Lecture 10 Oct 13, 2015 Math 241

$X \sim Bernoulli(p) := \begin{cases} 1 & wp\ p \\ 0 & wp\ 1-p \end{cases} = p^x (1-p)^{1-x} = p(x)$

(Bernoulli)

$X_1 \sim Bernoulli(p), \quad X_2 \sim Bernoulli(p)$

$X_1 \stackrel{d}{=} X_2$ by def, $p_1(x) = p_2(x)$ or $F_1(x) = F_2(x)$

general def. see why when we reach cont. r.v.'s

---

10 cards 4R, 6B

$P(2R \text{ out of } 3 \text{ cards}, 10 \text{ tot}, 4R) = \frac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}}$

$P(x\ R \text{ out of } 3 \text{ cards}, 10 \text{ tot}, 4R) = \frac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$

$P(x\ R \text{ out of } n \text{ cards}, 10 \text{ tot}, 4R) = \frac{\binom{4}{x}\binom{6}{n-x}}{\binom{10}{n}}$

$P(x\ R \text{ out of } n \text{ cards}, 10 \text{ tot}, K R) = \frac{\binom{K}{x}\binom{10-K}{n-x}}{\binom{10}{n}}$

$P(x\ R \text{ out of } n \text{ cards}, N \text{ tot}, K R) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$

Let r.v. X denote the # of successes in the situation of N total possibilities, n are drawn and K total successes drawn without replacement.

$$X \sim \text{Hypergeometric}(n, K, N) := \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

3 parameters!

3 Knobs you can twist

Parameter space of X

$N=0$? No... absurd, $n \neq 0$ either

$N=1$? $K=0$ or 1 if $K=0$ ... $\frac{\binom{0}{x}\binom{1-0}{1-x}}{\binom{1}{1}} = \frac{\binom{0}{x}\binom{1}{1-x}}{\binom{1}{1}} \Rightarrow x=0$ always

$\Rightarrow n=1$

$K=1$ ... $\Rightarrow x=1$ always

$\text{Hyper}(1,1,1) = \text{Deg}(1)$ $\text{Hyper}(1,0,1) = \text{Deg}(0)$ $\Rightarrow$ Degenerate!

$\Rightarrow N=2$, $K=0,1,2$, $n=1,2$

if $n=2$, $x=K$ (you take all balls) $\Rightarrow$ $n \neq K$ o/w degen

if $n=1$, $K=0$? No $\Rightarrow$ degen $\quad \text{Hyp}(1,0,2) = \text{Deg}(0)$

$K=2$? No $\Rightarrow$ degen $\quad \text{Hyp}(1,2,2) = \text{Deg}(1)$

$K=1$? $\text{Hyp}(1,1,2) = \frac{\binom{1}{x}\binom{1}{1-x}}{\binom{2}{1}} = \frac{\frac{1!}{(1-x)!\,x!}\,\frac{1!}{(1-x)!\,(x)!}}{2} \overset{d}{=} \text{Bern}\left(\frac{1}{2}\right)$

if $x=0 \Rightarrow \frac{1}{2}$

if $x=1 \Rightarrow \frac{1}{2}$

if $N=3 \Rightarrow K=1,2$, $n=1,2$

$N \in \mathbb{N} \setminus \{1\}$

$K \in \{1, 2, \ldots, N-1\}$

$n \in \{1, 2, \ldots, N-1\}$

} Parameter Space

$Supp[X]$:= the set of all possible values that X could be.

$$X \sim Hyper(1, K, N) := \frac{\binom{K}{x}\binom{N-K}{1-x}}{\binom{N}{n}} \overset{d}{=} Bern\left(\frac{K}{N}\right)$$

if sampling 1, ~~[illegible]~~ what could hyper? $\{0, 1\}$ → similar to which r.v.?

(0 = not get it, 1 = get it)

Why does this make sense?

$$\frac{\binom{K}{x}\binom{N-K}{1-x}}{\binom{N}{1}} = \frac{\frac{K!}{(K-x)!\,x!}\ \frac{(N-K)!}{(N-K-1+x)!\,(1-x)!}}{\frac{N!}{(N-1)!\,1!}}\Big\} N$$

if $x=0$: $\frac{\frac{K!}{K!}\ \frac{(N-K)!}{(N-K-1)!}\}N-K}{N} = \frac{N-K}{N}$

if $x=1$: $\frac{\frac{K!}{(K-1)!}\ \frac{(N-K)!}{(N-K)!}}{N} = \frac{K}{N}$

$X \sim Hyper(2, 4, 10) \Rightarrow Supp[X] = \{0, 1, 2\}$

→ $n < K$, $n < N-K$

$X \sim Hyper(n, K, N) \Rightarrow Supp(X) = \{0, \ldots, n\}$

$X \sim Hyper(5, 4, 10) \Rightarrow Supp[X] = \{0, 1, 2, 3, 4\}$

→ $n \geq K$, $n < N-K$

can't get more than actually exist!

$X \sim Hyper(n, K, N) \Rightarrow Supp(X) = \{0, \ldots, K\}$

$X \sim \text{Hypr}(6, 4, 10) \Rightarrow \text{Supp}(X) = \{2, 3, 4\}$

$\hookrightarrow n \geq K,\ n \geq N-K$

$X \sim \text{Hypr}(n, K, N) \Rightarrow \text{Supp}(X) = \{n-(N-K), \ldots, K\}$

$X \sim \text{Hypr}(5, 7, 10) \Rightarrow \text{Supp}(X) = \{2, 3, 4, 5\}$

$\hookrightarrow n < K,\ n \geq N-K$

$X \sim \text{Hypr}(n, K, N) \Rightarrow \text{Supp}(X) = \{n-(N-K), \ldots, n\}$

is X discrete? YES

	$n < K$	$n \geq K$
$n < N-K$	$\{0, \ldots, n\}$	$\{0, \ldots, K\}$
$n \geq N-K$	$\{n-(N-K), \ldots, n\}$	$\{n-(N-K), \ldots, K\}$

$\text{Supp}(X) = \{\max(0, n-(N-K)), \ldots, \min(n, K)\}$

$\sum p(x) = 1$? E.C. Hw

Equiv Parameterization: let $p := \frac{K}{N}$ i.e. the proportion of successes

$X \sim \text{Hypr}(n, p, N) := \dfrac{\binom{pN}{x}\binom{(1-p)N}{n-x}}{\binom{N}{n}}$

Param Space

$N \in \mathbb{N} \setminus \{2\}$

$n \in \{1, \ldots, N-1\}$

$p \in \{\frac{1}{N}, \frac{2}{N}, \ldots \frac{N-1}{N}\}$ → obtains non-integer values

let $N = 10$, $p = 0.5$, $n = 5$

$P(X = 3) = \dfrac{\binom{5}{3}\binom{5}{2}}{\binom{10}{5}} = .476$