Lecture 23, December

- p: P(MALE)

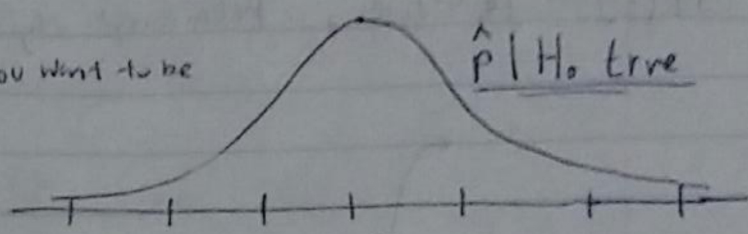
$H_0: p = 0.5$
$H_a: p \neq 0.5$
$\alpha = 5\%$
$n = 345$

default theory what believe.

How skeptical you want to be

can build retainment region

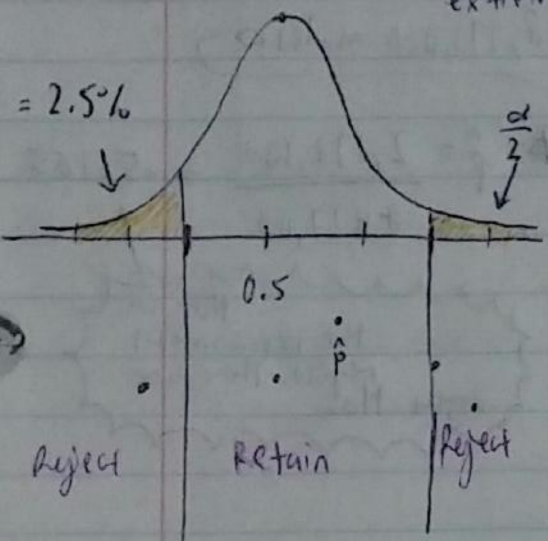
what percentage of distribution you view as extreme.



$\hat{p} \mid H_0$ true

0.5

$$\hat{p} \mid H_0 \text{ true} \sim N\left(p, \left(\sqrt{\frac{p(1-p)}{n}}\right)^2\right) = N(.5, .0269^2)$$



$\frac{\alpha}{2} = 2.5\%$

$\frac{\alpha}{2} = 2.5\%$

0.5

$\hat{p}$

Reject Retain Reject

Two sided test for one proportion - Two sided because can reject right or left

*Retainment region: $\left[p \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}\right]$

$= [0.5 \pm 2 * .0269]$

$= [.446, .554]$

* Probability of rejection is ~~alpha~~ alpha α.

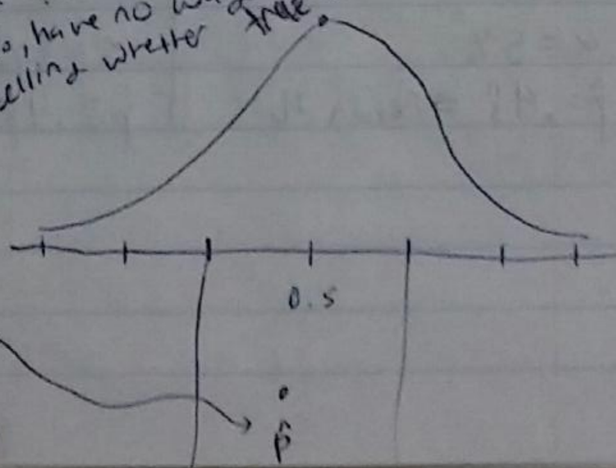
* experiment: collect data

169 males

$\Rightarrow \hat{p} = \frac{169}{345} = 0.48 \leftarrow$ Retain region $\Rightarrow$ Retain H_0

		Decision	
		Retain H_0	Reject H_0
Truth	H_0	✓	Type I error
	H_a	Type II error	✓

once retained H_0, have no way of telling whether true or false, whether H_0 or H_a.



0.5

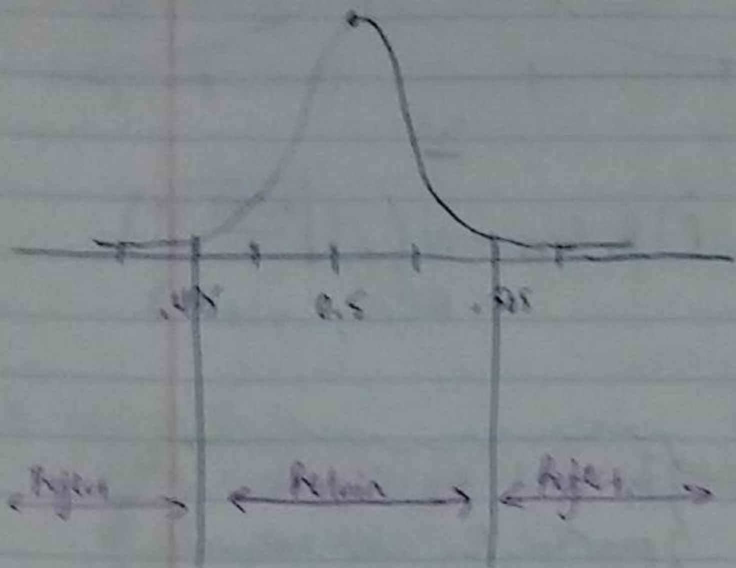
$\hat{p}$

* If $\alpha \downarrow$, P(Type II error) $\uparrow$
* If $\alpha \uparrow$, P(Type II error) $\downarrow$

Type II error, you're retained, but is false!

- I get more data, $n = 4,427,000$, 2008 census data (USA).

$$SE(\hat{p}) = \sqrt{\frac{.5(1-.5)}{4,427,000}} \; ; \quad \text{Retainment region} = \left[p \pm \frac{z_\alpha}{2}\sqrt{\frac{p(1-p)}{n}}\right]$$
$$= [.495, .505]$$



$\hat{P}_n \mid H_0$

2,173,000 males

$$\hat{p} = \frac{2,173,100}{4,427,00} = .51165$$

Not in the retainment region. Therefore reject H_0!

* Bigger n gives smaller SE.

~~changed Type I error~~
~~If α ...~~

* Type one error doesn't change as α goes up & down.

[illegible]

- Retain $H_0 \neq$ Accepting H_0

$H_0: p = 0.5$	$H_0: p = 0.50001$
$H_a: p \neq 0.5$	$H_a: p \neq 0.50001$
$n = 345$	$n = 345$
$\alpha = 5\%$	$\alpha = 5\%$
$\hat{p} = .48 \Rightarrow$ Retain H_0	$\hat{p} = .48 \Rightarrow$ Retain H_0

H_0: UFO's & Aliens do not exist

H_a: do exist

α low

↑ means can throw any evidence, data, proof, will always be a NO!!!

H_0: UFO's & Aliens don't exist

H_a: do exist

α high

H_0: Aliens do exist

H_a: don't

α low

↑ nothing can tell this guy to change his mind.

H_0: Aliens do exist.

H_a: don't

α hight

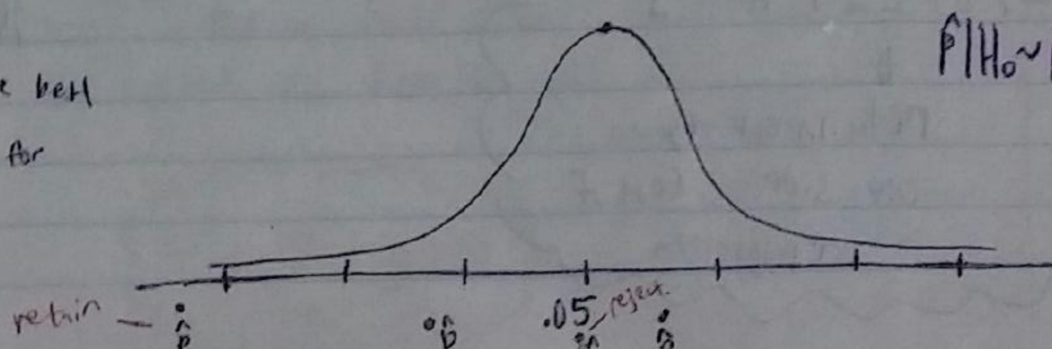
- Case study: If more than 5% of passengers complain, the Uber driver is fired. Uber makes a decision that is keep or fire after 1000 rides.

H_0: Uber driver is a good driver $\Rightarrow p \leq 0.05$ (less than 5% complaints)

H_a: Uber driver is a bad driver $\Rightarrow p > 0.05$ (more than 5% complaints)

$\hat{p} | H_0 \sim N\left(p, \left(\sqrt{\frac{p(1-p)}{n}}\right)^2\right)$

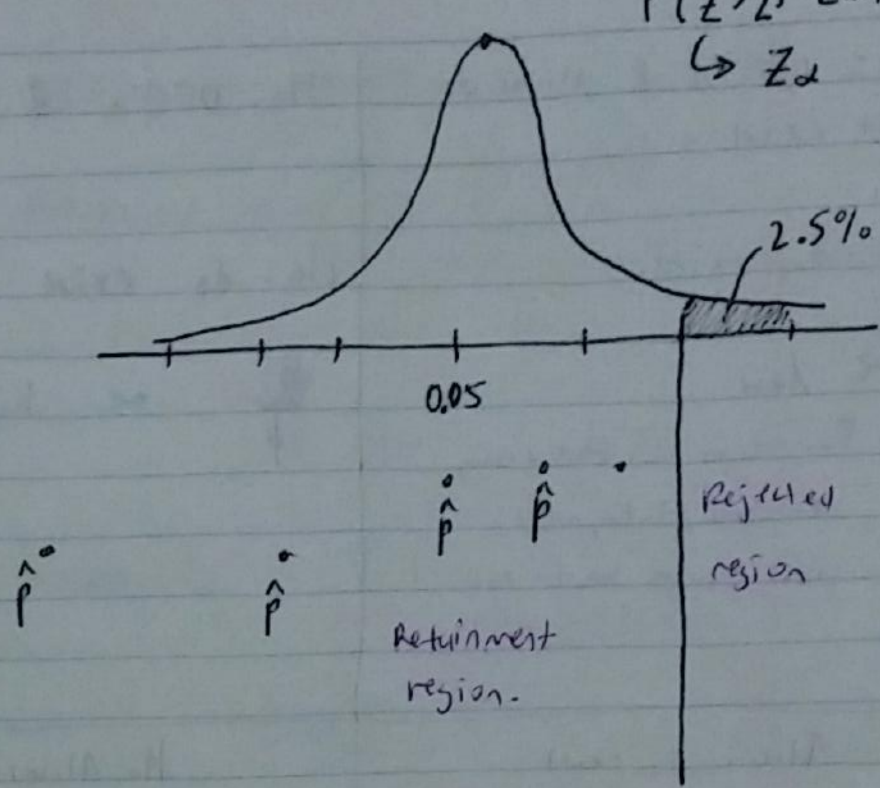
Under null the best possible null for the driver.



$P(Z > Z) = 2.5\%$
$\hookrightarrow Z_\alpha$

let $\alpha = 2.5\%$

$Z_\alpha := F_Z^{-1}(1-\alpha)$



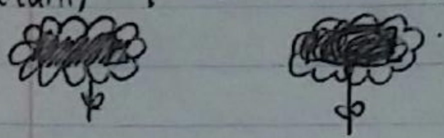
This is called one-sided (or right-sided) test of one proportion,

- Type I error ; Fire a good driver
- Type II error : You keep a bad driver

*Always two errors can make, Never know what/which error can make, random and stuff.

$P(\text{reject}) = \alpha$

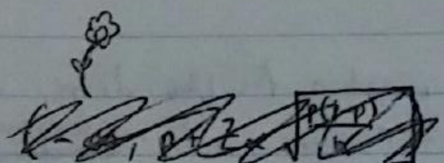
$P(\text{retain}) = 1-\alpha \Rightarrow P(Z \le Z_\alpha) = 1-\alpha \rightarrow P\left(\sqrt{\frac{p(1-p)}{n}}\, Z \le \sqrt{\frac{p(1-p)}{n}}\, Z_\alpha\right) = 1-\alpha$



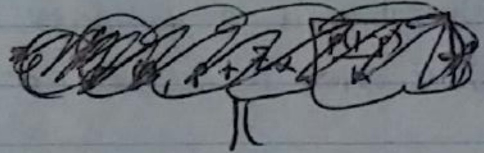
$\Downarrow$

$P\left(p + \sqrt{\frac{p(1-p)}{n}}\, Z \le p + Z_\alpha \sqrt{\frac{p(1-p)}{n}}\right) = 1-\alpha$

$\downarrow$



$\leftarrow P\Big(\underbrace{\hat{p} \le p + Z_\alpha \sqrt{\frac{p(1-p)}{n}}}_{\text{retainment region}}\Big)$



$\left[-\infty,\ p + Z_\alpha \sqrt{\frac{p(1-p)}{n}}\,\right]$

$\Downarrow$

retainment region
one-sided test of one proportion

$$(-\infty, p + z_\alpha\sqrt{\frac{p(1-p)}{n}}] = (-\infty, 0.5 + 2\sqrt{\frac{.05(.95)}{1000}}] = (-\infty, .0638]$$

$\hat{p} = \frac{71}{1000} = .071$ ← experiment
complaints = 71

↑
not in retainment
region ⇒ Reject H_0. Fire driver.

* Why does 5% ≠ 6.38% ?
* Will get closer to 0.05 as taken up.

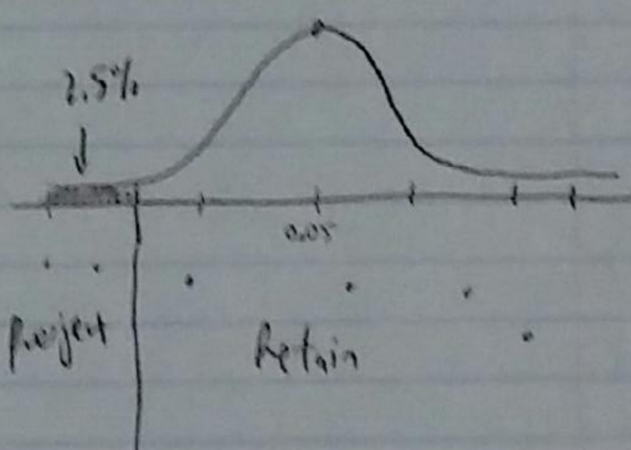
(remember $p > 0.05$)
critical probability of having a complaint.

The situation H_0: good driver, H_a: bad driver is one in which give driver benefit of the ~~doubt~~ doubt.

So, let's look at it the other way:

H_0: bad driver $p \geq 0.05$
H_a: good driver $p < 0.05$
everything else same.

$\alpha = 2.5\%$



Not a good idea because going to reject a lot of drivers.

* Type I error: Keep a bad driver.
* Type II error: A lot, ex: fire a good driver.

END OF MATH 241 :)