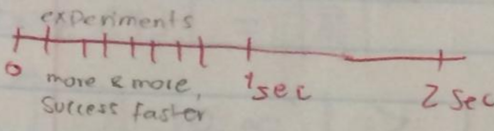
$T \sim Geom(p) := (1-p)^{t-1} p$, $F(t) = 1-(1-p)^t$, $E[t] = \frac{1}{p}$ experiments



What can t be?

$(0, \infty) \neq Supp(T)$

Within each second, n experiments

$p(t) = (1-p)^{nt-1} p$, $F(t) = 1-(1-p)^{nt}$ $E[t] = \frac{1}{pn}$

$= \frac{1}{p} exp \cdot \frac{1}{n} \frac{sec}{\cancel{exp}}$

$p = \frac{\lambda}{n}$

$\lambda := pn$

small, big

$$\lim_{n\to\infty} P(t) = \left(1-\frac{\lambda}{n}\right)^{nt-1} \frac{\lambda}{n} = \lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{nt} \lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{-1} \lim_{n\to\infty}\frac{\lambda}{n}$$

$$= \left(\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{n}\right)^{t} \underbrace{\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{-1}}_{1} \underbrace{\lim_{n\to\infty}\frac{\lambda}{n}}_{0}$$

$= 0$

We let experiments happen continously

is this a PMF? this doesn't have a PMF

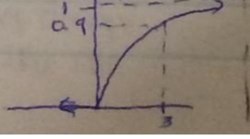
$\sum_T p^x(t) = 0 \neq 1$

Not a PMF

$$F^x(t) = \lim_{n\to\infty} 1-\left(1-\frac{\lambda}{n}\right)^{nt}$$

$$= 1-\left(\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{n}\right)^{t}$$

$$\underbrace{= 1-e^{-\lambda t}}_{\text{yes, it is a CDF}}$$



$e := \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$

$e := \sum_{i=0}^{\infty} \frac{1}{i!}$

$e := \int_1^e \frac{1}{x} dx = 1$

n	f(n)
10	2.519
100	2.705
1000	2.717
10,000	2.718

$$\lim_{n\to\infty}\left(1+\frac{a}{n}\right)^n = e^a \quad a \in \mathbb{R}$$

let $m = \frac{n}{a} \Rightarrow n = ma$

$\Rightarrow \frac{1}{m} = \frac{a}{n}$

$$\left(\lim_{m\to\infty}\left(1+\frac{1}{m}\right)^m\right)^a = e^a$$

$$F(0) = 1 - e^{-\lambda(0)} = 1 - 1 = 0$$

$$\lim_{t\to\infty} F(t) = 1 - \lim_{t\to\infty}\frac{1}{(e^{\lambda})^t} = 1 - 0 = 1 ✓$$

$\int 1 - e^{-\lambda t}$

Why it should be 1?

*Prove through the geometric series...

exam

$$F'(t) = \lambda e^{-\lambda t} = \frac{\lambda}{e^{\lambda t}} > 0$$ monotonically increasing function.

$\lambda > 0$

$$\begin{cases} p \in (0,1) \\ n \in \mathbb{N} \end{cases}$$

$F(x) := P(X \le x)$ ← CDF

$$\underbrace{1 - e^{-\lambda t}}_{\text{CDF}}$$

$|Supp[X]| \leq |\mathbb{N}|$

$|Supp[T]| = |\mathbb{R}|$ ^

$|[0,\infty)| > |\{1,2,3,..\}|$

Definition of a continuous r.v.

They do not have PMF's

$P^x(t) = 0$

$P(3) = 0$ at 3sec when experiments are happening continuously

Probability you stop a $\underbrace{3.0000}_{\infty \text{ 0's}}$ sec

$P(3.0000\ldots) = 0$

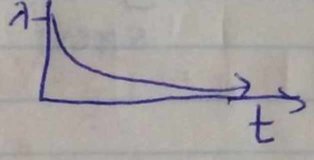
$P(3.0000)$

btw those #'s

$P(T \in [2.999995, 3.0004])$

$= F(3.00004) - F(2.9995) > 0$

$F(t) := F'(t) = \lambda e^{-\lambda t}$



Abstract Metric

Density of Probability in the r.v.

Prob. density function (PDF) rate change time

↳ support

CDF is the King

$\lambda = 1$

$F(3) = 0.05 \neq P(3) = 0$

rate of change of the CDF

$P(T \in [a,b])$ $\quad a,b \in Supp[T]$, $b > a$

$\parallel$

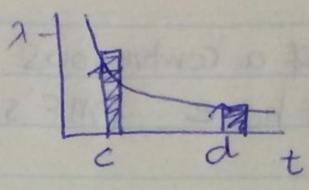
$F(b) - F(a) = \int_a^b f(t)\,dt$

antiderivative

Fundamental theory of calculous

$\frac{f(c)}{f(d)}$

how much likely is c or d to happen...



$P(T \in (-\infty, \infty)) = \lim_{t \to \infty} F(t) - \lim_{t \to -\infty} F(t)$

$\boxed{= \int_{-\infty}^{\infty} f(t)\,dt = 1}$

*why from $-\infty$ if not negative time?

Def: X is a continous r.v. if

1. CDF exits
2. PMF does not exist
3. $Supp[X] \subseteq \mathbb{R}$
4. $f(x)$ exits and $\int_{Supp[X]} f(x)\,dx$
5. $|Supp[X]| = |\mathbb{R}|$

$X \stackrel{d}{=} Y$

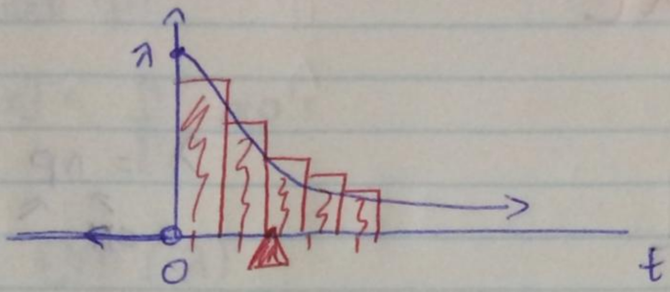
if $F_X(x) = F_Y(y)$

or $f_X(x) = f_Y(y)$

$E[X]$

$= \sum x\,p(x)$

Probability weithed avg.



$\sum x \,\text{rect}(x)$

$\vdots$

$$E[X] = \int_{Supp[X]} x\, f(x)\, dx$$

$\int$ = sum over uncoutable things

$$E[g(x)] = \int_{Supp[X]} g(x)\, f(x)\, dx$$

$$Var[X] = \int_{x \in Supp[X]} (x-\mu)^2\, f(x)\, dx$$

$$E[aX + c] = a\,E[X] + c$$

$$Var[aX + c] = a^2 \sigma^2$$

$$SE[aX + c] = |a|\,\sigma$$

$$E\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n} E[X_i] = n\mu$$

↳ if identically distributed

$$Var[\sum X_i] = \sum Var[X_i] = n\sigma^2 \leftarrow \text{if iid}$$

↑ if $X_1 \dots X_n$ are independent

no longer PMF
only CMF

pdf

$X \sim Exp(\lambda) := \lambda e^{-\lambda x}$

$Supp[X] = (0, \infty)$

Parameter Space

$\lambda \in (0, \infty)$

$\lambda = np$

$n \in \mathbb{N}$ $\quad p\,(0,1)$

$\int_0^\infty 1 = \infty$

idea

Solve to see when the mean is

$\bar{X} \to \mu$

$$E[X] = \int_0^\infty x \lambda e^{-\lambda x} dx$$

$u = x$

$du = dx$

$dv = e^{-\lambda x} dx$

$V = \int dv = -\frac{1}{\lambda} e^{-\lambda x}$

$\int u\,dv = uv - \int v\,du$

$\int v\,du = \int -\frac{1}{\lambda} e^{-\lambda x} dx = \frac{1}{\lambda^2} e^{\lambda x}$

$$E[X] = \lambda \int_0^\infty x e^{-\lambda x} dx = \lambda \left[-\frac{x}{\lambda} e^{-\lambda x} - \frac{1}{\lambda^2} e^{-\lambda x} \right]_0^\infty$$

$$= \left[\frac{1}{\lambda} e^{\frac{1}{\lambda x}} + e^{x \frac{1}{\lambda x}} \right]_0^\infty = 0 + \boxed{\frac{1}{\lambda}}$$

$X \sim Geo(p)$

$E[X] = \frac{1}{p}$ exp's

$X \sim Exp(\lambda)$

$E[X] = \frac{1}{\lambda}$ sec

	Single	multiple
Discrete	Geo	Neg Bin
Continous	Exp	Gamma

	PMF	PDF	CDF
Discrete	✓		✓
Continous		✓	✓

$$P(X > x_0 + x \mid X > x)$$

$$= \frac{P(X > x_0 + x)}{P(X > x)} = \frac{e^{\overbrace{-\lambda(x_0 + x)}^{e^{-\lambda x_0} e^{-\lambda x}}}}{e^{-\lambda x}} = e^{-\lambda x_0}$$

Memoryless

same idea as geometric