START OF SECOND UNIT

Lecture 8 - September 27, 2016

$\Omega = \{H, T\}$

H	T
$\uparrow$	$\uparrow$
1	0

$n = 3$
$\omega_1 = H$
$\omega_2 = H$
$\omega_3 = T$

what is the average of these 3 outcomes?
→ There is no average - can't average H, H, T. Can't make computations on H, H, T.

$$\mathbb{1}_{\omega = H} = \begin{cases} 1 & \text{if } \omega = H \\ 0 & \text{if } \omega = T \end{cases}$$

$1, 1, 0 \Leftarrow (H, H, T)$

$\Downarrow$

$$\bar{x} = \frac{1+1+0}{3} = \frac{2}{3}$$

• **<u>Definition</u>**: A random variable ("r.v.") is a function.

$$X : \underset{\text{sample space}}{\Omega} \longrightarrow \underset{\text{"values of r.v."}}{\mathbb{R}}$$

$X(H) = 1$
$X(T) = 0$

$\longrightarrow$ what is $P(X=1) := P(\{\omega : X(\omega) = 1\}) = P(\{H\}) = \frac{|\{H\}|}{|\Omega|} = \frac{1}{2}$ (due to it being equally likely)

$P : 2^{\Omega} \to [0, 1]$

• <u>The "support" of a r.v.</u>
→ ~~support~~ $supp[X]$ is the range of X, the set of all possible values.

$supp[X] = \{0, 1\}$

→ $supp[X] := \{x : P(X = x) > 0\}$

• <u>A discrete random variable is a $\overbrace{\text{random variable}}^{\text{r.v.}}$ $\overbrace{\text{such that}}^{\text{s.t.}}$:</u>

$$|supp[X]| \leq |\mathbb{N}|$$

↖ (countable infinity)

⇨

$\text{Supp}[X] := \{x : P(X=x) > 0\} \subseteq \mathbb{R}$

$\Omega = \{\omega_1, \omega_2 \dots\}$ s.t. $P(\{\omega\}) > 0$

Proof not on exam.

For discrete x

$$\sum_{\substack{x \in \text{Supp}[X] \\ \uparrow \text{value} \quad \uparrow \text{random variable}}} P(X=x) = 1$$

$$P\left(\bigcup_{x \in \text{Supp}(X)} \{\omega : X(\omega) = x\}\right) \overset{\text{Fact}}{=} P(\Omega) = 1$$

if not true, then,

ω s.t. $X(\omega) \notin \text{Supp}[X]$

$P(\{\omega\}) = 0$

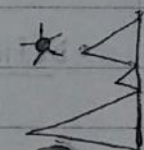
$= P(\{\omega : X(\omega) = x_1\}) + P(\{\omega : X(\omega) = x_2\}) + \dots = 1$

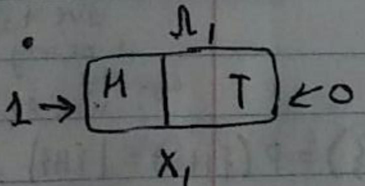
$= P(X = x_1) + P(X = x_2) + \dots = 1$

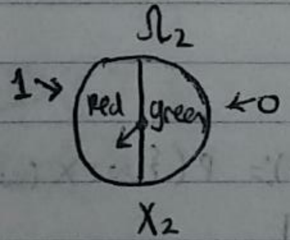
end of proof

$\{\omega : X(\omega) = x_1\} \cap \{\omega : X(\omega) = x_2\} \neq \phi$

$\exists\, \omega_0 \quad X(\omega_0) = x_1$ & $X(\omega_0) = x_2$







$$X \sim \begin{cases} 1 & \text{wp } \frac{1}{2} \\ 0 & \text{wp } \frac{1}{2} \end{cases}$$

↑ r.v. ↑ distributed as ↑ with probability

• $X \sim \text{Bernoulli}\left(\frac{1}{2}\right) := \begin{cases} 1 & \text{wp } \frac{1}{2} \\ 0 & \text{wp } \frac{1}{2} \end{cases}$

→ $\text{Supp}[X] = \{0, 1\}$

↑ possibilities that can pop out.

⇨

X is distributed Bernoulli with parameter p.

• $X \sim \text{Bern}(p) := \begin{cases} 1 & \text{w.p. } p \\ 0 & \text{w.p. } 1-p \end{cases}$ b/c $p + (1-p) = 1$

$\text{Supp}[X] = \{0, 1\}$

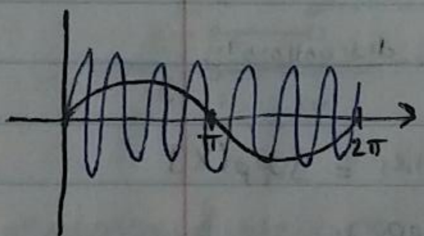
<u>Parametric Model</u> → <u>what's p? what can p be?</u>

$\text{Supp}(X) = \{0,1\}$

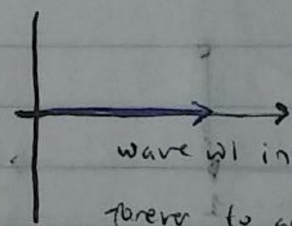
$p \in (0, 1)$ → <u>parameter space</u>

• $f(x) = \sin x$ $f(x,a) = \sin(ax)$

s.t. a is a constant



$a = 0$, $f(x) = 0$



wave w/ infinite period. Takes forever to go up & down.

In this class, <u>not consider a valid wave.</u>

• $p = 1$

1111111

• $p = 0$

00000000 → → $X \sim \{0 \text{ w.p } 1$

$X \sim \text{Deg}(c) := \{c \text{ w.p } 1$

$X \sim \text{Bernoulli}(0.9) := \begin{cases} 1 & \text{wp } \frac{1}{2} \\ 0 & \text{wp } \frac{1}{2} \end{cases}$ (90% of outcomes 1) (More ones than zeroes)

$X \sim \text{Bernoulli}(0.1) := \begin{cases} 1 & \text{wp } \frac{1}{2} \\ 0 & \text{wp } \frac{1}{2} \end{cases}$ (More zeros than 1)

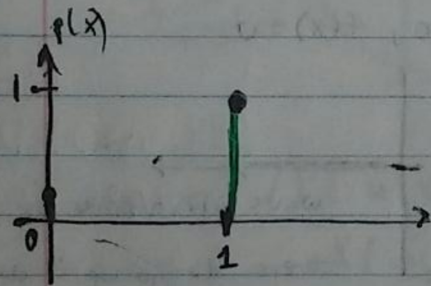
$X \sim \text{bernoulli}(0.99999) := \begin{cases} 1 & \text{wp } \frac{1}{2} \\ 0 & \text{wp } \frac{1}{2} \end{cases}$ (Now, lot's of ones)

• **Definition**: The probability mass function (PMF) is: $p(x) := P(X=x)$.

$$\sum_{x = supp[X]} p(x) = 1$$

→ If $x \notin Supp[X]$ $\quad p(x)=0$

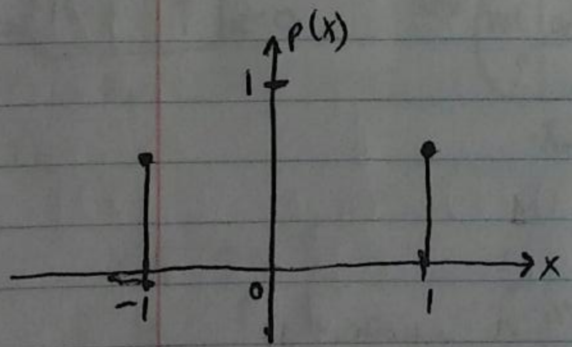
$X \sim$ Bernoulli(0.75)



2 ~~discontinutyies~~ discontinuity

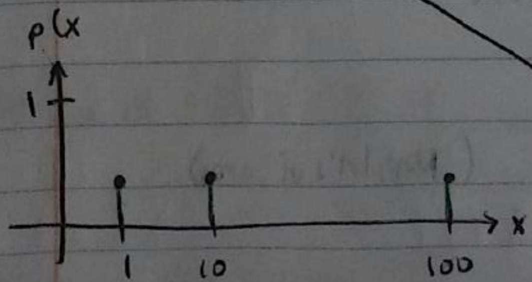
# of discontinuities = supp[X]
are the # of support.

• $X \sim$ Rademacher $:= \begin{cases} +1 & \text{wp } \frac{1}{2} \\ -1 & \text{wp } \frac{1}{2} \end{cases}$



• $X \sim$ Uniform $(\{1, 10, 100\}) = \begin{cases} 1 & \text{wp } \frac{1}{3} \\ 10 & \text{wp } \frac{1}{3} \\ 100 & \text{wp } \frac{1}{3} \end{cases}$

3 discontinuities



- $X \sim \text{Uniform}(A)$

$\text{Supp}[X] = A$

$p(x) = \dfrac{1}{|A|}$

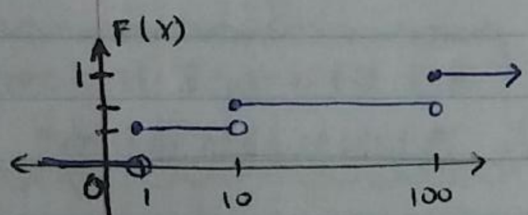
Parameter Space

$A \subset \mathbb{R}$

$|A| \in \mathbb{N} \setminus \{1\}$

- The cumulative distribution function (CDF) ("distribution fn") is:

$F(x) := P(X \leq x)$



$F(-32)$

$F(5)$

$F(y) = P(X \leq y) = $

$P(X \leq x) + P(X \in (x, y])$

$F(x)$

- Properties of the CDF

① $F(x) \in [0, 1]$

② $\lim_{x \to \infty} F(x) = 1$

③ $\lim_{x \to -\infty} F(x) = 0$

④ $\underbrace{x \leq y \Rightarrow F(x) \leq F(y)}_{\text{monotonically increasing}}$

$$X \sim \text{Bern}(p) := \begin{cases} 1 & \text{wp } \overset{P(A)}{\overset{\parallel}{p}} \\ 0 & \text{wp } 1-p \end{cases} = p^x (1-p)^{1-x}$$

$X_1 \sim \text{Bern}(p)$, $X_2 \sim \text{Bern}(p)$

→ They share the same PMF.

→ X_1 & X_2 are "equal in distribution" $X_1 \overset{d}{=} X_2$ if $P_1(x) = P_2(x)$ OR $F_1(x) = F_2(x)$

- 10 cards
  - 4 R
  - 6 B

$P(2 \text{ R in } 3 \text{ cards}) = \dfrac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}}$ w/o replacement

$P(x \text{ red in 3 cards}) = \dfrac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$

$P(x \text{ reds in } n \text{ cards}) = \dfrac{\binom{4}{x}\binom{6}{n-x}}{\binom{10}{n}}$

• 10 cards, k reds & 10-k blues

$$P(x \text{ reds in } n \text{ cards}) = \frac{\binom{k}{\cancel{x}}\binom{10-k}{n-\cancel{k}}}{\binom{10}{n}}$$

• N cards, k red & N-k blue

$X\sim$

$$P(x \text{ Red in } n \text{ cards}) = \frac{\binom{K}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$

~~[illegible]~~

Lecture 9 - September 29, 2016

• 10 cards, 4 R & 6 B

$$P(2R \text{ drawing } 3) = \frac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}}$$

2 reds ; 1 blue

$\Downarrow$

$$P(XR \text{ drawing } 3) = \frac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$$

$\Downarrow$

$$P(XR \text{ drawing } n) = \frac{\binom{4}{x}\binom{6}{n-x}}{\binom{10}{n}}$$

$\Downarrow$

10 cards

K R "successes"

10-K B "failures"

$$P(x \text{ Red drawing } n) = \frac{\binom{K}{x}\binom{10-K}{n-x}}{\binom{10}{n}}$$

$\Downarrow$

N cards

K R "successes"

N-K B "failures"

$$\boxed{P(x R \text{ drawing } n) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}} \Rightarrow$$