11/3/16

$$W \sim \begin{cases} 7 & wp\ 0.7 \\ 12 & wp\ 0.3 \end{cases}$$

$E[W] = 7 \cdot 0.7 + 12 \cdot 0.3 = 8.5 \text{min}$

$$B = \$0.40/\text{min} \cdot W \sim \begin{cases} \$2.80 & wp\ 0.7 \\ \$4.80 & wp\ 0.3 \end{cases}$$
$= g(W)$

$E[W] = 2.80 \cdot 0.7 + 4.80 \cdot 0.3 = \$3.12$

if i spend money on many cabrides, I will spend on average approximatly $3.12.

$E[B] = \$0.40/\text{min}\ E[W]$

r.v X, r.v $Y = \underbrace{aX}_{g(X)}$, $a \in \mathbb{R}$

$E(Y) = a E[X] \rightarrow E[aX] = \sum_{x \in Supp[X]} ax\,p(x) = a \sum_{x \in supp[X]} x\,p(x) = a E[X]$

$E[g(X)]$ $\qquad$ $E[aX]$

$T = B + \$3 \}\ g(B)$

$E(T) = E[B] + C$

$E[T] = \$6.12$

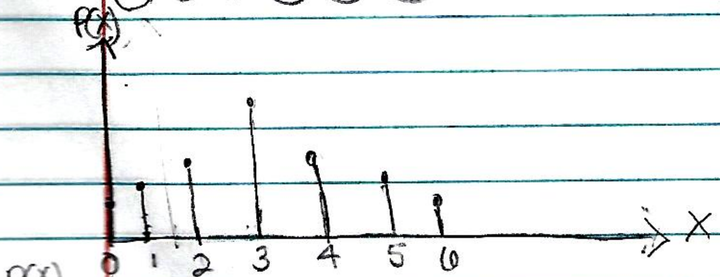
$Y = X + C$, $c \in \mathbb{R}$

$E(Y) = E[\underbrace{X + C}_{g(X)}] = \sum_{X \in supp[X]} (X + C)\,p(X)$

$\hookrightarrow \sum_{X \in supp[X]} X\,p(X) + C \sum_{X \in supp[X]} p(X)$

$E[X + C] = E[X] + C$

$E[aX + c] = a E[X] + C$

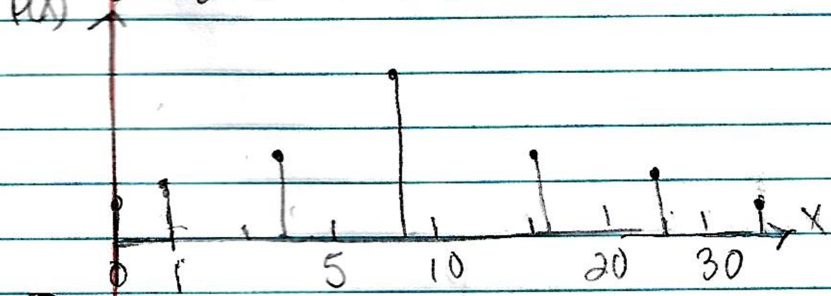


$X \sim \text{Binom}(6, \frac{1}{2})$

$Y = g(X) = X^2$

$E[X] = 3$

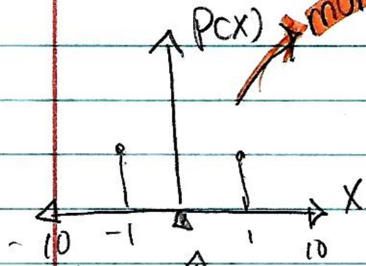
$E[X^2] \neq (E[X])^2$



$E[X^2] = \sum_{X \in supp[X]} X^2\,p(X)$

$\sum_{X=0}^{6} X^2 \binom{6}{X} \frac{1}{2^6}$

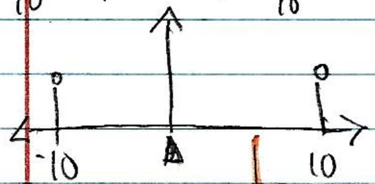
$= 17.5$



more concentrated around expected value

$X \sim \text{Radenm} := \begin{cases} 1 & \text{wp } \frac{1}{2} \\ -1 & \text{wp } \frac{1}{2} \end{cases}$

$E[X] = 0$



$Y = 10X$

$E[Y] = 10E[X] = 10 \cdot 0 = 0$

less concentrated around expected value

$e(x, \mu) = x - \mu \rightarrow$ not a loss function

$e(x, \mu) = |x - \mu| \rightarrow$ absolute loss, L1 loss

$e(x, \mu) = (x - \mu)^2 \rightarrow$ square loss, L2 loss

$$\boxed{\sigma^2 = Var[X] = L = (\underbrace{x - \mu}_{g(x)})^2 \rightarrow E[L] = E[(x-\mu)^2] = \sum_{x \in Supp[X]} (x-\mu)^2 p(x)}$$

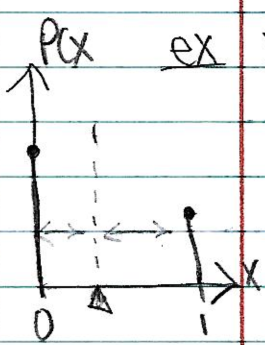
↳ variance

$Var[X] = ((-1) - (0))^2 p(-1) + ((1) - (0))^2 p(1)$

$= 1 \cdot 0.5 + 1 \cdot 0.5 = \boxed{1}$

$Var[10X] = (-10 - 0)^2 p(-10) + (10 - 0)^2 p(10)$

$= 100 \cdot 0.5 + 100 \cdot 0.5 = \boxed{100}$

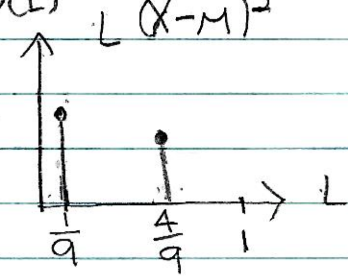


ex $X \sim Bern(\frac{1}{3})$

$E[X] = \frac{1}{3}$

$(0 - \frac{1}{3})^2 = \frac{1}{9}$

$(1 - \frac{1}{3})^2 = \frac{4}{9}$



$Var[X] = (0 - \frac{1}{3})^2 (\frac{2}{3}) + (1 - \frac{1}{3})^2 (\frac{1}{3})$

$= \frac{1}{9} \cdot \frac{2}{3} + \frac{4}{9} \cdot \frac{1}{3} = \frac{2}{9} = .259$

ex $X \sim Bern(p)$ $E[X] = p$

$Var[X] = (0 - p)^2 \cdot (1 - p) + (1 - p)^2 p$

$p^2 (1-p) + (1-p)^2 p$

$= 1 - p\,(p^2 + (1-p)\,p)$

$= (1-p)\,(\cancel{p^2} + p - \cancel{p^2}) \rightarrow \boxed{p(1-p)}$

Variance also called deviation

Bet on #7 $\quad X \sim \begin{cases} \$35 & wp\ \frac{1}{38} \\ -\$1 & wp\ \frac{37}{38} \end{cases}$ $\quad E[X_7] = -\$.053$

Bet on Black $\quad X \sim \begin{cases} \$1 & wp\ \frac{18}{38} \\ -\$1 & wp\ \frac{20}{30} \end{cases}$ $\quad E[X_B] = -\$.053$

→ more concentraded

$Var[X] = (35-(-.053))\cdot\frac{1}{38} + (-(-1)-(-.053))\cdot\frac{37}{38} = 33.207\$^2$

$Var[X] = (1-(-.053))\cdot\frac{18}{38} + (-(-1)-(-.053))\frac{20}{30} = 0.997\$^2$

$\sqrt{Var[X_7]} = \$5.79 \qquad \sqrt{Var[X_B]} = \$1.00$

$\sigma = SE[X] = \sqrt{Var[X]}$

↳ Standard error or Standard deviation.

$\bar{X}_7 = -\$0.053$

$X_B = -\$0.053$ → (faster)

$T_2 = X_1 + X_2$

$F[T_2] = f(E[X_1], E[X_2]$