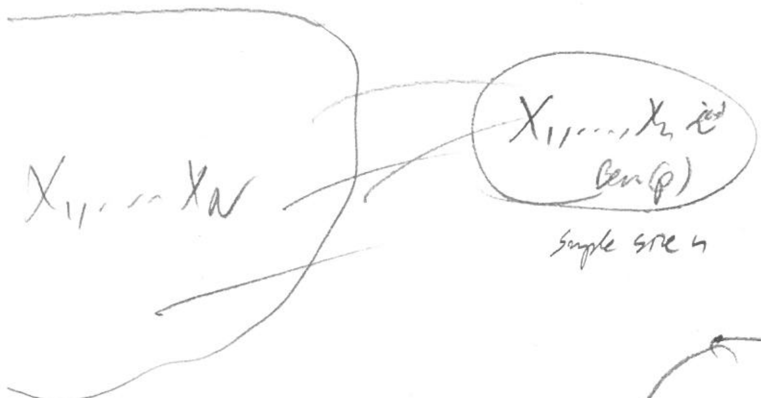
Lecture 22 Math 241 12/6/16

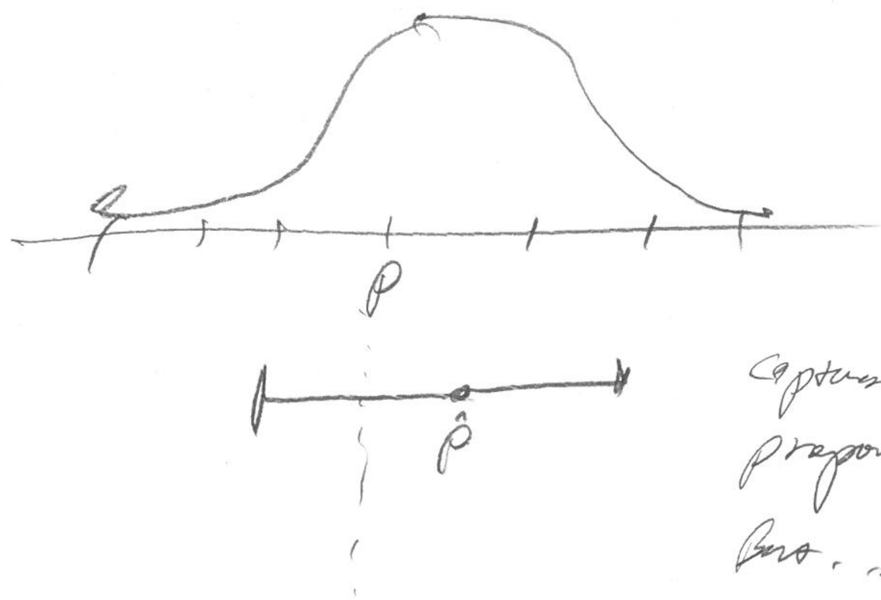
Use sample to make inference about pop.

Previously... Stat Inference: p unknown. ① Best guess $\hat{p}$

② Interval guess



$$CI_{\text{for } p} = \left[\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]$$



Captures it $1-\alpha$ proportion of the time.

But... erase it...

Any interval unknown if captures.

So what does it mean?

Interpretation of Coverage

objective

(1) Before taking sample

$P(p \in CI) = 1-\alpha$

(2) If you take many samples....

$\frac{\# p \in CI}{n} \longrightarrow p$ by LLN

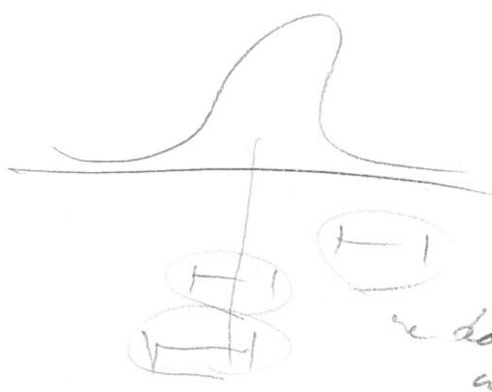
(3) But... $P(p \in CI)$ after sample taken.. $= \{0, 1\}$

why?

$P\left(p \in \left[\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]\right)$

no r.v.'s here!!

technically illegal statement



we don't know which we have...

subjective

(4) But as a subjectivist, if you have certain prior ideas about p,

$P(p \in CI) = 1-\alpha$ take Math 341

So $1-\alpha$ confidence $\neq$ $1-\alpha$ prob. unless you are subjective

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mushrooms

$\hat{p} = \ldots$ $CI = \ldots$ 14 targets....

What if I want greater coverage $\alpha \uparrow$ $\Rightarrow$ width of interval goes up! cost:

How can I do it without making interval large? $n \uparrow$ But that in the real world is costly...

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100 people polled, 55 for Clinton $CI_{p,95\%} = [\ ,\ ]$

New question:

I have a theory about what p is.

Consider births males and females

My theory is $p(\text{male}) \neq p(\text{female})$

this is a crazy theory which I only want to entertain if there's a "lot" of evidence

$p = 0.5$

What theory is the default/null theory? The one we don't want to throw out unless we are "sure"?

$p(\text{male}) = p(\text{female})$

"Simple model"

"Occam's Razor"

↳ we value simple things

Call this the null hypothesis H_0

$H_0 : p = 0.5$

the alternative hypothesis in this class is the complement of the null:

$H_a : p \neq 0.5$ (my theory)

We will discuss another type of setup next time.

We will see... $P(\text{rejecting } H_0 \mid H_0 \text{ is true})$ is controlled by us

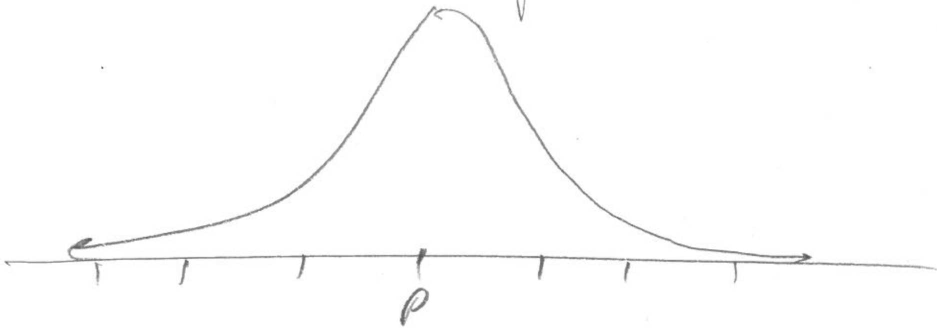
We are careful not to throw away the null model

unless there is a "lot" of evidence.

$\Rightarrow$ We assume H_0 is true and see how "normal"

the sample is relative to it.

$\hat{p} \mid H_0$ true $(p=0.5)$

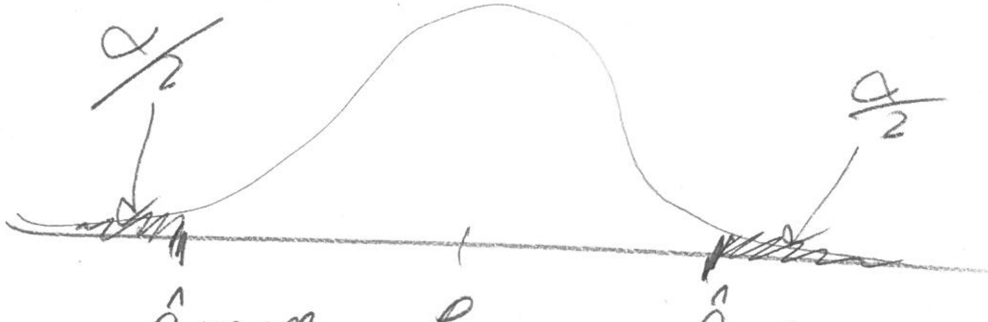


at some pt.. reject...

At what pt? Highway... too fast, too slow. You need to make a decision. Some states 65MPH, 70MPH, 75MPH...

Let $\alpha := P\left(\hat{p} \text{ "too rare"}\right)$

Make one rejection region for $\hat{p}$'s ~~too~~ big and another for $\hat{p}$'s "too small"



Find these $\hat{p}$'s

from previous class

$$\alpha = P\left(\hat{p} \in \left[\hat{p}_{small}, \hat{p}_{big}\right]\right) = P\left(\hat{p} \in \left[p \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}\right]\right)$$

$\Rightarrow \left[p \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}\right]$ is called the Retention Region

$\Rightarrow \left[p \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}\right]^c$ is called the Rejection Region

If $\hat{p} \in$ Retention Region $\Rightarrow$ Retain H_0. Not enough evidence to reject default model!

If $\hat{p} \notin$ Retention Region

i.e. $\hat{p} \in$ Rejection region $\Rightarrow$ Reject H_0. Thereby accepting H_a. There is enough evidence to reject default model.

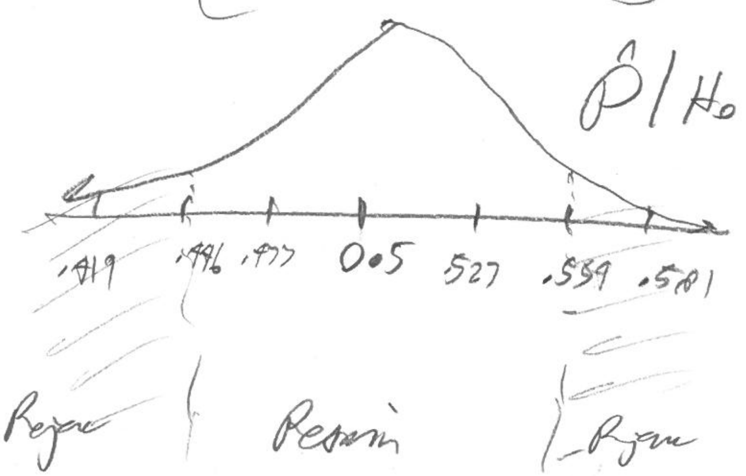
e.g. $H_0: p = 0.5$, $H_a: p \neq 0.5$, $\alpha = 5\%$.

sample: $n = 345$

$\Rightarrow z_{\frac{\alpha}{2}} = z_{2.5\%} = 2$

$.0269$

$$\text{Retention region} = \left[0.5 \pm 2\sqrt{\frac{0.5(1-0.5)}{345}}\right] = [.446, .554]$$



Ultrasound: 169 babies male $\Rightarrow \hat{p} = \frac{169}{345} = .48 \in$ Retaining Region

$\Rightarrow$ Retain H_0

Why do we need this??

Flip coin 100 times

H_0: coin is fair $\Rightarrow p_H = 0.5$

H_1: coin is not fair $\Rightarrow p_H \neq 0.5$

Exp 1: coin flips H 51x $\Rightarrow$ fair? Yes!

Exp 2: " " " " " " 98x $\Rightarrow$ fair? No!

Exp 3: " " " " " " 61x $\Rightarrow$ fair?

Not so clear!!

Run test at $\alpha = 5\%$

Retention region $= \left[0.5 \pm 2\sqrt{\frac{0.5(1-0.5)}{100}}\right] = [0.4, 0.6]$

$\hat{p} = \frac{61}{100} = 0.61 \notin$ Retention Region $\Rightarrow$ Reject H_0. Coin is not fair!

But what if $\alpha = 1\% \Rightarrow z_{\frac{\alpha}{2}} = 2.58$

Ret. Reg $= \left[0.5 \pm 2.58\sqrt{\frac{0.5(1-0.5)}{100}}\right] = [.371, .629]$

$\hat{p} = 0.61 \in$ Retention Region $\Rightarrow$ Retain H_0! Not enough evidence to say coin is unfair.

Choice of α matters! Must be selected beforehand! We will return to this...

M & M sample.

H_0: 20% are Blue! $p = 0.2$

H_a: Not so $p \neq 0.2$

$\alpha = 1\% \Rightarrow z_{\frac{\alpha}{2}} = 2.58$

$$\text{Retention Region} = \left[0.2 \pm \underbrace{2.58\sqrt{\frac{0.2(1-0.2)}{n}}}_{=\frac{1.029}{\sqrt{n}}}\right]$$

Do experiment!

Where does α come from and why should it matter?

You are making a random decision (due to random data). You could be wrong!

DECISION

TRUTH	Retain H_0	Reject H_0
H_0 true	✓	Type I error
H_0 false	Type II error	✓

P(Type I error) $= \alpha$. You choose this!

P(Type II error) ... math 242

P(Reject H_0 | H_0 false) = POWER! Not covered!

P(Type I err) ↑ ⇒ P(Type II err) ↓

P(Type I err) ↓ ⇒ P(Type II err) ↑

whack-a-mole!

Clinical trial

H_0: drug does not work (want to be sure it does)

H_a: drug works

decision: release drug

Type I err: reject H_0 if true: release drug which doesn't work to market. Cost?

Type II error: retain H_0 when false: fail to release working drug to market. Cost?

Alarm System

H_0: No fire

H_a: Fire

Decision: alarm

Type I err: "false alarm", Cost?

Type II err: fire but no alarm, Cost?

α should be high here!

Court

H_0: Innocent

H_a: Guilty

Decision: punish or not

Type I err: punish innocent person

Type II err: guilty person goes free

α?