$P(A \cup B \cup C) = P(A) + P(B) + P(C)$

$= P(AB) - P(AC) - P(BC) + P(ABC)$

$P(\bigcup_{i=1}^{n} A_i) = \sum P(A_i) - \sum_{i \neq j} P(A_i \cap A_j) + - + \cdots -^{n+1} P(\bigcap_{i=1}^{n} A_i)$

If $|\Omega| < \infty$ and $\forall w \in \Omega$, $P(\{w\}) = \frac{1}{|\Omega|}$

$\Rightarrow P(A) = \frac{|A|}{|\Omega|}$

$\Omega = \{w_1, w_2 \cdots w_n\}$ $\quad A = \{\alpha_1, \alpha_2 \cdots \alpha_n\}$ s.t. $|A| = n$.

$A = \emptyset \Rightarrow P(A) = 0 = \frac{|\emptyset|}{|\Omega|} = \frac{0}{|\Omega|} = 0$

$A = \Omega \quad P(\Omega) = \frac{|\Omega|}{|\Omega|} = \frac{|A|}{|\Omega|}$

$A = \underbrace{\{\alpha_1\}}_{A_1} \cup \underbrace{\{\alpha_2\}}_{A_2} \cup \cdots \cup \underbrace{\{\alpha_n\}}_{A_n}$ disjoin

$P(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i) = \sum_{i=1}^{n} \frac{1}{|\Omega|} = \frac{n}{|\Omega|} = \frac{|A|}{|\Omega|}$

9/24 Lec 6

10 flower

4 Red $\quad$ 3 Blue $\quad$ 3 Green

$\frac{10!}{4!3!3!} = \binom{10}{4}\binom{10}{3}\binom{10}{3}$

$= \binom{10}{4\,3\,3}$

$(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i$

$(a+b+c)^3 = (a+b+c)(a+b+c)(a+b+c) = \binom{3}{3,0,0} a^3 + \binom{3}{2,1,0} a^2 b + \cdots \binom{3}{1,1,1} abc$

$= \sum^{3} \binom{3}{i,j,k} a^i b^j c^k$

$(a_1 + a_2 + \cdots a_k)^n = \sum_{\sum_{k=1} i_k = n} \binom{n}{i_1, i_2 \cdots i_k} a^{i_1} a^{i_2} \cdots a_k^{i_k}$ multimomial expansion

$P\left(\begin{matrix}\text{at least one b-day}\\ \text{pair in the room}\end{matrix}\right) = P(\{\text{one pair}\} \cup \{\text{two pair}\} \cup \cdots \cup \{\binom{18}{2} \text{ pairs}\})$

$= P(\text{one pair}) + P(\text{two pair}) + \cdots P(\binom{18}{2} \text{ pairs})$

$= 1 - P(C^{c})$ complement

$= 1 - P(\text{no birday share})$

$= 0.346$

$$P(\text{no b-day share}) = \frac{\frac{365}{A} \cdot \frac{364}{B} \cdot \frac{363}{} \cdots 348}{\underbrace{\frac{365}{A} \cdot \frac{365}{B} \cdots \frac{365}{}}_{18}} = \frac{{}_{365}P_{18}}{365^{18}} = 0.653$$

n People.

$$P\left(\geq \begin{matrix}1\text{ shared}\\ \text{b-day}\end{matrix}\right) = 1 - \frac{{}_{365}P_n}{365^n} \quad = 0.5 \quad n \approx 23$$
$$= 0.99 \quad n \approx 60$$

$P(H_2$ ~~knowing the first~~ $H_1) = P(H_2) = \frac{1}{2}$

being given

being conditional on ....

"given" "pipe"

$P(A|B) = P(A)$ A, B informationally ~~independent~~ irrelevant or independent.

conditional probability.

$P($IBM ~~stock~~ $\uparrow$ | Raining in Brense Ales)

$= P($IBM stock$\uparrow)$

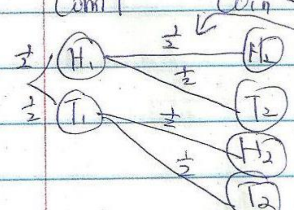
Def. on independent. $P(B|A) = P(B)$

$P(H_2|H_1) = \frac{1}{2}$

$\frac{1}{4} = P(H_2 \cap H_1) = \cancel{P(H_2) + P(H_1)}\ P(H_2)P(H_2) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$

$P(A, B) = P(A)P(B)$ if A B independent.

Coin 1 Coin 2



$P(H_1, H_2) = \frac{1}{4}$

$P(H_1, T_2) = \frac{1}{4}$

$P(H_2 | H_1)$

if A_i, A_j independent $\forall i,j$.

$P(A_1, A_2 \ldots A_i)$ multiplication Rule for independent.

$$= P\left(\bigcap_{i=1}^{n} A_i\right) = \prod_{i=1}^{n} P(A_i)$$

$P(\geq 1 \text{ double } 6 \text{ in } 24 \text{ rolle})$

$= P(\text{one double } 6 \cup \text{two double } 6\text{'s} \dots \cup 24 \text{ double } 6\text{'s})$

$= P(\text{one double } 6) + P(\text{two double } 6) \dots + P(24 \text{ double } 6\text{'s})$

$= 1 - P(A^C)$ ← no double six's in 24 Roll

$= 1 - P(\text{no double } 6\text{'s in } 24 \text{ Roll}) = 0.4914.$

$P(\text{no double } 6 \text{ first Roll} \cap \text{no double six second Roll} \dots \cap \text{no double } 6 \ 24^{th} \text{ Roll})$

$= \prod_{i=1}^{24} (P \text{ no } 6\text{-}6 \text{ on } i \text{ roll}) = P(\text{no double } 6)^{24}$

$= (1 - P(\text{double } 6))^{24} = \left(1 - \frac{1}{36}\right)^{24} = \left(\frac{35}{36}\right)^{24}$

$P(6 \cap 6) = P(6)P(6) = \frac{1}{36}$

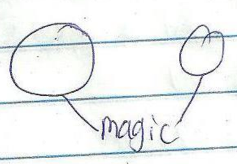
Dependence.

A, B dependence if $P(A|B) \neq P(A)$ and $P(B|A) \neq P(B)$

$1\% = P(\text{Buying apple} \mid \text{the person is allergic}) \overset{?}{<} P(\text{Buying apple}) - 15\%$ unconditional prob.

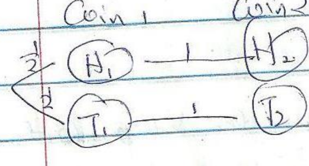
$15\% = P(Q64 \text{ on time} \mid \text{good weather}) > P(Q64 \text{ on time}) = 10\%$

A, B disjoint are A, B independent? (No) $0 = P(A|B) \overset{?}{=} P(A) = 0.2$



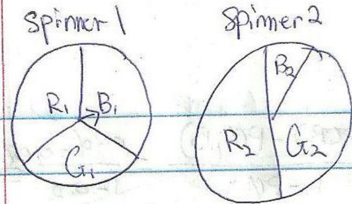
H_1 head and H_2 head

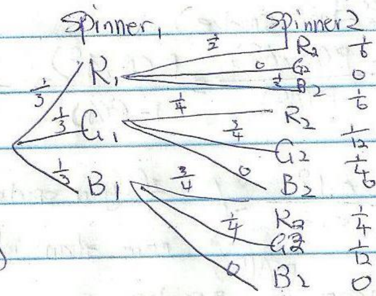
T_1 Tail T_2 Tail too



$1 = P(H_2|H_1) \neq P(H_2) = \frac{1}{2}$

Spinner 1 Spinner 2





$P(R_1 R_2)$

$= P(R_1)P(R_2)$

$\frac{1}{6} = \frac{1}{3}\left(\frac{1}{6}+\frac{1}{12}+\frac{1}{4}\right) = \frac{1}{6}$

$R_1\ R_2$ is independent.

$P(R_1 G_2) = P(R_1)P(G_2)$

$0 \neq \frac{1}{3}\left(0+\frac{3}{4}+\frac{1}{12}\right) = \frac{5}{18}$

$R_1\ G_2$ is dependent

9/25

Lec 7

$P(A) = \frac{4}{52} = \frac{1}{13}$ $\Rightarrow$ A, ♡ independent

$P(A|♡) = \frac{1}{13} = P(A)$

$n = 1000$

200 ~~student~~ smoker (A)

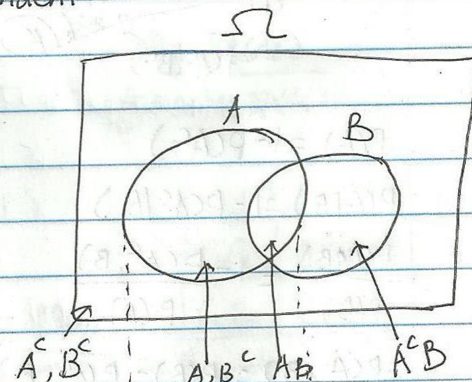
60 lung cancer (B)

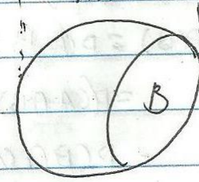
36 smoker & lung cancer (A, B)

$P(A) = 0.2$

$P(B) = 0.06$

$P(A,B) = 0.036$





P(lung cancer | smoker)

$P(B|A)$

$P(B|A) \propto P(B,A)$

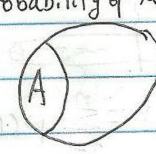
$P(B|A) = \cancel{\propto} P(B,A) = \frac{P(\Omega)}{P(A)}$

~~Zoom~~

$\frac{0.036}{0.2} = 0.18 \leftarrow P(B|A) = \frac{P(B,A)}{P(A)}$ ~~Bayes Rule~~ Bayes's Basic Rule

$P(B,A) = P(B|A)P(A)$

$P(A|B) = \frac{P(A,B)}{P(B)} = \frac{0.36}{0.06} = 0.6$ Probability of lung cancer giving smoker



$$\boxed{\begin{aligned} P(A,B) &= P(A|B)P(B) \\ P(B|A) &= \frac{P(A|B)P(B)}{P(A)} \end{aligned}}$$

double Bayes's ~~basic~~ Rule.