November 10, 2016

$Y \sim \text{Geom}(p)$

$\text{Var}[Y] = E[(Y-\mu)^2] = E[Y^2] - \mu^2 = E[Y^2] - (\frac{1}{p})^2$

$E[Y^2] = \sum_{y=1}^{\infty} y^2 (1-p)^{y-1} p \qquad z = y-1 \Rightarrow y = z+1$

$\qquad y = 1 \ldots \infty$

$= \sum_{z=0}^{\infty} (z+1)^2 (1-p)^z p \qquad z = 0 \ldots \infty$

$= p\left(\sum_{z=0}^{\infty} (z^2 + 2z + 1)(1-p)^z\right)$

$= \underbrace{\sum_{z=0}^{\infty} z^2 (1-p)^z p}_{\infty} + 2p \underbrace{\sum_{z=0}^{\infty} z(1-p)^z}_{\infty} + p \underbrace{\sum_{z=0}^{\infty} (1-p)^z}_{p \cdot \frac{1}{p} = 1}$

$= \underbrace{(1-p) \sum_{z=1}^{\infty} z^2 (1-p)^{z-1} p}_{E[Y^2]} + 2(1-p) \underbrace{\sum_{z=1}^{\infty} z(1-p)^{z-1}}_{\text{Expectation of Geom} = \frac{1}{p}} + 1$

$\Rightarrow E[Y^2] = (1-p) E[Y^2] + \frac{2(1-p)}{p} + 1$

$-(1-p)E[Y^2] \quad -(1-p)E[Y^2]$

$\Rightarrow pE[Y^2] = \frac{2(1-p)}{p} + 1$

$\Rightarrow E[Y^2] = \frac{2(1-p)}{p^2} + \frac{1}{p}$

$\Rightarrow \text{Var}[Y] = \frac{2(1-p)}{p^2} + \frac{1}{p} - \frac{1}{p^2}$

$= \frac{2-2p-1}{p^2} + \frac{p}{p^2}$

$\text{Var}[Y] = \frac{1-p}{p^2}$

> $X \sim NegBin(r, p)$

$Var[X] = \sum_{x=r}^{\infty} (x - \frac{r}{p})^2 \binom{x-1}{r-1} (1-p)^{x-r} p^r$

$E[X] = \frac{r}{p}$

> Review of rules:

$E[aX + C] = a\mu + C$

$Var[aX + C] = a^2 \sigma^2$

$SE[aX + C] = |a| \sigma$

$E[T] = \sum_{i=1}^{n} E[X_i] = n\mu$ (if identically distributed)

$Var[T] = \sum_{i=1}^{n} Var[X_i] = n\sigma^2$ (if iid)

$E[\bar{X}] = \mu$ (if $X_1, \ldots, X_n$ are identically distributed)

$Var[\bar{X}] = \frac{\sigma^2}{n}$ (if $X_1, \ldots X_n$ are iid)

$SE[\bar{X}] = \frac{\sigma}{\sqrt{n}}$

$X_1, \ldots, X_r \overset{iid}{\sim} Geom(p)$

$= X = X_1 + \ldots + X_r$

$E[X] = \frac{r}{p}$

$= Var[X] = r \cdot \frac{1-p}{p^2}$

> Memorylessness Property of the Geometric:

$X \sim Geom(p)$ means $X_1, X_2, \ldots \overset{iid}{\sim} Bern(p)$

$P(X = 17 \mid X > 10) \overset{?}{=} P(X = 7)$

Is "I know X is greater than 10, meaning you failed 10 times. What is the probability that your success is at $X = 17$ knowing you failed 10 times."

the same as

"What is the probability you succeed at $X = 7$?"

?

Yes! Because of the iid property. In roulette, if you spin 10 times and get black, the probability of getting red on the 11th spin is still the same. You don't need to factor in the $X=1$ to $X=10$.

$$P(X=17 \mid X>10) \overset{\checkmark}{=} P(X=7)$$

$$= \frac{P(X=17 \;\&\; X>10)}{P(X>10)}$$

$$= \frac{P(X=17)}{1-F(10)}$$

$$= \frac{(1-p)^{16}p}{(1-p)^{10}}$$

$$= (1-p)^6 p$$

$$P(X=7) = (1-p)^6 p$$

General Case

$$P(X=a+b \mid X>a)$$

$$= \frac{P(X=a+b \;\&\; X>a)}{P(X>a)} = \frac{P(X=a+b)}{1-F(a)}$$

$$= \frac{(1-p)^{a+b-1}p}{(1-p)^a} = (1-p)^{b-1}p = P(X=b)$$

End of Midterm 2

Nate Silver - 538

Model for predicting if clinton wins:

$X \sim \text{Bern}(0.75)$

$E[X] = 0.75$

↳ Many many realizations ⇒ average of the realizations is close to 0.75. (close to b/c law of large #s). $E[X]$ is only useful for an event that can occur many times. However, in the case of the 2016 election, $E[X]$ is meaningless b/c the same election cannot happen again.

Prediction based on the most probable outcome

↳ Mode $[X] = 1$

Nate Silver came up with a really large model with a bunch of parameters, which were built on different models with their own parameters.

↳ Heirarchical Model