Lecture #14

Thursday, November 3, 2016 12:16 PM

Uber Ride

$$W \sim \begin{cases} 7\text{ min} & \text{w.p. } 0.7 \\ 12\text{ min} & \text{w.p. } 0.3 \end{cases}$$

$$E[W] = (7 \cdot 0.7) + (12 \cdot 0.3) = 8.5\text{ min}$$

Expectation is long term property

$$B = \$0.40/\text{min} \cdot W = g(W) \qquad W \sim \begin{cases} \$2.80 & \text{w.p. } 0.7 \\ \$4.80 & \text{w.p. } 0.3 \end{cases}$$

$$E[B] = (\$2.80 \cdot 0.7) + (\$4.80 \cdot 0.3) = \$3.12$$

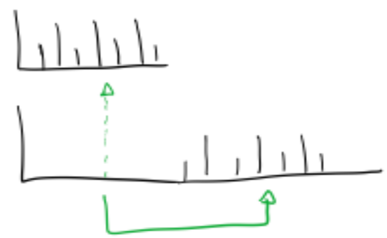
Need unit for HW and test

OR

$$E[B] = \$0.40/\text{min} \; E[W]$$

$$T = \underbrace{B + \$3}_{g(B)}$$

$$E[T] = E[B] + \$3 = \$6.12$$



r.v X, r.v $Y = aX$ $\quad E[Y] = aE[X]$

$$E[g(X)] = \int_\Omega g(X(\omega))P(d\omega)$$

If X discrete

$\text{supp}[X] = \{x_1, x_2, \ldots\}$

$\Omega = A_1 \cup A_2 \cup \ldots$

$$\int g(X(\omega))P(d\omega) + \int g(X(\omega))P(d\omega) + \ldots$$

$$\underbrace{\{\omega : X(\omega) = x_1\}}_{P(x_1)} \qquad \underbrace{\{\omega : X(\omega) = x_2\}}_{P(x_2)}$$

$$= g(x_1)\int X(\omega)P(d\omega) + g(x_2)\int X(\omega)P(d\omega) + \ldots = x_1 P(X = x_1) + x_2 (P(X = x_2)) + \ldots$$

NOT COVERED

$$\boxed{E[g(X)] = \sum_{x \in \text{supp}[X]} x\,p(x)}$$

$$\boxed{\begin{aligned} E[aX] &= \sum_{x \in \text{supp}[X]} ax\,p(x) = a\sum_{x \in \text{supp}[X]} x\,p(x) \\ E[aX] &= a\,E[X] \end{aligned}}$$

$$Y = X + c, \quad c \in \mathbb{R}$$

$$E[Y] = E[\underbrace{X + c}_{g(x)}] = \sum_{x \in \text{supp}[X]} (x + c)\,p(x) = \underbrace{\sum_{x \in \text{supp}[X]} x\,p(x)}_{} + c\underbrace{\sum_{x \in \text{supp}[X]} p(x)}_{}$$

$$\boxed{\begin{aligned} E[X + c] &= E[X] + c \\ E[aX + c] &= a \cdot E[X] + c \end{aligned}}$$

The picture above was rotated for people to see better ex $X \sim \mathrm{Bin}\left(6, \frac{1}{2}\right)$

$Y = g(X) = X^2$

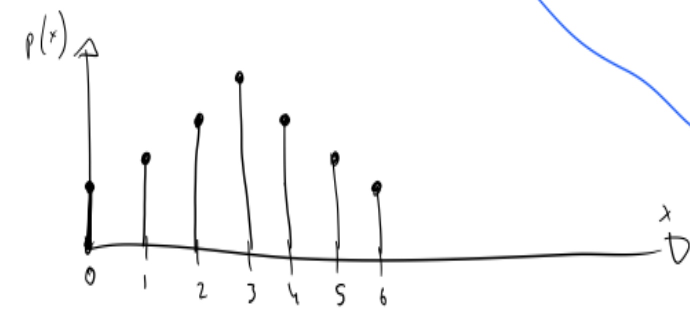
$E[X] = 3$

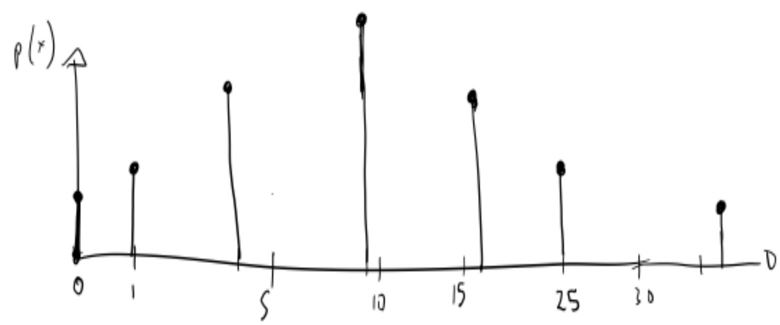
$E[X^2] \neq (E[X])^2$

$$E[X^2] = \sum_{x \in \mathrm{supp}[X]} x^2 \, p(x)$$

$$= \sum_{x=0}^{6} x^2 \binom{6}{x} \frac{1}{2^6}$$

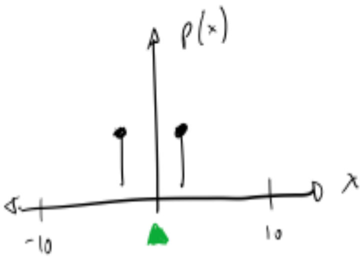
$$= 17.5$$





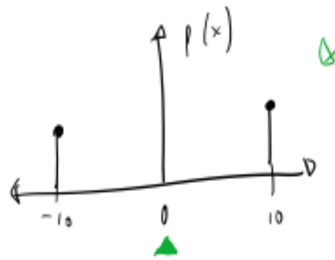
check spelling

Ex $X \sim \text{Redermaker?} := \begin{cases} 1 & \text{wp } \frac{1}{2} \\ -1 & \text{wp } \frac{1}{2} \end{cases}$ $E[x] = 0$



supp close to balance point (expected value)

supp far from . . . . -

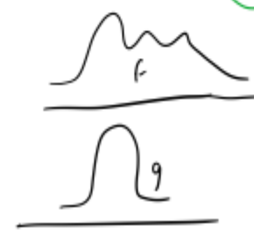


$y = 10x$

$E[y] = 10\,E[x] = 10 \cdot 0 = 0$

$\int_{\mathbb{R}} f(x)\,dx = 17$

$\int_{\mathbb{R}} g(x)\,dx = 17$



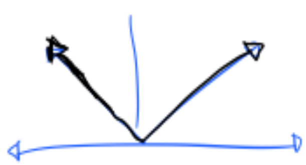
$L = (\underbrace{x-\mu}_{g(x)})^2$

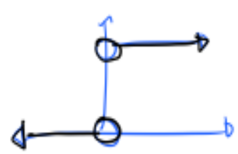
$\sigma^2 := Var[x] := E[L] = E[(x-\mu)^2] = \sum_{x \in supp[x]} (x-\mu)^2 p(x)$

$e(x,\mu) : x - \mu$ ← not a loss function

$e(x,\mu) = |x-\mu|$ ← abs loss, L1 loss

$f(x) = |x|$





$e(x,\mu) = (x-\mu)^2$ ← squared loss, L2 loss $\sqrt{\int_{\mathbb{R}} f(x)^2\,dx}$

the picture above was rotated to help you see better

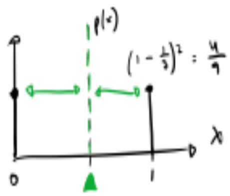
$$\mathrm{Var}[x] = \left((-1)-(0)\right)^2 p(-1) + \left((1)-0\right)^2 p(1)$$

$$= (1 \cdot 0.5) + (1 . 0.5) = \boxed{1}$$

$$\mathrm{Var}[10x] = \left((-10)-(0)\right)^2 p(-10) + \left((10)-(0)\right)^2 p(10)$$

$$= (100 \cdot 0.5) + (100 \cdot 0.5) = \boxed{100}$$

$X \sim \mathrm{Bern}\left(\frac{1}{3}\right) \quad E(x) = \frac{1}{3}$



$\left(0-\frac{1}{3}\right)^2 = \frac{1}{9}$

$$\mathrm{Var}[x] = \left(0-\frac{1}{3}\right)^2 \cdot \frac{2}{3} + \left(1-\frac{1}{3}\right)^2 \frac{1}{3}$$

$$= \frac{1}{9} \cdot \frac{2}{3} + \frac{4}{9} \cdot \frac{1}{3}$$

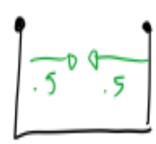
$$= \frac{6}{27} = \frac{2}{9} = .259$$

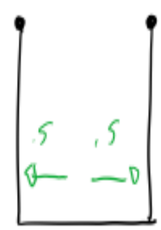
$X \sim \mathrm{Bern}(p) \quad E[x] = p$

$$\mathrm{Var}[x] = (0-p)^2 \cdot \overset{p(0)}{(1-p)} + (1-p)^2 \overset{p(1)}{p}$$

$$= p^2 (1-p) + (1-p)^2 p$$

$$= (1-p)\left(\cancel{p^2} + \overset{p-p^2}{(1-p)p}\right) = \boxed{p(1-p)}$$





Bet on #7

$$X_7 \sim \begin{cases} \$35 & \text{w.p. } \frac{1}{38} \\ -\$1 & \text{w.p. } \frac{37}{38} \end{cases} \qquad E[X_7] = -\$0.053$$

$$Var[X_7] = (\$35 - -\$0.053)^2 \cdot \frac{1}{38} + (-\$1 - -\$0.053)^2 \frac{37}{38} = 33.207\ \$^2$$

$$\sqrt{Var[X_7]} = \$5.79$$

Bet on Black

$$X_B \sim \begin{cases} \$1 & \text{w.p. } \frac{18}{38} \\ -\$1 & \text{w.p. } \frac{20}{38} \end{cases} \qquad E[X_B] = -\$0.053$$

$$Var[X_B] = (\$1 - -\$0.053)^2 \cdot \frac{18}{38} + (-\$1 - -\$0.053)^2 \frac{20}{38} = 0.997\ \$^2$$

$$\sqrt{Var[X_B]} = \$1.00$$

$$\boxed{\sigma := SE[X] = \sqrt{Var[X]}}$$

standard error or

standard deviation

$$\bar{X}_7 \to -\$0.053$$

$$\bar{X}_B \to -\$0.053$$

(faster)