09/13/2016

Theorem: $P(A) = 1 - P(A^c)$

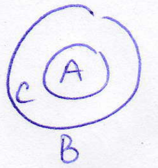
$\Omega = A \cup A^c$

$P(\Omega) = P(A \cup A^c)$

$P(\Omega) = P(A) + P(A^c)$ by ⓒ

$1 = P(A) + P(A^c)$ by ⓐ

Theorem $A \subseteq B \Rightarrow P(A) \leq P(B)$



$C := B \setminus A$

$B = A \cup C$

$P(B) = P(A \cup C)$

$P(B) = P(A) + P(C)$ by ⓒ

$P(B) - P(A) = P(C) \geq 0$ by ⓑ

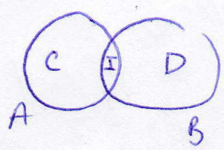
$P(B) \geq P(A)$

$P(A \cup B) = P(A) + P(B) - P(AB)$

$C = A \setminus B$

$D = B \setminus A$

$I = BA$



$P(A) = P(C) + P(I) \Rightarrow P(C) = P(A) - P(I)$

$P(B) = P(D) + P(I) \Rightarrow P(D) = P(B) - P(I)$

$P(A \cup B) = P(C) + P(D) + P(I)$

$= (P(A) - P(I)) + (P(B) - P(I)) + P(I)$

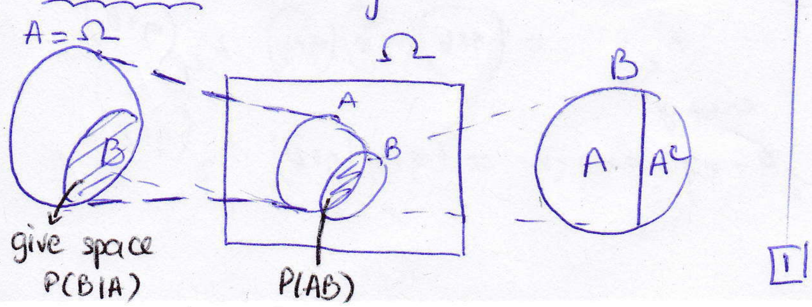
$= P(A) + P(B) - P(AB)$

Question: 1000 people, 200 smokers (A), 60 (B) lung cancer, 36 lung cancer & smokers (AB)

$P(A) = 0.2$, $P(B) = 0.06$, $P(AB) = 0.036$

What is probability of lung cancer among smokers

conditional smoking ?

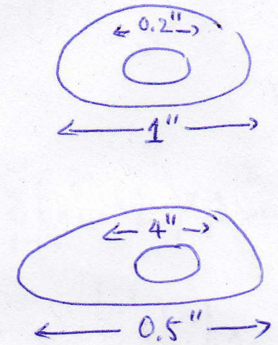


$x \propto y \Leftrightarrow x = cy$ s.t $c \in \mathbb{R}$

$P(B|A) = P(BA) \cdot \text{Zoom}$

$= P(BA) \cdot \dfrac{P(\Omega)}{P(A)}$

Note:



$\text{Zoom} = \dfrac{\text{begin size}}{\text{end size}} = \dfrac{1''}{0.5''} = 2$

$$\boxed{P(B|A) = \frac{P(BA)}{P(A)}}$$

Definition of conditional Probability

$P(A|B) = \dfrac{P(AB)}{P(B)}$

$\Rightarrow P(AB) = P(A|B) \cdot P(B)$

$$\Rightarrow \boxed{P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A)}}$$

$P(B|A) = \dfrac{P(BA)}{P(A)} = \dfrac{0.036}{0.2} = 0.18$

$P(A|B) = \dfrac{P(AB)}{P(B)} = \dfrac{0.036}{0.06} = 0.6$

$P(\text{l.c given didn't smoke}) = P(B|A^c)$

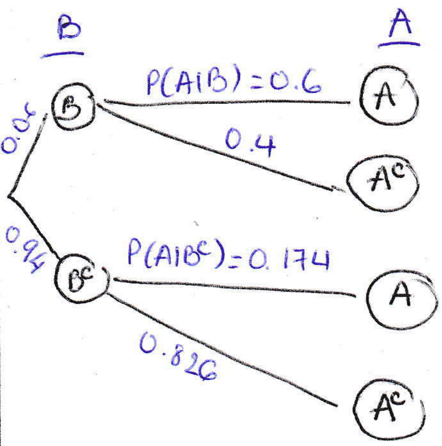
$\dfrac{P(BA^c)}{P(A^c)} = \dfrac{P(BA^c)}{1 - P(A)} = \dfrac{0.024}{0.8} = 0.03$

$P(B) = P(BA) + P(BA^c)$

$P(BA^c) = P(B) - P(BA)$

$= 0.06 - 0.036 = 0.024$

$\dfrac{P(B|A)}{P(B|A^c)} = \dfrac{0.18}{0.03} = 6$ Risk Ratio



Joint events

$P(AB) = 0.36$

$P(A^cB) = 0.024$

$P(AB^c) = 0.164$

$P(A^cB^c) = 0.776$

$P(A|B) = 1 - P(A^c|B)$

$$\frac{P(AB)}{P(B)} = 1 - \frac{P(A^cB)}{P(B)}$$

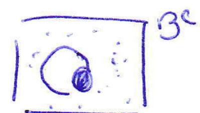
$P(AB) = P(B) - P(A^cB) \Rightarrow P(B) = P(AB) + P(A^cB)$

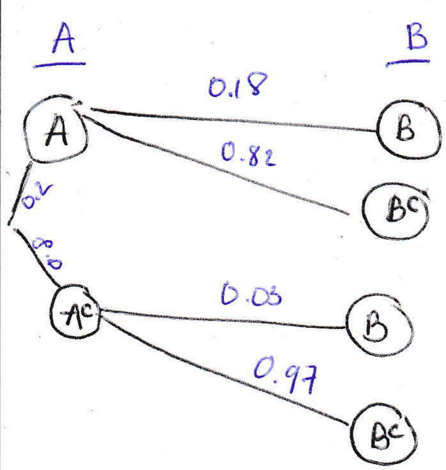
Joint events

$P(A) = P(AB) + P(AB^c)$

$P(AB^c) = P(A) - P(AB) = 0.2 - 0.036 = 0.164$

$P(A^cB^c) = 0.776$





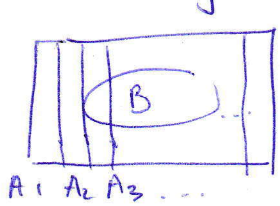
$P(AB) = 0.036$

$P(AB^c) = 0.164$

$P(A^cB) = 0.024$

$P(A^cB^c) = 0.776$

Consider event B and disjoint and collectively exhausted



$P(B) = P(B \cap \Omega)$

$= P(B \cap (A_1 \cup A_2 \cup \dots))$

$= P((B \cap A_1) \cup (B \cap A_2) \cup \dots)$

$= P(B \cap A_1) + P(B \cap A_2) + \dots$

$= \sum_{i=1}^{\infty} P(BA_i)$ Law of Total Probability

$A_1 = A$

$A_2 = A^c$

$P(B) = P(BA) + P(BA^c)$

$$P(A_\ell|B) = \frac{P(B|A_\ell)\cdot P(A_\ell)}{\sum_{i=1}^{\infty} P(B|A_i)P(A_i)}$$ Bayes Theorem

P(other is girl | one is girl) → P(—∩—) → P(

$= P(\{GG\} \mid \{GG, BG, GB\})$

$$= \frac{P(GG)}{P(GG, BG, GB)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

Monte Hall Game

1. Pick door
2. Game show host opens a door
3. You have option to switch