$W \sim \begin{cases} 7\text{min wp } 0.7 \\ 12\text{min wp } 0.3 \end{cases}$

$E[X] = 7 \cdot 0.7 + 12 \cdot 0.3 = 8.5 \text{ min}$

$B = 0.40/\text{min} \cdot W = g(W) \sim \begin{cases} \$2.8 \text{ wp } 0.7 \\ \$4.8 \text{ wp } 0.3 \end{cases}$

$E[X] = \$2.80 \cdot 0.7 + \$4.8 \cdot 0.3 = \$3.12$

$= 0.4 \cdot 8.5 = \$3.12 = \$0.4\, E[W]$

$E[X] := \int_{\Omega} X(\omega) P(d\omega) = \int_{\{\omega : X(\omega) = x_1\}} X(\omega) P(d\omega) + \int X(\omega) P(d\omega) + \dots$

$= X_1 \int_{\{\omega : X(\omega) = x_1\}} P(d\omega) + X_2 \int_{\{\omega : X(\omega) = x_2\}} P(d\omega)$

$= x_1 P(X = x_1) + x_2 P(X = x_2) + \dots = \sum_{x \in Supp[X]} x p(x)$

$E[g(x)] := \int_{\Omega} g(X(\omega)) P(d\omega) = \int_{\{\omega : X(\omega) = \omega_1\}} g X(\omega) P(d\omega) + \int_{\{\omega : X(\omega) = \omega_2\}} g X(\omega) P(d\omega) + \dots$

$= g(x_1) \int P(d\omega) + g(x_2) \int P(d\omega) + \dots$

$= g(x_1) P(X = x_1) + g(x_2) P(X = x_2) + \dots$

$E[g(x)] = \sum_{x \in Supp[X]} g(x) p(x)$

$E[aX] = \sum_{x \in Supp[X]} a x p(x) = a \sum x p(w) = a E[W] = E[aX]$

$T = B + \$3 = 3.12 + 3 = 6.12$

$\underbrace{Y = X + c}_{g(x)}, \quad c \in \mathbb{R}, \quad X \text{ is a r.v}$

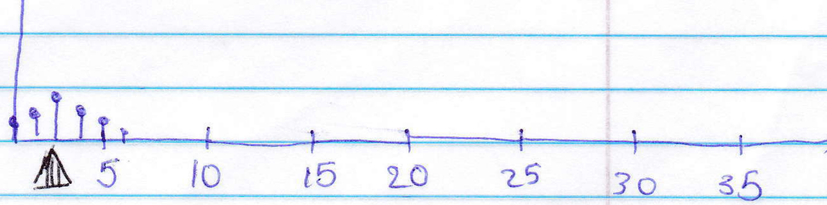
$E[X + c] = \sum_{x \in Supp[X]} (x + c) p(x) = \underbrace{\sum x p(x)}_{E[X]} + c \underbrace{\sum p(x)}_{(1)}$

$E[X] + c$

$\boxed{E[aX + c] = a E[X] + c}$

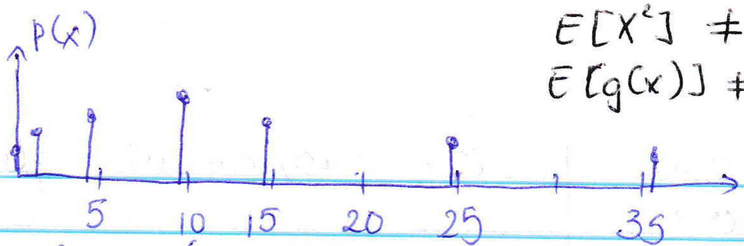
$X \sim Bin(6, \frac{1}{2})$

$E[X] = 6 \cdot \frac{1}{2} = 3$



$E[X^2] \neq (E[X])^2$

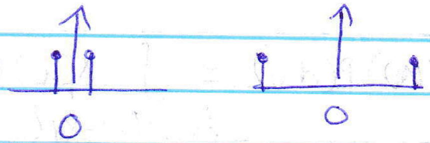
$E[g(x)] \neq g(E[X])$



$Y = X^2$ $\tilde{g}(x)$

$E[X^2] = \sum_{x=0}^{6} x^2 \binom{6}{x} \frac{1}{2^6} = 17.5 \overset{?}{=} h(E[X])$

$X \sim \text{Rademacher} = \begin{cases} 1 & wp\ \frac{1}{2} \\ -1 & wp\ \frac{1}{2} \end{cases}$ $E[X] = 0$



$E[10X] = 10E[X] = 10.0 = 0$

$E(X, \mu) = |x - \mu|$ - Absolute loss

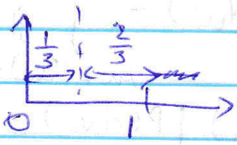
$E(X, \mu) = (X - \mu)^2$

$L := (X - \mu)^2$ $\sigma^2 := Var[X] = E[L] = E[(X-\mu)^2] = \sum_{x \in Supp[X]} (x - \mu)^2 p(x)$

$Var[X] = ((-1) - (0))^2 \frac{1}{2} + ((1) - (0))^2 \frac{1}{2} = 1 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} = 1$

$Var[Y] = ((-10) - (0))^2 \frac{1}{2} + ((10) - (0))^2 \frac{1}{2} = 100 \frac{1}{2} + 100 \frac{1}{2} = 100$

$X \sim \text{Bernoulli}(\frac{1}{3})$ $E[X] = \frac{1}{3}$



$Var[X] = (0 - \frac{1}{3})^2 \cdot \frac{2}{3} + (1 - \frac{1}{3})^2 \frac{1}{3} = \frac{2}{9}$

$Var[X] = (0 - p)^2 (1-p) + (1-p) = p = (1-p)(p^2 + (1-p)p) = p(1-p)$

Bet on # 7 : $X \sim \begin{cases} \$35 & wp\ \frac{1}{38} \\ -\$1 & wp\ \frac{37}{38} \end{cases}$

$Var[X] = (35 + 0.053)^2 \frac{1}{38} + (-1 + 0.053)^2 \frac{37}{38}$

$= 33.207\ \$^2$

$E[X] = -0.053$

Bet on Black : $X \sim \begin{cases} \$1 & wp\ \frac{18}{38} \\ -\$1 & wp\ \frac{20}{38} \end{cases}$

$Var[X] = (1 + 0.053)^2 \frac{18}{38} + (-1 - -0.053)^2 \frac{20}{38}$

$= 0.997\ \$^2$

σ : $SE[X] = \sqrt{Var[X]}$

$\bar{X}_{⑦} \to -\$0.053$ LLN

$\bar{X}_{Ⓑ} \to -\$0.053$ LLN