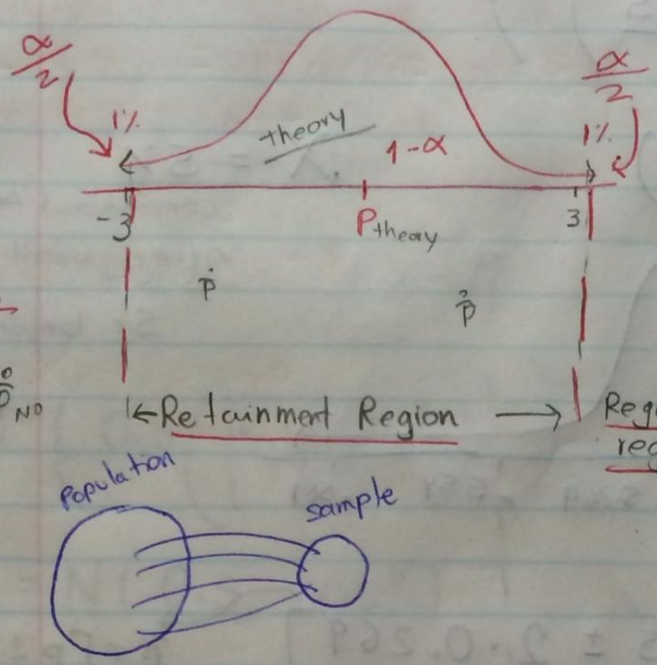
Theory M/F gender ratio is

Test theory

$p := P(\text{Male}) = 0.5$



$$\hat{P} \sim N\left(p, \left(\sqrt{\frac{p(1-p)}{n}}\right)^2\right)$$

at some point, people won't believe it anymore

α is %

α 2%

$\alpha := P(\hat{p} \text{ being "too" rare})$

$\hat{p} \in$ Retainment region $\Rightarrow$ Retain theory

$\hat{p} \notin$ Retainment region $\Rightarrow$ Reject theory

* α Probability of error

- $\alpha = 5\%$ → time theory sucks

going btw z & probability & vice versa

$$\alpha = P(|Z| \leq z_{\frac{\alpha}{2}})$$

$$= P(|Z| \leq 2)$$

$$= P\left(\left|\frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}}\right| \leq 2\right)$$

$$= P(-2 \leq \cdots \leq 2)$$

$$= P\left(-2\sqrt{\frac{p(1-p)}{n}} \leq \hat{p}-p \leq 2\sqrt{\frac{p(1-p)}{n}}\right)$$

$$= P\left(p-2\sqrt{\frac{p(1-p)}{n}} \leq \hat{p} \leq p+2\sqrt{\frac{p(1-p)}{n}}\right)$$

$$= P\left(\hat{p} \in \underbrace{\left[p \pm z_{\frac{\alpha}{2}}\sqrt{\frac{p(1-p)}{n}}\right]}_{\text{Retaining region}}\right)$$

* $n = 345$ babies

$$\hat{P} \sim N\left(0.5, \left(\sqrt{\frac{0.5 \times 0.5}{354}}\right)^2\right)$$

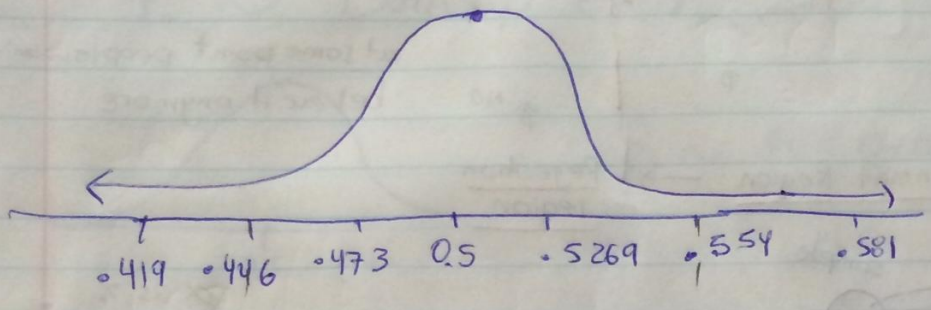
$$= N(0.5, 0.269^2)$$

$\alpha = 5\%$

conventional α

α time you don't cover

5% becomes a 2.



Retainment Region $= [0.5 \pm 2 \times 0.269]$ → $\hat{p} \in [p \pm z_{\frac{\alpha}{2}} \sqrt{p(1-p)/n}]$

$= [.446, .554]$

169 male babies

$\hat{p} = \frac{169}{345} = .48 \in$ Retain region

$\Rightarrow$ Retain theory

reject - No believe anymore

retain - No evidence sujects that the gender ratio is even

Is a coin fair? ($p = 0.5$) → "Null Hypothesis"

Flip coin 100 times.

statement about a theoretical parameter range. denoted "H_0"

Scen #1: 51 H ⇒ fair

Scen #2: 98 H ⇒ no fair

Scen #3: 61 H ⇒ ambiguous case

H_0: $p = 0.5$

H_a: Parameter Space \ $H_0: p \neq 0.5$

$(p \neq 0.5)$

Alternative Hypothesis

"H_0^c"

$\alpha = 5\%$

$$\hat{p} \sim N\left(0.5, \left(\sqrt{\frac{0.5 \cdot 0.5}{100}}\right)^2\right)$$

$$= N(0.5, .05^2)$$

Retaining region $= [0.5 \pm 2 \cdot .05]$

$= [0.4, 0.6]$

$\hat{p} = 0.61 \notin$ Retai region → Reject H_0 (accept H_a) "coin is not Fair"

If $\alpha = 1\%$ | lower – coin is fair higher – coin is unfair

$$\hat{p} \sim N\left(0.5, \left(\sqrt{\frac{0.5 \cdot 0.5}{100}}\right)^2\right)$$

$$= N(0.5, .05^2)$$

Retain region $= [0.5 \pm 2.56 \cdot 0.5]$

$= [0.35, 0.65]$

$\hat{p} = 0.61$ now would be part of accepting region

Exam - Say why he chose the α

- Blue M&M.

$H_0: p = 0.2$ company is giving right #
$H_a: p \neq 0.2$

$\alpha = 1\%$ This make it harder to reject it

$\Rightarrow z_{0.5\%} = 2.56$

$\hat{p} = \frac{69}{380} = .1816$ ← # of blues

$n = 380$

22

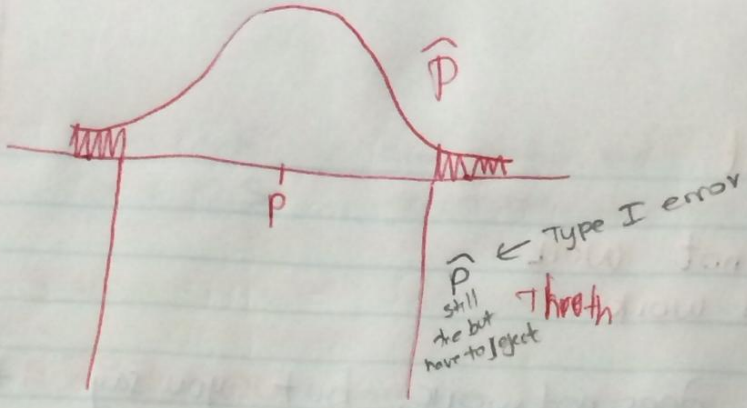
$$\hat{p} \sim N\left(0.2, \underbrace{\left(\sqrt{\frac{0.2 \times 0.8}{380}}\right)^2}_{.0205^2}\right)$$

$$\text{Rej Region} = [0.2 \pm \underbrace{2.56 \times 0.0205^2}_{0.0525}]$$

$$= [.1475, .2525]$$

$\hat{p} \in$ Rej Region $\Rightarrow$ retain H_0 ✓

∴ There is no evidence to suggests that the company is lying about their proportion



What % of the time you reject under the null hypothesis

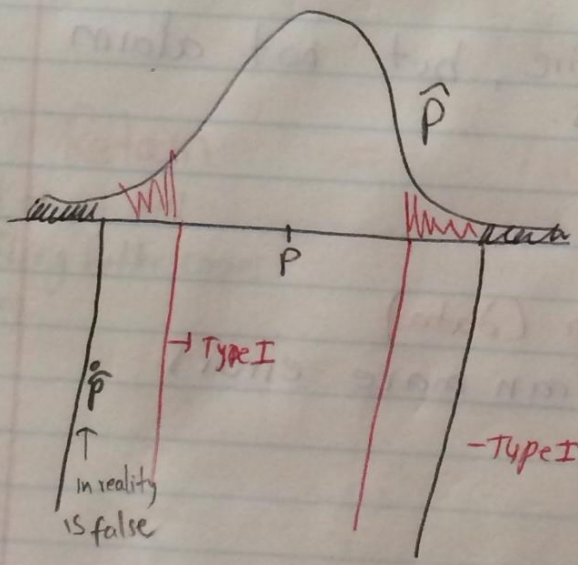
Our decision

	Retain H_0	Reject H_0
H_0 should be retained	✓	Type I error
H_0 false (should be rejected)	Type II error	✓

α: P(Type I error)

$\alpha\downarrow \Rightarrow$ P(Type II error) $\uparrow$

$\alpha\uparrow \Rightarrow$ P(Type II error) $\downarrow$



Cost

Type I, Type II errors

$n\uparrow \Rightarrow$ P(Type II error) $\downarrow$

getting more & more sensitive

more clarity. (Range is smaller)

P(Type I error) no change cause it's α

→ More "power"

↳ prob. of failing

no effect

- Clinical trial

Ho: drug does not work
Ha: drug does work

Type I error: drug does not work, but you say it does work
p is α
Cost - placebo,

Type II error: drug does work, but you say it doesn't
Cost - everyone that could be cured, won't be

- Big building Fire alarm System

Ho: Not fire
Ha: fire

Type I error: There is no fire, but the alarm goes off
Cost - Firemen & cost for city.

Type II error: There is a fire, but not alarm
Cost - lives.

* $\hat{p}$ is a r.v

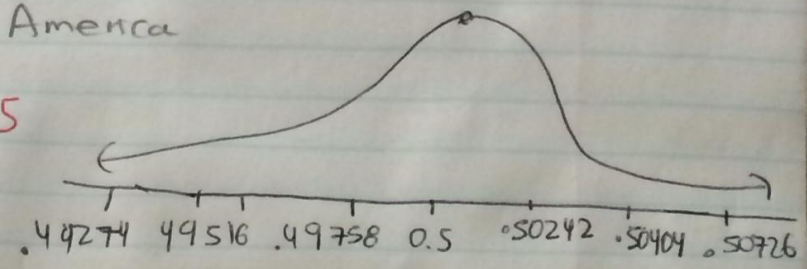
$\hat{p}$ is a random realization (data)
With random data, you can make errors

H_0: $p = 0.5$ gender ratio even
H_a: $p \neq 0.5$ gender ratio not even ✓ accepted it

$\alpha = 5\%$

$n = 4{,}247{,}000$ born in America

$\hat{p} = \frac{2{,}173{,}000}{4{,}247{,}000} = .51165 \neq 0.5$



Evidence suject that gender ratio is not even

$\hat{p} \sim N\left(0.5, \left(\sqrt{\frac{0.5 \times 0.5}{4247000}}\right)^2\right)$

$.00242^2$

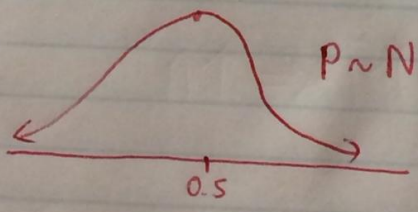
Retaining Region = $[.49516, .50484]$

Why more males than females?
- just don't know.

$\hat{p} \notin$ Retaining region $\Rightarrow$ Reject H_0

"Retain" = "Accept"

Keep it around till something better comes along.



$P \sim N(0.5, 0.269)$

Ret Region = $[.446, .554]$
$\hat{p} = 0.48 \in$ ↙ $\Rightarrow$ Retain