$Supp[T] = \{r, r+1 \ldots\}$

$= \mathbb{N} \setminus \{1, \cdots r-1\}$..

$\sum_{x \in supp(X)} P(x) = 1 \qquad \sum_{X=r}^{\infty} \binom{x-1}{r-1}(1-p)^{x-r} p^r$

10/22 $X_1, X_2, \cdots \overset{iid}{\sim} Ber(p)$

Lec 13 $X \sim NegBin(r, p) = \binom{x-1}{r-1}(1-p)^{x-r} p^r$

$= \binom{x-1}{r-1}(1-p)^{x-r} p^{r-1} \cdot p$

$Y \sim Binomial(x-1, p)$

$P(Y = r-1) = \binom{x-1}{r-1} p^{r-1} (1-p)^{x-r}$

$\overbrace{000 | 000 | \cdots}^{x-1} \underset{x}{1}$

$\underbrace{000|}_{r-1 \text{ success}}$

$Supp[x] = \{r, r+1 \ldots\}$

$= \mathbb{N} \setminus \{1, \cdots r-1\}$

$\sum_{x \in supp(x)} P(x) = 1 \qquad 1 = \sum_{x=r} \binom{x-1}{r-1}(1-p)^{x-r} p^r$

$\forall |x| < 1 \ldots (x-1)^{-1} = \sum_{i=1}^{\infty} x^{i-1}$

$\Rightarrow (-1)(-2)(x-1)^{-3} = \sum_{i=3}^{\infty} (i-2)(i-1) x^{i-3}$

$(r-1)!(1-x)^{-r} = \sum_{x=r}^{\infty} (i-1)(i-2) \ldots (i-r+1) x^{i-r}$

$= \frac{(i-1)!}{(i-r)!}$ mutiphy $(r-1)!$ both side.

$= \frac{(i-1)!}{(r-1)!(i-r)!}$ let $x = 1-p$

$= \binom{i-1}{r-1}$ $\qquad 1-x = p$

$x = i$.

X = # failures.

$p(x) = \binom{x+r-1}{x}(1-p)^x p^r$

$$\Rightarrow \frac{(x+r-1)!}{x!\,(r-1)!} = \frac{(x+r-1)(x+r-2)\cdots r}{x!} = \underbrace{(-1)^x \frac{-r(-r-1)\cdots(-r-x+1)}{x!}}_{\binom{-r}{x}}$$

Roll dice until you get 17 6's. chance that you rolled 107 times?

dice are independent, bec they don't talk to each other, if you get a 5 in first, you would not know about next one.

$X \sim \text{NegBino}(17, \frac{1}{6})$

$P(X=107) = \binom{106}{16}\left(\frac{5}{6}\right)^{90}\left(\frac{1}{6}\right)^{17}$

$X \sim \text{NegBin}(1, p) = \binom{x-1}{0}(1-p)^{x-1}p = \text{Geom}(p)$

$X_1, X_2, \ldots X_r \overset{iid}{\sim} \text{Geom}(p)$

$T = X_1 + X_2 + \cdots X_r \sim \text{NegBin}(r, p)$

$$\underbrace{0\ 0\ 0\ 0\ 1}_{X_1=5}\ \underbrace{0\ 0\ 1}_{X_2=3}\ \underbrace{0\ 0\ 0\ 0\ 0\ 1}_{X_3=6} \quad T=14.$$

$F(x) = P(X \le x) = \sum_{i=r}^{x}\binom{i-1}{r-1}(1-p)^{i-r}p^r$

$= 1 - P(X > x)$ Surviul function

$\cancel{= 1 - \sum_{i=x}^{\infty}\binom{i-1}{r-1}(1-p)^{i}}$

$= 1 - P(\le r-1 \text{ success by } x)$

$= 1 - P(0 \text{ success or } 1 \text{ success} \ldots \text{ or } r-1 \text{ success})$

$= 1 - (P(0 \text{ success}) + P(1 \text{ success}) \cdots + P(r-1 \text{ success}))$

$= 1 - (P(0) + P(1) \cdots P(r-1))$ Bin prob's $Y \sim \text{Binomial}(x, p)$

$P(Y = r-1) = \binom{x}{r-1}p^{r-1}(1-p)^{x-r+1}$

$= 1 - F(r-1)$ $\sim \text{Bin}(x, p)$

$$\begin{tikzcd}
\text{Hyper} \arrow[r, "\text{special case}"] \arrow[rr, bend left, "\text{limit}"] & \text{Bernoulli} \arrow[r, "\text{sum}"'] \arrow[dl, "\text{stopping}"'] \arrow[d] & \text{Binomial} \arrow[l, bend right, "\text{special case}"'] \\
\text{Geom} \arrow[r, "\cancel{\text{step}}", "\text{special case}"'] & \text{NegBin} &
\end{tikzcd}$$

$X_1, \ldots X_n \overset{iid}{\sim} p(x)$

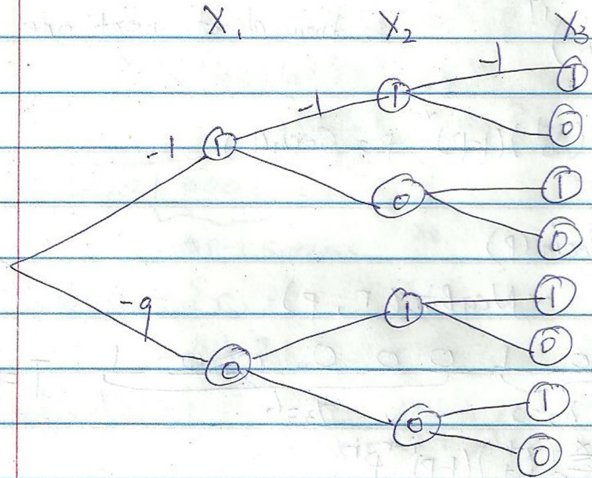
Define

$T_n = \sum_{i=1}^{n} X_i$

$\overline{X}_n = \frac{1}{n}\sum X_i = \frac{T_n}{n}$

"Avg. r.v"

$X_1, X_2, X_3 \overset{iid}{\sim} \text{Bern}(\frac{1}{10})$



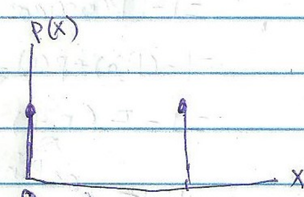
T	$\overline{X}$	Prob.
3	1	0.001
2	2/3	0.009
2	2/3	0.009
1	1/3	0.081
2	2/3	0.009
1	1/3	0.081
1	1/3	0.081
0	0	0.729
		1

$$\overline{X} \sim \begin{cases} 0 & \text{wp } \cancel{0.001}\ 0.729 \\ 1/3 & \text{wp } \cancel{0.0243}\ \cancel{0.027}\ 0.243 \\ 2/3 & \text{wp } \cancel{0.027}\ \cancel{0.243}\ 0.027 \\ 1 & \text{wp } \cancel{0.729}\ 0.001 \end{cases}$$

$X_1, X_2 \overset{iid}{\sim} \text{Ber}(\frac{1}{2})$

$X_1 = 0, \ X_2 = 1$

is a "realization" of X_1



$p(X_1 = x_1) = \frac{1}{2}$

Datum: is a realization of a r.v.

Data ---- s ---- s

$P(X = 5'6")$

height ~~high~~ of a random person

Sample Average

$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ is a realization of $\bar{X}$.

$X_1, \dots, X_6 \sim \text{Hyper}(3, 4, 8)$ 8 nicle in cup, 4 of them was mark (Success)

$x_1 = 3$

$x_2 = 2$ ← mark

$x_3 = 1$

$x_4 = 1$ $\bar{x} = \frac{3+2+1+1+0+2}{6}$

$x_5 = 0$

$x_6 = 2$ $= 1.5$

You can't get 8 success out of 4.
$X \sim \text{Hyper}(8, 4, 8)$.
without replacement

$X_1 \dots X_7 \sim \text{Binomial}(8, \frac{1}{2})$ ← Head

You can get 8 head
with replacement

$x_1 = 5$

$x_2 = 4$

$x_3 = 6$ $\bar{x} = \frac{5+4+6+4+6+3+7}{7}$

$x_4 = 4$

$x_5 = 6$ $= 5$

$x_6 = 3$

$x_7 = 7$

$X_1 \dots X_7 \overset{iid}{\sim} \text{Geom}(\frac{1}{2})$ ← until you get head

$x_1 = 1$ ← first time get Head

$x_2 = 1$

$x_3 = 2$ ← second time get head

$x_4 = 1$

$x_5 = 1$ $\bar{x} = \frac{1+1+2+1+1+2+1}{7}$

$x_6 = 2$

$x_7 = 1$ $= 1.\overline{285714}$