09/13/16

P is a set function

$\exists\ \Omega$

ⓐ $P(\Omega) = 1$

ⓑ $P(A) \geq 0\ \forall A$

ⓒ If $A_1, A_2, A_3, \ldots,$ disjoint $\Rightarrow P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} (P(A_i))$

$$\boxed{\begin{aligned} &\circ \text{ THEOREM: } P(A) = 1 - P(A^c) \\ &\Omega = A \cup A^c \\ &P(\Omega) = P(A \cup A^c) \\ &P(\Omega) = P(A) + P(A^c) \quad \text{by ⓒ} \\ &1 = P(A) + P(A^c) \quad \text{by ⓑ} \end{aligned}}$$

THEOREM: $A \subseteq B \Rightarrow P(A) \leq P(B)$

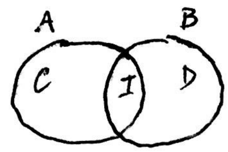
- $C := B \setminus A$
- $B = A \cup C$
- $P(B) = P(A \cup C)$
- $P(B) = P(A) + P(C)$ by ⓒ
- $P(B) - P(A) = P(C)$ by ⓑ

$P(C) \geq 0$ by ⓑ

$P(B) \geq P(A)$

• ~~THEOREM~~

$P(A \cup B) = P(A) + P(B) - P(AB)$



$C = A \setminus B$

$D = B \setminus A$

$I = AB$

- $P(A) = P(C) + P(I) \Rightarrow P(C) = P(A) - P(I)$
- $P(B) = P(D) + P(I) \Rightarrow P(D) = P(B) - P(I)$
- $P(A \cup B) = P(C) + P(D) + P(I) \Rightarrow$

$(P(A) - \cancel{P(I)}) + (P(B) - \cancel{P(I)}) + P(I)$

$= P(A) + P(B) - P(AB)$

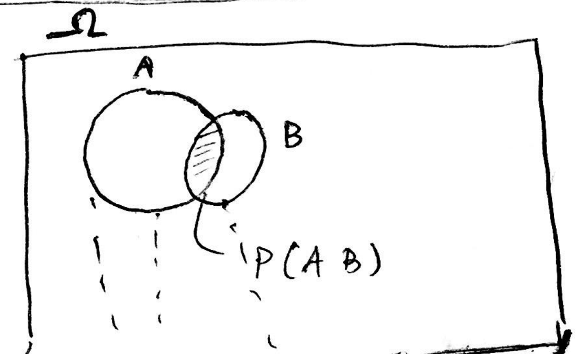
→ $n = 1{,}000$ people

(A) 200 smokers

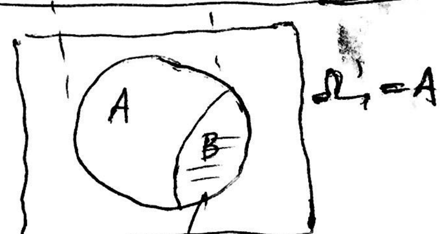
(B) 60 has lung cancer

(AB) 36 smoke & have lung cancer

- $P(A) = \frac{200}{1000} = 0.2$
- $P(B) = \frac{60}{1000} = 0.06$
- $P(AB) = \frac{36}{1000} = 0.036$



• What is the probability of lung cancer among smokers / given smoking / conditional on smoking?



"given"

proportional $P(B|A)$

$x \propto y$

$x = cy$ s.t. $c \in \mathbb{R}$

$\circ\ P(B|A) = P(BA) \cdot \text{Zoom}$

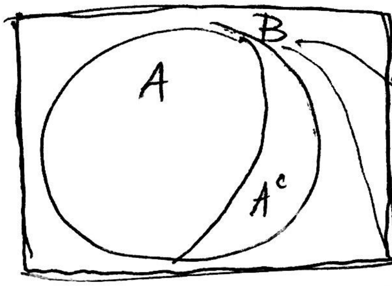
$P(B|A) \propto P(AB)$

$$\boxed{P(B|A) = \frac{P(BA)}{P(A)}}$$

$P(BA) = \frac{P(\Omega)}{P(A)} = \frac{P(BA)}{P(A)}$

Definition of conditional probability

- $P(B|A) = \frac{P(BA)}{P(A)} = \frac{0.36}{0.2} = \boxed{.18}$



$P(B) = P(BA) + P(BA^c)$

- $P(A|B) = ?$

↳ probability of people w/ lung cancer that's a smoker.

$$\frac{P(BA)}{P(B)} = \frac{0.36}{0.06} = \boxed{.6}$$

$$\boxed{P(B|A) = \frac{P(A|B)P(B)}{P(A)}}$$

Baye's Rule (1765)

- Probability of lung cancer, given didn't smoke

$$P(B|A^c) = \frac{P(BA^c)}{\underbrace{P(A^c)}_{1-P(A)=0.8}} = \frac{.024}{0.8} = .03$$

$\underbrace{P(B) = P(BA)}_{} + P(BA^c)$

$P(BA^c) = P(B) - P(BA)$

$.06 - .036 = .024$

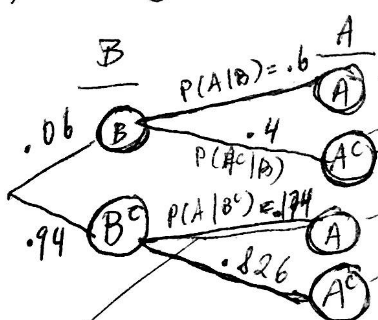
probability smoking

RISK RATIO

$$\frac{P(B|A)}{P(B|A^c)} = \frac{.18}{.03} = 6$$

probability nonsmoking

: ~~What is the probab~~ How likely is it that you will get cancer if you smoke ~~as~~ suppose to not smoking? $\boxed{\text{6 times more likely}}$



Joint events

$P(AB) = .036$

$P(A^cB) = .024$

$P(AB^c) = .164$

$P(A^cB^c) = .776$

$\overline{}$ 1

$P(A|B) = 1 - P(A^c|B)$

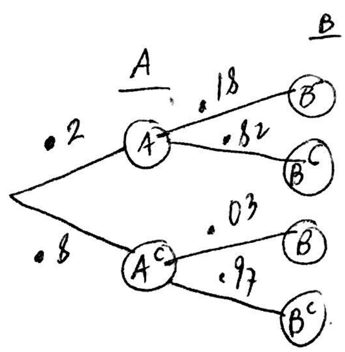
$$\frac{P(AB)}{P(B)} = 1 - \frac{P(A^cB)}{P(B)}$$

$P(AB) = P(B) - P(A^cB) \Rightarrow$

$P(B) = P(AB) + P(A^cB)$

$P(A) = P(AB) + P(AB^c)$

$P(AB^c) = P(A) - P(AB) = .2 - .036 = .164$



$P(AB) = 0.36$

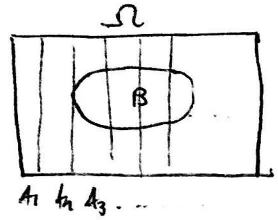
$P(AB^c) = .164$

$P(A^cB) = .024$

$P(A^cB^c) = .776$

Consider event B and disjoint and collectively exhaust.. events..

$A_1, A_2, A_3, \ldots$



- $P(B) = P(B \cap \Omega)$

$$= P(B \cap (A_1 \cup A_2 \cup A_3 \cup \ldots))$$
$$= P((B \cap A_1) \cup (B \cap A_2) \cup (B \cap A_3) \cup \ldots)$$

mutually exclusive

$$= P(B \cap A_1) + P(B \cap A_2) + P(B \cap A_3) + \ldots$$

- $P(B) = \sum_{i=1}^{\infty} P(BA_i) \longrightarrow$ Law of Total Probability

$$\boxed{P(A_k \mid B) = \frac{P(B \mid A_k)(P(A_k))}{\sum_{i=1}^{\infty} P(B \mid A_i) P(A_i)}} \longrightarrow \text{BAYE'S THEOREM}$$

2 children

Ω

GG	GB
BG	BB

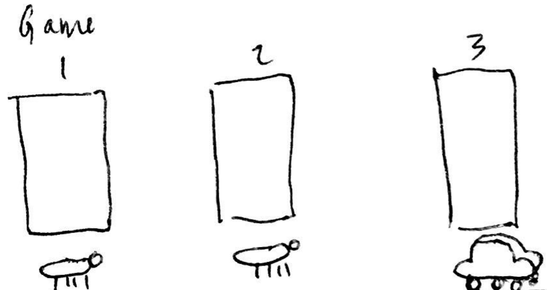
- $P(\text{other is girl} \mid \text{one is girl})$

$$P(\{GG\} \mid \{GG, GB, BG\})$$

$$\frac{P(GG)}{P(GG, GB, BG)}$$

$$\frac{\frac{1}{4}}{\frac{3}{4}} = \boxed{\frac{1}{3}}$$

- Monte Hall Game

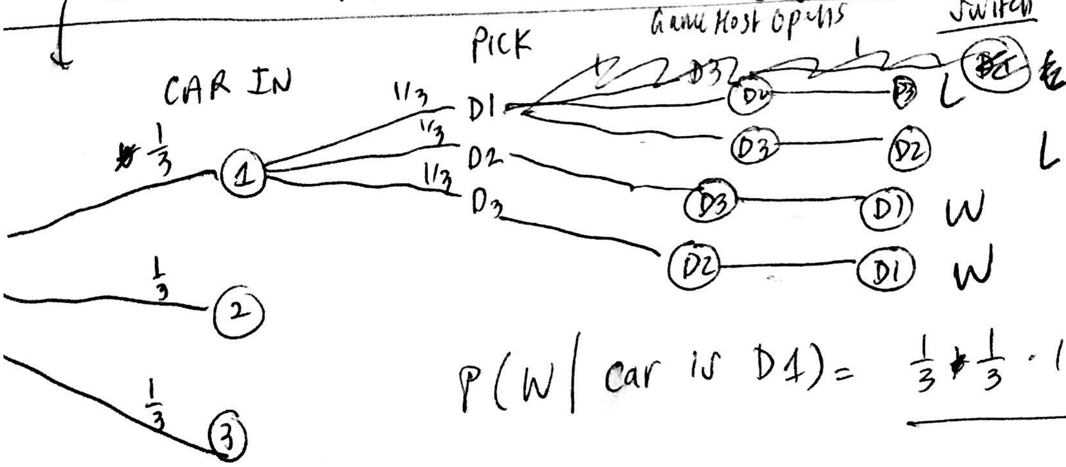


① Pick a DOOR

② Game Host OPENS DOOR to level game

③ You have option to switch.

Imagine car in 1



$$P(W \mid \text{car is } D1) = \frac{\frac{1}{3} \cdot \frac{1}{3} \cdot 1 \cdot 1 \cdot \frac{1}{3} \cdot \frac{1}{3} \cdot 1 \cdot 1}{\frac{1}{3}} = \frac{2}{3} \neq \frac{1}{2}$$