9/15

$P(A) = \frac{4}{52} = \frac{1}{3}$

$P(A|\heartsuit) = \frac{1}{13}$

This info Irrelevant ↳ information known

$P\begin{pmatrix}\text{IBM Stock} \\ \text{in a day} \uparrow\end{pmatrix} = P\left(\begin{matrix}\text{IBM Stock} \\ \text{in a day} \uparrow\end{matrix} \middle| \begin{matrix}\text{rains in} \\ \text{Buenos Aires}\end{matrix}\right)$ Irrelevant

Probibalistic Independence

Def of A, B being idependent

$P(A) = P(A|B)$

$P(B) = P(B|A)$

Under A, B being Indep.

$P(A) = P(A|B) = \frac{P(AB)}{P(B)}$

$\rightarrow P(AB) = P(A)\,P(B)$ multiplication Rule

$A_1, A_2 \ldots$ Independent

$P\left(\bigcap_{i=1}^{\infty} A_i\right) = \prod_{i=1}^{\infty} P(A_i)$

$P(H_2|H_1) = P(H_2) = \frac{1}{2}$

$P(H_1, H_2\, H_3\, H_4\, H_5) = \left(\frac{1}{2}\right)^5$

$= \prod_{i=1}^{5} P(H_i)$

$P(\{\geq 1\ 6\text{-}6 \text{ in } 24 \text{ rolls } 2 \text{ dice}\}) < \frac{1}{2}$

$= P(1\ 6\text{-}6) + P(2\ 6\text{-}6) + \ldots P(24\ 6\text{-}6)$

$= 1 - P\begin{pmatrix}\text{zero } 6\text{-}6 \\ \text{in } 24 \text{ rolls}\end{pmatrix}$ → compliment of ↗

$= 1 - P(\text{Not } 6\text{-}6\ 1^{st}\text{roll} \cap \text{Not } 6\text{-}6\ 2^{nd}\text{roll} \cap \ldots \cap \text{Not } 6\text{-}6\ 24^{th}\text{roll})$

↳ $P(\text{Not } 6\text{-}6\ 1^{st}) \cdot P(\text{Not } 6\text{-}6\ 2^{nd}) \ldots \cdot P(\text{not } 6\text{-}6\ 24^{th})$

$= P(\text{Not } 6\text{-}6)^{24} = (1 - P(6\text{-}6))^{24} = (1 - P(6) + P(6))^{24}$

$= (1 - (\frac{1}{6})^2)^{24} = 1 - (1 - \frac{1}{6})^2)^{24} = .4914039$

If $P(A|B) \neq P(A)$

$P(B|A) \neq P(B)$

or $P(AB) \neq P(A)P(B)$,

$\Rightarrow$ A, B NOT independent i.e "dependent"

$P(\text{Q64 bus is late}) < P(\text{Q64 bus is late} \mid \text{rain})$

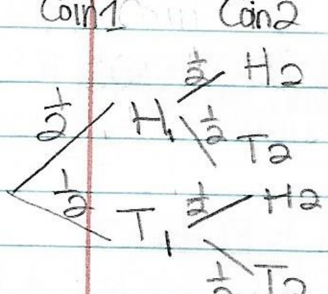
$P(\text{Q64 bus is late}) > (P\ \text{Q64 bus is late} \mid \text{sun + no traffic})$

↳ there are no conditions here (margial-prob)

A, B disjoint ⇒ A, B independent   disjoint - ultimate dependence

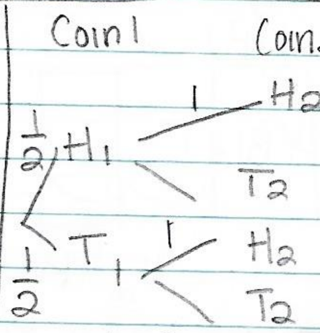
$P(A \mid B) = P(A)$

$0 \neq P(A)$



$P(HH) =$
$P(HT) =$ $\frac{1}{4}$
$P(TH) =$
$P(TT) =$

Done Midterm 1



$P(HH) = \frac{1}{2}$
$P(TT) = \frac{1}{2}$

$P(H_2 \mid H_1) = P(H_2)$

$1 \neq \frac{1}{2}$ → dependent

P(Shared Birthday)

$= P(\geq 1 \text{ share bday})$

$= P(1 \text{ shared bday}) + P(2 \text{ shared bday}) \ldots P(48 \text{ shared bday})$

$= 1 - P(\text{No bday}) = 1 - .04 = 96\%$

$$\frac{365 \cdot 364 \cdot 363 \ldots 365-48+1}{365^{48}} = \frac{{}_{365}P_{48}}{(365)^{48}} = 0.039$$

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* n people randomely administer their hats

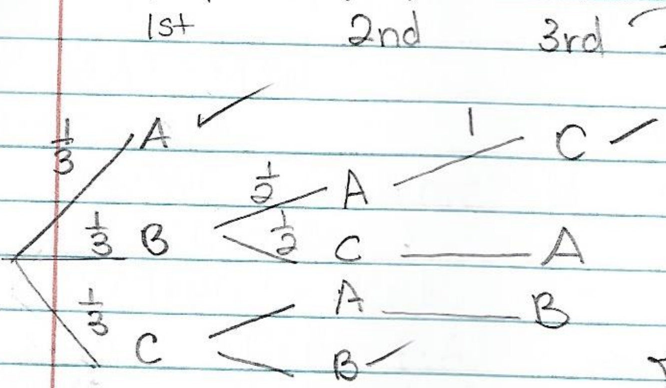
$P = P(\text{zero people get their hat})$

$1 - P = P(\text{at least 1 person gets hat})$

$= P(1p \text{ gets hat}) + P(2p \text{ gets hat}) + \ldots P(n \text{ people get hat})$

$$= P\left(\bigcup_{i=1}^{n} A_i\right)$$

3 people A, B, C



$P(A \cup B) = P(A) + P(B) - P(AB)$

$P(A_1 \cup A_2) = \sum_{i=1}^{2} P(A_i) - P(\bigcap_{i=1}^{2} A_i)$

$1 \quad - \quad \frac{1}{2!}$

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) - \sum_{i \neq j} P(A_i \cap A_j) + \sum_{i \neq j \neq k} P(A_i \cap A_j \cap A_k) \; . + - + - + - \; - (-1)^{n+1} P\left(\bigcap_{i=1}^{n} A_i\right)$$

$$\sum_{i=1}^{n} P(A_i) = 1$$

$$P(A_1 \cap A_2) = \frac{1 \cdot 1 \cdot (n-2)(n-3) \cdots 1}{n!} = \frac{(n-2)!}{n!}$$

$$P(A_1 \cap A_3) = \frac{1 \, (n-2) \, 1 \, (n-3) \cdots 1}{n!} = \frac{(n-2)!}{n!}$$

$$\sum_{i \neq j} P(A_i \cap A_j) = \sum_{i \neq j} \frac{(n-2)!}{n!} = \binom{n}{2} \frac{(n-2)!}{n!} = \frac{\cancel{n!}}{\cancel{(n-2)!}\,2!} \cdot \frac{\cancel{(n-2)!}}{\cancel{n!}} = \frac{1}{2!}$$

$$P\left(\bigcup_{i=1}^{n} A_i\right) = 1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \frac{1}{5!} \cdots \approx \frac{1}{3}$$

$$f(x) = \sum_{i=0}^{\infty} \frac{f^{(i)}(c)}{i!} (x-c)^i \quad \forall c \in \mathbb{R}, f \text{ continous}$$