- Make up class on 11/11 (Fri) 2.5 hrs.
- Exam in 2 weeks.
-

Review...

11/01 • $X \sim Geom(p) := (1-p)^{x-1}\,p$ (P.M.F).

$X \sim Geom(0.2) = 0.8^{x-1}\,0.2$

☆ will never hit zero!

| X | p(x) | F(X) |
|---|---|---|
| 1 | 0.200 | 0.200 |
| 2 | 0.160 | 0.360 |
| 3 | 0.128 | 0.488 |
| 4 | 0.102 | 0.590 |
| 5 | 0.082 | 0.672 |
| 6 | 0.066 | 0.738 |
| 7 | 0.052 | 0.790 |
| 8 | 0.042 | 0.832 |
| 9 | 0.034 | 0.866 |
| 10 | 0.027 | 0.827 |
| 11 | 0.021 | 0.914 |
| 12 | 0.017 | 0.931 |
| 13 | 0.014 | 0.945 |
| 14 | 0.011 | 0.956 |
| 15 | 0.009 | 0.965 |
| 16 | 0.007 | 0.972 |
| 17 | 0.006 | 0.978 |
| 18 | 0.005 | 0.983 |
| 19 | 0.004 | 0.987 |
| 20 | 0.003 | 0.990 |
| 21 | 0.002 | 0.992 |
| 22 | 0.001 | 0.993 |
| 23 | 0.001 | 0.994 |
| 24 | ⋮ | 0.995 |
| 25 | ⋮ | 0.996 |
| 26 | ⋮ | 0.997 |
| 27 | ⋮ | 0.998 |
| 28 | 0.000 | 0.999 |

Not necessary to show... so small & not gonna happen!

# 1 b/c Supp keeps going (infinite).
C.D.F → 1 when add up infinite p(x)

Approximate / Effective Support

$\{x : p(x) > .001\} \subset Supp[X] = \mathbb{N}$

$A \subset Supp[X]$ = smallest subset A s.t. $\sum_{x \in A} p(x) = 0.999$.

• $\bar{X} \rightarrow E[X] = M$ (simple avg r.v.)

Expectation w/ regard to P.M.F

⇒

• $X \sim Bern(p) \Rightarrow E[X] = p$

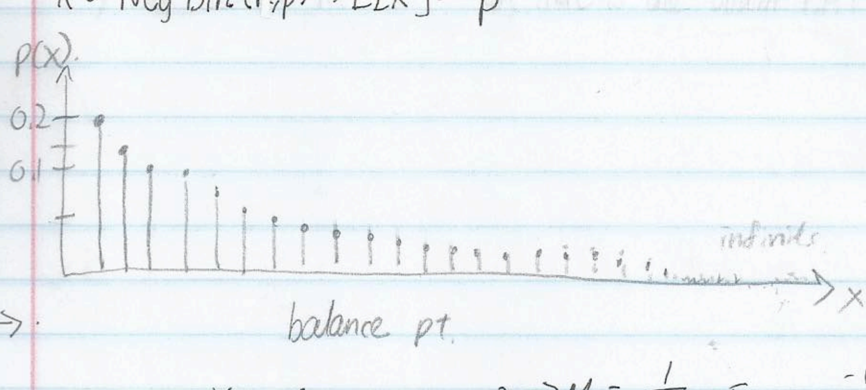
$X \sim Bin(n,p) \Rightarrow E[X] = np$

$X \sim Hyper(n, K, N) \Rightarrow E[X] = n\frac{K}{N}$ (wait).

$X \sim Geom(p) \Rightarrow E[X] = \frac{1}{p}$

$X \sim Neg\,Bin(r,p) \Rightarrow E[X] = \frac{r}{p}$

☆ Expectation with regard to Data




balance pt.

Important aspect = $\bar{X} \rightarrow M$

$X \sim Geom(p = 0.2) \Rightarrow M = \frac{1}{0.2} = 5$.

- $E[X] = 5$
- $Mode[X] = 1$

$X \sim \text{Geom}(p) := (1-p)^{x-1}p.$

- Definition of Expectation

$$E[x] = \sum_{x \in Supp[x]} x\, p(x)$$

(Sometimes, there's no balance pt). (It goes to infinity).

$$M = \underbrace{\sum_{x=1}^{\infty} x(1-p)^{x-1} p}_{\text{solve it.}}$$

$$= p\sum_{x=1}^{\infty} x(1-p)^{x-1} = p\sum_{y=0}^{\infty}(y+1)(1-p)^{y}$$

Let $y = x-1 \Rightarrow x = y+1$ $\quad x=1\ldots\infty$, $y=0\ldots\infty$

$$= p\Big(\sum_{y=0}^{\infty} y(1-p)^{y} + \underbrace{\sum_{y=0}^{\infty}(1-p)^{y}}_{\frac{1}{1-(1-p)} = \frac{1}{p}}\Big)$$

recall $\sum_{i=0}^{\infty} q^{i} = \frac{1}{1-q}.$

- Expected Value = Balance point.

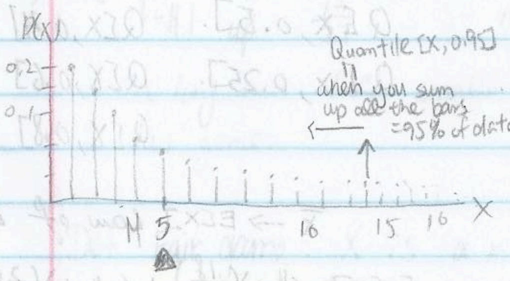
$$= p\Big(\sum_{y=0}^{\infty} y(1-p)^{y} + \frac{1}{p}\Big)$$

$$= \sum_{y=0}^{\infty} y(1-p)^{y}p + 1$$

$$= (1-p)\sum_{y=0}^{\infty} y(1-p)^{y-1}p + 1$$

ignore the zero

$$= (1-p)\underbrace{\sum_{y=1}^{\infty} y(1-p)^{y-1}p}_{\text{Expected value} = M} + 1$$




$$\Rightarrow M = (1-p)Mp + 1 \Rightarrow \cancel{M} = \cancel{M} - pM + 1 \Rightarrow \boxed{M = \frac{1}{p}.}$$

- $E[x]$ is a function

$$G[f] = \int_{\mathbb{R}} f(x)\,dx = 17.$$

**Questions**

→ $E[X] = 5$

→ Mode $[x] = 1$.

→ 95% quantile ⇒ 95% of a supp min 17 that

Quantile $[X, 0.95] = 14$.

→

Function of a function should come abt.

- Mode $[X] = \text{argmax}\{p(x)\}$

→ most likely value = mode.

- Quantile $[X, p] = \underset{x \in Supp[x]}{\text{argmin}} \{F(x) \geq p\}$

"percentile" if measured as a %.

Mean ≠ Median
- both measure the central balance point. (central tendency)

☆ Quantile $[X, 0.5]$ = Median $[X]$

meaning 50% below this data
50% below that

Expectation to measure things

• classifying r.v. = "Distri. Type / r.v type"

| If | |
|---|---|
| $E[X] = Median[X]$ | "symmetric" |
| $E[X] > Median[X]$ | skew right |
| $E[X] < Median[X]$ | skew left. |
| If one mode $E[X] = Median[X] = Mode[X]$ | unimodel<br>symmetric unimodel |

Tertiles
$Q[X, 0.33]$
$Q[X, 0.66]$

• Interquantile range.
$IQR[X] = Q[X, 0.25]$

| Quantiles | Quintiles | Deciles |
|---|---|---|
| $Q[X, 0.25]$ | $Q[X, 0.2]$ | $Q[X, 0.1]$ |
| $Q[X, 0.5]$ | $Q[X, 0.4]$ | $Q[X, 0.2]$ |
| $Q[X, 0.25]$ | $Q[X, 0.6]$ | $\vdots$ |
| | $Q[X, 0.8]$ | $Q[X, 0.9]$ |

• Scenario: Roulette in America

$\bar{X} \to E[X]$ Law of Large #'s.

Why we care abt Expected values?

Do this on exam

① give situation - Bet on Black pays 1:1

$E[X] = (\$1)(\frac{18}{38}) + (-\$1)(\frac{20}{38}) = -\$0.053$

② Built the Model.

$X \sim \begin{cases} \$1 & wp\ \frac{18}{38} \\ -\$1 & wp\ \frac{20}{38} \end{cases}$

Pro: → On avg, if you play many times, you will lose per play.
Overall on avg, $n \Rightarrow -\$0.053$ (on the long run).

③ calculate the $E[X]$

$X_1, \ldots, X_n \overset{iid}{\sim} \begin{cases} \$1 & wp\ \frac{18}{38} \\ -\$1 & wp\ \frac{20}{38} \end{cases} \to \lim_{n \to \infty} T = -\infty$

= If you keep playing, you can't tell how much you lose.
= the more you play, the \$ you lose = $-\infty$.
= in short term, if lucky win few times but ↓.
Play $\infty$, No limit of how much you lose

• Bet on Lucky #7.
Payout = 35:1

$E[X] = \$35 \cdot \frac{1}{38} + -\$1 \cdot \frac{37}{38} = -\$0.053\ldots$

$X \sim \begin{cases} \$35 & wp\ \frac{1}{38} \\ -\$1 & wp\ \frac{37}{38} \end{cases}$

• Bet on Dozen 1-12

Payout 2:1

$$X \sim \begin{cases} \$2 & \text{w.p. } \frac{12}{38} \\ -\$1 & \text{w.p. } \frac{26}{38} \end{cases}$$

$E[X] = 2 \cdot \frac{12}{38} + (-1) \cdot \frac{26}{38} = -\$0.053$

- Could win in the short term.
- In long term, $-\infty$. you can lose...

- No system, no magic → you will lose 5 cents in a long run.
- On the avg, you will lose the same amount for black / # 7 / Dozen...
- Every betting, they make 5 cents

• Roulette in Europe.

Bet on Black pays 1:1

$$X_1, \ldots, X_n \text{ iid } \begin{cases} \$1 & \text{w.p. } \frac{18}{37} \\ -\$1 & \text{w.p. } \frac{19}{37} \end{cases}$$

$E[X] = -\$0.027$. → "much fairer"

$\lim_{n \to \infty} T = -\infty$

in long run lose smaller amount of $

fair game.

• Def : Fair game : X is a r.v modelling payout.

$E[X] = 0$.

time

7mins • Van Weck

12mins Streets (traffic)

$p(\text{traffic}) = 0.3$

$$W \sim \begin{cases} 7 \text{ min} & \text{w.p. } 0.7 \\ 12 \text{ min} & \text{w.p. } 0.3 \end{cases}$$

over avg trips, my avg time in the taxi is ≈ 8.5 min.

$E[W] = 7 \cdot 0.7 + 12 \cdot 0.3$

$= \cancel{7.8}\ 8.5$ min

- On avg, 8.5 min approximately!
- Need infinite trials to get exact value.

• Uber charges $.40/min

What is my expected bill for time?

r.v.

$$B = \$0.4/\text{min} \cdot W \sim \begin{cases} \$2.80 & \text{w.p. } 0.7 \\ \$4.80 & \text{w.p. } 0.3 \end{cases}$$

depends on W