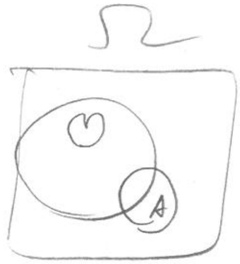
Lecture 7 Math 281 9/25/15

Ω



$$P(A) = \frac{4}{52} = \frac{1}{13}$$

$$P(A|♡) = \frac{1}{13} \quad \Rightarrow A, ♡ \text{ independent events}$$

$\frac{4}{52} = \frac{1}{13}$ numerically but not conceptually ... they are completely different events in different universes

$$P(A) = \frac{|A|}{|\Omega|}$$ all cards

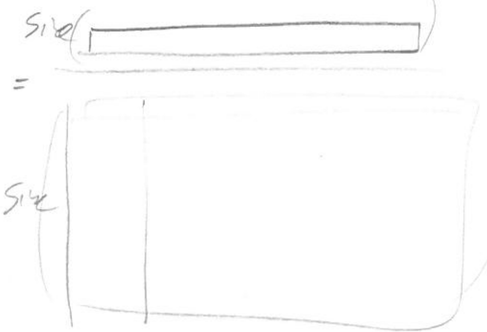
$$P(A|♡) = \frac{|A \cap ♡|}{|♡|}$$ all hearts

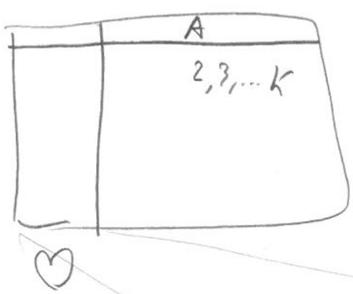
this event is now the universe, a subuniverse $\Omega' \subseteq \Omega$

$♡ \subseteq \{\text{all cards}\}$

How does this look like?

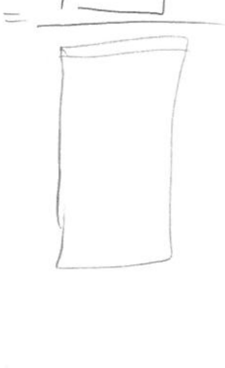
$P(A) =$





Zooming in

$P(A|♡) =$



I now live in this new universe

$A \cap ♡$

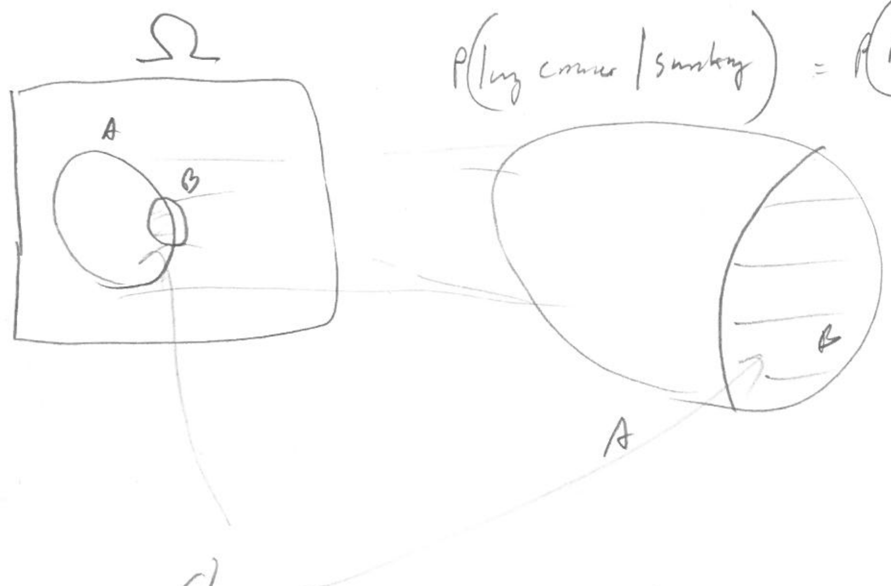
$n = 1000$ people

200 smokers (A) $\qquad P(A) = \frac{200}{1000} = 0.2$

60 lung cancer (B) $\qquad P(B) = \frac{60}{1000} = 0.06$

36 smokers & lung cancer $\qquad P(A,B) = \frac{36}{1000} = 0.036$

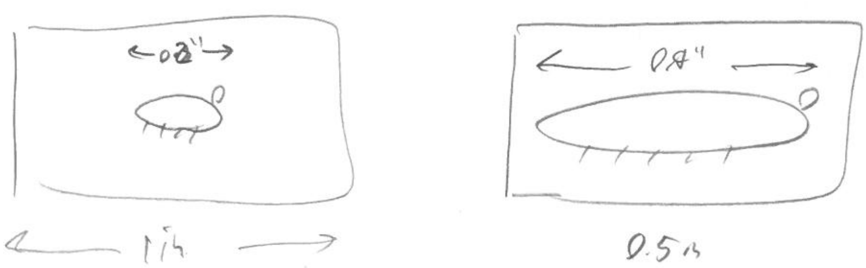
$P(\text{lung cancer} \mid \text{smoking}) = P(B|A)$



$P(B$ in a new universe$)$
$= P$

Shape is the same: $P(B \cap A)$
but now its bigger

$P(B|A) = P(B,A) \cdot$ Zoom factor

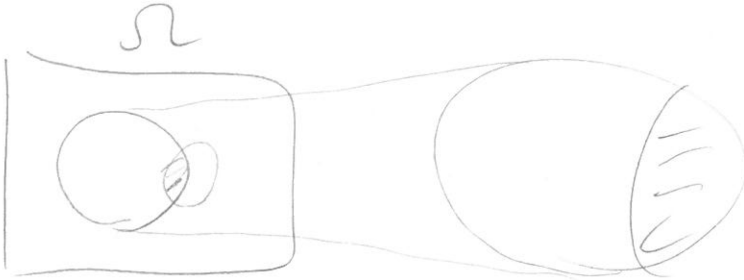


Zoom: 2

the bug is twice as big since the scope size is half

$$\text{Zoom factor} = \frac{\text{prior scope size}}{\text{zoomed scope size}} = \frac{1}{0.5} = 2$$

Likewise...



"proportional"

$$\text{zoom} = \frac{\text{bigger size}}{\text{smaller size}} = \frac{P(\Omega)}{P(A)} = \frac{1}{P(A)}$$

(1763)

$$\Rightarrow \boxed{P(B|A) = P(B,A) \cdot \frac{1}{P(A)} = \frac{P(B,A)}{P(A)}}$$

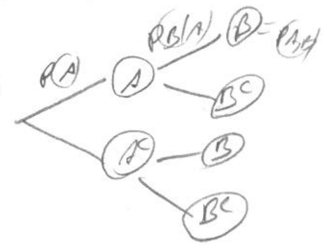
"Bayes Rule" AKA def of cond. prob.

$$\Rightarrow P(B,A) = P(B|A)\,P(A)$$

if B, A ind. $\Rightarrow P(B|A) = P(B)$

$$\Rightarrow \boxed{P(B,A) = P(B)\,P(A)}$$

derivation of multiplication Rule



Likewise $P(A|B) = \frac{P(A,B)}{P(B)} \Rightarrow P(A,B) = P(A|B)\,P(B)$

$$\Rightarrow P(B|A) = \frac{P(A|B)\,P(B)}{P(A)}$$

"Double Bayes Rule" $P(B|A) \propto P(A|B)$

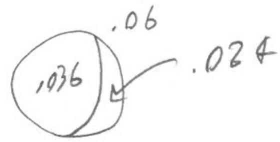
$P(B|A) = \frac{.036}{.2} = .18 \approx 20\%$. If you smoke, these are bad odds!

Low? High?

$P(\text{smoking} \mid \text{l.c})$

$P(A \mid B) = \frac{P(A,B)}{P(B)} = \frac{.036}{.06} = .6$ good chance he smoked...

$P(\text{l.c.} \mid \text{did not smoke}) = P(B \mid A^c) = \frac{P(A^c, B)}{P(A^c)} = \frac{.024}{1 - 0.2} = .03$

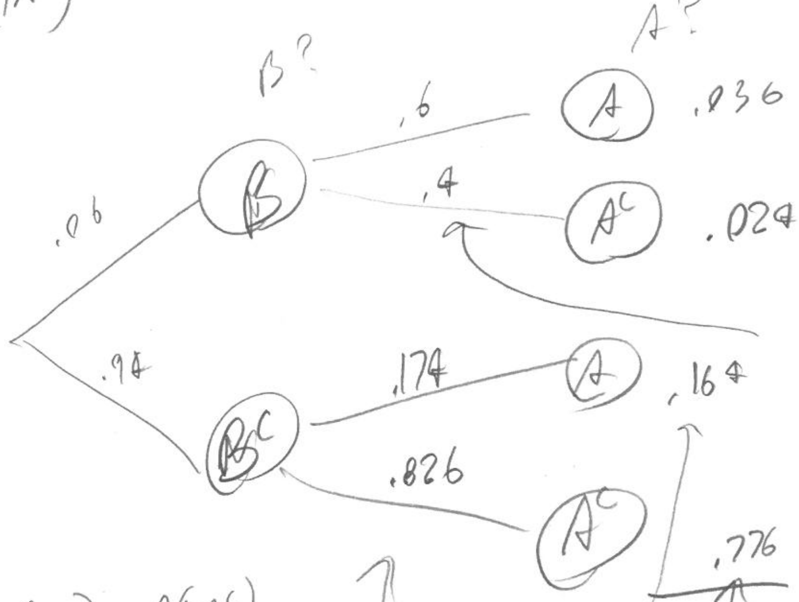


$P(B) = P(B, A) + P(B, A^c)$ ✓A

$.06 = .036 + ?$

solve...

$\frac{P(B \mid A)}{P(B \mid A^c)} = \frac{.18}{.03} = 6$ why?



$P(A^c \mid B) = \frac{P(A^c, B)}{P(B)}$

or $P(A \mid B) = 1 - P(A^c \mid B)$

complement rule

... for cond. events

$\frac{P(A,B)}{P(B)} = 1 - \frac{P(A^c, B)}{P(B)}$

$P(A) = P(A,B) + P(A,B^c)$

$P(A,B^c) = .2 - .036 = .164$

$P(A,B) = P(B) - P(A^c, B)$

do these make practical sense?

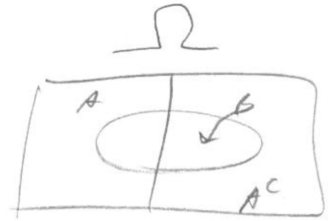
$P(B) = P(A,B) + P(A^c, B)$

the best road to prove it...

disjoint

WTS $P(B) = P(B, A) + P(B, A^c) \quad \forall A$

$P(B) = P\big((B\cap A) \cup (B \cap A^c)\big)$ (C)



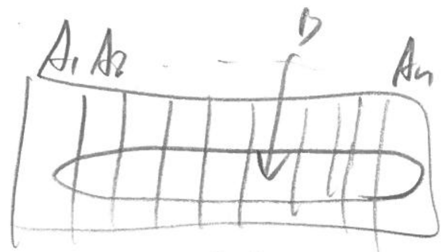
$= P\big(B \cap (A \cup A^c)\big)$

$= P(B \cap \Omega)$

$= P(B)$ ✓

proof from set theory (E.C.)

How about in general $A_1, \dots, A_n$ mutually exclusive, coll. exhaustive



$$P(B) = \sum_{i=1}^{n} P(B, A_i)$$ Law of Total Prob.

disjoint

$P(B, A_1) + P(B, A_2) + \dots + P(B, A_n)$

$(B \cap A_2) \cap (B \cap A_3)$

$B \cap A_2 \cap B = B \cap A_2 \cap A_3 = B \cap \phi = \phi$

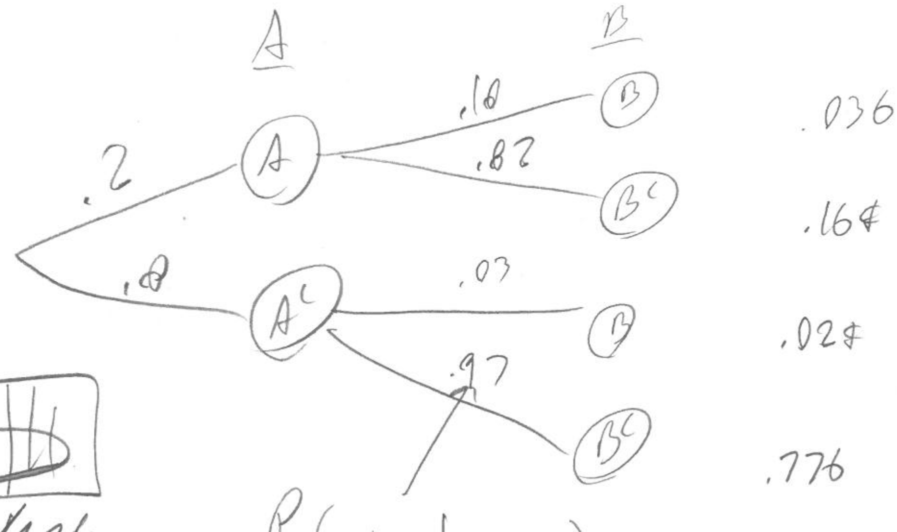
$= P\big((B\cap A_1) \cup (B \cap A_2) \cup \dots \cup (B \cap A_n)\big)$

$= P\big(B \cap (A_1 \cup \dots \cup A_n)\big) = P(B \cap \Omega) = P(B)$ ✓

What about $P(B^c \mid A^c)$? Readable from tree? No...

Need to "invert" the tree...

How many questions can be asked $P(\_ \mid \_) = ?$ 8 questions!



Back to ~~[illegible]~~

$P(\text{no l.c.} \mid \text{no smoke}) \approx 1$

Double

Bayes Rule ~~[illegible]~~

$$P(A_\ell \mid B) = \frac{P(B \mid A_\ell)\, P(A_\ell)}{P(B)}$$

$$= \frac{P(B \mid A_\ell)\, P(A_\ell)}{\sum_{i=1}^{n} P(B, A_i)} = \frac{P(B \mid A_\ell)\, P(A_\ell)}{\sum_{i=1}^{n} P(B \mid A_i)\, P(A_i)}$$

"Bayes Thm"

Cond. Prob. is super weird!

Let's say I know you have 2 kids and one is a girl.

P(other is a girl)? Knee-jerk reaction: $\frac{1}{2}$

$$P(GG \mid \text{one girl}) = P(\{GG\} \mid \{GB, BG, GG\}) = \frac{P(\{GG\} \cap \{GB, BG, GG\})}{P(\cdot)} = \frac{P(GG)}{P(\cdot)} = \frac{1}{3}$$

$\frac{1}{4}$ equally likely!