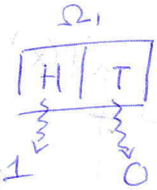
09/27/2016



$\Omega = \{H, T\}$

$n = 3$
$\omega_1 = H$
$\omega_2 = H$
$\omega_3 = T$

No average!!

$\mathbb{1}_{\omega = H} = \begin{cases} 1 & \text{if } \omega = H \\ 0 & \text{if } \omega = T \end{cases}$

Definition: A random variables ("r.v") is a function $X : \Omega \to \mathbb{R}$ $\to$ Values of r.v

Ω $\to$ sample space

$X(H) = 1$ What is $P(X = 1) := P(\{w : X(w) = 1\})$
$X(T) = 0$ $= P(\{H\}) = \frac{1}{2}$ $\Rightarrow \text{Supp}[X] = \{0, 1\}$

- The "Support" of a r.v $\text{Supp}[X]$ is the range of X, the set of all possible values $\text{Supp}[X] := \{x : P(X = x) > 0\} \subseteq \mathbb{R}$
- A "discrete" of a r.v is a r.v such that $|\text{Supp}[X]| \leq |\mathbb{N}|$ $\to$ countable infinity
- For discrete X: $\sum_{x \in \text{Supp}[X]} P(X = x) = 1$ $\quad \Omega = \{w_1, w_2, \dots\}$ such that $P(\{w\}) > 0$

$f(x) = x^2$
$f(9) = 9^2$

value of free variable

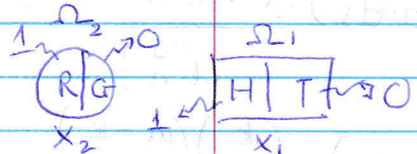
$\exists w$ such that $X(w) \notin \text{Supp}[X]$
$P(\{w\}) = 0$

$\bigcup_{x \in \text{Supp}[X]} \{w : X(w) = x\} \stackrel{?}{=} \Omega \Rightarrow$ Collectively exhaustive

$\{w : X(w) = x_1\} \cap \{w : X(w) = x_2\} \neq \emptyset \quad \exists w_0 \, X(w_0) = x_1 \;\&\; X(w_0) = x_2$

$P(X = x_1) + P(X = x_2) + \dots = 1$

$P(\{w : X(w) = x_1\}) + P(\{w : X(w) = x_2\}) + \dots = 1$



$X \sim \begin{cases} 1 & \text{wp } \frac{1}{2} \\ 0 & \text{wp } \frac{1}{2} \end{cases}$

X $\to$ r.v; $\sim$ $\to$ distribution as

$X \sim \text{Bernouilli}\left(\frac{1}{2}\right) = \begin{cases} 1 & \text{wp } \frac{1}{2} \\ 0 & \text{wp } \frac{1}{2} \end{cases}$ $\quad \text{Supp}[X] = \{0, 1\}$

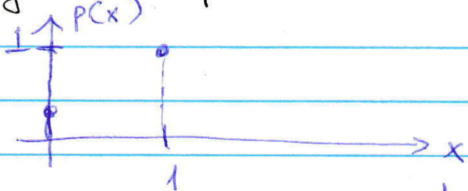
$X \sim \text{Bernouilli}(p) := \begin{cases} 1 & \text{wp } p \\ 0 & \text{wp } 1 - p \end{cases}$ $\quad \text{Supp}[X] = \{0, 1\}, \underbrace{p \in (0, 1)}_{\text{parameter space}}$

$X \sim \{0 \text{ wp } 1\}$; $X \sim \text{Deg}(c) := \{c \text{ wp } 1\}$

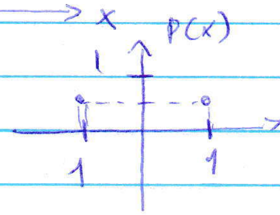
Definition: The probability mass function (PMF) $\quad P(x) := P(X = x)$

$\sum_{x \in \text{Supp}[X]} p(x) = 1$

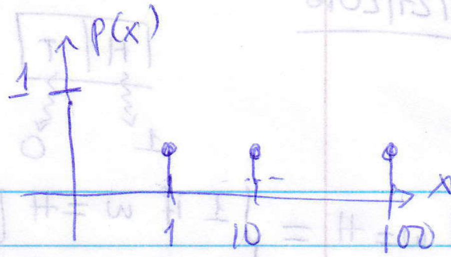
If $x \notin \text{Supp}[X]$, $p(x) = 0$



$X \sim \text{Rademacher} := \begin{cases} +1 & \text{wp } \frac{1}{2} \\ -1 & \text{wp } \frac{1}{2} \end{cases}$



$$X \sim \text{Uniform}(\{1,10,100\}) = \begin{Bmatrix} 1 & wp \ \frac{1}{3} \\ 10 & wp \ \frac{1}{3} \\ 100 & wp \ \frac{1}{3} \end{Bmatrix}$$

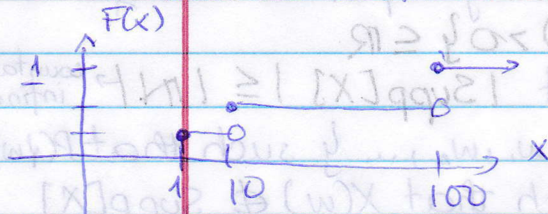


$X \sim \text{Uniform}(A)$ $\quad \text{Supp}[X] = A$, $p(x) = \frac{1}{|A|}$

Parameter Space: $A \subset \mathbb{R}$, $|A| \in \mathbb{N} \setminus \{1\}$

$P(X \leq x) + P(X \in [x, y])$ ″ Monotically increasing

The cumulative distribution function (CDF) is $F(x) := P(X \leq x)$



$F(-32) = 0$

$F(5) = \frac{1}{3}$

Properties of the CDF:

1. $F(x) \in [0, 1]$
2. $\lim_{x \to \infty} F(x) = 1$
3. $\lim_{n \to -\infty} F(x) = 0$
4. $x \leq y \Rightarrow F(x) \leq F(y)$

$$X \sim \text{Bern}(P) := \begin{Bmatrix} 1 & wp \ P \\ 0 & wp \ 1-P \end{Bmatrix} = P^x (1-P)^{1-x}$$

10 cards - 4R, 3B, $P(2R \text{ in } 3 \text{ cards}) = \dfrac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}}$

without replacement

$P(x\ R \text{ in } 3 \text{ card})$ $\quad \dfrac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$ $\quad P(x\ R \text{ in } n \text{ cards}) = \dfrac{\binom{4}{x}\binom{6}{n-x}}{\binom{10}{n}}$

10 cards, K R, $(10-k)$ B $\rightarrow P(X\ R \text{ in } n \text{ cards}) = \dfrac{\binom{k}{x}\binom{10-k}{n-x}}{\binom{10}{n}}$

N cards, K R, $(N-k)$ B $\rightarrow P(X\ R \text{ in } n \text{ cards}) = \dfrac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$