11/3

From last class

Just a model from a scenario. → $W \sim \begin{cases} 7 \text{ min} & \text{up } 0.7 \\ 12 \text{ min} & \text{up } 0.3 \end{cases}$

-Long term property for r.v → $E[W] = 7 \cdot 0.7 + 12 \cdot 0.3 = 8.5$ mins.

• Charge for min

– Uber charges \$0.40/min

$B = \$0.40/\text{min} \cdot W \sim \begin{cases} \$2.80 & \text{up } 0.7 \\ \$4.80 & \text{up } 0.3 \end{cases}$

$= g(W)$

$E[B] = \$2.80 \cdot 0.7 + \$4.80 \cdot 0.3 = \$3.12$.

simple way ⇒ $E[B] = \$0.40/\text{min} \cdot E[W]$

• r.v. X, r.v. $Y = aX$. $E[Y] = aE[X]$?

– Not Cover on Exam – HW.

$E[g(X)] = \int_\Omega gX(w)P(dw) = \int_{\{w:X(w)=x_1\}} gX(w)P(dw) + \int_{\{w:X(w)=x_2\}} gX(w)P(dw) + \dots$

If X discrease $Supp[X] = \{x_1, x_2, \dots\}$

$\Omega = A_1 \cup A_2 \cup \dots$

$= g(x_1)\int_{\{w:X(w)=x_1\}} P(dw) + g(x_2)\int_{\{w:X(w)=x_2\}} P(dw) + \dots$

The expectation $g(x)$ = the sum of what new X become.

$= g(x_1)\underbrace{P(X=x_1)}_{P(x_1)} + g(x_2)\underbrace{P(X=x_2)}_{P(x_2)} + \dots$

Cover! Rmb. ☆ $\therefore \boxed{E[g(x)] = \sum_{x \in Supp[X]} g(x)p(x)}$

$= \sum_{x \in Supp[X]} g(x)p(x)$

$-E[aX] = \sum_{x \in Supp[X]} a x p(x) = a \sum_{x \in Supp[X]} x p(x) = aE[X]$.

$\Rightarrow \boxed{E[aX] = aE[X]}$, by doing this, it's no longer necessary to do $\sim \begin{cases} x & \text{up} - \\ x & \text{up} - \end{cases}$

• Charge for Base Fare + Price for miles.

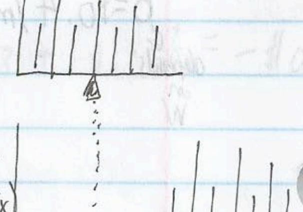
($= \$3$ above Base Fare)

New r.v $T = \underbrace{B + \$3}_{g(B)}$.

$E[T] = E[B] + \$3$

$= \$6.12$

☆ Adding \$3 to everything = moving the balance pt!



$Y = X + C, C \in \mathbb{R}$

$E[Y] = E[X+C]$

$= \sum_{x \in Supp[X]} (x+c)p(x)$

$= \underbrace{\sum_{x \in Supp[X]} x p(x)}_{} + c\overbrace{\sum_{x \in Supp[X]} p(x)}^{=1}$

$E[X+C] = E[X] + C$

$E[aX+C] = aE[X] + C$.

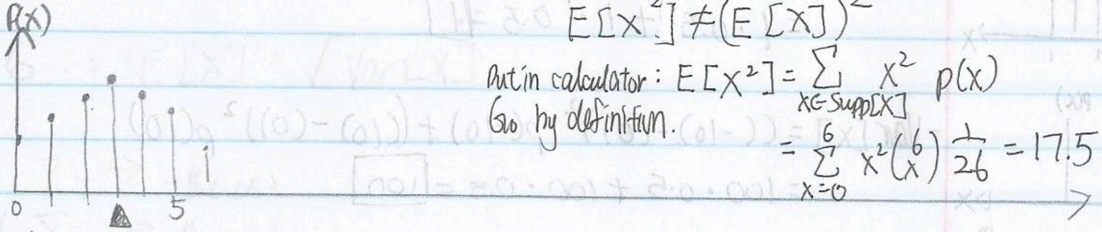
No Easy Way Out
by formula.

- $X \sim Bin(6, \frac{1}{2})$

$E[X]=3$

$E[X^2] \neq (E[X])^2$

☆ Plotting P.M.F from formula

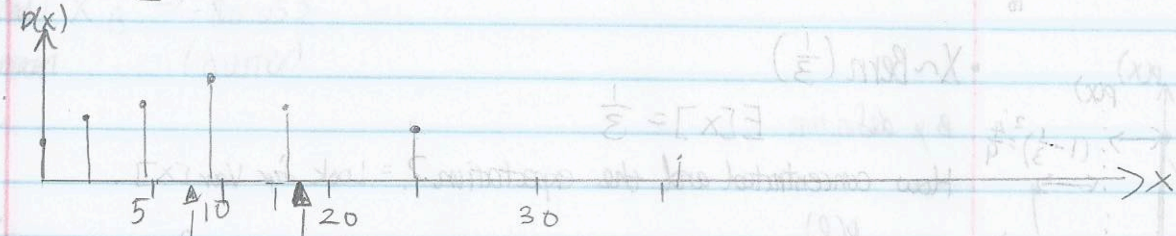


Put in calculator: $E[X^2] = \sum_{x \in Supp[X]} x^2 p(x)$

Go by definition.

$= \sum_{x=0}^{6} x^2 \binom{6}{x} \frac{1}{2^6} = 17.5$

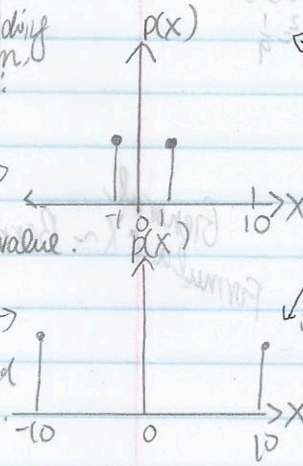
square it = $Y = g(X) = X^2$



~~No~~ ✓ by definition.

- $X \sim$ Radonmarker : $= \begin{cases} 1 & \text{up } \frac{1}{2} \\ -1 & \text{up } \frac{1}{2} \end{cases}$

Without doing calculation $E[X]=0$.



☆ more concentrated → supp = close to expected value.

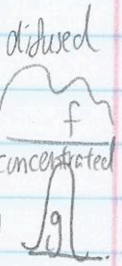
☆ diffused → supp further from expected value.

Different r.v. ! But in the long run = same

diffused

$\int_{\mathbb{R}} f(x)dx = 17$

$\int_{\mathbb{R}} g(x)dx = 17$



concentrated

$E[X]=0$

$Y = 10X$

$E[Y] = 10\, E[X] = 10 \cdot 0 = 0$

Distance from the mean - Theory of loss function

- $e(x, m) = x - m$ Not a loss function.

↓ make it love (add absolute value).

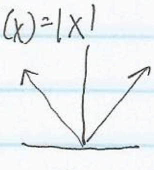
$- e(x,m) = |x-m|$ "Absolute Loss" L1 loss.

↓ could only optimize it! square it

→ to $\frac{d}{dx}$ to find the maxi & min. (CAN'T!)

$- e(x,m) = (x-m)^2$ "Square Loss" L2 loss $\sqrt{\int_{\mathbb{R}} f(x)^2 dx}$

$f(x) = |x|$



Could not minimize stuff!

could only optimize

$\overbrace{g(x)}$

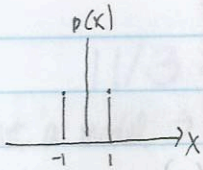
☆ $L = (X-m)^2$

In the long run, $E[L] = E[(X-m)^2] = \sum_{x \in Supp[X]} (x-m)^2 p(x)$. ☆ Variance[X].

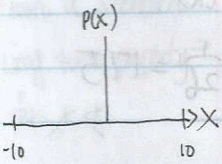
= how far away from balance point.

6^2

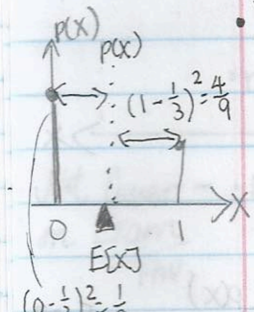
$6^2 = Var[X] = E[L] = E[X-m]^2 = \sum_{x \in Supp[X]} (x-m)^2 p(x)$ Big Definition.



- $Var[X] = ((-1)-(0))^2 p(-1) + ((1)-(0))^2 p(1)$
  $= 1 \cdot 0.5 + 1 \cdot 0.5 = \boxed{1}$



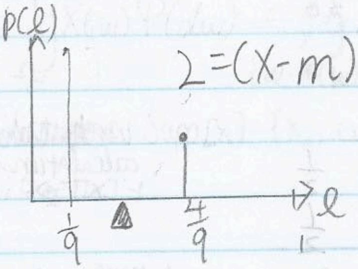
- $Var[X] = ((-10)-(0))^2 p(-10) + ((10)-(0))^2 p(10)$
  $= 100 \cdot 0.5 + 100 \cdot 0.5 = \boxed{100}$



- $X \sim Bern(\frac{1}{3})$

By definition $E[X] = \frac{1}{3}$

How concentrated and the expectation? = Look for $Var[X]$.



$l = (X-m)^2$.

$Var[X] = (0-\frac{1}{3})^2 \cdot \frac{2}{3} + (1-\frac{1}{3})^2 \cdot \frac{1}{3}$
$= \frac{1}{9} \cdot \frac{2}{3} + \frac{4}{9} \cdot \frac{1}{3}$
$= \frac{6}{27} = \frac{2}{9} = 0.259$

General Formula.

- $X \sim Bern(p)$

$E[X] = p.$

$Var[X] = (\underset{\text{part of Bern}}{0} - p)^2 \cdot \overset{p(0)}{(1-p)} + (\underset{\text{part of Bern}}{1} - p)^2 \overset{p(1)}{p}$

$= p^2(1-p) + (1-p)^2 p = 1-p(p^2 + (1-p)p) = \boxed{p(1-p)}$

- Roulette Bet on #7. Bet on Black

$X_7 \sim \begin{cases} \$35 & \text{wp } \frac{1}{38} \\ -\$1 & \text{wp } \frac{37}{38} \end{cases}$ $X_B \sim \begin{cases} \$1 & \text{wp } \frac{18}{38} \\ -\$1 & \text{wp } \frac{20}{38} \end{cases}$

- Expectations are the same
- Betting on Black = more concentrated

$E[X_7] = -\$0.053$ $E[X_B] = -\$0.053$

use calculator. $Var[X_7] = (\$35 - -\$0.053)^2 \cdot \frac{1}{38} + (-\$1 - -\$0.053)^2 \frac{37}{38} = 33.207.$ ☆ Unit $\$^2$ ← doesn't make sense! take the square root out.

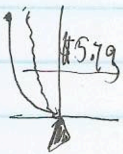
$\sqrt{Var[X_7]} = \$5.79$

$Var[X_B] = (\$1 - -\$0.053)^2 \cdot \frac{18}{38} + (-\$1 - -\$0.053)^2 \frac{20}{38} = 0.997\ \$^2.$

$\sqrt{Var[X_B]} = \$1.00$

"Standard error / Standard deviation"

$$\sigma := SE[X] = \sqrt{Var[X]}$$



$\bar{X}_7 \to -\$0.053$

$\bar{X}_{13} \to -\$0.053$

(faster)

☆ The more concentrated
the closer to the mean
‖
the smaller the
varience
will
get to the mean
faster!!