$\mathrm{Supp}[X_1] = \{1, 7, 19\}$

$\mathrm{Supp}[X_2] = \{5, 23, 88\}$

$p(x)$

X_2 \ X_1	1	7	19	
5	$\frac{1}{15}$	$\frac{1}{3}$	$\frac{2}{15}$	$\frac{16}{30}$
23	$\frac{1}{30}$	$\frac{1}{10}$	$\frac{1}{30}$	$\frac{5}{30}$
88	$\frac{1}{30}$	$\frac{1}{5}$	$\frac{1}{15}$	$\frac{9}{30}$
	$\frac{4}{30}$	$\frac{19}{30}$	$\frac{7}{30}$	1

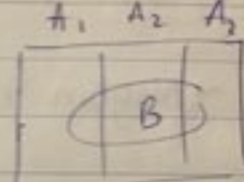
$\underbrace{\qquad\qquad}_{p(x_1) \to \text{marginal}}$

$$\sum_{x_1}\sum_{x_2} p(x_1, x_2) = 1$$

$P(X_1 = 1, X_2 = 5) = \frac{1}{15}$

$$P(X_1 = 1) = P(X_1 = 1, X_2 = 5) + P(X_1 = 1, X_2 = 23) + P(X_1 = 1, X_3 = 88)$$

$$= \sum_{x_2} p(x_1 = 1, x_2)$$



$$p(B) = p(B, A_1) + p(B, A_2) + p(B, A_3)$$

$$p(x_1) = \sum_{x_2} p(x_1, x_2)$$ "X_2 is margined out"

$$\underset{\frac{1}{30}}{p(1, 88)} \stackrel{?}{=} p(1)\,p(88) \Rightarrow \neq$$

$$\frac{1}{30} \quad \frac{4}{30} \cdot \frac{9}{30} \Rightarrow D$$

$$\underbrace{\tfrac{1}{30}}_{30} \qquad \underbrace{\tfrac{4}{30}\cdot\tfrac{9}{30}}_{36}$$

$$\boxed{E[X_1 + X_2] = E[X_1] + E[X_2]}$$

$X_1, X_2, \ldots, X_n$ r.v.'s $\Rightarrow E[T] = E[X_1 + \ldots + X_n]$

$$= E\left[\sum_{i=1}^{n} X_i\right]$$

$$= \sum_{i=1}^{n} E[X_i] = E[X_1] + \ldots + E[X_n]$$

$$= \mu_1 + \ldots + \mu_n$$

X_1, X_2 are r.v's $\quad T = X_1 + X_2$

$$E[T] = \sum_{t \in Supp[X]} t\, p(t)$$

$$\bar{X} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \quad E[g(\bar{x})] = \sum_{\bar{x} \in Supp[X]} g(\bar{x}) p(\bar{x})$$

$$E[X_1 + X_2] = \sum_{\langle x_1\, x_2 \rangle \text{ pairs}} (x_1 + x_2) \underbrace{p(x_1, x_2)}_{\text{joint mass function}}$$

$$\to \sum_{x_1 \in Supp[X_1]} \sum_{x_2 \in Supp[X_2]} (x_1 + x_2)\, p(x_1 + x_2)$$

$$= \sum_{x_1 \in Supp[X_1]} \sum_{x_2 \in Supp[X_2]} x_1\, p(x_1, x_2) + \sum_{x_2 \in Supp[X_2]} \sum_{x_1 \in Supp[X_1]} x_2\, p(x_1, x_2)$$

$$= \sum_{x_1} x_1 \underbrace{\sum_{x_2} p(x_1, x)}_{p(x_1)} + \sum_{x_2} x_2 \underbrace{\sum_{x_1} p(x_1, x_2)}_{p(x_2)} = \overbrace{\sum_{x_1} x_1 p(x)}^{E[X_1]} + \overbrace{\sum_{x_2} x_2 p(x_2)}^{E[X_2]}$$

X_1, X_2 are indep. r.v's $\quad T = X_1 + X_2 \Rightarrow p(x_1, x_2) = p(x_1)\, p(x_2)$

$$E[T] = \sum_{x_1} x_1 \sum_{x_2} p(x_1) \cdot p(x_2) + \sum_{x_2} x_2 \sum_{x_1} p(x_1)\, p(x_2)$$

$$= \underbrace{\sum_{x_1} x_1 p(x_1)}_{E[X_1]} \underbrace{\sum_{x_2} p(x_2)}_{1} + \underbrace{\sum_{x_2} x_2 p(x_2)}_{E[X_2]} \underbrace{\sum_{x_1} p(x_1)}_{1}$$

$$= E[X_1] + E[X_2]$$

$$\mathrm{Supp}[X_1] = \{1, 7, 19\}$$
$$\mathrm{Supp}[X_2] = \{5, 23, 88\}$$

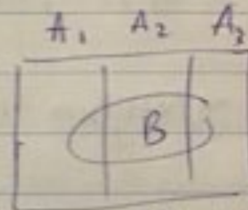
X_2 \ X_1	1	7	19	
5	$\frac{1}{15}$	$\frac{1}{3}$	$\frac{2}{15}$	$\frac{16}{30}$
23	$\frac{1}{30}$	$\frac{1}{10}$	$\frac{1}{30}$	$\frac{5}{30}$
88	$\frac{1}{30}$	$\frac{1}{5}$	$\frac{1}{15}$	$\frac{9}{30}$
	$\frac{4}{30}$	$\frac{19}{30}$	$\frac{7}{30}$	1

$p(x_1) \to$ marginal

$$\sum_{x_1}\sum_{x_2} p(x_1, x_2) = 1$$

$$P(X_1 = 1, X_2 = 5) = \frac{1}{15}$$

$$P(X_1 = 1) = P(X_1 = 1, X_2 = 5) + P(X_1 = 1, X_2 = 23) + P(X_1 = 1, X_3 = 88)$$
$$= \sum_{x_2} p(x_1 = 1, x_2)$$



$$p(B) = p(B, A_1) + p(B, A_2) + p(B, A_3)$$

$$p(x_1) = \sum_{x_2} p(x_1, x_2)$$ "x_2 is margened out"

$$p(1, 88) \stackrel{?}{=} p(1)\,p(88) \Rightarrow \neq \Rightarrow D$$

$$\frac{1}{30} \qquad \frac{4}{30}\cdot\frac{9}{30}$$

$$\underbrace{}_{30} \qquad \underbrace{}_{36}$$

$$\boxed{E[X_1 + X_2] = E[X_1] + E[X_2]}$$

$X_1, X_2, \ldots, X_n$ r.v.'s $\Rightarrow E[T] = E[X_1 + \ldots + X_n]$

$$= E\left[\sum_{i=1}^{n} X_i\right]$$

$$= \sum_{i=1}^{n} E[X_i] = E[X_1] + \ldots + E[X_n]$$

$$= \mu_1 + \ldots + \mu_n$$

$X_1, X_2, \dots X_n$ are ~~independent~~ identically distributed (not necessary independent)

$$E[T] = E[X_1 + X_2 + \dots + X_n] = n\mu$$

$$X \sim \text{Binom}(n,p)$$
$$X = X_1 + \dots + X_n$$
$$X_1, \dots, X_n \overset{iid}{\sim} \text{Bern}(p) \qquad E[X] = np$$

$$X \sim \text{NegBin}(r,p) \quad E[X] = \sum_{x=r}^{\infty} x \binom{x-1}{r-1} (1-p)^{x-r} p^r$$

$$X_1, X_2, \dots, X_r \overset{iid}{\sim} \text{Geom}(p) \Rightarrow X = X_1 + \dots + X_r \sim \text{NegaBin}(r,p)$$

$$\boxed{E[X] = n\mu = \frac{r}{p}}$$

$$X \sim \text{Hyper}(n, K, N) \qquad E[X] = \sum_{x \in Supp[X]} x \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

$$X \sim \text{Hyper}(1, K, N) = \text{Bern}\left(\frac{K}{N}\right)$$

$$X = X_1 + X_2 + \dots + X_n \quad s.t \quad X_1, X_2, \dots X_n$$
identical distributed $\text{Bern}\left(\frac{K}{N}\right)$

got it

mark coin = 0

$E[X]$ first moment
$E[X^2]$ 2^{nd} moment

$E[X^k]$ k^{th} moment

$$\boxed{E[X] = n\frac{K}{N}}$$

$$Var[X] := E[(X-\mu)^2] = E[X^2 - 2\mu X + \cancel{X}\mu^2]$$
$$= E[X^2] + E[-2\mu X] + E[\mu^2]$$
$$= E[X^2] - 2\mu^2 + \mu^2 = E[X^2] - \mu^2 = \sigma^2$$
$$\Rightarrow \boxed{E[X^2] = \sigma^2 + \mu^2}$$

$E[|X-\mu|]$ 1^{st} centered moment
$\underbrace{E[(X-\mu)^2]}_{Var[X]}$ 2^{nd} centered moment

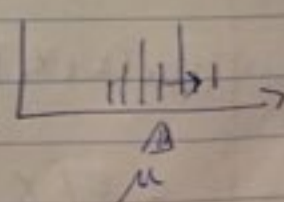
$E[|X-\mu|^k]$ 3^{rd} centered moment

$\frac{E[|X-\mu|]}{\sigma}$ 1^{st} standard moment

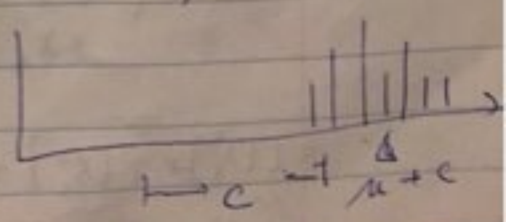
$1 = \frac{E[(X-\mu)^2]}{\sigma^2}$ 2^{nd} "

$Skew[X] = \frac{E[(X-\mu)^3]}{\sigma^3}$ 3^{rd} "

Kurt ~~Skew~~ $[X] = \frac{E[(X-\mu)^4]}{\sigma^4}$ 4^{th} "



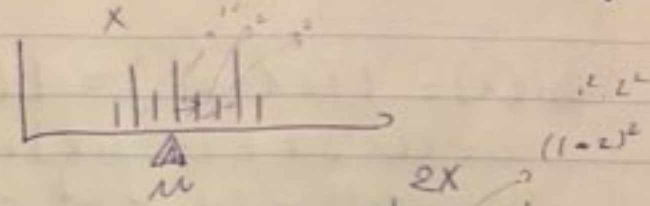
$Var[X]$
$Var[X+i]$ s.t. $c \in \mathbb{R}$



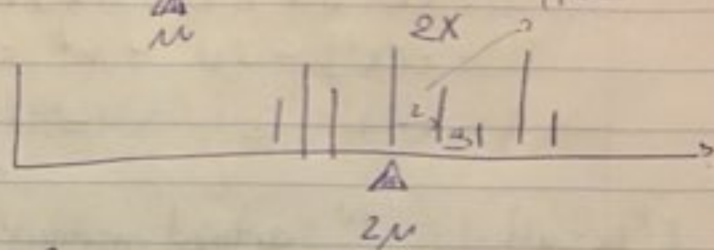
$$Var[X+c] = E[((X+c) - (\mu+c))^2] = E[(X-\mu)^2] = Var[X]$$

s.t $a \in \mathbb{R}$ $\quad Var[X]$

$Var[aX]$



multiply by 2



$$Var[aX] = E[(aX - a\mu)^2] = E[(a(X-\mu))^2]$$
$$= E[a^2(X-\mu)^2] = a^2 E[(X-\mu)^2]$$
$$= a^2 Var[X]$$

$$\boxed{Var[aX] = a^2\sigma^2}$$

$$\boxed{SE[aX] = \sqrt{Var[aX]} = \sqrt{a^2\sigma^2} = |a|\sigma}$$

$$\boxed{\begin{array}{l} Var[aX+c] = a^2\sigma^2 \\ SE[aX+c] = |a|\sigma \end{array}}$$

X_1, X_2 rv's

$$Var[X_1 + X_2] = E[[(X_1 + X_2) - (\mu_1 + \mu_2)]^2]$$

$$Var[X_1+X_2] = E[((X_1+X_2) - (\mu_1+\mu_2))^2]$$

$$= E[X_1^2 + X_2^2 + \mu_1^2 + \mu^2 + 2X_1X_2 - 2X_1\mu_1 - 2X_2\mu_2 - 2X_1\mu_2 - 2X_2\mu_1 + 2\mu_1\mu_2]$$

$$= \underbrace{E[X_1]}_{\sigma_1^2+\mu_1^2} + \underbrace{E[X_2^2]}_{\sigma_2^2+\mu_2^2} + \mu_1^2 + \mu_2^2 + 2E[X_1X_2] - 2\mu_1^2 - 2\mu_2^2 - 2\mu_1\mu_2 - 2\mu_1\mu_2 + 4\mu_1\mu_2$$