

$$X \sim \text{Geom}(p) := (1-p)^{x-1} p$$

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$$\text{Supp}(X) = \mathbb{N}$$

$$P(X=1)$$

$$X_1, X_2, \dots \stackrel{\text{iid}}{\sim} \text{Bern}(p)$$

Play poker until Royal Flush

$$P(X) = \frac{4}{\binom{52}{5}} = 1.53/\mu = 0.0000153$$

(a) Build a model

$$X \sim \text{Geom}(0.0000153)$$

(b) What's the probability you get R.F. on a million 5s?

$$P(X=1000000) = 1 - P = 0.9999985999999999 \cdot 0.0000153$$

(c) What's the probability of getting flash on million hand on 500000?

$$P(X \leq 1000000) = F(1000000) = 1 - 0.999985^{1000000}$$

$$X = \min \{t: X_t = 1\} \quad 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 0 \ 1$$

$$X = \min \left\{ t: \sum_{i=1}^t X_i = r \right\}$$

$$P(X=0) = 0$$

$$P(X=1) = 0$$

$$P(X=2) = 0$$

$$P(X=3) = p^3$$

$$P(X=4) = \cancel{p^3 (1-p)} \quad \cancel{\binom{3}{2} p^3 (1-p)}$$

$$> \quad 1 \ 1 \ 1$$

$$\binom{3}{2} (1-p) p^3$$

$$\begin{matrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{matrix}$$

$$P(X=5) = \cancel{p^3 (1-p)^2} \binom{4}{2} (1-p)^2 p^3$$

$$P(X=x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$$

$$X \sim \text{Neg Bin}(r, p) = \binom{x-1}{r-1} (1-p)^{x-r} p^r$$

$$\text{Supp}[X] = \{r, r+1, r+2, \dots\}$$

$$\text{Parameter space: } p \in (0, 1), r \in \mathbb{N}$$

$$f(x) = x^2$$

$$f(q) = q^2$$

$$X \sim \text{Neg Bin}(1, p) = \binom{x-1}{1-1} (1-p)^{x-1} p^1$$

$$= (1-p)^{x-1} p = \text{Geom}(p)$$

$$X_1, X_2, \dots, X_r \stackrel{\text{iid}}{\sim} \text{Geom}(p)$$

$$X_1 + X_2 + \dots + X_r \sim \text{Neg Bin}(r, p)$$

$$1 = \sum_{X \in \text{Supp}(X)} P(X)$$

$$\sum_{x=r}^{\infty} \binom{x-1}{r-1} (1-p)^{x-r} p^r$$

$$\sum_{i=0}^{\infty} (1-p)^{x-1} = p^{-1}$$

$$\Rightarrow \sum_{i=1}^{\infty} (-1)(x-1)(1-p)^{x-2} = (-1) p^{-2}$$

$$\sum_{i=3}^{\infty} (-1)^2(x-1)(x-2)(1-p)^{x-3} = (-1)(-2) p^{-3}$$

$$\sum_{x=r}^{\infty} (x-1)(x-2) \dots (x-(r+1))(1-p)^{x-r} = (-1)(-2) \dots (-r) p^{-r}$$

$$p^r \cdot p^{-r} = 1$$

$$X \sim \text{Neg Bin}(r, p) = (1-p)^x p^r = \binom{x+r-1}{r-1} (1-p)^r p^r$$

Failure

$$\frac{(x+r-1)!}{(r-1)! x!} = \frac{(x+r-1)(x+r-2) \dots (x+1)}{x!}$$

