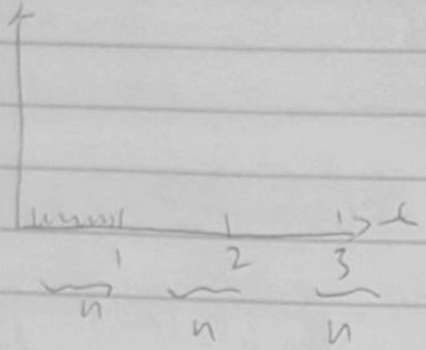
lec 17

11/11/2016

def $T \sim \text{geometric}(p) = \underbrace{[(1-p)^{t-1}]p}_{p(t)}$ $\quad F(t) = 1-(1-p)^t$

$F(t) = \frac{1}{p} \cdot \text{exp} \cdot \frac{\text{sec}}{\text{exp}}$

$\underbrace{1-F(t) = (1-p)^t}_{[\text{kap noks}]}$



$\Rightarrow E(T) = \frac{1}{p} \text{exp} \cdot \frac{\text{sec}}{n \cdot \text{exp}} = \frac{1}{np} \text{sec}$

Imagine n large but p small

let $\lambda = np \Rightarrow p = \frac{\lambda}{n}$ reparametrization

$p(t) = (1-\frac{\lambda}{n})^{nt-1} \frac{\lambda}{n}$

$f(t) = 1-(1-\frac{\lambda}{n})^{nt}$

let $n \to \infty$; but λ remain λ

$\lim_{n\to\infty} p(t) = \lim_{n\to\infty} (1-\frac{\lambda}{n})^{nt} \frac{\lambda}{n} = \lim_{n\to\infty} (1-\frac{\lambda}{n})^{nt} \underbrace{\lim_{n\to\infty} \frac{\lambda}{n}}_{0} = 0 \, \forall t$

$\sum p(t) = 0 \Rightarrow p(t)$ is not valid

$t \in \text{Supp}[t]$ $\quad T$ is not a discrete r.v.

$\lim_{n\to\infty} f(t) = \lim 1-(1-\frac{\lambda}{n})^{nt} = 1 - \lim (1-\frac{\lambda}{n})^{nt}$

$\hookrightarrow e = 1 - (\lim_{n\to\infty} \underbrace{(1-\frac{\lambda}{n})^{n}}_{e^{-\lambda}})^t$

$= 1 - e^{-\lambda t}$

$e = \sum_{i\ge 0}^{\infty} \frac{1}{i!}$ $\qquad \lim_{n\to\infty} \underbrace{(1+\frac{1}{n})^n}_{f(n)}$

$i \ge 0$

n	f(n)
10	2.814
100	2.705
1000	2.717
10000	2.78

$\int_1^e \frac{1}{x}dx = 1$

$\lim_{n\to\infty}(1+\frac{a}{n})^n, a\in\mathbb{R}$ $\qquad$ Let $\frac{1}{m} = \frac{a}{n} \to n = ma$

at $n\to\infty \to m = \infty$

$\lim_{m\to\infty}(1+\frac{1}{m})^{ma} = (\lim_{m\to\infty}(1+\frac{1}{m})^m)^a = e^a$

CDF'S

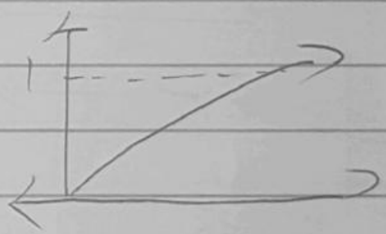
① $f(t)\in[0,1] \to$ prabability $0 > x > 1$ $\qquad t\in(0,\infty)$

$1-e^{-\lambda t}$ $\qquad 1-e^{\lambda t}\le 1$

$1 \ge e^{-\lambda t}$ $\qquad -\underbrace{e^{-\lambda t}}_{t} \le 0$

$0 \ge -\lambda t$

$-\lambda t \le 0$ ✓



λ is positive

t is positive

$-(+)(+) = -$

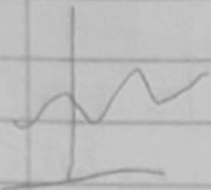
(2) $\lim_{t\to-\infty} F(t) = 0$ ✓ $\qquad$ ③ $\lim_{t\to-\infty} F(t) = 1$ ✓

Taken geometic, modified it $\qquad \lim_{t\to\infty} 1-e^{-\lambda t} = 1-\lim e^{-\lambda t}$

PMF not valid, CDF still valid.

$1-\lim \frac{1}{e^{\lambda t}} = 1-0 = 1$

$f(t)\ \frac{d}{dt}[F(t)] = \frac{d}{dt}[1-e^{-\lambda t}] = \underbrace{\lambda}_{np(t)} e^{-\lambda t} \ge 0$ Yes



need monotinict
means slope
is positive or 0.

This random

$supp[T] = (0, \infty)$

$|supp[t]| = \mathbb{R}$

↳ size $\mathbb{R}$ > size $\mathbb{N}$

$|supp[t]| = |\mathbb{R}| > |\mathbb{N}|$ → for a discrete R.V

T is a continous r.v

supp for geometric

$supp[X] = \{1 \dots\}$

No in exam

If quanton granty is real

$|*$——$*|$ Plank lenght 1.62×10^{-35} m

cannot distingyish between here and here

light $3E\ 8$ m/s, plank time: 5.3×10^{-44} s

$P(T=3) = p(3) = 0$

$P(T=3) = P(\underbrace{T = 3.0000 \dots}_{\text{contains infinite information}})$ → no interval

$p(T = 3.00\underset{\text{stop}}{0}) = P[T \in [2.99980, 3.00049]]$

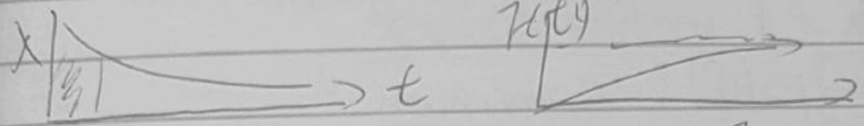
↳ $F(3.00049) - F(2.99980)$

$P(T \in [a,b]) = \int_a^b f(t)\,dt = F(b) - F(a)$

important

$f(t) := \frac{d}{dt}(F(t)) = \frac{d}{dt}[1 - e^{-\lambda t}] = \lambda e^{-\lambda t} \geq 0$? ↳ Yes

Probability density function (PDF)



~~f(t)~~ exponential decay function

$\lambda=2 \quad f(1)=2e^{-2\cdot 1}\approx 0.27 \neq P(1)=0$

$f(0.1)=2e^{2\cdot .1}\approx 1.43>1$

↳ CDF moving very quickly

collecting prob. quickly.

→ not measuring prob. measuring prob. density

PDF is an abstract metric good for things

(1) integrating to get prob (region) via F.T.C.

(2) compare two points relative likelihood

$6\approx \dfrac{f(0.1)}{f(1)}=\dfrac{P(T\in[0.1,\,0.1+\varepsilon])\,\frac{1}{\varepsilon}}{P(T\in[1,\,1+\varepsilon])\,\frac{1}{\varepsilon}}$ → realization around .1 / → realization around 1

Consider $\lim_{\varepsilon\to 0}\dfrac{P(T\in[0.1,\,0.1+\varepsilon)}{\varepsilon}\to F(0.1+\varepsilon)-F(0.1)$

$\dfrac{P(T\in[1,\,1+\varepsilon])}{\varepsilon}\to F(1+\varepsilon)-F(1)$

$1=P(T\in(-\infty,\infty))=\int_{-\infty}^{\infty}f(t)\,dt=1$ → PDF property

if I sum up PDF have to get 1

* → the prob. that numbers up in the supp. is the integral of supp.

→ it has to realize something

a/n → $\sum_{x\in Supp[X]} p(x)=1$ for discrete r.v.s X.

properities of continous r.v X

(1) $|supp[X]| = |\mathbb{R}| \} \to$ continuum

(2) has valid CDF $F(x)$ with no jumps, but gaps ok

(3) PMF does not exist.

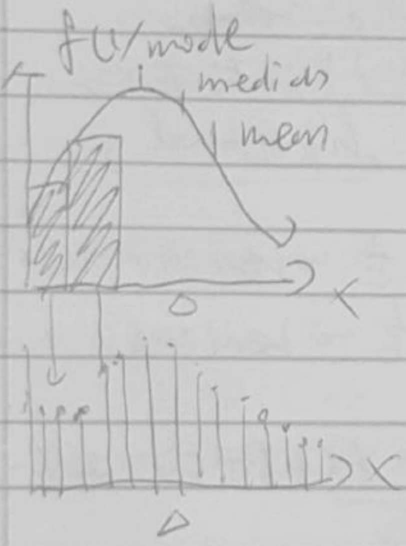
(4) PDF exist $f(x)$ (a) $f(x) \geq 0$

(b) $\int_{supp[X]} f(x)\,dx = 1$

X_1, X_2 cont. r.v and $X_1 \overset{d}{=} X_2$ if

$f_1(x) = f_2(x)$ PDFs the same or

$F_1(x) = F_2(x)$ CDFs the same



$$E[X] \approx \sum_{x \in \text{Approx } supp[X]} x\,p(x)$$

$$\downarrow$$

$$E[X] = \int_{supp[X]} x f(x)\,dx$$

$$E[g(x)] = \int_{supp[X]} g(x) f(x)\,dx$$

$$var[X] = E[(X-\mu)^2]$$

$$\sigma^2 = \int_{supp(x)} (x-\mu)^2 f(x)\,dx$$

$E[aX+c] = a\mu + c$

$var[aX+c] = a^2\sigma^2 \to SD[aX+c] = |a|\sigma$

$E[\sum X_i] = \sum E[X_i] = n\mu$ to if iid

$var[\sum X_i] = \sum var[X_i] = n\sigma^2$ $\to$ if iid

"Brand name r.v"

$\underbrace{X \sim EXP(\lambda)}$ $= \lambda e^{-\lambda x}$

↳ exponential r.V

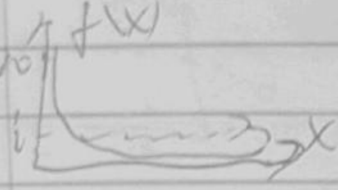
$supp[X] = (0, \infty)$

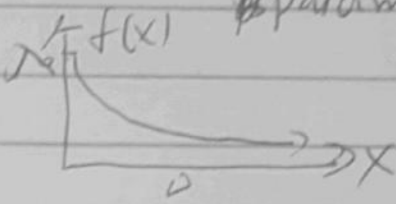
$\lambda = np$, $n \to \in \mathbb{N}$

p ↳ $(0,1)$

param space: $\lambda \in (0, \infty)$

↳ rate





$\int u\,dv = uv - \int v\,du$

↳ integration by parts

$E[X] = \int_0^\infty \underbrace{x\lambda e^{-\lambda x}}_{f(x)}\,dx$

$= \lambda \int_0^\infty x e^{-\lambda x}\,dx$

Let $u = x$ Let $dv = e^{-\lambda x}$

$du = dx$ $v = -\frac{1}{\lambda} e^{-\lambda x}$

$= \lambda \left[-\frac{1}{\lambda} x e^{-\lambda x} - \frac{1}{\lambda^2} e^{-\lambda x} \right]_0^\infty$

$\int v\,du = \int -\frac{1}{\lambda} e^{-\lambda} dx = \frac{1}{\lambda^2} e^{-\lambda x}$

$-\left(\lim_{x\to\infty} \underbrace{x e^{-\lambda x}}_{\frac{x}{e^{\lambda x}}} + \lim_{x\to\infty} \frac{1}{\lambda} e^{-\lambda x} \right) - \left(0 e^{-\lambda(0)} + \frac{1}{\lambda} e^{-\lambda(0)} \right)$

$-((0+0) - (0 + \frac{1}{\lambda})) = \boxed{\frac{1}{\lambda}}$

$X \sim Geom(p)$

$E[X] = \frac{1}{p} \to = \frac{1}{np} \Rightarrow \frac{1}{\lambda}$

Exponential has the memorylessness property

$P(X > a+b \mid X > b)$

$= P(X > a) = e^{-\lambda a}$

$P(X \le x) = F(x) = 1 - e^{-\lambda x}$

$P(X > x) = 1 - F(x) = e^{-\lambda x}$

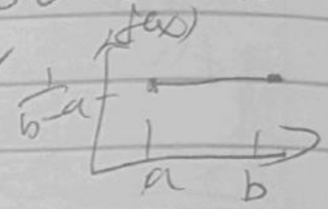
$P(X > a+b) \Rightarrow e^{-\lambda(a+b)}$

$\frac{P(X > a+b, \& X > b)}{\underbrace{P(X > b)}_{e^{-\lambda b}}} = \frac{e^{-\lambda a} \cancel{e^{-\lambda b}}}{\cancel{e^{-\lambda b}}} = e^{-\lambda a}$

	single stop	mult. stop
discrete	Geom	NegBinom
continue	Exponential	Erlang (Gamma

$X \sim$ uniform $(\{1, 7, 283\}) \to$ discrete uniform

$X \sim (a,b) \to$ "uniform" density $= \frac{1}{b-a} = f(x)$

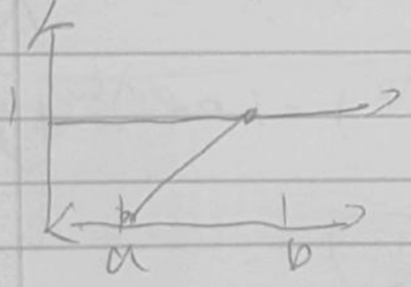


CDF constant levels
differentiate to 0

$supp[X] = [a,b]$

param space
$a \in \mathbb{R}$
$b \in \mathbb{R}$ but $a<b$

$F(x) = \int f(x)\,dx + c$
$= \int \frac{1}{b-a}\,dx + c$
$= \frac{x}{b-a} + c$



$F(a) = 0 \Rightarrow \frac{a}{b-a} + c = 0$
$\Rightarrow c = \frac{-a}{b-a}$

$\boxed{\Rightarrow F(x) = \frac{x-a}{b-a}}$

$E[X] = \int_a^b x \frac{1}{b-a}\,dx = \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b = \frac{b^2-a^2}{2(b-a)} = \frac{(b-a)(b+a)}{2(b-a)} = \frac{b+a}{2}$

$Var[X] = \sigma^2 = E[X^2] - \mu^2 = \int_a^b x^2 \frac{1}{b-a}\,dx - \left(\frac{a+b}{2}\right)^2 \Rightarrow \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b = \frac{1}{3}\frac{b^3-a^3}{a-b}$

$= \frac{b^2+ab+a^2}{3} - \frac{a^2+2ab+b^2}{4} \Rightarrow \frac{a^2-2ab+b^2}{12} = \frac{(b-a)^2}{12} = \boxed{\sigma = \frac{b-a}{\sqrt{12}}}$

$a=0,\ b=1$
$X \sim$ uniform$(0,1)$