N cards

K R "success"

N-K B "failure"

$X \sim Hyp \quad (n, K, N) \cdot p(\ ) = \dfrac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$ → general model

N = 0 // not interesting model

$N = 1$
$\left.\begin{array}{l} \\ K = 0 \text{ or } 1 \end{array}\right\} n = 1$

Param. Space

$N \in \mathbb{N} \setminus \{1\}$

$K \in \{1, 2, \ldots N-1\}$

$n \in \{1, 2, \ldots N-1\}$

$N = 2$

$K = 0$

$K = 1$

$K = 2$

$$X \sim Hyp(\underset{\underset{n}{\parallel}}{1}, K, N) = Bern\left(\frac{K}{N}\right)$$

$Supp[x] = \{0, 1\}$

$$= \frac{\binom{K}{x}\binom{N-K}{1-x}}{\binom{N}{1}}$$

$$P(x=1) = \frac{K}{N}$$

$$p(x=0) = \frac{N-K}{N} = 1 - \frac{K}{N}$$

limit to 4 successes

$X \sim Hyp(2, 4, 10)$ → 4 s, 6 failures $\qquad Supp[x] = \{0, 1, 2\}$ → max # of success when 2 balls.

$X \sim Hyp(5, 4, 10) \qquad Supp[x] = \{0, 1, 2, 3, 4\}$

$X \sim Hyp(8, 4, 10) \qquad Supp[x] = \{2, 3, 4\}$ → you can only fail 6 times out 8 tries.

$X \sim Hyp(5, 7, 10) \qquad Supp[x] = \{\qquad\}$

MIQUELRIUS

// "How to influence / friends" // schuab

$n < K$ (successes), $n < N - K$ (failures) $\quad Supp[x] = \{0, \dots n\}$

$n \geq K$, $n < N - K$ $\quad Supp[x] = \{0, \dots k\}$

$n \geq K$, $n \geq N - K$ $\quad Supp[x] = \{n - (N-K) \dots K\}$

$n < K$, $n \geq N - K$ $\quad Supp[x] = \{n - (N-K) \dots n\}$

	$n < K$	$n \geq K$
$n < N - K$	$\{0, \dots, n\}$	$\{0 \dots k\}$
$n \geq N - K$	$\{n - (N-K) \dots n\}$	$\{n - (N-K) \dots K\}$

$$Supp[x] = \{ \max(0, n-(N-k)), \dots, \min(n, k) \}$$

n - sample size ;
N - pop. size ;
K - # successes ;
p : proportion of successes $\quad p = \frac{K}{N} \Rightarrow K = pN$

$$x \sim Hypo(n, p, N) = p(x) = \frac{\binom{pN}{x}\binom{(1-p)N}{}}{\binom{N}{n}}$$

$p = 0.5$, $n = 6$, $N = 100$

$$p(x=3) = \frac{\binom{50}{3}\binom{50}{3}}{\binom{100}{3}} = 0.3223$$

$p = 0.5$, $n = 6$, $N = 1000$

$$p(x=3) = \frac{\binom{500}{3}\binom{500}{3}}{\binom{1000}{6}} = 0.3134$$

$p = 0.5$ $\quad n = 6$ $\quad N = 10000$

$$p(x=3) = \frac{\binom{5000}{3}\binom{5000}{3}}{\binom{10000}{6}} = 0.3126$$

converging as $N \uparrow$

NOT ON THE EXAM:

$$\lim_{N\to\infty} \frac{1}{\frac{N!}{n!(N-n)!}}$$

$$= \binom{n}{x} \underbrace{\lim \frac{pN}{N}}_{p} \underbrace{\lim \frac{pN-1}{N-1}}_{p} \cdots \underbrace{\lim \frac{pN-x+1}{N-x+1}}_{p} \cdot \underbrace{\lim \frac{(1-p)N}{N-x}}_{1-p} \cdot \underbrace{\lim \frac{(1-p)N-1}{N-x-1}}_{1-p} \cdots \underbrace{\lim \frac{(1-p)N-n+x+1}{N-n+1}}_{1-p}$$

p^x $\quad$ $(1-p)^{n-x}$

$$= \binom{n}{x} p^x (1-p)^{n-x}$$

$X \sim \text{Binomial}(n, p) := p(x)$

$\text{Supp}[X] = \{0, 1, 2, \ldots, n\}$

Param Space

$n \in \mathbb{N}$

$p \in (0,1)$

$p = 0 \Rightarrow X \sim \text{Deg}(0)$

$p = 1 \Rightarrow X \sim \text{Deg}(n)$

$\lim_{x\to 0} x^x = 1$

$X \sim \text{Binom}(1, p) = \cancel{\binom{1}{x}}^{1} p^x (1-p)^{1-x} = \text{Bern}(p)$

$\text{Supp}[X] = \{0, 1\}$

$\binom{1}{0} = 1$

$\binom{1}{1} = 1$