On exam: real life model question
→ give model
→ calculate.

10/20 • $X \sim Geom(p) := (1-p)^{x-1} p \rightarrow$ P.M.F
$Supp[X] = \mathbb{N}$

– Ex: Play poker until Royal Flush (Real Life Model Question)
$P(\text{Royal Flush}) = \frac{4}{\binom{52}{5}} = 1.53/\text{million} = 0.00000153.$

☆ The probability of a certain stage = P.M.F situation.

→ Getting R.F when playing A million times?

Model: $X \sim Geom(0.00000153)$

(use the P.M.F for answer) Calculation: $P(X = 1000000) = (0.9999985)^{999999} \cdot .000000153.$

– A million times or sooner.

This time or less or more ↓ Do C.D.F.

$P(X \leq 1000000) = F(1000000)$
$= 1 - (1-p)^x$
$= 1 - 0.9999985^{1000000} \approx 77\%.$

☆ Less than ↓ C.D.F

• $X_1, X_2, \ldots \overset{iid}{\sim} Berm(p)$
$X = \min\{t : X_t = 1\}$

Wait until you get r successes. (meaning) $\Leftarrow T = \min\{t : \sum_{i=1}^{t} X_i = r\}$

$r = 3$ $\frac{0}{1}$ $\frac{0}{2}$ $\frac{0}{3}$ $\frac{0}{4}$ $\frac{0}{5}$ $\frac{0}{6}$ $\frac{0}{7}$ $\frac{0}{8}$ $\frac{1}{9}$ $\frac{0}{10}$ $\frac{1}{1}$

Just like flipping coin: you don't know what happened previously. (Looking for 3 successes)

$P(T=1) = 0$ → not possible, can't get 3 successes in 1 flip.
$P(T=2) = 0$ → not possible either.
$P(T=3) = p^3$ → you need to have success 3 times. (they are independent & identically distributed).
$\frac{1}{1}$ $\frac{1}{2}$ $\frac{1}{3}$

$P(T=4) = 3(1-p)p^3$ → many ways to do it
$\frac{1}{1}$ $\frac{1}{2}$ $\frac{0}{3}$ $\frac{1}{4}$
$\frac{0}{1}$ $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$
$\frac{1}{1}$ $\frac{0}{2}$ $\frac{1}{3}$ $\frac{1}{4}$
→ The last one is fix!
→ The prob = these 3 situations.
→ you add each possible way (they are disjoint)

☆ if $\underline{1}\,\underline{1}\,\underline{1}\,\underline{0}$ ↓ not T=4 ($P(T=3)$)

☆ $\binom{4}{2}$ successes.

p^3 = 3 successes.

$P(T=5) = \binom{4}{2}(1-p)^2 p^3$ (= 6)
→ $\underline{1}\ \underline{0}\ \underline{1}\ \underline{0}\ \underline{1}$
$\underline{1}\ \underline{1}\ \underline{0}\ \underline{0}\ \underline{1}$
$\underline{1}\ \underline{0}\ \underline{0}\ \underline{1}\ \underline{1}$ you stop here, fifth must be a success otherwise you will keep going.
$\underline{0}\ \underline{1}\ \underline{1}\ \underline{0}\ \underline{1}$
$\underline{0}\ \underline{0}\ \underline{1}\ \underline{1}\ \underline{1}$
$\underline{0}\ \underline{1}\ \underline{0}\ \underline{1}\ \underline{1}$

$T = X$.

the failures. successes.

$\binom{X-1}{2}$ (2 successes) they could be anywhere

$r=3$ $\quad P(T=x) = \binom{x-1}{2}(1-p)^{x-3} p^{3}$

$\overbrace{\_\ \_\ \_\ \ldots\ \_}^{\binom{X-1}{2}}\ \ 1$
$1\ \ 2\ \ 3\ \ \ldots\ \ X-1\ \ X$

Model = "Negative Binomial"

- $X \sim \text{Neg Bin}(r,p) := \binom{x-1}{r-1}(1-p)^{x-r}p^{r}$

- all the possible outcomes. $\Leftarrow$ $\text{Supp}[X] = \{r, r+1, r+2, \ldots\}$ → you can't get less than r!

- Param. Space: $p \in (0,1)$, $r \in \mathbb{N}$

☆ Stopping at 1 success = Geometric

- $X \sim \text{Neg Bin}(1,p) = \binom{x-1}{1-1}(1-p)^{x-1}p^{1}$
  $= (1-p)^{x-1}p = \text{Geometric}(p)$

☆ Proof Not cover on exam.

- $1 = \sum_{x \in \text{Supp}[X]} p(x) \qquad 1 = \sum_{x=r}^{\infty} \binom{x-1}{r-1}(1-p)^{x-r}p^{r}$ (P.M.F.)

Geom series $\sum_{i=0}^{\infty} a^{i} = \frac{1}{1-a}$ if $a \in (0,1)$

$\downarrow$

$\downarrow \sum_{x=0}^{\infty}(1-p)^{x} = \frac{1}{p} = p^{-1}$

$\downarrow$ same exact thing

$\downarrow \sum_{x=1}^{\infty}(1-p)^{x-1} = p^{-1}$

$\downarrow$ Take $\frac{d}{dp}[\,\cdot\,]$ both sides.

Technically starts at $x=1$ but $=0$ so starts at $x=2$.

$\sum_{x=2}^{\infty}(-1)(x-1)(1-p)^{x-2} = (-1)p^{-2}$

forget abt $x=2 \to =0$.

$\sum_{x=3}^{\infty}(-1)^{2}(x-1)(x-2)(1-p)^{x-3} = (-1)(-2)p^{-3}$

$\sum_{x=4}^{\infty}(-1)^{3}(x-1)(x-2)(x-3)(1-p)^{x-4} = (-1)(-2)(-3)p^{-4}$

pattern: $d \quad d-1 \quad d \quad d-1 \quad d! \quad d-1$

$\sum_{x=r}^{\infty} \underbrace{(x-1)(x-2)\ldots(x-(r-1))}_{\frac{(x-1)!}{(x-r)!}}(1-p)^{x-r} = (r-1)!\,p^{-r}$

$\binom{x-1}{r-1}$

cancelling r-1 on both sides. $\sum_{x=r}^{\infty}\frac{(x-1)!}{(x-r)!\,(r-1)!}(1-p)^{x-r}p^{r} = \cancel{p^{-r}} \cdot \cancel{p^{r}} = 1$

$\downarrow$

$((x-1)-(x-r))!$

Not on exam.
great for cow farmer
→ help for calculating temp of the cows.

• $X \sim$ Neg Binomial (r, p)

pmf $X :=$ # failure.

$$\overbrace{\_\ \underbrace{0\ \ 0\ \ 0\ \ 0}_{x}\ \dots\ \_\ 1}^{x+r-1}$$

$$:= \binom{x+r-1}{r-1}(1-p)^x p^r = (-1)^x \binom{-r}{x}(1-p)^x p^r$$

$$\frac{(x+r-1)!}{x!(r-1)!} = \frac{\overbrace{(x+r-1)(x+r-2)\dots x}^{x}}{x!} = (-1)^x \frac{(-r)(-r-1)\dots(-r-x-1)}{x!}$$

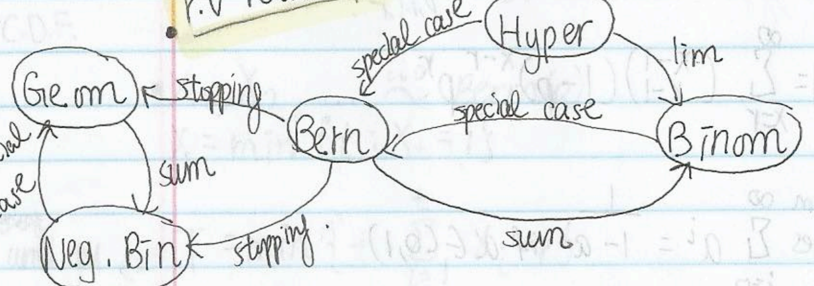
$$= (-1)^x \binom{-r}{x}$$

Another Fact

• $X_1, X_2, \dots, X_r \overset{iid}{\sim} Geom(p)$ = waiting for r successes.

$\Leftarrow X_1 + X_2 + \dots + X_r \sim NegBin(r, p)$

• r.v relationships



- Random Variable = a model
  not real
  (waiting to be happened)

- Realization = happened! Real
  = could happen.
  = must live in the support.
  x (small) $\in Supp[X]$ (Big)

• $X \sim Bern(\frac{1}{2}) = \begin{cases} 1 & wp\ \frac{1}{2} \\ 0 & wp\ \frac{1}{2} \end{cases}$

$X \neq 1$ (model) $\quad x = 1$ (realization = to make real)

$X=1$ means nothing by itself. (OK) → $P(X=1)$
X = function

Datum: Realization of a r.v. — realization of your height could be shorter / higher...

HW Data: Realization of r.V is.

iid data:

connected

Hyper geometric 8 coins
special spotted nickles $=4$.
Took 3 coins (drawn).
got 1 spotted. ($\overset{\text{small}}{x}=1$) came to realization.

From model to Data. (Got 1 spotted coin) $x=1$

- $X \sim Hyper(\underset{n}{\overset{\shortparallel}{3}}, \underset{K}{\overset{\shortparallel}{4}}, \underset{N}{\overset{\shortparallel}{8}})$

Supp $= 0, 1, 2, 3$

→ 7 ppl 7 identical cups 8 coins in each.

$$X_1, \dots, X_7 \overset{iid}{\sim} Hyper(3, 4, 8)$$

→ Drawing 3 from the cups. (7 ppl, 7 cups) iid.

$X_1 = 2, X_2 = 2, X_3 = 2, X_4 = 1, X_5 = 1, X_6 = 0, X_7 = 1$ (No one got 3) but could have it next time.

(Each Nickels are same, prob of head $= \frac{1}{2}$, not cared abt spotted)

Not care about the spotted ones.

→ 1 ppl drawing coins from 1 cup.

$$X \sim Binom(8, \tfrac{1}{2})$$

Heads $= x = 6$

→ 7 ppl doing the same thing as before.

$$X_1, \dots, X_7 \overset{iid}{\sim} Binom(8, \tfrac{1}{2})$$

# of heads $= x_1 = 4$, $X_2 = 4$, $X_3 = 0$, $X_4 = 4$, $X_5 = 4$, $X_6 = 4$, $X_7 = 4$. any legal things could happen.