- I take Uber
  Van Wyck 7 min (no traffic)
  Sevel ave 17 min (traffic)

$p(\text{traffic}) = 0.3$

low
↓
large $_{\#'s}$

$\bar{x} \to M$

$$W = \begin{cases} 7 \text{ min wp } 0.7 \\ 12 \text{ min wp } 0.3 \end{cases}$$

$E(w) = 7 \cdot 0.7 + 12 \cdot 0.3 = 8.5$

Uber charges \$ 0.40 /min

$$B = \underbrace{\$ 0.70/m - W}_{g(w)}$$

$E(B) = E(g(w))$

r: $g(x)$ x is discourse → $\text{supp}(x) = \{x_1, x_2 \ldots$

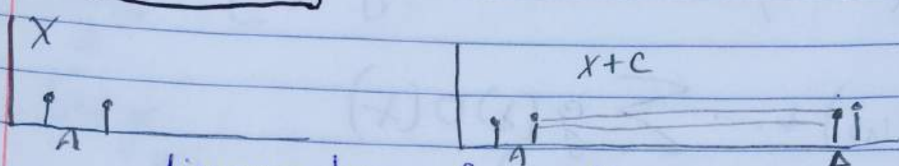
$$E(y) := \int_{\Omega} Y(\omega)\, dP(\omega)$$

$Y = X + c, \quad c \in \mathbb{R}$

$$E[X+c] = \sum_{g(x)} g(x)\,p(x) = \sum (x+c)\,p(x) =$$

$$= \underbrace{\sum x\,p(x)}_{} + \overbrace{\sum c\,p(x)}^{c \sum px \Rightarrow 1}$$

$$\boxed{E(X) + c}$$

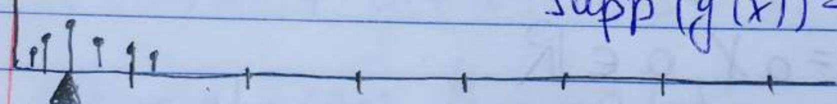


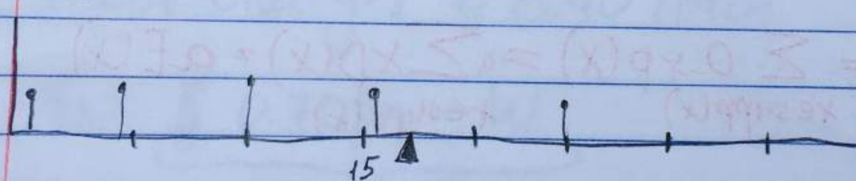
Linear transformation

$$\boxed{E[aX+c] = a\,E(X) + c}$$

$X \sim Bin(6, \frac{1}{2}) \quad Y = X^2 = g(X)$

$supp(g(x)) = \{g(x) : x \in supp(x)\}$

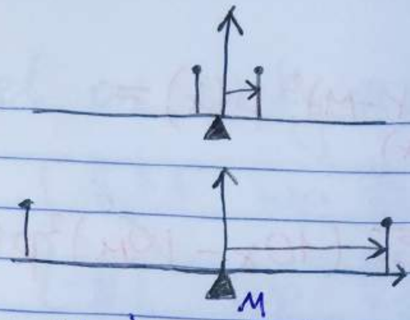




$$E(X) = \sum_{x \in supp(x)} x = p(x)$$

$$= \sum_{x=0}^{6} x^2 \binom{6}{x} \frac{1}{2^6} = \frac{1}{2^6}\left(\binom{6}{1} + 9\binom{6}{3} + 16\binom{6}{4} + 25\binom{6}{5} + 36\right)$$

$= 17.5$



$X \sim$ Random
$E(X) = 0$

$Y = 10X$

$E(X) = E(Y) \not\Rightarrow X \overset{d}{=} Y$

dispersion

distance

error

cost

lost

variance

deviance

- $e(X, M) := X - M$
- $e(X, M) := |X - M|$
- $e(X, M) := (X - M)^2$ – square error loss function

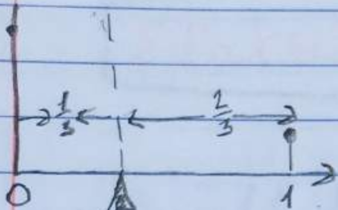
Quadratic loss or Square error Loss
(you can defirentiate)

$$\sigma^2 := Var(X) = E(X-\mu)^2 = \sum_{x \in supp(x)} (x-\mu)^2 \, p(x) =$$

$$= \sum_{x \in \{-1,1\}} x = \frac{1}{2} = 1 = \sum (10x - \cancel{10\mu}^{\,0})^2 p(x) =$$

$$= 100 \sum x^2 \, p(x) = 100$$

• $X \sim Bern\left(\frac{1}{3}\right)$



• $\sigma^2 = \left(0-\frac{1}{3}\right)^2 \frac{2}{3} + \left(1-\frac{1}{3}\right)^2 \cdot \frac{1}{3} = \frac{1}{9} \cdot \frac{2}{3} + \frac{7}{9} \cdot \frac{1}{3} =$

$= 0.259$

• $\sigma^2 = (0-p)^2(1-p) + (1-p)^3 p =$

$= p^2(1-p) + (1-p)^3 p$

$= p(1-p)(p+1-p)$

$= \boxed{p(1-p)}$

• Bet on lucky #7.

$$X \sim \begin{cases} \$35 & \text{wp } \frac{1}{38} \\ -\$1 & \text{wp } \frac{37}{38} \end{cases}$$

$E(X) = -\$0.053$

$E(X-M)^2$

$Var(X) = (\$35 - (-\$0.053))^2 \frac{1}{38} + (-\$1 - (-\$0.053)\frac{37}{38}$

$= 33.21\ \$^2 = \$5.76$

$$\boxed{SE(X) = \sqrt{Var(X)} = \$5.76}$$

$T = \underbrace{X_1 + X_2}_{g(X_1, X_2)}$

X_1	X_2	T	$p(X_1 X_2)$
x_{11}	x_{21}	$x_{11} + x_{21}$	$p(X_1)\,p(X_{21} \mid X_{11})$
x_{12}	x_{22}	$x_{12} + x_{22}$	
x_{13}	$\vdots$		
$\vdots$			

$$E(T) = \sum_{t \in supp(T)} t\, p(t)$$

$$\boxed{E(g(X_1, X_2)) = \sum_{\langle x_1, x_2 \rangle \in supp(X_1) \times supp(X_2)} g(X_1, X_2)\, p(X_1 X_2) = \\ = \sum_{x_1 \in supp(X_1)} \cdot \sum_{x_2 \in supp(X_2)} g(X_1, X_2)\, p(X_1, X_2)}$$

- $E[X_1+X_2] = \sum_{x_1} \cdot \sum_{x_2} (x_1+x_2)\, p(x_1,x_2) = \sum_{x_1} \sum_{x_2} x_1\, p(x_1,x_2)$

$+ \sum_{x_1} \cdot \sum_{x_2} x_2\, p(x_1,x_2) = \sum_{x_1} \sum_{x_2} x_1 p(x_1) p(x_2) +$

if X_1, X_2 independent $\Rightarrow p(X_1,X_2) = p(X_1) \cdot p(X_2)$

$+ \sum_{x_1} \sum_{x_2} x_2\, p(x_1)\, p(x_2) \rightarrow \underbrace{\sum_{x_1} x_1\, p(x_1)}_{E(X_1)} \cdot \underbrace{\sum_{x_2} p(x_2)}_{1} +$

$+ \underbrace{\sum_{x_1} p(x_1)}_{1} \cdot \underbrace{\sum_{x_2} x_2\, p(x_2)}_{E(X_2)}$

- $\sum_{x_1} x_1 \sum_{x_2} p(x_1,x_2) + \sum_{x_2} x_2 \sum_{x_1} p(x_1,x_2)$

$Supp(X_1) = \{1, 7, 19\}$

$Supp(X_2) = \{5, 23, 80\}$

	X_1: 1	7	19
5	$\frac{1}{15}$	$\frac{1}{3}$	$\frac{2}{15}$
23	$\frac{1}{30}$	$\frac{1}{10}$	$\frac{1}{30}$
80	$\frac{1}{30}$	$\frac{1}{5}$	$\frac{1}{15}$