Definition of Prob. (the three conditions)

9/13 P is a set function and $\exists\ \Omega$ s.t

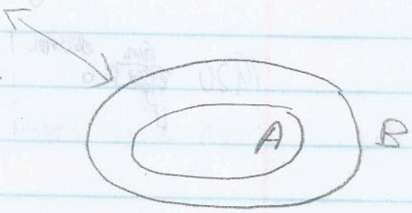
ⓐ $P(\Omega)=1$

ⓑ $P(A)\geq (A)\ \forall A$

ⓒ If $A_1, A_2, \ldots$ disjoint $\Rightarrow P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$

Thm: $P(A) = 1 - P(A^c)$

$$P(\Omega) = P(A \cup A^c)$$
$$P(\Omega) = P(A) + P(A^c)$$
$$1 = P(A) + P(A^c)$$
$$P(A) = 1 - P(A^c)$$



Thm: $A \subseteq B \Rightarrow P(A) \leq P(B)$

$C := B \setminus A$

$\left.\begin{array}{l} A, C \text{ are disjoint} \\ B = A \cup C \end{array}\right\}$ by construction.

$P(B) = P(A \cup C)$

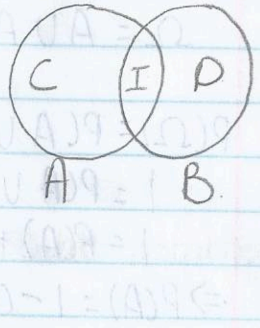
$P(B) = P(A) + P(C)$ by ⓒ condition

$P(B) - P(A) = P(C) = 0$ by ⓑ condition.

$P(0) \geq P(A)$

Thm: $P(A \cup B) = P(A) + P(B) - P(AB)$

Law of Inclusion-Exclusion.



$C = A \setminus B$

$D = C \setminus A$

$I = AB$

$P(A) = P(C) + P(I) \Rightarrow P(C) = P(A) - P(I)$

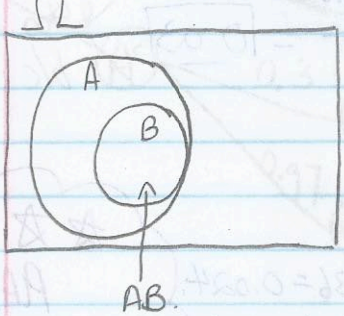
$P(B) = P(D) + P(I) \Rightarrow P(D) = P(B) - P(I)$

$$P(A \cup B) = P(C) + P(D) + P(I)$$
$$= (P(A) - P(I)) + (P(B) - P(I)) + P(I)$$
$$= P(A) + P(B) - P(I)$$
$$= P(A) + P(B) - P(AB)\ \checkmark$$

Note:

- $n = 1000$ people
  - 200 smokers (A)
  - 60 lung cancers (B)
  - 36 smoke & lung cancers (AB)

(Long Run Freq)
By LRF def.
$P(A) = 0.2$
$P(B) = 0.06$
$P(AB) = 0.036$.
↓
3.6% = the population of both at the same time

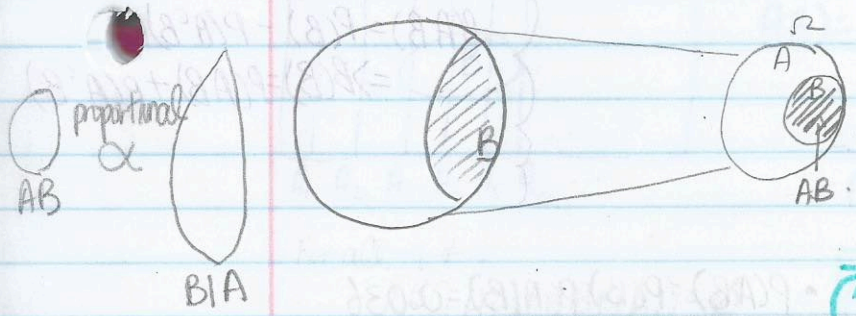


$$☆ = P(B \overset{\text{given}}{|} A) = \frac{36}{200}$$

Questions: The probability of lung cancer among smokers
given person in a smoker
conditional or the person smoking.

• Scoping in different ways.

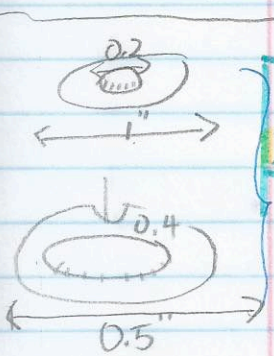
☆ $A = \Omega \subseteq \Omega$



$\rightarrow P(B|A) = P(BA) \cdot$ Zoom factor
$= P(BA) \cdot \frac{P(\Omega)}{P(A)}$

$$\boxed{P(B|A) = \frac{P(BA)}{P(A)}}$$

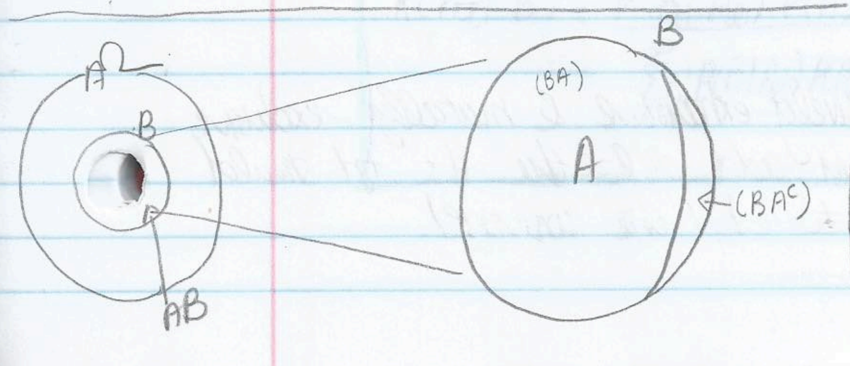
Def. conditional probability.



$$\boxed{\text{Zoom factor} = \frac{\text{begin scope}}{\text{end scope}}}$$

$$= \frac{1''}{0.5''} = 2$$

$$\boxed{☆\ P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}}$$ Bayes Rule (1763)



$$P(BA) = P(B|A)\,P(A) \Leftarrow P(B|A) = \frac{P(BA)}{P(A)}.$$

these numbers seem too low

$-P(B|A) = \frac{P(BA)}{P(A)} = \boxed{0.036}$

☆ $-P(A|B) = \frac{P(BA)}{P(B)} = \frac{0.036}{0.06} = \boxed{0.6}$

= among

On Exam: What does it mean? The prob of a smoker among those who have lung cancer

$-P(\text{lung cancer among non-smoker})$

$= P(B|A^c) = \frac{P(B|A^c)}{\underbrace{P(A^c)}_{1-P(A)}} = \frac{? = 0.024}{0.8} = \boxed{0.03}$

seems too high b/c

Note: $P(B) = P(BA) + P(BA^c)$

$P(BA^c) = P(B) - P(BA) = 0.06 - 0.036 = 0.024.$

☆☆☆☆☆☆☆

$P(A) = 1 - P(A^c)$

$P(A|B) = 1 - P(A^c|B)$

$\frac{P(AB)}{P(B)} = 1 - \frac{P(A^cB)}{P(B)}$

$P(AB) = P(B) - P(A^cB)$

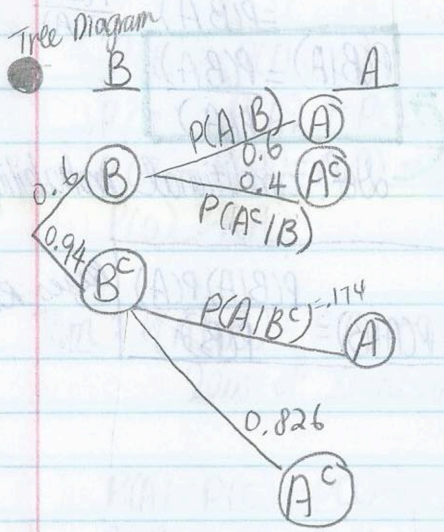
$\Rightarrow P(B) = P(AB) + P(A^cB)$

How likely is it that you will get cancer if you smoke suppose to not smoking?

$\boxed{\text{Risk Ratio} = \text{Ratio Prob} = \frac{P(B|A)}{P(B|A^c)}} = \frac{.18}{.03} = 6.$ times more likely.

$P\left(\begin{matrix} A \\ A^c \\ B \\ B^c \end{matrix} \middle| \begin{matrix} B \\ B^c \\ A \\ A^c \end{matrix}\right)$ 8 of them...

Tree Diagram



- $P(AB) = P(B)\,P(A|B) = 0.036$
- $P(A^cB) = 0.024$

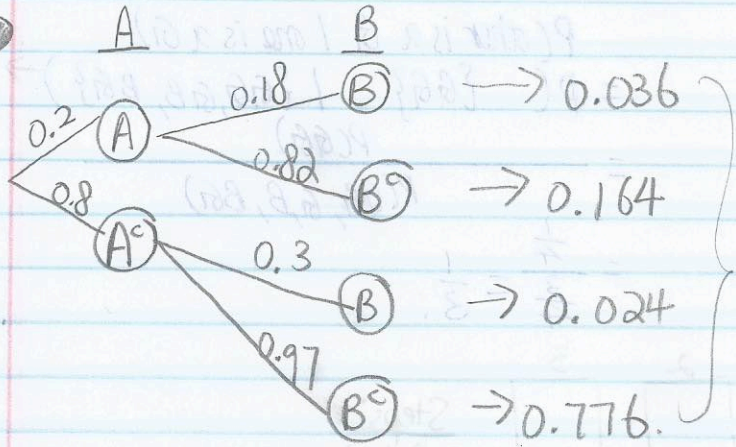
+

- $P(AB^c) = 0.164$
- $P(A^cB^c) = 0.776$

$= 1$

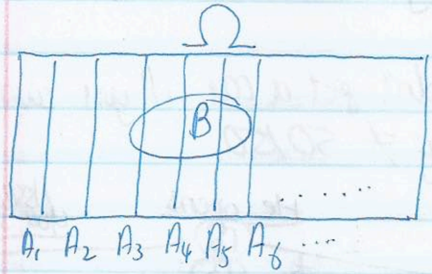
They are collectively exhaustive & mutually exclusive. when you add all the #s tgt should add up to 1 (the universe).

- Set up the tree as the following. (1st effect → 2nd effect ...).



→ Once both trees are built all answers are answered.

- Consider event B and events $A_1, A_2, \ldots\ldots$ mutually exclusive, collectively exhaustive



$$
\begin{aligned}
P(B) &= P(B \cap \Omega) \\
&= P(B \cap (A_1 \cup A_2 \cup A_3 \cup A_4 \cup \ldots)) \\
&= P((B \cap A_1) \cup (B \cap A_2) \cup (B \cap A_3) \cup \ldots\ldots) \\
&= P(B \cap A_1) + P(B \cap A_2) + \ldots\ldots
\end{aligned}
$$

For all $i \neq j$

$(B \cap A_i) \cap (B \cap A_j) = \emptyset$

$B \cap B \cap A_i \cap A_j$

$B \cap (A_i \cap A_j) = B \cap \emptyset = \emptyset.$

$$\boxed{P(B) = \sum_{i=1}^{\infty} P(BA_i)}$$ Law of Total Probability.

- Consider $P(A_l \mid B) = \dfrac{P(B \mid A_l)\, P(A_l)}{P(B)}$ Bayes Rule

$$\boxed{P(A_l \mid B) = \frac{P(B \mid A_l)\, P(A_l)}{\sum_{l=1}^{\infty} P(B \mid A_l)\, P(A_l)}}$$ Bayes Thm.

• Ω Assume boy & girl are equally likely.

Boy & Girl. —

GG	BG
GB	BB

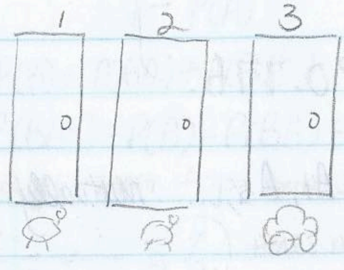
$P(\text{other is a G} \mid \text{one is a G})$

$= P(\{GG\} \mid \{GG, GB, BG\}) \rightarrow \frac{P(\cap)}{P(\ )}$ $\{GG\}$ $\{GG, GB, BG\}$

$= \frac{P(GG)}{P(GG, GB, BG)}$ (Conditional Prob)

$= \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}.$

— Monte Hall Game



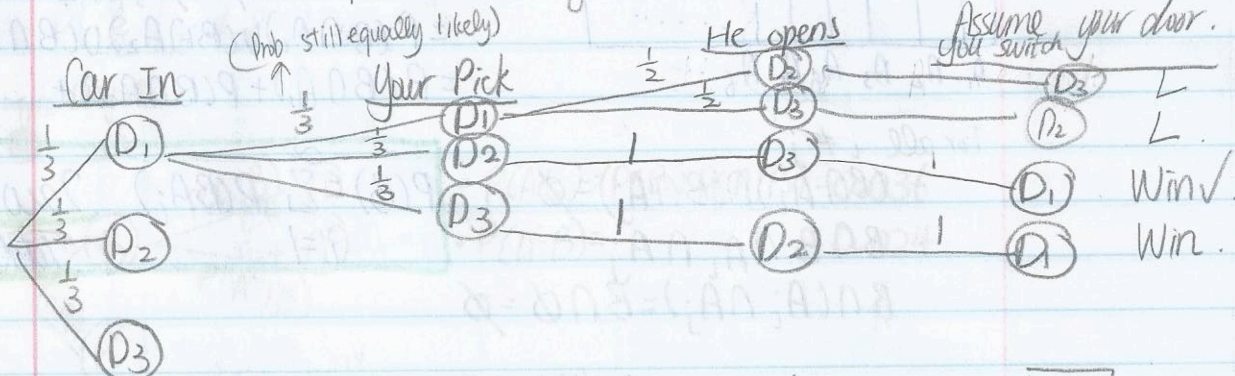
Steps:

① Pick Door

② Game show host opens a door with goat. (not your door)

③ You decide to switch from initial choose.

→ If you open door 1, you didn't get a car, if you switch ∴ the prob of winning ≠ 50/50.



$$P(W \mid \text{Car in } D_1) = \frac{\frac{1}{3}\cdot\frac{1}{3}\cdot 1\cdot 1 + \frac{1}{3}\cdot\frac{1}{3}\cdot 1\cdot 1}{\frac{1}{3}} = \boxed{\frac{2}{3}} \neq \frac{1}{2}$$

You should definitely switch, the prob of winning will be doubled

The pattern repeats each time