10/13 $X \sim \text{Bernoulli}(p) := \begin{cases} 1 & \text{wp } p \\ 0 & \text{wp } 1-p \end{cases} = p^x(1-p)^{1-x} = P(x)$

lec 10. $\text{supp}[X] = \{0, 1\}$, $p \in (0, 1)$

$X_1 \sim \text{Bernoulli}(p)$ $\quad X_1 \overset{d}{=} X_2$ ← equal distribution $\quad$ since CDF $F_1(x) = F_2(x)$

$X_2 \sim \text{Bernoulli}(p)$ $\quad$ and with discrete r.v.s

~~PMF~~ PMF $P_1(x) = P_2(x)$

4 Red ~~ar~~ balls $\quad$ P(getting 2 R when drawing 3 when 4 R balls, 10 balls) $= \dfrac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}}$

10 total balls

P(getting x R while drawing 3, 4 R, total 10 balls) $= \dfrac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$

P(getting x R while drawing n balls, 4 R of 10 total) $= \dfrac{\binom{4}{x}\binom{6}{n-x}}{\binom{10}{n}}$

P(getting x R while drawing n with K R out of 10 total) $= \dfrac{\binom{K}{x}\binom{10-K}{n-x}}{\binom{10}{n}}$

P(getting x R drawing n with K R out of N total)

$= \dfrac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$ ← sampling success without replacement $= P(x)$

超几何

$X \sim \text{Hypergeometric}(n, K, N) =$

$X \sim \text{Bernoulli}(p) \quad p \in (0, 1)$

Parameter Space $\quad N \neq 0, n \neq 0 \quad$ ✱ $X \sim \text{Hyper}(1, 1, 2) = \dfrac{\binom{1}{x}\binom{1}{1-x}}{\binom{2}{1}} \overset{d}{=} \text{Bern}(\tfrac{1}{2})$

$N = 3 \quad\quad N = \mathbb{N} \setminus \{1\} \quad\quad P(0) = \tfrac{1}{2}$

$K \in \{1, 2\} \quad\quad K \in \{1, \ldots N-1\} \quad\quad P(1) = \tfrac{1}{2} \quad\quad \text{supp}[X] = \{0, 1\}$

$n \in \{1, 2\} \quad\quad n \in \{1, \ldots N-1\}$

✱ $X \sim \text{Hyper}(1, K, N) = \dfrac{\binom{K}{x}\binom{N-K}{1-x}}{\binom{N}{1}} = \dfrac{\frac{K!}{(K-x)!\,x!}\;\frac{(N-K)!}{((N-K)-(1-x))!\,(1-x)!}}{N} \overset{d}{=} \text{Bern}(\tfrac{K}{N})$

$\text{supp}[X] = \{0, 1\}$

$x = 0$, $\dfrac{\frac{K!}{K!\,0!}\;\frac{(N-K)!}{(N-K-1)!\,0!}}{N} = \dfrac{N-K}{N}$

$x = 1 \quad \dfrac{K!}{(K-1)!\,1!} \cdot \dfrac{(N-K)!}{(N-K)!\,0!} = \dfrac{K}{N}$

4S, 6F.

$X \sim Hyper(5, 2, 10)$ $supp[X] = \{0, 1, 2\}$

$n < k$ & $n < N-k$, $supp[X] = \{0, \dots n\}$

$X \sim Hyper(5, 4, 10)$ $supp[X] = \{0, 1, 2, 3, 4\}$

$n \geq k$, $n < Nk$ $supp[X] = \{0, \dots k\}$

$X \sim Hyper(8, 4, 10)$ $supp[X] = \{2, 3, 4\}$

$n \geq k$ & $n \geq N-k$ $supp[X] = \{n-(N-k), \dots k\}$

$X \sim Hyper(5, 7, 10)$ $supp[X] = \{2, 3, 4, 5\}$

$n < k$ & $n \geq N-k$ $supp[X] = \{n-(N-k) \dots, n\}$

	$n < k$	$n \geq k$
$n < N-k$	$\{0, \dots n\}$	$\{0, \dots k\}$
$n \geq N-k$	$\{n-(N-k), \dots n\}$	$\{n-(N-k), \dots k\}$

$supp[X] = \{\max(0, n-(N-k)), \dots, \min(n, k)\}$

$\sum_{x \in supp(x)} p(x) = 1$

$X \sim Bernoulli(p) = \begin{cases} 1 & wp\ p \\ 0 & wp\ 1-p \end{cases}$ Equivalent parametriation

$X \sim Bernoulli(q) \begin{cases} 1 & wp\ 1-q \\ 0 & wp\ q \end{cases}$

$X \sim Hyper(n, p, N)$

$p := \frac{k}{N}$ (Probability of success ball)

$N \in \mathbb{N} \setminus \{1\}$

$n \in \{1, \dots, N-1\}$

$p \in \{\frac{1}{N}, \dots, \frac{N-1}{N}\}$

$X \sim Hyper(6, \overset{0.5}{\cancel{0.5}}, 10)$ there is 10 total ball, half percent ~~probability~~ probability of success ball)

$P(X=3) = 0.476$

$X \sim Hyper(6, 0.5, 100)$ Bag of 100 balls, 50% of success

$P(X=3) = 0.3223$

$X \sim Hyper(6, 0.5, 1000)$

$P(X=3) = 0.3134$

$F\, X \sim Hyper(6, 0.5, 10000)$

$P(X=3) = 0.3126$