$X_1, X_2, \ldots \overset{iid}{\sim} \text{Bern}(p)$

$X \sim \text{Neg Bin}(r, p) := \binom{x-1}{r-1}(1-p)^{x-r}p^r = p(x)$ PMF

Stop at r successes $= \underbrace{\binom{x-1}{r-1}(1-p)^{x-r}p^{r-1}}\cdot p$

$\underbrace{\overset{\downarrow}{0}\ 1\ 0\,0\,1\ 0}_{x-1}\ \overset{\downarrow}{1}_{x^{th}}$ $\quad x-r \quad r-1$

x − 1 trials prob p
iid

We get r − 1 successes

$Y \sim \text{Binomial}(X-1, p)$

$P(Y = r-1) \longrightarrow \binom{x-1}{r-1}p^{r-1}(1-p)^{x-r}$

$\text{Supp}[X] = \{r, r+1, \ldots\}$

$= \mathbb{N} \setminus \{1, \ldots, r-1\}$

$\sum_{x \in \text{Supp}[X]} p(x) = 1$

$\boxed{\text{Show}}\ 1 = \sum_{x=r}^{\infty} \binom{x-1}{r-1}(1-p)^{x-r}p^r$

no exam

$\frac{1}{1-x} = \sum_{i=1}^{\infty} x^{i-1} \rightarrow$ if $|x| < 1$

$(1-x)^{-1} = \sum_{i=1}^{\infty} x^{i-1}$

$(r-1)!\,(1-x)^{-r} = \sum_{i=r}^{\infty} \underbrace{(i-1)(i-2)\ldots(i-r+1)}_{\frac{(i-1)!}{(i-r)!}} x^{i-r}$

$(1-x)^{-r} = \sum_{i=r}^{\infty} \underbrace{\frac{(i-1)!}{(r-1)!\,(i-r)!}}_{\binom{i-r}{r-1}}$

cont behid...

$$1 = (1-x)^r (1-x)^{-r} = \sum_{i=r}^{\infty} \binom{i-1}{r-1} x^{i-r} \quad (1-x)^r$$

$i = x$
$x = 1-p$

Stop at r successes, count # of trials (x)

Equivalent Parametrization

" " " " ", count # failures

• Producing 100 computer chips
How many failures we have.

$$X \sim \text{NegBin}(r,p) := \binom{x+r-1}{x} (1-p)^x p^r$$

$$\frac{(x+r-1)!}{x!\,(r-1)!} = \frac{(x+r-1)(x+r-2)\ldots r}{x!}$$

$$= \frac{(-1)^x \, -r(-r-1)\ldots(-r-x+1)}{x!}$$

$$= (-1)^x \binom{-r}{x}$$

• Roll a die until you get 17 6's. What is the chance you roll 107 times?

- $\frac{1}{6}$ iid
- die speak to each other

$$X \sim \text{NegBin}(\overset{r}{17}, \overset{p}{\tfrac{1}{6}})$$

$$P(X=107) = \binom{106}{16}\left(\frac{5}{6}\right)^{90}\left(\frac{1}{6}\right)^{17}$$

waiting for 1 success

$$\text{NegBin}(1,p) = \underbrace{\binom{x-1}{0}}_{=\,1}(1-p)^{x-1}p = \text{Geom}(p)$$

wait for 1 success

$$X_1 \ldots X_r \overset{iid}{\sim} \text{Geom}(p)$$

$$T = X_1 + \ldots + X_r \sim \text{NegBin}(r,p)$$ adding Geometric

Keep going till i get it
count # of times I have done

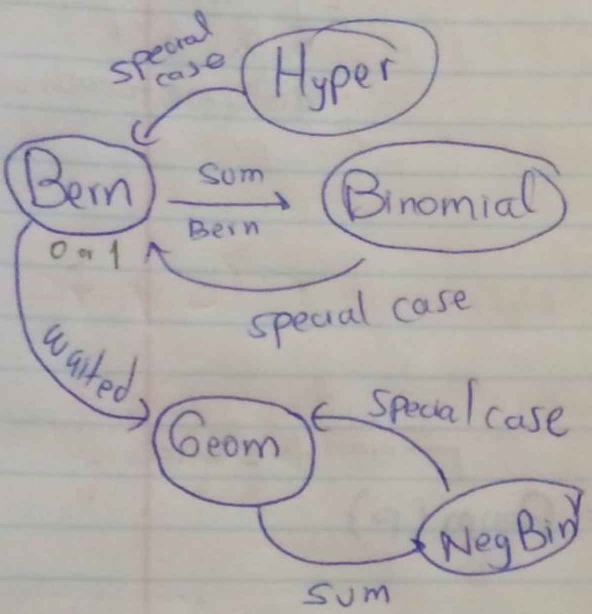
Wait until you get sucess

$$\underbrace{\_\,\_\,\_\,\_\,\_\,\_\,\_\,1}_{X_1=6}\ \underbrace{\_\,\_\,\_\,1}_{X_2=F}\ \underbrace{\_\,\_\,\_\,\_\,\_\,\_\,\_\,\_\,\_\,1}_{X_3=9} \quad r=3$$

Geometric + Geometric + Geometric

Done discrete random Var:

Test
- Supp
- PMF
- Parameter space



$$F(x) := P(X \le x) \stackrel{?}{=} \sum_{i=r}^{x} \binom{i-r}{r-1} (1-p)^{i-r} p^{i}$$

Survival function $= 1 - P(X > x)$

— — — … $\underline{x}$ $\underline{x+1}$ $\underline{x+2}$ - - -

< r successes

Support can go on Forever.

$= 1 - p(< r \text{ successes by } x)$

Made it pass X you have less than r successes

$= 1 - P\left(\begin{matrix}0 \text{ or } 1 \text{ or} \dots \text{or } r-1 \\ \text{Successes}\end{matrix}\right)$

$= 1 - \big(P(0 \text{ success}) + \dots + P(r-1 \text{ successess})\big)$

$Y \sim Bin(x, p)$

$P(Y = r-1)$

$= 1 - \left(\binom{x}{0} p^{0} (1-p)^{x} + \dots + \binom{x}{r-1} p^{r-1} (1-p)^{x-r+1}\right)$

r-1 successes in x trials

$= 1 - \sum_{i=0}^{x} P(x)$ ← Bin

0 successes in x trials

$= 1 - F(x)$ ← Binomial

$X_1, \dots, X_n \overset{iid}{\sim} p(x)$

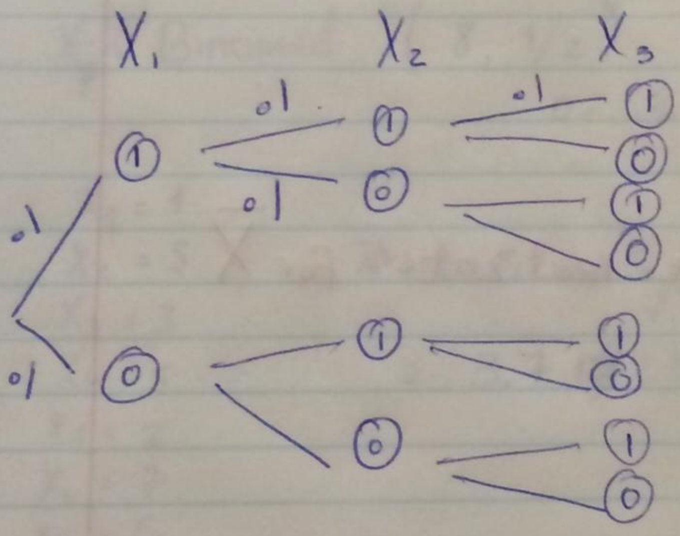
$T_n := X_1 + \dots + X_n$

$\bar{X}_n := \frac{X_1 + \dots + X_n}{n} = \frac{T_n}{n}$

"average r.v"

$\bar{X} \sim$
- 0 wp .729
- 1/3 wp .027
- 2/3 wp .243
- 1 wp .001

$X_1, X_2, X_3 \overset{iid}{\sim} \text{Bern}\left(\frac{1}{10}\right)$



T	$\bar{X}$	Prob
3	1	.009
2	2/3	.081
2	2/3	.081
1	1/3	.009
2	2/3	.081
1	1/3	.009
1	1/3	.009
0	0	.729

Models

child waiting to be born

$X_1, X_2 \overset{iid}{\sim} \text{Bern}\left(\frac{1}{2}\right)$

diff btw X and x

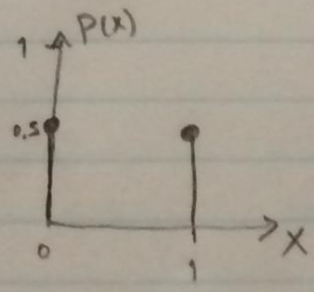
baby is born

$X_1 = 0$ Tails → datum

realization $X_2 = 0$ Tails

Data

$P(X=0) = \frac{1}{2}$



Model X_1

Datum: Realization of a r.V

Data: Realizations of r.V

$P(X = 6'1'')$

Process that spits out the height for people

r.v - "Data generator Processes"

* What else could this data had been?

$\bar{X}$ i.e the "sample avg"

$\bar{x} := \frac{1}{n} \sum_{i=1}^{n} x_i$ is a realization for $\bar{X}$

Sampling without replacement ramdomly

Coin experiment

- $X_1, \dots, X_6 \overset{iid}{\sim} Hyp(3, 4, 8)$ get 3 marked

Data
$X_1 = 1$
$X_2 = 2$
$X_3 = 0$
$X_4 = 1$
$X_5 = 1$

0 or 1 or 2 or 3 could have gotten

$$\bar{X} = \frac{1+2+0+1+1}{5} = 1.0$$

$(8, 4, 8)$
4

→ 8 coins count # of heads we get on 8.

- $X_1 .. X_7 \overset{iid}{\sim} Binomial(8, 1/2)$

0 or ... 8

count how many successes in 8 trials
Sampling with replacement
8 birnuly iid

$X_1 = 1$
$X_2 = 5$
$X_3 = 3$
$X_4 = 5$
$X_5 = 2$
$X_6 = 7$
$X_7 = 6$

$$\bar{X} = \frac{1+5+3+5+2+7+6}{7}$$

$$= 3.\overline{7\cancel{1}\cancel{4}285}$$

Take 1 coin out of time and see how many times it takes to get head

- $X_1, \dots X_7 \overset{iid}{\sim} Geom(\frac{1}{2})$

$X_1 = 1$
$X_2 = 3$
$X_3 = 2$
$X_4 = 2$
$X_5 = 1$
$X_6 = 2$
$X_7 = 2$

$$\bar{X} = \frac{1+3+2+2+1+2+2}{7} =$$

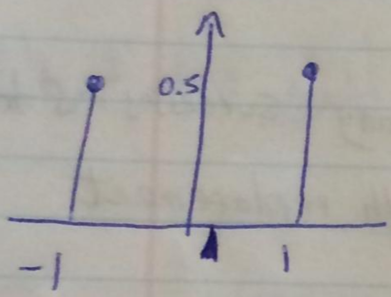
$$= 1.\overline{857142}$$

Neg Bin   Pull out till get 2 heads

- $X_1, \dots, X \overset{iid}{\sim}$ Rademaker $= \begin{cases} 1 & wp\ 1/2 \\ -1 & wp\ 1/2 \end{cases}$

heads $\dfrac{4+2+16+4+\cancel{4}4+5+6}{81} = \dfrac{41\cdot(1)+40(-1)}{81}$

$= \dfrac{1}{81}$



When to Sum or Multiply