uniform dones

$$X \sim \text{Bernolli}(p) = \underbrace{\begin{cases} 1 & wp \quad p \\ 0 & wp \quad 1-p \end{cases}}_{\text{inconvniente}}$$

Support $(x) = \{0,1\}$

$$\| $$

$$p^x (1-p)^{1-x} = P(x)$$

PMF

probability of mass fun

$X_1 \sim \text{Bern}(p)$, $X_2 \sim \text{Bern}(p)$

$X_1 \overset{d}{=} X_2$ equals in distribution - by definition is $F_1(x) = F_2(x)$ and for discrete r.v's $p_1(x) = p_2(x)$

10 cards, 4R, 6B

$$P\left(\begin{matrix}\text{getting 2R out of} \\ \text{a draw of 3 cards,} \\ \text{4R, 10 total}\end{matrix}\right) = \frac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}}$$

all possible ways of getting 3 cards

6 non Red

$$P\left(\begin{matrix}\text{getting x R cards,} \\ \text{draw of 3 with} \\ \text{4R, 10 total}\end{matrix}\right) = \frac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$$

$$P\left(\begin{matrix}\text{getting x Rcards} \\ \text{drawing } \underline{n} \text{ with} \\ \text{4R, 10 total}\end{matrix}\right) = \frac{\binom{4}{x}\binom{6}{n-x}}{\binom{10}{n}}$$

limits to what n and x can be.

$$P\left(\begin{array}{l}\text{getting } x \text{ R cards,} \\ \text{drawing } n \text{ with } K \\ \text{total R, 10 total} \\ \text{cards}\end{array}\right) = \frac{\binom{K}{x}\binom{10-K}{n-x}}{\binom{10}{n}}$$

$$P\left(\begin{array}{l}\text{getting } x \text{ R out} \\ \text{of } n \text{ draw, } K \text{ R,} \\ \text{and } N \text{ total cards}\end{array}\right) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

Sampling without replacement with successess and faikors

$$P\left(\begin{array}{l}\text{getting } x \text{ success} \\ \text{out of } n \text{ draws, } K \text{ total} \\ \text{success, } N \text{ total} \\ \text{possibilities}\end{array}\right) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

$X \sim p^x(1-p)^{1-x}$ $\quad p(x) = P(X=x)$

$X \sim \text{Hyppegeometric}(n, K, N) :=$

p btw 0 and 1.

Parameter Space

$N \in \mathbb{N} \setminus \{1\}$ any # exept 1

$K \in \{1, 2, \ldots, N-1\}$

$n \in \{1, 2, \ldots, N-1\}$

- N - total items can't be 0 or 1
- n - ballsthat can take out
- K - K of them that can be called sucesses (Marked as)

$$\text{Hyper}(1,1,2) := \frac{\binom{1}{x}\binom{1}{1-x}}{\binom{2}{1}} = \frac{\frac{1}{(1-x)!\,x!}\;\frac{1}{(1-x)!\,x!}}{2} \overset{d}{=} \text{Ben}\left(\frac{1}{2}\right)$$

Support[X]

everything that can pop out

$X \sim \text{Hyper}(1, K, N)$

Support[X] = {0, 1} → because only takes 1

$$\frac{\binom{K}{x}\binom{N-K}{1-x}}{\binom{N}{1}} = \frac{\frac{K!}{(K-x)!\,(x)!}\;\frac{(N-K)!}{((1-x)!}}{N} \overset{d}{=} \text{Ben}\left(\frac{K}{N}\right)$$

$$X = 0 \quad \frac{K!}{K!\,0!}\;\frac{(N-K)!}{(N-K-1)!\,1!} = \frac{N-K}{N}$$

$$X = 1 \quad 1 \qquad = \frac{K}{N}$$

1 or 0 → Bernuly

*

- $X \sim Hyper(2, 4, 10)$ — grab 2; 4 of those are success; total balls — Support[X] = {0, 1, 2} (R R R)

4S 10f

$n < K$, $n < N-K$ ← # of failures

general Support(x) = {0, ..., n}

- $X \sim Hyper(5, 4, 10)$ — grab; 4 S, 6 F; balls — n, K, N — Support(x) = {0,1,2,3,4} — 4 of them that exits

$n \geq K$, $n < N-K$

general Support(x) = {0, ..., K}

- $X \sim Hyper(8, 4, 10)$ Support(x) = {2, 3, 4}

4S 6F

1 S 7 F
but only have 6 F

$n \geq K$, $n \geq N-K$

general Support(x) = {n-(N-K), ..., K} — # of failures

- $X \sim Hyper(5, 7, 10)$ Support(x) = {2, 3, 4, 5} — only taking 5 of those

7S 3F

$n < K$, $n \geq N-K$ — # of failures

Supp(x) = {n-(N-K), ..., n}

failures	$n < K$	$n \geq K$
$n < N-K$	$\{0, \dots, n\}$	$\{0, \dots, K\}$
$n \geq N-K$	$\{n-(N-K), \dots, n\}$	$\{n-(N-K), \dots, K\}$

$$\text{Supp}(X) = \{ \text{Max}\left(0, n-(N-K)\right), \dots, \min(n, K) \}$$

$\sum p(x) = 1$ E.C

$x \in \text{Supp}(X) = 1$

equivalent parametirazation

$$X \sim \text{Bernuly}(p) := \begin{cases} 1 & \text{w.p. } p \\ 0 & \text{w.p. } 1-p \end{cases}$$

$\hookrightarrow p$ probability of Success

$$X \sim \text{Bernu}(q) := \begin{cases} 1 & \text{w.p. } 1-q \\ 0 & \text{w.p. } q \end{cases}$$

//

S.t $p := \frac{K}{N}$

$$X \sim \text{Hypp}(n, p, N) := \frac{\binom{pN}{x}\binom{(1-p)N}{n-x}}{\binom{N}{n}}$$

bag with N total balls

with p proportion of balls that are successful

& draw n balls out

- $N = 100$, $p = 0.5$, $n = 6$ $P(X=3) = 0.3223$
  $k = 50$

- $N = 1000$, $p = 0.5$, $n = 6$ $P(X=3) = 0.3134$

- $N = 10{,}000$, $p = 0.5$, $n = 6$ $P(X=3) = 0.3126...$

Sampling without replacement will converge to sampling with replacement after a huge number.

X - Hypper (, ,) Sampling without replacement