$\bar{X} \to \mu$ LLN

$X \sim \text{Geom}(p) := (1-p)^{x-1}\, p$

$X \sim \text{Geom}(0.2) = 0.8^{x-1} \cdot 0.2$

$\mu = E[X] = \sum_{x \in \text{Supp}[X]} x\, p(x)$

x	p(x)	F(x)
1	0.2	0.2
2	0.16	0.36
3	0.12	0.488
4	0.102	0.59
5	0.082	0.672
6	0.066	0.738
7	0.052	0.790
8	0.042	0.832
⋮	⋮	

$X \sim \text{Bern}(p)$

$E[X] = p$

$X \sim \text{Bin}(n,p)$

$E[X] = np$

$X \sim \text{Hyper}(n,K,N)$

$E[X] = n\frac{K}{N}$

$X \sim \text{Geom}(p)$

$E[X] =$

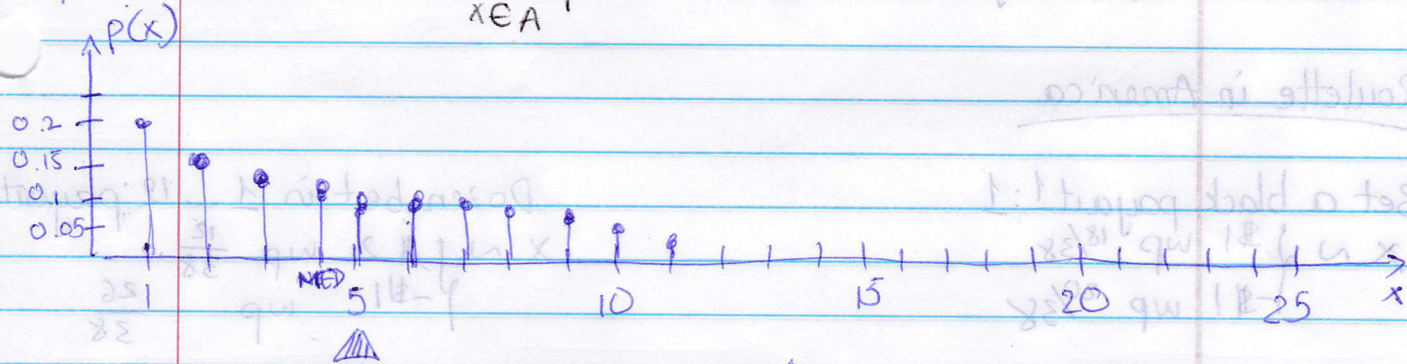
$X \sim \text{NegaBin}(r,p)$

$E[X]$

Approximate/Effective Support $:= \{x : p(x) \geq 0.001\} \subset \text{Supp}[X]$

= smallest subset of Supp[X] call in A such that

$\sum_{x \in A} p(x) = 0.999$



$X \sim \text{Geom}(p) \Rightarrow \mu = \sum_{x \in \text{Supp}[X]} x\, p(x) = \sum_{x=1}^{\infty} x(1-p)^{x-1} p$ Let $y = x-1$, $x = y+1$, $x = 1 \ldots \infty$, $y = 0 \ldots \infty$

$= p\sum_{x=1}^{\infty} x(1-p)^{x-1} = p\sum_{y=0}^{\infty}(y+1)(1-p)^{y} = p\left(\sum_{y=0} y(1-p)^{y} + \underbrace{\sum_{y=0}^{\infty}(1-p)^{y}}_{\frac{1}{1-(1-p)} = \frac{1}{p}}\right)$

$= p\left(\sum_{y=0}^{\infty} y(1-p)^{y-1} + \frac{1}{p}\right) = \sum_{y=0}^{\infty} y(1-p)^{y} p + 1$

$= (1-p)\sum_{y=1}^{\infty} y(1-p)^{y-1} + 1 \Rightarrow \mu = (1-p)\mu + 1 \Rightarrow \mu = \mu - p\mu + 1$

$\Rightarrow \mu = \frac{1}{p}$

$X \sim \text{Geom}(0.2) \Rightarrow E[X] = \frac{1}{0.2} = 5$

mod [X] = 1

$\text{Range}(X) = \text{Max}[X] - \text{Min}[X]$

$\text{Max}[X] = \text{Max}(\text{Supp}[X])$

function f $\quad G[f] = \int_{\mathbb{R}} f(x)\,dx = 17$

$\text{Min}[X] = \min(\text{Supp}[X]) = 1$

Quantile $[X, p]$ "~~perna~~" "perantile" if p is a % $:= \underset{x}{\text{argmin}} \{F(x) \geq p\}$

Median $[X]$ = Quantile $[X, 0.5]$

Condition	Distribution / r.v type
$E[X] = \text{Median}[X]$	Symmetric
$E[X] > \text{Median}[X]$	skew right
$E[X] < \text{Median}[X]$	skew left
one mode	
$\text{Mode}[X] = E[X]$	unimodel
$\text{Mode}[X] = \text{Median}[X]$	unimodel and symmetric

Tertiles

Quantile $[X, 0.33]$

Quantile $[X, 0.66]$

Quantiles

$Q[X, 0.25]$

$Q[X, 0.5]$

$Q[X, 0.75]$

Quintiles

$Q[X, 0.2]$

$Q[X, 0.4]$

$Q[0.6]$

$Q[0.8]$

Dentile

$Q[X, 0.1]$

$Q[X = 0.2]$

$Q[X = 0.i]$

$\text{IQR}[X] = Q[X, 0.75] - Q[X, 0.25]$

Roulette in America

Bet a black payout 1:1

$X \sim \begin{cases} \$1 & \text{wp } 18/38 \\ -\$1 & \text{wp } 20/38 \end{cases}$

$E[X] = \$1 \cdot \frac{18}{38} + -\$1 \cdot \frac{20}{38} = -0.053$

Dozen bet in 1 ... 12 payout 2:1

$X \sim \begin{cases} \$2 & \text{wp } \frac{12}{38} \\ -\$1 & \text{wp } \frac{26}{38} \end{cases}$

$E[X] = 2 \cdot \frac{12}{38} + -\$1 \frac{26}{38} = -0.053$

Bet on "lucky" #7 payout 35:1

$X \sim \begin{cases} \$35 & \text{wp } \frac{1}{38} \\ -\$1 & \text{wp } \frac{37}{38} \end{cases}$

$E[X] = \$35 \frac{1}{38} + -\$1 \frac{37}{38} = -0.053$

Roulette in Europe

$X_1, \ldots, X_n \overset{iid}{\sim} \begin{cases} \$1 & \text{wp } \frac{18}{37} \\ -\$1 & \text{wp } \frac{19}{37} \end{cases}$

$E[X] = \$1 \cdot \frac{18}{37} + -\$1 \frac{19}{37}$

$= -0.027$