• $\boxed{\in, \subseteq, \subset, \overset{\text{union}}{\cup}, \overset{\text{intersection}}{\cap}, \setminus, |\cdot|}$ ← "pipes" / cardinality

Lecture 2, August 30

set inclusion ($\in$) subset ($\subseteq$) proper subset ($\subset$) set difference/subtraction ($\setminus$)

• Sets $A, B \ldots$ ← sets are denoted with capital letters.

• Special sets: ϕ , Ω

null/empty set (ϕ) universe - everything care about right now. (Ω)

Pg 3

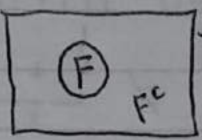
Pg 4 ^♡ sum of probabilities of all mutually exclusive & collectively exhaustive events is 1 exhaustive. That is, if A, B & C are mutually exclusive & collectively exhaustive even, then (the entire sample space):

$P(A)+P(B)+P(C)=1$.

- If A & B ME, then $P(A \text{ and } B)=0$ so can be simplified to $P(A)+P(B)=P(A \text{ or } B)$
- If A & B are CE, then $P(A \text{ or } B)=P(A)+P(B)=1$

- Example: what does this mean?

* $F \cap \Omega$ (Intersection of F & Ω)



so $F \cap \Omega = F$

* $\Omega \setminus F = F^c$

set complement: everything not in F

* $F \cup \Omega = \Omega$ (everything in F & Ω)

- $\phi \cap \Omega = \phi$ (intersection null set & Ω) nothing here, so nothing in common.

* $\phi \cup \Omega = \Omega$ (union empty set in Ω) add Ω to everything in null set

* $F \setminus \Omega = \phi$ because all element of F are in Ω, so ϕ.

$F \subseteq \Omega$

$F \in 2^{\Omega}$

- $(F^c)^c = F$ complement of a complement is itself

- $A \cup A^c = \Omega$ because union of A & everything not in A, so has to be Ω.

- $\{A, A^c\}$ are called collectively exhaustive.

One of the events must occur. Set of events covers entire sample space.

A group of sets $A_1, A_2 \ldots A_n$ is collectively exhaustive iff $A_1 \cup A_2 \cup A_3 \ldots \cup A_n = S$.

-Definition of collectively exhaustive: $\{A_1, A_2, A_3 \ldots\}$ ↳ put them together to get everything.

If $\bigcup_{i=1}^{\infty} A_i = \Omega$ ← Just means summing elements together $A_1 \cup A_2 \cup A_3 \cup \ldots$

similar to $\sum_{i=1}^{3} i = 6$ sumations & stuff

- $A \cap A^c = \phi$

↳ intersection of A & everything not in A is ϕ because have nothing in common.

* $\{A, A^c\}$ are mutually exclusive ("disjoint") → Two sets A & B are disjoint if $A \cap B = \phi$

☆ Events that cannot occur together (ME)

* $\{A_1, A_2, \ldots\}$ are mutually exclusive if $A_i \cap A_j = \phi \;\forall\; i \neq j$ (and)

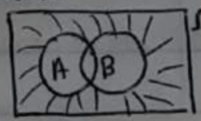
- $|\Omega| = |A| + |A^c|$ for $|\Omega|$ finite

$(|A| = |\Omega| - |A^c|)$

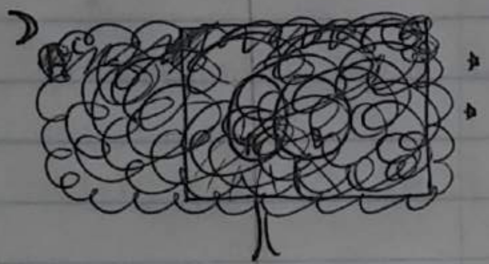
~~[illegible]~~

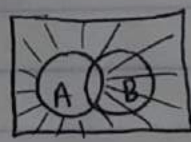
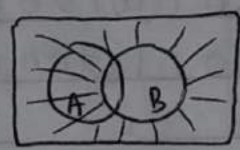
- Probability of any event must be between 0 & 1, inclusively. That is $0 \leq P(A) \leq 1$. ^♡ → Next page

• Consider $A, B \subset \Omega$

* $(A \cup B)^c =$  ← can $(A \cup B)^c = \underbrace{(A^c) \cup (B^c)}$? → No, not the same thing.

Not the same thing!



* $A^c =$  & $B^c =$ 

$(A \cap B)^c =$ 

can $(A \cup B) = A^c \cap B^c$ ✓ T

can $(A \cap B) = A^c \cap B^c$ X F

can $(A \cap B) = A^c \cup B^c$ ✓ T

$\boxed{\begin{array}{l} \text{So we have DeMorgan's Law} \\ *\ (A \cup B) = A^c \cap B^c \\ *\ (A \cap B) = A^c \cup B^c \end{array}}$

• $\mathbb{N} = \{1, 2, 3 \ldots\}$ ← natural numbers

$|\mathbb{N}| = \aleph_0$ (countable infinity)

• $\mathbb{Z} = \underbrace{\{\ldots -2, -1, 0, 1, 2 \ldots\}}_{\text{integers}}$

$|\mathbb{Z}| = \aleph_0$ because

$\mathbb{Z} = \{\ldots -2, -1, 0, 1, 2 \ldots\}$

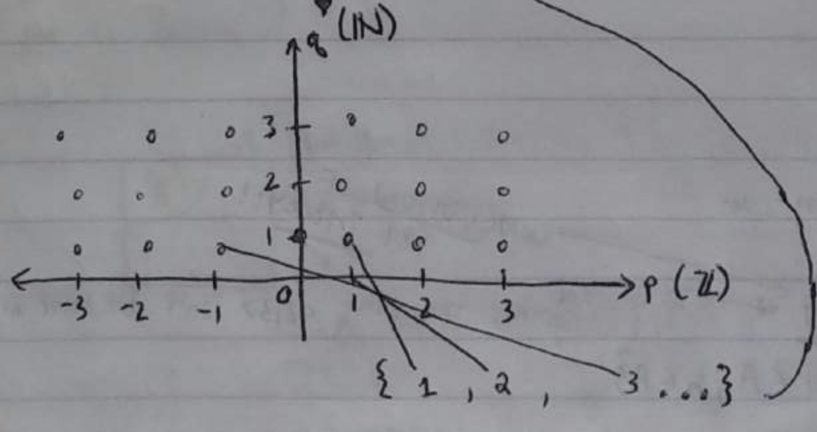
"mapping" $\{1 \; 2 \; 3 \; 4 \; 5\}$ (same as natural #s)

• $\mathbb{Q} = \{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{N}\}$

↑ symbol for "rationals"

all fractions / decimals.

$|\mathbb{Q}| = \aleph_0$



Pg 6

Consider $(0,1) \subset \mathbb{R}$

$|0,1| \leq |\mathbb{R}|$

Assume $|(0,1)| \neq \aleph_0$ ← Wrong, can't enumerate them!

$0.01011010\ldots$

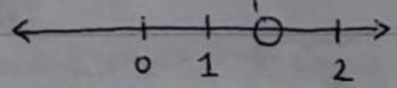
$0.10101011\ldots \Rightarrow 0.1101\ldots$

$0.01101000\ldots$

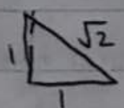
$0.01101001\ldots$

uncountable ∞ $|\mathbb{R}| = \mathfrak{c} > \aleph_0$

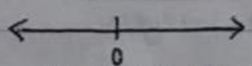
- $\mathbb{R} := \mathbb{Q} \cup \{\text{all holes}\}$ (irrationals)



"reals"
(real #s)
("continuum")



$\sqrt{2} \notin \mathbb{Q}$



No holes, all plugged up

- $[3,7] := \{x : x \geq 3, x \leq 7\} \subset \mathbb{R}$
- $(10,17] := \{x : x > 10, x \leq 17\} \subset \mathbb{R}$

- Ordered pair
  - $\langle a,b \rangle$ element a, element b in that order
  - $\langle a,b \rangle := \{\{a\},\{a,b\}\}$
  - $\langle a,b \rangle \neq \langle b,a \rangle$
  - $\langle a,b \rangle \neq \{a,b\}$

here care about order — here don't care about order

CONCLUSION OF SET THEORY!!

- CARTESIAN PRODUCT ← Think like graph. The point $\langle 2,3 \rangle$ isn't the same as $\langle 3,2 \rangle$.
  - $A \times B := \{\langle a,b \rangle : a \in A, b \in B\}$
  - Example

$A = \{1,2\}$

$B = \{3,4\}$

$A \times B = \{\langle 2,3 \rangle, \langle 1,4 \rangle, \langle 1,3 \rangle, \langle 2,4 \rangle\}$

$|A \times B| = 4$ — $|A| = 2$, $|B| = 2$ — $|A \times B| = |A| \cdot |B|$ IF FINITE!!! ★★

- USING SET THEORY

* Ω = sample space, experimental outcome space
* $\Omega = \{w_1, w_2, \dots\}$

little omega — The elements are called "outcomes"

* Experiment... $w \in \Omega$ is chosen

* COIN FLIP: $\Omega = \{H, T\}$ $|\Omega| = 2$

# of outcomes

- $A \subseteq \Omega$

"event": set of outcome

$$\underset{\substack{\text{domain} \\ \text{can only ask probability of the 4 things}}}{A \in 2^{\Omega}} = \{\phi, \underbrace{\{H\}, \{T\}, \{H,T\}}_{\text{event space - ie, all events}}\}$$

trivial events

- PROBABILITY: WORKING DEFINITION

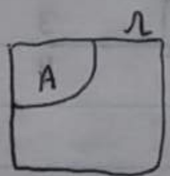
* $P(A) = \dfrac{|A|}{|\Omega|}$ if Ω is finite.

* $P(\{H\}) = \dfrac{|\{H\}|}{|\Omega|} = \dfrac{1}{2}$ } probability of flipping a coin & getting heads.

$\left[P(H) = \dfrac{|H|}{|\Omega|} \right.$ ← No good, Has to be a set to compute $\left. \right]$

does not $= \Omega$

* $P: \underset{\text{events}}{2^{\Omega}} \to [0,1]$



→ $P\phi = 0$

→ $P(\Omega) = 1$

→ $P(\{H,T\}) = P(\{H\} \cup \{T\}) = 1$

or

→ $P(\{H \cap T\}) = \phi$

probability of heads & tail not possible!

$$\cdot\ P(A^c) = \frac{|A^c|}{|\Omega|} = \frac{|\Omega| - |A|}{|\Omega|} = 1 - \frac{|A|}{|\Omega|} \Rightarrow \boxed{\begin{array}{c}\text{complement Rule} \\ \hline P(A) = 1 - P(A^c)\end{array}}$$

$$= 1 - P(A)$$

· Two Coin Flips

$\Omega^1 = \Omega^2$

$= \{\langle H,H\rangle, \langle H,T\rangle, \langle T,H\rangle, \langle T,T\rangle\}$

4 possible things can happen

$P(\cdot)$, 16 things probability can ask of.