09/01/16

2 coin flip $|2^{\Omega}|$

$\Omega' = \Omega^2$, $\Omega = \{H,T\}$

$|\Omega'| = 4$

Ω'

$\langle H,H\rangle$	$\langle H,T\rangle$
$\langle T,H\rangle$	$\langle T,T\rangle$

$P(A) = \frac{|A|}{|\Omega|}$

$\left[\text{if } \forall_\omega \; P(\{\omega\}) = \frac{1}{|\Omega|} \right.$

→ Every outcome is equally likely to happen

$P(\{\langle H,H\rangle\}) = P(H,H) = \frac{1}{4}$

- A: at least one H

$P(A) = \frac{|A|}{|\Omega|} = \frac{|A|}{4} = \frac{|\{\langle H,T\rangle, \langle T,H\rangle, \langle H,H\rangle\}|}{4} = \frac{3}{4}$

- B: at least one T

$P(B) = \frac{|B|}{|\Omega|} = \frac{|\{\{T,T\}, \{H,T\}, \{T,H\}\}|}{4} = \frac{3}{4}$

- $P(A \cup B) = \frac{4}{4}$
- $P(A \cap B) = \frac{|\{\langle H,T\rangle, \langle T,H\rangle\}|}{4} = \frac{2}{4} = \frac{1}{2}$

- $\Omega' = \Omega^4$

$|\Omega'|$ → # of outcomes that can happen

Ω'

$\langle H,H,H,H\rangle$	$\langle T,H,H,H\rangle$	$\langle H,T,H,H\rangle$	$\langle H,H,T,H\rangle$
$\langle H,H,H,T\rangle$	. . .	.	- -
. .	.	.	.
.	.		

- $P(H,H,H,H) = \frac{1}{16}$
- $P(H,T,H,T) = \frac{1}{16}$

~~[illegible]~~

$= \frac{|\{HHTT\}, \{TTHH\}, \{HTTH\}, \{THHT\}, \{HTHT\}, \{THTH\}|}{16} = \frac{6}{16}$

A: at least one head (H) → $\{\geq 1 H\}$

$P(A) = \frac{|A|}{16} = \frac{15}{16}$

$A^c = \{< 1H\} = \{\text{zero } H\}$

$1 - P(A^c) = 1 - \frac{1}{16} = \frac{15}{16}$

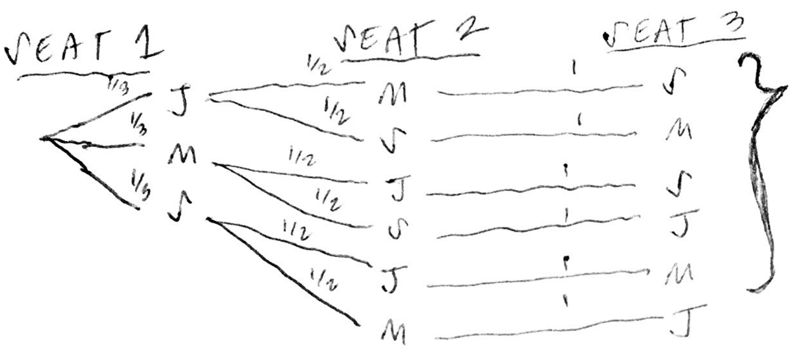
$\Omega' = \Omega^{10}$ with $\Omega = \{H, T\}$ $\qquad 2^x \approx 1000^{\frac{x}{10}}$

- $|\Omega'| = 2^{10} \approx 1000 \qquad 2 \approx 1000^{\frac{1}{10}} \approx 1.9953$
- $\underbrace{P(5H, 5T) = \frac{\quad}{2^{10}}}_{\text{NO OBVIOUS WAY TO DO THIS}} = .2460838$

$\Omega = \{J, M, S\}$ CHAIRS

$\{\langle J, M, S\rangle, \langle M, J, S\rangle, \langle S, J, M\rangle, \langle S, M, J\rangle, \langle J, S, M\rangle, \langle M, S, J\rangle\}$

How many ways to order these people?



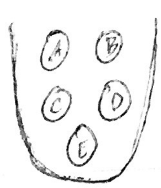
sampling without replacement $\qquad$ sampling with replacement

$|\Omega| = 6 \neq |\Omega^3| = 27$

$\underset{\text{SEAT 1}}{3} \quad \underset{\text{SEAT 2}}{2} \quad \underset{\text{SEAT 3}}{1} = 3!$ ← Factorial

$n! := \prod_{i=1}^{n} i$ ← PRODUCT

MARBLES



$5! = 120$

$10! = 3.6m$

$20! = 2.7 \times 10^{32}$

10 people 3 chairs

- # of ORDER $= \underset{\text{SEAT 1}}{10} \times \underset{\text{SEAT 2}}{9} \times \underset{\text{SEAT 3}}{8} = \frac{10!}{7!} = \boxed{\frac{10!}{(10-3)!}}$ $\boxed{{}_{10}P_3}$

permutation

$$\boxed{{}_nP_k := \frac{n!}{(n-k)!}}$$

$$10! \stackrel{?}{=} {}_{10}P_{10} = \frac{10!}{(10-10)!} = \frac{10!}{0!}$$

$$\boxed{0! = 1}$$

- 100 balls, take out 7... $\Rightarrow {}_{100}P_7$

Bob – Jane
Richard – Susan
Max – Alice

couples

$P(\text{couples sit together}) \overset{A}{=} \frac{|A|}{|\Omega|} = \frac{}{6!}$

WAY 1

$\underset{\text{SEATS 1 \& 2}}{3}$ $\underset{\text{SEATS 3 \& 4}}{2}$ $\underset{\text{SEATS 5 \& 6}}{1}$ $\overset{?}{=} 3! \cdot 2^3$

no within a bench, a couple can swap.

$3! \cdot 8 = \frac{48}{6!}$

WAY 2

$\underset{1}{6}\ \underset{2}{1}\ \underset{3}{4}\ \underset{4}{1}\ \underset{5}{2}\ \underset{6}{1} = \frac{48}{6!}$

$P(\text{alternating gender}) = \frac{(3!)^2}{6!}$

$\frac{3}{6}\ \frac{3}{5}\ \frac{2}{4}\ \frac{2}{3}\ \frac{1}{2}\ \frac{1}{1} \overset{?}{=} \frac{(3!)^2}{6!}$

No, cause we can start off with a girl.

$P(\text{alternating gender}) = P(BGBGBG) + P(GBGBGB)$

$P(A) = P(A_1) + P(A_2) = P(A_1 \cup A_2)$

if A_1 & A_2 are mutually exclusive.

$P\left(\overset{A}{\text{Bob Jane sit together}}\right)$ $\boxed{\frac{4! \cdot 2 \cdot 5}{6!}}$

$\underset{\text{SEAT 1 \& 2}}{1}\ \underset{3}{4}\ \underset{4}{3}\ \underset{5}{2}\ \underset{6}{1}$

$\overset{?}{=} \boxed{4! \cdot 2 \cdot 5}$

can move the bench multiple ways .. 5

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100 balls select 3 without replacement

$\frac{100P_3}{} = .9997$

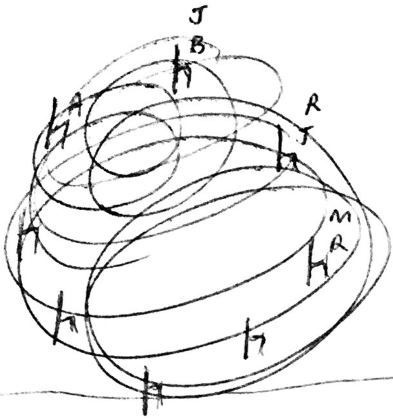
" " " with replacement

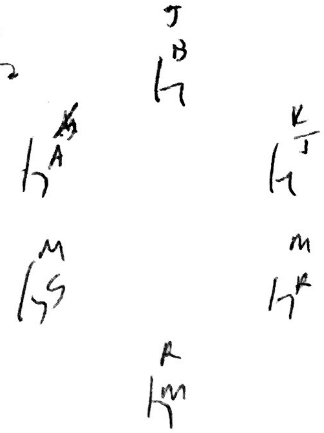
$100\ 100\ 100 = 100^3$

$\lim_{n\to\infty} \frac{nP_k}{n^k} = \frac{\frac{n!}{(n-k)!}}{n^k} = \frac{\overbrace{n(n-1)(n-2)\dots(n-k+1)}^{k}}{\underbrace{n \cdot n \cdot n \cdots n}_{k}} =$

$$= \lim \overset{1}{\frac{n}{n}} \cdot \lim_n \overset{1}{\frac{n-1}{n}} \cdot \lim \overset{1}{\frac{n-2}{n}} \cdot \cdots \lim_n \overset{1}{\frac{n-k+1}{n}} = 1$$

6 chairs, 6 people





$$\frac{6!}{6}$$

"same"
"indistinguishable"

principle of dividing out invariance

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5 flowers
3 orchid (O)
2 chrysant.. (X)

$O_1\ O_2\ O_3\ X_1\ X_2$
$O_1\ O_3\ O_2\ X_1\ X_2$
$O_2\ O_1\ O_3\ X_1\ X_2$
$O_2\ O_3\ O_1\ X_1\ X_2$
$O_3\ O_1\ O_2\ X_1\ X_2$
$O_3\ O_2\ O_1\ X_1\ X_2$

$$\frac{5!}{3!}$$