lec 22

Inverse Problem

-Parameter unknown

-use sample to draw inference above parameters.

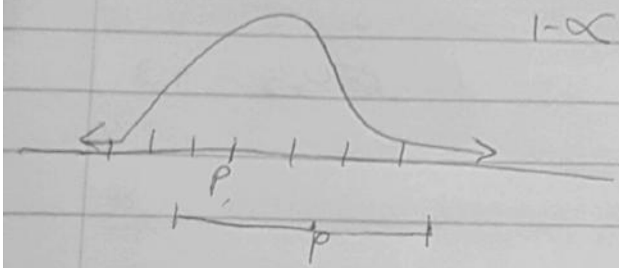
Statistical inference

① Point Estimation: Best guess: $\hat{p}$

② Interval Estimation: confidence Interval

$$CI_{1-\alpha,\,p} := \left[\hat{p} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-p)}{n}}\right]$$

③ Parameter value Testing (Hypothesis Testing)



$1-\alpha$ converge prob if $\alpha = 5\% = 95\%$

what does it mean?

interpretation converge objective

① Before taking simple

$P(p \in CI) = 1-\alpha$

② If you take my simple

$\frac{\# p \in CI}{n} \to p$ by LLN

③ But .. $P(p \in CI)$ after ~~Sych~~ sample ~~tdm~~ then $= \{0, 1\}$

why?

$P(p \in [\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-\hat{p})}{n}}])$

no r.v here! technically illegal statement.

④ But as n subjectness, if you have opinion prior ideas about p, $P(p \in CI) = 1-\alpha$

So $1-\alpha$ confidence $\neq 1-\alpha$ prob. unless you are subjective

~~Cost:~~

~~greater coverage $\alpha \uparrow \Rightarrow$~~

· $\alpha \downarrow \Rightarrow 1-\alpha \uparrow \Rightarrow Z_{\frac{\alpha}{2}} \uparrow$

- if the interval gets too big, usefulness goes ↓
- The interval will be more useful when ...

· Gender Ratio in Human Births $p = P(\text{male})$

$P(\text{male}) \neq P(\text{female})$

my theory $p \neq 0.5$

ie unequal girls ratio

crazy?

Default is "Null" Hypothesis

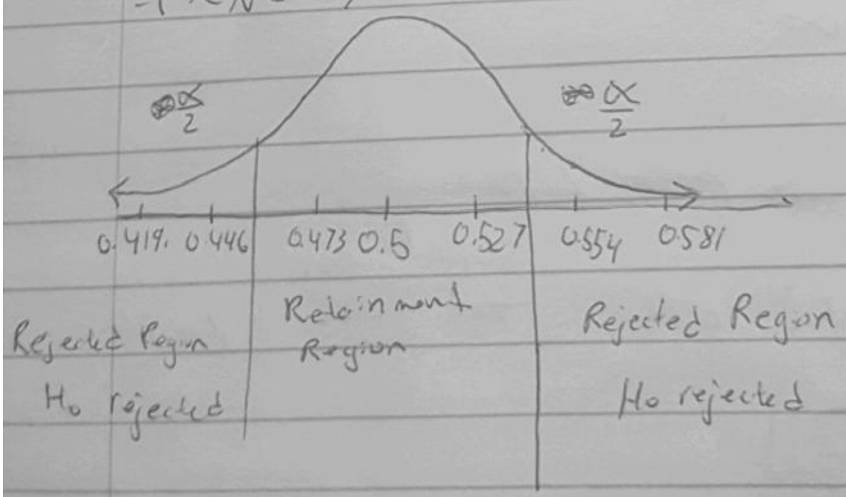
denoted,

$H_0: p = 0.5$

we take a Sample $n=345$

$$\hat{p} \sim N\left(\underset{0.5}{p}, \left(\underbrace{\sqrt{\frac{p(1-p)}{n}}}_{0.5 \quad 0.5, \; n=345}\right)^2\right)$$

$$= \hat{p} \sim N(0.5, 0.0269^2). \; 0.069$$



final only $\alpha = 1\%$ or 5%

let $\alpha = P(\text{too rare})$

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Retainment Region $= \left[p \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}\right]$

Rejected Region $= \left[p \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}\right]^c$

$1-\alpha = P(H_0 \text{ retained})$

$= P(\hat{p} \in [\hat{p}\ \text{small}, \hat{p}\ \text{large}])$

$= P(\hat{p} \in [p \pm \text{margin}])$

$= P\left(\hat{p} \in \left[p \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}\right)\right.$

Calculate $\hat{p}$

① If $\hat{p} \in$ Retained Region $\Rightarrow$ Retained H_0

But we do not have sufficient evidence to reject

② If $\hat{p} \in$ Rejected Region $\Rightarrow$ Rejected H_0 then we accept H_a. we have sufficent evidence to reject the null Hypothesis

Example. $n = 345$, $\alpha = 5\%$

retainment Region $= \left[0.5 \pm 2 \underbrace{\sqrt{\frac{0.5(1-0.5)}{345}}}_{0.0269}\right] = [.446, .554]$

if 169

babies were male $\Rightarrow \hat{p} = \frac{169}{345} = .48 \in$ retainment region

$\Rightarrow$ retain H_0

$\Rightarrow$ we do not have sufficent evidence to reject human girls rate equality.

range of $\quad CI \; p, 1-\alpha = \left(\hat{p} \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{p^2(1-p}{n}} \right.$

Flip a coin 100 times.

test the theory the ~~coin~~ coin is unfair.

fair means $p = P(H) = .5$

I $\quad$ 51 H $\Rightarrow \hat{p} = .51$ Fair? Yes. $\quad$ don't need 241

II $\quad$ 98 H $\Rightarrow \hat{p} = .98$ Fair? No $\quad$ " "

III $\quad$ 61 H $= \hat{p} = .61$ Fair? ~~Yes~~ need 241

$n = 100, \quad \alpha = 5\%$

$H_0: p = 0.5, \quad H_a: p \neq 0.5$

Ret Region $= \left[0.5 \pm 2 \sqrt{\frac{0.5(1-0.5)}{100}} \right] = [0.4, 0.6]$

61 H $= \hat{p} = .61$ fair? No Since $.61 > 0.6$

$\hat{p} \notin$ Retain Region $\Rightarrow$ Reject Ha, we have ~~empty~~ enough ~~reject~~ evidence to reject the theory that the coin is fair.

mors (the

claims the prop of blue is 20% less

$p($

$\alpha = 1\%$

$H_0 : p = 0.2$

$H_a : p \neq 0.2$

$n = 636$

Retainment

Region $= \left[p \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}} \right]$

$= \left[0.2 \pm \underbrace{2.84 \sqrt{\frac{0.2(1-0.2)}{636}}}_{.0450} \right]$

.0150

$[.155, .245]$ retainment region

$\hat{p} = \frac{168}{636} = .264 \notin$ Ret. Reg $\Rightarrow$ Reject H_0

We have enough evidence
the reject the claim made
by the this candy co or
think the prop of
Blue is d this is 20%