9/15/16

Paradox trials

Lecture 7

W	1
L	1 2 3

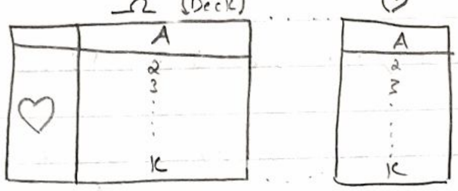
$P(\text{Ace in a deck of cards}) = \frac{4}{52} = \frac{1}{13}$

$P(\text{Ace given that we have a heart}) = P(A \mid \heartsuit) = \frac{1}{13}$ of 13 ♡s total one is an A

Information Known

This information is "irrelevant"

Ω (Entire Deck)



$P(\text{IBM Stock} \uparrow \text{ in a day}) = \text{let's say } \frac{1}{2}$

$= P(\text{IBM Stock} \uparrow \text{ in a day} \mid \text{rains in Buenos Aires}) = \frac{1}{2}$

irrelevant information

Def. of A, B being "independent" (<u>Probabalistic independence</u>)

$P(A) = P(A \mid B)$

equivalent to

$P(B) = P(B \mid A)$

Under A, B being independent

$P(A) = P(A \mid B) = \frac{P(AB)}{P(B)}$

$\Rightarrow P(AB) = P(A) \cdot P(B)$

Multiplication Rule

$A_1, A_2, \dots$ independent

$P\left(\bigcap_{i=1}^{\infty} A_i\right) = \prod_{i=1}^{\infty} P(A_i)$

$P(H_2 \mid H_1) = P(H_2) = \frac{1}{2} = \prod_{i=1}^{5} P(H_i)$

$P(H_1, H_2, H_3, H_4, H_5) = \left(\frac{1}{2}\right)^5 = \frac{1}{|\Omega^5|} = \frac{1}{2^5}$

$$P(\{\geq 1 \text{ 6-6 in 24 rolls of 2 dice}\}) < \frac{1}{2}$$

$= P(1 \text{ 6-6}) + P(2 \text{ 6-6}) + \ldots + P(24 \text{ 6-6})$

P of 1 double 6's + P of 2 double 6's + ... + P of 24 double 6's

$= 1 - P(\underbrace{\text{zero double 6 in 24 rolls}}_{\text{complement of this}}) = 1 - P(\text{zero 6-6}) =$

$= 1 - P(\text{Not 6-6 } 1^{st} \text{ roll} \cap \text{Not 6-6 } 2^{nd} \text{ roll} \cap \ldots \cap \text{Not 6-6 } 24^{th} \text{ roll})$

$\rightarrow P(\text{Not 6-6 } 1^{st}) \cdot P(\text{Not 6-6 } 2^{nd}) \cdot \ldots P(\text{Not } \cancel{\text{6-6}} \; 24^{th})$

$= P(\text{Not 6-6})^{24} = (1 - P(6\text{-}6))^{24} = (1 - P(6)P(6))^{24}$

$= (1 - (\frac{1}{6})^2)^{24}$

$= 0.4914039$

If $P(A|B) \neq P(A)$

or $P(B|A) \neq P(B)$

or $P(AB) \neq P(A)P(B)$

$\Rightarrow$ A, B <u>Not</u> independent i.e. "dependent"

Marginal Probability

$P(\text{Q64 bus is late}) < P(\text{Q64 bus is late} \mid \text{rain})$

$> P(\text{Q64 bus is late} \mid \text{Sun and no traffic})$

$P(\text{lung cancer} \mid \text{smoking}) > P(\text{lung cancer})$

dependent events

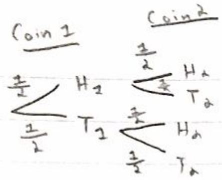
A, B disjoint $\overset{?}{\Rightarrow}$ A,B independent? NO

(does this imply)

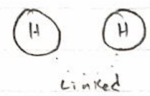
$P(A|B) \overset{?}{=} P(A)$ $\qquad$ $P(H \mid \text{coin loaded tails}) \neq P(H)$

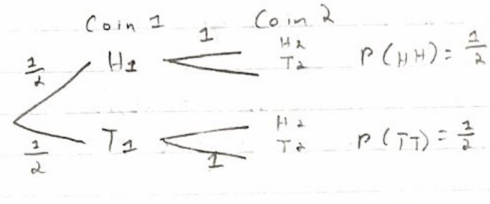
$0 \neq P(A)$ $\qquad$ $0 \neq P(H)$

9/15/16



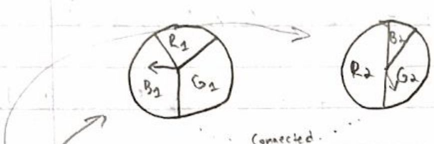
$P(H,H) =$
$P(H,T) =$
$P(T,H) =$
$P(T,T) =$ $\Big\}\ \frac{1}{4}$





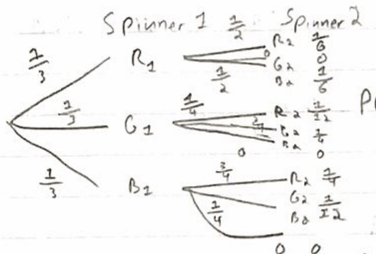
$P(H_2 \mid H_1) \overset{?}{=} P(H_2)$

$1 \neq \frac{1}{2}$



$P(R_1) = P(B_1) = P(G_1) = \frac{1}{3}$

$P(R_2) = \frac{1}{2}$ $P(G_2) = \frac{1}{3}$ $P(B_2) = \frac{1}{6}$



R_1, R_2 independent?

$P(R_2 \mid R_1) \overset{?}{=} P(R_2)$

$\frac{1}{2} = \frac{1}{2}$

Yes

R_1, G_2 independent

$P(G_2 \mid R_1) \overset{?}{=} P(G_2)$

$0 \neq \frac{1}{3}$

$P(\text{Shared Birthday}) = P(\geq 1 \text{ Shared Birthday})$

$= P(1 \text{ Shared Bday}) + P(2 \text{ Shared Bday}) + \ldots + P(48 \text{ Shared bday})$

$= 1 - P(\text{No shared Bdays})$

$= 1 - 0.04 = \boxed{96\%}$

$= \dfrac{\overset{2/15}{365} \cdot \overset{\text{can't be } 2/15}{364} \cdot \overset{\text{can't be } 2/15 \text{ or } 2/13}{363} \cdots 365-48+1}{365^{48}} =$

$= \dfrac{{}_{365}P_{48}}{365^{48}} = 0.039$

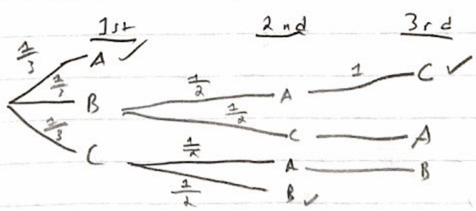
N people randomly administer their hats

$P = P(\text{zero people get their hat})$

$1 - P = P(\text{at least one person gets hat})$

$= P(1\ p \text{ gets hat}) + P(2\ p \text{ get hat}) + \ldots + P(n \text{ people get hat back})$

3 people A, B, C



$\frac{1}{3} + \frac{1}{6} + \frac{1}{6} = \frac{2}{3}$

Let A_1 : 1st person gets hat
A_2 : 2nd person gets hat
$\vdots$
A_n : nth person gets hat

$$P(A \cup B) = P(A) + P(B) - P(AB)$$
$$P(A_1 \cup A_2) = \sum_{i=1}^{2} P(A_i) - P\left(\bigcap_{i=1}^{2} A_i\right)$$

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) - \sum_{i \neq j} P(A_i \cap A_j) + \sum_{i \neq j \neq k} P(A_i \cap A_j \cap A_k)$$

$$= P(A_1 \cup A_2 \cup \ldots \cup A_n) = 1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \frac{1}{5!} \cdots + - + - (-1)^{n+1} P\left(\bigcap_{i=1}^{n} A_i\right)$$

Probability 1st person gets hat $\rightarrow$ $P(A_1) = \frac{1}{n}$ $\qquad \sum_{i=1}^{n} P(A_i) = 1$

$P(A_2) = \frac{1}{n}$

$\vdots$

gets hat

$$P(A_1 \cap A_2) = \frac{\underline{1}\ \underline{1}\ \underline{n-2}\ \underline{n-3} \ldots \underline{1}}{n!} = \frac{(n-2)!}{n!}$$

$$P(A_1 \cap A_3) = \frac{\underline{1}\ \underline{n-2}\ \underline{1}\ \underline{n-3}\ \ldots\ \underline{1}}{n!} = \frac{(n-2)!}{n!}$$

$$\sum_{i \neq j} P(A_i \cap A_j) = \sum_{i \neq j} \frac{(n-2)!}{n!} = {}_nC_2 \frac{(n-2)!}{n!} = \binom{n}{2} \frac{(n-2)!}{n!} = \frac{\cancel{n!}}{\cancel{(n-2)!}\, 2!} \frac{\cancel{(n-2)!}}{\cancel{n!}} = \frac{1}{2!}$$

$$P(A_1 \cap A_2 \cap A_3) = \frac{\underline{1}\ \underline{1}\ \underline{n-3}\ \underline{n-4}\ \underline{1} \ldots \underline{\ }}{n!} = \frac{(n-3)!}{n!}$$

$$\sum_{i \neq j \neq k} = \sum_{i \neq j \neq k} \frac{(n-3)!}{n!} = \binom{n}{3} \frac{(n-3)!}{n!} = \frac{n!}{(n-3)!\, 3!} \frac{(n-3)!}{n!} = \frac{1}{3!}$$

$$f(x) = \sum_{i=0}^{\infty} \frac{f^{(i)}(c)}{i!} (x-c)^i \quad \forall c \in \mathbb{R},\ f \text{ cont.}$$

if $x \approx 0 \Rightarrow c = 0$

$$f(x) = \sum_{i=0}^{\infty} \frac{f^{(i)}(0)}{i!} x^i \approx f(0) + f'(0)x + \frac{f''(0)}{2!}$$

$$\underset{(c=0)}{e^x} = e^0 + e^0 x + \frac{e^0 x^2}{2!} + \ldots$$

$$= 1 + x + \frac{x^2}{2!} + \ldots \qquad 1 - e^{-1} = 1 - (\ \ ) = 1 - \frac{1}{2!} + \frac{1}{3!} - \ldots$$

$$e^{-1} = 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} \ldots \qquad 1 - e^{-1} \approx 0.368$$

$P(\text{zero ppl get hat}) = e^{-1} \approx 0.368 \approx \frac{1}{3}$ $\qquad \uparrow P(\text{at least one person gets hat})$