9/24/15 Lec 6 Math 241

Correction

Thm 3

$A \neq \emptyset \Rightarrow P(A) > 0$ is false!

unless $P(\{\omega\}) > 0 \ \forall \omega$ this is a technical pt.

$\Rightarrow$ Thm 4

$A \subset B \Rightarrow P(A) < P(B)$ is only true if $P(\{\omega\}) > 0$

but $A \subset B \Rightarrow P(A) \le P(B)$ is ALWAYS TRUE (see book/hw)

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10 flowers 4R, 3G, 3B

$$\binom{10}{4,3,3} := \frac{10!}{4!\,3!\,3!} =$$

$$(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i$$

$(a+b+c)^3 =$

$$(a+b+c)(a+b+c)(a+b+c) = \binom{3}{3,0,0} a^3 + \binom{3}{2,1,0} a^2 b + \binom{3}{2,0,1} a^2 c + \dots$$

$$(a+b+c)^n = \underbrace{\sum_{i+j+k=3}}_{\text{all ways to do this}} \binom{n}{i,j,k} a^i b^j c^k$$

$$(a_1 + a_2 + \dots + a_k)^n = \sum_{\sum_{i=1}^{k} i_k = n} \binom{n}{i_1, i_2, \dots, i_k} a_1^{i_1} a_2^{i_2} \cdots a_k^{i_k}$$ multinomial expansion formula

Thm 6

$P(A \cup B) \le P(A) + P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (Thm 4)

$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cup B) \ge 0$ (?)

$\Rightarrow P(A) + P(B) \ge P(A \cup B)$ ✓

Boole's Inequality

$$P\left(\bigcup_{i=1}^{n} A_i\right) \le \sum_{i=1}^{n} P(A_i)$$

on the HW

More review of counting

Birthday Problem

P(at least one pair of you share same bday) $= P$(one pair or two pairs or, ...) $= P\left(\bigcup_{i=1}^{\binom{25}{2}} \{i \text{ pairs}\}\right)$

all of you

$= P$(one pair shares bday) + P(two pairs share bday) + ... + $P\left(\binom{25}{2} \text{ share same bday}\right)$

disjoint sets

HARD calculation

$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum P(A_i)$ (c)

but

recall $P(A) = 1 - P(A^c)$

$= 1 - \underline{P(\text{none of you share same bday})}$

$= \dfrac{|\cdot|}{|S|} = \dfrac{365 \cdot 364 \cdots}{365 \cdot 365 \cdots 365} = \dfrac{{}_{365}P_{25}}{365^{25}} = ?$

all possible bday configurations for 25 people

In general when $\dfrac{{}_{365}P_n}{365^n} = 0.5$? $n \approx 23$

$= 0.99$? $n \approx 60$

Flip two coins

$P(H_2 \text{ knowing } H_1 \text{ true}) = \frac{1}{2}$

We say the knowledge of the outcome of H_1 is informally irrelevant to the $P(H_2)$.

"Knowing" or "being given" or "being conditional on" is very important "Pipe symbol"

$P(A|B) = P(A)$ if B is informally irrelevant

or if A and B are "independent". Def of ind: $P(A|B) = P(A)$ or $P(B|A) = P(B)$

$P(\text{IBM stock} \uparrow \text{ today} \mid \text{raining in Buenos Aires}) = P(\text{IBM stock} \uparrow \text{ today})$

seems reasonable of..

$P(H,H) = P(H)\,P(H) = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$

and in general $A_1, A_2, A_3, \ldots A_n$ independent

$$P\left(\bigcap_{i=1}^{n} A_i\right) = \prod_{i=1}^{n} P(A_i)$$

"multiplication rule"

proven later in class

Chevalier de Mere..

$P(\geq 1 \text{ double } 6\text{'s in } 24 \text{ rolls}) = P(1 \text{ double } 6\text{'s in } 24 \text{ rolls or } 2 \text{ double } 6 \text{ or } 3 \text{ double } 6\text{'s or} \ldots \text{ or } 24 \text{ double } 6\text{'s})$

$= P(1 \text{ double } 6) + \ldots + P(24 \text{ double } 6\text{'s})$ (c)

but tedious - 24 calc's

don't know how to do these (yet)

Thm 1
$P(A) = 1 - P(A^c)$

$= 1 - P(0 \text{ double } 6\text{'s in } 24 \text{ rolls}) = P(\text{not double } 6 \text{ first roll} \cap \text{not double } 6 \text{ } 2^{nd} \text{ roll} \cap \ldots \cap \text{not double six } 24^{th} \text{ roll})$

$= 1 - \prod_{i=1}^{24} P(\text{not double six } i^{th} \text{ roll})$ by mult. rule

$= 1 - P(\text{not double six on a roll of two dice})^{24}$ assume repeatable conditions

$= 1 - P(\text{not a six} \cap \text{not a six})^{24}$

$= 1 - \left(P(\text{not a six})\,P(\text{not a six})\right)^{24}$ multiplication rule AGAIN

$= 1 - \left(\frac{5}{6}\cdot\frac{5}{6}\right)^{24} = \boxed{.4914}$

Opposite of independence: dependence

$P(A|B) \neq P(A)$ or $P(A \cap B) \neq P(A)P(B)$

$P(\text{Q64 late} \mid \text{rainy overnight}) \neq P(\text{Q64 late})$

$>$

$P(\text{lung cancer} \mid \text{smoker}) > P(\text{lung cancer})$

"unconditional" prob's

dependent events

"conditional prob's"

A, B disjoint. Are A, B independent?

$P(A|B) \stackrel{?}{=} P(A)$ No... if B happened, A could not have happened!

Proof

$P(A \cap B) \stackrel{?}{=} P(A)P(B)$

$= P(\phi) = 0 = \underset{\geq 0}{P(A)}\,\underset{\geq 0}{P(B)}$ true only if A, B have prob 0.

so anytime it's not trivial, disjoint $\Rightarrow$ dependence

If dependent $\Rightarrow$ disjoint? (HW)

Example of this ridiculous...

$P(H|T) \stackrel{?}{=} P(H)$

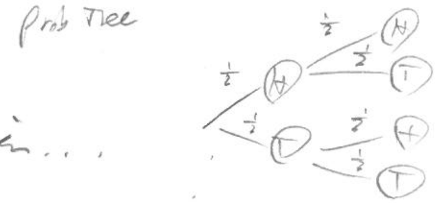
$0 \neq \frac{1}{2} \Rightarrow$ dependent

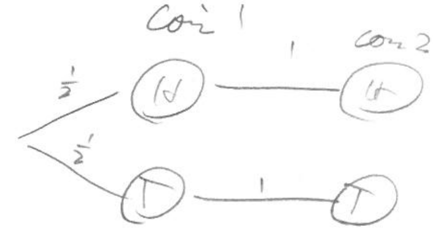
absurd!

$P(H_2|T_1) = P(H_2)$ Yes (independent)

unless...

Dependent Coin...





$P(H_1 \mid H_2) \overset{?}{=} P(H_1)$

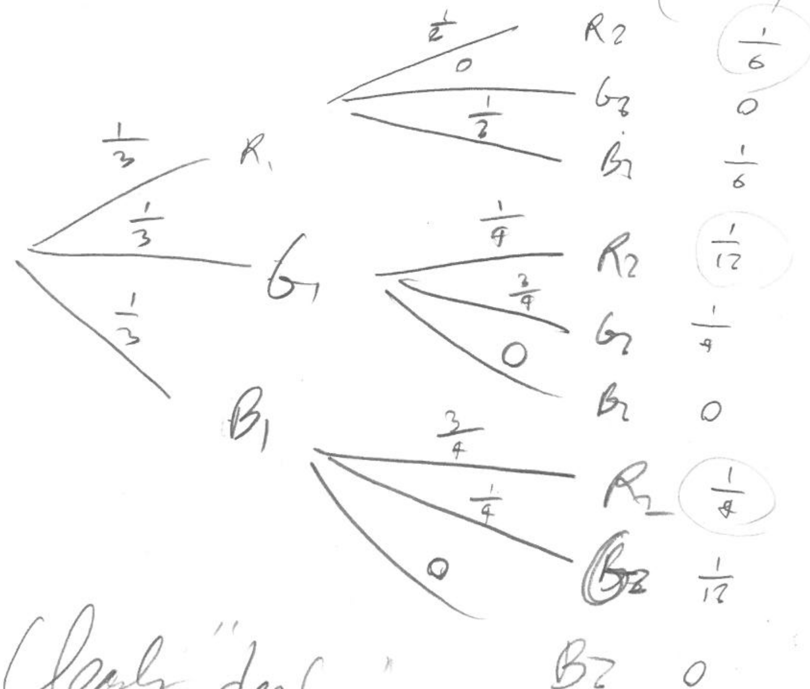
no temporal relationship implied

$1 \neq \frac{1}{2} \Rightarrow$ dependent

Sometimes "independent" does not mean "independence"

$P(R_1) = \frac{1}{3}$ $\qquad$ $P(R_2) = \frac{1}{2}$

If ind., $P(R_1 \cap R_2) = P(R_1)P(R_2) = \frac{1}{2}\frac{1}{3} = \frac{1}{6}$



$P(R_1)\,P(R_2)$

$= \frac{1}{3}\left(\frac{1}{6} + \frac{1}{12} + \frac{1}{4}\right)$

$= \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}$

Clearly "dependent" in English but independent probabilistically

$P(R_1, G_2) \overset{?}{=} P(R_1)P(G_2)$

$0 \neq \frac{1}{3} \cdot \frac{1}{3}$

R_1, G_2 dependent