Discreete	Geometric	NegBinomial
Continuous	Exponential	Erlang

	PMF	PDF	CDF
Discreete	✓		✓
Continuous		✓	✓

→ Has the property Of Memorylessness

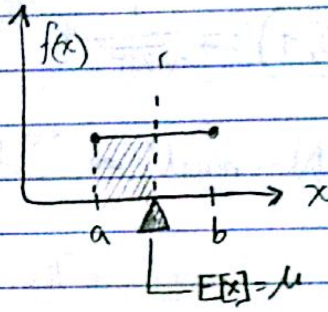
$$P(X > x_0 + x \mid X > x_0) = \dots = P(X > x)$$

$$= \frac{P(X > x_0 + x)}{P(X > x_0)} = \frac{e^{-\lambda(x_0+x)}}{e^{-\lambda x_0}} = e^{-\lambda x}$$

Nov 17 Lecture 18

Continuous R.V's.

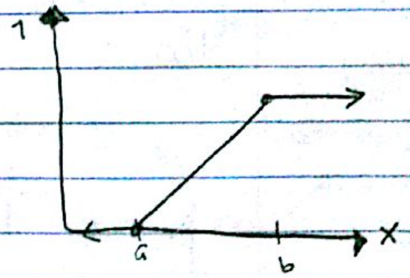
$\text{Supp}[X] = [a, b]$



$X \sim \text{Uniform}(a, b) := \frac{1}{b-a}$

Parameter Space:

$a \in \mathbb{R},\ b \in \mathbb{R},\ a < b$



$$F(x) = \int f(x)\,dx + C = \int \frac{1}{b-a}\,dx + C = \frac{x}{b-a} + C$$

→ To find "C" we need a point i

$$0 = F(x) = \frac{a}{b-a} + C \Rightarrow C = \frac{a}{b-a}$$

$$\text{Median}[X] = [X : F(x) = 0.5] \Rightarrow \frac{x-a}{b-a} = \frac{1}{2} \Rightarrow x - a = \frac{b-a}{2}$$

$$E[X] = \int_{x \in Supp[X]} x f(x)\,dx = \int_a^b \frac{1}{x(b-a)}\,dx = \frac{1}{ba}\left[\frac{x^2}{2}\right]_a^b = \frac{b^2-a^2}{2(b-a)} = \frac{\cancel{(b-a)}(b+a)}{2\cancel{(b-a)}} = \boxed{\frac{b+a}{2}}$$

$$\boxed{E[X] = \frac{b+a}{2}}$$

$$Var[X] = E[X^2] - \mu^2 = \int_a^b x^2 \frac{1}{b-a}\,dx - \left(\frac{b+a}{2}\right)^2 = \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b - \frac{b^2+2ab+a^2}{4} = \frac{b^3-a^3}{3(b-a)} - \ldots$$

$$= \frac{\cancel{(b-a)}(b^2+ab+a^2)}{3\cancel{(b-a)}} - \frac{b^2+2ab+a^2}{4} = \frac{4b^2+4ab+4a^2-3b^2-6ab-3a^2}{12}$$

$$= \frac{b^2-2ab+a^2}{12} = \frac{(b-a)^2}{12} = \sigma^2$$

$$Z \sim \mathbb{N}(0,1) := \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

"Standard Normal" = "Standard Gaussian" = "Std. Bell Curve"

$V(0,1) = 1$ $\quad$ $Supp[Z] = \mathbb{R}$ $\quad$ The only r.v that could be anything.

$F(x) = x$

$$\int_{Z \in Sup[Z]} f(z)\,dz = 1 \longrightarrow 1 = \int_{\mathbb{R}} \frac{e^{-x^2/2}}{\sqrt{2\pi}}\,dx \Rightarrow \int_{\mathbb{R}} e^{-x^2/2} = \sqrt{2\pi} = \int_{\mathbb{R}} e^{-n^2}\sqrt{2}\,dn = \sqrt{2\pi}$$

$$= \int_{\mathbb{R}} e^{-n^2}\,dn = \sqrt{\pi}$$

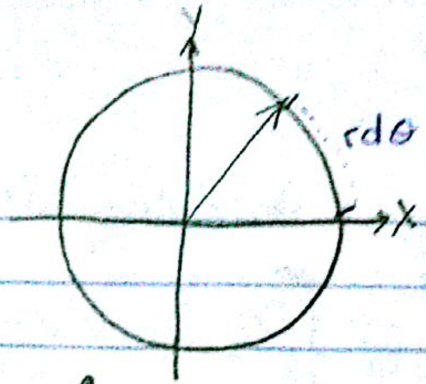
Gaussian Integral

$x^2+y^2=r^2$

$x=r\cos\theta$

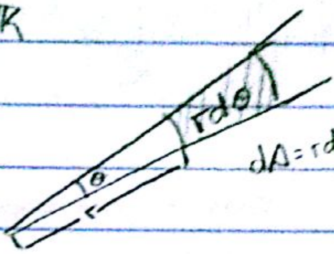
$y=r\sin\theta$

Gaussian Integral



$$\iint_{\mathbb{R}\,\mathbb{R}} e^{-(x^2+y^2)} \frac{dx\,dy}{dA} = \int_{\mathbb{R}} e^{-x^2}dx \int_{\mathbb{R}} e^{-y^2}dy = \sqrt{\pi}\times\sqrt{\pi} = (\sqrt{\pi})^2 = \pi$$

$$\iint e^{-r^2}dr$$



$$dA = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} drd\theta$$

$$= \begin{vmatrix} \cos(\theta) & -r\sin(\theta) \\ \sin(\theta) & r\cos(\theta) \end{vmatrix} drd\theta$$

$$= \int_{0}^{2\pi}\int_{0}^{\infty} e^{-r^2}\frac{dA}{r\,dr} = 2\pi\int_0^\infty e^{-r^2}r\,dr$$

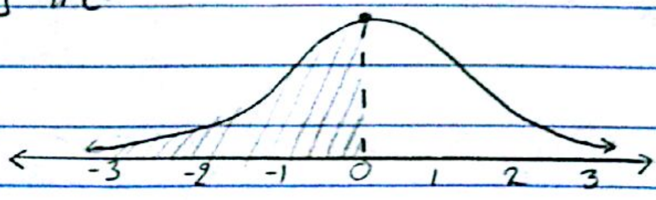
$\theta \quad r$

$$= r\sin^2\theta + r\cos^2\theta = r\,dr\,d\theta$$

$$= \cancel{2}\pi\int_{u_1}^{u_2} e^{-u}\frac{\cancel{r}\,du}{\cancel{2r}} = \pi\left[-e^{-u}\right]_{u_1}^{u_2}$$

$$= \pi\left(-e^{-\infty} - -e^{-0}\right) = \pi(0--1) = \pi$$

$\mathrm{Supp}[Z] = \mathbb{R}$



$\boxed{\text{The Point foreseen is: } \left(\overset{z}{0}, \overset{F_z(0)}{1/2}\right)}$

$$F_Z(x) = \int_{-\infty}^{x} \frac{e^{-x^2/2}}{\sqrt{2\pi}}dx \Longrightarrow F_Z(0) = 1/2 \longrightarrow (0, 1/2)$$

Not Possible
(Risch Algorithm)

Let $u = \frac{x^2}{2}$, $\frac{du}{dx} = x \Rightarrow dx = \frac{du}{x}$

$$E[X] = \dots = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} e^{-x^2/2} x \, dx = \frac{1}{\sqrt{2\pi}} \int_{u_1}^{u_2} e^{-u} \cancel{x} \frac{du}{\cancel{x}} = \frac{1}{\sqrt{2\pi}} \left(-e^{-u}\right)\Big|_{u_1}^{u_2}$$

$$= \frac{1}{\sqrt{2\pi}} \left(-e^{-\frac{x^2}{2}}\right)\Big|_{-\infty}^{+\infty} = \frac{1}{\sqrt{2\pi}} \left(-e^{-\frac{\infty^2}{2}} - -e^{-\frac{(-\infty)^2}{2}}\right) = \frac{1}{\sqrt{2\pi}}(0 - -0) = 0$$

Comes Pre-Standardized ("Standard Normal")

$$\left.\begin{aligned} P(Z \in [-1, 1]) &= 0.68 \\ P(Z \in [-2, 2]) &= 0.35 \\ P(Z \in [-3, 3]) &= 0.997 \end{aligned}\right\}$$ "68-35-99.7" Rule

"Empirical Rule"

$X \sim Exp(\lambda) = \lambda e^{-\lambda x}$

$Y = 2X \sim ?$ $\quad F_Y(x) = P(Y \leq x) = P(2X \leq x) = P\left(X \leq \frac{x}{2}\right) = F_X\left(\frac{x}{2}\right)$

$$f_Y(x) = \frac{d}{dy}[F_Y(x)] = \frac{d}{dx}\left[F_X\left(\tfrac{x}{2}\right)\right] = \frac{1}{2} f_X\left(\tfrac{x}{2}\right) = \frac{1}{2}\lambda e^{-\lambda(x/2)} = \frac{\lambda e^{-\lambda x/2}}{2} =$$

$Exp\left[\frac{\lambda}{2}\right]$

$$\boxed{Y = aX \Rightarrow E[Y] = E\left[\frac{\lambda}{a}\right] \quad a > 0}$$

$$X = \sigma Z + \mu \Rightarrow E[X] = \sigma E[Z] + \mu = \mu$$

$$Var[X] = \sigma^2 Var[Z] = \sigma^2$$

$$F_X(x) \Rightarrow P(X \le x) \Rightarrow P(\sigma Z + \sigma \le x) = P\left(Z \le \frac{x-\mu}{\sigma}\right) = F_Z\left(\frac{x-\mu}{\sigma}\right)$$

$$f_X(x) = \frac{d}{dx}\left[F_X(x)\right] = \frac{d}{dx}\left[F_Z\left(\frac{x-\mu}{\sigma}\right)\right] = f_Z\left(\frac{x-\mu}{\sigma}\right)\frac{1}{\sigma} = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}}{\sigma\sqrt{2\pi}}$$

$$X \sim N(\mu, \sigma) := \longrightarrow = \frac{e^{-\frac{(x-\mu)^2}{2\sigma}}}{\sqrt{2\pi\sigma^2}}$$

General Normal R.V.

Param Space:

$\mu \in \mathbb{R}$

$\sigma^2 \in (0, \infty)$

↳ Zero Excluded

$N(\mu, \sigma^2) = \text{Degenerate}(\mu)^{mu}$

$F_Z(x)$ ↗ Very Big Deal

$$P(X \in [\mu - \sigma, \mu + \sigma])$$
$$= P(\mu - \sigma \le X \le \mu + \sigma)$$
$$= P\left(-1 \le \frac{X-\mu}{\sigma} \le 1\right) = P(Z \in [-1, 1])$$