$P(A) = \frac{|A|}{|\Omega|}$ * equal likely

if $\forall a$, $P(\{a\}) = \frac{1}{\Omega}$

$\Omega = \{H, T\}$ $|\Omega'| = 4$

$|2^{\Omega'}| = 16$

$\Omega' = \Omega^2$

Ω'

$\{H,H\}$	$\{H,T\}$
$\{T,H\}$	$\{T,T\}$

$P(\cdot)$

$P(\{\langle H,H\rangle\}) = \frac{1}{4}$

A at least one H

$P(A) = \frac{|A|}{|\Omega|} = \frac{3}{4} = \frac{|\{(H,H),(H,T),(T,H)\}|}{4}$ $P(H,H)$

B At least one T

$P(B) = \frac{|B|}{|\Omega|} = \frac{3}{4} = \frac{|\{(T,T),(H,T),(T,H)\}|}{4}$

$P(A \cup B) = \frac{4}{4}$ $P(A \cap B) = \frac{1}{2}$ (at least one head or at least one tail)

$P(A \text{ and } B)$

$P(A \,\&\, B)$

$P(A, B) = \frac{|\{(H,T),(T,H)\}|}{4}$

$P(AB)$

$\Omega' = \Omega^4 \; (\Omega \times \Omega \times \Omega \times \Omega)$

$|\Omega'| = 16$

$|2^{\Omega'}| = 2^{16}$

$P(H,H,H,H) = P(H,T,H,T)$

$\frac{1}{16} = \frac{1}{16}$

$\frac{|A|}{|\Omega|} = \frac{|A|}{16} = \frac{|\{(H,H,T,T),(H,T,T,H),(H,T,H,T),(T,T,H,H),(T,H,H,T),(T,H,T,H)\}|}{16}$

$= \frac{6}{16} = .375$

$P(\text{at least one } h) = \frac{|H|}{|\Omega|} = \frac{|\{(H,H,H,H),(H,T,T,T)\cdots\}|}{16}$

make compliment

$A'(\text{noheads}) = \frac{1}{16} = \{(T,T,T,T)\} = 1 - P(\text{zero head}) = \frac{15}{16}$

$\Omega' = \Omega^{10}$ $\qquad 2^x \approx 1000^{\frac{x}{10}}$

$|\Omega| = 2^{10} = 1024$

$|2^\Omega| = 2^{1024}$

$p(5H, 5T) = \frac{10!}{1024} \qquad \frac{|\{(H,H,H,H,H,T,T,T,T,T)\ldots\}|}{1024}$

$|\Omega'| = 6 \neq |\Omega^3|$

$\Omega = \{J, M, S\}$

n people, n number of replace seat them $= n!$

n seats without replace # of order $= n!$ $\qquad$ 3 chairs $\quad \underline{3}\ \underline{2}\ \underline{1} = 3!$ factorials

n " WR " " $= n^n$

Seat 1		Seat 2		Seat 3
$\frac{1}{3}$ J	(taken) →	$\frac{1}{2}$ M	$\xrightarrow{1}$	S
		$\frac{1}{2}$ S		
$\frac{1}{3}$ M		$\frac{1}{2}$ J	$\xrightarrow{1}$	J
		$\frac{1}{2}$ S		
$\frac{1}{3}$ S		$\frac{1}{2}$ J	$\xrightarrow{1}$	M
		$\frac{1}{2}$ M		

$20! = 2.7 \times 10^{32}$

$10! = 3.6m$

$5! = 120$

10 people, 3 chairs

$\underset{\text{Seat 1}}{\underline{10}}\ \underset{2}{\underline{9}}\ \underset{3}{\underline{8}} = \frac{10!}{7!} = \frac{10!}{(10-3)!}$ $\quad {}_{10}P_3$ Parameter # of order mutation

sampling without replacement

$_nP_k = \frac{n!}{(n-k)!}$

10 people, 10 chairs $= \frac{10!}{(10-10)!} = 10!$

Bob - Jane,
Richard - Susan,
Max - Alice.

P(all three couples sits together)

$$\text{seats: } \frac{6}{1}\ \frac{1}{2}\ \frac{4}{3}\ \frac{1}{4}\ \frac{2}{5}\ \frac{1}{6} = \frac{48}{6!}$$

couples $\frac{3}{1 \text{ and } 2}$ $\frac{2}{3 \text{ and } 4}$ $\frac{1}{5 \text{ and}}$ $= 3! = 2^3 = 6 \cdot 8 = 48$

variation: 2 2 2

P(Bob, Jane sit together) $\frac{5}{\text{seat } 1 \text{ and } 2}\ \frac{4}{3}\ \frac{3}{4}\ \frac{2}{5}\ \frac{1}{6}$

5 configurate B and J can sit

flip around BJ or JB

$$\frac{5(4!)\,2}{6!}$$

$$P(A) = P(A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5) = P(A_1) + P(A_2) \ldots + P(A_5)$$

$\{A_1 \ldots A_5\}$ are naturally exclusive

Sample 3 balls from a bag of 100

without replacement

$$\# \text{ of ways} = 100 \cdot 99 \cdot 90 = \frac{100!}{(100-3)!} = \frac{100!}{97!}$$

with replacement

$$\# \text{ of ways} = 100 \cdot 100 \cdot 100$$

$$\lim_{n \to \infty} \frac{nP_k}{n^k} = \lim \frac{\frac{n!}{(n-k)!}}{n^k} = \frac{\overbrace{n(n-1)(n-2)\cdots(n-k+1)}^{k}}{\underbrace{n \cdot n - n \cdots n}_{k}} =$$

$$= \lim \frac{n-k+1}{n} = 1$$

principle of dividing at invariance

3 couples

h
h h
h h
h

$\frac{6!}{6}$

some permitation are "the same"
"indistingable"
"non-unique"
"non-distinct"
"invarious"

5 flowers = 3 orchits (O)
2 chry steme (X)

A distinct number order

$\frac{5!}{3!}$

$\frac{5!}{2!}$