★ 8 coins, 4 dots, 4 no dots, 7 people

- $X \sim Hyper(\overset{n}{3}, \overset{K}{4}, \overset{N}{8})$

  $x=2$

- $X_1, \ldots, X_7 \overset{iid}{\sim} Hyper(\overset{n}{3}, \overset{k}{K}, \overset{N}{8})$

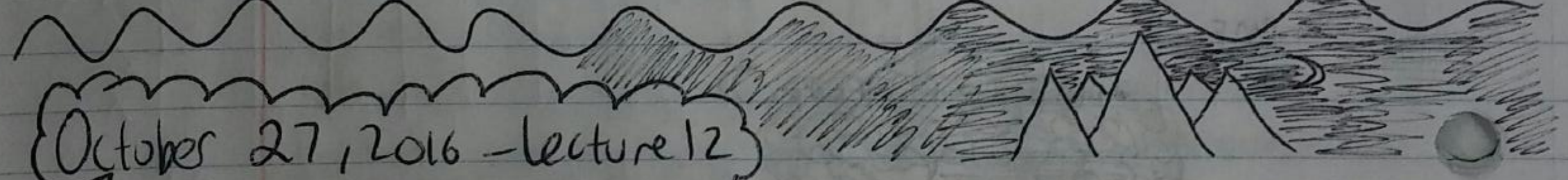
  all cups independent from each, 7 iid distributions

★ $x_1 = 1,\ x_2 = 1,\ x_3 = 3,\ x_4 = 3,\ x_5 = 1,\ x = 2,\ x_7 = 2$

$Supp[X] = \{0, 1, 2, 3\}$

- $X_1, \ldots X_7 \overset{iid}{\sim} Binomial(8, \frac{1}{2})$

  $x_1 = 3,\ x_2 = 3,\ x_3 = 4,\ x_4 = 4,\ x_5 = 5,\ x_6 = 6,\ x_7 = 5$



October 27, 2016 - lecture 12

- $X_1, X_2, \ldots, X_n$
- $T := X_1 + X_2 + X_3 + \ldots + X_n$

"total r.v."

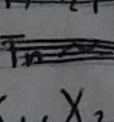
"sum random variable"

$$\boxed{\bar{X}_n = \frac{T_n}{n} = \frac{x_1 + \ldots + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i}$$

~~$X_1, X_2, X_3 \ldots X_n \sim Bern(\ldots)$~~

~~T_n~~

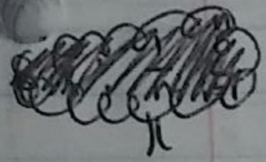
- $X_1, X_2, X_3 \sim Bern(0.1)$

 $T \sim Binom(3, 0.1) \rightarrow T \sim \begin{cases} 0 & wp\ .729 \\ 1 & wp\ .243 \\ 2 & wp\ .027 \\ 3 & wp\ .001 \end{cases}$ ← got #s by using PMF

$$\bar{X} \sim \begin{cases} 0 & wp\ .729 \\ \frac{1}{3} & wp\ .243 \\ \frac{2}{3} & wp\ .027 \\ 1 & wp\ .001 \end{cases}$$

Average

$X \sim \text{Binom}(n,p) = \binom{n}{x} p^{x} (1-p)^{n-x}$



SAMPLE AVERAGE

- $\bar{X} : \frac{1}{n} \sum_{i=1}^{n} X_i$, n (sample size)

Ex) $x = 5'11''$ is a piece of data & 5'11" is a realization, so small x.

X is a r.v model for adult height

random variable model for human height & 5'11 is in support.

* This a realization of this random variable model

$P(X = 5'11'')$: probability if take realization of this, get 5'11".

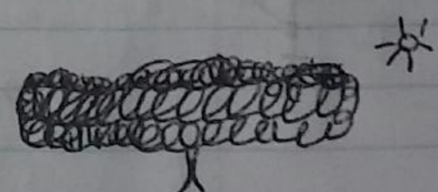
$P(X = 5'11'') = \ldots$ need the PMF

Ex) $X \sim \text{Binom}(8, \frac{1}{2}) = \binom{8}{x} (\frac{1}{2})^{x} (1-p)^{8-x}$ ← PMF

x	p(x)	F(x)
0	.004	.004
1	.031	.004 + .031 = .035
2	.109	.035 + .109 = .145
3	.219	.363
4	.273	.637
5	.219	.855
6	.109	.965
7	.031	.976
8	.004	1
	1	

← X must be less than or equal to 8

↑ doesn't sum to anything special



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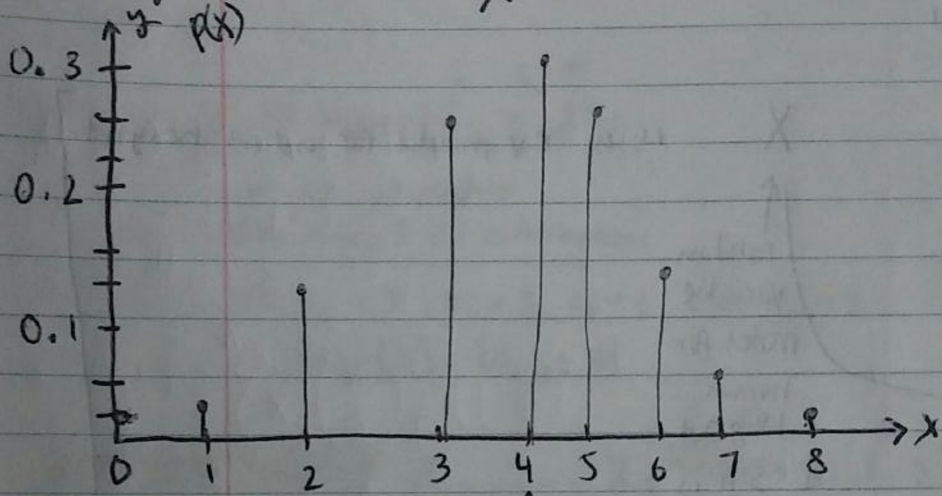
* * * ▷ COIN EXAMPLE

• <u>Drawing PMF</u>

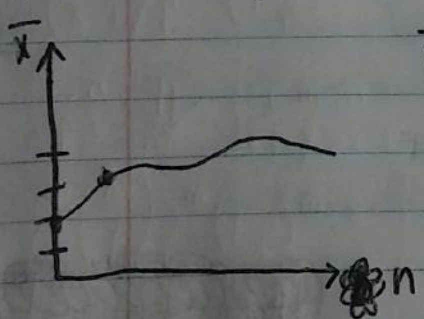


balance point *

Ex)

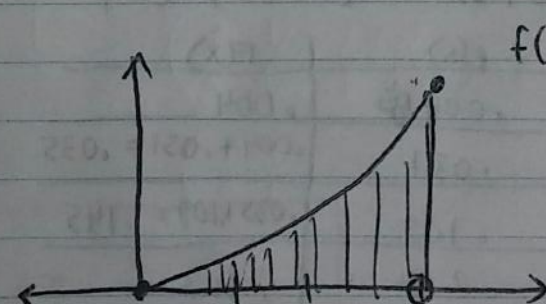
n	1	2	3	4																	21
X	2	5	4	4	3	5	4	5	6	5	7	4	5	3	5	5	1	4	3	3	4
$\bar{X}$	2	3.5	3.67				3.857							4.429							4.095

$\bar{X} \longrightarrow$ going towards 4



$f(x) = x^2$

$x \in \{0, 3\}$



$G[f] = \int f(x)\,dx = \underline{\underline{9}}$

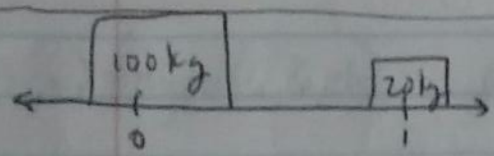
$\uparrow$ $\mathbb{R}$

function of a function

$G: F \longrightarrow \mathbb{R}$

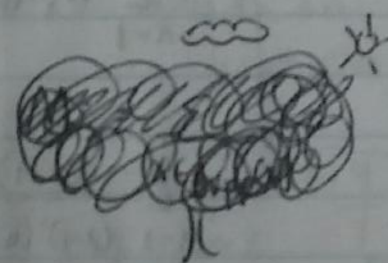
space of function — scalar

⇨ Next page

- 

$$\sum_i w_i(d_i-d^*)=0 \Rightarrow \sum w_i d_i - \sum w_i d^* = 0 \Rightarrow \sum w_i d_i = d^* \sum w_i \Rightarrow$$

$$d^* = \frac{\sum_i w_i d_i}{\sum_i w_i d_i}$$ → balance point

- $M := \dfrac{\sum_{x \in Supp[X]} p(x)\,x}{\underbrace{\sum_{x\in Supp[X]} p(x)}_{1}}$ ⇨ 

$E[X] := M$

$$\boxed{M := \sum_{x \in Supp[X]} x\,p(x)}$$

↑ mean, expected value, expectation, first moment.

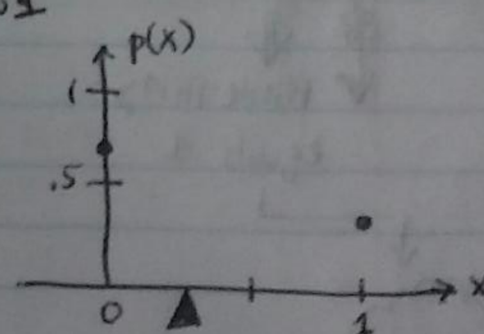
- **Law of large #s**

$$\bar{X} \to M$$

Ex) $X \sim Bern(0.3) \longrightarrow Supp[0,1] \leftarrow (0 \text{ \& } 1)$ - not 0 to 1

$E[X] = 0 \cdot p(0) + 1 \cdot p(1) = p(1) = 0.3$

→ Is 0.3 in $Supp[X]$? NO



(* In general $M \notin Supp[X]$)

- $X \sim Bern(p)$

$E[X] = 0 \cdot p(0) + 1 \cdot p(1) = p(1) = \boxed{p}$

• Proof that in coin example, balance is 4

supp[0, 1, 2 . . . . , 8]

$\mu := 0 \cdot p(0) + 1 \cdot p(1) + \dots 8 \cdot p(8)$

$= .031 + 2(.109) + 3(.219) + 4(.273) + 5 \cdot (.219) + 6(.109) + 7(.031) + 8(.004)$

$= \boxed{4}$

$$\boxed{\mu := \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x} \qquad X \sim Binom(n,p)}$$

(Not on exam)

$$= \sum_{x=1}^{n} x \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x} =$$

$$n \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!(n-x)!} p^x (1-p)^{n-x}$$

$((n-1)-(x-1))!$

$\binom{n-1}{x-1}$

$$np \sum_{x=1}^{n} \binom{n-1}{x-1} p^{x-1} (1-p)^{(n-1)-(x-1)}$$

x = 1 . . . . n ⇓ let m = n-1

y = 0 . . . n-1 = 0 . . . m let y = x-1

$$np \boxed{\sum_{y=0}^{m} \binom{m}{y} p^{m} (1-p)^{m-y}}$$

⇓ whole thing equals 1

np ← BALANCE POINT

• X ~ Hyper (n, K, N)

$$\mu = \sum_{x \in Supp[x]} x \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

• $X \sim Hyper(n, K, N)$ $\qquad \mu = \sum_{x \in Supp(X)} x \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$

n							7							14							
x	1	2	1	2	2	2	2	2	3	2	1	2	1	2	2	3	1	2	2	1	0
$\bar{X}$							1.857							1.857							1.762

$X \sim Hyper(3, 4, 7)$
$\quad n \; K \; N$

This is an iid because same PMF

$$\mu = \sum_{x \in Supp(X)} x \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}} = n \underbrace{\frac{K}{N}}_{p} = 3 \cdot \frac{4}{7} = \underline{1.714}$$

• $X \sim Geom\left(\frac{1}{3}\right)$ ← go until get a spot

n	1	3	3	4	5	6	7
x	3	2	1	3	3		
$\bar{X}$					2.4		

2.4 → 3

← go until we get 3 spots

• $X_{\dots} \overset{iid}{\sim} NegBin\left(3, \frac{2}{3}\right)$

n		
x	5	9
$\bar{X}$		7

7 → 4.5