95% size.

Confidence Interval for the time program of randomness lites:

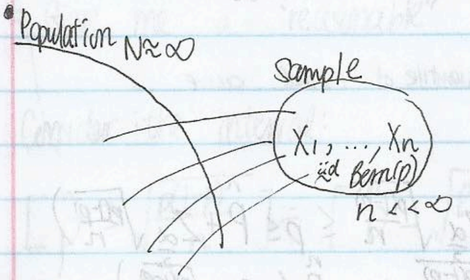
Best interval guess for...

$CI_{95\%} = [.59 \pm Z\sqrt{\frac{0.59(1-.59)}{32}}]$

$= [.416, .764]$

12/06

Bern case



"Inverse Problem"

- parameter unknown
- Use sample to draw inference above parameters.

Statistical Inference.

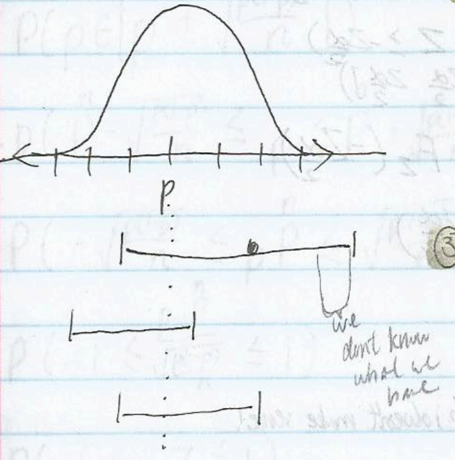
① Point Estimation: Best guess: $\hat{p}$

② Interval Estimation: Confidence Interval

$CI_{1-\alpha, p} := [\hat{p} \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}]$

③ Parameter Value Testing (Hypothesis Testing)

$1-\alpha$ coverage prob.
if $\alpha = 5\%$.
$= 95\%$
(catches)



Interpretations of a CI.

if only shown.

only 1. If you take my sample

① $\frac{\#\{p \in CI\}}{n} \xrightarrow{} 1-\alpha$ (cover)

$\rightarrow p$ by LLN.

objective.

② Before experiment,

$P(p \in CI) = 1-\alpha \longrightarrow$

③ If you believe in subjective prob ( )
then under prior information, you can say
$P(p \in CI) = 1-\alpha$

But $P(p \in CI)$ after sample then $= \{0, 1\}$

WHY? $P(p \in [\hat{p} \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}])$

no r.v's here!
technically illegal statement.

④ But as n subjectness, if you have opinion prior ideas abt p,
$P(p \in CI) = 1-\alpha$
So $1-\alpha$ confidence $\neq 1-\alpha$ prob. unless you are subjective.

cost:

- greater coverage $\alpha \uparrow \Rightarrow$ width of interval $\uparrow$
usefulness $\downarrow$

- making interval large $\Rightarrow n \uparrow$
(but that in the real mode is costly

- $\alpha \downarrow \Rightarrow 1-\alpha \uparrow \Rightarrow Z_{\frac{\alpha}{2}} \uparrow$

- If the interval gets too big, usefulness goes $\downarrow$.
- The interval will be more useful when...

- Gender Ratio in Human Births $p = P(\text{male})$

$P(\text{male}) \neq P(\text{female})$

We know how the world works. $p = 0.5$ so we know how the distribution looks like.

My theory: $p \neq 0.5$
i.e. unequal girls ratio
crazy?

we take a sample of $n = 345$

$$\hat{p} \sim N\left(\underset{0.5}{p}, \left(\sqrt{\frac{p(1-p)}{n}}\right)^2\right)$$

$0.5 \quad 0.5$, $n = 345$, 0.0269

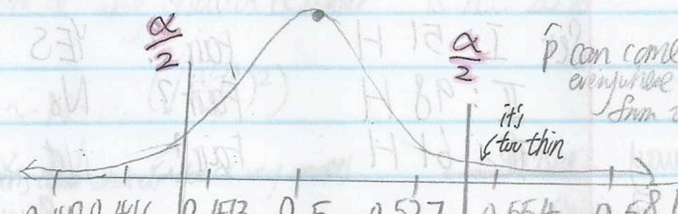
$$= \hat{p} \sim N(0.5, 0.0269^2).\ 0.0269$$

- Default / "Null" Hypothesis denoted.

$H_0: p = 0.5$

The crazy theory is the alternate Hypothesis.

Take this $\Rightarrow$ $H_a: H_0$ is false: $p \neq 0.5$ (my theory).



$\hat{p}$ can come from everywhere but mostly from the center

Rejected Region H_0 rejected

reasonable $\hat{p}$

Rejected Region H_0 rejected.

$\hat{p}$ too small

Retainment Region "H_0 retained"

$\hat{p}$ too large

$\hat{p} \leftarrow$ not likely. too rare

$\hat{p}$ way are very unlikely.

from previous class

don't believe this line (not believe).

Let $\alpha = P(\text{too rare})$

$$\begin{aligned} 1-\alpha &= P(H_0 \text{ retained}) \\ &= P(\hat{p} \in [\hat{p}_{\text{small}}, \hat{p}_{\text{large}}]) \\ &= P(\hat{p} \in [p \pm \text{margin}]) \\ &= P\left(\hat{p} \in \left[p \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}\right]\right) \end{aligned}$$

* on final only given $\alpha = 1\%$ or 5%

$\Rightarrow$ Retainment Region $= \left[p \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}\right]$

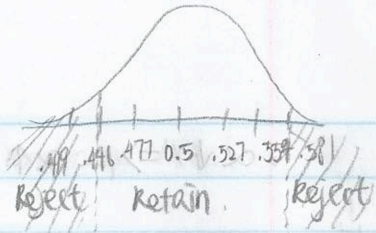
$\Rightarrow$ Rejected Region $= \left[p \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}\right]^C$.

Calculate $\hat{p}$

☆ The null Hypothesis ☆

(I) If $\hat{p} \in$ Retained Region $\Rightarrow$ Retained H_0
But we do not have sufficient evidence to reject

(II) If $\hat{p} \in$ Rejected Region $\Rightarrow$ Rejected H_0. then we accept H_a. We have sufficient evidence to reject the null Hypothesis.



Examples: $n = 345, \alpha = 5\%$

Retainment Region $= [0.5 \pm 2 \underbrace{\sqrt{\frac{0.5(1-0.5)}{345}}}_{0.0269}] = [.446, .554]$.

If 169 babies were male $\Rightarrow \hat{p} = \frac{169}{345} = .48 \in$ Retainment Region $\Rightarrow$ we do not have sufficient evidence to reject human girls rate equality. $\Rightarrow$ Retain H_0.

• Flip a coin 100 times, you want to know if coin is fair.

Ser I: 51 H Fair? YES

II: 98 H Fair? No.

III: 61 H Fair? Not so clear!!! Run test at $\alpha = 5\%$.

Too far away from what we expect.

Logical $H_0: p = 0.5$

$H_a: p \neq 0.5$

$\alpha = 5\%$

Retainment Region $= [p \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}] = [0.5 \pm 2 \sqrt{\frac{0.5(1-0.5)}{100}}] = [0.4, 0.6]$

$\hat{p} = \frac{61}{100} \Rightarrow \hat{p} \notin$ Retainment Region. $\Rightarrow$ Reject $H_0 \Rightarrow$ coin is unfair

- Mars (the candy co). says the prop of blue M&M's is 20%. You think otherwise.

Let $p := P(\text{Blue})$ $n = 615$ m&m's.

only 2 values on final.

$Z_{\frac{\alpha}{2}} = 2.84$ when $\alpha = 1\%$

or $= 2$ when $\alpha = 5\%$

$H_0 : p = 0.2$

$H_a : p \neq 0.2$

Retainment Region $= [p \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}]$

$= [0.2 \pm 2.84 \sqrt{\frac{0.2(1-0.2)}{615}} = [.1542, .2458]$

$\hat{p} = \frac{158}{615}$ (blue M&MS) $= 0.2569 \Rightarrow \hat{p} \notin$ Retainment Region

$\Rightarrow$ Reject H_0.

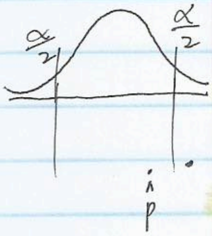
The prob of blue M&M's is not 20%!

$\rightarrow$ Where does α come from & why should it matter?

- $\hat{p}$ is draw from $\hat{p} \mid H_0 \sim N(p, (\sqrt{\frac{p(1-p)}{n}})^2)$

meaning := rejecting the no if it works

you pick how often it happens (your choice) (make α very small)



$P(\text{Type I error}) = \alpha$

$\rightarrow$ reject H_0 if true

$P(\text{Type II error})$ = beyond this class

$\rightarrow$ retain H_0

$P(\text{Reject } H_0 \mid H_0 \text{ false})$ = POWER.

Decision

Truth

	Retain H_0	Reject H_0
H_0 true	✓	Type I error
H_0 false	Type II error	✓

- 2 types of mistakes you can make.

You have a Trade off to make

- $\alpha \uparrow \Rightarrow P(\text{Type I error}) \uparrow \Rightarrow P(\text{Type II error}) \downarrow$.
- $\alpha \downarrow \Rightarrow P(\text{Type I error}) \downarrow \Rightarrow P(\text{Type II error}) \uparrow$

Trial I: Clinical

H_0: drug does not work

H_a: drug works

Decision: release drug to market

- Type I error: release a drug that does not work.

cost: possible deaths.

- Type II error: not releasing drug that works.

cost: people could be helped.

base on what we choose
we will pick our α

Trail II: Court Case

H_0 : Innocent

H_a : Guilty

Decision: punish or not.

Type I error: punish an innocent person.

Type II error: let a guilty person go free.

Trail III: Fire alarm

H_0 : No fire

H_a : Fire

Decision: set off alarm

Type I error: false alarm

Type II error: fire but no alarm.