Ramdon variables ("r.v.")

9/27. $\Omega = \{H, T\}$ $n=3$

$1 \leftarrow$ | H | T | $\rightarrow 0$

$w_1 = H$
$w_2 = H$
$w_3 = T$
(not taking avg)

$\mathbb{1}_{w=H} = \begin{cases} 1 & \text{if } \omega = H \\ 0 & \text{if } w = T \end{cases}$

Taking avg $\rightarrow 1, 1, 0$ $\bar{X} = \frac{1+1+0}{3} = \frac{2}{3}$

Domain $= H, T = \Omega$
Range $= \{0, 1\}$

$P(\{w : X(\omega) = 1\}) = P(\{H\}) = \frac{|\{H\}|}{|\Omega|} = \frac{1}{2}$

$\underset{\text{r.v.}}{X} : \underset{\text{sample space of outcomes}}{\Omega} \rightarrow \underset{\text{values}}{\mathbb{R}}$

$X(H) = 1$
$X(T) = 0$

• What is the $P(X=1)$? $= \frac{1}{2}$

Note: $P : 2^{\Omega} \rightarrow [0,1]$

Support of a variable

Support $[X] = \{0, 1\} \neq \mathbb{R}$

Def: Support $[X] = \{x : P(\underset{\text{r.v}}{X} = \underset{\text{arbitrary value of the r.v}}{x}) > 0\} \subseteq \mathbb{R}$

b/c $17 \notin$ Supp $[X]$
$P(X=17) = 0$

Def: A (finite) discrete r.v. X,
$|\text{Supp}[X]| \leq |\mathbb{N}|$

The amt of things in that you can count to..

the sum of that gonna pop out $\rightarrow$ • $\sum_{x \in \text{Supp}[X]} P(X=x) = 1$

FACT

• $\bigcup_{x \in \text{Supp}[X]}^{\text{omega}} \{w : X(w) = x\} = \underset{\text{True}}{\Omega}$ / collectively ~~exclusive~~? = mutually exhastive...

Recall
$\Omega = \{w_1, w_2, w_3, ...\}$
s.t. $P(\{w_i\}) > 0$

$\exists \omega \; X(\omega) \notin \text{Supp}[X] \Rightarrow P(\{w\}) = 0$

• $\{w : X(w) = x_1\} \cap \{w : X(w) = x_2\} \neq \emptyset$ $X(w_0) = x_1$ & $X(w_0) = x_2$.
= mutually exclusive $= \emptyset$

$$\boxed{\sum_{x \in Supp[x]} P(X=x)=1}$$

Proof continue...

$P(\underbrace{\quad}_{X \in Supp[x]}) = P(\Omega) = 1$

☆ something gotta happen = 1

$P(\{w : X(w)=x_1\}) + P(\{w : X(w)=x_2\}) + \ldots\ldots = 1 \rightarrow P(X=y_1)+P(X=y_2)+\ldots=1.$

Ω_1

$1 \leftarrow$ | H | T | $\rightarrow 0$

Ω_2

1 ←  → 0

$X_1 = \begin{cases} 1 & \text{if } w = H \\ 0 & \text{if } w = T \end{cases}$ $X_2 = \begin{cases} 1 & \text{if } w = \text{Red} \\ 0 & \text{if } w = \text{Green.} \end{cases}$

- X_1 & X_2 are logically equ. (the same) (prob equ). No matter if we flip coin / spin) when we only care abt the values popped out from the experiment.

$X \sim \boxed{\begin{cases} 1 & w.p \ \frac{1}{2} \\ 0 & w.p \ \frac{1}{2} \end{cases}}$

$\sim$ → distribute as

w.p → with prob.

- $Supp[X] = \{0, 1\}$

"Brandy Name"

- $\underbrace{X \sim \text{Bernouilli}}\left(\tfrac{1}{2}\right) := \begin{cases} 1 & \text{up } \frac{1}{2} \\ 0 & \text{up } \frac{1}{2} \end{cases}$

- $Supp[X] = \{0, 1\}$

(R)(G) More generally,

☆ $X \sim \text{Bernouilli}(p) := \begin{cases} 1 & \text{up } p \\ 0 & \text{up } 1-p \end{cases}$

"Paramaser Space."

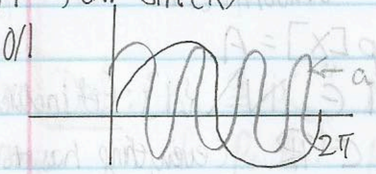
$p \in (0,1)$

→ p exists between anything from 0, 1 not not including 0/1

- More generally, X is distributed Bernouilli w/ "paramaser" p. "Parametric model"

$f(x) = \sin(x)$ $f(x; a) = \sin(ax)$

a is a constant.



- Let $a = 0$

$f(x) = 0$

☆ $X \sim \text{Bernouilli}(1) := \{1 \text{ up } 1 \longrightarrow X \sim Deg(c) := \{C : \text{up } 1$

Deg → "degenerase"

☆ $X \sim \text{Bernouilli}(0) := \{0 \text{ up } 1.$

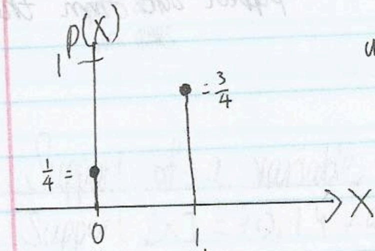
• Probability • (PMF)
Mass Function X is a discrete r.v.

$$\sum_{x \in Supp[X]} p(x) = 1 \quad p(x) := P(X = x)$$

random variable model ↑ free variable ↑

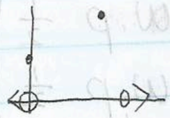
☆ Domain: $p: \mathbb{R} \to [0,1]$

- $X \sim \text{Bernouilli}\left(\frac{3}{4}\right)$



☆ $1-p$ $\quad p$

usualey do it this way →

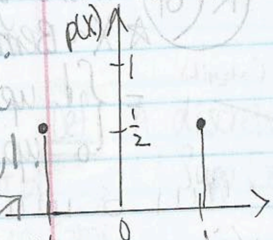


How many cases can happen (arbitrary)? $= Supp[X] = 2$ cardin

Random walk in 1 dementim

- $X \sim \text{Rademacher} := \begin{cases} +1 & \text{up } \frac{1}{2} \\ -1 & \text{up } \frac{1}{2} \end{cases}$

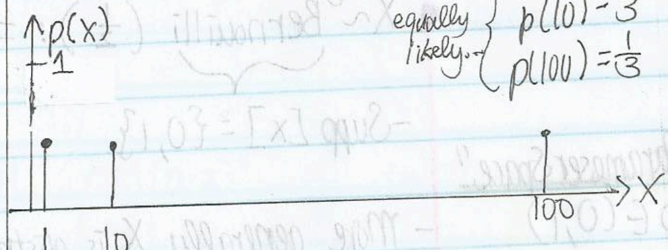
This is not Bern. b/c support is not 0, 1.



Plot of PMF

Discrete Uniform of r.v

- $X \sim \text{Uniform}(\{1, 10, 100\})$ equally likely $\begin{cases} p(1) = \frac{1}{3} \\ p(10) = \frac{1}{3} \\ p(100) = \frac{1}{3} \end{cases}$



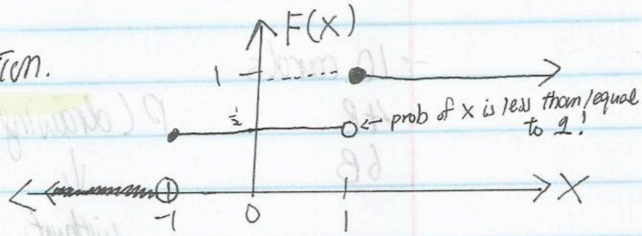
☆ Some Concepts:
- $X \sim \text{Uniform}(A)$
- $\to Supp[X] = A$
- $\to |A| \in \mathbb{N} \setminus \{1\}$ (if included A = disgenerous).
- $\to A \subset \mathbb{R}$ (everything has to be countable; once we have # we can find their avg). ∴ we are not concerning abt the w.

(CDF)

Def: Cumulasie Distribution Function.

$F(x) := P(X \leq x)$ prob

"distr. function"



$F(-32) = 1$

$F(138) = 1$

Properties of the CDF

In general: How many discontinuity? # of elements in the suppt.

① $F(x) \in [0, 1] \quad \forall x \in \mathbb{R}$

② $\lim_{x \to \infty} F(x) = 1$

③ $\lim_{x \to -\infty} F(x) = 0$ (getting lower & lower support).

④ $x \leq y \Rightarrow F(x) \leq F(y)$ → Monotonically Increasing. (strictly ≠)

$x < y \Rightarrow F(x) < F(y)$ → Strictly Monotonically Increasing

$-X \sim Bern(p) := p(x) = \begin{cases} 1 & \text{up } p \\ 0 & \text{up } 1-p \end{cases}$ EXTREMELY ugly!!! → Possible theory $= p^x(1-p)^{1-x}$ = PMF of Benaurilli

Extremely difficult to work with.

PMF only suppt. 0, 1..?

Equally Likely? NO. 2 completely different things. but there are something between them are in common =Equal in Distribution.

$-X_1 \sim Bern(p)$

$X_2 \sim Bern(p)$.

Def: "$X_1 \overset{d}{=} X_2$" X_1 is equal in distribution to X_2 if $p_1(x) = p_2(x)$ & $F_1(x) = F_2(x)$.

- 10 cards
  4R
  6B

$P(\text{drawing } 2R \text{ in } 3 \text{ cards}) = \dfrac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}}$.

↓ without replacement.

$P(\text{drawing } xR \text{ in } 3 \text{ cards}) = \dfrac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$

$P(\text{drawing } x\,R \text{ in } n \text{ cards}) = \dfrac{\binom{4}{x}\binom{6}{n-x}}{\binom{10}{n}}$

- 10 cards
  K R
  10-K B

$P(x\,R \text{ in } n \text{ cards}) = \dfrac{\binom{K}{x}\binom{10-K}{n-x}}{\binom{10}{n}}$

- N cards
  K Reds
  N-K Blue.

$P(\text{drawing } x\,R \text{ in } n \text{ cards}) = \dfrac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}} \Bigg\} = X \sim \text{Hypergeometric}(n, K, N)$

↓ without replacement