50%

$X \sim \text{Hypper}(6, 0.5, 100)$ How many sucesses

$P(X=3) = 0.3223$

$$\lim_{N\to\infty} (n, p, N) = \lim_{N\to\infty} \frac{\binom{pN}{x}\binom{(1-p)N}{n-x}}{\binom{N}{n}}$$

$$= \lim_{N\to\infty} \frac{\frac{(pN)!}{(pN-x)!\,x!} \frac{((1-p)N)!}{((1-p)N-(n-x))!\,(n-x)!}}{\frac{N!}{(N-n)!\,n!}}$$

$\lim f(x)\, g(x)$

$\lim g(x) \cdot \lim g(x)$

$$= \frac{n!}{(n-x)!\,x!} \lim_{N\to\infty} \frac{(N-n)!}{N!} \frac{(pN)!}{(pN-x)!} \frac{((1-p)N)!}{((1-p)N(n-x))!}$$

$$= \binom{n}{x} \lim_{N\to\infty} \frac{\overbrace{(pN)(pN-1)\cdots(pN-x+1)}^{x}\overbrace{((1-p)N)((1-p)N-1)\cdots((1-p)N-(n-x)+1)}^{n-x}}{\underbrace{(N)(N-1)(N-2)\cdots(N-n+1)}_{n}}$$

$$= \binom{n}{x} \overbrace{\underbrace{\lim_{N\to\infty} \frac{pN}{N}}_{p} \underbrace{\lim \frac{pN-1}{N-1}}_{p} \cdots \underbrace{\lim \frac{pN-x+1}{N-x+1}}_{p}}^{p^x} \cdot \overbrace{\underbrace{\lim \frac{(1-p)N}{N-x}}_{1-p} \underbrace{\lim \frac{(1-p)N-1}{N-x+1}}_{1-p} \cdots \underbrace{\lim \frac{(1-p)N-(n-x)+1}{N-n+1}}_{1-p}}^{(1-p)^{n-x}}$$

$$= \binom{n}{x} p^x (1-p)^{n-x} = p(x)$$

$N = \infty$ $\quad K = p.n$ $\quad n = \infty$

∞

$X \sim \text{Binomial}(n, p) :=$

Sampling with replacement

$n < K, \quad n < N-K$

$\text{Supp}[X] = \{0, \ldots, n\}$

$X \sim hypp$

Parameter space $\quad p \in \{\frac{1}{N} \cdots \frac{N-1}{N}\}$

$n \in \mathbb{N}$

$p \in (0,1)$

$\binom{n}{x} 0^x (1)^{n-x}$

$P(X=0) = \binom{n}{0} 0^0 1 \qquad 0^0 = ?$ undefined

$0^0 := \lim_{x \to 0} x^x = 1$

$\text{Binom}(n, 0) = \overset{\text{Degenerate}}{\text{Deg}}(0)$

$\text{Binom}(n, 1) = \text{Deg}(n)$

$$\boxed{X \sim \text{Deg}(c) := \{c \text{ wp } 1}$$

$p(x) > 0 \quad \forall x \in \text{Supp}[X]$ PMF

$P(\Omega) = 1$ because something has to happen

$$\sum_{x \in \text{Supp}[X]} p(x) = 1 = \sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x}$$

$$(a+b)^n = \sum_{x=0}^{n} \binom{n}{x} a^{n-x} b^x$$ Binomial theorem

$$\sum_{x \in Supp(x)} P(x) = 1 = \sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x}$$

$$= ((1-p)+p)^n = \sum_{x=0}^{n} \binom{n}{x} (1-p)^{n-x} p^x$$

Binomial Random variable

$\uparrow$ a $\uparrow$ b

$$1 = 1^n$$

* $X \sim \text{Binomial}(1, p)$ choosing 1 balls out of ∞ bag where p proportion # of successes

$$= \binom{1}{x} p^x (1-p)^{1-x}$$

$X \in \{0,1\}$ $\qquad \dfrac{1!}{x!(1-x)!}$

$$= p^x (1-p)^{1-x} = \text{Bernolling}(p)$$

- r.V. X_1 is "independent" of r.V X_2

if

$$P(X_1 = x_1 \mid X_2 = x_2) = P(X_1 = x_1)$$

Bayle Rule or

$$P(X_2 = x_2 \mid X_1 = x_1) = P(X_2 = x_2)$$

$$P(X_1 = x_1, X_2 = x_2) = P(X_1 = x_1) P(X_2 = x_2)$$

$\{0,1\} \times \{0,1\}$ $\forall x_1 \in Supp[X_1]$ $\forall x_2 \in Supp[X_2]$

Joint mass Function $\rightarrow$ $P_{1,2}(X_1, X_2) = P_1(x) P_2(x)$ $\leftarrow$ no for exam

$X_1 \sim \text{Bern}\left(\frac{1}{3}\right)$

$X_2 \sim \text{Bern}\left(\frac{1}{3}\right)$

$\text{Supp}[X_1, X_2] = \{0,1\}$

$X_1 \overset{d}{=} X_2$ && X_1, X_2 independent

have the same PFM
same support
identical distribution

Independent

$P(X_1 = 1 \mid X_2 = 1) = P(H_1 \mid H_2) = P(H_1) = P(X_1 = 1)$

$P(X_1 = 1 \mid X_2 = 0) = P(H_1 \mid T_2) = P(H_1) = P(X_1 = 1)$

$P(X_1 = 0 \mid X_2 = 1) = P(T_1 \mid H_2) = P(T_1) = P(X_1 = 0)$

$P(X_1 = 0 \mid X_2 = 0) = P(T_1 \mid T_2) = P(T_1) = P(X_1 = 0)$

$X_1 \overset{d}{=} X_2$ && X_1, X_2 independent

independent and identically distributed iid

$X_1, X_2 \overset{iid}{\sim} \text{Bern}\left(\frac{1}{3}\right)$

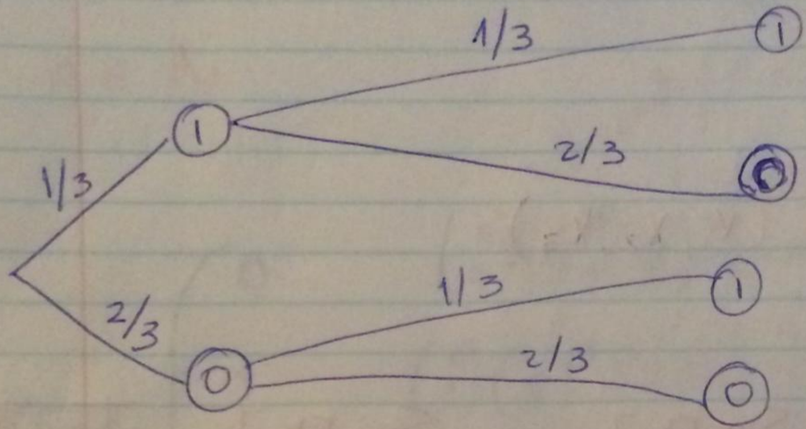
Same PFM
both are spitting out 0's and 1's
and they are identically distributed

$X_1, X_2 \overset{iid}{\sim} \text{Bern}\left(\frac{1}{3}\right)$

Independence

$X_1, X_2 = 1$ happen at the same time for both

Supp[X_1] Supp[X_2]



JMF	T
$\frac{1}{9}$	2
$\frac{2}{9}$	1
$\frac{2}{9}$	1
$\frac{4}{9}$	0
1	

$T_2 := X_1 + X_2$ new r.v

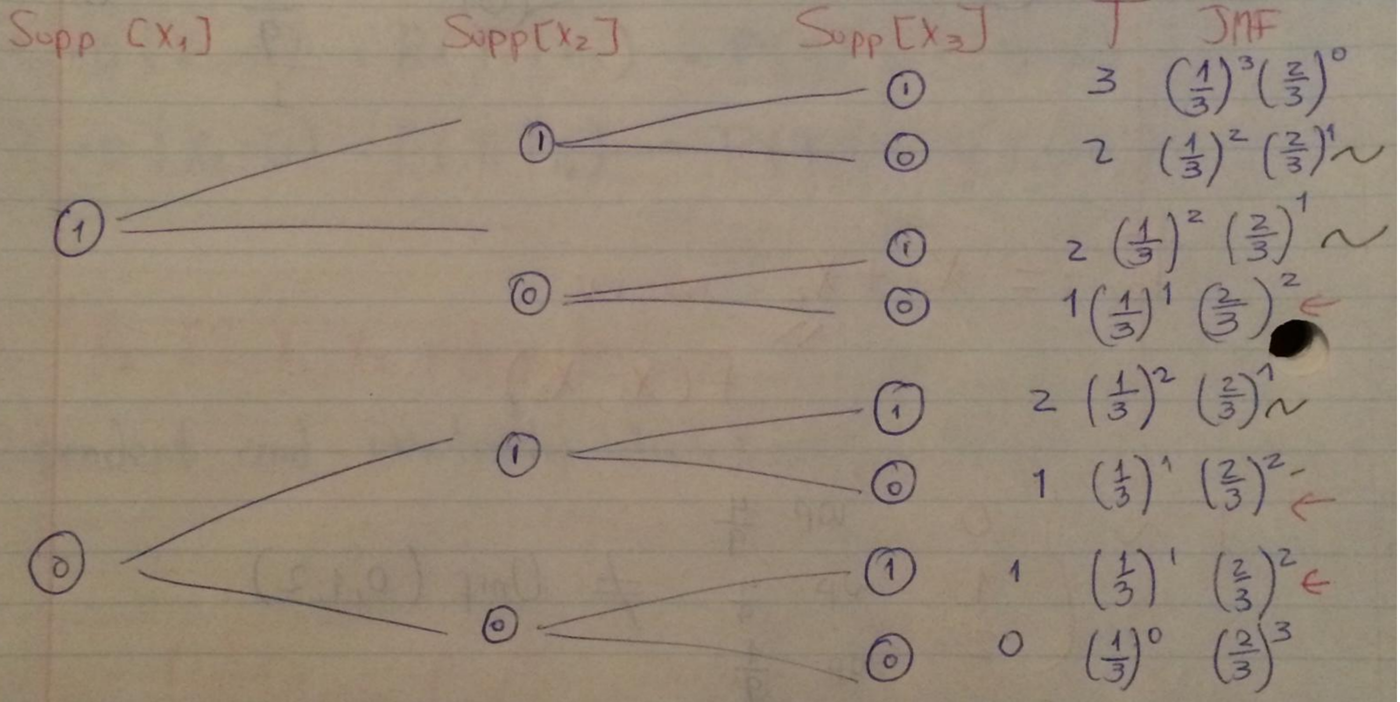
$= f(X_1, X_2)$

$$T \sim \begin{cases} 0 & \text{wp } \frac{4}{9} \\ 1 & \text{wp } \frac{4}{9} \\ 2 & \text{wp } \frac{1}{9} \end{cases} \neq \text{Unif}(0,1,2)$$

PMF of T

$X_1, X_2, X_3 \overset{iid}{\sim} \text{Bern}\left(\frac{1}{3}\right)$

$T_3 = X_1 + X_2 + X_3$
$= f(X_1, X_2, X_3)$



$$T \sim \begin{cases} 0 \quad \binom{3}{0} 1 \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^3 & \underline{0}\ \underline{0}\ \underline{0} \\ 1 \quad \binom{3}{1} 3 \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^2 & \underline{1}\ \underline{0}\ \underline{0},\ \underline{0}\ \underline{1}\ \underline{0},\ \underline{0}\ \underline{0}\ \underline{1} \leftarrow \\ 2 \quad \binom{3}{2} 3 \left(\frac{1}{3}\right)^2 \left(\frac{2}{3}\right)^1 & \underline{1}\ \underline{1}\ \underline{0},\ \underline{1}\ \underline{0}\ \underline{1},\ \underline{0}\ \underline{1}\ \underline{1} \sim \\ 3 \quad \binom{3}{3} 1 \left(\frac{1}{3}\right)^3 \left(\frac{2}{3}\right)^0 & \underline{1}\ \underline{1}\ \underline{1} \end{cases}$$

$X_1, \dots, X_n \overset{iid}{\sim} \text{Bern}(\frac{1}{3})$ $\quad T_n := \sum_{i=1}^{n} X_i$

$n \in \mathbb{N}$

succeed

Fail

$$T \sim \begin{cases} 0 & \binom{n}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^n \leftarrow \text{succeded 0 times, failed } n \text{ times} \\ 1 & \binom{n}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^{n-1} \\ 2 & \binom{n}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^{n-2} \\ \vdots & \\ n-2 & \binom{n}{n-2}\left(\frac{1}{3}\right)^{n-2}\left(\frac{2}{3}\right)^2 \\ n-1 & \binom{n}{n-1}\left(\frac{1}{3}\right)^{n-1}\left(\frac{2}{3}\right)^1 \\ n & \binom{n}{n}\left(\frac{1}{3}\right)^n\left(\frac{2}{3}\right)^0 \end{cases}$$

PFM for T

$$T \sim \begin{cases} 0 & \binom{n}{0}(p)^0(1-p)^n \\ 1 & \binom{n}{1}(p)^1(1-p)^{n-1} \\ 2 & \binom{n}{2}(p)^2(1-p)^{n-2} \\ \vdots & \\ n-2 & \binom{n}{n-2}(p)^{n-2}(1-p)^2 \\ n-1 & \binom{n}{n-1}(p)^{n-1}(1-p)^1 \\ n & \binom{n}{n}(p)^n(1-p)^0 \end{cases} = \binom{n}{x} p^x (1-p)^{n-x}$$

CDF

$$F(x) := P(X \leq x)$$

$$= \sum_{i=0}^{x} p(i)$$

$$= \sum_{i=0}^{x} \binom{n}{i} p^i (1-p)^{n-i}$$

$$I_{1-p}(n-k, 1+k)$$

$$:= (n-k)\binom{n}{k}\int_0^{1-p} t^{n-k-1}(1-t)^k dt$$

reg incomplete