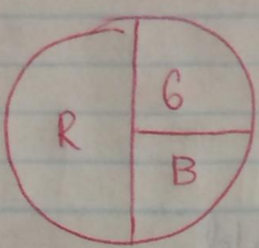


$P(R) \neq \frac{1}{3}$

$\forall w \in \Omega$

$P(\{w\}) \neq \frac{1}{|\Omega|}$

$P(A) = \frac{|A|}{|\Omega|}$

(I) Long Run Frequency L.r.f (1600's)

$P(A) := \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \mathbb{1}_{w_i \in A}$

lacking in that definition

$\lim_{x \to \infty} \sin(x)$ does not exist

Problems

1. $n \neq \infty$. You can never compute $P(A)$
2. How does this limi exist?
3. Repeatable conditions (stationarity) who says the coin doesn't deapear
4. Not general

that my favorite color is purple / that someone is guilty or not

coins
cards
dice
→ same

chevalier de mere.

$P\left(\begin{array}{l}\geq 1 \ \ 6\text{-}6 \\ \text{on two dice in} \\ 24 \text{ rolls}\end{array}\right) < \frac{1}{2}$ .4914

little bit less than half the time.

L.R.F - "Objective" property of the physical World

L.R.F → probability

II Propensity theory (1950's)

Objects have inherent probabilities

Prob ⟶ L.r.f

"Objective" - property of the physical world.

Problems

① No computable base on believe

② No general

III Logical theory (late1800's)

Given same evidence, all people will admite to same probability.

- Silly

IV Subjective (1950's)

Given same evidence
but difference of opinion is allowed.

Weakness - No definition of probability if all have different opinions

I, II ⇒ Objective

III, IV ⇒ epistemic
Consider/deal with human knowledge

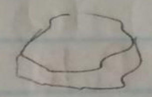
$= (0,1)$ ← epistemic ⟶ Represents "degree of belief"
btw 0 or 1
"degree of corraboration"
Ask all sorts of questions
"ignorant"

$P\left(\begin{matrix}\text{OJ Simpson} \\ \text{guilty}\end{matrix}\right)$

$= \{0,1\}$ Objectivist
did or didnot
Very limited

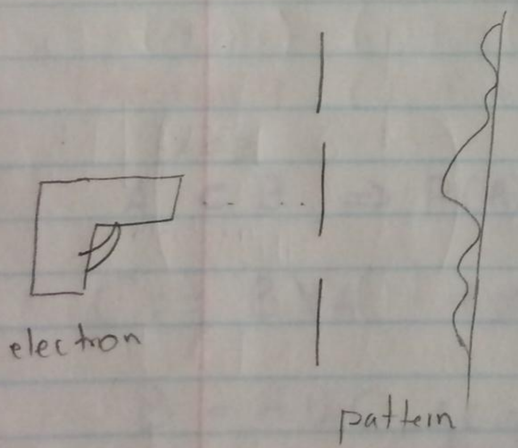
$P(F = ma) =$

→ Ufair die - Flip it over & over Can't see any pattern.



$P(H) = \frac{1}{2}$ → objective
if I knew all about my finger, air etc
I would know for sure that is heads.

Determinism - probability doesn't exists if we knew everything.



Randomness is inherent in the universe

* Assume Ω non-empty.

"P" (Probability) is a set function with domain $A \subseteq \Omega$
Such that:

a) $P(A) \geq 0 \quad \forall A \subseteq \Omega$

b) $P(\Omega) = 1$

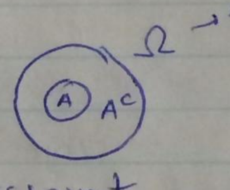
countable additing

c) Given $A_1, A_2, \ldots$ disjoint $\Rightarrow P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$

Theorem I Complement Rule

$P(A) = 1 - P(A^c)$

$A \subseteq \Omega$



$A \cup A^c = \Omega$. A, A^c are disjoint - Set theory

$P(A \cup A^c) = P(\Omega)$ def. of function

$P(A \cup A^c) = 1$ by (b)

$P(A) + P(A^c) = 1$ by (c) sums.

$P(A) = 1 - P(A^c)$

Theorem 2

$P(\emptyset) = 0$ only way to happen is if it's empty -

$P(\emptyset) = 1 - P(\emptyset^c)$ thm 1

$P(\emptyset) = 1 - P(\Omega)$ "set theory" $\emptyset^c = \Omega$

$P(\emptyset) = 1 - 1$ (b)

$= 0$

Theorem 3

If A is non-empty then $P(A) > 0$ $p \to q \Leftrightarrow \sim q \to \sim p$

$A \neq \emptyset \Rightarrow P(A) > 0$

$\Leftrightarrow P(A) \leq 0 \Rightarrow A = \emptyset$

$P(A) \leq 0$ AND $P(A) \geq 0$

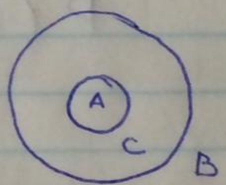
$\Rightarrow P(A) = 0 \Rightarrow A = \emptyset$

$A \neq 0 \Rightarrow P(A) \neq 0$ Must be > 0

Theorem 4.

$A \neq B$

$A \subset B \Rightarrow P(A) < P(B)$



$C := B \setminus A \neq \emptyset$ set theo

$B = A \cup C$, A, C is disjoint Set theory

$P(B) = P(A \cup C)$

$P(B) = P(A) + P(C)$ (c)

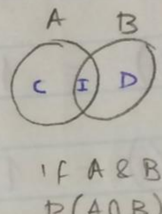
$P(B) - P(A) = P(C) > 0$ Thr 3

$P(B) > P(A)$

Thm 5.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

getting rid of double-counting



if A & B are disjoint, then
$P(A \cap B) = 0$

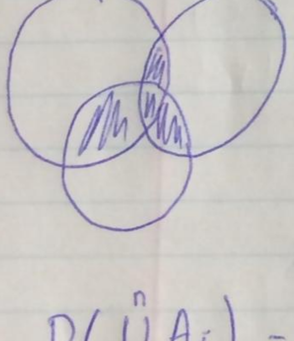
$I := A \cap B$
$C := A \setminus B$
$D := B \setminus A$

$A = C \cup I$, C; I disjoint
$B = D \cup I$, D, I disjoint

$$P(A \cup B) = P(C \cup I \cup D) = P(C) + P(I) + P(D) \quad (C)$$

$$P(A) = P(C) + P(I) \Rightarrow P(C) = P(A) - P(I)$$
$$P(B) = P(D) + P(I) \Rightarrow P(D) = P(B) - P(I)$$

$$= P(A) - \cancel{P(I)} + \cancel{P(I)} + P(B) - P(I)$$
$$\Rightarrow P(A) + P(B) - P(A \cap B)$$



$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A,B) - P(B,C) - P(A,C) - P(A,B,C)$$

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) - \sum_{i \neq j} P(A_i \cap A_j) + \dots + (1)^{n-1} P\left(\bigcap_{i=1}^{n} A_i\right)$$

Theorem 6

If $|\Omega| < \infty$

and $\forall \omega \in \Omega \quad P(\{\omega\}) = \frac{1}{|\Omega|} \Rightarrow P(A) = \frac{|A|}{|\Omega|}$

$A = \emptyset \Rightarrow P(A) = \frac{|\emptyset|}{|\Omega|} = \frac{0}{|\Omega|} = 0$ ✓

$A = \Omega \Rightarrow P(A) = \frac{|\Omega|}{|\Omega|} = 1$ ✓

$\Omega = \{\omega_1 \dots \omega_n\}$

$A \subset \Omega \Rightarrow A = \{r_1, \dots, r_n\}$

$A = A_1 \cup A_2 \cup A_3 \dots \cup \dots \cup A_n$ all disjoint

$A_1 = \{r_1\}$, $A_2 = \{r_2\}$, $A_n = \{r_n\}$

$P(A) = P\left(\bigcup_{i=1}^{n} A_i\right)$

$P(A) = \sum_{i=1}^{n} P(A_i) = \sum_{i=1}^{n} \frac{1}{|\Omega|}$

$= \frac{n}{|\Omega|} = \frac{|A|}{|\Omega|}$

$P(2,3,6) = \frac{3}{6}$

* 10 flowers

4R, 3B, 3G

10! to put 10 flowers in 10 pots.

$$\frac{10!}{3!\,4!\,3!} \rightarrow \text{indistinct}$$

$$\frac{10!}{3!\,4!\,3!} : \binom{10}{4,3,3}$$ Multi Choose Notation

$$\binom{n}{k_1 \ldots k_n} := \frac{n!}{k_1! \ldots k_n!}$$

$\underline{R}$ $\underline{\bar{R}}$ $\underline{\bar{R}}$ $\underline{R}$ $\underline{\bar{R}}$ $\underline{\bar{R}}$ $\underline{R}$ $\underline{\bar{R}}$ $\underline{\bar{R}}$ $\underline{\bar{R}}$ reds and nonreds

Equal

$$\frac{10!}{3!\,7!} = \binom{10}{4}\binom{6}{3} = \frac{10!}{3!\,\cancel{6}!} \quad \frac{\cancel{6}!}{3!\,4!}$$

$\underline{G}$ $\underline{B}$ $\underline{B}$ $\underline{G}$ $\underline{B}$ $\underline{G}$ 3 Green