December 8, 2016

* Review:

Let $p := P(\text{Male})$

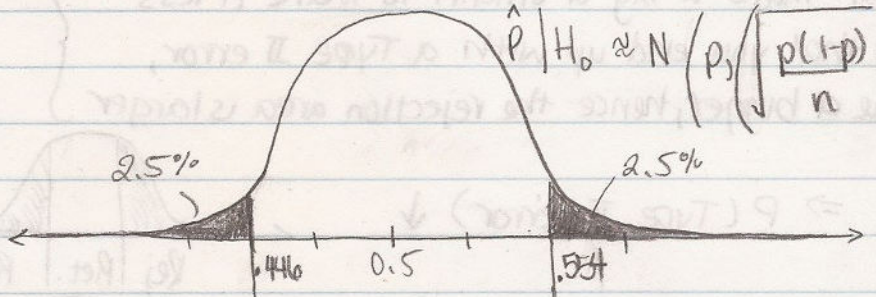
H_0: $p = 0.5$

H_a: $p \neq 0.5$

$\alpha = 5\%$ ← we pick this.

$n = 345$

"I believe I'll defy extremes 5% of the time."



$\hat{p}$ - This did not come from big $\hat{p}$.

↑

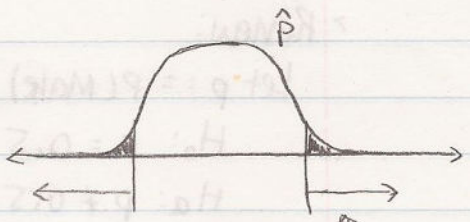
"Two sided test of one proportion"

= Two ways to reject a theory.

165 births male

$\hat{p} = \frac{165}{345} = 0.48$ ∈ Retainment Region ⇒ Retain H_0

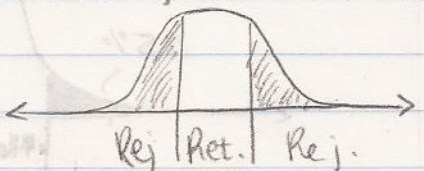
Truth	Decision: Retain H_0	Decision: Reject H_0
H_0 True	✓	Type I Error
H_0 False	Type II Error	✓



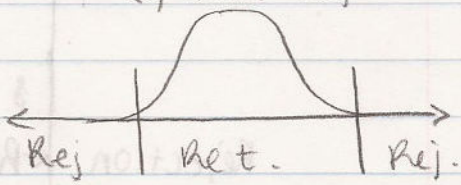
$P(\text{Type I Error}) = \alpha$

You can make α big or small. To make it less likely that you end up with a Type II error, make α bigger, hence the rejection area is larger

$\alpha \uparrow \Rightarrow P(\text{Type II error}) \downarrow$



$\alpha \downarrow \Rightarrow P(\text{Type II error}) \uparrow$



> What if we do the human gender experiment again with more data?

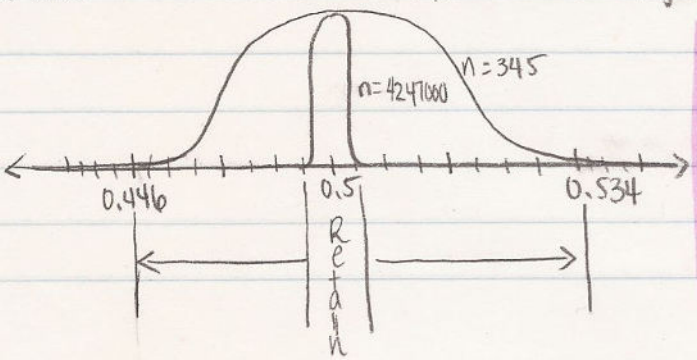
$n = 4{,}247{,}000$
= every baby born in America in 2008.

$\alpha = 0.5$ (same).

Retainment Region $= \left[0.5 \pm 2\sqrt{\frac{0.5(1-0.5)}{4{,}247{,}000}}\right] = [0.499, 0.500]$

Run experiment: 2,173,000 male

$\hat{p} = \frac{2173000}{4247000} = 0.51165$ ∉ Retainment Region. We reject H_0.



* $n \uparrow \Rightarrow \alpha$ does not change.

so

$n \uparrow \Rightarrow P(\text{Type II error}) \downarrow$

$H_0: p = 0.5$
$H_a: p \neq 0.5$
$\alpha = 5\%$
$n = 345$
$\hat{p} = 0.48$
$\Rightarrow$ Retain H_0

$H_0: p = 0.50001$
$H_a: p \neq 0.50001$
$\alpha = 5\%$
$n = 345$
$\hat{p} = 0.48$
$\Rightarrow$ Retain H_0.
This is an acceptable theory because it is approximately this.

> Aliens.

H_0: UFO's and aliens don't exist
H_a: UFO's and aliens do exist
α low
⇓
Skeptical. Will not allow any arguments to sway his opinion.

H_0: UFO's and aliens don't exist
H_a: UFO's and aliens do exist
α High
⇓
Gullible. Will allow any argument to sway his opinion.

H_0: UFO's and aliens do exist
H_a: UFO's and aliens don't exist
α low
⇓
You won't convince this guy that aliens don't exist.

H_0: UFO's and aliens do exist
H_a: UFO's and aliens don't exist
α High
⇓
You can easily convince him that aliens don't exist.

> Business Case Study

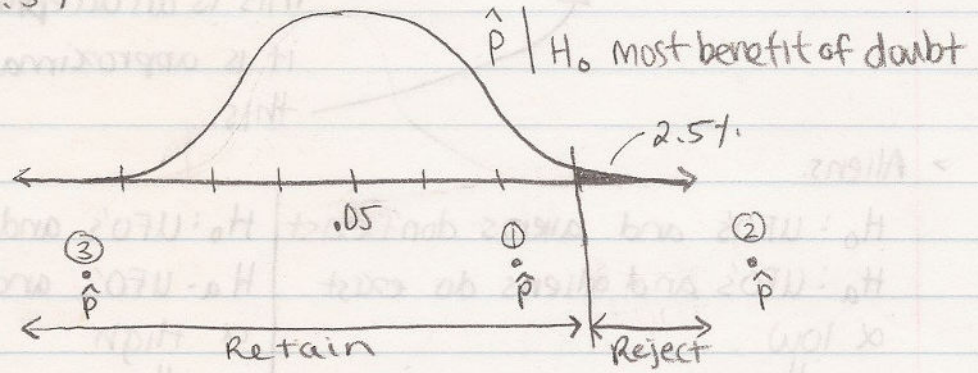
Uber fires drivers if more than 5% of passengers complain. Decision is made after 1000 rides.

p: P(complaint)

H_0: Good driver = $p \leq 5\%$

H_a: Bad driver = $p > 5\%$

$\alpha = 2.5\%$



"One-sided Test of One proportion" or "Right-Tailed"

① and ③ are retained because we do not care if complaints are < 5%.

② is rejected because it would mean way too many complaints.

$P(Z > a) = 1 - 2.5\%$

If two-sided	Z	If one-sided	Z
5%	2	2.5%	2
1%	2.84	0.5%	2.84

$\alpha = P(\text{reject})$

$1-\alpha = P(\text{retain})$

$= P(Z < Z_\alpha)$

$= P\left(\sqrt{\frac{p(1-p)}{n}} < Z_\alpha \sqrt{\frac{p(1-p)}{n}}\right)$

$= P\left(p + \sqrt{\frac{p(1-p)}{n}} < p + Z_\alpha \sqrt{\frac{p(1-p)}{n}}\right)$

$1-\alpha = P\left(\hat{p} < p + Z_\alpha \sqrt{\frac{p(1-p)}{n}}\right)$

> Experiment

71 complaints.

$\hat{p} = \frac{71}{1000} = .071$

Retainment Region $= \left(-\infty, 0.05 + 2\sqrt{\frac{.05(1-.05)}{1000}}\right] = (-\infty, .0638]$

$\Rightarrow .071 \notin$ Retainment region $\Rightarrow$ Reject H_0. Fire driver.

"There is sufficient evidence to reject that Bob is a good driver."

Uber claims they fire at 0.5%, but why isn't Bob fired at 5.1%? Why is he fired at 6.38%? Because at 5.1%, 5.7%, etc, Bob could just have bad luck. At 6.38%, Bob isn't just unlucky, he's a bad driver.

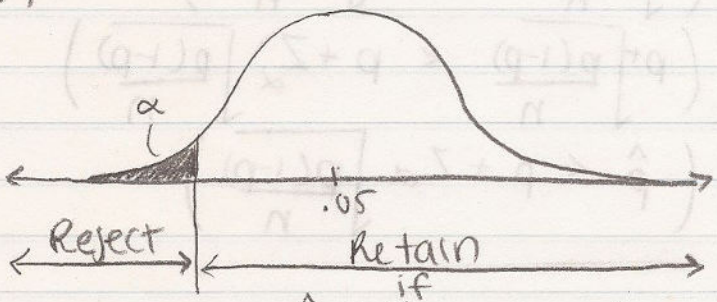
> What if you are firing everybody except the good drivers?

H_0: Bad driver $\quad p \geq 0.5$

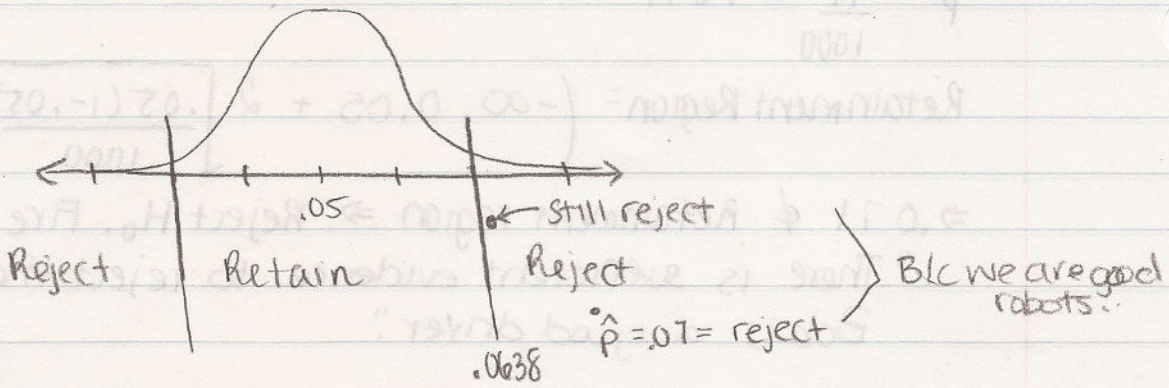
H_a: Good driver $\quad p < 0.5$

↑ (what you need to dispute)

α: 2.5%



End of Math 241!



P value := $P(\hat{p}$ is more extreme $\mid H_0$ true)

$= P(\hat{p} > 0.071)$

$= P\left(\frac{\hat{p} - .05}{.0069} > \frac{.071 - .05}{.0069}\right)$

$= P(Z > 3.04) = 0.11\% < 2.5\%$