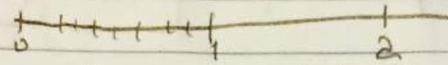
$T \sim \text{Geom}(p)$: $(1-p)^{t-1}p$, $F(t)=1-(1-p)^t$
$E(T)=\frac{1}{p}$ exp

what can't be? $(0,\infty)$



within each second n, in expression

$p(t)=(1-p)^{nt-1}p$, $F(t)=1-(1-p)^{nt}$

$=\frac{1}{p}\,\cancel{\text{exp}}\cdot\frac{1}{n}\frac{\text{sec}}{\cancel{\text{exp}}}$

$E(T)=\frac{1}{pn}(\text{sec})$

$\Rightarrow p=\frac{\lambda}{n}$

$\lambda = pn$

small big

$\lim_{n\to\infty} p(t) = \left(1-\frac{\lambda}{n}\right)^{nt-1}\frac{\lambda}{n} = \left(\lim_{n\to 0}\left(1-\frac{\lambda}{n}\right)^n\right)^t$

$\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{-1}$ $\lim_{n\to\infty}\frac{\lambda}{n}=0$

$\sum p^*(t)=0\neq 1$

not PMF

$F^*(t)=\lim_{n\to\infty} 1-\left(1-\frac{\lambda}{n}\right)^{nt}$ e: $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$

$=1-\left(\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^n\right)^t = 1-e^{\lambda t}$

yes, it is a CDF

$$e = \sum_{i=0}^{\infty} \frac{1}{i!}$$

$$\int_1^{e} \frac{1}{x}\,dx = 1$$

n	$f(n)$
10	2.517
100	2.705
1000	2.717
10000	2.718

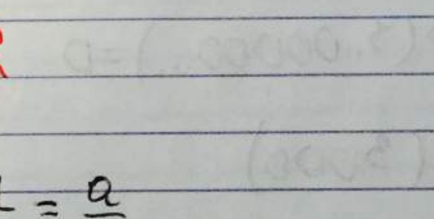
$$\left(\lim_{n\to\infty}\left(1+\frac{1}{m}\right)^{m}\right)^{a} = e^{a}$$

$$\lim_{n\to\infty}\left(1+\frac{a}{n}\right)^{n} = e^{a} \quad a \in \mathbb{R}$$

Let $m = \frac{n}{a} \Rightarrow n = ma \Rightarrow \frac{1}{m} = \frac{a}{n}$

- $F(0) = 1 - e^{-\lambda(0)} = 1 - 1 = 0$
- $\lim_{t\to\infty} F(t) = 1 - \lim_{t\to\infty} \frac{1}{(e^{\lambda})^{t}} = 1 - 0 = 1$

$F'(t) = \lambda e^{-\lambda t} \cdot \frac{\lambda}{e^{\lambda t}} > 0$ monotonically increasing



$$\begin{cases} p \in (0,1) \\ n \in \mathbb{N} \end{cases}$$

$\lambda > 0$

$|supp(X)| \leq |\mathbb{N}|$

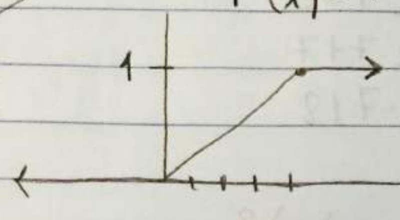
$|Supp(T)| = |\hat{\mathbb{R}}|$

they do not have PMF's

$|(0,\infty)| > |\{1,2,3,\dots,\}|$

(Def of ontomous r.v.)

$F(x) = P(X \leq x)$



$p^*(t) = 0$

$P(3) = 0$

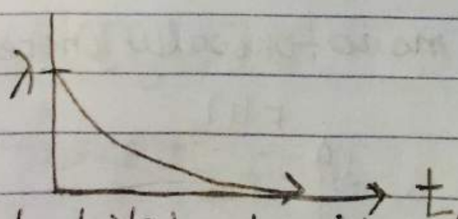
$P(3.00000\ldots) = 0$

$P(3000)$

$P(T \leftarrow [2.9995, 3.0004])$

$= F(3.0004) - F(2.9995) > 0$

$f(t) = F'(t) = \lambda e^{-\lambda t}$



Abstract metric
$\lambda = 1$

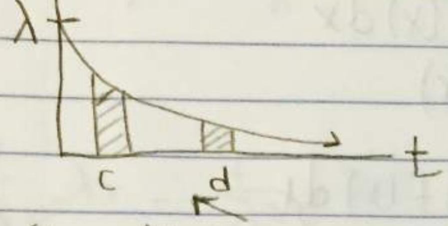
probability density function (PDF)

$f(3) = 0.05 \neq p(3) = 0$

$$P(T\in(a,b)) = F(b) - F(a) = \int_a^b f(t)\,dt$$

fundamental theorem of calculus

$$\frac{f(c)}{f(d)}$$



(how likely c happen versus d)

* Def. X is a continuous r.v if

1. CDF exist
2. PMF does not exist
3. $Supp[X] \subseteq \mathbb{R}$
4. $f(x)$ exists and
5. $|supp(x)| = |\mathbb{R}|$

$$P(T\in(-\infty\ \infty)) = \lim_{t\to\infty} F(t) - \lim_{t\to-\infty} F(t) = \boxed{\int_{-\infty}^{\infty} f(t)\,dt = 1}$$

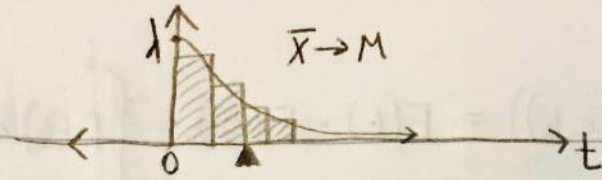
$$X \overset{d}{=} Y$$

if $F_X(x) = F_Y(x)$

or

$f_X(x) = f_Y(x)$

$E(T)$



$\sum x \,\text{rect}(x)$

$\vdots$

$$E(X) = \int_{Supp(X)} x f(x)\, dx$$

$$F(g(x)) = \int_{Supp(x)} g(x) f(x)\, dx$$

$$Var(X) = \int_{x \in supp[X]} (x \cdot a)^2 f(x)\, dx$$

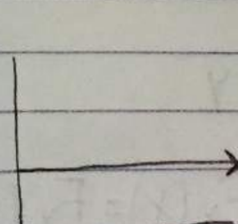
$$F(aX + C) = a\, E(X) + C$$
$$Var[aX + c] = a^2 6^2$$
$$SE\,[aX + c] = |a| 6$$
$$E\left(\sum_{i=1}^{n}\right] = \sum_{i=1}^{n} E[X_i] = nM$$ if ident dis r.v.

$$Var[\sum X_i] = \sum Var(X_i) = n 6^2$$ if iid

if X_1, X_2 are independent

$X \sim Exp(\lambda) := \lambda e^{-\lambda x}$

$Supp(X) = (0, \infty)$



Parameter Space:

$\lambda \in (0, \infty)$

$$E(X) = \int_0^\infty x \lambda e^{-\lambda x} dx = \lambda \int_0^\infty x e^{-\lambda x} dx =$$

$u = x \qquad dv = e^{-\lambda x}$

$du = dx$

$v = \int dv = -\frac{1}{\lambda} e^{-\lambda x}$

$$= \lambda \left[x - \frac{1}{\lambda} e^{-\lambda x} - \frac{1}{\lambda^2} e^{-\lambda x} \right]_0^\infty = \left[\frac{1}{\lambda} e^{\frac{1}{\lambda x}} + e^{\frac{1}{\lambda x}} \right]_0^\infty = 0 + \frac{1}{\lambda}$$

$$\int v\, du = \int -\frac{1}{\lambda} e^{-\lambda x} dx = \frac{1}{\lambda^2} e^{-\lambda x}$$

$X \sim \text{Geom}(p)$

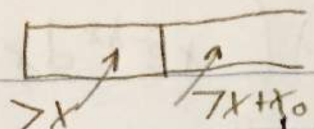
$E(X) = \frac{n}{p}$ exp's

$X \sim \text{Exp}(\lambda)$

$E(X) = \frac{1}{\lambda}$ sec

	stopping Single	Multiple
Discrete	Gem	Neg Bin
continuous	Exp	Erling

	PMF	PDF	CDF
discr	✓		✓
cont		✓	✓



$$P(X > x_0 + x \mid X > x) = \underline{P(X > x_0 + x)} = \frac{\overbrace{e^{-\lambda(x_0+x)}}^{e^{-\lambda x}\, e^{-\lambda x}}}{e^{-\lambda x}}$$

$$= e^{-\lambda x_0}$$