MATH241 H.W#3 → Class Fri tomorrow & next week 9/24/15

10 flowers ; 3 B , 3 G , 4 R → $\frac{10!}{4!3!3!} = \binom{10}{4\,3\,3}$

→ $(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i$

→ $(a+b+c)^3 = (a+b+c)(a+b+c)(a+b+c) = \binom{3}{3,0,0} a^3 + \ldots + \binom{3}{1,1,1} abc$

$(a+b+c)^n = \sum_{i+j+k=3} \binom{n}{i,j,k} a^i b^j c^k$

→ $(a_1+a_2+\ldots a_k)^n = \sum_{\sum_{k=1}^{k} i_k = n}^{n} \binom{n}{i_1, i_2, \ldots i_k} a_1^{i_1} \cdot \ldots \cdot a_k^{i_k}$

// $P\left(\bigcup_{i=1}^{\binom{20}{2}} \{i \text{ pairs}\}\right)$

Birthday Problem - (Probability of at least one birthday is shared by a pair in 20 people)

↳ = P(one pair) + P(two pairs) + ... + P($\binom{20}{2}$ pairs)

↳ = 1 - P(^{c}) = 1 - P(no pairs)

$P(\ ) \quad \underbrace{\frac{1}{\underset{\text{Jasmine}}{365}} \quad \frac{1}{\underset{\text{Katherine}}{365}} \quad \cdots\cdots}_{20} = \frac{365 \cdot 364 \cdot \ldots \cdot 346}{365^{20}} = \frac{{}_{365}P_{20}}{365^{20}}$

$1 - \quad = .411 = 41\%$

A room with n people $P(\ ) = 1 - \frac{{}_{365}P_n}{365^n} \overset{?}{=}$

coin flip - $P(H_2 \text{ knowing } H_1) = \frac{1}{2}$ because the second flip will still be H or T → $\frac{1}{2}$.

↳ $P(H_2 \mid H_1)$

given (pipe)

→ $P(A \mid B) = P(A)$ → this means A + B ~~are~~ "independent" or "informationally irrelevant"

→

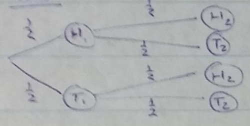
9/24/15 cont

ex. $P(\text{IBM stock} \uparrow \mid \text{raining in Buenos Aires}) = P(\text{IBM stock} \uparrow)$

ex. $P(H_2, H_1) = P(H_2) \cdot P(H_1)$

demonstration of independence

coin 1 coin 2 Joined Events



$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ H_2, H_2

H_1, T_2

T_1, H_2

T_1, T_2

General : $A_1, A_2, \ldots A_n$

$$P\left(\bigcap_{i=0}^{n} A_i\right) = \prod_{i=0}^{n} P(A_i) = P(A_1) \cdot P(A_2) \cdot P(A_3) \ldots P(A_n)$$

DeMare → $P(\geq 1 \text{ double } 6 \text{ in } 24 \text{ rolls of dice})$

$$P\left(\bigcup_{i=1}^{24} \{i \text{ double } 6\}\right) = P(1 \text{ double } 6) + P(2 \text{ double } 6) + \ldots + P(24 \text{ double } 6)$$

$$= 1 - P(\underset{\text{in 24 rolls}}{\text{zero double 6}})$$

↳ compound event → $P(\underset{\text{in 1st roll}}{\text{no double 6}} \cap \underset{\text{in 2nd roll}}{\text{no double 6}} \cap \ldots \cap \underset{\text{in 24th roll}}{\text{no double 6}})$

$$= \prod_{i=1}^{24} P(\underset{\text{in } i\text{th rolls}}{\text{no double 6}})$$

↳ $P(\underset{\text{in 1st}}{\text{no double 6}}) = P(\underset{\text{in 2nd}}{\text{no double 6}})$ why? (besides independence)

because of common sense! they are the same die.

$$= P(\text{no double } 6)^{24} = (1 - P(\text{double } 6))^{24} = \left(\frac{35}{36}\right)^{24}$$

$P(6 \cap 6) = \frac{1}{6} \cdot \frac{1}{6}$

$1 - \left(\frac{35}{36}\right)^{24} = .4914 = 49\%$

A is dependent → $P(A \mid B) \neq P(A)$, $P(B \mid A) \neq P(B)$

ex. $P(\text{A person purchases an apple} \mid \text{that person has allergy}) < P(\text{A person purchases an apple}) = 5\%$

↳ $= 1\%$

→

ex. $P(\text{Q64 on time} \mid \underbrace{\text{raining}}_{\text{dependent}}) \leftarrow < \quad P(\text{Q64 on time}) = 5\%$

↳ 1%

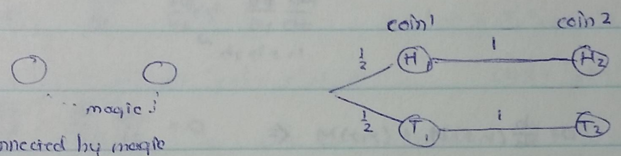
ultimate dependence

ex. A, B disjoint, are A + B independent?

Ω: [A B]

lets say you flip + get heads, what is P(T)? it's 0.

↳ $P(H_1 \mid T_1) = 0$!!

ex. 

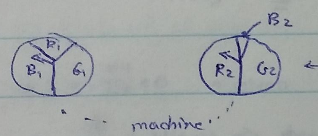
two coins connected by magic

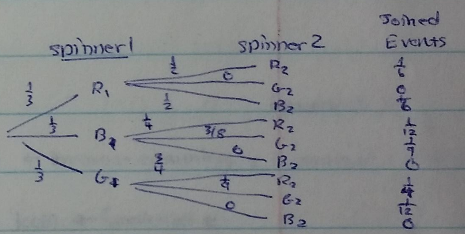
unconditional probability
↓
only care about H_2

Are they dependent? $1 = P(H_2 \mid H_1) \overset{?}{\neq} P(H_2) = \frac{1}{2}$

100%

prob of H_2 given H_1

ex. spinner1 →  ← spinner 2



R_1, R_2 dependent? → then

$\overset{?}{=} P(R_1, R_2) \overset{?}{\neq} P(R_1)P(R_2) \overset{?}{=}$

$\frac{1}{6}\ P(R_1, R_2) \overset{?}{=} P(R_1)P(R_2) = \frac{1}{3}\left(\underbrace{\frac{1}{6} + \frac{1}{12} + \frac{1}{4}}_{\frac{1}{2}}\right) = \frac{1}{6}$

↓ they are equal ← ↳ (can get R_2 a bunch of ways)

↳ they are independent

↳ $0 = P(R_1, G_2) \overset{?}{\neq} P(R_1)P(G_2) = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9}$

↳ these are dependent.