P is a set function and $\exists\, \Omega$ such that:

(a) $P(\Omega) = 1$

(b) $P(A) \geq 0 \;\forall A$

(c) If $A_1, A_2, \ldots$ disjoint $\Rightarrow P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$

__Theorem:__ $P(A) = \underset{P(\Omega)}{1} - P(A')$

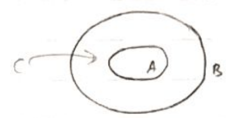
$P(A) \subseteq P(B)$

$P(\Omega) = P(A \cup A^c)$

$P(\Omega) = P(A) + P(A^c)$

$1 = P(A) + P(A^c)$

$P(A) = 1 - P(A^c)$



__Theorem:__ $A \subseteq B \Rightarrow P(A) \leq P(B)$

$C := B \setminus A$

A, C are disjoint } by construction

$B = A \cup C$

$P(B) = P(A \cup C)$

$P(B) = P(A) + P(C)$ by (c)

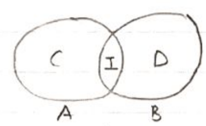
$P(B) - P(A) = P(C) \geq 0$ by (b)

__Theorem:__ $P(A \cup B) = P(A) + P(B) - P(AB)$ → Law of Inclusion-Exclusion

$C := A \setminus B$

$D := B \setminus A$

$I := AB$



$P(A) = P(C) + P(I) \Rightarrow P(C) = P(A) - P(I)$

$P(B) = P(D) + P(I) \Rightarrow P(D) = P(B) - P(I)$

$P(A \cup B) = P(C) + P(D) + P(I)$

$= ((P(A) - P(I)) + (P(B) - P(I)) + P(I)$

$= P(A) + P(B) - P(AB)$

$n = 1000$ people

(A) 200 - smokers

(B) 60 - lung cancer

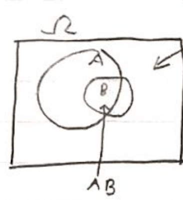
(AB) 36 - smoker and lung cancer

By L.R.F. def.

$P(A) = 0.2$

$P(B) = 0.06$

$P(AB) = 0.036$



A^cB^c: people who don't smoke and don't have lung cancer

The probability of lung cancer __among__ smokers. __given__ a person is a smoker. __Conditional__ on the person smoking.

Page 2

$n = 1000$ people
A - 200 Smokers
B - 60 Lung Cancer
AB - 36 Both

LRF: $P(A) = 0.2$
$P(B) = 0.06$
$P(AB) = 0.036$ } approximates

$A = \Omega' \subseteq \Omega$ ← don't care

$P(B|A) = \frac{36}{200}$

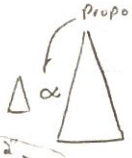
↑ Given

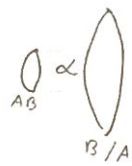
Probability of B given A

The Prob. of Lung Cancer among smokers, given person is a smoker, conditional on the person smoking

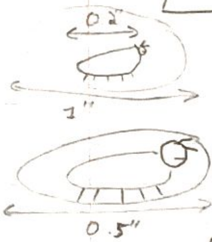
zoom = $\frac{\text{begin scope}}{\text{end scope}}$

proportional





$$P(B|A) = P(BA) \cdot \text{zoom}$$
$$= P(BA) \cdot \frac{P(\Omega)}{P(A)} \begin{matrix} \leftarrow \text{beg-scope} \\ \leftarrow \text{end scope} \end{matrix}$$



$$\text{zoom} = \frac{\text{begin scope}}{\text{end scope}} = \frac{1''}{0.5''} = 2$$

Def: conditional probability

$$P(BA) = P(B|A)\,P(A) \Leftarrow \boxed{P(B|A) := \frac{P(BA)}{P(A)}}$$

Bayes Rule (1763) $\boxed{P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}}$

$$P(BA) \cdot \frac{P(\Omega)}{P(A)} = \frac{0.036}{0.2} = 0.18 \text{ (approximately correct)}$$

$$P(A|B) = \frac{P(BA)}{P(B)} = \frac{0.036}{0.06} = \boxed{0.6}$$

Among those with lung cancer how many smoke? ↑ 60%? Seems too low

$$P(\text{lung cancer among non-smokers}) = P(B|A^c) = \frac{P(BA^c)}{\underbrace{P(A^c)}_{1-P(A)}} = \frac{?}{0.8} = \frac{0.024}{0.8} = 0.03$$

$$P(B) = P(BA) + P(BA^c)$$
$$P(BA^c) = P(B) - P(BA) = 0.06 - 0.036 = 0.024$$

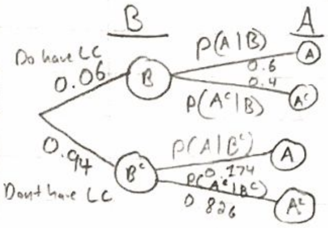
Risk Ratio: $\frac{P(B|A)}{P(B|A^c)} = \frac{0.18}{0.03} = 6$

6 times more likely to ge lung cancer

9/13/16.

$P\left(\frac{\begin{matrix} A \\ A^c \\ B \\ B^c \end{matrix}}{} \quad \frac{\begin{matrix} B \\ B^c \\ A \\ A^c \end{matrix}}{} \right)$

8 total



joint event

$P(AB) = P(B)\ P(A|B) = 0.06 \cdot 0.6 = 0.036$

$P(A^cB) = \quad 0.06 \cdot 0.4 = 0.024$

$P(AB^c) = \quad 0.164$

$P(A^cB^c) = \quad 0.776$

$+ \quad 1.000$

$P(A) = 1 - P(A^c)$

$P(A|B) = 1 - P(A^c|B)$

$\frac{P(AB)}{P(B)} = 1 - \frac{P(A^cB)}{P(B)}$

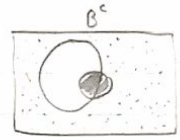
$P(AB) = P(B) - P(A^cB)$

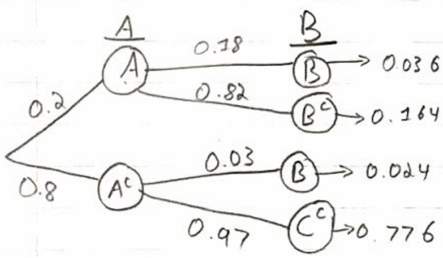
$\Rightarrow P(B) = P(AB) + P(A^cB)$

$P(A) = P(AB^c) + P(AB)$

$P(AB^c) = P(A) - P(AB) = 0.2 - 0.036 = 0.16$

B^c





$P(A^c|C^c)$

Consider event B and events $A_1, A_2, \dots$ Mutually exclusive and collectively exhaustive.

$P(B) = P(B \cap \Omega)$

$= P(B \cap (A_1 \cup A_2 \cup A_3 \cup \dots))$

$= P((B \cap A_1) \cup (B \cap A_2) \cup (B \cap A_3) \cup \dots)$

$= P(B \cap A_1) + P(B \cap A_2) + \dots$

$x \in \bigcup_{i=1}^{\infty} A_i \Rightarrow \forall A_i \text{ s.t. } x \in A_i$

$P(B) = \sum_{i=1}^{\infty} P(BA_i)$ Law of total probability

$\hookrightarrow P(B|A_i)\,P(A_i)$

$\forall i \neq j \quad (B \cap A_i) \cap (B \cap A_j) = \emptyset$

$B \cap B \cap A_i \cap A_j$

$B \cap A_i \cap A_j = B \cap \emptyset = \emptyset$

A, A^c

$P(B) = P(BA) + P(BA^c)$

Baye's Rule

$$P(A_\ell \mid B) = \frac{P(B \mid A_\ell)\, P(A_\ell)}{P(B)}$$

$$\boxed{\text{Baye's Theorem: } P(A_\ell \mid B) = \frac{P(B \mid A_\ell)\, P(A_\ell)}{\sum_{i=1}^{\infty} P(B \mid A_i)\, P(A_i)}}$$

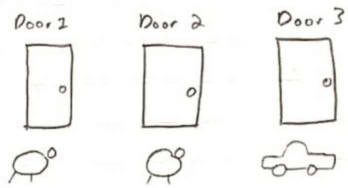
G = Girl
B = Boy

Ω

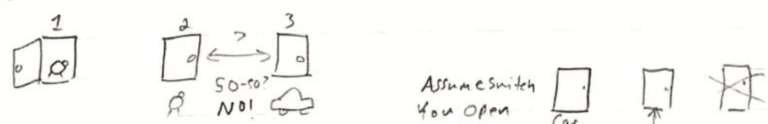
G G	B G
G B	B B

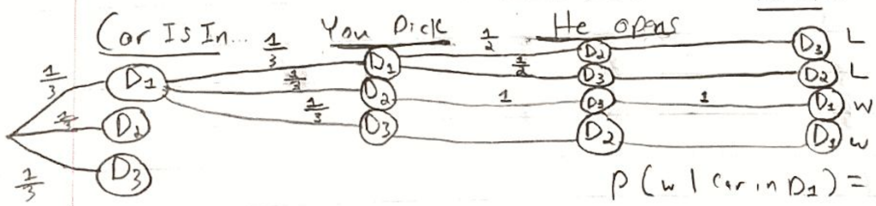
$$P(\text{Other is a Girl} \mid \text{One is a Girl})$$
$$= P(\{GG\} \mid \{GG, GB, BG\}) = \frac{P(GG \cap GG, GB, BG)}{P(GG, GB, BG)}$$
$$= \frac{P(GG)}{P(GG, GB, BG)}$$
$$= \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{4}{12} = \boxed{\frac{1}{3}}$$

Monte Hall Game



① Pick door
② Game show host opens a door w/ goat (not your door)
③ You decide to switch from initial choice





$$P(w \mid \text{Car in } D_1) = \frac{\frac{1}{3}\cdot\frac{1}{3}\cdot 1\cdot 1 + \frac{1}{3}\cdot\frac{1}{3}\cdot 1\cdot 1}{\frac{1}{3}}$$

Probability of winning given the car is in D_1