9/15/2016

~~I~~ Information irrelavant

$P(A) = \frac{4}{52} = \frac{1}{13} = P(A|\heartsuit) = \frac{1}{13}$

$P(\text{IBM stock} \uparrow \text{ in a day}) = P\left(\text{IBM stock} \uparrow \text{ in a day} \,\middle|\, \text{rains in Buzorare}\right)$

Definition: A, B are independent events if

+ $P(A|B) = P(A)$
+ $P(B|A) = P(B)$

$P(A) = P(A|B) = \frac{P(AB)}{P(B)} \Rightarrow P(AB) = P(A)P(B)$

Multiplication Rule

$A_1, A_2, \ldots$ independent events

$$\boxed{P\left(\bigcap_{i=1}^{\infty} A_i\right) = \prod_{i=1}^{\infty} P(A_i)}$$

$P(H_2|H) \overset{?}{=} P(H_2)$

$P(H_1H_2H_3H_4H_5) = \frac{1}{|\Omega^5|} = \frac{1}{2^5} = P(H_1).P(H_2)..P(H_5) = \frac{1}{2^5}$

$P(\{\geq 1 \text{ 6-6 in 24 rolls}\}) < \frac{1}{2}$

$= P(1 \text{ 6-6 in } 24) + P(2 \text{ 6-6 in } 24) + \ldots + P(\geq 4)$

$= 1 - P(\text{zero 6-6}) = 1 - P(\text{not 6-6 in roll 1} \cap \text{not 6-6 roll} \cap \ldots)$

$= 1 - P(\text{no 6-6 in 24})^{24} = 1 - (1 - P(66))^{24}$

$= 1 - (1 - P(6)P(6))^{24} = 1 - (1 - (\frac{1}{6})^2)^{24}$

$= 0.4914039$

Definition: A, B are dependent $P(A|B) \neq P(A)$ or $P(B|A) \neq P(B)$, $P(AB) \neq P(A).P(B)$

$P(\text{Q64 is late}) < P(\text{Q64 is late} \mid \text{raining \& traffic})$

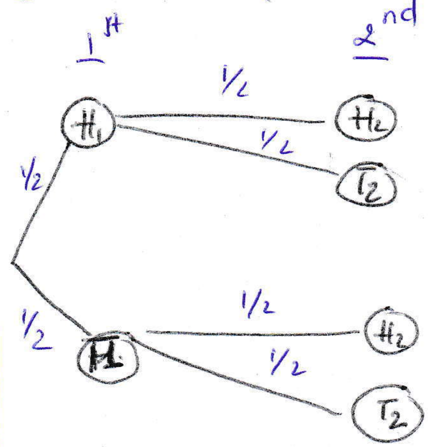
$P(\text{Q64 is late}) > P(\text{Q64 is late} \mid \text{sunny \& no traffic})$

Marginal Prob. Comb Prob.

$P(\text{lung cancer}) < P(\text{lung cancer} \mid \text{smoking})$

A, B disjoint $\overset{?}{\not\Rightarrow}$ A, B independent

$0 = P(H|T) \neq P(H)$

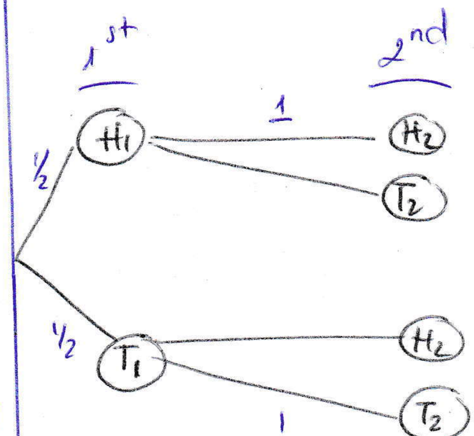


$P(HH) = \frac{1}{4}$

$P(HT) = \frac{1}{4}$

$P(TH) = \frac{1}{4}$

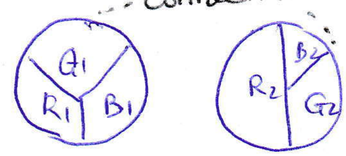
$P(TT) = \frac{1}{4}$



$P(HH) = \frac{1}{2}$

$P(TT) = \frac{1}{2}$

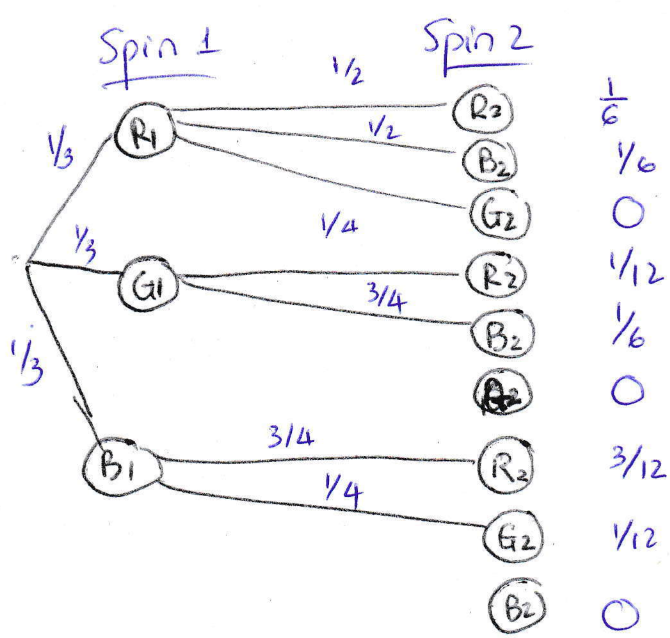
Physical connection



$P(R_1) = P(G_1) = P(B_1) = \frac{1}{3}$

$P(R_1) = \frac{1}{2}$, $P(G_2) = \frac{1}{3}$

$P(B_2) = \frac{1}{6}$



Consider R_1, R_2: $P(R_2 \mid R_1) = P(R_2) = \frac{1}{3}$

Consider R_1, G_2: $0 = P(G_2 \mid R_1) \neq P(G_2) = \frac{1}{3}$

P(shared birthday)

$= P(\geq 1$ shared bday among 49 people$)$

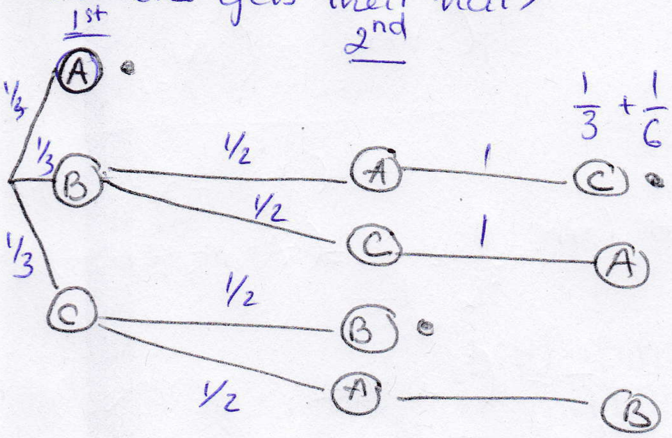
$= P(1$ share$) + P(2$ share$) \ldots P(49)$

$= 1 - P($no shared bday$)$

$$= 1 - \frac{365 \cdot 364 \cdot 363 \ldots (365-49+1)}{365^{49}}$$

$$= 1 - \frac{365P49}{365^{49}} \approx 97\%$$

P(no one gets their hat)



$\frac{1}{3} + \frac{1}{6} + \frac{1}{6} = \frac{2}{3}$

$= 1 - P($someone gets their hat$)$

$= 1 - (P($one get hat$) + P(2) + \ldots + P(n))$

$= 1 - P(A_1 \cup A_2 \ldots \cup A_n) = 1 - P(\bigcup_{i=1}^{n} A_i)$

$P(A \cup B) = P(A) + P(B) - P(AB)$

$\Rightarrow P(\bigcup_{i=1}^{2} A_i) = \sum_{i=1}^{2} P(A_i) - \cancel{2} P(\bigcap_{i=1}^{2} A_i)$

$$P(\bigcup_{i=1}^{n} A_i) = \underbrace{\sum_{i=1}^{n} P(A_i)}_{\sum_{i=1}^{n} \frac{1}{n} = 1} - \underbrace{\sum_{i \neq j} P(A_i \cap A_j)}_{\binom{n}{2} \frac{(n-2)!}{n!} = \frac{1}{2}}$$

$$+ \underbrace{\sum_{i \neq j \neq k} P(A_i \cap A_j \cap A_k)}_{\binom{n}{3} \frac{(n-3)!}{n!} = \frac{1}{3}} - + - + (-1)^{n+1} P(\bigcap_{i=1}^{n} A_i)$$

$P(A_1) = \frac{1}{n}$, $P(A_2) = \frac{1}{n} \Rightarrow \sum_{i=1}^{n} \frac{1}{n} = 1$

$$P(A_1 \cap A_2) = \frac{1 \cdot 1 \cdot (n-2) \cdot (n-3) \ldots 1}{n!} = \frac{(n-2)!}{n!}$$

$$P(A_2 \cap A_4) = \frac{(n-2) \cdot 1 \cdot (n-3) \cdot 1 \ldots}{n!} = \frac{(n-2)!}{n!}$$

$$e^x = e^0 + e^0 x + \frac{e^0}{2!} x^2 + \frac{e^0}{3!} x^3 + \ldots$$

$$= 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \ldots = \sum_{i=0}^{\infty} \frac{1}{i!}$$

$$e^{-1} = 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \ldots$$

$$1 - e^{-1} = 1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \ldots$$

$$= 1 - P(\bigcup_{i=1}^{n} A_i) = 1 - (1 - e^{-1}) = e^{-1}$$

≈ 0.36