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$$(\text{CDF}) = 1 - \frac{\binom{n}{k+1}\binom{N-n}{K-k-1}}{\binom{N}{K}} \, {}_3F_2(a,b,c;d,e;1)$$

where $a=1$, $b=k+1-K$, $c=k+1-n$, $d=k+2$, and $e=N+k+2-K-n$.

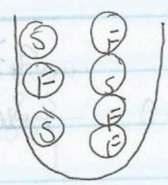
(PMF)
(PDF) Probability Density Functions

Hypergeometric = Sampling without replacement

N cards
K success
N-K failure

$$X \sim \text{Hypergeneric}(n, K, N) = P(X) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

3 params
n: sample size
N: population size
K: # of success



N=7
K=3
n=2

\# of things you are drawing
\# of success
total # of items

$$X \sim \text{Hyper}(2, 3, 7)$$

$$P(X=1) = \frac{\binom{3}{1}\binom{4}{1}}{\binom{7}{2}}$$

★ can't pull 3.7 ball.

Question
not interesting: N=0? N=1?
K=0.. K=1

N=2
K=1
n=2

• Param Space

legal things that can pop out.

$N \in \mathbb{N} \setminus \{1\}$
$K \in \{1, \dots, N-1\}$
$n \in \{1, \dots, N-1\}$

Random model

- $X \sim Hyper(1, K, N) = Bern\left(\frac{K}{N}\right)$
  Support $[x] = \{0, 1\}$
  - could be success/failure
  - when you see $\{0,1\}$ = Bernouilli

$X \sim Hyper(1,K,N) = P(x) = \dfrac{\binom{K}{x}\binom{N-K}{1-x}}{\binom{N}{1}}$

$\boxed{X=0}$: $\dfrac{\binom{K}{0}\binom{N-K}{1}}{N}$ = itself

$\boxed{X=1}$: $\dfrac{\binom{K}{1}\binom{N-K}{0}}{N}$

$\boxed{P(X=0)} = \dfrac{N-K}{N} = \underbrace{1 - \dfrac{K}{N}}_{1-p}$

$\boxed{P(X=1)} = \underbrace{\dfrac{K}{N}}_{p}$

Situation: 4S, 6F

$\overset{N-K}{\{0\}}$ = no successes only failure

N-K = # of failures.

		Cases
	• $X \sim Hyper(2, 4, 10)$	$n<K,\ n<N-K$
Drawing 2 out	Support $[x] = \{0, 1, 2\}$	$Supp[x] = \{0, 1, \ldots, n\}$
Drawing 5 out.	• $X \sim Hyper(5, 4, 10)$	$n \geq K,\ n < N-K$
	Support $[x] = \{0,1,2,3,4\}$	$Supp[x] = \{0, 1, \ldots, K\}$
Drawing 8 out	• $X \sim Hyper(8, 4, 10)$	$n \geq K,\ n \geq N-K$
	$Supp[x] = \{2, 3, 4\}$	$Supp[x] = \{n-(N-K), \ldots, K\}$
	Situation 7S, 3F	
Drawing 5 out	• $X \sim Hyper(5, 7, 10)$	$n<K,\ n \geq N-K$
	$Supp[x] = \{2,3,4,5\}$	$Supp[x] = \{n-(N-K), \ldots, n\}$

- can't get more than K b/c only K success

Now limited to # of failure ∴ no {0}

Table: ①

The cases can see in 2 ways

	$n<K$	$n \geq K$
$n<N-K$	$\{0, \ldots, n\}$	$\{0, \ldots, K\}$
$n \geq N-K$	$\{n-(N-K), \ldots, n\}$	$\{n-(N-K), \ldots, K\}$

② one line

$Supp[x] = \{\max(0, n-(N-K)), \ldots, \min(N,K)\}$

★ $\boxed{True} \rightarrow \sum_{x \in Supp[x]} p(x) = 1$

- n: sample size $\quad p = \frac{K}{N} \to K = pN$

N: population size

p: prop of success

Same model seen in different way. -Re-Parm.

$$X \sim \text{Hyper}(n, p, N) = p(x) = \frac{\binom{pN}{x}\binom{(1-p)N}{n-x}}{\binom{N}{n}}$$

$\to N=100 \downarrow N=1000 \downarrow N=10000$

change of popu size.

Prob matters.

- $p=0.5,\ n=6,\ N=100$

$$P(X=3) = \frac{\binom{50}{3}\binom{50}{3}}{\binom{100}{6}} = 0.3223$$

- $p=0.5,\ n=6,\ N=1000$

$$P(X=3) = \frac{\binom{500}{3}\binom{500}{3}}{\binom{1000}{6}} = 0.3134$$

- $p=0.5,\ n=6,\ N=10000$

$$p(X=3) = \frac{\binom{5000}{3}\binom{5000}{3}}{\binom{10000}{6}} = 0.3126$$

<u>Proof</u>

- $$\lim_{N\to\infty} \frac{\binom{pN}{x}\binom{(1-p)N}{n-x}}{\binom{N}{x}} = \lim_{N\to\infty} \frac{\frac{(pN)!}{x!(pN-x)!}\cdot\frac{((1-p)N)!}{(n-x)!((1-p)N-n+x)!}}{\frac{N!}{n!(N-n)!}} =$$

$$= \frac{n!}{x!(n-x)!} \lim \frac{\frac{(pN)!}{(pN-x)!}\cdot\frac{((1-p)N)!}{((1-p)N-n+x)!}}{\frac{N!}{(N-n)!}}$$

$$\frac{n!}{x!(n-x)!} = \binom{n}{x}$$

Recall

- $\lim af(x) = a \lim f(x)$.
- $\lim f(x)g(x) = \lim f(x) \lim g(x)$.

Doing this here

$$\frac{\overset{N}{7!}}{(\underset{N}{7}-\underset{n}{3})!} = \frac{7!}{4!} = \underbrace{7\cdot6\cdot5}_{3}$$

$$= \binom{n}{x}\lim_{N\to\infty} \frac{\overbrace{pN(pN-1)\cdots(pN-x+1)}^{x}\overbrace{((1-p)N)((1-p)N-1)\cdots((1-p)N-n+x+1)}^{n-x}}{\underbrace{N(N-1)\cdots(N-x+1)\cdot(N-x)\cdots(N-n+1)}_{n}}$$

$$= \underbrace{\underbrace{\lim \frac{p\cancel{N}}{\cancel{N}}}_{p} \cdot \underbrace{\lim \frac{pN-\cancel{1}}{N-\cancel{1}}}_{p} \cdots}_{p^x} \underbrace{\lim \frac{pN-\cancel{x+1}}{N-\cancel{x+1}}}_{p} \cdot \underbrace{\underbrace{\lim \frac{(1-p)N}{N-x}}_{1-p} \underbrace{\lim \frac{(1-p)N-1}{N-x-1}}_{1-p} \cdots \underbrace{\lim \frac{((1-p)N-n+x+1)}{N-n+1}}_{1-p}}_{(1-p)^{n-x}}$$

Final result $= \binom{n}{x} p^x (1-p)^{n-x}$

$\parallel$

Sampling N balls with replacement.

$\downarrow$

when # too big

$X \sim \text{Binomial}(n,p) = p(x) = \binom{n}{x} p^x (1-p)^{n-x}$ $\boxed{0^0 = 1}$

- Supp $[X] = \{0, 1, \dots, n\}$
- Parameser Space $= n \in \mathbb{N}$

$p \in (0,1)$

0 and 1 not included. otherwise it will include the deg.

$p=1$ $X \sim \text{Binom}(n, \underset{=}{1}) = \binom{n}{x} 1^x 0^{n-x} = \binom{n}{n} 1^n 0^0 = 1$ Deg(n)

$p=0$ $X \sim \text{Binom}(n, \underset{=}{0}) = \binom{n}{x} 0^x 1^{n-x}$ Deg(0)

~~X~~ taking 1 out $\leftarrow$ • $X \sim \text{Binom}(1,p) = \binom{1}{x} p^x (1-p)^{1-x} = \text{Berm}(p)$

from $p =$ Berm $=$ - Supp $[X] = \{0,1\}$

$\binom{1}{x}$

$\binom{1}{0} = 1$

$\binom{1}{1} = 1$