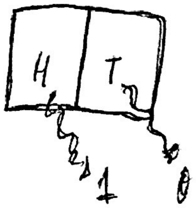
09/27/16 Coin review

$\Omega = \{H, T\}$



flip 3 times

$n = 3$

$\left.\begin{array}{l} \omega_1 = H \\ \omega_2 = H \\ \omega_3 = T \end{array}\right\} \longrightarrow 1, 1, 0 = \bar{x} = \frac{1+1+0}{3} = \frac{2}{3}$ AVERAGE.

$$\mathbb{1}_{\omega = H} = \begin{cases} 1 & \text{if } \omega = H \\ 0 & \text{if } \omega = T \end{cases}$$

Definition: A random variable ("r.v") is a function.

$X : \Omega \to \mathbb{R}$

Ω: sample space; $\mathbb{R}$: "value" of r.v.

- $X(H) = 1$
- $X(T) = 0$

$Supp[X] = \{0, 1\}$

- what is probability X equals one?

$P(X=1) := P(\{\omega : X(\omega) = 1\}) = P(\{H\}) = \frac{|\{H\}|}{|\Omega|} = \boxed{\frac{1}{2}}$

Illegal $P \in X^2 \to [0,1]$

- The "support" of a r.v.
  - $Supp[X]$ is the range of X, $\to Supp[X] := \{x : P(x) > 0\} \subseteq \mathbb{R}$

the set of all possible values.

- A "discrete r.v" is a r.v. such that $|Supp[X]| \leq |\mathbb{N}|$ countable ∞

Discrete X

$$\sum_{x \in Supp[X]} P(X = x) = 1$$

x: value (free variable); X: r.v.

$$\bigcup_{x \in Supp[X]} \{\omega : X(\omega) = x\} \overset{\text{FACT}}{\underset{?}{=}} \Omega$$

$\exists \omega$ s.t. $X(\omega) \notin Supp[X]$

$P(\{\omega\}) = 0$

$\Omega = \{\omega_1, \omega_2, \dots\}$ s.t. $P(\{\omega\}) > 0$

collectively exhaustive

$\{\omega : X(\omega) = x_1\} \cap \{\omega : X(\omega) = x_2\} \cdots \neq \emptyset$

$\exists \omega_0 \quad X(\omega_0) = x_1$

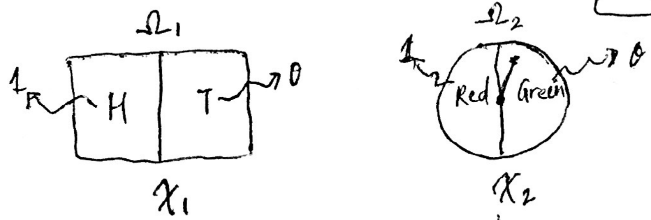
&

$X(\omega_0) = x_2$

$$P\Big(\bigcup_{x \in Supp[X]} \{\omega : X(\omega) = x\}\Big) = P(\Omega) = 1$$

$= P(\omega : X(\omega) = x_1) + P(\omega : X(\omega) = x_2) \cdots = 1$

$P(X = x_1) + P(X = x_2) + \dots = 1$



Are these equal?

$$X \sim \begin{cases} 1 & \text{wp } \frac{1}{2} \\ 0 & \text{wp } \frac{1}{2} \end{cases} \iff X \sim \text{Bernoulli}\left(\tfrac{1}{2}\right)$$

"standard"

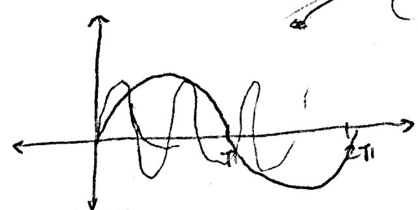
X: r.v; $\sim$: distribute as; wp: with probability

$Supp[X] = \{0, 1\}$

$X \sim \text{Bern}(p) := \begin{cases} 1 & \text{w.p. } p \\ 0 & \text{w.p. } 1-p \end{cases}$

$f(x) = \sin(x)$

$f(x) = \sin(ax)$
s.t. a is a constant
increases frequency



$a = 0$
↳ Trivial case

Parametric Model $\begin{cases} \text{Supp}[X] = \{0,1\} \\ p \in [0,1] \to \underbrace{p \in (0,1)}_{\text{PARAMETER SPACE}} \end{cases}$

Ex. $X \sim \{0 \text{ w.p. } 1$

$X \sim \text{Deg}(c) := \{c \text{ w.p. } 1$

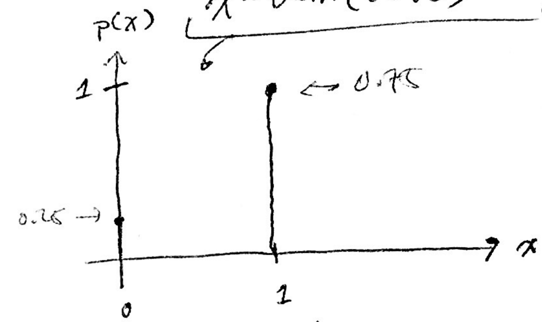
If $x \notin \text{Supp}[X]$
$p(x) = 0$

X is distributed Bernoulli with parameter p

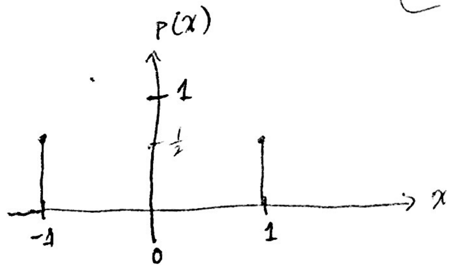
Definition: the probability mass function (PMF) is $p(x) := P(X = x)$

$\sum_{x \in \text{Supp}[X]} p(x) = 1$

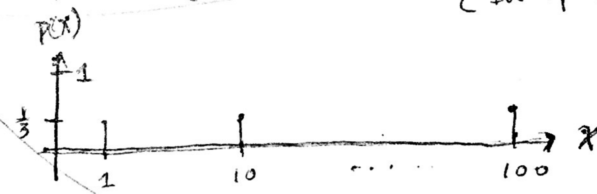
$X \sim \text{Bern}(0.75)$



- $X \sim \text{Rademacher} = \begin{cases} 1 & \text{w.p. } \frac{1}{2} \\ -1 & \text{w.p. } \frac{1}{2} \end{cases}$
  Not Bern. b/c -1, not 0.



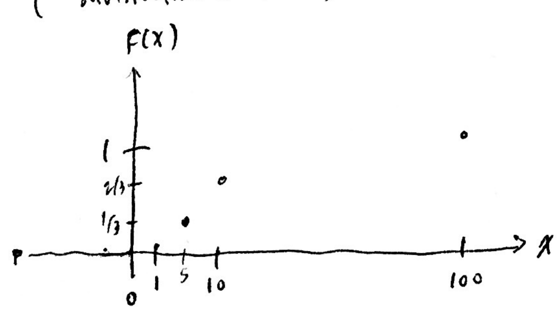
- $X \sim \text{Uniform}(\{1, 10, 100\}) = \begin{cases} 1 & \text{w.p. } \frac{1}{3} \\ 10 & \text{w.p. } \frac{1}{3} \\ 100 & \text{w.p. } \frac{1}{3} \end{cases}$



- $X \sim \text{Uniform}(A)$
  $\text{Supp}[X] = A$
  $p(x) := \frac{1}{|A|}$

Parameter Space
- $A \subset \mathbb{R}$
- $|A| \in \mathbb{N} \setminus \{1\}$

- The Cumulative Distribution Function (CDF) ("distribution function") is $F(x) := P(X \leq x)$



$F(5) = \frac{1}{3}$
$F(10) = \frac{2}{3}$
$F(100) = 1$
$F(1{,}000) = 1$
$F(-32) = 0$

Properties of the CDF

1. $F(x) \in [0,1]$
2. $\lim_{x \to \infty} F(x) = 1$
3. $\lim_{x \to -\infty} F(x) = 0$ — monotonically increasing
4. $x \leq y \Rightarrow F(x) \leq F(y)$

$F(y) = P(X \leq y) = P(X \leq x) + P(X \in (x, y])$
↳ ≥ 0
$\therefore F(x)$

$X \sim \text{Bern}(p) := \begin{cases} 1 & \text{w.p. } p \\ 0 & \text{w.p. } 1-p \end{cases} = p^x (1-p)^{1-x}$

$P(A)$

$X_1 \sim \text{Bern}(p)$, $X_2 \sim \text{Bern}(p)$

Def: X_1, X_2 are "equal" in distribution $X_1 = X_2$
if $P_1(x) = P_2(x)$ OR $F_1(x) = F_2(x)$

10 cards
4 Red
6 Blue.

- $P(2\ R \text{ in } 3 \text{ cards}) = \dfrac{\binom{4}{2}\binom{6}{1}}{\binom{10}{3}}$

without replacement

- $P(x \text{ Red's in } 3 \text{ cards}) = \dfrac{\binom{4}{x}\binom{6}{3-x}}{\binom{10}{3}}$

- $P(x\ Rs \text{ in } n \text{ cards}) = \dfrac{\binom{k}{x}\binom{10-k}{n-k}}{\binom{10}{n}}$

N cards
K red
N- K blue

$$\boxed{P(x\ R \text{ in } n \text{ cards}) = \dfrac{\binom{k}{x}\binom{N-k}{n-k}}{\binom{N}{n}}}$$

$X \sim \text{Hyper}\ldots(n, K, N)$