Lecture 2 08/30/16 A. Kapelner

$F \cap \Omega = F$ $F \in 2^{\Omega}$

$F \cup \Omega = \Omega$ $F \subseteq \Omega$

$\phi \cap \Omega = \phi$ $F \setminus \Omega = \phi$

$\phi \cup \Omega = \Omega$ $\Omega \setminus F = F^c$ (set complement)

$(F^c)^c = F$

$A \cup A^c = \Omega$

$\{A, A^c\}$ are called collectively exhensive

$\{A_1, A_2, A_3, \dots\}$

if $\bigcup_{i=1}^{\infty} A_i = \Omega$ $A_1 \cup A_2 \cup A_3 \dots$

$A \cap A^c = \phi$

$\{A, A^c\}$ - mutually exclusive. ("disjoint")

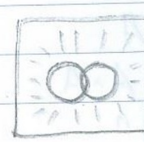
$\{A_1, A_2, \dots\}$ are mutually exclusive if

$A_i \cap A_j = \phi \quad \forall_{i \neq j}$

$|\Omega| = |A| + |A^c|$ for $|\Omega|$ finite.

$|A| - |\Omega| - |A^c|$

Consider $A, B \subseteq \Omega$ $(A \cup B)^c$



$(A \cup B)^c = A^c \cap B^c$

$(A \cap B)^c = A^c \cup B^c$ } D'Morgan's Law.

holes = irrational #s.

rat. $\mathbb{Q} = \left\{ \frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{N} \right\}$ "real" $\mathbb{R} = \mathbb{Q} \cup \{\text{all numbers}\}$

$|\mathbb{Q}| = \aleph_0$ $[3,7] := \{x : x \geq 3, x \leq 7\} \subset R$ *

MIQUELRIUS

Lecture 2a

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$\mathbb{R} \setminus \mathbb{Q}$

Real numbers; can't count them;

Consider $(0,1) \subset \mathbb{R}$

$|(0,1)| \leq |\mathbb{R}|$

Assume $|(0,1)| \neq \aleph_0$

0.01001...
0.00101
0.01101
0.111...

$|\mathbb{R}| = c > \aleph_0$

uncountable infinity.

(no need to know)

Ordered Pair

$\langle a, b \rangle := \{\{a\}, \{a, b\}\}$

element a, element b in the order.

Courtesan Product

$A \times B := \{\langle a, b \rangle : a \in A, b \in B\}$

Let's say $A = \{1, 2\}$ $\quad A \times B = \{\langle 2,3 \rangle, \langle 1,4 \rangle, \langle 1,3 \rangle, \langle 2,4 \rangle\}$

$B = \{3, 4\}$ $\quad$ not $\langle 3,2 \rangle$

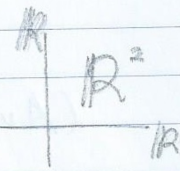
$|A \times B| = 4 \qquad |A \times B| = |A||B|$ if finite.

$A^2 := A \times A$

$A^3 := A \times A \times A$

$|A^n| := |A|^n$

$n \in \mathbb{N}$



$\Omega = \{\omega_1, \omega_2, \ldots\}$ "outcome"

Sample Space

Experiment outcome Space

Experiment $\omega \in \Omega$ is chosen

when coin flips $\longrightarrow$ # of outcomes.

$\Omega = \{H, T\} \quad |\Omega| = 2$

Lecture 2b

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→ "event" : set of outcomes

$A \subseteq \Omega$

$A \in 2^\Omega = \{\phi, \{H\}, \{T\}, \{H,T\}\}$

event space ie all events.

Probability :

$P(A) = \frac{|A|}{|\Omega|}$ if Ω finite.

$\cancel{P(H) = \frac{|H|}{|\Omega|}}$ $\quad P(\{H\}) = \frac{|\{H\}|}{|\Omega|} = \frac{1}{2}$

→ correct grammar

$\underset{\text{is a Function}}{P} : \overset{\text{events}}{2^\Omega} \longrightarrow \underset{\text{range of } P}{[0,1]}$ $\qquad \boxed{A}^{\Omega}$ (?)

$P(\phi) = 0 \longrightarrow P(\underbrace{\{H\} \cap \{T\}}_{\phi})$ Probability of getting not both heads & tails.

$P(\Omega) = 1$

$P(\{H,T\}) = 1$

$P(\{H\} \cup \{T\}) = 1$

$P(A^c) = \frac{|A^c|}{|\Omega|} = \frac{|\Omega| - |A|}{|\Omega|} = 1 - \frac{|A|}{|\Omega|}$

$= 1 - P(A)$

Complement Rule.

$\Rightarrow P(A) = 1 - P(A^c)$

Two coin flips

$\Omega' = \Omega^2$

$= \{\langle H,H\rangle, \langle H,T\rangle, \langle T,H\rangle, \langle T,T\rangle\}$

Lecture 2c