^ Any set is a set of itself, but not a proper subset. ($X \subseteq X$ true and $X \subset X$ is F for any set X).

Empty set is also subset of any given set X. It is also always a proper subset of any set except itself.

Lecture 1 - August 25, 2016

• SET THEORY: by 20^{th} century, ALL OF MATH CONSTRUCTED FROM SET THEORY.

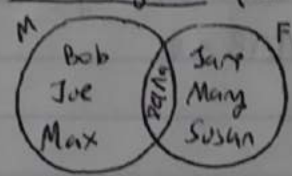
$\underset{\text{Assignment}}{F :=} \{$ Jane, Mary, Susan, Dana $\}$ → There is no order

^begin; Objects/Elements; ^end

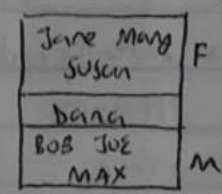
Set name F for female Names

M := { Bob, Joe, Max, Dana }
↳ Male named

Venn Diagram (for M & F)



or



$\mathbb{N} := \{1, 2, 3, \ldots\}$ (ellipses)
↑ set of natural (real) #s.

$\mathbb{Z} := \{\ldots, -1, 0, 1, \ldots\}$
↑ symbol for integers

$\underbrace{\text{JANE} \in F}_{\text{"set inclusion"}}$ → Reads as "Jane is in the set F"

(element, set)

• SUBSETS^ → A set w/ n elements has 2^n subsets.

{Jane, Mary} $\subseteq$ F (subset)

- All elements in L.H.S $\in$ R.H.S
(left hand side, right hand size)

D = {1, 4}
D is not even subset of A since 4 is not in A.

{Jane, Mary} $\subset$ F
proper subset

- All elements in l.h.s ≠ r.h.s

C = {1, 3, 5} then ← A = {1, 3, 5} B = {1, 5}
$C \subseteq A$ b/c $C = A$. the $B \subset A$

Proper subset of set A is a subset of A not equal to A. In other words, if B is a proper subset of A ($B \subset A$), then all elements of B are in A but A contains at least one element not in B.

EXAMPLES

{Jane} $\subseteq$ F → True

{JANE} $\in$ F → True
Singleton set

Jane $\in$ F → True

Jane $\subseteq$ F → False because its not a set, so makes no sense.
(subset)

• UNION ($\cup$)

F $\cup$ M = COMBINE → $\underbrace{\{\{\text{Jane, Mary, Susan, Dana}\}, \{\text{Bob, Joe, Max, Dana}\}\}}_{\text{"Flattens" or "collapses"}}$

(set, set, sets combined)

English "and/or"

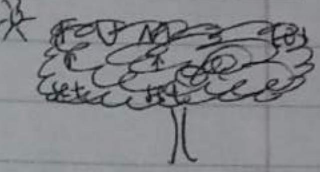
↳ {Jane, Mary, Susan, Dana, Bob, Joe, Max}

↳ • The second Dana in the 'M' set is removed.

Pg 2

- Dana $\in F \Rightarrow$ Dana $\in F \cup M$
- To add 0 into natural number set, you do: $\mathbb{N} \cup \{0\}$ must include brackets because you can only UNION 2 sets.

- <u>Intersection ($\cap$)</u>



$F \cap M$ set of all elements $\in$ L.H.S & $\in$ R.H.S

$F \cap M = \{Dana\}$

$F \cap \{Bob, Max\} = \{\}$ ← empty/null set.

- A, B are <u>mutually exclusive</u> if $A \cap B = \phi$
- <u>EXAMPLE PROBLEMS</u>

  $\phi \subseteq F \rightarrow$ True

  $\phi \subset F \rightarrow$ True

  $\phi \in F \rightarrow$ False

- <u>SET DIFFERENCE/SUBTRACTION</u>

set of all elements of L.H.S <u>save</u> elements of R.H.S

-Elements in both sides get removed

- $F \setminus M = \{Jane, Mary, Susan\}$

  (Dana got removed)

- $A \cap B = \phi$ so $A \setminus B = A$

  $B \setminus A = B$

- $A \subseteq B \Rightarrow A \setminus B = \phi$

  ↳ all elements in A has to be in so if you do difference, wipe out everything.

- $\{2n : n \in \mathbb{Z}\}$ — : "such that"

  ↳ All elements of $2 \times n$, such that n is an element of integers.

  $E := \{\ldots -2, 0, 2 \ldots\}$ ($n=-1$, $n=0$, $n=1$)

  ↳ even #s

- $A = \{1, 2, 3\}$

  $\{B : B \subseteq A\}$

  ↳ All elements of B, such that B is a subset of A.

POWER SET $\rightarrow 2^A := \{\{\phi\}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, A\}$ (1, 2, 3, 4, 5, 6, 7, 8)

• **SET SIZE/CARDINALITY** →

$A = \{1, 2, 3\}$

$|A| = 3$

Absolute value symbol

All this means is # of elements in set

"PIPE"

EXAMPLES:

$|F \cup M| \neq |F| + |M|$ } Not same thing. Not =.

$7 \neq \underbrace{4 + 4}_{8}$

$|2^A| = 8$ $|2^{F \cup M}| = 128$

$|F \setminus M| = 3$

gets rid of ben, so 3

• **Special Set "Ω" (capital omega)**

"universe" "sample space" "space of discourse"

- set of all elements under consideration.
- COIN FLIP:

  $\Omega = \{H, T\} = \{T, H\}$
- DICE ROLL:

  $\Omega = \{1, 2, 3, 4, 5, 6\}$
- EXAMPLE

  $\Omega = F \cup M$, what is the probability a random # is female, assuming Ω.

  ↑ a name chosen among elements where all are "equally likely"

  • 4/7...

  • 3/7... ? $\frac{|F|}{|\Omega|}$ ← (female names / All elements under consideration.)

  • etc...

Working Definition

$P(A) = \frac{|A|}{|\Omega|}$

↑ probability of A