Lecture #13

Tuesday, November 1, 2016 12:14 PM

$$X \sim Geom(p) := (1-p)^{x-1} \cdot p$$

$$X \sim Geom(0.2) := (0.8)^{x-1} \cdot 0.2$$

x	p(x)	F(x)	
1	0.200	0.200	
2	0.160	0.360	
3	0.128		
4	0.102		
5	0.082		
6	0.066		
7	0.052		
8	0.042		
9	0.034		
10	0.027		
11	0.021		
12	0.017		
13	0.014		
14	0.011		
15	0.008		
16	0.007		
17	0.006		
18	0.005		
19	0.004		
20	0.003		
21	0.002		
22	0.001		
23	0.001		
24	0.001		
25	0.001		
26	0.000		.499

Approximate / Effective Support

$\{x : p(x) > .001\} \subset \mathrm{Supp}[X] := \mathbb{N}$

smaller subset A s.t $\sum_{x \in A} p(x) = 0.999$

$\bar{X} \to E[X] = M$

simple avg r.v

$X \sim Bern(p) \Rightarrow E[X] = p$

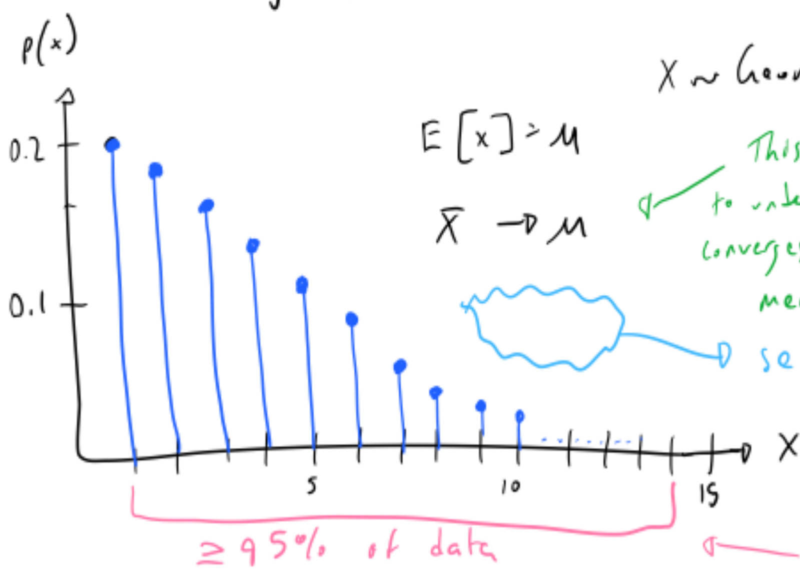
$X \sim Binom(n,p) \Rightarrow E[X] = np$

$X \sim Hyper(n, K, N) \Rightarrow E[X] = n \cdot \frac{K}{N}$ (wait for proof)

$X \sim Geom(p) \Rightarrow E[X] = \frac{1}{p}$

$X \sim NegBin(r,p) \Rightarrow E[X] = \frac{r}{p}$

$X \sim Geom(p = 0.2) = \mu = \frac{1}{0.2} = 5$



$E[X] > \mu$

$\bar{X} \to \mu$

This is important to understand how it converges and what it means.

see below

see below Quantile[x, 0.95]

$$X \sim \text{Geom}(p) := (1-p)^{x-1} p$$

$$E[X] = \sum_{x \in \text{Supp}[X]} x\, p(x)$$

doesn't converge

$$= \sum_{x=1}^{\infty} x (1-p)^{x-1} p$$

$$= p \sum_{x=1}^{\infty} x (1-p)^{x-1} = p \sum_{y=0}^{\infty} (y+1)(1-p)^{y}$$

$$= p \left(\sum_{y=0}^{\infty} y (1-p)^{y} + \sum_{y=0}^{\infty} (1-p)^{y} \right)$$

recall: $\sum_{i=0}^{\infty} a^{i} = \frac{1}{1-a}$ (geom series)

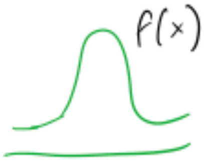
$$= p \left(\sum_{y=0}^{\infty} y (1-p)^{y} + \frac{1}{p} \right) = \sum_{y=0}^{\infty} y (1-p)^{y} p + 1$$

$$= (1-p) \sum_{y=0}^{\infty} y (1-p)^{y-1} p + 1$$

$$\mu = (1-p)\mu + 1 \Rightarrow \cancel{\mu} = \cancel{\mu} - p\mu + 1$$

$$\boxed{\mu = \frac{1}{p}}$$

$E[x]$ is a functionale



$$G[F] = \int_{\mathbb{R}} f(x)\,dx = 17$$

$$\text{Mode}[x] = \text{argmax} \{ p(x) \}$$

$$\boxed{E[x] = 5 \ , \ \text{Mode}[x] = 1}$$

$$\text{Quantile}[X, p] = \underset{x \in \text{Supp}[x]}{\text{argmin}} \{ F(x) \geq p \}$$

"percentile" if measured as a %

point where CDF is greater than p

$$\boxed{\text{Quantile}[x, 0{,}95] = 14}$$

← . . . . . . greater than 0,95

$$\text{Median}[x] := \text{Quantile}[X, 0.5]$$

If	Distribution Type / r.v type
$E[x] = \text{Median}[x]$	"Symmetric"
$E[x] > \text{Median}[x]$	skew right
$E[x] < \text{Median}[x]$	skew left
If one mode	unimodal
$E[x] = \text{Median}[x] = \text{Mode}[x]$	symmetric unimodal

Interquartile range

$$IQR[x] := Q[X, 0.75] - Q[X, 0.25]$$

Quartile

$Q[X, 0.25]$

$Q[X, 0.5]$

$Q[X, 0.75]$

Tertile (incorrect spelling)

$Q[X, 0.33]$

$Q[X, 0.66]$

Quintile

$Q[X, 0.2]$

$Q[X, 0.4$

$Q[X, 0.6]$

$Q[X, 0.8]$

Decile

$Q[X, 0.1]$

$\vdots$

$Q[X, 0.9]$

ex **Roulettes in America**

Bet on black pays 1:1

$X_1, \dots\dots X_n \overset{iid}{\sim} \begin{cases} \$1 & \text{wp } \frac{18}{38} \\ -\$1 & \text{wp } \frac{20}{38} \end{cases}$

$\bar{X} \to E[X]$ Law of Large #s

$E[X] = (\$1)\left(\frac{18}{38}\right) + (-\$1)\left(\frac{20}{38}\right)$

$= -\$0.053$ → do calculation

$\lim_{n \to \infty} T = -\infty$

EXAM Questions

situation

model

give $E[X]$

explain $\boxed{E[X]}$

ex Bet on Lucky #7

Payout 35:1

$X \sim \begin{cases} 35 & \text{wp } \frac{1}{38} \\ -\$1 & \text{wp } \frac{37}{38} \end{cases}$

$E[X] = \$35 - \left(\frac{1}{38}\right) + (-\$1)\left(\frac{36}{38}\right)$

$= -\$0.053$

ex Bet on dozen 1-12 Payout 2:1

$X \sim \begin{cases} \$2 & \text{wp } \frac{12}{38} \\ -\$1 & \text{wp } \frac{26}{38} \end{cases}$

$E[X] = (2)\left(\frac{12}{38}\right) + (-1)\left(\frac{26}{38}\right) = -\$0.053$

Def: Fair game: X is a r.v modeling payment

$E[X] = 0$

(ex) Uber Ride

Time

7 min Van Wyck

12 min Streets (traffic)

p(traffic) = 0.3

$$W \sim \begin{cases} 7 \text{ min} & \text{wp } 0.7 \\ 12 \text{ min} & \text{wp } 0.3 \end{cases}$$

$$E[W] = (7 \cdot .07) + (12 \cdot .03)$$

(ex) Uber charges \$0.4 / min

What is my expected bill for time?

$$B = \$0.40/\text{min} \cdot W \sim \begin{cases} \$2.80 & \text{wp } 0.7 \\ \$4.80 & \text{wp } 0.3 \end{cases}$$

To be continued