Lec 16 11/10/16 Math 241

Rules recap

$E[aX+c] = a\mu+c$

if ident. distr.

$X_1,\ldots,X_n$

$E[T] = \sum_{i=1}^{n} E(X_i) = n\mu \quad \Rightarrow \quad E[\bar{X}] = \mu$

$Var[aX+c] = a^2\sigma^2$

$SE[aX+c] = |a|\sigma$

$Cov[X_i, X_j]$ not covered in this class!

$X_1,\ldots,X_n$ ind.

$Var(T) = \sum_{i=1}^{n} Var(X_i) = n\sigma^2$

If ident. distr. too (iid)

$\Rightarrow Var[\bar{X}] = \frac{\sigma^2}{n} \Rightarrow SE[\bar{X}] = \frac{\sigma}{\sqrt{n}}$

---

$X \sim Binom(n,p)$

$X = X_1 + \ldots + X_n$ s.t. $X_1,\ldots,X_n \overset{iid}{\sim} Bern(p)$

$\Rightarrow Var(X) = n\sigma^2 = np(1-p)$ or $\sum_{x=0}^{n} (x-np)^2 \binom{n}{x} p^x (1-p)^{n-x}$

you choose...

$X_1,\ldots,X_r \overset{iid}{\sim} Geom(p)$

$X = X_1 + \ldots + X_r \sim NegBin(r,p)$

$Var(X) = r\sigma^2$

$\sigma^2$ is the variance of a Geom$(p)$

$Y \sim$ Geom$(p)$

$$Var(Y) = E(Y^2) - \left(\frac{1}{p}\right)^2$$

$$E(Y^2) = \sum_{y=1}^{\infty} y^2 (1-p)^{y-1} p$$

let $z = y-1 \Rightarrow y = z+1$

$y = 1 \ldots \infty$

$z = 0 \ldots \infty$

$$= \sum_{z=0}^{\infty} (z+1)^2 (1-p)^z p$$

$$= p\left(\sum_{z=0}^{\infty} (z^2 + 2z + 1)(1-p)^z\right)$$

$$= \underbrace{\sum_{z=0}^{\infty} z^2 (1-p)^z p} + \underbrace{2p \sum_{z=0}^{\infty} z (1-p)^z} + \overbrace{p \overbrace{\sum_{z=0}^{\infty} (1-p)^z}^{\frac{1}{p}}}^{1}$$

$$= (1-p) \underbrace{\sum_{z=1}^{\infty} z^2 (1-p)^{z-1} p}_{E(Y^2)} + 2(1-p) \underbrace{\sum_{z=1}^{\infty} z (1-p)^{z-1}}_{\text{Exp Geom } \frac{1}{p}}$$

$$\Rightarrow (1) E(Y^2) = (1-p) E[Y^2] + \frac{2(1-p)}{p} + 1$$

$1 - (1-p) = p$

$$\Rightarrow p\, E[Y^2] = \frac{2(1-p)}{p} + 1$$

$$\Rightarrow E(Y^2) = \frac{2(1-p)}{p^2} + \frac{1}{p}$$

$$\Rightarrow Var(Y) = \frac{2(1-p)}{p^2} + \frac{1}{p} - \frac{1}{p^2}$$

$$= \frac{2-2p-1}{p^2} + \frac{p}{p^2}$$

$$\boxed{\sigma^2 = \frac{1-p}{p^2}}$$

$$\Rightarrow \boxed{Var(X) = r\frac{1-p}{p^2}}$$

$X \sim$ Hyper$(n, K, N)$

$Var(X) = \ldots$ HARD... not covered!

$X_1, X_2, \ldots \stackrel{iid}{\sim} \text{Bern}(p)$

$X \sim \text{Geom}(p)$ if $X$ is stopping time

$$P(X=17 \mid X>10) = \frac{P(X=17 \;\&\; X>10)}{P(X>10)} = \frac{P(X=17)}{1-F(10)} = \frac{(1-p)^{16}p}{(1-p)^{10}}$$
$$= (1-p)^6 p$$
$$= P(X=7)$$

$$P(X=a+b \mid X>a) = \frac{P(X=a+b \;\&\; X>a)}{P(X>a)} = \frac{P(X=a+b)}{1-F(a)}$$
$$= \frac{(1-p)^{a+b-1}p}{(1-p)^a} = (1-p)^{b-1}p = P(X=b)$$

"Memorylessness" ... due to the iid Bernoullis. If you failed 1000 times, the 1001st begins a new geometric r.v. with parameter $p$.

---

Election wagering  Nate Silver said $P(\text{Clinton win}) = 0.75$

$X \sim \text{Bern}(0.75)$  Need a decision $\in \text{Supp}(X)$  choose $\text{Mode}[X]$!

$E(X) = 0.75$ Any meaning?

$P(\text{Clinton win} \mid \text{NS's models being correct})$

Models:

$$\underset{\text{electoral votes}}{Y} = f(\overbrace{X_1, \ldots, X_p}^{\text{covariates}} \mid \overbrace{\beta_1, \ldots, \beta_K}^{\text{params}}) + \varepsilon, \quad \varepsilon \sim h(\ldots)$$

MCMC Simulate all prior's many times...

$\beta_1 = g_1(X_1, \ldots X_p \mid \gamma_1, \ldots, \gamma_L)$

$\beta_2 = g_2(\ldots \mid \delta_1, \ldots \delta_J)$ etc...

Math 341