$X \sim Geom(p) = (1-p)^{x-1} p \quad Supp[X] = \mathbb{N} \quad , p \in (0,1)$

$X_1, X_2, \ldots \overset{iid}{\sim} Bern(p)$

play poker until Royal Flush : $P(\ ) = \frac{4}{\binom{52}{5}} = 1.53/\text{million} = 0.00000153$

Build model : $X \sim Geom(0.00000153)$

$P(X = 1000000) = 0.9999985^{999999} \cdot 0.00000153$

Prob of getting royal flush on millionth hand or sooner?

$P(X \leq 1000000) = F(1000000) = 1 - p = 1 - 0.9999985^{1000000} \approx 77\%$

$X = \min\left\{ t : \sum_{i=1}^{t} X_i = r \right\}$

$P(X=0) = 0 \qquad P(X=2) = 0 \qquad P(X=4) = p^3(1-p) \cdot 3$

$P(X=1) = 0 \qquad P(X=3) = p^3 \qquad P(X=5) = p^3(1-p)^2 \cdot 6$

$P(X=x) = p^3(1-p)^{x-3}\binom{x-1}{2}$ (PMF)

$X \sim NegBin(r,p) := \binom{x-1}{r-1} p^r (1-p)^{x-r}$

$Supp[X] = \{r, r+1, r+2, \ldots\}$

Parameter Space : $p \in (0,1)$, $r \in \mathbb{N}$

$X \sim NegaBin(1,p) = \binom{x-1}{1-1}(1-p)^{x-1} p^1 = (1-p)^{x-1} p = Geom(p)$

$X_1, X_2, \ldots, X_r \overset{iid}{\sim} Geom(p)$

$X_1 + X_2 + \ldots + X_r \sim NegaBin(r,p)$

$1 = \sum_{x \in Supp[X]} p(x) \qquad 1 = \sum_{x=r}^{\infty} \binom{x-1}{r-1}(1-p)^{x-r} p^r$

$$\sum_{i=0}^{\infty} q^i = \frac{1}{1-q}$$

$$\sum_{i=0}^{\infty} (1-p)^i = \frac{1}{p} = p^{-1}$$

$\sum_{i=0}^{\infty}(1-p)^{x-1} = p^{-1}$ take $\frac{d}{dp}$ ~~that~~ both sides

$\sum_{i=1}^{\infty}(1-p)^{x-1} = p^{-1} \Rightarrow \sum_{i=2}^{\infty}(-1)(x-1)(1-p)^{x-2} = (-1)p^{-2}$

$\iff \sum_{i=2}^{\infty}(-1)^2(x-1)(x-2)(1-p)^{x-3} = (-1)p^{-2}$

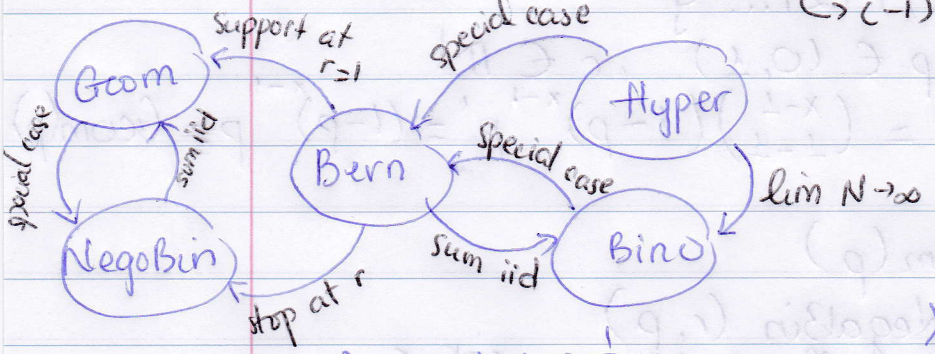
$\iff \sum_{i=3}^{\infty}\overbrace{(-1)^2}^{d}\overbrace{(x-1)}^{d-1}\overbrace{(x-2)}^{d}(1-p)^{x-3} = \underbrace{(-1)(-2)}_{(-1)^d d!}\overbrace{\underbrace{p^{-3}}_{p^{-d}}}^{d+1}$

$\iff \sum_{i=3}^{\infty}\cancel{x_7}\,(1-p) = (r-1)!\,p^{-r}$

$\sum_{x=r}^{\infty}\overbrace{(x-1)(x-2)\dots(x-(r+1))}^{\frac{(x-1)!}{(x-r)!}}(1-p)^{x-r} = (r-1)!\,p^{-r}$

$\frac{(x-1)!}{(x-r)!(r-1)!} \Rightarrow \binom{x-1}{r-1}$ $\qquad \frac{(x+r-1)!}{(r-1)!\,x!} = \frac{(x+r-1)(x+r-2)\dots r}{x!} = \binom{-r}{x}$

$X \sim NegaBin(r,p) = \binom{x+r-1}{r-1}\underbrace{(1-p)^x p^r}$ $\hookrightarrow (-1)^x\binom{-r}{x}(1-p)^x p^r$



$\begin{array}{c} n \quad K \quad N \\ '' \quad '' \quad '' \end{array}$

$X \sim Hyper(3,4,8)$

$X_1, \dots, X_7$

$X \overset{iid}{\sim} Hyper(3,4,8)$

$X \sim Bern\left(\frac{1}{2}\right) := \begin{cases} 1 & wp\ \frac{1}{2} \\ 0 & wp\ \frac{1}{2} \end{cases}$

$X = 1 \in Supp[X]$ realization of r.v

data: replication of r.v's