Lecture 13 : [redacted] November 1^{st}, 2016

→ Nov 11, 2016 9AM
$\underbrace{\text{Nov 11, 2016}}$ HW 6, due then
Exam up to HW6

- $X \sim Geom(p) := (1-p)^{x-1} \cdot p$

→ (EX) $X \sim Geom(0.2) = 0.8^{x-1} \cdot 0.2$

$$\mu = E(X) = \sum_{x \in Supp[X]} x \, p(x)$$

x	p(x)	F(x)
1	0.2	0.2
2	0.160	0.36
3	0.128	0.488
4	0.102	0.590
5	0.082	0.672
6	0.066	0.738
7	0.052	0.790
8	0.042	0.832
9	0.034	
10	.027	
11	.021	
12	.017	
13	.014	
14	.011	
15	.008	
16	.007	
17	.006	
18	.005	
19	.004	
20	.003	
21	.002	
22	.001	
23	.001	
24	.001	
25	.001	
26	.001	
27	.001	.999
28	.000	

$X \sim Bern(p)$ $\bar{X} \longrightarrow \mu$ L.L.N.

$E(X) = p$

$X \sim Binomial(n, p)$

$E(X) = np$

$X \sim Hyper(n, K, N)$

$E(X) = n \frac{K}{N}$

$X \sim Geom(p)$

$E(X) =$

$X \sim NegBin(r, p)$

$E(X) =$

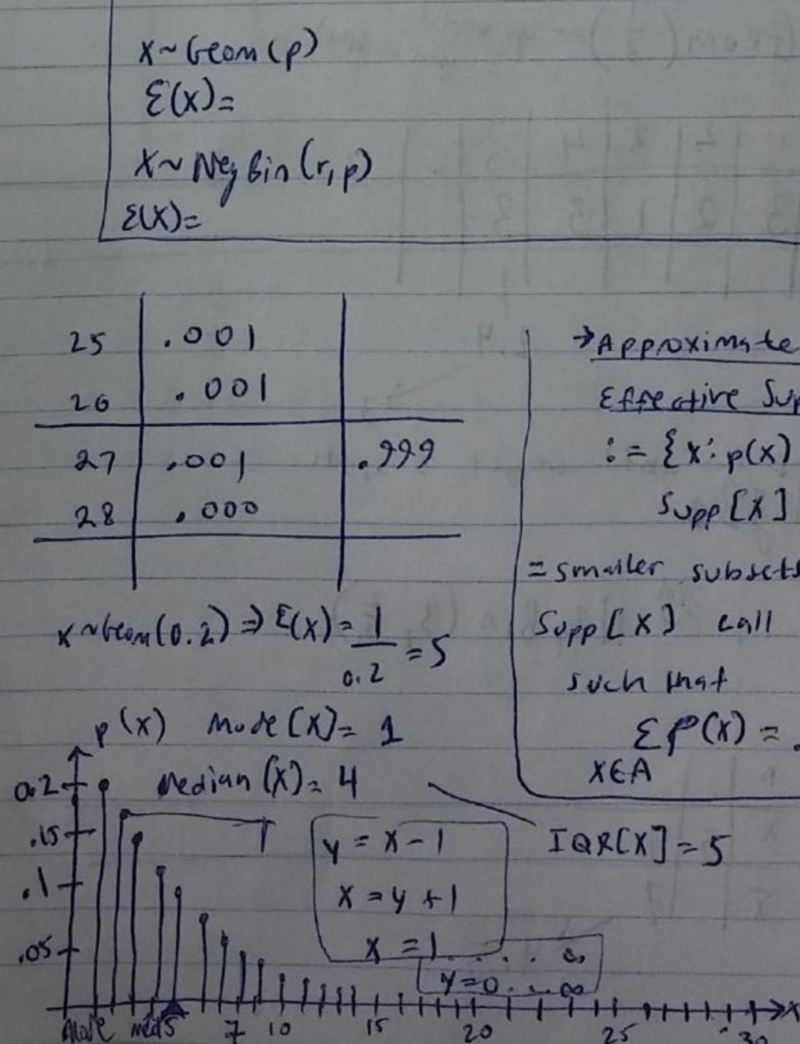
→ Approximate / Effective Support
$:= \{x : p(x) \geq .001\} \subset Supp[X]$
= smaller subsets of $Supp[X]$ call is A such that
$$\sum_{x \in A} p(x) = .999$$

$X \sim Geom(0.2) \Rightarrow E(X) = \frac{1}{0.2} = 5$

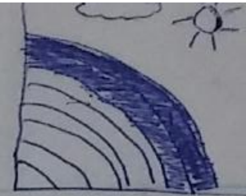
Mode(X) = 1

Median(X) = 4

IQR[X] = 5



$Y = X - 1$
$X = Y + 1$
$X = 1, \ldots, \infty$
$Y = 0, \ldots, \infty$



$X \sim Geom(p) \Rightarrow \mu = \sum_{x \in Supp(X)} x\,p(x) = \sum_{x=1}^{\infty} x(1-p)^{x-1} p = p \sum_{x=1}^{\infty} x(1-p)^{x-1}$

$\Leftarrow p \sum_{y=0}^{\infty} (y+1)(1-p)^{y}$

$p\left(\sum_{y=0}^{\infty} (1-p)^{y} + \underbrace{\sum_{y=0}^{\infty} (1-p)^{y}}_{\frac{1}{1-(1-p)} = \frac{1}{p}} \right)$

$p\left(\sum_{y=0}^{\infty} y(1-p)^{y-1} + \frac{1}{p} \right) \Rightarrow \sum_{y=0}^{\infty} y(1-p)^{y} p + 1 \Rightarrow (1-p) \sum_{y=1}^{\infty} y(1-p)^{y-1} + 1$

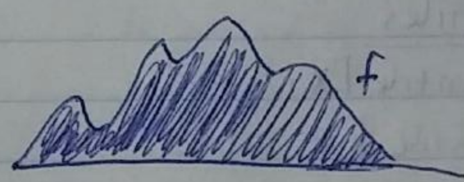
$\mu = \mu - p\mu + 1 \quad \Leftarrow \mu = (1-p)\mu + 1$

$\boxed{\mu = \frac{1}{p}}$

and birds go flying at the speed of sound to show you how it all began, And birds come flying from the nderground, f you see it, then you'll nderstand

→ function f

$G[f] = \int_{\mathbb{R}} f(x)\,dx = 17$



→ $mode[X] := \underset{x \in Supp[X]}{\arg\max} \{p(x)\}$

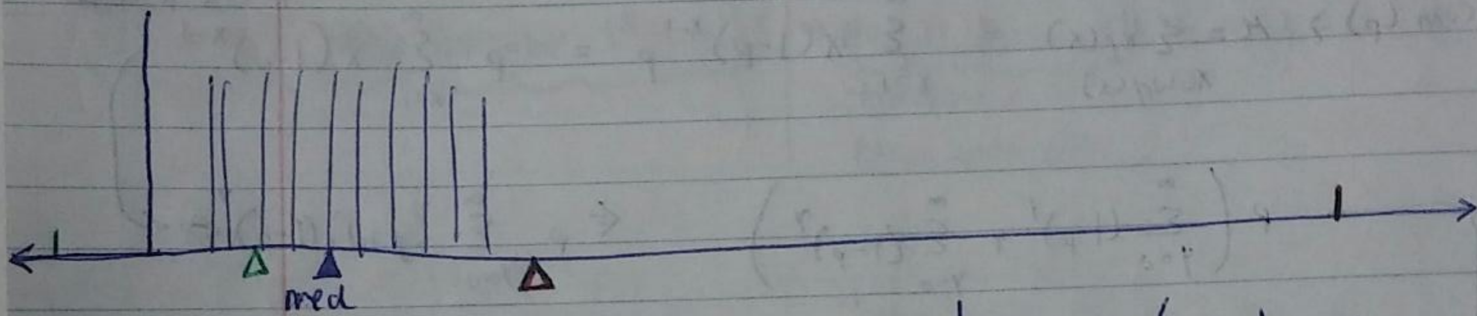
→ $Min[X] = min(Supp[X]) = 1$

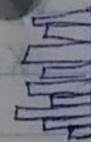
→ $Max[X] = max(Supp[X])$

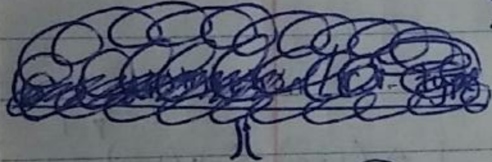
→ $Range[X] = Max[X] - min[X]$

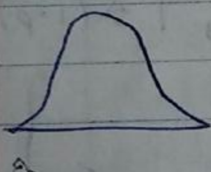
- $Quantile[X, p]$
  "Percentile" if p is a percent.
  $:= \arg\min \{F(x) \geq p\}$

→ $Median[X] := Quantile[X, 0.5]$









condition	distribution / r.v. type
$E[X] = Median[X]$	Symmetric
$E[X] > Median[X]$	Skew right
$E[X] < Median[X]$	Skew left
one mode	unimodel
$mode[X] = E[X] = Median[X]$	unimodal & symmetric

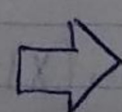
- Tertiles
  - Quantile [X, 0.33]
  - Quantile [X, 0.66]

- Quartiles
  - Quantiles [X, 0.25]
  - Quantiles [X, 0.50]
  - Quantiles [X, 0.75]

- Quintiles
  - Q[X, 0.2]
  - Q[X, 0.4]
  - Q[X, 0.6]
  - Q[X, 0.8]

- Deciles
  - Q[X, 0.1]
  - Q[X, 0.2]
  - Q[X, 0.3]
  - ⋮
  - Q[X, 0.9]

- IQR[X] = Q[X, 0.75] − Q[X, 0.25]



• Roulette in America

Bet a black.

$$X_1, \dots, X_n \overset{iid}{\sim} \begin{cases} \$1 & wp \quad \frac{18}{38} \\ -\$1 & wp \quad \frac{20}{38} \end{cases}$$

↑

custom r.v. model we just build

$$\bar{X} \xrightarrow[n \to \infty]{} E[X] \quad LLN$$

$$E[X] := \$1 \cdot \frac{18}{38} + -\$1 \cdot \frac{20}{38} = -\$.053$$

↑

If you play many many times, lose 5¢.
more you play, more you lose.

$$\lim_{n \to \infty} T_n = -\infty$$

↑

In long run will lose everything.

Can't beat this.

If you play, average winning is $-\$0.053$

$$\bar{X} \to \mu \quad LLN$$

exp. value is only irreputable as a "long run" or large single

$$T = X_1 + X_2 + \dots X_n$$

$$\lim_{n \to 0} T = -\infty$$

If I play forever, I lose ∞!