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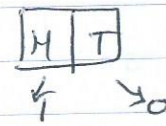
Discrete Random Variables — A. Kapelner — Post Exam 1

Def: A random variable ("r.v.") is a function.

$$X: \underset{\text{sample space}}{\Omega} \rightarrow \underset{\text{"values" of the r.v.}}{\mathbb{R}}$$

$\Omega = \{H, T\}$



$X(H) = 1$

$X(T) = 0$

What's the $p(\underbrace{X=1}_{\text{if it is "H"}}) := P\left(\{w : X(w) = 1\}\right) = P\left(\{H\}\right) = \frac{|\{H\}|}{|\Omega|} = \frac{1}{2}$

$P: 2^{\Omega} \rightarrow [0, 1]$

The "Support" of a r.v.

$Supp[X]$ is the range of x, the set of all possible values.

$Supp[X] = \{0, 1\}$

$Supp(x) := \{x : P(X = x) > 0\} \subseteq \mathbb{R}$, in other words, 0 and/or 1.

A "discrete r.v." is a r.v. s.t.

$$\underbrace{|Supp[x]|}_{\text{size of support}} \leq |\mathbb{N}| \longrightarrow \text{"countable infinity"}$$

size of support : number of things that can possibly happen

For Discrete X,

$$\sum_{x \in Supp[X]} P(X = x) = 1 \quad \} \text{something to happen.}$$

x → value, X → r.v.

Proof: $P\left(\bigcup_{x \in Supp[X]} \{w : X(w) = x\}\right) \overset{\text{Fact}}{=} P(\Omega) = 1$ where $\Omega = \{w_1, w_2 \dots\}$ s.t. $P(\{w\}) > 0$

$\left\{\begin{array}{l} \nexists\, w \text{ s.t. } X(w) \notin Supp[X] \\ P[\{w\}] = 0 \end{array}\right.$ } collectively exhaustive ; $\{w : X(w) = x_1\} \cap \{w : X(w_0) = x_2\} \neq \emptyset$

then $\exists$ w_0 $X(w_0) = x_1$ & $X(w_0) = x_2$

Violation!

$= P(\{w : X(w) = x_1\}) + P(\{w : X(w) = x_2\}) + \dots = 1$

$P(X = x_1) + P(X = x_2) + \dots = 1$

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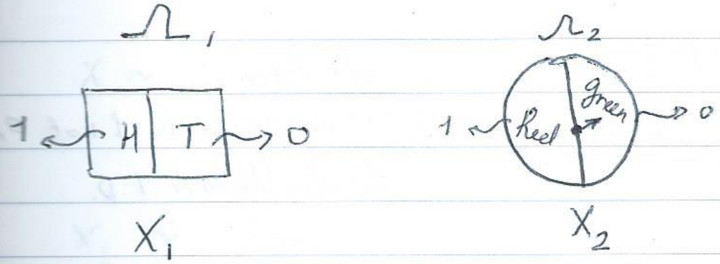
A. Kapelner



- Support is the same $(0,1) \leftrightarrow (0,1)$
- The Probabilities are the same.

$$X \sim \begin{cases} 1 & \text{wp } 1/2 \\ 0 & \text{wp } 1/2 \end{cases}$$

r.v. — distributed as — with probability

} dropping Ω out of picture.

"care about what & how often pops out."

"standard"

$$X \sim \text{Bernoulli}\left(\frac{1}{2}\right) := \begin{cases} 1 & \text{wp } 1/2 \\ 0 & \text{wp } 1/2 \end{cases}$$

} 0 & 1 are the only things that can pop out, i.e. not 10.

$\text{Supp}[X] = \{0,1\}$

X is distr. Bernoulli with param p.

$$X \sim \text{Bern}(p) := \begin{cases} 1 & \text{w.p. } p \\ 0 & \text{w.p. } 1-p \end{cases}$$

} Sum of ind. probabilities = 1.

// $p + 1 - p = 1$

$\text{Supp}[X] = \{0,1\}$

X is distr. with param. p.

// $X \sim \text{Bernoulli}(0.9) := \cancel{\begin{cases} 1 & \text{wp } 1/2 \\ 0 & \text{wp } 1/2 \end{cases}}$ → must change.

↳ 1111111......0 ...

cyliscope?

$p = 0$ // 1111

$X \sim \{0 \text{ w.p. } 1$

$X \sim \text{Deg}(c) := \{c \text{ w.p. } 1$

P lives in Parameter Space.

$$\underbrace{p \in (0,1)}_{\text{parameter space}}$$

...ition:

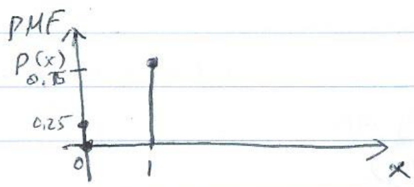
The prob. mass function (PMF) is $\underbrace{p(x)}_{\text{"what you will see"}} := P(X = x)$

If $x \notin \text{Supp}[X]$

$p(x) = 0$

$$\sum_{x \in \text{Supp}[X]} p(x) = 1$$

Let's see: $X \sim \text{Bern}(0.75)$



number of discont. ⇒ 2.

for general ⇒ size of support.

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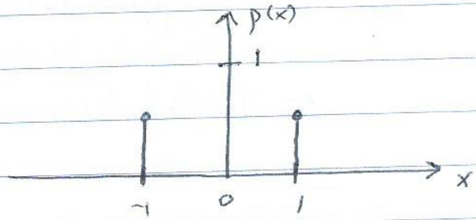
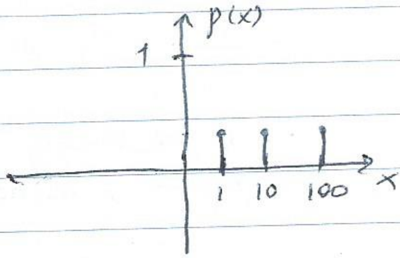
$X \sim \text{Rademacher} := \begin{cases} 1 & \text{wp } \frac{1}{2} \\ -1 & \text{wp } \frac{1}{2} \end{cases}$ } not Bern. It's supports $1, -1$ not $0, 1$. - Random Walks in 1-D.



"Another..."

$X \sim \text{Uniform}(\{1, 10, 100\}) = \begin{cases} 1 & \text{wp } 1/3 \\ 10 & \text{wp } 1/3 \\ 100 & \text{wp } 1/3 \end{cases}$



$X \sim \text{Uniform}(A)$

$\text{Supp}[X] = A$

$p(x) = \frac{1}{|A|}$

Parameter space (What could A possibly be?)

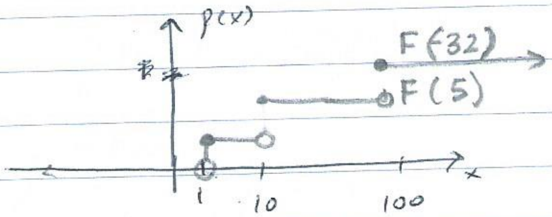
$A \subset \mathbb{R}$ "not equal to make sure it is discrete, else its not"

$|A| \in \mathbb{N} \setminus \{1\}$

// Let's say size of $A = 1$, $A = \{119\}$, it gonna print 119 wp 1, "uninteresting" case.

The Cumulative distribution function $\boxed{(\text{CDF})}$ ("distr. func") is

$$F(x) := P(X \leq x)$$



"pick up 1/3 everytime"

Properties of the CDF.

1. $F(x) \in [0, 1]$
2. $\lim_{x \to \infty} F(x) = 1$ "as $x \to \infty$, you "collect all the p's""
3. $\lim_{x \to -\infty} F(x) = 0$ "not able to pick up anything"
4. $\overbrace{x \leq y \Rightarrow F(x) \leq F(y)}^{\text{monotonically inc.}}$ Discont: $|A|$

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$X \sim Bern(p) = \begin{cases} 1 & \text{wp } p \\ 0 & \text{wp } 1-p \end{cases} \Rightarrow$ "nicer" $\Rightarrow \; = p^x (1-p)^{1-x}$

$X_1 \sim Bern(p), \quad X_2 \sim Bern(p)$ $\Big\{$ 2 diff. processes. They share the same PMF.

<u>Def:</u> X_1, X_2 are "equal" in distribution $X_1 \overset{!}{=} X_2$

if $p_1(x) = p_2(x)$ or $F_1(x) = F_2(x)$

// 10 cards
4R
6B

$$p(2R \text{ in } 3 \text{ cards}) = \frac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}}$$

without replacement

$$p(xR \text{ in } 3 \text{ cards}) = \frac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$$

$$P(xR \text{ in } n \text{ cards}) = \frac{\binom{4}{x}\binom{6}{n-x}}{\binom{10}{n}}$$

// 10 cards
K Reds
10-K Blue

$$p(xR \text{ in } n \text{ card}) = \frac{\binom{K}{x}\binom{10-K}{n-x}}{\binom{\cancel{X}}{n}}$$

// N cards
K Reds
N-K Blue

$X \sim$ Hypergenetic(n, K, N)

$$p(xR \text{ in } n \text{ cards}) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$