Lecture 9 Oct 2, 2015 M241

$$P(X=\$1) := P\left(\{\omega : X(\omega) = \$1\}\right) = P(\{H\}) = \frac{|\{H\}|}{|\Omega|} = \frac{1}{2}$$

is abuse of notation for

due to equally likely ...

"Support of a r.v." $Supp[X] := \{X(\omega) : \omega \in \Omega\} \subseteq \mathbb{R}$

since $X : \Omega \to \mathbb{R}$

$= \{x : P(X=x) > 0\}$

why not $\geq$?

r.v. (big) realiz (small)

$$\sum_{x \in Supp(X)} P(X=x) = 1 \,?$$

Why...

$$\Omega = \bigcup_{x \in Supp[X]} \{\omega : X(\omega) = x\}$$

↖ sets are disjoint due to X being a function

$$\Rightarrow \underset{1 =}{P(\Omega)} \underset{(a)}{=} \sum_{x \in Supp(X)} P(\{\omega : X(\omega) = x\}) = \sum_{x \in Supp(X)} P(X = x)$$

"Something has to happen"! $P(\Omega) = 1$!!

$$X \sim \begin{cases} \$1 & \text{wp } \frac{1}{2} \\ \$0 & \text{wp } \frac{1}{2} \end{cases}$$

There are many Ω's and situations that can generate the same r.v. X.

Thus... we don't care about Ω anymore!

~~B~~2

This r.v. X is actually called:

$$X \sim \underbrace{\text{Bernoulli}}_{\text{"brand name r.v."}}\left(\tfrac{1}{2}\right) := \begin{cases} 1 & \text{wp } \tfrac{1}{2} \\ 0 & \text{wp } \tfrac{1}{2} \end{cases} \qquad \text{Supp}(X) = \{0, 1\}$$

More generally..

$$X \sim \text{Bernoulli}(p) := \begin{cases} 1 & \text{wp } p \\ 0 & \text{wp } 1-p \end{cases}$$

"with parameter p"

What are the valid values of p?

$p \in (0,1)$ "the parameter space"

any #!

but not $0 \Rightarrow X = 0$ always } $X \sim \text{Deg}(0)$

or $1 \Rightarrow X = 1$ always } $X \sim \text{Deg}(1)$

degenerate r.v. — technically a r.v. but not interesting... since non-random!

Parameter spaces will be defined as all non-degenerate cases

If $|\text{Supp}(X)| \leq |\mathbb{N}|$ i.e. finite or ctbly infinite, then X is called a "discrete r.v."

$\rightarrow$ midterm 2 (all discrete)

If X is discrete... $p(x) := P(X = x)$

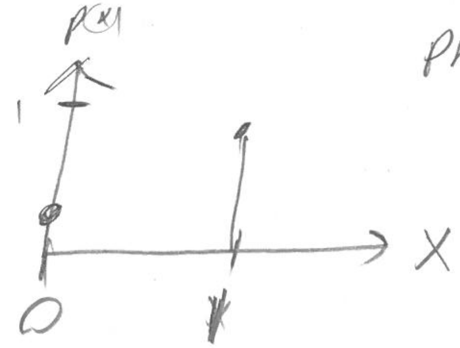
$\uparrow$ probability mass function (PMF)

$p(x) > 0 \ \forall x \in Supp(X)$
$p(x) = 0 \ \forall x \notin Supp(X)$

$p(x) \in [0, 1] \ \forall x$

$$\sum_{x \in Supp(X)} p(x) = 1 \text{ by def.}$$

$X \sim \text{Bernoulli}\left(\frac{3}{4}\right)$



PMF plot

$X \sim \text{Rademacher} := \begin{cases} -1 & \text{wp } \frac{1}{2} \\ 1 & \text{wp } \frac{1}{2} \end{cases}$

no params!

$X \sim \text{Unif}(\{1, 10, 100\}) := \begin{cases} 1 & \text{wp } \frac{1}{3} \\ 10 & \text{wp } \frac{1}{3} \\ 100 & \text{wp } \frac{1}{3} \end{cases}$

Uniform discrete

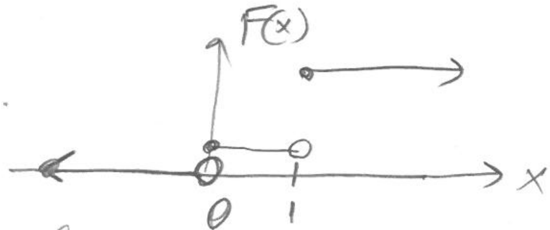
Parameters: $A \subset \mathbb{R}$, $|A| \in \mathbb{N} \setminus \{1\}$ if $|A| = 1 \overset{X\sim}{\Rightarrow} \text{deg}(a)$

$Supp(X) = A$

$F(x) := P(X \le x)$

Cumulative distr. function / distr. function

$X \sim \text{Bernoulli}\left(\frac{3}{4}\right)$



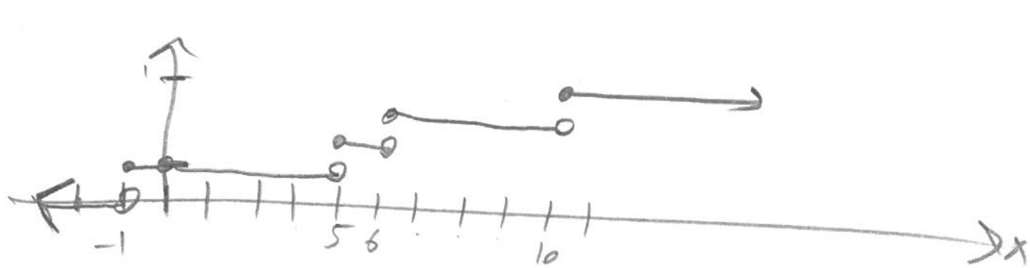
$F(-32) = 0$

$F(1001) = 1$

Consequences

1) $\lim_{x \to \infty} F(x) = 1$ 2) $\lim_{x \to -\infty} F(x) = 0$ 3) $F(x) \in [0,1] \ \forall x$ 4) $x \le y \Rightarrow F(x) \le F(y)$

$X \sim \text{Unif}(\{-1, 5, 6, 10\})$



For discrete r.v.'s $F(x)$ is always a discontinuous step function with

$|\text{Supp}(X)|$ # of discontinuities