Pg 14

HW read 1.7, 2.1-2.3

p 77/68 p160/1,5,9

p 107/30,34,37,39,53,55,57,65

p 151/1,5,23

Test #1
on 1.3-1.5, 1.7
2.1-2.4
3.2
~~Thursday, Sept 22~~
Tuesday, Sept 27

lecture 4 - September 6, 2016

• 5 Flowers
3 O's
2 X's

→ ways to arrange 5 distinct flowers: $5!$

If don't care about orchids → $\frac{5!}{3!}$

If don't care about chrysanthemums, but care about orchids → $\frac{5!}{2!}$
(chrysanthemums: indistinct; orchids: distinct)

Both Os & Xs indistinct: $\underbrace{}_{\text{No care about these}}$ $\frac{5!}{\underbrace{3!\cdot 2!}_{12}} = \frac{120}{12} = \boxed{10 \text{ ways to put 5 flowers in 5 pots where Os \& Xs indistinguishable}}$

10 ways

OOOXX	XOXOO
OOXOX	XXOOO
OXOOX	OXXOO
XOOOX	OOXXO
XOOXO	OXOXO

$O X_1 X_2 O O$
$O X_2 X_1 O O$
distinct

$\frac{5!}{12} \cdot 12 = 5!$ ← indistinct (order no matter)

↑ distinct (order matters)

• 10 coin flips

$H_1 H_2 H_3 H_4 H_5 \; T_1 T_2 T_3 T_4 T_5$

$$P(5H, 5T) = \frac{\frac{10!}{5!\cdot 5!}}{2^{10}} = .246093$$

$$P(A) = \frac{|A|}{|\Omega|}$$

• 1000 coin flip

$$P(600H, 400T) = \boxed{\frac{\frac{1000!}{600!\,400!}}{2^{1000}} \in [0, 1]}$$

$$P := P(600H, 400T) = \frac{\frac{1000!}{600!\,400!}}{2^{1000}} \in [0,1]$$

$$\ln(p) = \ln(1000!) - \ln(600!) - \ln(400!) - 1000\ln(2)$$

★Note $n! = \prod_{i=1}^{n} i$
→ $\ln(n!) = \sum_{i=1}^{n} \ln(i)$

★★ Sterling's Approx.: $n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$

Sterling's Approximation

$n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n \Rightarrow \boxed{\ln(n!) \approx \frac{1}{2}\ln(2\pi) + (n+\frac{1}{2})\ln(n) - n}$

$\ln(p) = \ln(1000!) - \ln(600!) - \ln(400!) - 1000\ln(2)$
$= (\cancel{\frac{1}{2}\ln(2\pi)} + 1000.5\ln(1000) - \cancel{1000}) - (\cancel{\frac{1}{2}\ln(2\pi)} + 600.5\ln(600) - \cancel{600}) - (\frac{1}{2}\ln(2\pi) +$
$400.5\ln(400) - \cancel{400}) - 1000\ln(2) \approx -23.79 \rightarrow p = e$

$\boxed{4.6 \times 10^{-11}}$

$p := P(600H, 400T) = \dfrac{\frac{1000!}{600!\,400!}}{2^{1000}} \in [0,1]$

* $\dfrac{n!}{k!(n-k)!}$ s.t. $n \in \mathbb{N}$, $k \in \mathbb{N}$, $k \le n$

• 6 people

$\{J, B, R, S, B, A\}$

→ 4 chairs, how many orders? $\Rightarrow {}_6P_4 = \dfrac{6!}{(6-4)!}$

→ How many orders such that order of the 4 doesn't matter? $\dfrac{{}_6P_4}{4!} = \dfrac{6!}{4!(6-4)!} = \dfrac{\cancel{6}^{3}\cdot 5\cdot \cancel{4!}}{\cancel{4!}\cdot \cancel{2!}} = 15$

$\binom{6}{4} := {}_6C_4$

"choose notation"

6 "choose" 4

* n objects
* k samples
* order doesn't matter

$\binom{n}{k} := \dfrac{n!}{k!(n-k)!}$

$\{J, B, S, R\}$

JBSM
JBSA
JBRM
JBRA
JBMA
JSRA
JSRM
JSMA
JRMA
BSRM
BSRA
BSMA
BRMA
SRMA = 15

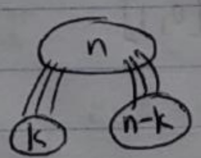
24 ways to order this.

① $\binom{n}{1} = \frac{n!}{1!(n-1)!} = n$

② $\binom{n}{n-1} = \dots = n$ → one person left out.

③ $\binom{n}{n} = \dots = 1$ → 10 people, 10 chairs, Only one way to have them sit, all of them sit (if don't care about order)

④ $\binom{n}{n-k} = \dots = \binom{n}{k}$ →



⑤ $\binom{n}{0} = \dots = 1$

↑ sit 0 people down, leave n of them standing.

• P(of sitting 4 people of 6 s.t. (such that) Jane sits) $= \frac{4({}_5P_3)}{{}_6P_4} \leftarrow \Omega$ (order matters)

$\underbrace{\frac{1}{J}\frac{5}{2}\frac{4}{3}\frac{3}{4} + \frac{\ }{1}\frac{\ }{J}\frac{\ }{3}\frac{\ }{4} + \frac{\ }{1}\frac{\ }{2}\frac{\ }{J}\frac{\ }{4} + \frac{\ }{1}\frac{\ }{2}\frac{\ }{3}\frac{\ }{J}}_{4({}_5P_3)}$

$= \frac{\binom{5}{3}}{\binom{6}{4}}$ (order doesn't matter) → $\{J, \_, \_, \_, \_\}$ ${}_5C_3$.

$\boxed{\text{Answer is } 2/3}$

• Set A s.t. (such that) $|A| = n$

$|2^A| = 2^n$

$2^A := \{B : B \subseteq A\} = \{B : B \subseteq A \;\&\; |B| = 3\} \cup \{B : B \subseteq A \;\&\; |B| = 6\} \cup \dots =$

$= \bigcup_{i=0}^{n} \{B : B \subseteq A \;\&\; |B| = i\}$ → Yes, these sets are mutually exclusive. Once |B| is 3, is 3, can't be anything else

→ Also are collectively exhaustive

$$\bigcup_{i=0}^{n} \{B : B \subseteq A \ \& \ |B| = i\} \Rightarrow |2^A| = \sum_{i=0}^{n} |\{B : B \subseteq A \ \& \ |B| = i\}|$$

Consider this...

$$2^A := \{B : B \subseteq A\} = \underbrace{\{B : B \subseteq A \ \& \ |B| = 3\}}_{\text{size of this is } {}_nC_3} \cup \underbrace{\{B : B \subseteq A \ \& \ |B| = 6\}}_{\text{size} = {}_nC_6} \cup \ldots \bigcup_{i=0}^{n} \{B : B \subseteq A \ \& \ |B| \ i\}$$

→ size of $\sum_{i=0}^{n} \underbrace{|\{B : B \subseteq A \ \& \ |B| = i\}|}_{{}_nC_i} = \boxed{\sum_{i=0}^{n} \binom{n}{i} = 2^n}$

- Example

$A = \{1, 2, 3\}$

$\{B : B \subseteq A \ \& \ |B| = 2\}$ $\{B : B \subseteq A \ \& \ |B| = 1\}$

$\{\{1,2\}, \{1,3\}, \{2,3\}\}$ $\{\{1\}, \{2\}, \{3\}\}$

→ $(1+x)^n = \sum_{i=1}^{n-1} \left(\binom{n-1}{i} + \binom{n-1}{i-1} \right) x^i \Rightarrow \binom{n}{i} = \binom{n-1}{i} + \binom{n-1}{i-1}$

pascal's Rule.

- $(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$
- $(a+b)^3 = (a+b)(a+b)(a+b) = a^3 + 3a^2b + 3ab^2 + 1b$
- $(a+b)^4 = (a+b)(a+b)(a+b)(a+b) =$

$\binom{4}{4} a^4 b^0 + \binom{4}{3} a^3 b^1 + \binom{4}{2} a^2 b^2 + \binom{4}{1} a^1 b^3 + \binom{4}{0} a^0 b^4$

$$\boxed{\star \ (a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^i b^{n-i}}$$

★ Binomial Theorem ★

★ Pascal's Δ

$$\begin{matrix} & & & \binom{0}{0} & & & \\ & & \binom{1}{0} & & \binom{1}{1} & & \\ & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} & \\ \binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3} \end{matrix} = \begin{matrix} & & & 1 & & & \\ & & 1 & & 1 & & \\ & 1 & & 2 & & 1 & \\ 1 & & 3 & & 3 & & 1 \\ & & & \vdots & & & \end{matrix}$$

- Back to 6 people, 4 chairs, June example

$\binom{6}{4} = 15$

$= \binom{5}{3} + \binom{5}{4}$

$= \binom{5}{3} + \binom{4}{3} \binom{4}{4}$

$= 10 + 4 + 1$

- Poker Game Example

ranks → $R = \{A, 2, 3, \ldots, 10, J, Q, K\}$ $D = R \times S$

suits → $S = \{♠, ♡, ♢, ♣\}$ $|D| = 52$

Next page →

Pg 18

HW Read 2.4, start 3.2

Pg 110/69
Pg 151/24, 26, 30
Pg 160/7, 19, 21, 28
171/8, 9, 55, 57

244/11, 13, 17, 23
31, 37, 41, 43, 71

Start doing review practice

Test #1
1.3-1.5
1.7, 2.1-2.4 3.2 ⇒ Tues, Sept 27

→ 5-card draw Poker

Dealt 5 cards s.t. order does not matter.

$$P(\text{win}) = \frac{|ms|}{|\Omega|} = \frac{\quad}{\binom{52}{5}}$$

Leiting