12/6/16

"Inverse Problem" parameters unknown use sample to draw inference above parameters

$X \sim Bern(p)$

Population $N \approx \infty$

sample

$X_1, \dots X_n$ Bern(p)

unknown

$n << \infty$

$\hat{p} = \frac{\# 1's}{n}$

Statistical Inference

1) Point estimation: Best guess $\hat{p}$

2) Interval Estimation

Confidence Interval

$CI_{1-\alpha, p} := \hat{p} \pm \frac{Z\alpha}{2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

3) Parameter valu testing (Hypothesis Testing)

Interpretation of a CI

1) $\frac{\# \text{ of times } \{p \in CI\}}{n} \rightarrow 1-\alpha$

2) Before the experiment, $P(p \in CI) = 1-\alpha$

3) If you Believe in subjective prob., then under prior information, you can say, $P(p \in CI) = 1-\alpha$

example Gender Ratio in Human Births

$p := p(\text{male})$

My theory : $p \neq 0.5$

i.e unequal gender ratio

Crazy ?

Default / "Null" Hypothesis denoted: $H_0 : p = 0.5$

The crazy theory is the alternative Hypothesis

$H_a : H_0$ is False: $p \neq 0.5$

Assume $H_0$ is true

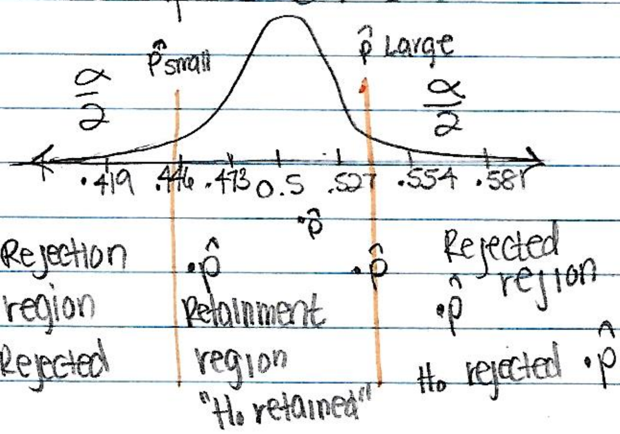
$\hat{p} \sim N\left(p, \left(\sqrt{\frac{p(1-p)}{n}}\right)^2\right)$

$\hat{p} \sim N\left(0.5, \left(\frac{\sqrt{.5(1-.5)}}{345}\right)^2\right)$

$\hat{p} \sim N(0.5, .02692)$

$p = .5$

sample size $n = 345$



For final z value
$\alpha = 1\%$
or
$\alpha = 5\%$

Let $\alpha := P(\text{too rare})$

$1-\alpha = P(H_0 \text{ retained})$

$= P(\hat{p} \in [\hat{p}_{small}, \hat{p}_{Large}])$

$= P(\hat{p} \in [p \pm \text{margin}])$

$= P\left(\hat{p} \in \left[p \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}\right]\right)$

Retainment region $= \left[p \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}\right]$

Rejection Region $= \left[p \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}\right]^c$

Calculate $\hat{p}$

I If $\hat{p} \in$ Retainment region $\rightarrow$ retain $H_0$

But, we do not have sufficient evidence to reject the null Hypothesis.

II If $\hat{p} \in$ Rejection region

$\rightarrow$ reject $H_0$, then accept $H_a$.

We have sufficient evidence to reject th null Hypothesis.

example

$n = 345$, $\alpha = 5\%$

Retainment region $= \left[0.5 \pm 2 \sqrt{\frac{0.5(1-0.5)}{345}}\right]$

$= [.446, .554]$

If 169 babies were male $\Rightarrow \hat{p} = \frac{169}{345} = .48 \in$ Retainment region

$\hookrightarrow$ We do not have sufficient evidence to reject human gender ratio equality.

example

Flip a coin 100 times

You want to know if coin is fair.

Sce. I : 51 H Fair? Yes

Sce. II : 98 H Fair ? No

Sce III : 61 H Fair?

$1\% = 2.84$

$H_0 : p = 0.5$

$H_a : p \neq 0.5$

$\alpha = 5\%$

Retainment Region $= \left[p \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}\right]$

$= 0.5 \pm 2 \sqrt{0.5 \pm 2 \left[\frac{0.5(1-.5)}{100}\right]}$

$= [0.4, 0.6]$

$\hat{p} = \frac{61}{100} = .61$ $\hat{p} \notin$ Retainment region

examples Mars (the candy co.) says the prob of blue m & M's is 20%. You think otherwise?

Let $p := P(\text{Blue})$

$H_0 : p = 0.2$

$H_a : p \neq 0.2$

$n = 615$ m & m's

$\alpha = 1\%$

Ret region $= \left[p \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}\right]$

$= \left[0.2 \pm 2.84 \sqrt{\frac{0.2(1-0.2)}{615}}\right] =$

$[.1542, .2458]$

$\hat{p} = \frac{158}{615} = .2569 \Rightarrow \hat{p} \in$ Ret. R

$\hat{p}$ is drawn from $\hat{p} \mid H_0 \sim N\left(p, \left(\sqrt{\frac{p(1-p)}{n}}\right)^2\right)$

Decision

| | Retain $H_0$ | Reject $H_0$ |
|---|---|---|
| $H_0$ True | ✓ | Type I error |
| $H_0$ False | Type II error | ✓ |

Prob(Type I error) $= \alpha$

P(Type II error) = Beyond scope of class

P(Reject $H_0$ | $H_0$ false) = POWER

$\alpha \uparrow \Rightarrow P(\text{Type I error}) \uparrow \Rightarrow P(\text{Type II error}) \downarrow$

$\alpha \downarrow \Rightarrow P(\text{Type I error}) \downarrow \Rightarrow P(\text{Type II error}) \uparrow$

<u>Clinical Trial</u>

$H_0$: drug does not work

$H_a$: drug works

Decision: Release drug to the market

Type I error:
releasing a drug that doesn't work

Cost: possible death

Type II error
not releasing a drug that does work.

Cost: people can't be helped

<u>Court case</u>

$H_0$: Innocense

$H_a$: Guilty

Decision: punishment or not

Type I error
Punish an innocent person

Type II error
Let guilty person go free