11/8 : "Standard error / standard deviation"

$$6 := SE[X] = \sqrt{Var[X]}$$

$\overline{X}_T \rightarrow -\$0.053$

☆ The more concentrated the closer to the mean, the smaller the varience will get to the mean faster!!

$\overline{X}_B \rightarrow -\$0.053$ (faster)

A Fuction of 2 variables that splits out 1 #

$$\overbrace{g(X_1, X_2)}$$

• $T = X_1 + X_2$: "Expectation of Sum"

$$E[T] = \sum_{\underbrace{t \in Supp[T]}_{\text{Hard}}} t \, \overbrace{p(t)}^{\text{hard}}$$

$$f(X_1, X_2) = X_1^2 - \sqrt{X_2}$$

$$\mathbb{R}^2 \rightarrow \mathbb{R}$$

$$E[g(\underbrace{X_1, X_2}_{\vec{X}})] = \sum_{\vec{x} \in Supp(\vec{x})} g(\vec{x}) \, p(\vec{x})$$

$$= \sum_{(X_1, X_2)} g(X_1, X_2) \, p(X_1, X_2)$$ ← joint pass function.

Proof: $E[X_1 + X_2] = \sum_{(X_1, X_2) \in Supp} (X_1 + X_2) \, p(X_1, X_2)$

$$= \sum_{X_1 = X \in Supp[X_1]} \sum_{X_2 \in Supp[X_2]} X_1 \, p(X_1, X_2) + \sum_{X_2 \in Supp[X_2]} \sum_{X_1 \in Supp[X_1]} X_2 \, p(X_1, X_2)$$

$$= \sum_{X_1} X_1 \sum_{X_2} p(X_1 X_2) + \sum_{X_2} X_2 \sum_{X_1} p(X_1, X_2)$$

☆ Assume X_1, X_2 independent. $\Rightarrow p(X_1, X_2) = p(X_1) \, p(X_2)$.

$T = X_1 + X_2$

$$= \sum_{X_1} X_1 \sum_{X_2} \underbrace{p(X_1)}_{\text{constant}} p(X_2) + \sum_{X_2} X_2 \sum_{X_1} p(X_1) \underbrace{p(X_2)}_{\text{constant}}$$

Sum of PMF = 1.

$$= \underbrace{\sum_{X_1} X_1 \, p(X_1)}_{E[X_1]} \underbrace{\sum_{X_2} p(X_2)}_{1} + \underbrace{\sum_{X_2} X_2 \, p(X_2)}_{E[X_2]} \underbrace{\sum_{X_1} p(X_1)}_{1}$$

X_1

X_2 \ X_1	1	7	19	$\leftarrow P(X_1, X_2)$
5	$\frac{1}{15}$	$\frac{1}{3}$	$\frac{2}{15}$	$\frac{16}{30}$
23	$\frac{1}{30}$	$\frac{1}{10}$	$\frac{1}{30}$	$\frac{5}{30}$
88	$\frac{1}{30}$	$\frac{1}{5}$	$\frac{1}{15}$	$\frac{9}{30}$
	$\frac{4}{30}$	$\frac{19}{30}$	$\frac{7}{30}$	1

$p(X_2)$.

1 b/c its sum of P.M.F (Joint). Joint Mass Function

Not cover in 2H but helpful to understand the material.

- $Supp(X_1) = \{1, 7, 19\}$
- $Supp(X_2) = \{5, 23, 88\}$

Note: $P(B) = P(B, A_1) + P(B, A_2) + P(B, A_3)$

"Law of Total Probability"

$P(X_1 = 1) = \frac{4}{30}$

$= \sum_{X_2 \in Supp[X_2]} p(X_1 = 1, X_2 = X_2)$

Mass Function: P.M.F of $X_1 = p(X_1)$. Marginal

marginal of P.M.F itself

$$P(X_1) = \sum_{X_2 \in Supp[X_2]} p(X_1, X_2)$$

$$P(X_2) = \sum_{X_1 \in Supp(X_1)} p(X_1, X_2)$$

Thus, $E[X_1 + X_2] = \sum_{(X_1, X_2) \in Supp} (X_1 + X_2) p(X_1, X_2) = \sum_{X_1 \in Supp[X_1]} \sum_{X_2 \in Supp[X_2]} X_1 \, p(X_1, X_2) + \sum_{X_2} \sum_{X_1} X_2 \, p(X_1, X_2)$

$= \sum_{X_1} X_1 \underbrace{\sum_{X_2} p(X_1, X_2)}_{P(X_1)} + \sum_{X_2} X_2 \underbrace{\sum_{X_1} p(X_1, X_2)}_{P(X_1)}$

$= E[X_1] + E[X_2]$.

Turns out it doesn't matter if they are independent or not!

$P(X_1 = 1, X_2 = 88) \overset{?}{=} P(X_1 = 1) P(X_2 = 88)$

$\frac{1}{30} \neq \frac{4}{30} \quad \frac{9}{30}$

No

for any r.v.'s $X_1, \ldots, X_n$.

$E[X_1 + \ldots + X_n] = E[X_1] + \ldots + E[X_n]$.

$$E(T) = E\left[\sum_{i=1} X_i\right] = \sum_{i=1} E[X_i]$$

If $X_1, \ldots, X_n$ identically distributed w/exp. M.
(not necessily independent!)

$E[T] = nM$ $\quad$ $E[\bar{X}] = M$

Further $E[\bar{X}] = E\left[\frac{T}{n}\right] = \frac{1}{n} E[T] = \frac{nM}{n} = M$

Proof for Expectation – Geom
– Hyper.

waiting for the first r^{th} success... • Recall $X_1, X_2, \ldots, X_r \overset{iid}{\sim} \text{Geom}(p)$

$$\underbrace{\qquad}_{\parallel} \quad T \sim \text{NegBin}(r,p) \qquad \boxed{E[T] = \overset{nM}{\overset{\shortparallel}{\frac{r}{p}}}}$$

$$E[T] = \sum_{x=r}^{\infty} x \binom{x-1}{r-1} (1-p)^{x-r} p^r$$

$X \sim \text{Hyper}(1,K,N) = \text{Bern}\left(\frac{K}{N}\right) \leftarrow$ • $X \sim \text{Hyper}(n, K, N)$

$$E[X] = \sum_{x \in Supp[X]} x \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}} \qquad X = \underbrace{X_1 + X_2 + \cdots + X_n}_{\text{identically distributed}} \ \text{Bern}\left(\frac{K}{N}\right)$$

$$\Rightarrow \boxed{E[X] = n\frac{K}{N}}$$ (without doing it from def of $E[X]$).

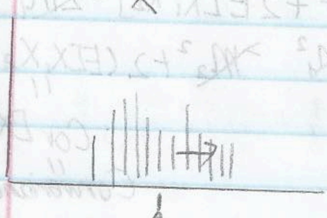
In each individual time, how far am I away... $\leftarrow$ • $Var[X] := E[(X-m)^2]$.

$= E[X^2 - 2mX + m^2]$
$= E[X^2] + E[-2mX] + E[m^2]$
$= E[X^2] - 2m^2 + m^2$
$\Rightarrow Var[X] = E[X^2] - m^2$
$\Rightarrow \boxed{E[X^2] = \sigma^2 + m^2}$

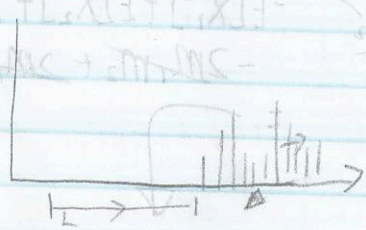
• $Var[X]$

$Var[X+c]$ $c \in \mathbb{R}$.

X



Var[X+c] don't change much.



$\boxed{Var[X+c]} = E[((X+c)-(m+c))^2]$
$= E[(X-m)^2]$
$= \boxed{Var[X]}$

They are pretty much the same...

Not cover just to see...

• – $E[X]$ 1^{st} moment
$E[X^2]$ 2^{nd} moment
$\vdots$
$E[X^k]$ k^{th} moment

– $E[|X-m|]$ 1^{st} center moment
$E[|X-m|^2]$ 2^{nd} center moment
$\vdots$
$E[|X-m|^k]$ k^{th} central moment.

– $\frac{E[|X-m|]}{\sigma}$ 1^{st} standardize moment
$\frac{E[|X-m|^2]}{\sigma^2} = 1$ 2^{nd} s.t.d moment
$\vdots$

Right light $\text{Skew}[X] = \frac{E[|X-m|^3]}{\sigma^3}$ 3^{rd} s.t.d moment.

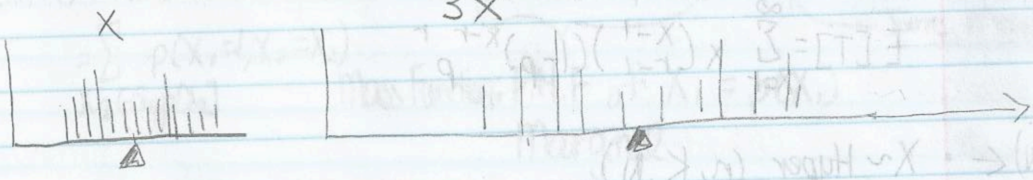
Tail fatness. $\text{Kurt}[X]$ $E\frac{[|X-m|^4]}{\sigma^4}$ 4^{th} s.t.d moment.

$\boxed{\begin{aligned} Var[aX] &= a^2 Var[X] \\ SE[aX] &= |a| SE[X] \end{aligned}}$

$Var[X] = E[(X-m)^2]$

- $Var[X]$

  $Var[aX]$ s.t. $a \in \mathbb{R}$.



$$Var[aX] = E[(\underbrace{aX-am}_{a(X-m)})^2] = E[a^2(X-m)^2]$$
$$= a^2 E[(X-m)^2]$$
$$= a^2 Var[X].$$

- $SE[aX] = \sqrt{Var[aX]} = \sqrt{a^2 Var[X]} = |a| SE[X]$

$\boxed{Var[aX+c] = a^2 \cdot \sigma^2}$

$\boxed{SE[aX+c] = |a| SE[X]}$

- Two r.v.'s X_1, X_2

$$Var[X_1+X_2] = E[((\overset{a}{X_1} + \overset{b}{X_2}) - (\overset{c}{m_1} + \overset{d}{m_2}))^2]$$
$$= E[X_1^2 + X_2^2 + m_1^2 + m_2^2 + 2X_1X_2 - 2X_1m_1 - 2X_2m_2 - 2X_1m_2 - 2X_2m_1 + 2m_1m_2]$$

$\sigma_1^2 + m_1^2$

$\sigma_2^2 + m_2^2$

$$= E[X_1^2] + E[X_2^2] + \cancel{m_1^2} + \cancel{m_2^2} + 2E[X_1X_2] - \cancel{2m_1^2} - \cancel{2m_2^2} - 2m_1m_2$$
$$- 2\cancel{m_1}m_2 + \cancel{2m_1m_2} - \cancel{m_1^2} - \cancel{m_2^2} + 2(\underbrace{E[X_1X_2] - m_1m_2}_{Cov[X_1,X_2] \,=\, \text{Covariance}})$$

$=$

Need to know

Rule $Var[X_1+X_2] = \sigma_1^2 + \sigma_2^2 + 2\,Cov[X_1, X_2]$

Special case:

- Two r.v.'s X_1, X_2 indep. $\Rightarrow Var[X_1 + X_2] = Var[X_1] + Var[X_2]$

$$E[X_1 X_2] = \sum_{x_1} \sum_{x_2} x_1 x_2 \, p(x_1, x_2)$$

$$= \sum_{x_1} \sum_{x_2} x_1 x_2 \, p(x_1) \, p(x_2)$$

$$= \sum_{x_1} x_1 p(x_1) \sum_{x_2} x_2 p(x_2) = E[X_1] \cdot E[X_2] = m_1 m_2.$$

$$Cov[X_1, X_2] = M_1 M_2 - M_1 M_2 = 0.$$

- If $X_1, X_2, \ldots, X_n$ independent. (not necessarily identically distributed).

$$\Rightarrow Var[T] = Var\left[\sum_{i=1}^{n} X_i\right] = Var[X_1 + \ldots + X_n] = Var[X_1] + \ldots + Var[X_n] = \sum_{i=1}^{n} \sigma_i^2$$

- $X_1, X_2, \ldots, X_n \overset{iid}{\sim}$

$$Var[T] = n\sigma^2 \Rightarrow SE[T] = \sqrt{n}\,\sigma$$

$$E[\bar{x}] = m$$

$$Var[\bar{x}] = Var\left[\frac{T}{n}\right] = \frac{1}{n^2} Var[T] = \frac{1}{n^2} n\sigma^2 = \boxed{\frac{\sigma^2}{n}} \Rightarrow \boxed{SE[\bar{x}] = \frac{\sigma}{\sqrt{n}}}$$