Lec 16

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Let $Y \sim$ geometric (p)

Then, $Var[Y] = E[(Y-\mu)^2] = E[Y^2] - \mu^2 = E[Y^2] - (\frac{1}{p})^2$

Note: $E[Y^2] = \sum_{y=1}^{\infty} y^2 (1-p)^{y-1} p$

$$= \sum_{z=0}^{\infty} (z+1)^2 (1-p)^z p$$

~~let $z = y-1 \rightarrow y = z+1$~~

$y = 1 \ldots \infty$ $z = 0 \ldots \infty$

$$= p \left(\sum_{z=0}^{\infty} (z+1)^2 (1-p)^z p \right.$$

$$= \underbrace{\sum_{z=0}^{\infty} z^2 (1-p)^z p}_{\infty} + \underbrace{2p \sum_{z=0}^{\infty} z (1-p)^z}_{\infty} + \underbrace{p \sum_{z=0}^{\infty} (1-p)^z}_{1}$$

$$= \underbrace{(1-p) \sum_{z=1}^{\infty} z^2 (1-p)^{z-1} p}_{E[Y^2]} + 2(1-p) \underbrace{\sum_{z=1}^{\infty} z (1-p)^{z-1}}_{E[X] = \frac{1}{p}} + 1$$

$\rightarrow$ geometric

$\Rightarrow E[Y^2] = (1-p) E[Y^2] + \frac{2(1-p)}{p} + 1$

$1 - (1-p) = p$

$\Rightarrow p E[Y^2] = \frac{2(1-p)}{p} + 1$

$\Rightarrow E[Y^2] = \frac{2(1-p)}{p^2} + \frac{1}{p}$

$\Rightarrow Var[Y] = \frac{2(1-p)}{p^2} + \frac{1}{p} - \frac{1}{p^2} = \frac{2 - 2p - 1 + p}{p^2}$. $Var[X] = \frac{1-p}{p^2}$

$\sigma^2 = \frac{1-p}{p^2}$

Let $X \sim$ Negative Binomial (r, p)

$Var[X] = \sum_{x=r}^{\infty} (x - \frac{r}{p})^2 \binom{x-1}{r-1} (1-p)^{x-r} p^r \Rightarrow Var[X] = r \frac{1-p}{p^2}$ (?)

$Var[X] = \frac{r(1-p)}{p^2}$

$E[X] = \frac{r}{p}$

Note $X_1 \ldots X_r$ iid Geo(p)

Recall: $E[X] = \frac{r}{p}$

Small: $X = X_1 + X_2 + \ldots + X_r$

Thus $Var[X] = r \cdot \frac{1-p}{p^2}$

Note: Geometric Distribution has memoryless property that is the probability of a success after n number of failures is the same as the p of success "any point of time" (in this case, at any trial)

$P(X=x)$ $P(X=a+b \mid X>a) = \frac{P(X=a+b)}{P(X>b)} = \frac{(1-p)^{a+b-1}}{(1-p)^{b}} p$

$= (1-p)^{a-1} p$