$$E[X_1 + X_2] = \underbrace{\sum_{x_n} x_1 \sum_{x_2} p(x_1, x_2)}_{E[X_1]} + \underbrace{\sum_{x_2} x_2 \sum_{x_1} p(x_1, x_2)}_{E[X_2]}$$

$E[X_1] + E[X_2]$

X_1

$P(X_1 = 1, X_2 = 5) \stackrel{?}{=} P(X_1 = 1)\, P(X_2 = 5)$

	X_1: 1	7	9	
X_2: 5	1/15	1/3	2/15	16/30
23	1/30	1/10	1/30	5/30
28	1/3	1/5	1/15	3/30
	4/30	19/30	7/30	

$P(X_1\ X_2)$ JMF

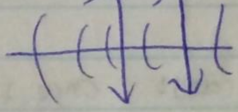
$P(X_2)$

$P(X_2 = 5)$

$P(X_2 = 5, X_1 = 1)$

$+ P(X_2 = 5, X_1 = 7)$

$P(X_2 = 5, X_1 = 9)$



$$g(x) = \int_{\mathbb{R}} f(x, x)\, dy$$

$\sum_{x_1} p(x_1, 5)$ ← margining out

$P(H) = P(A, B^c) + P(A, B)$

For all r.vs $X_1, \dots, X_n$ with finite expectations

$$E\left(\sum_{i=1}^{n} X_i\right) = \sum_{i=1}^{n} E[X_i]$$

$$E[\bar{X}] = E\left[\frac{T_n}{n}\right] = \frac{1}{n} E[T_n] = \frac{1}{n} \sum_{i=1}^{n} E[X_i] = \frac{1}{n} \sum_{i=1}^{n} \mu = \frac{1}{n} \mu \dots \mu = \boxed{\mu}$$

Under identical definition (distribution) of $X_1, \dots, X_n$

$E[T_n] = n\mu$

$\bar{X} \to \mu$ test

law of Large numbers

LLN

add up geometric get NegBin
↳ waiting for r thing to happen

Expectation of a geometric → $\frac{1}{p}$

$X_1, \ldots, X_r \overset{iid}{\sim} \text{Geom}(p)$

$T = X_1 + \ldots + X_r \sim \text{NegBin}(r, p)$

$E[T] = E[X_1 + \ldots + X_r] = r\,E[X_1] = \frac{r}{p}$

How many Success I get

$\cancel{X} \sim \text{Hyper}(n, K, N)$

identically distributed Bern(K/N)

$X = X_1 + \ldots + X_n$ ~ identically distributed Bernollies

$X_1 \sim \text{Bern}(K/N)$ $X_2 \sim \text{Bern}(K/N)$ ★

Success or Feulire # of successes total

iid ~ means look at the pmf. One is not equal to another.
coin 1 - pmf 1/2 coin 2 - pmf 1/2. Because PMF are the same, then they are iid.

Not independent, because once I get a successball: the other success is lower.

$X_1 \sim \text{Bern}\left(\frac{K}{N}\right)$ $X_2 \sim \left(\frac{K-1}{N-1}\right)$

$\star\ E[X] = E[X_1 + \ldots + X_n] = n\,E[X_1] = \boxed{n\,\frac{K}{N}}$

$$\sigma^2 = \text{Var}[X] := E[(X-\mu)^2]$$

$$= E[X^2 - 2\mu X + \mu^2]$$

$$= E[X^2] + E[-2\mu X] + E[\mu^2]$$

$$= E[X^2] - 2\mu \underbrace{E[X]}_{\mu} + \mu^2$$

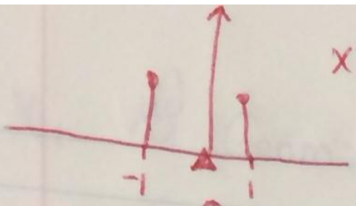
$$= E[X^2] - \mu^2$$

$$\Rightarrow E[X^2] = \sigma^2 + \mu^2$$

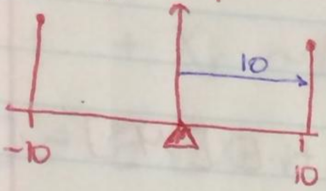
$$E[X-\mu] = E[X] - E[\mu]$$
$$= \underbrace{E[X]}_{\mu} - \mu = 0$$

$$\left.\begin{matrix} E[X] \\ E[X^2] \\ E[X^3] \end{matrix}\right\} \text{Moments} \qquad \left.\begin{matrix} E[(X-\mu)] \\ \sigma^2 = E[(X-\mu)^2] \\ E[(X-\mu)^3] \end{matrix}\right\} \text{central moments}$$

$$\left.\begin{matrix} \dfrac{E(X-\mu)}{\sigma} \\[2ex] \dfrac{E[(X-\mu)^2]}{\sigma^2} \\[2ex] \dfrac{E[(X-\mu)^3]}{\sigma^3} \\[2ex] \dfrac{E[(X-\mu)^4]}{\sigma^4} \end{matrix}\right\} \text{standarized moments}$$



X ~ Radom.



10 X

$a \in \mathbb{R}$

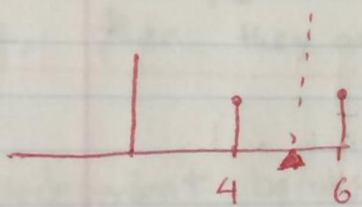
$$\text{Var}[\overbrace{aX}^{X}] = E[(\overbrace{aX - E[aX]}^{(X-\mu)})^2]$$

$$= a^2 \, \text{Var}[X]$$

$$= E[(aX - a\mu)^2]$$
$$= E[(a(X-\mu))^2]$$
$$= E[a^2(X-\mu)^2]$$
$$= a^2 E[(X-\mu)^2]$$
$$= a^2\sigma^2$$

$$SE[aX] = \sqrt{\text{Var}(aX)} = \sqrt{a^2\sigma^2} = |a\sigma| = |a|\sigma$$

Variance = σ mesures the distance from the pivot
Average sq distance from the pivot.



X+5

$c \in \mathbb{R}$

$$\sigma^2 = \text{Var}[X+c] = E[((X+c) - E[X+c])^2]$$
$$= E[((X+\cancel{c}) - (\mu+\cancel{c}))^2]$$
$$= \sigma^2$$

$$\text{Var}[aX + c]$$
$$= a^2 \, \text{Var}[X] = a^2\sigma^2$$

SE - Standard error

$$SE[aX + c]$$
$$= |a|\sigma$$

$$\mathrm{Var}[X_1 + X_2] = E\left[\left((X_1+X_2) - E[X_1+X_2]\right)^2\right]$$

$$= E\left[(X_1 + X_2 - \mu_1 - \mu_2)^2\right]$$

$$= E\left[X_1^2 + X_2^2 + \mu_1^2 + \mu_2^2 + 2X_1X_2 - 2X_1\mu_1 - 2X_2\mu_1 - 2X_1\mu_2 - 2X_2\mu_2 + 2\mu_1\mu_2\right]$$

$$= E[X_1^2] + E[X_2^2] + \cancel{\mu_1^2} + \cancel{\mu_2^2} + 2E[X_1X_2] - 2\cancel{\mu_1}^2 - 2\mu_1\mu_2 - 2\cancel{\mu_1\mu_2} - 2\cancel{\mu_2}^2 + 2\cancel{\mu_1}\mu_2$$

$$= \sigma_1^2 + \sigma_2^2 + 2\underbrace{\left(E[X_1X_2] - \mu_1\mu_2\right)}_{\mathrm{Cov}[X_1, X_2]}$$

$\sigma^2 = E[X^2] - \mu^2$

$\sum_{x_1}\sum_{x_2} x_1 x_2\, p(x_1, x_2)$ *

Covarience mesure the amount of dependence

if X_1, X_2 independent $p(a,b) = p(a)\cdot p(b)$

$\Rightarrow p(X_1, X_2) = p(X_1)\,p(X_2)$

$$\sum_{x_1}\sum_{x_2} x_1 x_2\, p(x_1, x_2)$$

$$= \sum_{x_1}\sum_{x_2} x_1 x_2\, p(x_1)\, p(x_2) = \sum_{x_1} x_1\, p(x_1) \sum x_2\, p(x_2)$$

$$= \mu_1 \mu_2$$

$$\boxed{\text{Under indp, } \mathrm{cov}[X_1, X_2] = 0 \Rightarrow \mathrm{Var}[X_1 + X_2] = \sigma_1 + \sigma_2^2}$$

If $X_1, \ldots X_n$ are indep.

$$Var\left[\sum_{i=1}^{n} X_i\right] = \sum_{i=1}^{n} Var[X_i] \overset{\text{if iid}}{=} n\sigma^2$$

<u>If indep</u>

$$SE[\Sigma X_i] = \sqrt{\Sigma Var[X_i]} \overset{\text{if iid}}{=} \sqrt{n\sigma} = \sqrt{n}\,\sigma$$

$$E[\bar{X}] = \mu$$

$$Var[\bar{X}] = Var\left[\frac{T_n}{n}\right] = \frac{1}{n^2} Var(T) \overset{\text{if } X_1 \ldots X_n \text{ indep}}{=} \frac{1}{n^2} \sum_{i=1}^{n} Var[X_i]$$

$$\overset{\text{if iid}}{=} \frac{1}{n^2} n\sigma^2 = \frac{\sigma^2}{n}$$

$$SE[\bar{X}] = \sqrt{\frac{\sigma^2}{n}} = \frac{\sigma}{\sqrt{n}}$$

if iid

$X_1, \dots, X_n \overset{iid}{\sim} \text{Bern}(p)$

$T = X_1 + \dots + X_n \sim \text{Binom}(n, p)$

$\text{Var}[T] = \sum_{i=1}^{n} \text{Var}[X_i] = \sum_{i=1}^{n} p(1-p) = np(1-p)$

$\text{Var}[X] = \sum_{x \in supp[X]} (x-\mu)^2 p(x) = \sum_{i=0}^{n} (n-np)^2 \binom{n}{x} p^x (1-p)^{n-x}$

$$E[X^2] - \mu^2 \qquad \sum_{i=0}^{n} x^2 \overbrace{\binom{n}{x}}^{\frac{n!}{x!(n-x)!}} p^x (1-p)^{n-x} - n^2 p^2$$

no for exam

$$= np \sum_{x=1}^{n} x \underbrace{\frac{(n-1)!}{(x-1)!(n-x)!}}_{\binom{n-1}{x-1}} p^{x-1} (1-p)^{n-x}$$

let $y = x-1$

$x = y+1$

$x = 1 \dots n$

$y = 0 \dots n-1$

$$= np \sum_{y=0}^{n-1} (y+1) \binom{n-1}{y} p^y (1-p)^{n-1-y}$$

let $m = n-1$

$$= np \left(\underbrace{\sum_{y=0}^{m} y \binom{m}{y} p^y (1-p)^{m-y}}_{E[Bin(n,p)] \;\; np} + \underbrace{\sum_{y=0}^{m} \binom{m}{y} p^y (1-p)^{m-y}}_{1} \right)$$

Bin(n,p)

$= np((n-1)p + 1) = np(np - p + 1)$

$= n^2p^2 - np^2 + np - n^2p^2 = \boxed{np(1-p)}$

$X \sim \text{Geom}(p)$

$$\text{Var}[X] = E[(X-\mu)^2]$$
$$= E[(X - \tfrac{1}{p})^2]$$
$$= \sum_{x=1}^{\infty} (x - \tfrac{1}{p})^2 (1-p)^{x-1} p = \dots = \boxed{\frac{1-p}{p^2}}$$

$X \sim \text{Geom}(p)$ $\quad P(X=17) = (1-p)^{16} p$

$$P(X=17 \mid X>10) = \frac{P(X=17 \,\&\, X>10)}{\underbrace{P(X>10)}_{(1-p)^x = (1-p)^{10}}} = \frac{P(X=17)}{(1-p)^{10}}$$

Failed 10 times

$$= \frac{(1-p)^{16} p}{(1-p)^{10}} = (1-p)^6 p = P(X=7)$$

$$P(X = b+x \mid X > b) = P(X=x) \quad \forall b \in \mathbb{N}$$

"Memorylessness"