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10 cards

4R, 6B

$P(2R \text{ in } 3 \text{ cards}) \; \frac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}}$

$P(X \text{ R in } 3 \text{ cards}) = \frac{\binom{4}{x}\binom{6}{x-3}}{\binom{10}{3}}$

N cards

K successes

N-K Failures

P (X succ. in n cards)

$\hookrightarrow p(x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$ → PMF

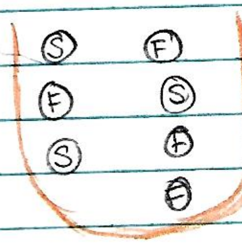
X ~ Hypergemetric↑

3 params

n : sample size

N : population size

K = # of successes



N = 7

K = 3

n = 2

$X \sim \text{Hyper}(2,3,7) \rightarrow P(X=1) = \frac{\binom{3}{1}\binom{4}{1}}{\binom{7}{2}}$

- Parameter Space

$N \in \mathbb{N} \setminus \{1\}$

$K \in \{1, \ldots, N-1\}$

$n \in \{1, \ldots, N-1\}$

ex $X \sim \text{Hyper}(1,K,N) = \text{Bern}\left(\frac{K}{N}\right)$

$\text{Supp}[X] = \{0,1\}$ — Hint: Bernouilli

$X \sim \text{Hyper}(1,K,N) = p(x) = \frac{\binom{K}{x}\binom{N-K}{1-x}}{\binom{N}{1}}$

If $x=0 \rightarrow \frac{\cancel{\binom{K}{0}}^{1}\binom{N-K}{1}}{\binom{N}{1}} = \underbrace{\frac{N-K}{N} = 1 - \frac{N}{K}}_{1-p}$

If $x=1 \rightarrow \frac{\binom{K}{1}\cancel{\binom{N-K}{0}}^{1}}{N} = \frac{K}{N} \Big\} p$

ex $X \sim \text{Hyper}(2, \underset{4S}{4}, \underset{6F}{10}) \rightarrow \text{Supp}[X] = \{0,1,2\} \rightarrow n<K, n<N-K \quad \text{supp}[X] = \{0 \ldots n\}$

$X \sim \text{Hyper}(5,4,10) \rightarrow \text{Supp}[X] = \{0,1,2,3,4\} \rightarrow n \geq K, n<N-K \quad \text{Supp}[X] = \{0 \ldots K\}$

$X \sim \text{Hyper}(8,4,10) \rightarrow \text{Supp}[X] = \{2,3,4\} \rightarrow n \geq K, n \geq K, n \geq N-K \quad \text{Supp}[X] = \{n-(N-K) \ldots K\}$

$X \sim \text{Hyper}(5, \underset{7S}{7}, \underset{3F}{10}) \rightarrow \text{Supp}[X] = \{2,3,4,5\} \rightarrow n<K, n \geq N-K \quad \text{Supp}[X] = \{n-(N-K) \ldots n\}$

	$n < K$	$n \geq K$
$n < N-K$	$\{0 \dots n\}$	$\{0 \dots K\}$
$n \geq N-K$	$\{n-(N-K) \dots n\}$	$\{n-(N-K) \dots K\}$

$\text{Supp}[X] = \{\max(0, n-N-K), \dots \min(n,k)\}$

$$\sum_{x \in \text{support}[X]} p(x) = 1$$

$p := \frac{K}{N} \quad \rightarrow K = pN$

$$X \sim \text{Hyper}(n, p, N) = p(x) = \frac{\binom{pN}{x}\binom{(1-p)N}{n-x}}{\binom{N}{n}}$$

ex $p = 0.5$, $n = 6$ $N = 100$

$$p(X=3) = \frac{\binom{50}{3}\binom{50}{3}}{\binom{100}{6}} = .3223$$

$p = 0.5$, $n = 6$, $N = 1000$

$$p(X=3) = \frac{\binom{500}{3}\binom{500}{3}}{\binom{1000}{6}} = .3134$$

$p = 0.5$, $n = 6$, $N = 10000$

$$p(X=3) = \frac{\binom{5000}{3}\binom{5000}{3}}{\binom{10000}{6}} = .3126$$

- $$\lim_{N \to \infty} \frac{\binom{pN}{x}\binom{(1-p)N}{n-x}}{\binom{N}{n}} = \lim_{N \to \infty} \frac{\frac{(pN)!}{x!(pN-x)!} \frac{((1-p)N)!}{(n-x)!((1-p)N-n+x)!}}{\frac{N!}{n!(N-n)!}}$$

- $\lim af(x) = a \lim f(x) \rightarrow$ $$\frac{n!}{x!(n-x)!} \lim_{N \to \infty} \frac{\frac{(pN)!}{(pN-x)} \frac{((1-p)N)!}{((1-p)N-n+x)!}}{\frac{N!}{(N-n!)}}$$

- $$\binom{n}{x} \lim_{N \to \infty} \frac{pN(pN-1)\dots(pN-x+1)((1-p)N)((1-p)N-1)\dots((1-p)N-n+x+1)}{N(N-1)\dots(N-x+1)(N-x)\dots(N-n+1)}$$

$$\binom{n}{x} \lim \frac{p\cancel{N}}{\cancel{N}} \cdot \lim \frac{p\cancel{N-1}}{\cancel{N-1}} \cdot \lim \frac{p\cancel{N-x+1}}{\cancel{N-x+1}} \cdot \lim \frac{(1-p)N}{N-x} \cdot \lim \frac{(1-p)N-1}{N-x-1} \dots \lim$$

$p \quad p \quad p \quad 1-p \quad 1-p \quad 1-p$

$\binom{n}{x} p^x (1-p)^{n-x}$

$X \sim \text{Binomial}(n,p) := p(x) = \binom{n}{x} p^x (1-p)^{n-x}$

→ Degeneret

$p=1 \quad X \sim \text{Binom}(n,1) = \binom{n}{x} 1^x 0^{n-x}$

$= \binom{n}{n} 1^n 0^0 = 1$

$p=0 \quad X \sim \text{Binom}(n,0) = \text{Deg}(0) = \binom{n}{x} 0^x 1^{n-x}$

$X \sim \text{Binom}(1,p) = \binom{1}{x} p^x (1-p)^{1-x} = \text{Bern}(p)$

$\text{Supp}[X] = \{0,1\}$

<u>Param space</u>

$n \in \mathbb{N}$

$p \in (0,1)$