L1

Math 281
Lecture 1 8/27/15

- Syllabus

Basic Set Theory & Vocabulary

Math is built on the foundation of sets, certainly prob & stats. Began in 1870's, formalized in 20th century

Set: A collection of distinct objects without order

none, one, many — unique / no duplicates — anything you can think of

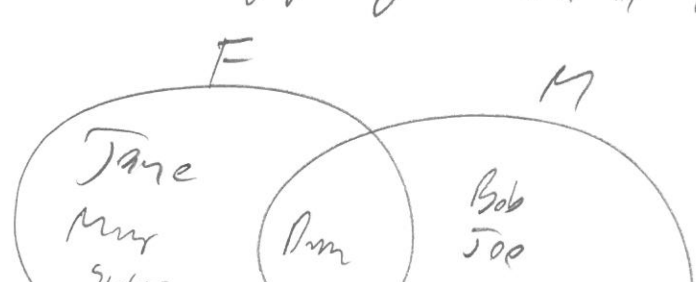
notation refers to set

F = {Jane, Mary, Susan, Dana} ← braces denote begin, end of "enumeration"; commas separate objects

M = {Bob, Joe, Mark, Dana}

= {Dana, Mary, ...} — ... ellipses useful for when it is understood what comes next

You can illustrate graphically with a "Venn Diagram"



Operators

Set equality: all elements shared $M \neq F$
Inequality: at least one

~~Set operations~~

Set Inclusion

object

is an element of

Jane $\in$ F ← sets

Exclusion

Jane $\notin$ M

i.e. Jane is = to one of the elements

Sets can have any # of elements

$$\mathbb{N} := \{1, 2, 3, \ldots\}, \quad \mathbb{N}_0 := \{0, 1, 2, \ldots\} = \mathbb{N} \cup \{0\}$$

$$\mathbb{Z} := \{\ldots, -2, -1, 0, 1, 2, \ldots\}$$

Come back to this

<u>Subsets</u>

denoted with "$\subset$" or "$\subseteq$"

$\{$Jane, Mary$\}$ $\subset$ or $\subseteq$ F

$A \subset B$ means "proper subset" i.e. $A \neq B$. There is at least one element in B not in A.

$A \subseteq B$ means "subset" i.e. $A = B$ or $A \subset B$ leaves the option of being equal.

$\{$Jane$\}$ $\in$ F?   Jane $\subset$ F?

PARSE! Math is like a compiler!

Combine sets using union operator

$\{Joe\} \cup \{Mary, Susan, Dana\} = F$

$\{Joe\} \cup \{Joe\} = \{Joe\}$ Why? Union is addition without double-counts.

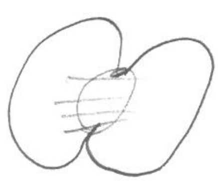
$Joe \cup \{Joe\} = ?$ Very strange question!

≈ non-exclusive or

if a == 3 || a < 4

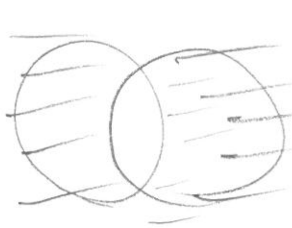
≡

$F \cup M$ is "male, female and/or both"



Set Intersection finds common elements of a set

$F \cap M = ?$ $\{Dana\} \neq Dana$



Intersection is the "And" operator

Male & Female at the same time!

$F \cap \{Bob, Joe\} = \{\}$, $\emptyset := \{\}$

"empty set or "null set""

if $A \cap B = \emptyset$ ... what does this mean? Share no elements

e.g. $Odds \cap Evens = \emptyset$ both infinite!!

$\emptyset \subset F$? $\emptyset \in F$?

We can subtract sets too

$F \setminus M$ 

females minus male

$= \{Jane, Mary, Susan\}$

$M \setminus F$ 

$= \{Bob, Joe, Max\}$

if $A \setminus B = A$ what is $A \cap B = ?$

$f(x) = x^2$
$f(q) = q^2$
free variable

We can also take the powerset: all sets subset

let $A = \{1, 2, 3\}$

$= \{B : B \subseteq A\}$ conditions

set builder notation

$P(A) = 2^A = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, A\}$

↑ set function set → set

We can also ask for the size of ~~set~~, called "cardinality"

$|A| = 3, \quad |F| = 7, \quad |M| = 5$

$|M \cup F| = ? \ 7 \qquad |F \cap M| = 1$

$|M \cup F| \neq |M| + |F|$

$|2^A| = 8$

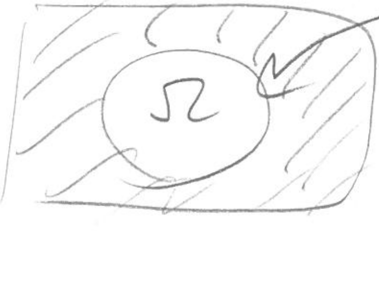
$|2^F| = ? \qquad \underset{\text{Joe}}{2} \cdot \underset{\text{Amy}}{2} \cdot \underset{\text{Susan}}{2} \cdot \underset{\text{Dan}}{2} = 2^4 = \underline{\underline{16}}$

X X X X

A special set is called Ω (Omega)

the "universe", "sample space", "space of discourse," current scope

here limited to. You define it!



scope. You don't see anything else

let $\Omega = F \cup M$

Note $F \subseteq \Omega, \ M \subseteq \Omega$ all sets are subsets of the universe in scope!

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Eg. Coin Flip $\Omega = \{H, T\}$ Can get a 7 on a coin! It's absurd!

Imagine draw a name from $\Omega = F \cup M$

What is the probability this name is female?

$$\frac{|F|}{|\Omega|} = \frac{4}{7} \ldots$$ more to come