11/01/16

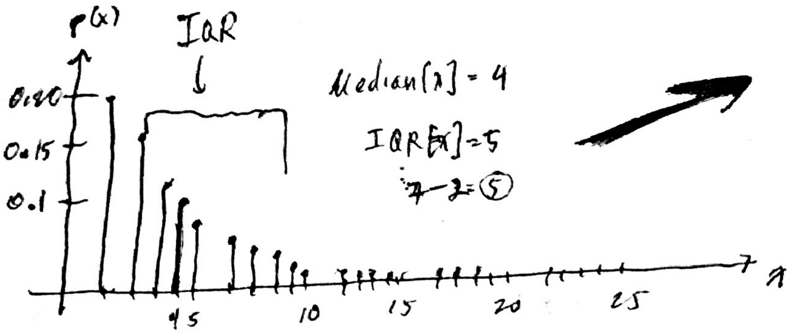
$X \sim Geometric(p) = (1-p)^{x-1} p$

$X \sim Geometric(0.2) = (0.8)^{x-1} 0.2$

x	p(x)	F(x)
1	0.2	0.2
2	0.160	0.360
3	0.128	0.488
4	0.102	0.590
5	0.082	.672
6	0.066	.738
7	0.052	0.710
8	0.042	0.832
⋮		
25	.002	
26	.002	
27	.001	
28	.000	

Approximate / Effective Support

$:: \{x : p(x) \geq 0.001\} \subset Supp[x]$



Median[x] = 4

IQR[x] = 5

~~7 - 2~~ = 5

let $y = x-1$ $x = 1 \dots \infty$

$x = y+1$ $y = 0 \dots \infty$

$$X \sim Geometric(p) \Rightarrow \mu = \sum_{x \in Supp[x]}^{\infty} x(1-p)^{x-1} p = p \cdot \sum_{x=1}^{\infty} x(1-p)^{x-1} = p \sum_{y=0}^{\infty} (y+1)(1-p)^{y} =$$

$$p\left(\sum_{y=0}^{\infty} y(1-p)^{y} + \sum_{y=0}^{\infty} (1-p)^{y}\right) = p\left(\sum_{y=0}^{\infty} y(1-p)\right) = \sum_{y=0}^{\infty} y(1-p)^{y} p + 1 \Rightarrow$$

$$(1-p)\sum_{y=1}^{\infty} y(1-p)^{y-1} + 1 \Rightarrow \mu = (1-p)\mu + 1 \Rightarrow \boxed{\mu = \frac{1}{p}}$$

Function f

$G[f] = \int_{\mathbb{R}} f(x)\,dx = 17$

Mode[x] := $\arg\max_{x \in Supp[x]} \{p(x)\}$

get the biggest one and return the x. (In the graph above, it'll be 1)

Min[x] = min(supp[x])
= 1

Max[x] = max(supp(x))
= DNE

Range(x) = Max[x] - Min[x]
= DNE

Quantile[x, p]
"percentile"

argmin $\{F(x) \geq p\}$

70% percentile of table is 6

$\text{Median}[X] = \text{Quantile}[X, 0.5] \longrightarrow$ for table, it'll be (4)

	distr / r.v
$E[X] = \text{Median}[X]$	symmetric
$E[X] > \text{Median}[X]$	skew right
$E[X] < \text{Median}[X]$	skew left
one mode	unimode
$\text{mode}[X] = E[X] = \text{Median}[X]$	unimode & symmetric

Tertiles

$\text{Quantile}[X, 0.33]$

$\text{Quantile}[X, 0.66]$

Quintiles

$\text{Quantile}[X, .2]$

$\vdots \quad [X, .4]$

$\vdots \quad [X, .6]$

$\text{Quantile}[X, .8]$

$$IQR[X] = Q[X, .75] - Q[X, 0.25]$$

Quartiles

$\text{Quantile}[X, 0.25]$

" $[X, 0.5]$

" $[X, 0.75]$

Deciles

$\text{Quantile}[X, 0.1]$

$\vdots$

" $[X, 0.9]$

EXAMPLE:

Roulette in America

Bet on black... Payout 1 : 1

$$X \sim \begin{cases} \$1 & \text{w.p. } \frac{18}{38} \\ -\$1 & \text{w.p. } \frac{20}{38} \end{cases}$$

win \$1, lose \$1

$$E[X] = \$1 \cdot \frac{18}{38} + -\$1 \cdot \frac{20}{38} = -\$0.053$$

If you play many times, you will on average lose

$$\bar{X} \xrightarrow[n \to \infty]{} E[X] \quad \text{LLN}$$

(long run property)

total pocket

$$\lim_{n \to \infty} T_n = -\infty$$

more you play, the more you'll lose!

still roulette

Bet on "lucky" #7 : Payout 35 : 1

$$E[X] = \$35 \cdot \frac{1}{38} + -\$1 \cdot \frac{37}{38} = -\$0.053$$

$$X \sim \begin{cases} \$35 & \text{w.p. } \frac{1}{38} \\ \$1 & \text{w.p. } \frac{37}{38} \end{cases}$$

Dozen bet is 1 ... 12 Payout is 2 : 1

$$X \sim \begin{cases} \$2 & \text{w.p. } \frac{12}{38} \\ -\$1 & \text{w.p. } \frac{26}{38} \end{cases} \qquad E[X] = \$2 \cdot \frac{12}{38} + -\$1 \cdot \frac{26}{38} = -\$0.053$$

Roulette in Europe = 37 : 1 green

$$X \sim \begin{cases} \overset{1:1}{\$1} & \text{w.p } \frac{18}{37} \\ -\$1 & \text{w.p. } \frac{19}{37} \end{cases} \qquad E[X] = \$1 \cdot \frac{18}{37} + -\$1 \cdot \frac{19}{37} = -\$0.027$$

If X model a fair game...

"Fair game" is if $E[X] = 0$