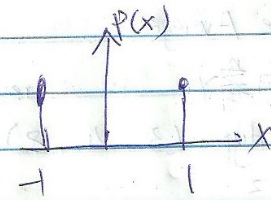
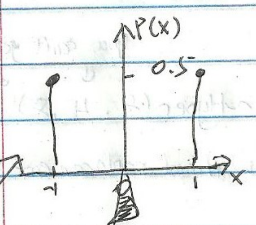
$X_1 \dots X_{81}$ iid Rademacher $= \begin{cases} 1 & \text{wp } \frac{1}{2} \\ -1 & \text{wp } 1/2 \end{cases}$

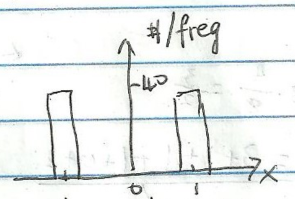


$5+7+3+2+5+4+13$

$\bar{X} = \frac{(1)39 + (-1)(42)}{81} = \frac{-3}{81}$

10/27
lec14



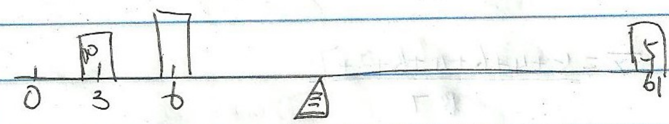


histogram.

$P(A) = \lim_{n\to\infty} \frac{1}{n} \sum \mathbb{1}_{w \in A}$.

Pivot called the "expectation", the "expected value" or the "mean." $E(X)$ or μ.

$\bar{X}_n \xrightarrow{n\to\infty} E(X)$ law of large #'s $\qquad \frac{d}{dx}[X^2] = 2x$



$X = \frac{X_1 W_1 + X_2 W_2 + X_3 W_3 \dots}{\sum w_i}$ weight avg.

$E(X) = \int_\Omega X(w)\, dP(w) = \int X(w)\, dP(w) + \int X(w)\, dP(w) + \dots$

X is discrete $\qquad \{w : X(w) = X_1\}, \{w : X(w) = X_2\}$

$\text{supp}(X) = \{X_1, X_2 \dots\}$ $\qquad = X_1 \int dP(w) + X_2 \int dP(w)$

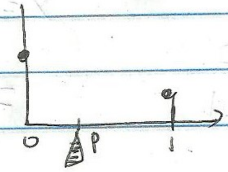
$\int_0^{10} f(x)dx = \int_0^3 f(x)dx + \int_3^{10} f(x)dx$ $\qquad = X_1 P(X_1) + X_2 P(X_2) + \dots$

$= \sum_{x \in \text{supp}(X)} x\, P(X)$

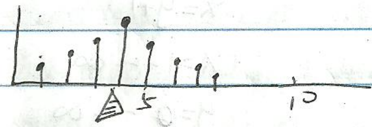
$E(X) = (-1)\frac{1}{2} + (1)\frac{1}{2} = 0$

$X \sim \text{Bernoulli}(\frac{1}{3})$

$E(X) = (0)P(0) + (1)P(1) = \frac{1}{3} = p$



$X \sim \text{Binomial}(8, \frac{1}{2})$



$$\sum_{x=0}^{8} x \binom{8}{x} \overbrace{\left(\tfrac{1}{2}\right)^x \left(\tfrac{1}{2}\right)^{8-x}}^{\frac{1}{2^8}}$$

$$= \frac{1}{256}\left(1\binom{8}{1} + 2\binom{8}{2} + 3\binom{8}{3} + 4\binom{8}{4} \cdots + 8\right)$$

$$= \frac{1}{256} \cdot 1024 = 4.$$

$X \sim \text{Binomial}(n, p)$

$$E(x) = \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=1}^{n} x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

$$= np \sum_{x=1}^{n} \underbrace{\frac{(n-1)!}{(x-1)!(n-x)!}}_{=\binom{n-1}{x-1}} p^{x-1} (1-p)^{n-x}$$

$y = x-1$

$\Rightarrow x = y+1$

$x = 1 \cdots n$

$y = 0 \cdots n-1$

$$= np \sum_{y=0}^{n-1} \binom{n-1}{y} p^y (1-p)^{n-1-y}$$

Let $m = n-1$

$$= np \underbrace{\sum_{y=0}^{m} \binom{m}{y} (p)^y (1-p)^{m-y}}_{=1}$$

$\leftarrow$ sum of Binomial(m, p)

$$= np$$

$X \sim \text{Unif}(1, 5, 6)$

$$Ex = 1p(1) + 5p(5) + 6p(6)$$

$$= \tfrac{1}{3}(1+5+6) = 4$$

$X \sim \text{Unif}(A)$

$$E(x) = \sum_{x \in A} x p(x) = \frac{1}{|A|} \sum_{x \in A} x.$$

$X \sim \text{Hyper}(n, K, N)$

$$E(X) = \sum_{x \in supp(x)} x \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{k}} = n\frac{K}{N}$$

$X \sim \text{Geometric}(p)$

$\frac{1}{p} = \mu = E(X) = \sum_{x=1}^{\infty} x(1-p)^{x-1} p$ Let $y = x-1$

$x = y+1$

$\mu = (1-p)\mu + 1 \qquad = \sum_{y=0}^{\infty} (y+1)(1-p)^y p$ $\qquad x = 1 \cdots \infty$

$\mu = \mu - p\mu + 1 \qquad\qquad\qquad y = 0 \cdots \infty$

$0 = -p\mu + 1 \qquad = \sum_{y=0}^{\infty} y(1-p)^y p + \underbrace{p\sum_{y=0}^{\infty}(1-p)^y}_{=1}$ $\qquad |q|<1 \Rightarrow \sum_{x=0}^{\infty} q^x = \frac{1}{1-q}$

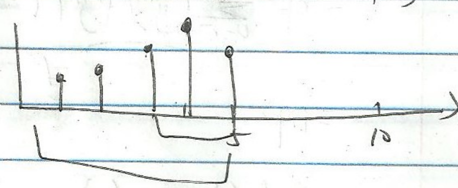
$p\mu = 1 \Rightarrow \mu = \frac{1}{p} \qquad = \sum_{y=1}^{\infty} y(1-p)^y p + 1$ $\qquad p \cdot \frac{1}{1-(1-p)} = 1$

$X \sim \text{NegBin}(r, p)$

$E(X) = \sum_{x=r}^{\infty} x \binom{x-1}{r-1}(1-p)^{x-r} p^r = \frac{r}{p}$

$\text{Quartile}[X, p] = \min_{x \in S_X} \{x : F(x) \geq p\}$. "Percentile".

$X \sim \text{Binomial}(10, 0.4)$



X	P(X)	F(X)
0	0.0060	0.0060
1	0.0404	0.0464
2	0.1200	0.1623
3	0.2150	0.3873
4	0.2508	0.6331
5	0.2007	0.8338
6	0.1115	0.9410
7	0.465	0.9877
8	0.0106	0.9983
9	0.016	0.9989
10	0.0001	1
	1	

IQR = 2

$\text{Median}(X) = \text{Quartile}[X, 0.5]$

$\text{IQR}[X] = \text{Quartile}[X, 0.75] - \text{Quartile}[X, 0.25]$

inter-quartile range

tertiles : 0, 33, 66, 100

quartiles : 0, 25, 50, 75, 100

quintiles. 0, 20, 40, 60, 80, 100

dectile : 0, 10, 20 --- 100 .

Model[X] = argument{P(x)} = 4 (biggest P(x) or bar).

American

~~gambling~~

$$X \sim \begin{cases} \$1 & wp\ \frac{18}{38} \\ -\$1 & wp\ \frac{20}{38} \end{cases} \qquad E(X) = 1\left(\frac{18}{38}\right) + (-1)\left(\frac{20}{38}\right) = -\frac{2}{38} \approx -\$0.05$$

If you bet on #7 .

$$X \sim \begin{cases} \$35 & wp\ \frac{1}{38} \\ \$1 & wp\ \frac{37}{38} \end{cases} \qquad E(X) = 35\,\frac{1}{38} + (-1)\,\frac{37}{38} = -\frac{2}{38} \approx -\$0.05$$

Bet on 1-12

$$X \sim \begin{cases} \$2 & wp\ \frac{12}{38} \\ -\$1 & wp\ \frac{26}{38} \end{cases} \qquad E(X) = 2\,\frac{12}{38} + (-1)\,\frac{26}{38} = -\frac{2}{38} \approx -0.05 .$$

Europe

$$X \sim \begin{cases} \$1 & wp\ \frac{18}{37} \\ -\$1 & wp\ \frac{19}{37} \end{cases} \qquad E(X) = 1\left(\frac{18}{37}\right) + (-1)\left(\frac{19}{37}\right) = -\frac{1}{37} \approx -\$0.03$$