Lecture 5 Math 241 9/8/16

5-card draw poker. You get 5 cards of the 52... what is p(win) for each way to win?

$$P(\text{Royal Flush}) = \frac{|\text{Roy Fl}|}{\binom{52}{5} = 2{,}598{,}960} = \frac{\binom{4}{1}}{\binom{52}{5}} \approx \frac{1}{649{,}740}$$

10, J, Q, K, A are suit

only 4 suits

$$P(\text{Str. Flush}) = \frac{\binom{4}{1}\binom{9}{1}}{\binom{52}{5}} = \frac{\binom{4}{1}\binom{9}{1}\binom{5}{5}}{\binom{52}{5}} = \frac{\binom{4}{1}\binom{10}{1} - 4}{\binom{52}{5}}$$

(Str. Flush: all combs in a row; all suits same suit)

A 2 3 9 5

2 3 9 5 6

3 9 5 6 7

⋮

9 10 J Q K

$\binom{10}{1}$ ~~10 J Q K A~~ roy. fl.

$\}$ 9

$$P(\text{4-of-a-kind}) = \frac{\binom{13}{1}\binom{12}{1}\binom{4}{1}}{\binom{52}{5}}$$

A A A 3 ♢

$$P(\text{full house}) = \frac{\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}}{\binom{52}{5}} \neq \frac{\binom{13}{2}\binom{4}{3}\binom{4}{3}}{\binom{52}{5}} = \frac{1}{2}(P(\dots))$$

AAA33

$\binom{13}{1}\binom{12}{1} = 13 \cdot 12 \qquad \binom{13}{2} = \frac{13 \cdot 12}{2}$ ← why?

AAA33 ≠ 333AA

$$P(\text{flush}) = \frac{\binom{4}{1}\binom{13}{5} - \binom{4}{1}\binom{10}{1}}{\binom{52}{5}}$$ → technically ....

$$P(\text{straight}) = \frac{\binom{10}{1}\binom{4}{1}^5 - \binom{4}{1}\binom{10}{1}}{\binom{52}{5}}$$ ← why not $\binom{40}{2}$?

$$P(3 \text{ of a kind}) = \frac{\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}^2}{\binom{52}{5}}$$

AA33θ ≠ 33AAθ

$$P(2 \text{ of a kind}) = \frac{\binom{13}{2}\binom{4}{2}^2\binom{11}{1}\binom{4}{1}}{\binom{52}{5}}$$ why not $\binom{13}{1}\binom{12}{1}$?

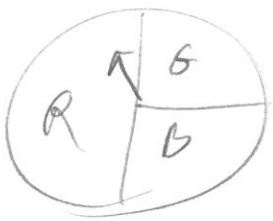
$$P(\text{no pair}) = \frac{\binom{13}{5}\binom{4}{1}^5 - \text{straights} - \text{flush} + \text{str flush}}{\binom{52}{5}}$$

$P(A) = \frac{|A|}{|\Omega|}$ if $\forall \omega \; P(\omega) = \frac{1}{|\Omega|}$ → all outcomes equally likely

How to guarantee equally likely with people sitting?

In reality, ... difficult

$\Omega = \{R, G, B\}$ experiment:



$P(\{R\}) = \frac{|\{R\}|}{|\Omega|} = \frac{1}{3}$? NO...

Weather $\Omega = \{$sun, cloudy, rain, snow$\}$

$P(\{snow\}) = \frac{1}{4}$? No...

$\Omega = \{$Trump, Clinton$\}$

$P(\{Trump\}) = \frac{1}{2}$? No...

$P(H) = P(T) = \frac{1}{2}$? Are you sure?

If $P(\omega) \neq \frac{1}{\Omega}\ \forall \omega$ ... need better def. of prob.

(I) Lim. Freq. Def. (most popular)

Indicator Function

$$\mathbb{1}_{\omega \in A} := \begin{cases} 1 \text{ if } \omega \in A \\ 0 \text{ if } \omega \notin A \end{cases}$$

$$\lim_{n \to \infty} \frac{\sum_{i=1}^{n} \mathbb{1}_{\omega_i \in A}}{n}$$ prop. of times event occurs

Problems
(1) $n \neq \infty$ ... we never see it! thus, $P(A) \approx \frac{1}{N}\sum_{i=1}^{N} \mathbb{1}_{\omega_i \in A}$
we only can get approx. probs. Need repeatable experiment!
(2) Is ω drawn in same way each time?
(3) Non generic $P($OJ Simpson guilty$)$? $P($Trump wins$)$?
(4) $P($flipped coin being $H) = 0$ or 1 No!

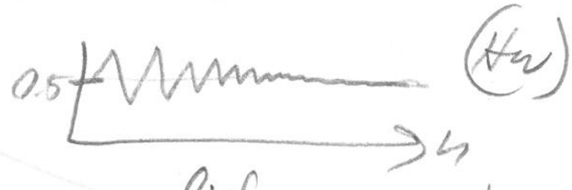
History of prob... began with gambling games... Same game repeated over and over

1654 M. de Chevalier de Mere wrote in a letter to Pascal, Fermat

$P(\{\geq 1$ double 6 in 24 rolls of two dice$\}) < \frac{1}{2}$

turns out... its .4914 (proof soon)

Also... LRF is an objective definition i.e. a property of the physical world. If we weren't here... $P(H) = \frac{1}{2}$ still.



lrf → prob.
defn by LRF

II Propensity Theory

Objects have an inherent property to go one way or another... thus the prob. which induces the l.r.f.

e.g. U238 has $\frac{1}{2}$ life of 4.5 Byr. Randomness is due to quantum mechanics. Still objective... if we're not here U238 still is radioactive..

Problems

① Calculate it? ② Not general.... P(OJ Simpson guilty) event already happened....

P(Trump)... stretch!!

"Astragali"

(III) Logical Theory – given a set of evidence... all people agree on prob.

— wrong — Math 341 discusses this

(IV) Subjective Theory – " " " " " " do not need to agree on prob

$P_1(\text{Trump}) = \ldots$

$P_2(\text{Trump}) = \ldots$

$P(\text{coin flip is H}) = \{0, 1\}$ objective

$\in [0,1]$ subjectively

$P(\text{OJ Simpson guilty}) \in [0,1]$

$P(F = ma \text{ is true}) =$ can be addressed "degree of belief"

"degree of corroboration"

<u>Problems</u>

(1) Everyone has different opinions... no ground truth...

$P(\text{Einstein's relativity is true})$

$= \{0, 1\}$ only

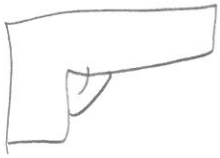
$\in [0,1]$ only because we don't know how the universe works

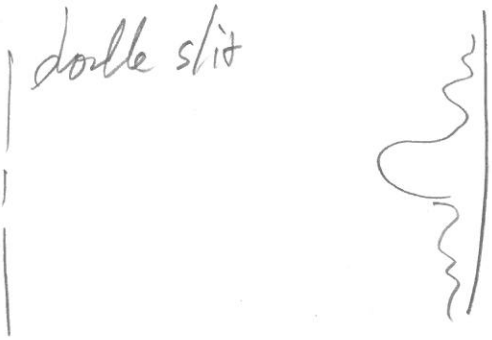
we are ignorant...

$P(H) = \frac{1}{2}$ ... is it really random?? If we understood everything about the universe wouldn't it be 0 or 1?? Certainly

Laplace ... classic ... determinism ...

Curie ... 1920's





wave-particle duality

Quantum theory ⇒

Randomness fundamental part of universe

(at least for very small + large)

Einstein "dice with the universe"

Why did this prob get invented before 1600's?

Greeks did geometry ... why not probability ??

< boxes prizes > not equally likely ...

even

Engineers ... dice, coins, cards ...

⇒ No way to define prob in the "real world"

But there is a mathematical theory of prob ...

In particular, we find the usual admiration for Newtonian mechanics, and the consequent belief in *universal determinism*. Indeed, Laplace's *Philosophical Essay on Probabilities* of 1814 gives one of the most famous formulations of the thesis of universal determinism. This is the formulation involving what is known as *Laplace's demon*. I will expound it in the next section.

Universal determinism and Laplace's demon

Laplace writes:

> We ought then to regard the present state of the universe as the effect of its anterior state and as the cause of the one which is to follow. Given for one instant an intelligence which could comprehend all the forces by which nature is animated and the respective situation of the beings who compose it – an intelligence sufficiently vast to submit these data to analysis – it would embrace in the same formula the movements of the greatest bodies of the universe and those of the lightest atom; for it, nothing would be uncertain and the future, as the past would be present to its eyes.
>
> (1814: 4)

The vast intelligence here described has come to be known as Laplace's demon. The idea is obviously founded on that of a human scientist (perhaps Laplace himself) using Newtonian mechanics to calculate the future paths of planets and comets. Extrapolating from this success, it was natural to suppose that a sufficiently vast intelligence could calculate the entire future course of the universe. Laplace himself relates his vast intelligence to human successes in astronomy. As he says:

> The human mind offers, in the perfection which it has been able to give to astronomy, a feeble idea of this intelligence. Its discoveries in mechanics and geometry, added to that of universal gravity, have enabled it to comprehend in the same analytical expressions the past and future states of the system of the world.
>
> (Laplace 1814: 4)

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"P" is a function s.t.

(a) $\exists\ \Omega \neq \phi$ s.t. $P(\Omega) = 1$

(b) $P(A) \geq 0 \quad \forall A \subseteq \Omega$

(c) If $A_1, A_2, \ldots$ disjoint $\Rightarrow P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$

prob of you can add disjoint

Thm 1 $P(A) = 1 - P(A^c)$

$\Omega = A \cup A^c$ set theory

$P(\Omega) = P(A \cup A^c)$ by def of function

$P(\Omega) = P(A) + P(A^c)$ via (c)

$1 = P(A) + P(A^c)$ via (a)

$\Rightarrow P(A) = 1 - P(A^c)$ (algebra)

Thm 2 $P(\phi) = 0$

$P(\Omega) = 1 - P(\Omega^c) = 1 - P(\phi) \Rightarrow 1 = 1 - P(\phi) \Rightarrow P(\phi) = 0$

Thm 3

$A \subseteq B \Rightarrow P(A) \leq P(B)$

$\Rightarrow C := B \setminus A$

$\Rightarrow A \cup C = B \ \& \ A \cap C = \phi$

$P(A \cup C) = P(B)$

$P(A) + P(C) = P(B)$ (c)

$P(C) = P(B) - P(A) \geq 0$ (b)

$P(B) \geq P(A)$ ✓