9/6 — $\Omega = \{B, J, R, S, M, A\}$ 6 people 6 seats

How many ways to ...

$P(\text{alt. gender}) = \dfrac{\quad}{6!}$

$\underline{3}\ \underline{2}\ \underline{1}$

$3!^2 \left\langle \begin{array}{cccccc} \underline{3} & \underline{3} & \underline{2} & \underline{2} & \underline{1} & \underline{1} \\ B & G & B & G & B & G \end{array} \right. = (3!)^2 \cdot 2$

x

$3!^2 \left\langle \begin{array}{cccccc} 3 & 3 & 2 & 2 & 1 & 1 = \\ G & B & \ldots \end{array} \right.$

— 5 flowers 5 pots 3 "O's", 2 "X's"

$\dfrac{5!}{3!}$ O's indistict

$\dfrac{5!}{2!}$ X's indistinct.

$\dfrac{5!}{3!\,2!} = \dfrac{5 \cdot \cancel{4} \cdot \cancel{3} \cdot 2}{\underbrace{\cancel{3} \cdot 2 \cdot 2}_{12}} = 10$

OOOXX
OOXOX
OXOOX
XOOXO
XOXOO
XXOOO
OXXOO
OOXXO
OXOXO

$\dfrac{5!}{12} \cdot 12 = 5!$

Caring abt the X's. $\left\langle \begin{array}{l} X_1 O O X_2 O \\ X_1 O O X_2 O \end{array} \right.$

$X_1 O_1 O_2 X_2 O_3$
$X_1 O_1 O_3 X_2 O_2$
$X_1 O_2 O_1 X_2 O_3$
$X_1 O_2 O_3 X_2 O_1$
$X_1 O_3 O_2 X_2 O_1$
$X_1 O_3 O_1 X_2 O_2$

12

6

120.

- 10 coin flips

Size: 5H's 5T's.

$H_1 H_2 H_3 H_4 H_5 T_1 T_2 T_3 T_4 T_5$

☆ Heads & Tails are all the same

$$P(5H, 5T) = \frac{|\{\ \ ,\ \dots\}|}{2^{10}}$$

$$= \frac{\frac{10!}{5!5!}}{2^{10}} = .2460938$$

$$\frac{10!}{5!5!}$$

↑ every possible ways/outcome 2^{Ω}

- 1000 coin flips

Note: it lives in $[0,1]$

$$P := P(600H, 400T) = \frac{\frac{1000!}{600!\,400!}}{2^{1000}} \in [0,1]$$

= the # will be too big (Boom!)

• Stirlys Approx. (Very) Famous

For approximation

$$\ln(n!) \approx \frac{1}{2}\ln(2\pi) - (n-\tfrac{1}{2})\ln(n) - n$$

$$n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$

Use (log) $\ln(p) = \ln(1000!) - \ln(600!) - \ln(400!) - 1000\ln(2)$.

$$\ln n! := \prod_{i=1}^{n} \ln(i!) = \sum_{i=1}^{n} \ln(i)$$

For solution → do log.

$$\ln(p) = \ln(1000!) - \ln(600!) - \ln(400!) - 1000\ln(2)$$
$$= \tfrac{1}{2}\ln(2\pi) - 1000.5\ln(1000) - 1000) -$$
$$(\tfrac{1}{2}\ln(2\pi) - 600.5\ln(600) - 600) -$$
$$\tfrac{1}{2}\ln(2\pi - 400.5\ln(400) - 400) - 1000\ln(2)$$

$$= 23.79 \Rightarrow p =$$

$$\frac{1000!}{600!\,400!} = \frac{n!}{k!\,(n-k)!} \qquad n \in k \quad k \le n \quad k \in \mathbb{N}_0$$

– How many ways to arrange 6 ppl in 4 spots (order no longer matters)
$\{J, B, S, R, M, A\}$

when order matters: $_6P_4 = \frac{6!}{(6-4)!} = \frac{6!}{2!}$

doesn't matter: "6 choose 4"
$\binom{6}{4}$: $_6C_4 = \frac{_6P_4}{4!} = \frac{6!}{4!\,2!} = 15$ ways

combination

15 ways.

JBSR BSRM
JBSM BSRA
JBSA BSMA
JBRM
JBRA
JBMA
JSRA

$4!\binom{6}{4} = {}_6P_4$

$\binom{n}{k} = {}_nC_k = \frac{n!}{k!(n-k)!}$

When order doesn't matter

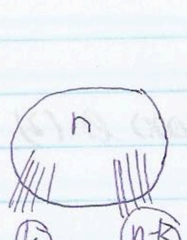
① $\binom{n}{1} = \frac{n!}{1!(n-1)!} = n$

② $\binom{n}{n-1} = \frac{n!}{(n-1)!\underbrace{(n-(n-1))}_{1!}!} = n$

③ $\binom{n}{n} = \dots\dots = 1$

④ $\binom{n}{0} = \frac{n!}{0!\,n!} \dots = 1$

⑤ $\binom{n}{n-k} = \dots\dots = \binom{n}{k}$



★ order matters.

$\{J\ B\ S\ R\ M\ A\}$

messy

seats

$-P(\text{sitting Jane in 4 seats})$ with 6 ppl. $= \dfrac{\frac{1}{J}\frac{5}{2}\frac{4}{3}\frac{3}{4} + \frac{5}{1}\frac{1}{J}\frac{4}{2}\frac{3}{4} + \ldots + \ldots}{{}_6P_4}$

2 ways to do it

$= 4\,({}_5P_3) = \dfrac{2}{3}$

★ order doesn't matter

easier or

order doesn't matter bracket $\rightarrow \{J, \_, \_, \_\}$

$= \dfrac{\binom{5}{3}}{\binom{6}{4}} = \dfrac{\frac{5\cdot 4\cdot 3!}{3!\,2!}}{15} = \dfrac{10}{15} \Rightarrow = \dfrac{2}{3}$

becomes.

- lock J in one seat. → "5 choose 3"

Cool Example (not useful)

- set A s.t $|A| = n$

$|2^A| = 2^n$

$2^A := \{B : B \subseteq A\} = \{B : B \subseteq A \;\&\; |B| = 3\} \cup \{B : B \subseteq A \;\&\; |B| = 6\} \cup \ldots$

$\bigcup_{i=0}^{n} \{B : B \subseteq A \;\&\; |B| = i\}$

mutually exclusive & collectively exhaustive

sizes

$|2^A| = \sum_{i=0}^{n} |\{B : B \subseteq A \;\&\; |B| = i\}| = \boxed{\sum_{i=0}^{n} \binom{n}{i} = 2^n}$

size of the subset.

(Helpful one)
☆Another Famous Example

$-(a+b)^2 = \underbrace{(a+b)}_{2}\underbrace{(a+b)}_{2} = a^2 + 2ab + b^2$

$-(a+b)^3 = (a+b)(a+b)(a+b) = a^3 + 3a^2b + 3ab^2 + b^3$

What's the pattern?

$-(a+b)^4 = (a+b)(a+b)(a+b)(a+b) = \binom{4}{4}a^4b^0 + \binom{4}{3}a^3b + \binom{4}{2}a^2b^2 + \binom{4}{1}ab^3 + \binom{4}{0}a^0b^4$

= The sum of this = the sum of $\sum_{i=0}^{n}\binom{n}{i} = 2^n$!

$\binom{n}{i} = \binom{n}{n-i}$

• Binomial Thm

$-(a+b)^n = \sum_{i=0}^{n}\binom{n}{i}a^i b^{n-i}$

☆Another Cool Example

(The proof is not so important) -only need to know how to use it's outcome

$-(1-x)^n = (1+x)(1+x)^{n-1}$

$\sum_{i=0}^{n}\binom{n}{i}x^i = (1-x)\sum_{i=0}^{n-1}\binom{n-1}{i}x^i$

$1 + \sum_{i=1}^{n-1}\binom{n}{i}x^i + x^n \quad = \underbrace{\sum_{i=0}^{n-1}\binom{n-1}{i}x^i + \sum_{i=0}^{n-1}\binom{n-1}{i}x^{i+1}}$ reindexing.

$\sum_{i=1}^{n}\binom{n-1}{i-1}x^i = 1 + \sum_{i=1}^{n-1}\binom{n-1}{i-1}x^i + x^n$

$= \sum_{i=1}^{n-1}\left(\binom{n-1}{i} + \binom{n-1}{i-1}\right)x^i$

• Pascal's Rule

$\Rightarrow \binom{n}{i} = \binom{n-1}{i} + \binom{n-1}{i-1}$

$-\binom{6}{4} = \binom{5}{3} + \binom{5}{4}$

Tie's everything tgt.

$= \binom{5}{3} + \binom{4}{4} + \binom{4}{3}$

bk to the question of "6 choose 4"

$= 10 + 1 + 4.$

Pascal △

$\binom{0}{0}$

$\binom{1}{0} \quad \binom{1}{1}$

$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$

$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$

||

1

1 1

1 2 1

1 3 3 1

(Helpful one)

☆Another Famous Example

- $(a+b)^2 = \underbrace{(a+b)}_{2}\underbrace{(a+b)}_{2} = a^2 + 2ab + b^2$

- $(a+b)^3 = (a+b)(a+b)(a+b) = a^3 + 3a^2b + 3ab^2 + b^3$

What's the pattern?

- $(a+b)^4 = (a+b)(a+b)(a+b)(a+b) = \binom{4}{4}a^4b^0 + \binom{4}{3}a^3b + \binom{4}{2}a^2b^2 + \binom{4}{1}ab^3 + \binom{4}{0}a^0b^4$

= The sum of this = the sum of $\sum_{i=0}^{n}\binom{n}{i} = 2^n$! $\quad \binom{n}{i} = \binom{n}{n-i}$

- **Binomial Thm**

- $(a+b)^n = \sum_{i=0}^{n}\binom{n}{i}a^i b^{n-i}$

☆Another Cool Example

(The proof is not so important) -only need to know how to use it's outcome

- $(1-x)^n = (1+x)(1+x)^{n-1}$

$\sum_{i=0}^{n}\binom{n}{i}x^i = (1-x)\sum_{i=0}^{n-1}\binom{n-1}{i}x^i$

$1 + \sum_{i=1}^{n-1}\binom{n}{i}x^i + x^n = \sum_{i=0}^{n-1}\binom{n-1}{i}x^i + \sum_{i=0}^{n-1}\binom{n-1}{i}x^{i+1}$

reindexing.

$\sum_{i=1}^{n}\binom{n-1}{i-1}x^i = X + \sum_{i=1}^{n-1}\binom{n-1}{i-1}x^i + x^n$

$= \sum_{i=1}^{n-1}\left(\binom{n-1}{i} + \binom{n-1}{i-1}\right)x^i$

- **Pascal's Rule**

$\Rightarrow \binom{n}{i} = \binom{n-1}{i} + \binom{n-1}{i-1}$

Pascal △

$\binom{0}{0}$

$\binom{1}{0} \quad \binom{1}{1}$

$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$

$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$

||

1

1 1

1 2 1

1 3 3 1

- $\binom{6}{4} = \binom{5}{3} + \binom{5}{4}$

Tie's everything tgt.

$= \binom{5}{3} + \binom{4}{4} + \binom{4}{3}$

$= 10 + 1 + 4.$

b/c to the question of "6 choose 4"

New Topic.

Decks & Cards.
(not on exams)

ranks

— $R := \{A, \cancel{1}, 2, \ldots, 10, J, Q, K\}$ $\quad \underset{\text{Size of R}}{|R|} = 13$

suits

$S := \{♠, ♡, ◇, ♣\}$ $\quad |S| = 4 \Rightarrow |D| = 52$

spade heart diamond club.

$D = R \times S = \{\langle\ , ◇\rangle, \langle K, ♡\rangle, \ldots\}$

— "5 cards draw poker"

$P(\text{win}) = \dfrac{|\text{win}|}{\binom{52}{5}}$ $\binom{52}{5} = 2{,}598{,}960.$

$P(\text{Royal Flush}) = \dfrac{\binom{4}{1}}{\binom{52}{5}}$ $\quad \binom{4 \text{ suits}}{1 \text{ suits}}$

10, J, Q, K, A