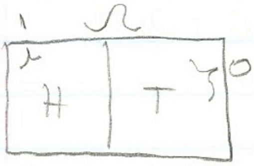
October 2, 2015 — Lecture 9



$$X \sim \begin{cases} 1 & \text{w.p. } \frac{1}{2} \\ 0 & \text{w.p. } \frac{1}{2} \end{cases}$$

$P(\{H\}) = \frac{1}{2}$

$X : \Omega \to \mathbb{R}$

$P : 2^{\Omega} \to [0,1]$

$$P(X=1) := P(\{\omega : X(\omega) = 1\}) = P(\{H\}) = \frac{1}{2}$$

Support of a r.v. $\text{Supp}[X] := \{x : P(X=x) > 0\} \subseteq \mathbb{R}$

$\text{Supp}(X) = \{0,1\}$ $\quad \sum_{x \in \text{Supp}(x)} P(X=x) = 1$

<u>Proof</u>: $\Omega = \dot{\bigcup}_{x \in \text{Supp}(x)} \{\omega : X(\omega) = x\}$

$$P(\Omega) = P(\bigcup \{\omega : X(\omega) = x\})$$
$$P(\Omega) = \sum P(\omega : X(\omega) = x) \quad (c)$$
$$(a) \quad 1 = \sum P(X=x)$$

$X \sim \text{Bernoulli}(\frac{1}{2}) := \begin{cases} 1 & \text{w.p. } \frac{1}{2} \\ 0 & \text{w.p. } \frac{1}{2} \end{cases}$ $\quad \text{Supp}(X) = \{0,1\}$

brand name random variables

"parameter"

$X \sim \text{Bernoulli}(p) := \begin{cases} 1 & \text{w.p. } p \\ 0 & \text{w.p. } 1-p \end{cases}$

$p \in [0,1)$

parameter space

Degenerate r.v.

$\text{Deg}(c) := \{ c \text{ w.p. } 1$

$X \sim \text{Bernoulli}(0) := \begin{cases} 1 & \text{w.p. } 0 \\ 0 & \text{w.p. } 1 \end{cases} \overset{d}{=} \deg(0)$

If $|\text{Supp}(X)| \leq |\mathbb{N}|$ $\Rightarrow$ X is called a discrete r.v.

countable infinity

For discrete random variables

$p(x) := P(X = x)$

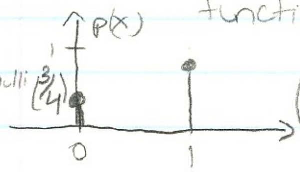
↑

probability mass function (pmf)

(I) $\sum_{x \in Supp[X]} p(x) = 1$

(II) $p(x) \in (0,1)$

$X \sim Bernoulli(\frac{3}{4})$

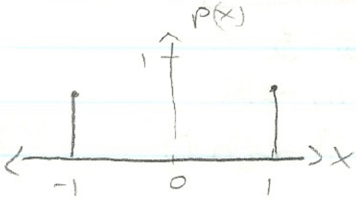


(pmf plot)

$X \sim Rademacher := \begin{cases} 1 & w.p. \ \frac{1}{2} \\ -1 & w.p. \ \frac{1}{2} \end{cases}$

↑ "random walk"

r.v.



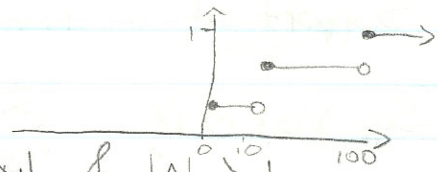
(uniform discrete)

$X \sim Unif(\overbrace{\{1, 10, 100\}}^{A}) := \begin{cases} 1 & w.p. \ \frac{1}{3} \\ 10 & w.p. \ \frac{1}{3} \\ 100 & w.p. \ \frac{1}{3} \end{cases}$

parameter: A

Parameter space: $A \subset \mathbb{R}$ s.t. $|A| \leq |\mathbb{N}|$ & $|A| \geq 1$

$Supp.(X) = A$



For any r.v. not necessarily discrete

$F(x) := P(X \leq x)$

↑

"Cummulative distribution function"

or "distribution function

(I) $F(x) \in [0,1]$

(II) $\lim_{x \to \infty} F(x) = 1$

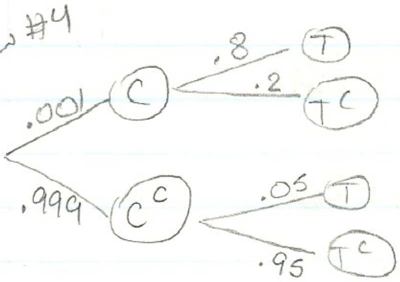
(III) $\lim_{x \to -\infty} F(x) = 0$

(IV) $x < y \Rightarrow F(x) \leq F(y)$

How many discontinuities in $F(x)$?

Review for Midterm 1

From HW #4

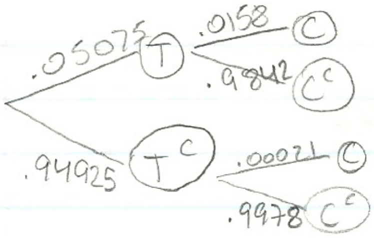


$P(T,C) = .0008$

$P(T^c,C) = .0002$

$P(T,C^c) = .04995$

$P(T^c,C^c) = .94905$



$P(T,C) = .0008$

$P(T,C^c) = .04995$

$P(T^c,C) = .0002$

$P(T^c,C^c) = .94905$

$P(T) = P(T,C) + P(T,C^c)$

$= .0008 + .04995$

$= .05075$

$P(C|T) = \frac{P(T,C)}{P(T)}$ $P(T,C) \to .0008$, $P(T) \to .05$

$.0008 + .05(.999)$

(h) $= 1.6\%$

(i) =

(j) $= P(T|C^c)P(C^c)$

(n) $= \frac{P(C|T)}{P(C|T^c)} = \frac{1.6\%}{.021\%} = 75$

HW #3

(2) $A = \{\omega_1, \omega_2, \ldots\}$ $\quad A = A_1 \cup A_2 \cup \ldots$ ($A_1 = \{\omega_1\}$, $A_2 = \{\omega_2\}$) $\quad \{A_1, A_2 \ldots\}$ are disjoint

$P(A) = P(A_1) + P(A_2) + \ldots$ axiom (c)

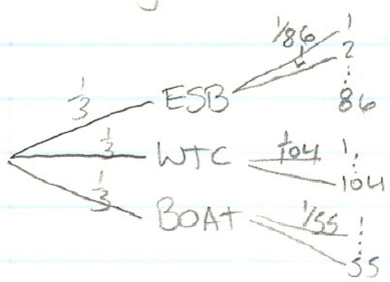
$= \sum_{i=1}^{\infty} P(A_i) = \sum_{i=1}^{\infty} P(\{\omega_i\})$

Midterm 1

Problem with the bridges

HW 4

Building Problem



$$P(ESB, 50) = \frac{1}{3} \cdot \frac{1}{86} = \frac{1}{258}$$

$$P(WTC, 50) = \frac{1}{3} \cdot \frac{1}{104} = \frac{1}{312}$$

$$P(BOA, 50) = \frac{1}{165}$$

$$P(BOAT \mid 50) = \frac{P(BOAT, 50)}{P(50)} = \frac{\frac{1}{165}}{\frac{1}{165} + \frac{1}{312} + \frac{1}{258}} = .355$$

$$P(50) = P(50, BOAT) + P(50, WTC) + P(50, ESB)$$

$$P(BOAT) = .333$$