No class
Tuesday
Class Thursday

Lecture 11 - October 20, 2016

- $X \sim \text{Geom}(p) := (1-p)^{x-1}\, p$

  $\text{Supp}[X] = \mathbb{N}$

  $p \in (0,1)$

  $X_1, X_2, \ldots \overset{iid}{\sim} \text{Bern}(p)$

- (Ex) Play poker until get a royal flush.

  $P(\,) = \dfrac{4}{\binom{52}{5}} = 1.53/\text{million} = 0.00000153$

  doesn't change, each shuffle doesn't make it independent.

  (a) Build a model.

  $X \sim \text{Geom}(0.00000153)$

  (b) What's probability get royal flush on millionth hand.

  $P(X = 1000000) = .9999985^{999999} * 0.00000153$

  (c) what's probability on millionth time or sooner.

  $P(X \leq 1000000) = F(1000000) =$ $\underbrace{1 - (1-p)^F}$

  $1 - .9999985^{1000000}$

  $= 77\%$

- $X = \min\{t : X_t = 1\}$

  $\Downarrow$

  $\boxed{X = \min\{t : \sum_{i=1}^{t} X_i = r\}}$

  $X_1, X_2 \ldots \overset{iid}{\sim} \text{Bern}(p)$

  3 successes; $r=3$

  $\overbrace{0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ 0\ 1}$

* $P(X=0) = 0$, would mean happened before tried
* $P(X=1) = 0$, can't get 3 successes in 1 experiment
* $P(X=2) = 0$, can't get 3 successes in 2 tries

* $P(X=3) = p^3$ $\quad \frac{1}{1}\ \frac{1}{2}\ \frac{1}{3}$

* $P(X=4) = p^3 \cdot 3(1-p)^1$

$\cancel{1\ 1\ 0\ 1}$ (1 2 3 4) $\quad \cancel{0\ 1\ 1\ 1}$ (1 2 3 4)

$\cancel{1\ 0\ 1\ 1}$ (1 2 3 4)

$\left(\underbrace{\frac{1}{1}\ \frac{1}{2}\ \frac{1}{3}}_{\text{this is } P(X=3)}\ \frac{0}{4}\right)$

has to be there because 3 successes must to have happen.

$= \boxed{\binom{3}{2}(1-p)p^3}$

1	2	3	4
1	1	0	1
0	1	1	1
1	0	1	1

F (last must be 1, so have 3 spots... 2 must be one, so it is $\binom{3}{2}$.)

* $P(X=5) = p^3\, 6\,(1-p)^2$

3 successes must happen

$= \boxed{\binom{4}{2}(1-p)^2 p^3}$

1	2	3	4	5
1	0	1	0	1
1	1	0	0	1
0	0	1	1	1
1	0	0	1	1
0	1	1	0	1
0	1	0	1	1

← fixed, has to be 1, or else keep going or will have happened before

* if $r=3$

$$P(X=x) = p^3 (1-p)^{x-3} \binom{x-1}{2}$$

free variable $\quad (1-p)^{x-3}$ failures

• Negative Binomial

$$X \sim NegBin(r, p) := p^r (1-p)^{x-r} \binom{\overbrace{x-1}^{\text{free variable}}}{r-1}$$

p^r: must happen $\quad (1-p)^{x-r}$: stop at x experiment, so need x-r failures.

$$Supp[X] := \{r, r+1, r+2 \dots\}$$

can't be less than r, can't get r successes in less than r tries.

* param space of Neg. binomial: $p \in (0,1)$, [redacted] $r \in \mathbb{N}$

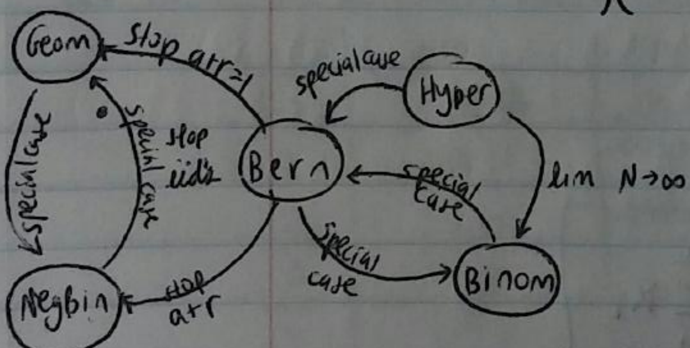
$$\boxed{* X \sim \text{Negbin}(1,p) = \underbrace{\binom{x-1}{1-1}}_{1}(1-p)^{x-1}p^1 = (1-p)^{x-1}p = \text{Geom}(p)}$$

$$\left[\begin{array}{l} X_1, X_2, \ldots X_r \overset{iid}{\sim} \text{Geom}(p) \\ \text{[redacted]} \\ X_1 + X_2 + \ldots + X_r \sim \text{Neg}(r,p) \end{array}\right]$$

• $1 = \sum_{x \in Supp[x]} p(x)$ $\quad 1 = \sum_{x=r}^{\infty} \binom{x-1}{r-1}(1-p)^{x-r}p^r$ ← sums to 1

PMF

[redacted])



• $X \sim \text{Bern}\left(\frac{1}{2}\right) := \begin{cases} 1 & wp\ \frac{1}{2} \\ 0 & wp\ \frac{1}{2} \end{cases}$

$X = 1$ $\quad x = 1 \in Supp[X]$

* realization of the random variable.

* It could have been different. This could have been zero.

* Now fixed in stone - realization

datum: realization of r.v.

data: realization of r.v.'s

* <u>8 coins, 4 dots, 4 no dots, 7 people</u>

- $X \sim \text{Hyper}(\overset{n}{\overset{\shortparallel}{3}}, \overset{K}{\overset{\shortparallel}{4}}, \overset{N}{\overset{\shortparallel}{8}})$

  $x = 2$

- $X_1, \dots, X_7 \overset{iid}{\sim} \text{Hyper}(\overset{n}{\overset{\shortparallel}{3}}, \overset{K}{\overset{\shortparallel}{K}}, \overset{N}{\overset{\shortparallel}{8}})$

  all cups independent
  from each, 7 iid distributions

* $x_1 = 1,\ x_2 = 1,\ x_3 = 3,\ x_4 = 3,\ x_5 = 1,\ x = 2,\ x_7 = 2$

$\text{Supp}[X] = \{0, 1, 2, 3\}$

- $X_1, \dots X_7 \overset{iid}{\sim} \text{Binomial}(8, \frac{1}{2})$

  $x_1 = 3,\ x_2 = 3,\ x_3 = 4,\ x_4 = 4,\ x_5 = 5,\ x_6 = 6,\ x_7 = 5$

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- $X_1, X_2, \dots, X_n$
- $T := X_1 + X_2 + X_3 + \dots + X_n$

  "total r.v."

  "sum random variable"

$$\boxed{\bar{X}_n = \frac{T_n}{n} = \frac{X_1 + \dots + X_n}{n} = \frac{1}{n}\sum_{i=1}^{n} X_i}$$

~~$X_1, X_2, X_3, \dots X_n \sim \text{Bern}(\dots)$~~

- $X_1, X_2, X_3 \sim \text{Bern}(0.1)$

  $T \sim \text{Binom}(3, 0.1) \rightarrow T \sim \begin{cases} 0 & \text{wp } .729 \\ 1 & \text{wp } .243 \\ 2 & \text{wp } .027 \\ 3 & \text{wp } .001 \end{cases}$ ← got #s by using PMF

$\bar{X} \sim \begin{cases} 0 & \text{wp } .729 \\ \frac{1}{3} & \text{wp } .243 \\ \frac{2}{3} & \text{wp } .027 \\ 1 & \text{wp } .001 \end{cases}$

Average