Lecture 14, NOV 3, 2016

$$W \sim \begin{cases} 7 \text{ min} & \text{wp } 0.7 \\ 12 \text{ min} & \text{wp } 0.3 \end{cases}$$

$E(W) = 7 \times .7 + 12 \times .3 = 8.5$ minutes

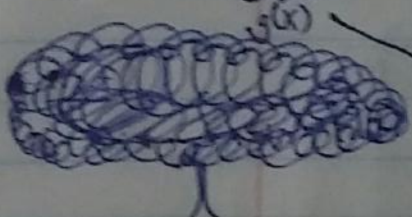
\$0.40/min

$B = \underbrace{\$0.40/\text{min} \cdot W}_{\text{new random variable that is fn of old random variable}} \sim \begin{cases} \$2.80 & \text{wp } 0.7 \\ \$4.80 & \text{wp } 0.3 \end{cases}$

$E(B) = \$2.80 * 0.7 + \$4.80 * 0.3 = \$3.12$

$= \$0.40/\text{min} \cdot 8.5 \text{ min} = \$3.12$ → $= \$0.40\,E[W]$

$Y = \underbrace{aX}_{g(x)}$, $a \in \mathbb{R}$, X random variable, does $E(Y) = aE(x)$?



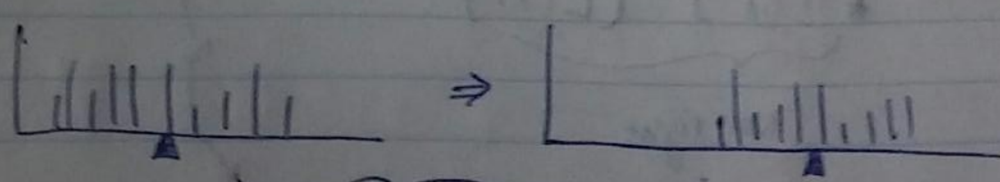
$$\boxed{E[g(x)] = \sum_{x \in Supp(X)} g(x)p(x)}$$

- $E[g(x)] = E[aX] = \sum_{x \in Supp(X)} ax\,p(x) \Rightarrow a \underbrace{\sum x p(x)}$ (a is a constant) $\Rightarrow \boxed{E[aX] = aE(x)}$

- BASE FARE

$T = B + \$3.00$

$= \$6.12$



shift over by a constant c.

$Y = x + C$

$g(x)$

$$\underbrace{Y = X + c}_{g(x)} \Rightarrow$$ derive :

$$\boxed{E[X+c] = \sum_{x \in supp[x]} (x+c)p(x) = \underbrace{\sum x\,p(x)}_{E[X]} + c \underbrace{\sum p(x)} = E[X] + c}$$

$$\boxed{E[\underbrace{aX+c}_{g(x)}] = aE[X]+c}$$

$T = B + \$3$

$E[T] = E[B] + \$3$

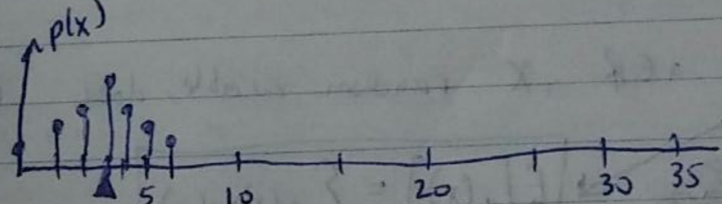
$= \$3.12 + \$3$

$= \underline{\$6.12}$

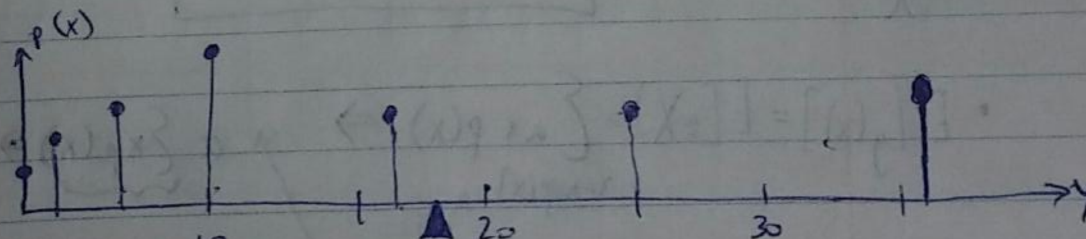
- $X \sim \text{Binomial}\left(6, \frac{1}{2}\right)$

$E(X) = n \cdot p$

$= 6 \cdot \frac{1}{2} = \underline{3}$



$Y = \underbrace{X^2}_{g(x)}$



$$* \underbrace{E(X^2) \stackrel{?}{=} (E(X))^2}_{\text{Not right}}$$

$$E[X^2] = \sum_{x=0}^{6} x^2 \binom{6}{x} \frac{1}{2^6} = \underline{17.5} \neq h(E(x))$$

$$E[g(x)] \neq g(E(x))$$

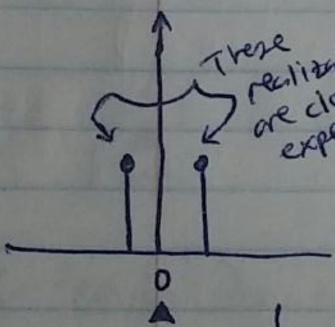
★ New Unit

- $X \sim$ Radomacher $= \begin{cases} 1 & wp\ \frac{1}{2} \\ -1 & wp\ \frac{1}{2} \end{cases}$

(expectation)

$E[X] = 0$

These realizations are closer to expected value

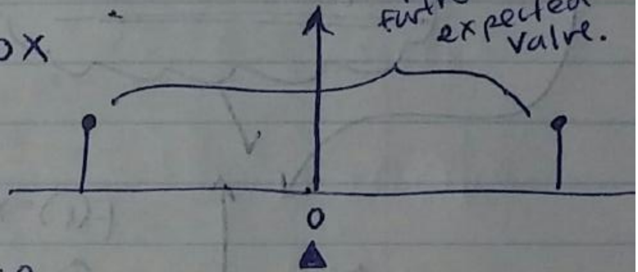


- $Y = 10X$

$\downarrow$

$E[10X] = 10 \cdot 0 = 0$

These realizations further away from expected value.

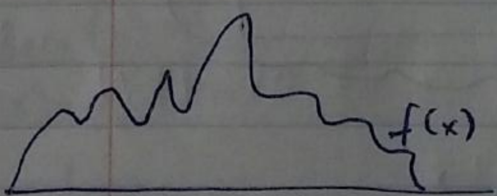


Are these 2 random variables identically distributed?

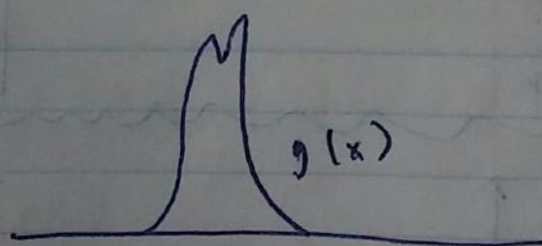
→ No, they are not identically distributed. They are ~~quickly~~ clearly different, have same expectation, but not identically distributed.

How are they different?

→ $Y = 10X$ is distributed farther apart than the top one. Top one more concentrated, bottom one more spread out.

- 

$\int_{\mathbb{R}} f(x)dx = 1?$

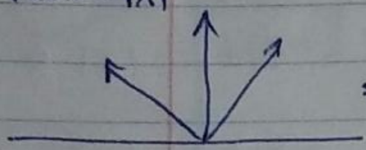


$\int_{\mathbb{R}} g(x) = 1?$

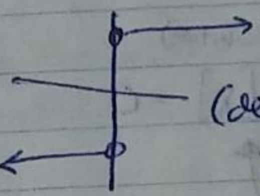
- $e(x, \mu) = x - \mu$ ← This is wrong

$e(x, \mu) = |x - \mu|$ ← This is right. CALLED ABSOLUTE LOSS or L1

• $f(x) = |x|$



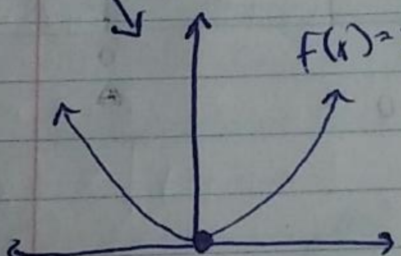
$\Rightarrow$



(derivative)

↑ don't like this

so $e(x,\mu) = (x-\mu)^2$ ← $\boxed{\text{square loss, L2}}$



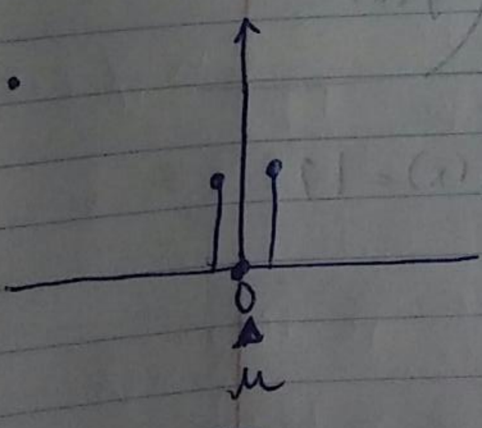
variance is an expectation

$L := (X-\mu)^2$

variance of r.v. X

$$\sigma^2 := Var(X) = E[L] = E[(X-\mu)^2] = \boxed{\sum_{x \in supp[X]} (x-\mu)^2 p(x)}$$

↑ greek letter sigma



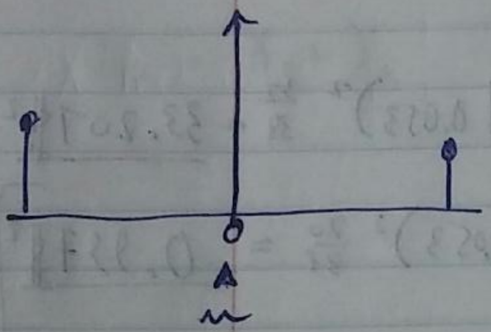
$X \sim \text{Rademacher} = \begin{cases} 1 & wp\ \frac{1}{2} \\ -1 & wp\ \frac{1}{2} \end{cases} \qquad E[X]=0$

$$Var[X] = ((-1)-(0))^2\,\tfrac{1}{2} + ((1)-(0))^2\,\tfrac{1}{2}$$

$$= 1\cdot\tfrac{1}{2} + 1\cdot\tfrac{1}{2} = \boxed{1}$$

⇨ Next page

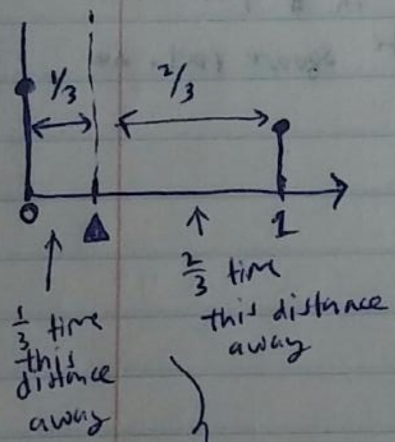
- $Y = 10X$

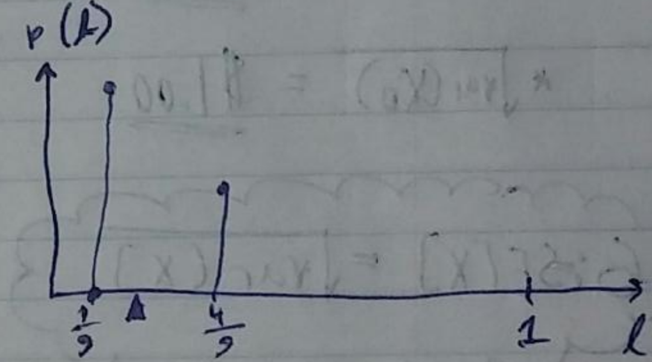


$$Var[X] = ((-10)-0)^2 \tfrac{1}{2} + ((10)-(0))^2 \tfrac{1}{2}$$
$$= 100 \cdot \tfrac{1}{2} + 100 \cdot \tfrac{1}{2} = \boxed{100}$$

- <u>$X \sim Bern(\frac{1}{3})$</u>

$E[X] = \frac{1}{3}$





$$Var(X) = (0-\tfrac{1}{3})^2 \tfrac{2}{3} + (1-\tfrac{1}{3})^2 \tfrac{1}{3} =$$
$$\tfrac{1}{9} \cdot \tfrac{2}{3} + \tfrac{4}{9} \cdot \tfrac{1}{3} = \tfrac{6}{27} = \boxed{\tfrac{2}{9}}$$

- <u>$X \sim Bern(p)$</u>

$E[X] = p$

$$\hookrightarrow Var[X] = (0-p)^2(1-p) + (1-p)^2 p \Rightarrow p^2(1-p) + (1-p)^2 p$$
$$\Downarrow$$
$$\boxed{p(1-p)} \Leftarrow (1-p)(p^2 + (1-p)p)$$

* <u>Roulette</u>

- Bet on #7

$$X \sim \begin{cases} \$35 & wp \ \frac{1}{38} \\ -\$1 & wp \ \frac{37}{38} \end{cases}$$

$E[X_7] = -\$0.053$

- Bet on Black

$$X \sim \begin{cases} \$1 & wp \ \frac{18}{38} \\ -\$1 & wp \ \frac{20}{38} \end{cases} \Rightarrow$$

$E[X_B] = -\$0.053$

which one do you think is more spread out?

→ $E[X_7]$

* $\mathrm{Var}(X_7) = (\$35 - -\$0.053)^2 \frac{1}{38} + (\$1 - - \$0.053)^2 \frac{37}{38} = 33.207\$^2$

* $\mathrm{Var}(X_B) = (\$1 - - \$0.053)^2 \frac{18}{38} + (-\$1 - -\$0.053)^2 \frac{20}{38} = 0.997\$^2$

* $\sqrt{\mathrm{var}(X_7)} = \$5.29$

* $\sqrt{\mathrm{var}(X_B)} = \$1.00$

** can't think in $\$^2$, so take the square root. **

$$\sigma := SE[X] = \sqrt{\mathrm{var}(X)}$$

expected square error loss of units that we care about

Bet on #7

$$X_7 \sim \begin{cases} \$35 & \text{wp } \frac{1}{38} \\ -\$1 & \text{wp } \frac{37}{38} \end{cases}$$

$E[X_7] = -\$0.053$

$\bar{X}_7 \to -\$0.053$ LLN

Bet on Black

$$X_B \sim \begin{cases} \$1 & \text{wp } \frac{18}{38} \\ -\$1 & \text{wp } \frac{20}{38} \end{cases}$$

$E[X_B] = -\$0.053$

$\bar{X}_B = -\$0.053$ LLN

converges faster than $\bar{X}_7$ b/c variance smaller

This one narrows in faster because the variance is smaller.

***Next unit**

$T_3 = X_1 + X_2$

$$E(T) \stackrel{?}{=} f(E(X_1), E(X_2))$$

$$= \sum_{t \in Supp(T)} t\, p(t)$$

Supp(X_1) Supp(X_2)

$p(x)$

x_{11}

x_{12}

x_{13}

$\vdots$