$X \sim Hyper(6, 0.5, 100)$ Bag of 100 balls, 50% of success.

$P(X=3) = 0.3223$

$X \sim Hyper(6, 0.5, 1000)$

$P(X=3) = 0.3134$

$X \sim Hyper(6, 0.5, 10000)$

$P(X=3) = 0.3126$

$\lim af(x)$
$= a \lim f(x)$

10/15
Lec11

$$\lim_{N\to\infty} Hyper(n,p,N) = \lim_{N\to\infty} \frac{\binom{pN}{x}\binom{(1-p)N}{n-x}}{\binom{N}{n}} = \lim_{N\to\infty} \frac{\frac{(pN)!}{x!(pN-x)!}\frac{((1-p)N)!}{(n-x)!((1-p)N-(n-x))!}}{\frac{N!}{n!(N-n)!}}$$

$$= \overbrace{\frac{n!}{x!(n-x)!}}^{\binom{n}{x}} \lim_{N\to\infty} \frac{(N-n)!}{N!}\,\frac{(pN)!}{(pN-x)!}\,\frac{((1-p)N)!}{((1-p)N-(n-x))!}$$

$$= \binom{n}{x} \lim_{N\to\infty} \frac{\overbrace{(pN)(pN-1)\cdots(pN-x+1)}^{x}\;\overbrace{((1-p)N)((1-p)N-1)\cdots((1-p)N-(n-x)+1)}^{(n-x)\text{ term}}}{\underbrace{(N)(N-1)(N-2)\cdots\cdots(N-n+1)}_{n \text{ term}}}$$

$$= \binom{n}{x} \overbrace{\underbrace{\lim_{N\to\infty}\frac{pN}{N}}_{p} \cdot \underbrace{\lim_{N\to\infty}\frac{pN-1}{N-1}}_{p} \cdots \underbrace{\lim_{N\to\infty}\frac{pN-x+1}{N-x+1}}_{p}}^{p^x}\; \overbrace{\underbrace{\lim_{N\to\infty}\frac{(1-p)N}{N-x}}_{1-p}\; \underbrace{\lim_{N\to\infty}\frac{(1-p)N-1}{N-x-1}}_{1-p}}^{(1-p)^{n-x}}$$

using L'Hôp rule $\Rightarrow \cdots \underbrace{\lim_{N\to\infty}\frac{(1-p)N-(n-x)+1}{N-n+1}}_{1-p} = \binom{n}{x}p^x(1-p)^{n-x} = p(x)$ Binomial

$\uparrow$

$X \sim Binomial(n,p) = \binom{n}{x}p^x(1-p)^{n-x}$ positive.

Parametric space.

$n \in \mathbb{N}$

$p \in (0,1)$

$Supp(X) = \{0, \dots, n\}$

$X \sim Binomial(n,0) = \binom{n}{x}0^x 1^{n-x}$

$P(0) = \underbrace{\binom{n}{0}}_{1} 0^0 \underbrace{1^n}_{1}$

$0^0 = \lim_{x\to 0} x^x = 1$

$0! = 1$

$P(\Omega) = 1$

$\sum_{x \in supp(x)} P(x) = 1$

$X \sim Binomial(n,1) = \binom{n}{x} 1^x 0^{n-x}$

$= Deg(n)$

$\sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x} = 1$ ← prove by Binomial.

$(a+b)^n = \sum_{x=0}^{n} \binom{n}{x} a^{n-x} b^x$

$((1-p)+p)^n = \sum_{x=0}^{n} \binom{n}{x} (1-p)^{n-x} p^x$

$\underset{a}{\uparrow} \quad \underset{b}{\uparrow}$

$= 1^n = 1$

success; infinite Bail

$X \sim \text{Binomial}(1, p)$ $\quad supp(x) = \{0, 1\}$.

$= \binom{1}{x} p^x (1-p)^{1-x} = p^x (1-p)^{1-x} = Ber(p)$

random Variable X_1 and r.v. X_2 are "independent" if

$P(X_1 = x_1 \mid X_2 = x_2) = P(X_1 = x_1)$ ← small x!

or $P(X_2 = x_2 \mid X_1 = x_1) = \cancel{P(X_2 = x_2)}\ P(X_2 = x_2)$

or $P(X_1 = x_1, X_2 = x_2) = P(X_1 = x_1) P(X_2 = x_2)$

$\forall (x_1, x_2) \in Supp(X_1) \times Supp(X_2)$

Ex: $X_1 \sim \text{Bernoulli}(\frac{1}{3})$ coin A

$X_2 \sim \text{Bernoulli}(\frac{1}{3})$ coin B.

$P(X_1 = 1 \mid X_2 = 1) = P(H_1 \mid H_2) = P(H_1) = P(X_1 = 1) = \frac{1}{3}$

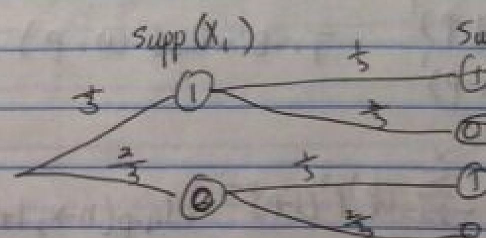
$P(X_1 = 1 \mid X_2 = 0) = P(H_1 \mid T_2) = P(H_1) = P(X_1 = 1)$ ✓

$P(X_1 = 0 \mid X_2 = 1) = P(T_1 \mid H_2) = P(T_1) = P(X_1 = 0) = \frac{2}{3}$

$P(X_1 = 0 \mid X_2 = 0) = P(T_1 \mid T_2) = P(T_1) = P(X_1 = 0)$ ✓

$X_1 \overset{d}{=} X_2$ and X_1, X_2 independent. (iid) independent identical distribution.

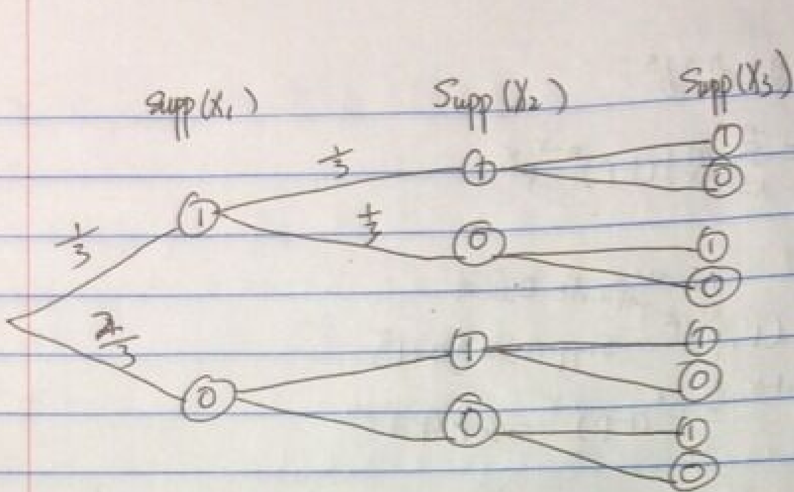
w/iid $X_1, X_2 \overset{iid}{\sim} Ber(\frac{1}{3})$



JMF.	T
$\frac{1}{9}$	2
$\frac{2}{9}$	1
$\frac{2}{9}$	1
$\frac{4}{9}$	0

$T_2 = X_1 + X_2 = f(X_1, X_2, X_3) = \frac{1}{3}$

$T \sim \begin{cases} 0 & wp\ \frac{4}{9} \\ 1 & wp\ \frac{4}{9} \neq \text{Unifor}(0,1,2) \\ 2 & wp\ \frac{1}{9} \end{cases}$



I ~~JmF~~	JmF
3	$(\frac{1}{3})^1(\frac{2}{3})^0$
2	$(\frac{1}{3})^2(\frac{2}{3})^1$
2	$(\frac{1}{3})^2(\frac{2}{3})^1$
1	$(\frac{1}{3})^1(\frac{2}{3})^2$
2	$(\frac{1}{3})^2(\frac{2}{3})^1$
1	$(\frac{1}{3})^1(\frac{2}{3})^2$
1	$(\frac{1}{3})^1(\frac{2}{3})^2$
0	$(\frac{1}{3})^0(\frac{2}{3})^3$

$$T \sim \begin{cases} 0 & \text{wp } \binom{3}{0}\ 1\ (\frac{1}{3})^0(\frac{2}{3})^3 \quad \underline{0}\ \underline{0}\ \underline{0} \\ 1 & \text{wp } \binom{3}{1}\ 3\ (\frac{1}{3})^1(\frac{2}{3})^2 \quad \underline{0}\,\underline{0}\,\underline{1},\ \underline{0}\,\underline{1}\,\underline{0},\ \underline{1}\,\underline{0}\,\underline{0} \\ 2 & \text{wp } \binom{3}{2}\ 3\ (\frac{1}{3})^2(\frac{2}{3})^1 \quad \underline{0}\,\underline{1}\,\underline{1},\ \underline{1}\,\underline{1}\,\underline{0},\ \underline{1}\,\underline{0}\,\underline{1} \\ 3 & \text{wp } \binom{3}{3}\ 1\ (\frac{1}{3})^3(\frac{2}{3})^0 \quad \underline{1}\,\underline{1}\,\underline{1} \end{cases}$$

$$T_n = \sum_{i=1}^{n} X_i$$

$$T \sim \begin{cases} 0 & \binom{n}{0}(\frac{1}{3})^0(\frac{2}{3})^n \\ 1 & \binom{n}{1}(\frac{1}{3})^1(\frac{2}{3})^{n-1} \\ \vdots & \vdots \\ n-1 & \binom{n}{n-1}(\frac{1}{3})^{n-1}(\frac{2}{3})^1 \\ n & \binom{n}{n}(\frac{1}{3})^n(\frac{2}{3})^0 \end{cases}$$

$X_1, X_2 \overset{iid}{\sim} Ber(p)$ ← anything between 0 & 1, except 0 & 1.

$$T \sim \begin{cases} 0 & \binom{n}{0}(p)^0(1-p)^n \\ 1 & \binom{n}{1}(p)^1(1-p)^{n-1} \\ \vdots & \vdots \\ n-1 & \binom{n}{n-1}(p)^{n-1}(1-p)^1 \\ n & \binom{n}{n}(p)^n(1-p)^1 \end{cases}$$

one of # between 0 to n.

$$P(T=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

~~$T \sim \text{Binomial}(n, p)$~~

$$F(x) := P(X \le x) = \sum_{i=0}^{x} p(i) = \sum_{i=0}^{x} \binom{n}{i} p^i (1-p)^{n-i} = I_{1-p}(n-x, 1+x)$$

$$= (n-x)\binom{n}{x} \int_0^{1-p} t^{n-x-1}(1-t)^x\,dt$$

regarlarized incomplete beta function