Lec #15

Math 241 A. Kapelner Nov 8 16

Let x_1, x_2 be RV.

Then $T = x_1 + x_2$

where $E[T] = \underbrace{\sum_{t \in Supp[T]} tp(t)}_{Hard}$ } gen. not a good strategy.

$\Rightarrow$ "not covered" part is skipped...

Let x_1, x_2 be independent r.v.

$T = x_1 + x_2$

Then, $p(x_1, x_2) = p(x_1)p(x_2)$ } prob. of joint event happ. given (both) at the same time.

$$E[T] = \sum_{x_1} x_1 \sum_{x_2} p(x_1)p(x_2) + \sum_{x_2} x_2 \sum_{x_1} p(x_1)p(x_2)$$

$$= \sum_{x_1} \underbrace{x_1 p(x_1)}_{E[x_1]} \underbrace{\sum_{x_2} p(x_2)}_{1} + \underbrace{\sum_{x_2} x_2 p(x_2)}_{E[x_2]} \underbrace{\sum_{x_1} p(x_1)}_{1}$$

OK.

$Supp[x_1] = \{1, 7, 19\}$

$Supp[x_2] = \{5, 23, 88\}$

x_2 \ x_1	1	7	19	
5	1/15	1/3	2/15	10/30
23	1/30	1/10	1/30	
88	1/30	1/5	1/15	
	4/30			1

$\Rightarrow$ JMF

$p(x_1 = 1, x_2 = 5) = 1/15$

$p(x_1 = 1) = 4/30$

Long - short: $\underline{E[T] = E[x_1 + x_2] = E[x_1] + E[x_2]}$.

Note: Events are dependent.

General Rule: $x_1, x_2, \ldots x_n$ rv's

$$E[T] = E[x_1, x_2, \ldots, x_n]$$

$$= E[\sum_{i=1}^{n} x_i] = \sum_{i=1}^{n} E[x_i] = E[x_1] + \ldots + E[x_n]$$

or $\underbrace{\mu_1 + \ldots + \mu_n}_{n \text{ terms.}}$

Thus: $E[T] = \sum_{i=1}^{n} E[x_i] = \mu n$

→ $X \sim \text{Binomial}(n, p)$

$X = X_1 + \dots + X_n$

$X_1, \dots, X_n \overset{iid}{\sim} \text{Bernoulli}(p)$

$E[X] = np.$ Note: n Bernoulli(p) terms.

→ $X \sim \text{Negative Binomial}(r, p)$

$E[X] = \frac{r}{p}$ Note: r geometric(p) terms.

→ $X \sim \text{Hypergeometric}(n, K, N)$

$$E[X] = \sum_{x \in \text{supp} X} x \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

$E[X] = n\mu = n\frac{K}{N}$ Note: $\text{Hyper}(1, K, N) = \text{Bern}(K/N)$

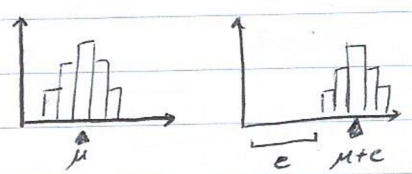
__Variance__: $\text{Var}[X] := E[(X-\mu)^2]$

$\text{Var}[X] = E[X^2] - \mu^2$

$\sigma^2 = E[X^2] - \mu^2$

Thus, $E[X^2] = \sigma^2 + \mu^2$.

Var[X]



Distance __from__ the balance pnt

shift.

is thus the same, $\text{Var}[X+c] = \text{Var}[X]$.

$\text{Var}[aX] = a^2 \text{Var}[X]$ ⟺ identity

$SE[aX] = \sqrt{\text{Var}[X]} = \sqrt{a^2\sigma^2} = |a|\sigma$

$\text{Var}[aX+c] = a^2\sigma^2$

$SE[aX+c] = \sqrt{\text{Var}[aX+c]} = \sqrt{\text{Var}[aX]} = |a|\sigma$

Note when events are dependent, then covariance comes into play

Let x_1, x_2 be r.v.

$Var[X_1+X_2] = E[x_1+x_2 - (\mu_1+\mu_2)^2]$

"steps skipped"

$\Rightarrow \sigma_1^2 + \sigma_2^2 + 2(\underbrace{E[X_1 X_2] - \mu_1\mu_2}_{\text{covariance.}})$

Note: $Cov[x_1, x_2] := E[x_1 x_2 - \mu_1\mu_2]$

If x_1, x_2 are indep. but not necessarily identically distributed, what is $E[x_1 x_2]$?

$$E[x_1 x_2] = \sum_{x_1}\sum_{x_2} x_1 x_2\, p(x_1 x_2) = \sum_{x_1}\sum_{x_2} x_1 x_2\, p(x_1) p(x_2)$$

$$= \sum_{x_1} x_1 p(x_1) \sum_{x_2} x_2 p(x_2) = E[x_1]E[x_2] = \mu_1\mu_2$$

$Cov[x_1, x_2] = E[x_1 x_2] - \mu_1\mu_2 = 0$ if $x_1 x_2$ are indep.

$Var[x_1 x_2] = Var[x_1] + Var[x_2]$ if $x_2 x_1$ are independent.

General Rule: (if $x_1 \ldots x_n$ are indepent)

$$Var[T] = Var[x_1 + \ldots + x_n] = Var[x_1] + \ldots + Var[x_n]$$

$$= \sum_{i=1}^{n} Var[x_i] = \sum_{i=1}^{n} \sigma_i^2$$

if $x_1, \ldots x_n$ iid

Then $Var[T] = n\sigma^2$

$$Var[\bar{x}] = Var[T/n] = \frac{1}{n^2} Var[T] = \frac{1}{n^2}(n\sigma^2) = \frac{\sigma^2}{n}$$

$$SE[\bar{x}] = \frac{\sigma}{\sqrt{n}}$$