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$X \sim \text{Bernoulli}(p)$

$X \sim \text{Binomial}(n,p)$

$X \sim \text{Hyper}(n,K,N)$

$\lim_{N\to\infty} \text{Hyper}(n,p,N) = \text{Binom}(n,p)$

$$\sum_{x \in \text{Supp}[X]} p(x) = 1 \qquad \sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x} = 1$$

Recall $(a+b)^n = \sum$ let $a = p$, $b = 1-p$, $i = x$

$$(p + (1-p))^n = \sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x}$$

X_1 and X_2 are independent $X_1, X_2 \overset{ind}{\sim}$

$P(X_1 = x_1 \mid X_2 = x_2) = P(X_1 = x_1) \quad \forall x \in \text{Supp}[X_1]$

-or- $P(X_2 = x_2 \mid X_1 = x_2) = P(X_2 = x_2) \quad \forall x \in \text{Supp}[X_2]$

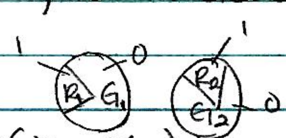
-or- $P(X_1 = x_1, X_2 = x_2) = P(X_1 = x_1) P(X_2 = x_2)$

↳ joint mass function or JMF

<u>Def</u> <u>Independent and identically distributed</u> means (iid)

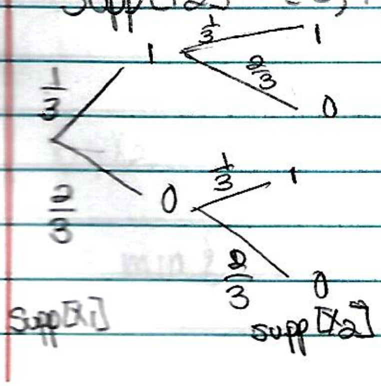
$X_1 \overset{d}{=} X_2$ and $X_1, X_2 \overset{ind}{\sim}$ denotes $X_1, X_2 \overset{iid}{\sim}$

$X_1, X_2 \overset{iid}{\sim} \text{Bern}(\frac{1}{3})$



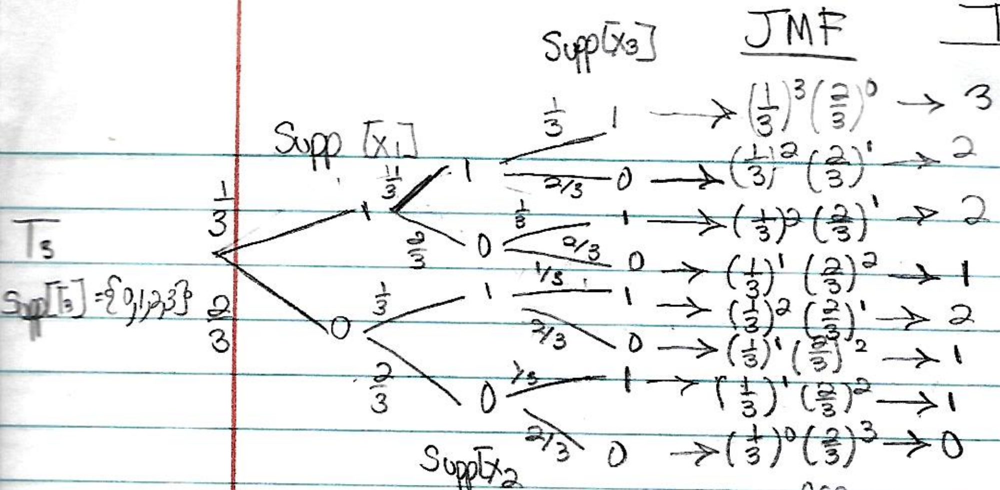
let $T_2 = X_1 + X_2 = g(X_1, X_2)$

$\text{Supp}[T_2] = \{0, 1, 2\}$



	T
$P(X_1=1, X_2=1) = \frac{1}{9}$	2
$P(X_1=1, X_2=0) = \frac{2}{9}$	1
$P(X_1=0, X_2=1) = \frac{2}{9}$	1
$P(X_1=0, X_2=0) = \frac{4}{9}$	0
1	

$$T_2 \sim \begin{cases} 0 \text{ wp } \frac{4}{9} \\ 1 \text{ wp } \frac{4}{9} \\ 2 \text{ wp } \frac{1}{9} \end{cases}$$



Third Row to pascals Triangle

$$T_3 \sim \begin{cases} 0 & \text{wp} \quad (1/3)^0(2/3)^3 \\ 1 & \text{wp} \quad (1/3)^1(2/3)^2 \\ 2 & \text{wp} \quad (1/3)^2(2/3)^1 \\ 3 & \text{wp} \quad (1/3)^3(2/3)^0 \end{cases}$$

$1 = \binom{3}{0}$

$3 = \binom{3}{1}$

$3 = \binom{3}{2}$

$1 = \binom{3}{3}$

$T=1$: 1 0 0 / 0 1 0 / 0 0 1 $\Big\}\binom{3}{1}$

$T=0$: 0 0 0 $\}\binom{3}{0}$

$T=3$: 1 1 1 $\}\binom{3}{3}$

$T=2$: 1 1 0 / 1 0 1 / 0 1 1 $\Big\}\binom{3}{2}$

$$T_3 \sim \binom{3}{x}\left(\frac{1}{3}\right)^x\left(\frac{2}{3}\right)^{3-x} = \text{Binom}\left(3, \frac{1}{3}\right)$$

$X_1, X_2 \ldots, X_n \overset{iid}{\sim} \text{Bern}(\frac{1}{3})$

$T_n = \sum_{i=1}^{n} X_i$

$$T_n \sim \begin{cases} 0 & \text{wp} \quad \binom{n}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^n \\ 1 & \text{wp} \quad \binom{n}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^{n-1} \\ 2 & \text{wp} \quad \binom{n}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^{n-2} \\ \vdots & \vdots \\ n-2 & \text{wp} \quad \binom{n}{n-2}\left(\frac{1}{3}\right)^{n-2}\left(\frac{2}{3}\right)^2 \\ n-1 & \quad \binom{n}{n-1}\left(\frac{1}{3}\right)^{n-1}\left(\frac{2}{3}\right)^1 \\ n & \quad \binom{n}{n}\left(\frac{1}{3}\right)^{n}\left(\frac{2}{3}\right)^0 \end{cases} \qquad \text{Binom}(n, \tfrac{1}{3})$$

$X_1, X_2 \ldots X_n \overset{iid}{\sim} \text{Bern}(p)$

$$T \sim \begin{cases} 0 & \text{wp} \quad \binom{n}{0} \\ 1 & \text{wp} \quad \binom{n}{1} \\ 2 & \text{wp} \quad \binom{n}{2} \; p^2(1-p)^{n-2} \\ \vdots & \vdots \\ n-2 & \text{wp} \quad \binom{n}{n-2} \\ n-1 & \text{wp} \quad \binom{n}{n-1} \\ n & \text{wp} \quad \binom{n}{n} \end{cases} \qquad \text{Binom}(n, p)$$

Binomial (n, p) is $\lim_{N \to \infty}$ Hyper (n, p, N)

- or - $X_1, X_2, \ldots X_n \overset{iid}{\sim} \text{Bern}(p)$

$T = X_1 + X_2 \ldots + X_n$

*$F(x) = P(X \leq x)$

$$\sum_{i=0}^{x} \binom{n}{i} p^i (1-p)^{n-i} = \underbrace{I}_{\text{regularized incomplete gamma Function}}{}_{1-p}(n-k, 1+k)$$

$$\parallel \quad (n-k)\binom{n}{k} \int_0^{1-p} t^{n-k-1} (1-t)^k \, dt$$

$X_1, X_2 \ldots \overset{iid}{\sim} \text{Bern}(p) \rightarrow$ infinite sequence of iid Bern(p)'s

$T := \min\{t : X_t = 1\} \rightarrow$ First success AKA "stopping time"

$p(1) = p(T=1) = p$ $\quad \underline{1}$

$p(2) = p(T=2) = P(X_1=0, X_2=1) = P(X_1=0)P(X_2=1) = (1-p)(p)$ $\quad \underline{0} \; \underline{1}$

$p(3) = p(T=3) = (1-p)^2 p$ $\quad \underline{0} \; \underline{0} \; \underline{1}$

$p(T=t) = (1-p)^{t-1} p$

$X \sim \text{Geometric}(p) := (1-p)^{x-1} p$

parameter space $\rightarrow p \in (0,1)$

$\text{supp}[X] = \mathbb{N}$

$$\sum_{x \in \text{Supp}[X]} p(x) = 1 \qquad \sum_{x=1}^{\infty} (1-p)^{x-1} \overset{?}{=} \frac{1}{p}$$

$$q = 1-p \quad \sum_{x=1}^{\infty} q^{x-1} = \frac{1}{1-q} \xrightarrow{\text{re-index}} \sum_{x=0}^{\infty} q^x = \frac{1}{1-q} =: S$$

$S = q^0 + q^1 + q^2 + q^3 + \ldots$

$S = 1 + q^1 + q^2 + q^3 + \ldots$

$S = 1 + q(1 + q + q^2 + \ldots)$

$S = 1 + qS$

$S - qS = 1$

$(1-q)S = 1$

$S = \frac{1}{1-q}$

$$F(x) = p(X \le x) = \sum_{i=1}^{x} (1-p)^{i-1} p$$

compliment $\rightarrow 1 - p(X > x) = 1 - (1-p)^x$

$$p(X > x) = P(X = x+1) + P(X = x_2) + \ldots = \sum_{i=x+1}^{\infty} (1-p)^{i-1} p$$