Sum up previous 2 lectures.

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- $E[aX+c]=am+c$
- $E[T_n]=\sum_{i=1}^{n} E[X_i]=nm$ (if identically distributted)
- $Var[aX+c]=a^2\sigma^2 \Rightarrow SE[aX+c]=|a|\sigma$
- If $X_1,\ldots,X_n$ independent.
  $Var[T_n]=\sum_{i=1}^{n} Var[X_i]=n\sigma^2$ (if iid)
  $p(1-p)$

- $E[\bar{X}]=m$ (if identically distributed)
- $Var[\bar{X}]=\frac{\sigma^2}{n}$ (if iid)
- $\Rightarrow SE[\bar{X}]=\frac{\sigma}{\sqrt{n}}$ (if iid)

- $Y\sim Geom(p):=(1-p)^{y-1}p$

$Var[Y]=E[(Y-m)^2]$
$=E[(Y-\frac{1}{p})^2]$
$=E[Y^2]-(\frac{1}{p})^2$ $=\frac{2-p}{p^2}-\frac{1}{p^2}$

$\therefore Var[Y]=\boxed{\frac{1-p}{p^2}}$

Proof: $Eg[x]=\sum_{x\in Supp[X]} g(x)p(x)$

No need to know.

$E[Y^2]=\sum_{y=1} y^2(1-p)^{y-1}p$

Let $z=y-1\Rightarrow y=z+1$
$y=1\ldots\infty \Rightarrow z=0\ldots\infty$

$p(\sum_{z=0}^{\infty}\underbrace{(z+1)^2}_{z^2+2z+1}(1-p)^z=\sum_{z=0}^{\infty} z^2(1-p)^z p+2p\sum_{z=0}^{\infty} z(1-p)^z+\underbrace{p\sum_{z=0}^{\infty}(1-p)^z}_{\frac{1}{p}}$ from Geom series.

$\underbrace{(1-p)\sum_{z=1}^{\infty} z^2(1-p)^{z-1}p}_{E[Y^2]}+\underbrace{(1-p)2\sum_{z=1}^{\infty} z(1-p)^{z-1}p}_{\frac{1}{p}}$

$=(1-p)E[Y^2]+2(1-p)\frac{1}{p}+1=E[Y^2]$

$\frac{2(1-p)}{p}+1=pE[Y^2]\rightarrow\frac{2(1-p)}{p^2}+\frac{1}{p}=E[Y^2]\Rightarrow\frac{2(1-p)+p}{p^2}=E[Y^2]\Rightarrow\frac{2-2p+p}{p^2}=\frac{2-p}{p^2}.$

Let $X_1, \dots, X_n \overset{iid}{\sim} \text{Geom}(p)$

$X = \sum_{i=1}^{r} X_i$

- $X \sim \text{NegBin}(r,p) = X = \sum_{i=1}^{r} X_i$

NegBin $\shortparallel$ sum of Geom.

$$\text{Var}[X] = r \underbrace{\frac{1-p}{p^2}}_{\text{from Geom Var!}}$$

Doing from def (not good strategy)!!

$$= \sum_{x=r}^{\infty} \left(x - \frac{r}{p}\right)^2 \binom{x-1}{r-1} (1-p)^{x-r} p^r$$

No need to know — Hyper Var

- The geometric r.v. has the Memorylessness property

Note: C.D.F of Geom $1-(1-p)^x$

☆ Geom = you run a bern iid. until you get a success!

$X \sim \text{Geom}(p)$

telling already failed 10 times

Is $P(X=7) \overset{?}{=} P(X=17 \mid X>10) = \dfrac{\overset{(1-p)^{16}p}{P(X=17} \,\&\, \overset{(1-p)^{10}}{X>10)}}{P(X>10)}$ $= 1-P(X\le 10) = 1-F(10) = 1-(1-(1-p)^x) = (1-p)^{10}$

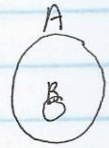
Yes. It's obvious when it's iid!

$$= \frac{(1-p)^{16}p}{(1-p)^{10}} = (1-p)^6 p = P(X=7) \checkmark.$$

$P(X=a+b) = (1-p)^{a+b-1}p$

$$P(X=a) = P(X=a+b \mid X>b) = \frac{P(X=a+b \,\&\, X>b)}{\underset{(1-p)^b}{P(X>b)}} = \frac{(1-p)^{a+b-1}p}{(1-p)^b} = (1-p)^{a-1}p$$

$B \subset A$



$A \cap B = B$

Note:

☆ Expectation = infinity times ... on average, ...

It's meaningless if $n=1$! Only works if n = big #.

Same for Variance

Unpredictable

$P(\text{Clinton}) = 0.75$

$X \sim \text{Bern}(0.75)$.

$E[X] = 0.75$

Only one time to run the election $n=1 \Rightarrow E[X]$ meaningless!!

— Only way to predict Mode $[X]=1$.

The parameter $p=0.75$

Doing this for billion times to get the p $\longrightarrow$ $K = f(X_1, \dots, X_p \mid \overbrace{B_1, \dots, B_k}^{\text{parameter.}}) + \Sigma \leftarrow$ unpredictable noise.

electoral voter $B_1 \sim g_1(X_1, \dots, X_p \mid \gamma_1, \dots \gamma_n)$

$B_2 \sim g_2(X_1, \dots, X_p \mid f_1, \dots f_a)$.

for further science