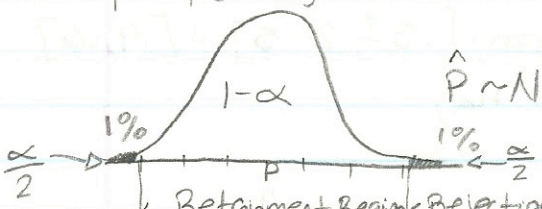
December 3, 2015

Math 241 Lecture 22

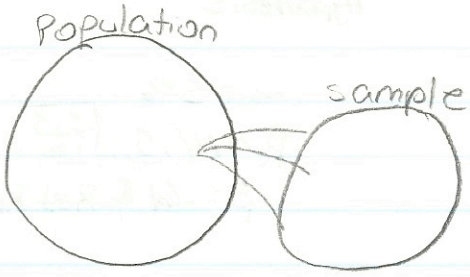
Male/Female gender ratio is even

$p := p(\text{male}) = .5$



$\hat{P} \sim N\left(p, \left(\sqrt{\frac{p(1-p)}{n}}\right)^2\right)$

$\alpha := P(\hat{p} \text{ being "too rare"})$



$\hat{p} \in$ Retainment Region $\Rightarrow$ Retain Theory

$\hat{p} \notin$ Retainment Region $\Rightarrow$ Reject Theory

$\alpha = 5\%$ $\quad 1-\alpha = P(|Z| \le \frac{2\alpha}{2}) = P(|Z| \le 2) = P\left(\left|\frac{\hat{P}-p}{\sqrt{\frac{p(1-p)}{n}}}\right| \le 2\right)$

$= P\left(-2 \le \left|\frac{\hat{P}-p}{\sqrt{\frac{p(1-p)}{n}}}\right| \le 2\right) = P\left(-2\sqrt{\frac{p(1-p)}{n}} \le \hat{P}-p \le 2\sqrt{\frac{p(1-p)}{n}}\right)$

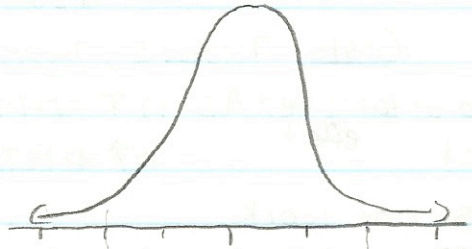
$= P\left(p-2\sqrt{\frac{p(1-p)}{n}} \le \hat{P} \le p+2\sqrt{\frac{p(1-p)}{n}}\right) = P\left(\hat{p} \in \underbrace{\left[p \pm \frac{2\alpha}{2}\sqrt{\frac{p(1-p)}{n}}\right]}_{\text{Retainment Region}}\right)$

$n = 345$ babies

$\hat{p} \sim N\left(0.5, \left(\sqrt{\frac{0.5-0.5}{345}}\right)^2\right)$

$= N(0.5, .0269^2)$

$\alpha = 5\%$



RetRegion = $[.5 \pm 2(.0269)]$

$= [.446, .554]$

169 male babies

$\hat{p} = \frac{169}{345} = .48 \in$ RetRegion

$\therefore$ Retain Theory

Null Hypothesis: statement about a theoretical possible range denoted H_0

Is a coin fair? ($p = .5$) Flip coin 100x

Sen #1: 51 H $\rightarrow$ fair

Sen #2: 98 H $\rightarrow$ not fair

Sen #3: 61 H $\rightarrow$

$H_0: p = .5$

H_a: parameter space | H_0: $p \neq .50$

↑ Alternative Hypothesis

(H_0^c = "=")

$\alpha = 5\%$

$\hat{p} \sim N(.5, (\sqrt{\frac{.5 \cdot .5}{100}})^2) = N(.5, .5^2)$ Ret Region $= [.5 \pm 2 \cdot .5] = [.4, .6]$

$\hat{p} = .61$ & Ret Region ⇒ Reject H_0 (Accept H_a)

$H_0: p = 0.2$

$H_a: p \neq .2$

$\alpha = 1\%$ ⇒ $z_{.5\%} = 2.56$

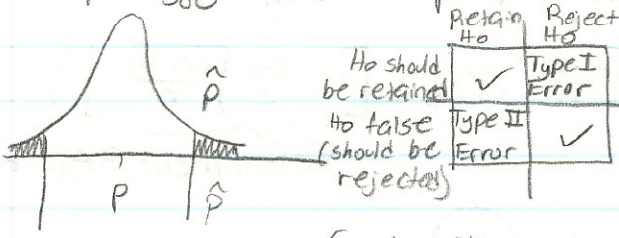
$\hat{p} \sim N(.2, (\sqrt{\frac{.2 \cdot .8}{380}})^2)$

$.0525$ $.0205^2$

Ret Region $= [.2 \pm 2.56 \cdot .0205]$

$= [.1475, .2525]$

$\hat{p} = \frac{69}{380} = .1816$ $\hat{p} \in$ Ret Region ⇒ Retain H_0



	Retain H_0	Reject H_0
H_0 should be retained	✓	Type I Error
H_0 false (should be rejected)	Type II Error	✓

$\alpha :=$ Prob(Type I Error)

As $\alpha \downarrow$ then P(type II error) ↑

As $\alpha \uparrow$ then P(type II error) ↓

Costs Type I, Type II error's

"more power" prob of failing the effect ⇒ As $n \uparrow$ ⇒ P(Type II error) ↓

$n \uparrow$ ⇒ P(Type I error) no change

Clinical Trial

H_0: drug does not work

H_a: drug does work

Type I error: drug doesn't work but you say it does

Type II error: drug does work but you say it doesn't

Fire Alarm System in Big Building

H_0: No fire

H_a: Fire

Type I Error: there's no fire but the alarm goes off

Type II Error: Fire but no alarm

$\hat{p}$ is a random variable

$\hat{p}$ is a <u>random</u> realization (data)

with random data, you can make errors

$H_0: p = .5$

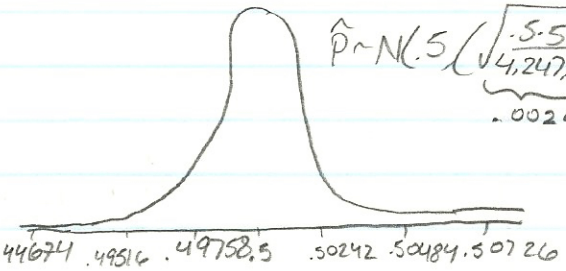
$H_a: p \neq .5$

$\alpha = 5\%$

$n = 4{,}247{,}000$ babies born in 2008

$$\hat{p} = \frac{2{,}173{,}000}{4{,}247{,}000} = .51165$$

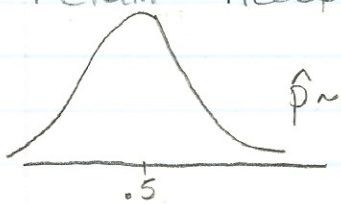
$\hat{p} \sim N\left(.5, \underbrace{\sqrt{\frac{.5 \cdot .5}{4{,}247{,}000}}}_{.00242}^{2}\right)$



Retainment Region $[.49516, .50484]$

$\hat{p}$ & Retainment Region $\Rightarrow$ Reject H_0

"Retain" = "Accept"



$\hat{p} \sim N(.5, .0269)$

$H_0: p = .501$ $H_a: \neq p .501$

RetRegion = $[.446, .554]$ $\hat{p} = .48 \in$ $\Rightarrow$ Retain H_0