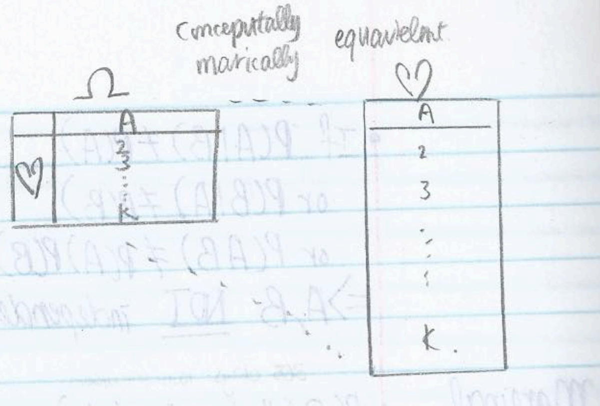
- Prob of Demo = Random



9/15 • Cards Game

Prob of # of Aces. $P(A) = \frac{4}{52} = \frac{1}{13}$

"

$P(A|♡) = \frac{1}{13}$

↑ information known.

→ the info is "irrelevant".

$P(\text{IBM stock}\uparrow \text{ in a day}) = P(\text{IBM stock}\uparrow \text{ in a day} \mid \text{rains in somewhere})$

irrelevant.

• Def of A, B being independent:

$P(A) = P(A|B)$ equav $P(B) = P(B|A)$.

• Under A, B being independent

$P(A) = P(A|B) = \frac{P(AB)}{P(B)}$

$\Rightarrow P(AB) = P(A)P(B)$

Multiplication Rule.

$- P(\underbrace{H_2 | H_1}_{\text{indep. events}}) = P(H_2) = \frac{1}{2} = \prod_{i=1}^{5} P(H_i) \rightarrow P(H_1 H_2 H_3 H_4 H_5) = (\frac{1}{2})^5$

$= \frac{1}{|\Omega^5|} = \frac{1}{2^5}$

• $P(\{\geq 1 \text{ 6-6 in 24 rolls of 2 dice}\}) < \frac{1}{2}$. (Double 6)

24 calculation $= P(1 \text{ 6-6}) + P(2 \text{ 6-6}) + \dots + P(24 \text{ 6-6})$

☆ easier to do it (use 1 minus). → $= 1 - P(\text{Zero 6-6 in 24 rolls})$. This is the compliment of

$= 1 - P(\text{Not 6-6 } 1^{st} \text{ roll} \cap \text{Not 6-6 } 2^{nd} \text{ roll} \cap \dots \cap \text{not 6-6 } 24^{th} \text{ roll})$.

↓

$P(\text{Not 6-6 } 1^{st}) \cdot P(\text{Not 6-6 } 2^{nd}) \cdot \dots \cdot P(\text{Not 6-6 } 24^{th})$

$= P(\text{Not 6-6})^{24}$

$= (1 - P(6\ 6))^{24} = (1 - P(6)P(6))^{24} = (1 - (\frac{1}{6})^2)^{24}$

$= 1 - (1 - (\frac{1}{6})^2)^{24} = .4914039$

- If $P(A|B) \neq P(A)$
  or $P(B|A) \neq P(B)$
  or $P(AB) \neq P(A)P(B)$

$\Rightarrow$ A, B <u>NOT</u> independent i.e. "dependent"

365 days. rain/snow/sun

Marginal Probabilty

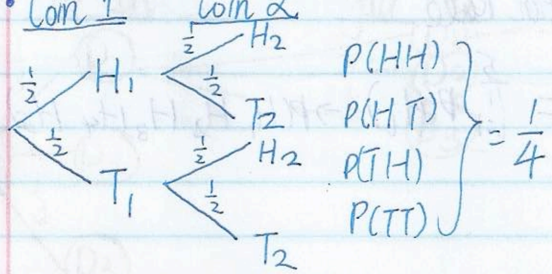
- $P(\text{Q64 bus is late}) < P(\text{Q 64 bus is late} \mid \text{rain})$
  $> P(\text{Q 64 bus is late} \mid \text{sun and no traffic})$ just a day of perfect

$= 1 - P(\text{Q64 on time})$

- $P(\text{lung cancer} \mid \text{smoke}) > P(\text{lung cancer})$.
  these 2 events are very dependent

- A, B disjoint $\overset{?}{\Rightarrow}$ A B independent?
  $P(A|B) \overset{?}{=} P(A)$
  $0 \neq P(A)$

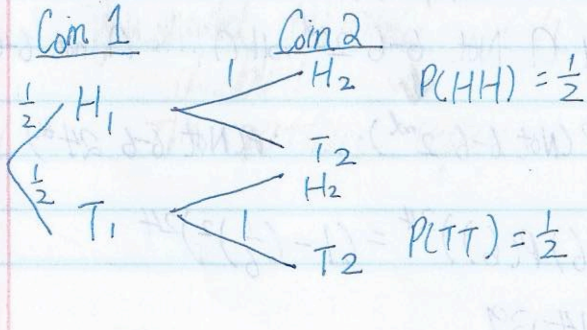
- <u>Coin 1</u> <u>Coin 2</u>



H H

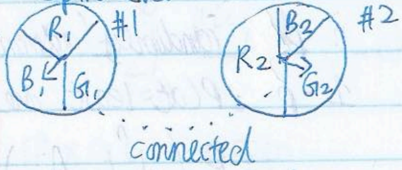
independent

<u>Coin 1</u> <u>Coin 2</u>



$P(H_2|H_1) = P(H_2)$
$1 \neq \frac{1}{2}$.
not independent?

Spinners

- 

Spinner 1: $P(R_1) = P(B_1) = P(G_1) = \frac{1}{3}$
Spinner 2: $P(R_2) = \frac{1}{2}$, $P(G_2) = \frac{1}{3}$, $P(B_2) = \frac{1}{6}$.

<u>Spinner 1</u> $\qquad$ <u>Spinner 2</u>

- R_1 ($\frac{1}{3}$)
  - R_2 ($\frac{1}{2}$) $\quad \frac{1}{6}$
  - G_2 (0) $\quad 0$
  - B_2 ($\frac{1}{2}$) $\quad \frac{1}{6}$
- G_1 ($\frac{1}{3}$)
  - R_2 ($\frac{1}{4}$) $\quad \frac{1}{12}$
  - G_2 ($\frac{3}{4}$) $\quad \frac{1}{4}$
  - B_2 (0) $\quad 0$
- B_1 ($\frac{1}{3}$)
  - R_2 ($\frac{3}{4}$) $\quad \frac{1}{4}$
  - G_2 ($\frac{1}{4}$) $\quad \frac{1}{12}$
  - B_2 (0) $\quad 0$

Total: 1.

- R_1, R_2 independent?
  $P(R_2 | R_1) = P(R_2)$
  $\frac{1}{2} \stackrel{?}{=} \frac{1}{2}$
  YES.

- R_1, G_2 independent?
  $P(G_2 | R_1) \stackrel{?}{=} P(G_2)$
  $0 \neq \frac{1}{3}$

- P(share b-day)
  = P(≥ 1 share b-day)
  = P(1 share b-day) + P(2 share b-day) + ... + P(48 share b-day)
  = 1 − P(no share b-days)
  = 1 − 0.04 = $\boxed{96\%}$!!!!

$$= \frac{365 \cdot 364 \cdot 363 \cdots (365-48+1)}{365^{48}} = \frac{{}_{365}P_{48}}{365^{48}} = 0.039$$

- n people. Randomly administer their hats

2 ways to calculate it:
P = P(Zero ppl get their hats) ← easier this way ☆. (but if more than 3 ppl X).
1 − P = P(at least one person gets hat)
$\quad$ = P(1 ppl gets hat) + P(2 ppl gets hat) + ... + P(n ppl get their hats)

3 people A, B, C

1st	2nd	3rd
A ✓ ($\frac{1}{3}$)	B ($\frac{1}{2}$)	C ✓ (1)
	C ($\frac{1}{2}$)	B
B ($\frac{1}{3}$)	A ($\frac{1}{2}$)	C ✓
	C ($\frac{1}{2}$)	A
C ($\frac{1}{3}$)	A ($\frac{1}{2}$)	B
	B ✓ ($\frac{1}{2}$)	

$\frac{1}{3} + \frac{1}{6} + \frac{1}{6} = \frac{2}{3}$

A_1 : 1^{st} person gets hat
A_2 : 2^{nd} person gets hat
$\vdots$
A_n : n^{th} person gets hat.

n ppl randomly administer their hat.
$\therefore$ $1-p = P(\text{at least one person gets hat})$
$= P(\bigcup_{i=1}^{n} A_i)$

$P(A \cup B) = P(A) + P(B) - P(AB)$

General inclusion exclusion Formula.

$P(A_1 \cup A_2) = \sum_{i=1}^{2} P(A_i) - P(\bigcap_{i=1}^{2} A_i)$

$P(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i) - \sum_{i \neq j} P(A_i \cap A_j) + \sum_{i \neq j \neq k} P(A_i \cap A_j \cap A_k) + - + - (-1)^{n+1} P(\bigcap_{i=1}^{n} A_i)$

$- P(A_1 \cup A_2 \cup \dots \cup A_n) = 1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \frac{1}{5!} - \dots$

$P(A_1) = \frac{1}{3}$
$P(A_2) = \frac{1}{3}$
$\vdots$

$\sum_{i=1}^{n} P(A_i) = 1$

possible hats, didn't get

Assuming A_1 & A_2 got their hats.

$- P(A_1 \cap A_2) = \frac{1 \; 1 \; n-2 \; n-3 \; \dots \; 1}{n!} = \frac{(n-2)!}{n!}$

n ppl n hats.

$P(A_1 \cap A_3) = \frac{1 \; n-2 \; 1 \; n-3 \dots 1}{n!} = \frac{(n-2)!}{n!}$

$\text{Sum}: (\sum_{i \neq j}) P(A_i \cap A_j) = \sum_{i \neq j} \frac{(n-2)!}{n!} = \binom{n}{2} \frac{(n-2)!}{n!} = \frac{\cancel{n!}}{(n-2)!\,2!} \frac{\cancel{(n-2)!}}{\cancel{n!}} = \frac{1}{2!}$

the only term left with

$\star$ $\sum_{i=1}^{n} P(A_i) = 1$

$- P(A_1 \cap A_2 \cap A_5) = \frac{1 \; 1 \; n-3 \; n-4 \; 1 \; \dots \; 1}{n!} = \frac{(n-3)!}{n!}$

these kind of problem only n- moving card.

$\sum_{i \neq j \neq k} = \sum_{i \neq j \neq k} \frac{(n-3)!}{n!} = \binom{n}{3} \frac{(n-3)!}{n!} = \frac{\cancel{n!}}{\cancel{(n-3)!}\,3!} \frac{\cancel{(n-3)!}}{\cancel{n!}} = \frac{1}{3!}$

Prob of At least 1 person gets his hat

$\therefore P(A_1 \cup A_2 \cup \dots \cup A_n) = 1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \frac{1}{5!} \dots$
$= 1 - e^{-1} \approx 0.368$

Taylor's

- $f(x) = \sum_{i=0}^{\infty} \frac{f^{(i)}(c)}{i!}(x-c)^i \quad \forall c \in \mathbb{R}$, f cont.

if $x \approx 0 \Rightarrow c = 0$

$$f(x) = \sum_{i=0}^{\infty} \frac{f^{(i)}(0)}{i!} x^i \approx f(0) + f'(0)x + \frac{f''(0)}{2!}x^2$$

$$\rightarrow e^x = e^0 + e^0 x + \frac{e^0 x^2}{2!} + \ldots\ldots$$

$$(c=0) = 1 + x + \frac{x^2}{2!} + \ldots$$

$$1 - e^{-1} = 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \ldots\ldots$$

$$= 1 - (\quad) = 1 - \frac{1}{2!} + \frac{1}{3!} \ldots\ldots$$

☆ No matter how many ppl in a room $P(\text{zero ppl gets their hat}) = e^{-1} \approx 0.365 \approx \boxed{\frac{1}{3}}$