Lecture #12

Thursday, October 27, 2016 12:15 PM

$$T_n = X_1 + X_2 + \dots + X_n$$

"sum r.v"

"total r.v"

$$\bar{X}_n = \frac{T_n}{n} = \frac{X_1 + X_2 + \dots + X_n}{n}$$

sample average r.v

→ "sample size"

ex $X_1, X_2, X_3 \overset{iid}{\sim}$ Bernoulli (0.1)

Binomial PMF

$$T_3 \sim \text{Binomial}(3, 0.1) = \begin{cases} 0 & \text{wp } .729 \\ 1 & \text{wp } .243 \\ 2 & \text{wp } .027 \\ 3 & \text{wp } .001 \end{cases}$$

$$= \bar{X}_3 = \begin{cases} 0 & \text{wp } .729 \\ 1/3 & \text{wp } .243 \\ 2/3 & \text{wp } .027 \\ 1 & \text{wp } .001 \end{cases}$$

$$\bar{x} = \frac{1}{n} \sum \quad \frac{1}{n} \sum_{i=1}^{n} x_i$$

realization from $\bar{X}$

what can $\bar{x}$ be?

0, 1/3, 2/3, 1

ex

$x = 5'8"$

X r.v. model for

$P(X = 5'8")$ → need model i.e $p(x)$

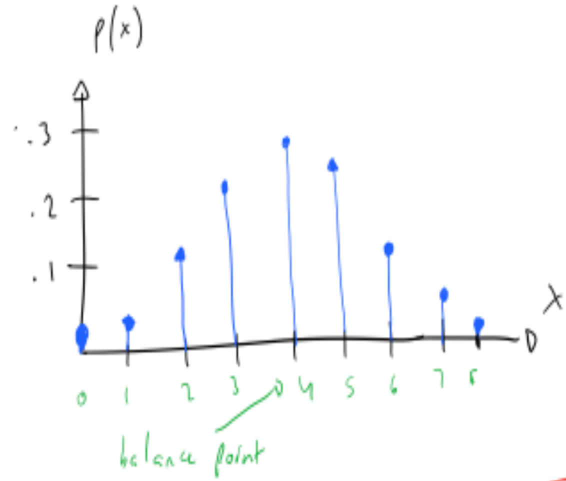
ex $X \sim \text{Binomial}(8, 1/2) = \binom{8}{x}\left(\frac{1}{2}\right)^x\left(1-\frac{1}{3}\right)^{8-x}$

$$= \frac{\binom{8}{x}}{2^8}$$

"Its a good test question"

x	p(x)	F(x)
0	.004	.004
1	.031	.035
2	.109	.145
3	.219	.363
4	.273	.637
5	.219	.8
6	.109	.965
7	.031	.996
8	.004	1

$P(8) = 1$?



Listen to lecture

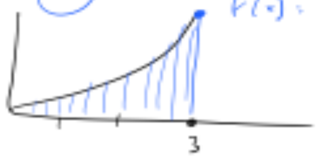
7 cups with coins

n	1	2	3	4	5	6	7							14							21
x	4	4	4	3	4	3	3	3	4	4	5	5	2	5	4	1	6	4	4	4	8
$\bar{X}_n$	4	4					3.714							3.857							4.048

$\frac{T_n}{n}$

$\bar{X}$ as n gets bigger $\lim_{n\to\infty} \bar{X}_n$

(ex)



$f(x) = x^2$

$G[f] := \int_{\mathbb{R}} f(x)\,dx = 9$

$G : \mathcal{F} \to \mathbb{R} \qquad x \in [0,3]$

space of function (#)

(ex)

$\frac{100 \cdot 0 + 20 \cdot 1}{100 + 20} = 0.17$

$$\sum_i w_i (d_i - d^*) = 0 \Rightarrow \sum w_i d_i = \sum w_i d^* \Rightarrow d^* = \sum \frac{w_i d_i}{\sum w_i}$$

$p(x)$'s +

$x \in supp[x]$

$$E[x] := \mu := \frac{\sum_{x \in Supp[x]} x\,p(x)}{\underbrace{\sum_{x \in Supp[x]} p(x)}_{1}} = \sum_{x \in supp[x]} x\,p(x)$$

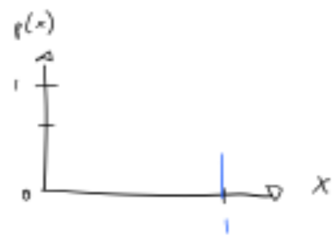
called "mean" "expectation" "first moment" "expected value"

E : all r.v's $\to \mathbb{R}$

(ex) $X \sim$ Bernoulli (0.3)

$E[x] = 0 \cdot p(0) + 1 \cdot p(1) +$

$= 0.3$



Is $E[x] \in Supp[x]$? No but $E[x]$ is in range of values within support. Inclusive if degenerate cases.

(ex) $x \sim Bern(p)$

$E[x] = 0 \cdot p(0) + 1 \cdot p(1)$

$= p$

ex $x \sim \text{Binomial}\left(8, \frac{1}{2}\right)$

$$E[x] = 0 \cdot p(0) + 1 \cdot p(1) + \ldots\ 8 \cdot p(8)$$

$$= (.031) + (2 \cdot .109) + (3 \cdot .219) + (4 \cdot .273)$$

$$+ (5 \cdot .219) + (6 \cdot .109) + (7 \cdot .031) + (8 \cdot .004)$$

Binomial PMF

$$E[x] = \sum_{x=0}^{n} x \left[\binom{n}{x} p^x (1-p)^{n-x}\right] = \sum_{x=1}^{n} x \frac{n!}{x!(n-x)!}$$

$$\binom{n-1}{x-1}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!\,(n-x)!}\, p^{x-1} (1-p)^{((n-1)-(x-1))}$$

$$(n-x) = (n-1)-(x-1)$$

Let $m := n-1$ $\quad x = 1, \ldots, n$

Let $y := x-1$ $\quad y = 0, \ldots, n-1 = 0, \ldots, m$

$$= np \sum_{y=0}^{m} \binom{m}{y} p^y (1-p)^{m-y}$$

$$= np \cdot 1 = \boxed{np}$$

listen to lecture and fill in notes

ex $X \sim \text{Hyper}(3,4,7)$

$\frac{T_n}{n}$

n	1	2	3	4	5	6	7							14							21
x	1	1	0	0	1	2	3	1	3	2	3	2	2	2	3	1	2	1	2	2	2
$\bar{x}_n$							1.143							1.643							1.714

① 2 spotted 2 unspotted
$X \sim \text{Bernoulli}(.5)$

② 2 spotted 4 unspotted
$X \sim \text{Bernoulli}(.3)$

$\frac{T_n}{n}$

n	1	2	3	4	5	6	7
x	2	1	2	1	1	6	1
$\bar{x}_n$							2

Take 2 spotted out

$\frac{T_n}{n}$

n	1	2	3	4	5	6	7
x	5	4	4	7			
$\bar{x}_n$				5			

$X \sim \text{NegBin}(2, 2/3)$

$$\boxed{\bar{x} \to E[x] \quad \text{Law of Large #s}}$$

in the long run, your mean value will be expected value