#: 10/13/16

Binomial Theorem

X ~ Bernulli (p)

X ~ Binomial (n, p)

X ~ Hypergeom (n, k, N)

$$\sum_{x \in Supp[X]} p(x) = 1$$

$$\sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x}$$

$$(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^i b^{n-i}$$

$a = p$; $b = 1-p$; $i = x$

$$(p+(1-p))^n = \sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x} = 1^n = \boxed{1}$$

X_1 and X_2 are indipendent random variable if $P(X_1 = x_1 \mid X_2 = x_2) = P(X_1 = x_1)$

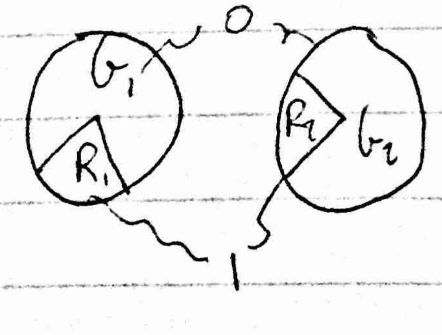
$P(X_2 = x_2 \mid X_1 = x_1) = P(X_2 = x_2)$

$P(X_1 = x_1, X_2 = x_2) = P(X_1 = x_1) P(X_2 \ldots$

$\forall x \in Supp[X_1]$; $\forall x \in Supp[X_2]$

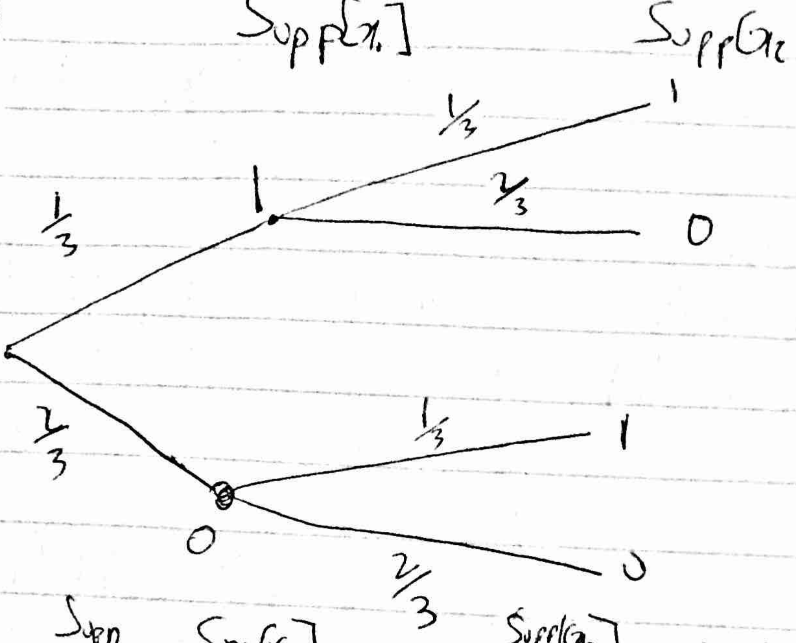
X_1 and X_2 are indipendent and identically distributed if X_1, X_2 are $X_1 \overset{d}{=} X_2$ and denote $(X_1, X_2 \overset{iid}{\sim}$

$X_1, X_2 \overset{iid}{\sim} Bern\left(\frac{1}{3}\right)$, $Supp[T] = \{0,1,2\}$ T

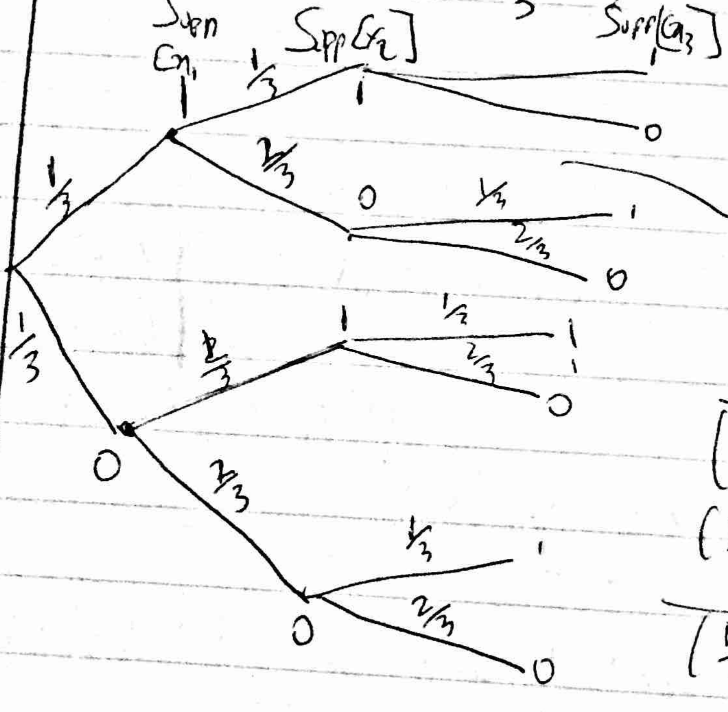


$$T_2 = \underbrace{X_1}_{\{0,1\}} + X_2 = g(X_1, X_2)$$

$$T \sim \begin{cases} 0 & wp \ \frac{4}{9} \\ 1 & wp \ \frac{4}{9} \\ 2 & wp \ \frac{1}{9} \end{cases}$$



	T
$P(x_1=1; x_2=1) = \frac{1}{9}$	2
$P(x_1=1; x_2=0) = \frac{2}{9}$	1
$P(x_1=0; x_2=1) = \frac{2}{9}$	1
$P(x_1=0, x_2=0) = \frac{4}{9}$	0



$\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^0$

$\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^1$

$\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^1$

$\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2$

$\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^1$

$\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^2$

$\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^2$

$\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^3$

$$T \sim \begin{cases} 0 & \text{wp } \binom{3}{0} \\ 1 & \\ 2 & \\ 3 & \end{cases}$$

$$T_n = \sum_{i=1}^{n} X_i$$

if $X_1, X_2, X_3, X_4 \overset{iid}{\sim} Bern(p)$

So

$$Supp[T_n] = \{0, 1, \dots, n\}$$

$$T \sim Binomial(n, \tfrac{1}{3}) \begin{cases} 0 & \binom{n}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^n \\ 1 & \binom{n}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^{n-1} \\ 2 & \binom{n}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^{n-2} \\ \vdots & \\ n-2 & \binom{n}{n-2}\left(\frac{1}{3}\right)^{n-2}\left(\frac{2}{3}\right)^2 \\ n-1 & \binom{n}{n-1}\left(\frac{1}{3}\right)^{n-1}\left(\frac{2}{3}\right)^1 \\ n & \binom{n}{n}\left(\frac{1}{3}\right)^n\left(\frac{2}{3}\right)^0 \end{cases}$$

p

p

p

1 − p

2 Concepts for Binomial

$$\lim_{n \to \infty} Hyper(n, p, N) = Binomial(n, p)$$

1000 Coin Flips $P(600H) =$ $X \sim Binomial(1000, \tfrac{1}{2})$

$P(600H) = (P(X = 600))$

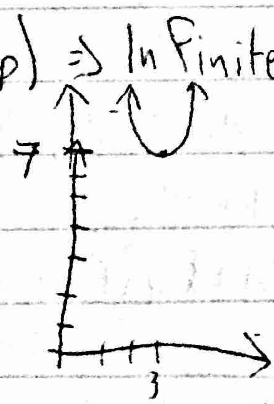
$F(x) = P(X \le x) \Rightarrow$ CDF of Binomial $F(x) = \sum_{i=0}^{x} p(i) = \sum_{i=0}^{x} \binom{n}{i} p^i (1-p)^{n-i}$

= Incomplete/regularized gamma function

$$I_{1-p}(n-k, 1+k) := (n-k)\binom{n}{k} \int_0^{1-p} t^{n-k-1} (1-t)\, dt$$ no closed form

$X_1, X_2, \dots \overset{iid}{\sim} Bern(p) \Rightarrow$ Infinite sequence of random variables

$f(x) = 7 + (x-3)^2$



$\min(f(x)) = 7$

$\arg\min(f(x)) = 3$

$\max(f(x)) =$

$\arg\max(f(x))$

$T = \min\{t : X_t = 1\}$

$\{3, 5, 8, \ldots\}$

$\uparrow$

First Success ("Stopping time

$P(1) = P(T=1) = P(X_1 = 1) =$

$\underline{1}$ _ _ _ _ _ _ _

$P(2) = P(T=2) = P(X_1 = 0)P(X_2 = 1)$

$\underline{0}\ \underline{1}$ _ _ _ _

$= \underbrace{(1-p)}_{1\text{ failure}}\ \underbrace{p}_{1\text{ success}}$

$P(3) = P(T=3) = (1-p)^2\ p$

$\qquad\qquad\qquad\quad f \quad s$

$P(x) = P(T=x) = (1-p)^{x-1}(p)$

$x \sim$ ~~Hyper~~ Geometric... $= (1-p)^{x-1}$

Para Space

$p \in (0,1)$

$\text{Supp}[x] = \mathbb{N}$

$\sum_{x \in \text{Supp}(x)} P(x) = 1 \qquad \sum_{x=1}^{\infty}(1-p)^{x-1} p \overset{?}{=} 1$

$\Rightarrow \sum_{x=1}^{\infty}(1-p)^{x-1} = \frac{1}{p}$

Let $q = 1-p$

$\sum_{x=1}^{\infty} q^{x-1} = \frac{1}{1-q}$

$\sum_{x=1}^{\infty} q^{x-1} = \sum_{x=0}^{\infty} q^x = q^0 + q^1 + q^2 + q^3$

$= 1 + q^1 + q^2 \ldots$

$= 1 + q(1 + q + q^2)$

$= 1 + qS$

$S - qS = 1$

$(1-q)S = 1$

$S = \frac{1}{1-q}$

$\sum_{i=0}^{\infty} q^i = \frac{1}{1-q}$

Geometric Series

$F(x) = P(X \le x) = 1 - P(X > x) \quad = (1-p)^x$

$\sum_{i=1}^{x} (1-p)^{x-1} p$

$\frac{}{1} \; \frac{}{2} \; \frac{}{3} \; \dots \; \frac{}{x} \Big| \; \frac{}{x+1} \; \frac{}{x+2} \; \frac{}{x+10} \; \frac{}{x+9}$

$(1-p)^{x-1}$
f

$P(X > x) = P(X = x+1) + P(X = x+2) + \dots$

$= \sum_{i=x+1}^{\infty} (1-p)^{i-1} p = \sum_{i=1}^{\infty} (1-p)^{x+i-1} p$

$P = (1-p)^x \sum_{i=1}^{\infty} (1-p)^{i-1} p$