• 10 cards, k reds & 10-k blues

$$P(x \text{ reds in } n \text{ cards}) = \frac{\binom{k}{x}\binom{10-k}{n-x}}{\binom{10}{n}}$$

• N cards, k red & N-k blue

$X \sim$

$$P(x \text{ Red in } n \text{ cards}) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

~~[illegible]~~

Lecture 9 - September 29, 2016

• 10 cards, 4 R & 6 B

$$P(2R \text{ drawing } 3) = \frac{\overset{2 \text{ reds}}{\binom{4}{2}}\overset{1 \text{ blue}}{\binom{6}{1}}}{\binom{10}{3}}$$

$\Downarrow$

$$P(XR \text{ drawing } 3) = \frac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$$

$\Downarrow$

$$P(XR \text{ drawing } n) = \frac{\binom{4}{x}\binom{6}{n-x}}{\binom{10}{n}}$$

$\Downarrow$

10 cards
K R "successes"
10-K B "failures"

$$P(x \text{ Red drawing } n) = \frac{\binom{K}{x}\binom{10-K}{n-x}}{\binom{10}{n}}$$

$\Downarrow$

N cards
K R "successes"
N-K B "failures"

$$\boxed{P(x\, R \text{ drawing } n) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}} \Rightarrow$$

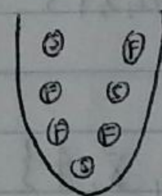
general model.

$$X \sim \text{Hypergeometric}(n, K, N) := p(x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$ ★

• (Ex)
$N = 118$
$K = 37$
$n = 23$
getting 16

$X \sim \text{Hyper}(23, 37, 118)$

$$\boxed{P(X=16) = \frac{\binom{37}{16}\binom{81}{7}}{\binom{118}{23}}}$$



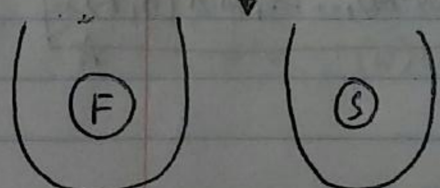
$N = 7$
$K = 3$

★ There are 3 parameters in hypergeometric model ★

• (Ex) Bag of balls

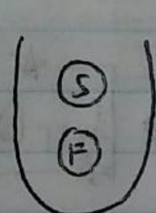
$N = 0$ $\quad n = 0$

$N = 1$ $\quad$ $\underbrace{K = 0 \text{ or } 1}_{n=1}$



$N = 2$
$K = 0$
$K = 1$
$K = 2$
$n = 1$

Random situation reached



Param Space ★ ★ ★
- $N \in \mathbb{N} \setminus \{1\}$
- $K \in \{1, 2, \ldots N-1\}$
- $n \in \{1, 2, \ldots N-1\}$

• (Ex) $X \sim \text{Hyper}(\underset{n}{\overset{\shortparallel}{1}}, K, N) = \text{Bern}\left(\frac{K}{N}\right)$

$$= \frac{\binom{K}{x}\binom{N-K}{1-x}}{\binom{N}{\underset{n}{1}}}$$

$\underbrace{\text{Supp}(X) = \{0, 1\}}$

[this thing is going to be equal to a Bernoulli because either have success or failure. Either have one or none.]

$$P(X=1) = \frac{K}{N} = p$$

$$P(X=0) = \frac{N-K}{N} = 1 - \frac{N}{K} = 1 - p$$

4 successes, 6 Failures

(successes)

(Ex) $X \sim Hyper(2,4,10)$

$Supp(X) = \{0,1,2\}$

(Ex) $X \sim Hyper(5,4,10)$ ✿

$supp[X] = \{0,1,2,3,4\}$

(Ex) $X \sim Hyper(8,4,10)$

$supp[X] = \{2,3,4\}$

(Ex) $X \sim Hyper(5,7,10)$

$Supp[X] = \{2,3,4,5\}$

These are the 4 situations that happen

$n < K$, $n < N-K$
$Supp(X) = \{0,....n\}$

$n \geq K$, $n < N-K$
$Supp[X] = \{0,...,K\}$

$n \geq K$, $n \geq N-K$
$Supp[X] = \{n-(N-K)..K\}$

$n < K$, $n \geq N-K$
$Supp[X] = \{n-(N-K)...n\}$

	$n<K$	$n \geq K$
★ $n<N-K$	$\{0,....,n\}$	$\{0,...K\}$
$n \geq N-K$	$\{n-(N-K)...n\}$	$\{n-(N-K)...K\}$

$\Rightarrow$ $\boxed{Supp(X) = \{\max(0,n-(N-K)),....,\min(n,K)\}}$

✿ $$\sum_{x \in Supp(X)} P(X) = 1$$

- n : sample size
- N : population size
- K : # successes

P : proportion of successes

$P = \frac{K}{N} \Rightarrow K = pN$

$X \sim Hypergeometric(n,p,N) :=$

$$p(x) = \frac{\binom{pN}{x}\binom{(1-p)N}{n-x}}{\binom{N}{n}}$$

- (Ex) $p=0.5$, $n=6$, $N=100$

$$P(X=3) = \frac{\binom{50}{3}\binom{50}{3}}{\binom{100}{6}} = .3223$$

$p=0.5$, $n=6$, $N=1000$

$$P(X=3) = \frac{\binom{500}{3}\binom{500}{3}}{\binom{1000}{6}} = .3134$$

$p = 0.5 \quad n = 6 \quad N = 10000$

$$P(X=3) = \frac{\binom{5000}{3}\binom{5000}{3}}{\binom{10000}{6}} = .3126$$

At some point, what matter of replacement or not, will be p.

Don't need to know for exam

$$\lim_{N \to \infty} \frac{\frac{(pN)!}{x!(pN-x)!} \cdot \frac{((1-p)N)!}{(n-x)!((1-p)N-n+x)!}}{\frac{N!}{n!(N-n)!}} = \underbrace{\frac{\frac{1}{x!(n-x)!}}{\frac{1}{n!}}}_{\frac{n!}{x!(n-x)!} = \binom{n}{x}} \lim \frac{\frac{(pN)!}{(pN-x)!} \cdot \frac{((1-p)N)!}{((1-p)N-n+x)!}}{\frac{N!}{(N-n)!}}$$

$$= \binom{n}{x} \lim_{N \to \infty} \frac{\overbrace{(pN)(pN-1) \cdots (pN-x+1)}^{x}(1-p)N((1-p)N-1) \cdots ((1-p)N-n+x+1)}{\underbrace{(N)(N-1) \cdots (N-n+1)}_{n}}$$

$$= \binom{n}{x} \overbrace{\lim \overbrace{\frac{pN}{N}}^{p} \lim \overbrace{\frac{pN-1}{N-1}}^{p} \cdots \lim \overbrace{\frac{pN-x+1}{N-x+1}}^{p}}^{p^x} \cdot \lim \overbrace{\frac{(1-p)N}{N-x}}^{1-p} \cdot \lim \overbrace{\frac{(1-p)N-1}{N-x-1}}^{1-p} \cdot$$

$$\cdots \underbrace{\lim \frac{(1-p)N-n+x+1}{N-n+1}}_{1-p}$$

$(1-p)^{n-x}$

$$X \sim \text{Binomial}(n,p) := p(x) = \binom{n}{x} p^x (1-p)^{n-x}$$

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* $X \sim \text{Binomial}(n,p) := p(x)$
$= \binom{n}{x} p^x (1-p)^{n-x}$

* $\text{Supp}[X] = \{0, 1, \dots, n\}$

* <u>Param. Space</u>
$n \in \mathbb{N}$
$p \in (0,1)$

$X \sim \text{Binomial}(n, 0)$
~~p=0 → X ~ Degenerate~~

$p = 0 \Rightarrow X \sim \text{Deg}(0)$

$p = 1 \Rightarrow X \sim \text{Deg}(n)$

$\binom{n}{x} 1^x 0^{n-x}$

$\binom{n}{x} 0^x 1^{n-x}$

$\boxed{0^0 := 1}$

$\lim_{x \to 0} x^x = 1$

* $X \sim \text{Binom}(1,p) = \binom{1}{x} p^x (1-p)^{1-x} = \underline{\text{Bern}(p)}$
$\text{supp}[X] = \{0, 1\}$ $\quad \binom{1}{0} = 1$
$\binom{1}{1} = 1$

<u>Lecture 10 – October 13, 2016</u>

Wasn't here