11/1/16

$X \sim Geom(p) := (1-p)^{x-1} p$

ex $X \sim Geom(0.2) = (0.8)^{x-1}(0.2)$

X	p(x)	F(x)	X	P(x)	F(x)	X	p(x)	F(x)
1	0.200	0.200	11	.021		21	.002	
2	0.160	0.360	12	.017		22	.001	
3	0.128		13	.014		23	.001	
4	0.102		14	.011		24	.001	
5	0.082		15	.009		25	.001	
6	0.066		16	.007		26	.001	
7	0.52		17	.006		27	.001	
8	0.42		18	.005		28	.000	.999
9	0.34		19	.004				
10	0.27		20	.003				

Approximate / Effective support

$\{x : p(x) \geq .001\} \subset Supp[X] = \mathbb{N}$

smallest subset A s.t $\sum_{x \in A} p(x) = 0.999$

True expectation = True balance point

$\bar{X} \rightarrow E[X] = \mu$ (single avg r.v)

$X \sim Bern(p) = E[X] = p$

$X \sim Binom(n,p) = E[X] = np$

$X \sim Hyper(n, K, N) = E[X] = n\frac{K}{N}$ (wait)

$X \sim Geom(p) = E[X] = \frac{1}{p}$

$X \sim NegBinom(r,p) = E[X] = \frac{r}{p}$

$E[X] = \sum_{x \in supp[X]} x\, p(x) = \sum_{x=1}^{\infty} x(1-p)^{x-1} p$

let $y = x-1$

$x = y+1$

$x = 1 \ldots \infty$

$y = 0 \ldots \infty$

$= p\sum_{x=1}^{\infty} x \cdot (1-p)^{x-1} = p\sum_{y=0}^{\infty} (y+1)(1-p)^{y}$

$$= p\left(\sum_{y=0}^{\infty} y(1-p)^y + \sum_{y=0}^{\infty} (1-p)^y\right)$$

recall: $\sum_{i=0}^{\infty} = \frac{1}{1-a}$

$$= p\left(\sum_{y=0}^{\infty} y(1-p)^y + \frac{1}{p}\right) = \sum_{y=0}^{\infty} y(1-p)^y p + 1$$

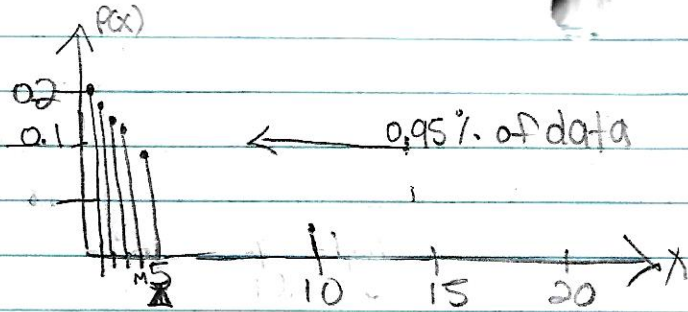
$$= (1-p)\sum_{y=0}^{\infty} y(1-p)^{y-1} p + 1 = (1-p)\sum_{y=1}^{\infty} y(1-p)^{y-1} p + 1$$

$$\mu = (1-p)\mu + 1 = \cancel{\mu} = \cancel{\mu} - p\mu + 1 = \boxed{\mu = \frac{1}{p}}$$

E[X] is a functional

$G[t] = \int_{\mathbb{R}} f(x)\, d(x) = 17$

Mode[X] = argmax $\{p(x)\}$



Quantile[X,p] = $\underset{x \in supp[X]}{\text{argmin}} \{F(x) \geq p\}$

"percentile" if measured as a %.

Quantile [X, .95]

Quantile [X, .5] → median[X] :=

E[X] = 5, mode[X] = 1

quantile [X, .95] = 14

median [4]

If	Distri. Type / r.v type
E[X] = median [X]	"symmetric"
E[X] > median[X]	skew right
E[X] < median [X]	skew left
If one mode	unimodal
E[X] = median[X] = mode[X]	symmetric unimodal

Interquantile range

$IQR[X] = Q[X, 0.75] - Q[X, 0.25]$

Quantiles

$Q[X, 0.25]$

$Q[X, 0.5]$ → interquantile

$Q[X, 0.75]$

Quintiles	Tertiles	Deciles
$Q[X, 0.2]$	$Q[X, .33]$	$Q[X, 0.1]$
$Q[X, 0.4]$	$Q[X, .66]$	$Q[X, 0.2]$
$Q[X, 0.6]$		$\vdots$
$Q[X, 0.8]$		$Q[X, 0.9]$

Roulette in America

Bet on black pays 1:1

$X \sim \begin{cases} \$1 \text{ wp } 18/38 \\ -\$1 \text{ wp } 20/38 \end{cases}$

$E[X] = (\$1)(18/38) + (-\$1)(20/38) = -\$.053$

$X_1 \ldots X_n \overset{iid}{\sim} \begin{cases} 1\$ \text{ wp } 18/38 \\ -1\$ \text{ wp } 20/38 \end{cases}$

$\lim_{n \to \infty} T = -\infty$

Bet on lucky #7

pay out 35:1

$X \sim \begin{cases} \$35 \text{ wp } 1/38 \\ -\$1 \text{ wp } 37/38 \end{cases}$

$E[X] = (\$35)(1/38) + (-\$1)(37/38) = -\$.053$

Bet on Dozen 1-12

Pay out 2:1

$X \sim \begin{cases} \$2 \text{ wp } 12/38 \\ \$-1 \text{ wp } 26/38 \end{cases}$

$E[X] = (\$2)(12/38) + (-\$1)(26/38) = -\$.053$

Def fair game - X is a r.v modeling payout

$E[X] = 0$

Time

7min Van Wyck

Streets (traffic)

12 min p(traffic) = 0.3

$$W \sim \begin{cases} 7 \text{ min} & \text{wp } 0.7 \\ 12 \text{ min} & \text{wp } 0.3 \end{cases}$$

$$E(W) = (7)(.7) + (12)(.3) = 7.8 \text{ min}$$

If you ride many times in a cab, on avg you will spend $\approx$ 7.8 min on cab

Uber charges \$.40/min

What is my expected bill for time?

$$B = \$.40(W) \sim \begin{cases} \$2.80 & \text{wp } 0.7 \\ \$4.80 & \text{wp } 0.3 \end{cases}$$