9/1

$- P(A) = \frac{|A|}{|\Omega|}$ if $\forall_w \; P(\{w\}) = \frac{1}{|\Omega|}$ $= \frac{1}{4}$?

Probability

$- \Omega = \{H, T\}$

$\Omega' = \Omega^2$

$|\Omega'| = 4$

Ω'

$\langle H,H \rangle$	$\langle H,T \rangle$
$\langle T,H \rangle$	$\langle T,T \rangle$

$- |2^{\Omega'}| = 16$

$- P(\cdot)$ = Probability of

function set ordered pair

$- P(\{\langle H,H \rangle\}) = \frac{1}{4}$.

short form

$P(H.H)$.

- A: at lease 1 Head

$P(A) = \frac{|A|}{|\Omega|} = \frac{|\{(H,H),(H,T),(T,H),(T,T)\}|}{4} = \frac{3}{4}$.

- B: At lease 1 Tail

$P(B) = \frac{|\{(H,T),(T,T),(T,H)\}|}{4} = \frac{3}{4}$.

☆ → at least 1 head & tail

$- P(A \cup B) = \frac{4}{4}$ $\rightarrow \neq P(A) + P(B)$ $\Rightarrow$ A&B are not mutally exclusive.

$- P(A \cap B) = P(A \text{ and } B) = P(A \& B) = P(A, B) = P(A\,B)$.

$= \frac{|\{\langle H,T \rangle, \langle T,H \rangle\}|}{4} = \frac{1}{2}$.

$\rightarrow \neq P(A) - P(B)$.

$\neq \frac{P(A)}{P(B)}$

- $\Omega = \{H, T\}$
  $\Omega' = \Omega^4$
  $|\Omega'| = 16$
  $|2^{\Omega'}| = 2^{16}$

  There are 2^{16} you could ask here.

$\Omega \times \Omega \times \Omega \times \Omega$

<H,H,H,H>	<H,H,H,D>		
	<T,H,H,H>		

— every outcome is the same.

← $P(HHHH) = P(HTHT)$
$= \frac{1}{16} \qquad = \frac{1}{16}$

Not equal to the prob of 2 Hs & 2 Ts.
$- \neq P(2H, 2T)$
because you could have different order.

☆ • $P(2H, 2T) = \frac{|A|}{|\Omega|} = \frac{|A|}{16} = \frac{|\{\langle H,H,T,T\rangle, \langle H,T,T,H\rangle, \langle T,T,H,H\rangle, \langle T,H,T,H\rangle, \langle H,T,H,T\rangle, \langle T,H,H,T\rangle\}|}{16}$

$= \boxed{\frac{6}{16}} = 0.375$

• $P(\text{at least one } H) = \frac{|A|}{|\Omega|} = \frac{|\{\langle H,H,H,H\rangle, \langle H,T,T,T\rangle \ldots\ldots\}|}{16}$ there's a lot

Easy way → $A: \geq 1H$
$A^c: < 1H = \underset{zero}{0}\, H = \{\langle T,T,T,T\rangle\}$.

☆complant rule $= 1 - P(\text{zero } H) = 1 - \frac{1}{16} = \boxed{\frac{15}{16}}$
(Answer)

• $\Omega' = \Omega^{10}$
$|\Omega| = 1024$
$|2^{\Omega}| = 2^{1024}$
$2^x \approx 1000^{\frac{x}{10}}$

$P(5H, 5T) = \frac{|A|}{|\Omega|} = \frac{|\{\langle H,H,H,H,H,T,T,T,T,T\rangle \ldots\ldots\ldots}{1024}$

→ we have to list out ….. to predict the possible strings.

→

Sampling without replacement. Different ways to do

made up ex: • $\Omega = \{J, m, S\}$
⊓, ⊓, ⊓ 3chairs.

Why $|\Omega'| = 6 \neq |\Omega^3|$?
It's not caterian product.

How many ways to have them seated?

1 way: $\{\langle J, m, S\rangle, \langle J, S, m\rangle, \langle m, S, J\rangle, \langle m, J, S\rangle, \langle S, J, m\rangle, \langle S, m, J\rangle\}$

3 way. Seat #1
$\frac{1}{3}$ J
$\frac{1}{3}$ m
$\frac{1}{3}$ S

Seat #2
$\frac{1}{2}$ m, $\frac{1}{2}$ S
$\frac{1}{2}$ J, $\frac{1}{2}$ S
$\frac{1}{2}$ J, $\frac{1}{2}$ m

Seat #3
1 S
1 m
1 S
1 J
1 m
1 J

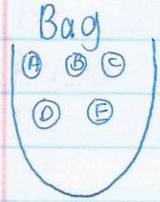
J < m — S, S — m
m < J — S, S — J
S < J — m, m — J

$\boxed{6}$ outcomes.

2 way.
How many ppl can set in

$\underset{\text{seat \#1}}{3} \quad \underset{\#2}{2} \quad \underset{\#3}{1} = \boxed{3!}$ factorial.

$n! = \prod_{i=1}^{n} i$

• 

- n people, n chairs # of ways to seat them $= n!$ > same!
- n sample without replacement # of orders $= n!$
- n sample with replacement # of orders $= n^n$ ☆

- $20! = 2.7 \times 10^{32}$
- $10! = 3.6$ m.
- $5! = 120$

• 10 people, 3 chairs.

$$\underset{\text{seat\#1}}{\underline{10}} \times \underset{\#2}{\underline{9}} \times \underset{\#3}{\underline{8}} = \frac{10!}{7!} = \underset{{}_{10}P_3}{} \frac{10!}{(10-3)!}$$

${}_{10}P_3$ ← permutation = # when "order matters".

- 10 people, 10 chairs.

$$= {}_{10}P_{10} = \frac{10!}{(10-10)!} = \frac{10!}{0!} = 10!$$

$\boxed{☆ 0! = 1}$

3 pairs of couples, 6 chairs.

- Bob - Jane, Richard - Susan, max - Alice.

$$P(\text{all 3 couples sit with each other}) = \frac{|A|}{|\Omega|} = \frac{|A|}{6!} = \frac{}{6!}$$

if Bob → must be Jane. R S M A

1 way ☆

$$\underset{\text{seat\#1}}{\underline{6}} \quad \underset{\#2}{\underline{1}} \quad \underset{\#3}{\underline{4}} \quad \underset{\#4}{\underline{1}} \quad \underset{\#5}{\underline{2}} \quad \underset{\#6}{\underline{1}} = 6 \cdot 4 \cdot 2 = 48 = |A| \to \frac{48}{6!}$$

2nd way ☆ Attention

BJ RS mA	RS mA	mA

$$\underset{1\&2}{\underline{3}} \quad \underset{3\&4}{\underline{2}} \quad \underset{5\&6}{\underline{1}} = 3! \cdot 2^3 = 6 \cdot 8 = \boxed{48}$$

each way → 2 2 2

has 2 ways to be seated!!

3rd way $\{B, J\}$

$$\frac{1}{\text{seat. 1 \& 2}} \quad \overbrace{\frac{4}{3} \quad \frac{3}{4} \quad \frac{2}{5} \quad \frac{1}{6}}^{4!} = \frac{5(4!)2}{6!}$$

$$\frac{4}{\text{seat } 1} \quad \frac{1}{2\&3} \quad \frac{3}{4} \quad \frac{2}{5} \quad \frac{1}{6}$$

$\vdots$

There are 5 ways how B, J could sit.

$\overline{1} \quad \overline{2} \quad \overline{3} \quad \overline{4} \quad \overline{5\&6}$

$\rightarrow P(A) = P(A_1 \cup A_2 \cup A_3 \cup A_4 \cup A_5)$

$= P(A_1) + P(A_2) + \dots P(A_5) = 5P(A_1)$

$\Uparrow$

$\{A_1, \dots A_5\}$ are mutalley exclusive!

- Sample 3 balls from a bag of 100 (without replacement).

$$\# \text{ of ways} = 100 \cdot 99 \cdot 98 = \frac{100!}{(100-3)!} = \frac{100!}{97!}$$

(with replacement)

$$\# \text{ of ways} = 100 \cdot 100 \cdot 100$$

$$= \frac{100P_3}{100^3} = .9702$$

$$\frac{100P_3}{100^3} = .9702$$

- 10,000 balls sample 3

$$\frac{10{,}000 \cdot 9{,}999 \cdot 9{,}998}{10{,}000 \cdot 10{,}000 \cdot 10{,}000} = .9997$$

- $$1 = \lim_{n \to \infty} \frac{nP_k}{n^k} = \lim \frac{\frac{n!}{(n-k)!}}{n^k} = \lim \frac{\overbrace{n(n-1)(n-2)\dots(n-k+1)}^{k}}{\underbrace{n \cdot n \cdot n \dots \cdot n}_{k}}$$

$$= \lim \frac{k}{n} \lim \frac{k-1}{n} \dots \lim \frac{n-k+1}{n} = 1$$

little fact

re $\lim f(x)g(x)$

$= \lim f(x) \lim g(x)$

if $f \cdot g$ contin

A B J R S M
B J R S M A.

- Bob - Jane $\sqcap \sqcap \sqcap \sqcap \sqcap \sqcap = 6!$
  Richard - Susan
  Max - Alice

$\frac{6!}{6}$ ← Principle of dividing out invarions.

6 chairs case. → Some permutations are "the same"
"indistinguishable"
"non - unique"
"non - distict"
"invarions"

- 5 flowers
  ||
  3 orchids (O)
  + 2 chrysanthemum (X)

$O_1\ O_2\ O_3\ X_1\ X_2$
all distict # order 5!

- care about the orchids (O) but not (X)

could have been:

$$\boxed{\begin{matrix} O_1 & O_2 & O_3 \\ O_1 & O_3 & O_2 \\ O_2 & O_1 & O_3 \\ O_2 & O_3 & O_1 \\ O_3 & O_1 & O_2 \\ O_3 & O_2 & O_1 \end{matrix}} \begin{matrix} X_1 & X_2 \\ X_1 & X_2 \\ X_1 & X_2 \\ X_1 & X_2 \\ X_1 & X_2 \\ X_1 & X_2 \end{matrix} = \frac{5!}{3!}$$

- care about the (X) not (O).

$$\begin{matrix} O_1\ O_2\ O_3 & X_1\ X_2 \\ & X_2\ X_1 \end{matrix} = \frac{5!}{2!}$$