$T \sim \text{Binomial}(n, p) := \binom{n}{x} p^x (1-p)^{n-x}$

$T = X_1 + \dots + X_n$ Such That $X_1, \dots, X_n \overset{iid}{\sim} \text{Bern}(p)$

- Exam
Think, are they Bernules, identical independent
If yes, then use binomial

* Flip 97 coins. Probability of 37 heads?

$$\frac{\frac{97!}{60!\,37!}}{2^{97}}$$

97 Bernulies
97 iid Bernulies - All identically distributed
no connected

$$T \sim \text{Binomial}(\underbrace{97}_{97\text{ coins}}, \underbrace{\tfrac{1}{2}}_{\text{Probability of heads}})$$

$$\underbrace{P(T = 37)}_{\text{Probability of 37 heads}} = \binom{97}{37} \overbrace{\left(\tfrac{1}{2}\right)^{37} \underbrace{\left(1 - \tfrac{1}{2}\right)}_{\frac{1}{2}}^{\overbrace{97-37}^{60}}}^{\frac{1}{2^{97}}}$$

Losing

* Bet 86x on Black

win or not - Bernuly
America - 18/38 to win
Bernuli
iid - independent

P(you win 52 Times)

$T \sim \text{Binomial}\left(86, \frac{18}{38}\right)$

$P(T = 52) = \binom{86}{52} \left(\frac{18}{38}\right)^{52} \left(\frac{20}{30}\right)^{34}$

* 5000 houses in Belle ..., NY

P(claim insurance for floding) $= \frac{1}{120}$

$T \overset{?}{\sim} \text{Binomial}\left(5000, \frac{1}{120}\right)$ No

home - Bernuly
* Not independent
Flood in one house, means flood on other houses.

* 96000 homes in the NE with property insurance

$P(\text{claim in NY}) = \frac{1}{250}$

$T \overset{?}{\sim} \text{Binomial}\left(96000, \frac{1}{250}\right)$ NO.

- Each home get claim or not ∴ Barnuly r.v.
- Each home identically distributed? share parameter p
  NO because p of claim is different in different areas
- Are they independent? - Mostly independent

Geometric

Wait until 1 success

$X_1, X_2, X_3 \ldots \overset{iid}{\sim} \text{Bern}(p)$ Keep flipping until you see 1 H.

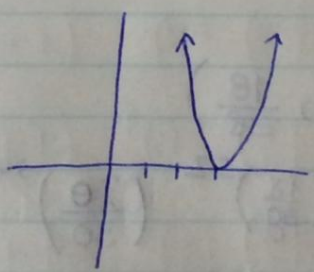
(Possible infinite)

At what point do we succeed?

$T := \underset{t}{\text{argmin}} \{X_t = 1\}$ 1 means Success

Stopping Time

$f(x) = (x-3)^2$



$\min_x (\{f(x)\}) = 0$

min over X.

$\underset{x}{\text{argmin}} (\{f(x)\}) = 3$

Which x so that min is true

$P(T=1) = P(X_1=1) = p$ $\underline{1}$

$P(T=2) = P(X_1=0, X_2=1)$ $\underline{0}$ $\underline{1}$

$= P(X_1=0) \cdot P(X_2=1)$

$= (1-p) \cdot p$

$P(T=3) = (1-p)^2 \cdot p$ $\underline{0}$ $\underline{0}$ $\underline{1}$

$\underbrace{\underline{0}\ \underline{0}\ \underline{0}\ \underline{0}\ \underline{0}}_{x-1}\ \overset{x}{\underline{1}}$ Last one

$P(T=4) = (1-p)^3 p$

$T \sim \text{Geometric}(p) := (1-p)^{x-1} p$

$P(T=x) = (1-p)^{x-1} p$

Parameter Space

$p \in (0,1)$

$p = 0$ illegal never success
$p = 1 \Rightarrow T \sim Deg(1)$

Support

whatever T can take on.

$Supp(T) = \mathbb{N}$

PMF – $\sum_{x \in Supp(x)} p(x) = 1$ Must be the case for the PMF all x in support must = 1

$$\sum_{x=1}^{\infty} (1-p)^{x-1} p = 1$$

$$\Rightarrow \sum_{x=1}^{\infty} (1-p)^{x-1} = \frac{1}{p}$$ substract 1

$$\Rightarrow \sum_{x=0}^{\infty} (1-p)^{x} = \frac{1}{p}$$ $q := 1-p$

show $\sum_{x=0}^{\infty} q^x = \frac{1}{1-q}$ Geometric Series.

$$S = 1 + \underbrace{q + q^2 + q^3 + \dots}$$

$$S = 1 + q\,(\underbrace{1 + q + q^2 + q^3 + \dots}_{S})$$

$S = 1 + qS \rightarrow S - qS = 1$

$(1-q)S = 1$

$S = \frac{1}{1-q}$

$$\sum_{x=0}^{\infty} a^x < \int_0^{\infty} a^x dx = \frac{a^x}{\ln(a)}\Big]_0^{\infty} = \frac{1}{\ln(a)}\left(\lim_{x \to \infty} a^x - 1\right)$$

if $|a| < 1$

$$F(x) := P(X \leq x) = \sum_{i=1}^{x} (1-p)^{i-1} p$$

$$= 1 - P(X > x)$$
$$= 1 - (1-p)^x$$

$$P(X \in A)$$
$$= 1 - P(X \notin A)$$
$$= 1 - P(X \in \text{Supp}(X) \setminus A)$$

Ex

$$F(3) = P(X \leq 3)$$
$$= P(X=1) + P(X=2) + P(X=3)$$
$$= 1 - P(X > 3)$$
$$= 1 - (P(X=4) + P(X=5) + \dots)$$

$$\left[\underline{0}\ \ \underline{0}\ \ \underline{0}\ \ \underline{0}\ \ \dots\ \ \underset{x}{\underline{0}}\right]\ \_\ \ \_\ \ \_$$

$T > x$

*Flip coin. P first H on 19th flip?

$X \sim \text{Geometric}\left(\frac{1}{2}\right)$ $\quad P(X=19) = \left(1-\frac{1}{2}\right)^{18}\frac{1}{2}$

PMF, drop it

iid bernulis

bernuli, idependent and stop on x flip they you get geometric

$$p(x) = (1-p)^{x-1} p$$

PMF

* Produce Chips

$P(\text{defective}) = \frac{1}{1000}$

Bernuly $\frac{1}{2}$ either defective or not.

$P\left(\begin{array}{l}\text{I see a defective chip} \\ \text{on } 873^{rd} \text{ run}\end{array}\right) = ?$

$(1-P)^{x-1} P$

$X \sim \text{Geometric}\left(\underset{P}{\frac{1}{1000}}\right) = P(x = 873) = \left(\frac{999}{1000}\right)^{873} \frac{1}{1000}$

Machine if machine is broken, then most chips can be broken $\therefore$ not totally independent.

* $P\left(\underset{\text{a defective chip}}{\text{I see}} \text{ before } 102^{nd} \text{ run}\right) = ?$

$1-(1-p)^x$

$P(101) = P(X \leq 101) = 1 - \left(\frac{999}{1000}\right)^{101}$

Wait until r successes $X_1, X_2, \ldots \overset{iid}{\sim} \text{Bern}(p)$

Wait until r successes

$T := \underset{t}{\text{argmin}} \left\{ \sum_{i=1}^{t} X_i = r \right\}$

Keep flippling until you get 17 heads.
Tell me how many flips you had.

$r=3$ 3 successes

$P(T=3) = p^3$ 1 1 1

$P(T=4) = 3(1-p)p^3$ _ _ _ 1   1 1 0 1   1 0 1 1   0 1 1 1

Fail once
got 3 successes

$P(T=5) = \binom{4}{2}(1-p)^2 p^3$ $\underbrace{1\ 1\ 0\ 0}_{x-1}$ 1   1 0 1 0 1   1 0 0 1 1

3 successes   0 1 0 1 1   0 1 1 0 1   0 0 1 1 1

order doesn't matter.

$$P(T=x) = \binom{x-1}{r-1}(1-p)^{x-r} p^{r}$$

$x-r$ → Fail

p^r → succeed r times

— PMF

$T \sim \text{Neg Bin}(r, p)$

p → how likely to get a success

Negative Binomial

Parameter Space

$r \in \mathbb{N}$

$p \in (0, 1)$

Bernoly

$p = 0 \Rightarrow$ Illegal (never win)

$p = 1 \Rightarrow$ Deg (r)

Bernoly | hyper | geometric | binomal | Neg binomial

hyper → 3 white marvels