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indicator function

$\Omega = \{H, T\}$ $\quad$ 1 ← | H | T | → 0

$n=3$

$\omega_1 = H$, $\omega_2 = H$, $\omega_3 = T$

$\mathbb{1}_{\omega=H} = \begin{cases} 1 & \text{if } \omega = H \\ 0 & \text{if } \omega = T \end{cases}$

$1, 1, 0 \to \bar{X} = \frac{1+1+0}{3} = \frac{2}{3}$

$X : \Omega \to \mathbb{R}$ values

r.v (sample space of outcome)

↳ Random variables

$X(H) = 1$

$X(T) = 0$

$P(X=1) = \frac{1}{2} = P(\{\omega : X(\omega) = 1\})$

$P : 2^{\Omega} \to [0,1] = P(\{H\}) = \frac{|\{H\}|}{|\Omega|} = \frac{1}{2}$

Support of r.v

$\text{Supp}[X] = \{0, 1\} \neq \mathbb{R}$ → arbitrary value

Def: $\text{Supp}[X] = \{x : P(X = x) > 0\} \subseteq \mathbb{R}$

r.v

Def - A discrete random variable X,

$|\text{Supp}[X]| \leq |\mathbb{N}|$

$\Omega = \{\omega_1, \omega_2, \omega_3 \ldots\}$ s.t $P(\{\omega_i\}) > 0$

$\sum_{x \in \text{Supp}[X]} P(X=x)$ $\qquad$ $\bigcup_{x \in \text{Supp}[X]} \{\omega : X(\omega) = x\} \stackrel{?}{=} \Omega$ → Fact

$\exists \omega \;\; X(\omega) \notin \text{supp}[X] \to P(\{\omega\}) = 0$

$\{\omega : X(\omega) = x_1\} \cap \{\omega : X(\omega) = x_2\} = \emptyset$

$P(\{\omega : X(\omega) = x_1\}) + P(\{\omega : X(\omega) = x_2\}) + \ldots = 1$

↳ $P(X = x_1) + P(X = x_2) + \ldots = 1$

$$\boxed{\sum_{x \in \text{Supp}[X]} P(X=x) = 1}$$

1 ← | H | T | → 0

$X_1 = \begin{cases} 1 & \text{if } \omega = H \\ 0 & \text{if } \omega = T \end{cases}$

1 ← (Red | blue) → 0

$X_2 = \begin{cases} 1 & \text{if } \omega = \text{red} \\ 0 & \text{if } \omega = \text{blue} \end{cases}$

distribute as

with probability

$$X \sim \begin{cases} 1 & \text{w.p. } \frac{1}{2} \\ 0 & \text{w.p. } = \frac{1}{2} \end{cases}$$

* "Brand name"

$supp[X] = \{0,1\}$

$X \sim \text{Bernouilli}\left(\frac{1}{2}\right) := \begin{cases} 1 & wp\ \frac{1}{2} \\ 0 & wp\ \frac{1}{2} \end{cases}$

$supp[X] = \{0,1\}$

more generally

$$X \sim \text{Bernouilli}(p) := \begin{cases} 1 & wp \quad p \\ 0 & wp \quad 1-p \end{cases}$$

X is distributed Bernouilli w 1 "parameter" p. → "Parametric Mode"

1111111... → $X \sim \text{Bern}(1) := \{1\ wp\ 1$ → degenerate r.v

00000... $X \sim \text{Bern}(0) := \{0\ wp\ 1$ ↳ $X \sim \text{Deg}(c) := \{c\ wp\ 1$

$p \in (0,1)$ → "parameter space"

X is a discrete random variable

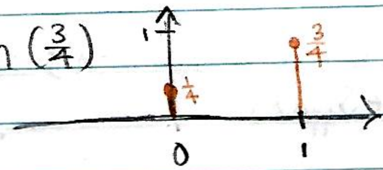
$p(x) := P(X = x)$

r.v free variable

Probability Mass Function (PMF) → $p(x)$

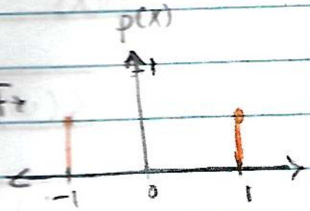
$p : \mathbb{R} \to [0,1]$

ex $X \sim \text{Bern}\left(\frac{3}{4}\right)$



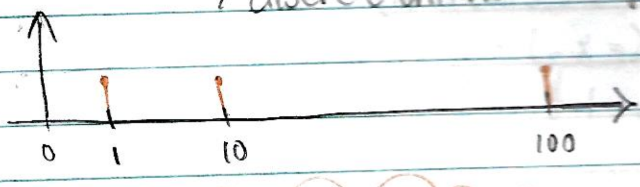
$X \sim \text{Rademacher} := \begin{cases} 1 & wp\ \frac{1}{2} \\ -1 & wp\ \frac{1}{2} \end{cases}$

PMF:



$X \sim \text{uniform}(\{1, 10, 100\})$ $p(1) = \frac{1}{3}$ $p(100) = \frac{1}{3}$

↳ discrete uniform $p(10) = \frac{1}{3}$



$X \sim \text{Uniform}(A)$

$supp[X] = A$

$|A| \in \mathbb{N} \setminus \{1\}$

$A \subset \mathbb{R}$

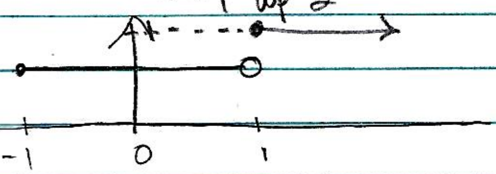
Cummolative distribution function

$F(x) := P(X \leq x)$

(" distribution function)

$X \sim \text{Rademacher} := \begin{cases} 1 & \text{wp } \frac{1}{2} \\ -1 & \text{wp } \frac{1}{2} \end{cases}$ $\qquad F(x) := P(X \leq x)$

$F(-32)$



Properties of CDF

1) $F(x) \in [0,1] \ \forall x \in \mathbb{R}$
2) $\lim_{x \to \infty} F(x) = 1$
3) $\lim_{x \to -\infty} F(x) = 0$
4) $x \leq y \to F(x) \leq F(y)$ → monotonically increasing

$x < y \overset{?}{\Rightarrow} F(x) < F(y)$

$X \sim \text{Bern}(p) := p(x) = \begin{cases} 1 & \text{wp } p \\ 0 & \text{wp } p \end{cases} \rightarrow p^x(1-p)^{1-x}$

PMF of Bernoulli

$X_1 \sim \text{Bern}(p)$
$X_2 \sim \text{Bern}(p)$ $\qquad$ Def: "$X_1 \overset{d}{=} X_2$" X_1 is equal in distribution to X_2 if $p_1(x) = p_2(x)$ or $F_1(x) = F_2(x)$

10 cards, 4 red 6 blue
P drawing 2 red in 3 cards $= \frac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}}$

$P\left(\text{drawing } x\ R \text{ in } 3 \text{ cards}\right) = \frac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$ $\qquad P\left(\text{drawing } x\ R \text{ in } n \text{ cards}\right) = \frac{\binom{4}{x}\binom{6}{n-x}}{\binom{10}{n}}$

10 cards
$K = R$
$10-K = B$

$P(x\ R \text{ in } n \text{ cards}) = \frac{\binom{K}{x}\binom{10-K}{n-x}}{\binom{10}{n}}$

N cards
K reds
N-K blue

$P(\text{drawing } x\ R \text{ in } n \text{ cards}) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$