

5 flower's 5 flower pots

3 B, 2 R

If the blue and red indistinct  $\frac{3!}{1!1!} = 10$

$$P(RBRBB) = \frac{1}{10} = \frac{2 \cdot 3 \cdot 1 \cdot 2 \cdot 1}{5!} = \frac{3!2!}{5!} = \frac{1}{10}$$

$$\text{Other way } = \frac{1}{10}$$

$$|A| = n$$

$$2^A = \left\{ B : B \subseteq A \right\} |K^A| = 2^{|A|} = 2^n.$$

$$2^A = \left\{ B : B \subseteq A, |B|=0 \right\} \cup \left\{ B : B \subseteq A, |B|=1 \right\} \cup \dots \cup \left\{ B : B \subseteq A, |B|=n \right\}$$

$$= \bigcup_{i=0}^n \left\{ B : B \subseteq A, |B|=i \right\}$$

(gt can't share any element because all set size different)

$$|A_1 \cup A_2| = |A_1| + |A_2| \text{ if } A_1, A_2 \text{ max}$$

$$|2^A| = \sum_{i=0}^n \left| \left\{ B : B \subseteq A, |B|=i \right\} \right| = \sum_{i=0}^n (i) = 2^n$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a+b)^3 = (a+b)(a+b)(a+b) = a^3 + 3a^2b + 3ab^2 + b^3$$

$$2 \cdot 2 \cdot 2 = 8$$

$$(a+b)^4 = (a+b)(a+b)(a+b)(a+b) = \binom{4}{0} a^4 b^0 + \binom{4}{1} a^3 b^1 + \binom{4}{2} a^2 b^2 + \binom{4}{3} a^1 b^3 + \binom{4}{4} a^0 b^4$$

$$(a+b)^n = \sum_{i=0}^n \binom{n}{i} a^i b^{n-i} = \sum_{i=0}^n \binom{n}{i} a^{n-i} b^i$$

Binomial theorem.

$$\bullet (1+x)^n = \sum_{i=0}^n \binom{n}{i} 1^{n-i} x^i = \sum_{i=0}^n \binom{n}{i} x^i = 1 + \sum_{i=1}^n \binom{n}{i} x^i$$

$$\bullet (1+x)(1+x)^{n-1} = (1+x) \sum_{i=0}^{n-1} \binom{n-1}{i} x^i = \sum_{i=0}^{n-1} \binom{n-1}{i} x^i + 1$$

$$\begin{aligned} & \text{Left: } i = i+1 \quad \sum_{i=1}^n \binom{n-1}{i-1} x^i \\ & i = j-1 \quad \sum_{i=1}^{n-1} \binom{n-1}{i-1} x^i + \sum_{i=1}^{n-1} \binom{n-1}{i-1} x^i + x^n \\ & = i + \sum_{i=1}^{n-1} \binom{n-1}{i-1} x^i + \sum_{i=1}^{n-1} \binom{n-1}{i-1} x^i + x^n \\ & = \sum_{i=1}^{n-1} \binom{n}{i} x^i = \sum_{i=1}^{n-1} \left( \binom{n-1}{i} + \binom{n-1}{i-1} \right) x^i \end{aligned}$$

$$\Rightarrow \boxed{\binom{n}{i} = \binom{n-1}{i} + \binom{n-1}{i-1}}$$

(0)

$$\begin{array}{ccc} \binom{1}{0} & \binom{1}{1} \\ \binom{2}{0} & \binom{2}{1} & \binom{2}{2} \\ \binom{3}{0} & \binom{3}{1} & \binom{3}{2} & \binom{3}{3} \end{array}$$

$$\begin{array}{cccc} & & 1 & \\ & 1 & 2 & 1 \\ 1 & 3 & 3 & 1 \\ 1 & 4 & 6 & 4 & 1 \end{array} \quad \text{Pascal's law}$$

5 cards of 52 order does not matter How many?

$$P(\text{Royal flush}) = \frac{\binom{4}{1} \binom{1}{1}}{\binom{52}{5}} = 2598960$$

10, J, Q, K, A

same suits

$$P(\text{Straight flush}) = \frac{\binom{9}{1} \binom{10}{1}}{\binom{52}{5}}$$

All same suit

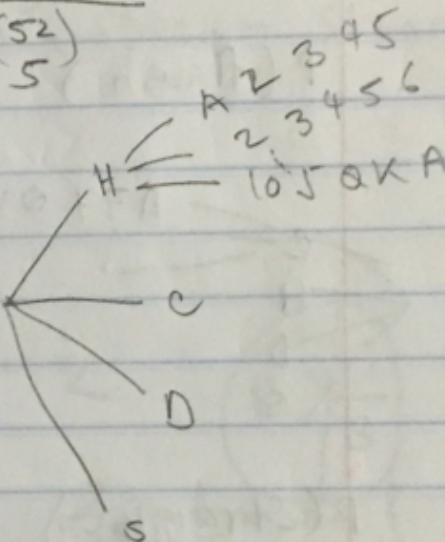
A, 2 3 4 5,  
2 3 4 5 6

6 7 8 9 10

7 8 9 10 J

8 9 10 J R

9 10 J R K



10 J R K A  
of a kind      odd card leftover.

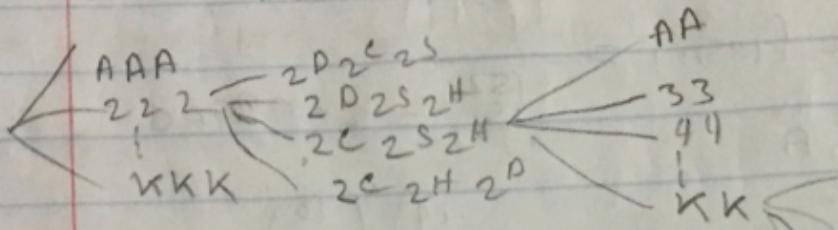
$$P(4\text{ of a kind}) = \frac{\binom{13}{1} \binom{4}{1} \binom{12}{1} \binom{4}{1}}{\binom{52}{5}}$$

2 2 2 2 K  
A A A A  
2 2 2 2

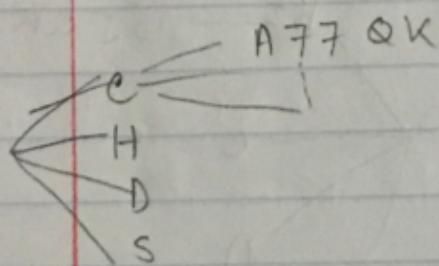
K K K K

$$P(\text{full house}) = \frac{\binom{13}{1} \binom{4}{3} \binom{12}{1} \binom{4}{2}}{\binom{52}{5}}$$

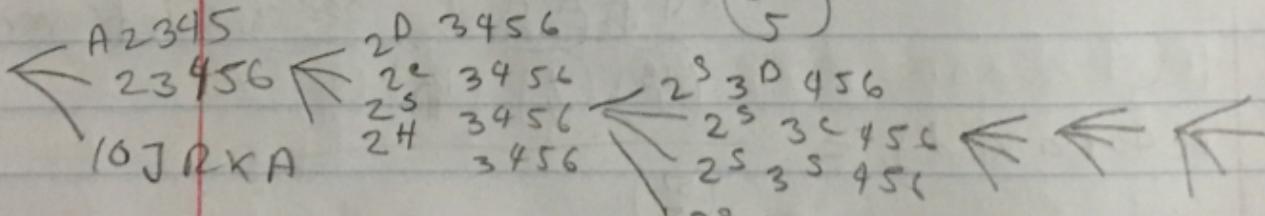
QQQ77



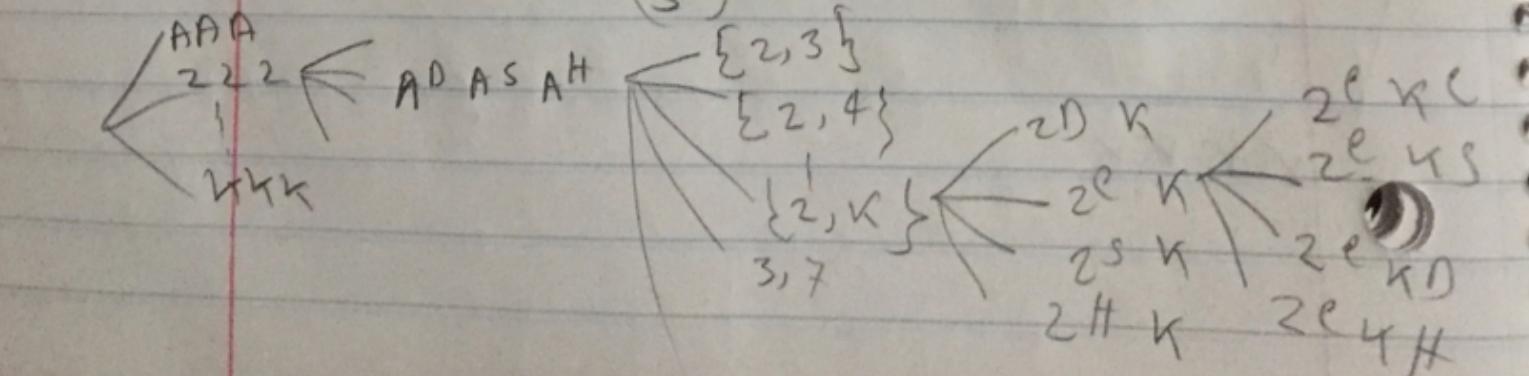
$$P(\text{flush}) = \frac{\binom{4}{1} \binom{13}{5}}{\binom{52}{5}}$$



$$P(\text{straight}) = \frac{\binom{10}{1} \binom{4}{1}^5}{\binom{52}{5}}$$



$$P(\text{trips}) = \frac{\binom{13}{1} \binom{4}{3} \binom{12}{2} \binom{4}{1}^2}{\binom{52}{5}}$$



$$P(\text{2 pair}) = \frac{\binom{13}{2} \binom{4}{2} \binom{11}{2} \binom{4}{2}}{\binom{52}{5}}$$

77 KK 4  
AA 22  
AA KK

22 33

22 KK

33 KK

$$P(\text{no Pair}) = \frac{\binom{13}{5} \binom{4}{1}^5 - \text{flash} - \text{straights}}{\binom{52}{5}}$$

$$\Omega = \{H, T\}$$

$$P(\{H\}) = \frac{|\{H\}|}{|\Omega|} = \frac{1}{2}$$

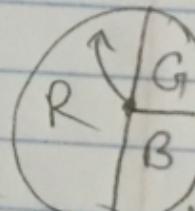
$$P(A) = \frac{|A|}{|\Omega|}$$

$$P(\{R\}) = \frac{|\{R\}|}{|\Omega|} = \frac{1}{3}$$

$$\forall E \in \Omega \quad P(E) = \frac{1}{|\Omega|}$$

$$\Omega = \{ \text{sunny, cloudy, dry} \}$$

$$P(E \approx 3) \neq \frac{1}{3}$$



$\Omega = \{R, G, B\}$   
All outcomes equally likely.

$\mathbb{1}_{\omega \in A} := \begin{cases} 1 & \omega \in A \\ 0 & \text{otherwise} \end{cases}$  indicator function  
proposition of the event A occurs

$P(A) := \lim_{n \rightarrow \infty}$

$$\sum_{i=1}^n \frac{\mathbb{1}_{\omega_i \in A}}{n}$$

long run frequency (lrf)