Special set Ω

universal sample space $\quad \forall A \quad A \subseteq \Omega$

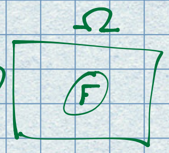
Space of discourse $\quad Joe \in \Omega$

$\Omega = F \cup M \qquad \forall w \quad w \in \Omega$

little omega

$F \cap \Omega = F$

$F \cup \Omega = \Omega$



Select a name at random

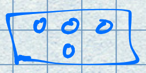
$P(F) = \frac{|F|}{|\Omega|}$ ← size of set we care about ← size of universe

$A_1 \dots A_n$ and $\bigcup_{i=1}^{n} A_i = \Omega$

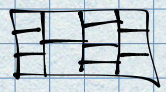
$A_1 \cup A_2 \cup \dots A_n$

$\sum_{i=1}^{\infty} i \Rightarrow A_1 \dots A_n$ called "collectively exhausted

$A_1 \dots A_n \quad A_i \cap A_j = \emptyset \quad \forall i \neq j \Rightarrow A_1 \dots A_n$ are disjoint "mutually exclusive"



mutually exclusive & coll. exhausted.



for finite set $|\Omega| = |A_1| + |A_2| + \dots = \sum_{i=1}^{n} |A_i|$

Set operator compliment

$A^c = \Omega / A$

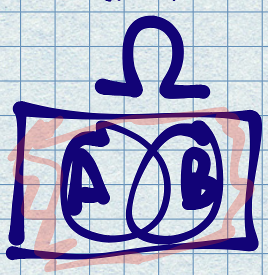
$F^c = \{Bob, Joe, Max\}$ based on Lecture 1 notes

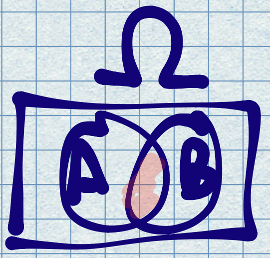
$F \cup F^c = \Omega$
$F \cap F^c = \emptyset$

disjoint, mutually exclusive, collectively exhausted

Consider A, B

$(A \cup B)^c$





$$(A \cap B)^c = A^c \cap B^c$$

De Morgans Law

$\mathbb{N} := \{1, 2, \ldots\}$

$\mathbb{Q} := \{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{N}\}$

$\mathbb{Z} := \{\ldots, -1, 0, 1, \ldots\}$

$\sqrt{2} \notin \mathbb{Q}$

$* \; \frac{p}{q} = \sqrt{2}$

p and q have no common factors

$\longleftrightarrow \mathbb{Q}$ (0)

$\cancel{\$}\,6\,0\,0\,\cancel{0}$

$\frac{p^2}{q^2} = 2$ $\quad p = p_1 \cdots p_k$

$q = q_1 \cdots q_n$

$2 \in \{p_1, \ldots p_k\} \leftarrow \frac{p_1^2 \cdots p_k^2}{2} = \frac{p^2}{2} = q^2 \in \mathbb{N}$

$p_1^2 \cdots p_k^2;\ 2 = q_1^2 \cdots q_r^2$

$\in \mathbb{Z} \quad p_1^2 \cdots p_{k-1}^2 = \frac{q_1^2 \cdots q_r^2}{2}$

$\mathbb{R} := \mathbb{Q} \cup \{\text{all holes in } \mathbb{Q}\}$ 1870's

$|\mathbb{N}| = \aleph_0$ countable "countable infinite

$|\mathbb{Z}| = \aleph_0$ countable ∞

$|\mathbb{Q}| = \aleph_0$ countable ∞

$|\mathbb{R}| = \underset{\text{uncountable } \infty}{c} > \aleph_0$

$[1,2] := \{x : x \geq 1 \;\&\; x \leq 2\}$

$(1,2) := \{x : x > 1 \;\&\; x < 2\}$

$[a,b]$ set $a < b := \{x : x \geq a \;\&\; x \leq b\}$

(a,b) set $a < b := \{x : x > a \;\&\; x < b\}$

$(-\infty, \infty) = \mathbb{R}$

Set operator : Cartesian Product

ordered pair $\langle a, b \rangle \neq \{a, b\}$

$A \times B := \{\langle a, b\rangle : a \in A, b \in B$

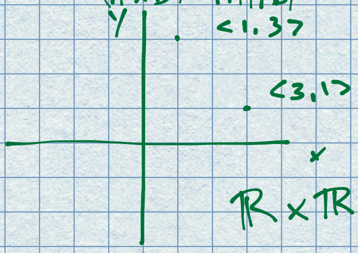
$\{1,2\} \times \{3,4\}$

$= \{\langle 1,3\rangle, \langle 2,4\rangle, \langle 2,3\rangle, \langle 2,4\rangle, \langle 1,4\rangle\}$

$A^2 := A \times A$

$A^n = A \times A \times \ldots \times A.$

$|A \times B| = |A||B|$



$\mathbb{R} \times \mathbb{R}$

Descartes

cartesian plane

Elements are now o/comes of experiments

$\Omega = \{H, T\}$

$\omega_1 \quad \omega_2$

$\{H\}, \{T\}$ are mutually exclusive & col. exhausted? Yes

$2^{\Omega} = \{\emptyset, \{H\}, \{T\}, \{H,T\}\}$

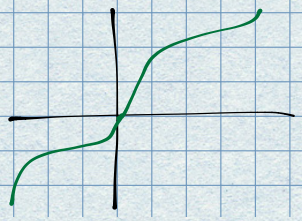
Element of the 2^{Ω} Are called events

$$P(\{H\}) = \frac{|\{H\}|}{|\Omega|} = \frac{1}{2}$$

$\cancel{P(H)}$ → √

$f(x) = x^3$

$f : \mathbb{R} \to \mathbb{R}$



$P : 2^{\Omega} \to [0,1]$

$P(A)$

$P(\emptyset) = \frac{|\emptyset|}{|\Omega|} \quad \frac{0}{2} = 0$

$P(\{H\}) = 1/2$

$P(\{T\}) = \frac{|\{T\}|}{|\Omega|} = 1/2$

$P(\{H, T\}) = \frac{|\{H,T\}|}{|\Omega|} = \frac{2}{2} = 1$

$\Omega^2 = \Omega \times \Omega$

$\langle H,H\rangle$	$\langle H,T\rangle$
$\langle T,T\rangle$	$\langle T,H\rangle$

$P(H,H) = \frac{|\{H,H\}|}{|\Omega^2|} = \frac{1}{4}$

Let A be the event of atleast one H

$P(A) = \frac{|C|}{|\Omega 2|} = \frac{3}{4} \qquad = \{HH, HT, TH\}$

Ω^4

HHHH HHTH

$P(HTHT) > P(HHHH)$

$P(HTHTHT) > P(H \dots H)$

$\frac{\{HTHT\}}{|\Omega^4)} = \frac{1}{16} = \frac{\{HHHH\}}{16}$

$A := \{2H \; 2T\}$

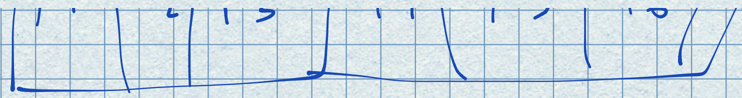
$P \left(\begin{Bmatrix} HHTT, HTHT, TTHH \\ HTTH, THHT, THTH \end{Bmatrix} \right) \frac{6}{16} \Rightarrow \frac{1}{16}$

Flip a coin & roll a die

Ω

H1	H2	H3	H4	H5	H6
T1	2	T3	T4	T5	T6

$\frac{1}{2} \cdot \frac{3}{6} = \frac{3}{12}$



Jane, Mary, Susan

$\{J, M, S\}$ seat them in front where chairs are distint

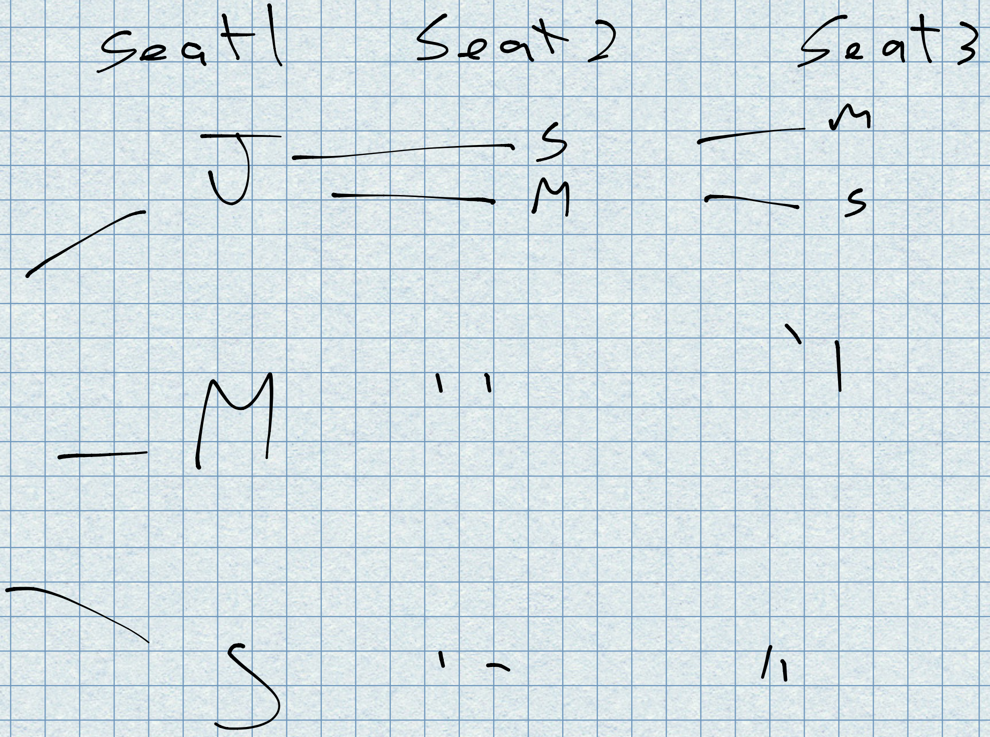
$\Omega = \{J, M, S\}$

$|\Omega^3| = 27$

$P(\langle J, M, S\rangle) = \frac{|\langle J, M, S\rangle|}{|\Omega|} = \frac{1}{6}$

sampling w/o replacement

$\frac{3}{\text{seat1}} \; \frac{2}{\text{seat2}} \; \frac{1}{\text{seat1}} = 6$



Permutations "order matters".

n people, n seats how many permutatio?

$\underset{\text{seat1}}{\underline{n}}$ $\underset{\text{seat2}}{\underline{n-1}}$ ... ... $\underset{\text{seat n}}{\underline{1}} = n! := \prod_{i=1}^{n} i$

n - factorial

$10! = 3,600,00$
$20! = 2.7 \times 10^{23}$

= diam. universe in ft.

10 ppl 3 seats = $\underset{\#1}{\underline{10}} \cdot \underset{\#2}{\underline{9}} \cdot \underset{\#3}{\underline{8}} = \frac{10!}{7!} = \frac{10!}{(10-3)!}$

How many perm. of n objects k positions?

$\frac{n!}{(n-k)!}$ $\quad {}_3P_3 = \frac{3!}{(3-3)!} = \frac{6}{0!} = 6 \quad 0! := 1$