09/15/16

$P(A) = \frac{4}{52} = \frac{1}{13}$ $\quad P(A|\heartsuit) = \frac{1}{13}$

$P(\text{IBM stock}\uparrow \text{ in a Day}) = P(\text{IBM stock}\uparrow \text{ in a day} \mid \text{rains in Buenos Aires})$

"Informationally Irrelevant"

Def: A, B are independent if
$$\boxed{P(A|B) = P(A) \quad \text{OR} \quad P(B|A) = P(B)}$$

$P(A) = P(A|B)$

$P(A) = \frac{P(A|B)}{P(B)}$

$P(A \cap B) = P(A)\,P(B)$
↳ multiplication rule

$$P\left(\bigcap_{i=1}^{\infty} A_i\right) = \prod_{i=1}^{\infty} P(A_i)$$

- $P(H_2|H_1) \stackrel{?}{=} P(H_2) \rightarrow$ yes, b/c knowing H_1 does not effect H_2. (independent)

- $P(H_1, H_2, H_3, H_4, H_5) = \frac{1}{|\Omega^5|} = \frac{1}{2^5} = P(H_1) \cdot P(H_2) \cdot \ldots P(H_5)$
  $\frac{1}{2} \cdot \frac{1}{2} \cdot \ldots \frac{1}{2} = \left(\frac{1}{2}\right)^5$

$P(\{\geq 1 \text{ 6-6 in 24 rolls}\}) < \frac{1}{2}$

$= P(1 \text{ 6-6 in } 24) + P(2 \text{ 6-6 in } 24) + \ldots + P(24 \text{ 6-6 in } 24)$

OR $= 1 - P(\text{zero 6-6 } \cancel{\text{in 24 rolls}}) = 1 - P(\text{not 6-6 in 1} \cap \text{not 6-6 in 2} \cap \ldots \cap \text{not 6-6 in 24})$

$1 - (P(\text{no 6-6}))^{24} = 1 - (1 - P(6\text{-}6))^{24}$

$1 - (1 - P(6)P(6))^{24} = 1 - \left(1 - \left(\frac{1}{6}\right)^2\right)^{24} = \boxed{.4914039}$

- Definition A, B are dependent
  $P(A|B) \neq P(A)$ & $P(B|A) \neq P(B)$

  $P(A \cap B) \neq P(A)\,P(B)$

• P(Q64 is late) < P(Q64 is late | raining + traffic)

• P(Q64 is late) > P(Q4 is late | sunny + no traffic)

• P(Lung cancer) < P(Lung cancer | smoking)

↳ marginal probability

↳ Across all possibilities

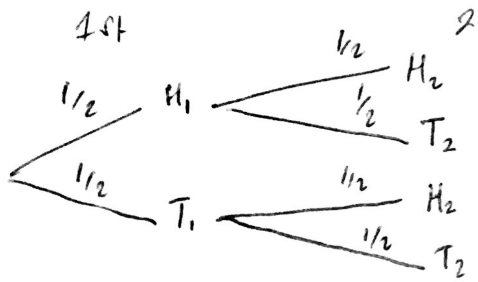
Conditional probability

• A, B disjoint $\overset{?}{\not\Rightarrow}$ A, B independent

$P(A|B) \overset{?}{=} P(A)$

• $0 = P(H|T) \neq P(H) = \frac{1}{2}$

<u>2 coins</u>



$P(HH) = \frac{1}{4}$

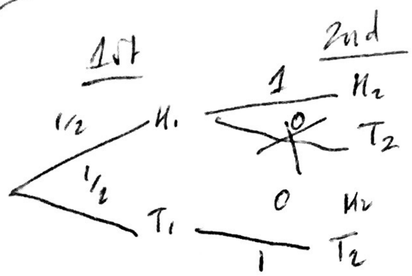
$P(HT) = \frac{1}{4}$

$P(TH) = \frac{1}{4}$

$P(TT) = \frac{1}{4}$

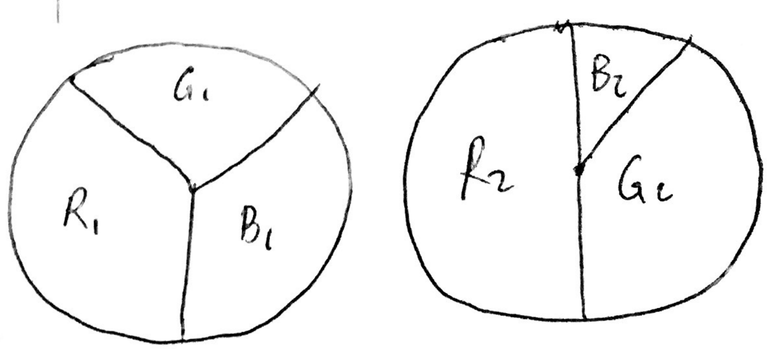
Magic coin: whatever side the first lands on, second will land on same thing.

$1 = P(H_2 | H_1) \overset{?}{\neq} P(H_2) = \frac{1}{2}$



$P(H,H) = \frac{1}{2}$

$P(T,T) = \frac{1}{2}$

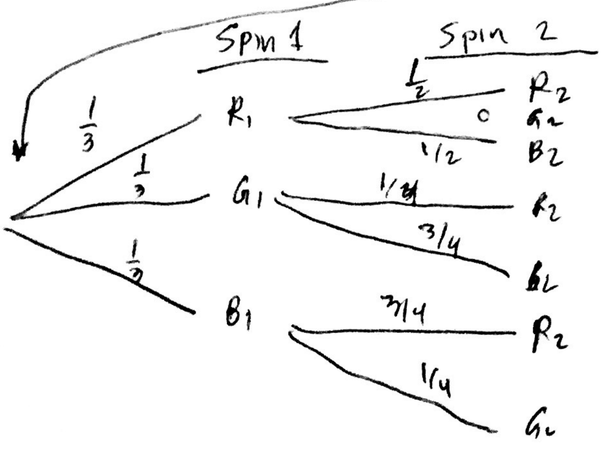


• $P(R_1) = P(G_1) = P(B_1) = \frac{1}{3}$

• $P(R_2) = \frac{1}{2}$ $P(G_2) = \frac{1}{3}$

$P(B_2) = \frac{1}{6}$

These 2 spinners



$\frac{1}{6}$

0

$\frac{1}{6}$

$\frac{1}{6}$

$\frac{1}{4}$

$\frac{3}{12}$

Consider R_1, R_2

$\frac{1}{2} = P(R_2 \mid R_1) \neq P(R_2) = \frac{1}{2}$

Consider R_1, G_2

$0 = P(G_2 \mid R_1) \neq P(G_2) = \frac{1}{3}$

END OF TEST 1 INFO.

Famous Question 1: $P(\text{shared Birthday})$

$= P(\geq 1 \text{ shared birthday among } 49 \text{ people})$

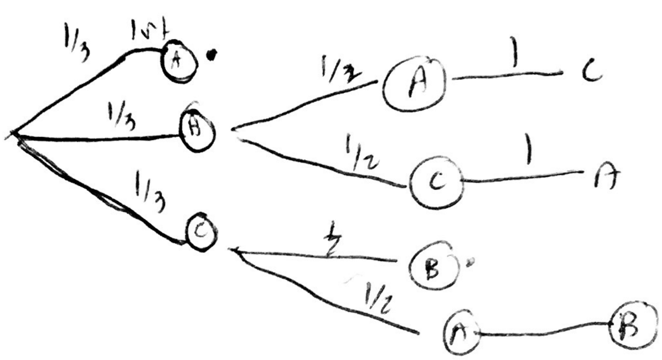
$= P(1 \text{ shared b-day}) + P(2\text{-shared b-day}) + \dots + P(49 \text{ shared b-day})$

$= 1 - P(\text{no shared birthdays}) = 1 - \frac{\underline{365}\ \underline{364}\ \underline{363}\ \underline{\quad} \dots \underline{365-49+1}}{(365)^{49}}$

$\frac{{}_{365}P_{49}}{365^{49}} = 97\%$

2: $P(\text{No one gets their HAT})$

$n = 3$



$\frac{1}{3} + \frac{1}{6} + \frac{1}{6} = \frac{2}{3}$

A_1 = person 1 got their hat

A_2 = person 2 got their hat

$\vdots$

A_n = person n got their hat

$= 1 - (P(\text{someone gets their hats}))$

$= 1 - (P(\text{one gets hat}) + P(\text{two}) + \dots + P(\text{all hats}))$

$= 1 - P(A_1 \cup A_2 \cup \dots \cup A_n)$

$= 1 - P\left(\bigcup_{i=1}^{n} A_i\right)$

Remember...

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) - P\left(\bigcap_{i=1}^{n} A_i\right)$

GIVE UP

$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) - \sum_{i \neq j} P(A_i \cap A_j) + \sum_{i \neq j \neq k} P(A_i \cap A_j \cap A_k) + \dots + (-1)^{n+1} P\left(\bigcap_{i=1}^{n} A_i\right)$

$P(A_1) = \frac{1}{n}, \quad P(A_2) = \frac{1}{n} \quad \sum_{i \neq j} \frac{(n-2)!}{n!}$

$\sum_{i=1}^{n} \frac{1}{n}$

$f(x) = \sum_{i=n}^{\infty} \frac{f^{(i)}(i)}{i!}(x-i)^i \quad \forall c \in \mathbb{R}$

Taylor series