{\sum_{y=0}^{m} \overbrace{\binom{m}{y} p^y (1-p)^m}^{\text{PMF of binomial}}}_{1 \rightarrow \text{sum of support, you get } 1} = \boxed{np}$$

- $X \sim \text{Hypp}(n, K, N)$

$$E[X] = \sum_{x \in \text{Supp}(x)} x \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

$$\boxed{= n \frac{K}{N}}$$

- $X \sim \text{Geom}(p)$

$$E[X] = \frac{1}{p}$$

Pivot represents what you'll have if you had infinate data.

$$p = \frac{1}{100}$$

$$\boxed{E[X] = \sum_{x \in \text{Supp}(X)} x\, p(x)}$$

support of Geometric $\mathbb{N}$

$$E(X) = \sum_{x=1}^{\infty} x\,(1-p)^{x-1} p$$

$E\langle x \rangle = \mu$

$y = x-1$

$\Rightarrow x = y+1$

$x = 1 \dots \infty$

$y = 0 \dots \infty$

$$= \sum_{y=0}^{\infty} (y+1)(1-p)^y p$$

$|q| < 1 \ldots \sum_{x=0}^{\infty} q^x = \frac{1}{1-q}$

Geometric Series

$$= \sum_{y=0}^{\infty} y (1-p)^y p + \underbrace{p \underbrace{\sum_{y=0}^{\infty} (1-p)^y}_{\frac{1}{p}}}_{1}$$

$$= \sum_{y=1}^{\infty} y (1-p)^y p + 1$$

$$= (1-p) \sum_{y=1}^{\infty} y (1-p)^{y-1} p + 1$$

$$\mu = (1-p)\mu + 1 \longrightarrow \mu = \mu - p\mu + 1 \rightarrow 0 = -p\mu + 1 \rightarrow \mu p = 1$$

$$\boxed{\mu = \frac{1}{p}}$$

- $X \sim \text{Neg Bin}\ (r, p)$

$$E[X] = \sum_{x=r}^{\infty} x \binom{x-1}{r-1} (1-p)^{x-r} p^{r}$$

$$\vdots$$

$$= \frac{r}{p}$$

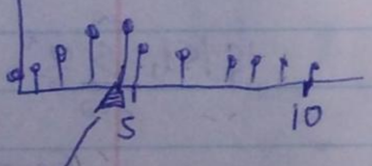
A.K.A "percentile"

Quantile $[X, p]$

$$:= \min_{x \in Supp[X]} \{ x : F(x) \geq p \}$$

Data Quantile have to converge to real quantile.

$X \sim \text{Bin}\ (10, 0.4)$



Pivot : 10x4

when adding all # when will I get 70%.

X	P(x)	F(x)
0	.0060	.0060
1	.0404	.0464
2	.1209	.1623
3	.2150	.3673
4	.2506	.6331
5	.2007	.8338
6	.1115	.9450
7	.0465	.9877
8	.0106	.9983
9	.0016	.9999
10	.0001	1
	1	

25 (F(3) = .3673), 75 (F(5) = .8338) — IQR 75-25

F(5) = .8338 → 70 percentile

F(6) = .9450 → 90 percentile

F(10) = 1 → cause all sum is 1 anyways

$Median(X) := Quantile[X, 0.5)$

$IQR(X) := Quantile[X, 0.75] - Quantile[X, 0.25]$ 75% - 25%

interquantile range

Tertiles: 0, 33, 66, 100 quatiles

Quartiles: 0, 25, 50, 75, 100

Quintales: 0, 20, 40, 60, 80, 100

Deciles: 0, 10, 20, ..., 100

- $Mode[X] = \underset{x}{\arg\max}\{p(x)\}$ give me back the most likely value.

Medium
Mode
Quantile
expectation
very useful

questions to ask about r.v

$$X \sim \begin{cases} \$1 & \text{wp } \frac{18}{38} \\ -\$1 & \text{wp } \frac{20}{38} \end{cases}$$

black

Red

Support $\{1, -1\}$

$$E[X] = 1\left(\frac{18}{38}\right) + (-1)\left(\frac{20}{38}\right) = -\frac{2}{38} = -\$0.053$$

- Casino involves a negative expectation

If you play a bunch of times, the average losing is 0.05

- $$X \sim \begin{cases} \$35 & \text{wp } \frac{1}{38} \\ -\$1 & \text{wp } \frac{37}{38} \end{cases}$$

Bet on #7

$$E(X) = 35 \frac{1}{38} + (-1)\frac{37}{38} = -\frac{2}{38} = -\$0.05$$

- Bet on 1-12

$$X \sim \begin{cases} 2 & \text{wp } 12/38 \\ 1 & \text{wp } 26/38 \end{cases}$$

$$E[X] = 2\,\frac{12}{38} + (-1)\,\frac{26}{38} = \frac{-2}{38} = -\$0.05$$

lose less than American one.

Europe

$$X \sim \begin{cases} \$1 & wp\ 18/37 \\ -\$1 & wp\ 19/37 \end{cases}$$

$$E(X) = 1\,\frac{18}{37} + (-1)\,\frac{19}{37} = -\frac{1}{37} = \$-0.027$$