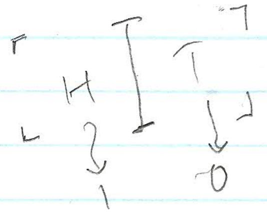
Lecture 8

$\Omega = \{H, T\}$

$n=3$

$\left.\begin{array}{l} \omega_1 = H \\ \omega_2 = H \\ \omega_3 = T \end{array}\right\} \to 1, 1, 0 = \bar{x} = \frac{1+1+0}{3} = \frac{2}{3}$ average

$\mathbb{I}_{\omega}\ H = \begin{cases} 1 & \text{if } \omega = H \\ 0 & \text{if } \omega = T \end{cases}$



Def.

Random Variable ("r.v")

$\underset{\uparrow}{X} : \underset{\uparrow}{\Omega} \longrightarrow \mathbb{R}$ values

r.v.   sample space of outcomes

$X(H) = 1$
$X(T) = 0$

$P(X=1) = \frac{1}{2}$

$P : 2^n \to [0,1]$

$P(\{\omega : X(\omega) = 1\})$
$= P(\{H\}) = \frac{|\{H\}|}{\Omega} = \frac{1}{2}$

<u>Support of a r.v.</u>

$\text{Supp}[X] = \{0,1\} \neq \mathbb{R}$

$17 \notin \text{Supp}[X]$
$P(X=17) = 0$

r.v.   arbitary value of the r.v.

Def. $\text{Supp}[X] = \{x : P(X = x) > 0\} \subseteq \mathbb{R}$

Def: A discrete r.v. X,

$|\text{Supp}(X)| = |\mathbb{N}|$

$1 = \sum_{x = \text{Supp}[X]} P = (X = x)$

fact: $\bigcup_{x \in \text{Supp}[X]} \{\omega : X(\omega) = x\} = \Omega$

$\Omega = \{w_1, w_2, w_3 \ldots\}$ s.t. $P(\{w_i\}) > 0$

$\neg \exists w \; X(w) \notin Supp[X] \Rightarrow P(\{w\}) = 0$

$\{w : X\{w\} = x_1\} \cap \{w : X(w) = x_2\} \neq \cancel{\emptyset}$

$X(w_0) = x_1 \;\&\; X(w_0) = x_2$

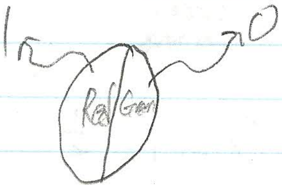
$p(X \in Supp[X]) = P(\Omega) = 1$

$P(\{w : X(w) = x_1\} + \{w : X(w) = x_2\} + \ldots = 1$

$P(X = x_1) + P(X = x_2) + \ldots = 1$

Ω

$1 \leadsto [H][T] \leadsto 0$



$X_1 = \begin{cases} 1 \text{ if } & w = H \\ 0 \text{ if } & w = T \end{cases}$

$X_2 = \begin{cases} 1 \text{ if } & w = Red \\ 0 \text{ if } & w = Green \end{cases}$

distribute as

with probability

$X \sim \begin{cases} 1 & w.p. \; \frac{1}{2} \\ 0 & w.p \; \frac{1}{2} \end{cases}$

* Don't need to consider the underline environment (Ω)

$\underline{supp[X] = \{0, 1\}}$

$X \sim Bernoulli(\frac{1}{2}) := \begin{cases} 1 & w.p. \; \frac{1}{2} \\ 0 & w.p. \; \frac{1}{2} \end{cases}$

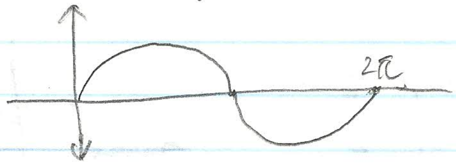
"Brand name"

More generally, $X \sim$ Bernoulli $(p) := \begin{cases} 1 & w.p. \; p \\ 0 & w.p \; 1-p \end{cases}$

X is distribut Bernoulli with "parameter" p. "Parametric model"

* possibility of 1 and 0 has to sum up to 1

$f(x) = \sin(x)$



$f(x, a) = \sin(ax)$

a is a constant

$a = 0$ * uninteresting trivial case

$f(x) = 0$

"degenerate"

$X \sim \text{Bern}(1) = \{ 1 \text{ w.p } 1$

$X \sim \text{Deg}(c) := \{ c \text{ w.p } 1$

$p \in (0,1)$ * everything between 0 and 1 but not "0" and "1"

"parameter space"

X is a distribute r.v.

$p(x) := P(X = x)$

r.v. free variable

Probability Mass Function (PMF)

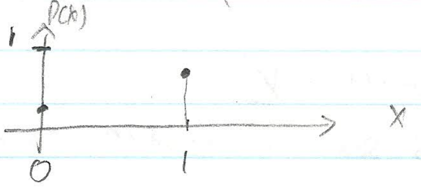
$p : \mathbb{R} \to [0,1)$

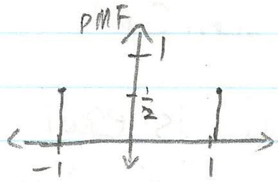
$$\sum_{x \in \text{supp}[X]} p(x) = 1$$

"standard random walk in 1-dimension"

$X \sim \text{Rademacher} : \begin{cases} +1 & \text{wp } \frac{1}{2} \\ -1 & \text{wp } \frac{1}{2} \end{cases}$

$X \sim \text{Bern}(\frac{3}{4})$

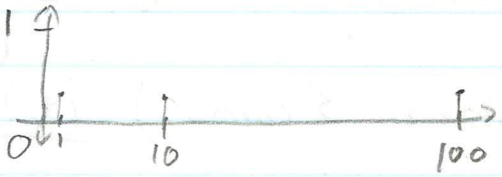




Discrete Uniform

$X \sim \text{uniform}(\{1, 10, 100\})$

$p(1) = \frac{1}{3}$
$p(2) = \frac{1}{3}$
$p(3) = \frac{1}{3}$



$X \sim \text{uniform}(A)$, $\text{supp}[X] = A$, $|A| = \mathbb{N} \setminus \{1\}$

subtraction

$A \subset \mathbb{R}$

subset

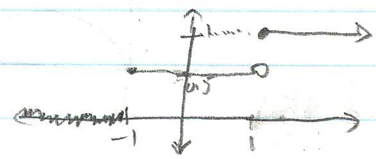
* Answer got to countable.

CDF

Def. Cummulative Distribution Function, "Distribute function"

$F(x) := P(X \le x)$

$X \sim \text{Redreulous} := \begin{cases} 1 & wp\ \frac{1}{2} \\ -1 & wp\ \frac{1}{2} \end{cases}$



Properties of the CDF

① $F(x) \in [0,1] \quad \forall x \in \mathbb{R}$

② $\lim_{x \to \infty} f(x) = 1$ ③ $\lim_{x \to -\infty} F(x) = 0$

④ $x \le y \Rightarrow F(x) \le f(y)$ * "monotonically increasing"

$x < y \Rightarrow f(x) < f(y)$

"strickly monotonically increasing"

$X \sim \text{Bern}(p) := p(x) = \begin{cases} 1 & wp\ p \\ 0 & wp\ 1-p \end{cases} = p^x (1-p)^{1-x}$

"PMF Bernolli"

* only valid for the support, other places

$X_1 \sim \text{Bern}(p)$

$X_2 \sim \text{Bern}(p)$

Def. X_1 is equal in distribution X_2

"$X_1 \overset{d}{=} X_2$", X_1 is same distribution to X_2

$P_1(x) = P_2(x) \sim F_1(x) = F_2(x)$

P(drawing x R in n card) $= \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$

"$X \sim \text{Hypergenermetric}(n, k, N)$"

10 cards

4R

6B

P(drawing 2 red in 3 cards) $\frac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}}$

P(drawing x red in 3 cards) $\frac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$

P(drawing x R in n cards) $\frac{\binom{4}{x}\binom{6}{n-x}}{\binom{10}{n}}$

10 cards

K = R

10 − K = B

P(x R in N cards) $= \frac{\binom{K}{x}\binom{n-k}{n-x}}{\binom{10}{n}}$