Lect # 22 Prof Kapelner

Statistical Inference

① Point Estimation => best guess $= \hat{p}$

② Interval Estimation (range of value of p)

Confidence Int. $CI_{1-\alpha, p} := \left[\hat{p} \pm z_{\alpha} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right]$ // How often does p appear.

$\hat{p} \sim N(p, (\sqrt{\frac{p(1-p)}{n}})^2)$

Confidence Intervals:

①

$\frac{\# \cdot \{p \in CI\}}{n} \to 1-\alpha$

② $p(p \in CI) = 1-\alpha$ } Before experiment

③ Say $p(p \in CI) = 1-\alpha$ after the experiment

Statistical Inference (cont...)

③ Hypothesis Theory : Testing theories about a parameter.

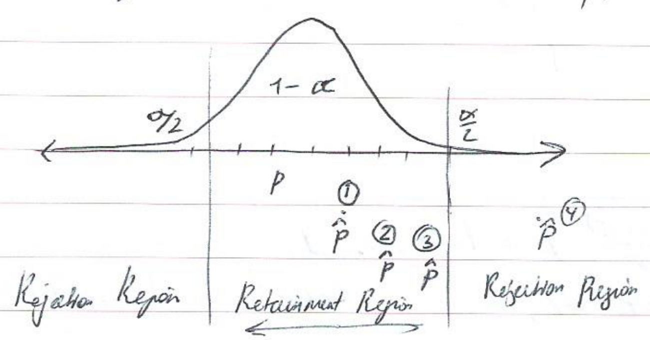
Human Gender Ratio:

$p := p(\text{male})$ is not 50% (Assumed by prof. Kap.) (?)

=> True statement.

At $p = 0.5$, we call this "null hypothesis" denoted as $H_0 := p = 0.5$

The alternate hypothesis is denoted $H_a := p \neq 0.5$.



$\hat{p} | H_0 \approx N(p, (\sqrt{\frac{p(1-p)}{n}})^2)$

① and ② are reasonable. Both are close to p.

③ is questionable.

④ Is not reasonable, rare to happen

α: p(too rare)

$1-\alpha :=$ p(not too rare)

= p(retain)

= p($\hat{p} \in$ [p ± margin])

$$1-\alpha := P\left(\hat{p} \in \left[p \pm Z_{\frac{\alpha}{2}}\sqrt{\frac{p(1-p)}{n}}\right]\right)$$ } Retainment Region.

- If $\hat{p} \in RR \Rightarrow$ Retain H_0 : Not enough evidence to reject the Null hyp.
- If $\hat{p} \notin RR \Rightarrow$ Reject H_0. Reject Null hypt $\Leftrightarrow$ enough Evidence.

Let $n = 345$ births $\alpha = 5\%$ $Z_{\frac{\alpha}{2}} = 2$ *

$$RR = \left[0.5 \pm 2\sqrt{\frac{0.5\,(1-0.5)}{345}}\right] = [.446, .554]$$

Do the exp. and get 169 males

$\hat{p} = \frac{169}{345} = 0.48 \in RR \Rightarrow$ Retain H_0

Testing an unfair coin:

Flip a coin 100x.

Fair $:\Rightarrow$ $P(H) = 0.5$

Scen. I : You get 51 H $\Rightarrow$ $\hat{p} = 0.51$, fair? Yes

Scen. II : You get 98 H $\Rightarrow$ $\hat{p} = 0.98$ Fair No!

Scen. III : You get 0.61 $\Rightarrow$ $\hat{p} = 0.61$ Fair? We cannot tell!

$n = 100$ $\alpha = 5\%$

$H_0 = p = 0.5$

$H_a = p \neq 0.5$

$$RR = \left[0.5 \pm 2\sqrt{\frac{0.5\,(1-0.5)}{100}}\right] = [0.4, 0.6]$$

$\hat{p} = 0.61 \notin RR \Rightarrow$ Reject H_0. Conclusion: Coin **IS NOT** Fair.