$Y \sim \text{Geom}(p)$

$$\text{Var}[Y] = E[(Y-\mu)^2] = E[Y^2] - \mu^2 = E[Y^2] - \frac{1}{p^2}$$

$$E[Y^2] = \sum_{y=1}^{\infty} y^2 (1-p)^{y-1} p$$

Let $z = y-1 \Rightarrow y = z+1$

$y = 1 \ldots \infty$

$z = 0 \ldots \infty$

$$= \sum_{z=0}^{\infty} (z+1)^2 (1-p)^{z} p$$

$$= \sum_{z=0}^{\infty} z^2 (1-p)^z p + \sum_{z=0}^{\infty} 2z (1-p)^z p + \sum_{z=0}^{\infty} (1-p)^z p$$

$$= (1-p) \underbrace{\sum_{z=1}^{\infty} z^2 (1-p)^{z-1} p}_{E[Y^2]} + 2(1-p) \underbrace{\sum_{z=0}^{\infty} z (1-p)^{z-1} p}_{E[Y] = \frac{1}{p}}$$

$$+ \, p \underbrace{\sum_{z=0}^{\infty} (1-p)^z}_{\frac{1}{p}}$$

$$\Rightarrow E[Y^2] = (1-p) E[Y^2] + \frac{2(1-p)}{p} + 1$$

$$\Rightarrow p E[Y^2] = \frac{2(1-p) + p}{p} \Rightarrow E[Y^2] = \frac{2(1-p)+p}{p^2}$$

$$= \frac{2 - 2p + p}{p^2} = \frac{2-p}{p^2}$$

$$Var[X] = \sum_{x=r}^{\infty} \left(x - \frac{r}{p}\right)^2 \binom{x-1}{r-1} (1-p)^{x-r} p^r$$

$X \sim NegaBin(r, p)$ $\qquad$ $E[X] = \frac{r}{p}$

$$\boxed{Var[X] = r\frac{1-p}{p^2}}$$

$E[aX + c] = ar + c$

$Var[aX + c] = a^2\sigma^2 \Rightarrow SE[aX + c] = |a|\sigma$

$E[T] = \sum_{i=1}^{n} E(X_i) = n\mu$

if identical distribution

$\underbrace{Var[T] = \sum_{i=1}^{n} Var[X_i]}_{\text{if independent}} = n\sigma^2$

if iid

$E[\bar{X}] = \mu$ $\quad X_1 \dots X_n$

identical distribution

$Var[\bar{X}] = \frac{\sigma^2}{n}$ if $X_1, \dots X_n \overset{iid}{\sim}$

$\Rightarrow SE[\bar{X}] = \frac{\sigma}{\sqrt{n}}$

$X_1, \dots X_r \overset{iid}{\sim} Geo(p)$

$X = X_1 + \dots + X_r$

$Var[X] = r$

[illegible]

$X \sim Geom(p)$ $\qquad$ $X_1, X_2, \dots \overset{iid}{\sim} Bern(p)$ $\qquad$ $(1-p)^6 p$

Memorylessness $\qquad$ $P(X = 17 \mid X > 10) \overset{?}{=} P(X = 7)$

you know you fail $\qquad$ You fail 6 times

$$\text{"} \quad P(X=17) = (1-p)^{16} p$$

$$\frac{P(X=17 \,\&\, X>10)}{P(X>10)} = (1-p)^{6} p \qquad P(X \leq x) = 1-(1-p)^{x}$$

$$P(X>x) = 1-F(x) = (1-p)^{x}$$

$$\text{"} \quad (1-p)^{10}$$

$$P(X=x) = P(X=a+b \mid X>b) = \frac{P(X=a+b)}{P(X>b)} = \frac{(1-p)^{a+b-1}}{(1-p)^{b}} p$$

$$= (1-p)^{a-1} p$$