11/07/16

UBER example

$$W \sim \begin{cases} 7 \text{ min. w.p. } 0.7 \\ 12 \text{ min w.p } 0.3 \end{cases} \longrightarrow E[X] = 7 \cdot 0.7 + 12 \cdot 0.3 = \boxed{8.5 \text{ min.}}$$

Transformation of variables

uber rate : \$0.40 /min.

$$B = \$0.40/\text{min} \cdot W \sim \begin{cases} \$2.80 \text{ w.p. } 0.7 \\ \$4.80 \text{ w.p. } 0.3 \end{cases}$$

B is a function of W., $g(x)$

Average, how much you spend

$$E[B] = \$2.80 \cdot 0.7 + \$4.80 \cdot 0.3 = \boxed{\$3.40}$$

$$E[B] = \$0.40 \cdot 8.5 =$$

This is what we did here

$$\boxed{E[aX] = a \cdot E[X]}$$

$$E[X] := \int_\Omega X(\ )P(\ )$$ * NOT NEEDED *

X is $\Rightarrow Supp[X] = \{x_1, x_2, \ldots\}$

$$E[g(X)] = E[aX] = \sum_{x \in Supp[X]} ax\,p(x) = a \cdot \underline{\sum x\,p(x)}$$

$$\boxed{E[g(x)] = \sum_{x \in Supp[x]} g(x)p(x)}$$

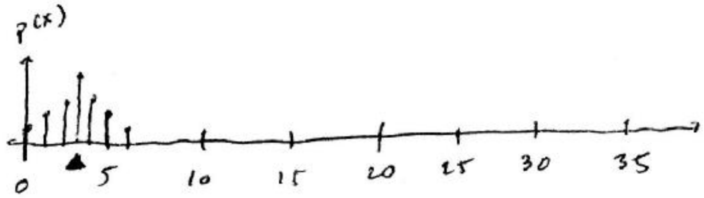
AMOUNT SPENT ON TIME + BASE FARE (UBER)

$T = B + \$3 \Longrightarrow$ for the example above, it's just $\$3.40 + \$3 = \boxed{\$6.40}$

$$E[X+c] = \sum_{x \in Supp[x]} (x+c)p(x) = \underbrace{\sum x\,p(x)}_{E[X]} + c \underbrace{\sum p(x)}_{1} = E[X] + c \cdot 1 = E[X] + c$$

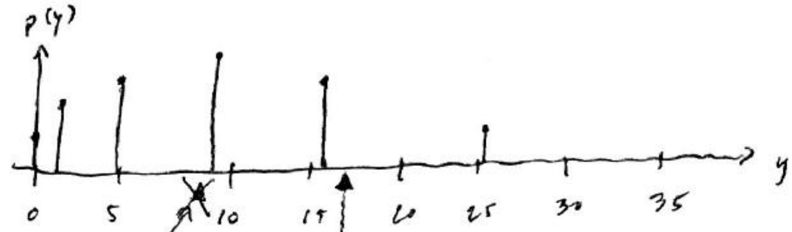
$$\star\ \boxed{E[\underbrace{aX+c}_{g(x)}] = a E[X] + c}$$

- $X \sim \text{Binom}(6, \frac{1}{2}) \longrightarrow$



$E[X] = 6 \cdot \frac{1}{2} = 3$

- $Y = X^2$, $\underbrace{X^2}_{g(x)}$



$E[X^2] \neq (E[X])^2$

$E[X^2] = \sum_{x=0}^{6} x^2 \binom{6}{x} \cdot \frac{1}{2^6} = 17.5$

$\hookrightarrow \overset{?}{=} h(E[X])$ → Got to just go thru with definition... No formula for this.

NO

1

$X \sim \text{Rademacher} = \begin{cases} +1 & \text{w.p } \frac{1}{2} \\ -1 & \text{w.p } \frac{1}{2} \end{cases}$ $\quad E[X] = 1 \cdot \frac{1}{2} + -1 \cdot \frac{1}{2} = 0$

2

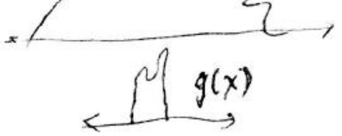
Same expectation, but NOT I.D.

$\downarrow$

$y = 10x$

$\hookrightarrow$ still 0.

$\hookrightarrow E[10X] = 10 \cdot E[X] = 10 \cdot 0 = 0$



$\int_{\mathbb{R}} f(x)\,dx = 17$ & $\int_{\mathbb{R}} g(x)\,dx = 17$

of

$Var[X] = [(-1)-(0)]^2 \frac{1}{2} + [(1)-(0)]^2 \frac{1}{2}$

1 $= 1 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} = 1$

$Var[X] = ((-10)-(0))^2 \frac{1}{2} + ((10)-(0))^2 \frac{1}{2}$

2 $= 100 \cdot \frac{1}{2} + 100 \cdot \frac{1}{2} = \boxed{100}$

$e(X, \mu) = \cancel{X - \mu}$ Absolute loss / L1

$e(x, \mu) = |x - \mu|$ → Not good...

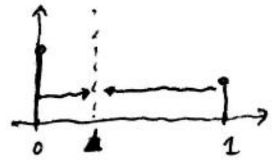
$e(x, \mu) = (x, \mu)^2$ = square loss / L2

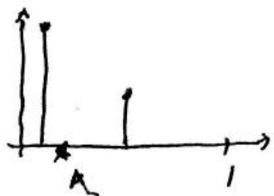
$L := (X - \mu)^2$

$Var[X] = E[L] = E[(X-\mu)^2] = \sum_{x \in \text{supp}(X)} (x-\mu)^2 p(x)$

variance or σ^2

$X \sim Bern(\frac{1}{3})$





$E[X] = \frac{1}{3}$

$1 \cdot (.3) + 0 \cdot (.7) =$

$Var[X] = (0-\frac{1}{3})^2 \frac{2}{3} + (1-\frac{1}{3})^2 \frac{1}{3} = \frac{2}{9}$

$X \sim Bern(p)$

$E[X] = p$

$Var[X] = (0-p)^2(1-p) + (1-p)^2(p) = p^2(1-p) + (1-p)^2 p =$

$(1-p)(\cancel{p^2} + (1-p)\cancel{p}) = \boxed{p(1-p)}$

Roulette Example

- Bet on #7

$X_7 \sim \begin{cases} \$35 & w.p. \ \frac{1}{38} \\ -\$1 & w.p. \ \frac{37}{38} \end{cases}$

$E[X_7] = -\$0.053$

- Bet on black

$X_B \sim \begin{cases} \$1 & w.p. \ \frac{18}{38} \\ -\$1 & w.p. \ \frac{20}{38} \end{cases}$

$E[X_B] = -\$0.053$

much more spread out b/c it has a 37

$Var[X_7] = (\$35 - \$0.053)^2 \frac{1}{38} + (\$1 - \$0.053)^2 \frac{37}{38} = 33.207 \$^2$

$Var[X_B] = (\$1 - 0.053)^2 \frac{18}{38} + (-\$1 - \$0.053)^2 \frac{20}{38} = 0.997 \$^2$

we can't use $\$^2$ so $\sqrt{\$^2}$ it.

$\sqrt{33.207} = \$5.29$

$\sqrt{.997} = \$1.00$

$$\boxed{\begin{aligned} \sigma &:= SE[X] = \sqrt{Var[X]} \\ \sigma^2 &:= Var[X] = E[L] = E[(X-\mu)^2] = \sum_{x \in supp(X)} (x-\mu)^2 p(x) \end{aligned}}$$

$\bar{X}_7 \to -\$0.053$ LLN

$\bar{X}_B \to -\$0.053$ LLN

Which one will converge faster to the $E[X]$?

- X_B will narrow in faster b/c the variance is smaller.