

$$\Omega = \{B, T, R, S, M, A\} \quad P(\text{gathering garden}) = \frac{\frac{3}{2} \frac{3}{2} \frac{2}{2} \frac{2}{2} \frac{1}{1} \frac{1}{1}}{(5!)} = \frac{2(3!)^2}{6!} = \frac{72}{6!} = \frac{1}{10}$$

5 flowers 30's, 2x's

If all distances # orderings = 5!

but if orchids indistinguishable, $\frac{5!}{3!}$

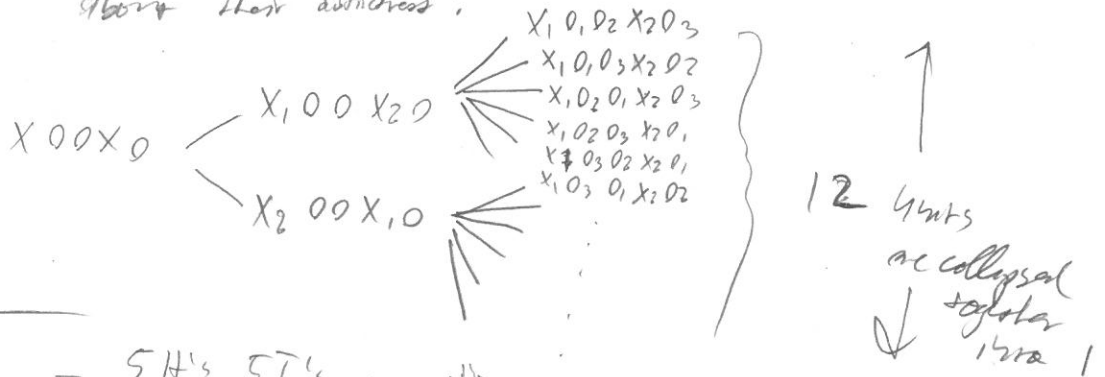
... chrys " $\frac{5!}{2!}$

If orchids and chrys indistinguishable $\frac{5!}{3!2!} = \frac{5 \cdot 4 \cdot 3 \cdot 2}{3 \cdot 2 \cdot 2} = 10$

000XX	X0X00
00X0X	XX000
0X00X	0XX00
X000X	00XX0
X00X0	0X0X0

orderings

Does $\frac{5!}{12} \cdot 12 = 5!$? Does each of these orderings become 12 if you "begin to care" about their distances?



10 coin flips - 5 H's, 5 T's s.t. H's > T's

$$P(5H, 5T) = \frac{\frac{10!}{5!5!}}{2^{10}} = .24609375$$

1000 coin flips

calculator bump!

$$P(600H, 400T) = \frac{\frac{1000!}{600!400!}}{2^{1000}} \in [0, 1] \text{ easily handled \#!}$$

Solution to question... use logs

2

$$\ln(p) = \ln(1000!) - \ln(600!) - \ln(400!) - 1000 \ln(2)$$

since $n! = \prod_{i=1}^n i \Rightarrow \ln(n!) = \sum_{i=1}^n \ln(i)$

no more blarney but tedious!!!

$$n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n \quad \text{Stirling's Approx.}$$

$$\Rightarrow \ln(n!) \approx \frac{1}{2} \ln(2\pi n) + \left(n + \frac{1}{2}\right) \ln(n) - n$$

$$\begin{aligned} \Rightarrow \ln(p) &= \left(\frac{1}{2} \ln(2\pi n) + 1000.5 \ln(1000) - 1000\right) - \left(\frac{1}{2} \ln(2\pi n) + 600.5 \ln(600) - 600\right) \\ &\quad - \left(\frac{1}{2} \ln(2\pi n) + 400.5 \ln(400) - 400\right) - 1000 \ln(2) \\ &= 1000.5 \ln(1000) - 600.5 \ln(600) - 400.5 \ln(400) - \frac{1}{2} \ln(2\pi) - 1000 \ln(2) \\ &= -23.79 \end{aligned}$$

$$\Rightarrow p = e^{\ln(p)} = 4.6 \times 10^{-11} \quad (\text{rare})$$

$$\frac{1000!}{600! 400!} \leftarrow \frac{n!}{k!(n-k)!} \quad \text{s.t. } k \leq n, n \in \mathbb{N} \quad \text{is a quantity that comes up a lot!}$$

Imagine $\{J, B, S, R, M, A\}$ 6 chairs $\Rightarrow 6!$

But Chris $\Rightarrow {}^6P_4 = \frac{6!}{(6-4)!} = \frac{6!}{2!} \quad \underline{6} \quad \underline{5} \quad \underline{4} \quad \underline{3}$

What if... I didn't care about the order of the 4 people? How many outcomes?

Sample without replacement (but order doesn't matter)

$\{J, B, S, R\}$	$\{B, S, R, M\}$
$\{J, B, S, M\}$	$\{B, S, R, A\}$
$\{J, B, S, A\}$	$\{B, S, M, A\}$
$\{J, B, R, M\}$	$\{B, R, M, A\}$
$\{J, B, R, A\}$	$\{S, R, M, A\}$
$\{J, B, M, A\}$	
$\{J, S, R, M\}$	
$\{J, S, R, A\}$	
$\{J, S, M, A\}$	
$\{J, R, M, A\}$	

$\Rightarrow 15$

Unique

S.t.

Order

doesn't

matter

$$15 \cdot 4! = \text{all online}$$

$$6 \cdot 5 \cdot 4 \cdot 3$$

"6 choose 4"

$$\Rightarrow \frac{6P_4}{4!} = \frac{6!}{2! \cdot 4!} = \frac{6!}{4! \cdot 2!} = 6C_4 \text{ or } \binom{6}{4}$$

↓
combinations of 6

$$nC_k = \binom{n}{k} = \frac{n!}{(n-k)!k!}$$

Identities

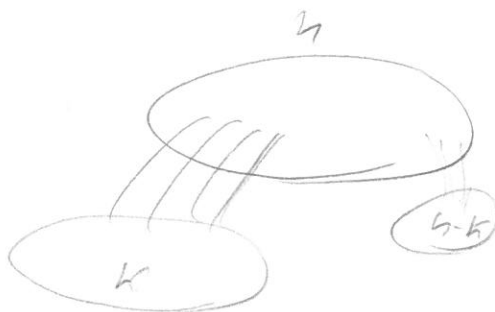
$$① \binom{n}{1} = \frac{n!}{(n-1)!1!} = n$$

$$② \binom{n}{n-1} = \frac{n!}{(n-(n-1))!(n-1)!} = \frac{n!}{1!(n-1)!} = n$$

$$③ \binom{n}{n} = \frac{n!}{n!0!} = 1$$

$$④ \binom{n}{0} = \frac{n!}{0!n!} = 1 \text{ weird...}$$

$$⑤ \binom{n}{n-k} = \frac{n!}{(n-k)!k!} = \binom{n}{k}$$



each choice of k leaves $n-k$ so there must be exact same #!

Consider order matters

$$P(4 \text{ cars, I get Jane}) = \frac{\frac{1}{5} \frac{5}{2} \frac{4}{3} \frac{3}{1} + \frac{5}{1} \frac{1}{5} \frac{4}{3} \frac{3}{4}}{6P_4} = \frac{4(5 \cdot 4 \cdot 3)}{6 \cdot 5 \cdot 4 \cdot 3} = \frac{2}{3}$$

Consider order doesn't matter $\{J, -, -, -\}$

$$= \frac{\binom{5}{3}}{\binom{6}{4}} = \frac{\frac{5!}{3!2!}}{\frac{6!}{4!2!}} = \frac{10}{15} = \frac{2}{3}$$

all possible ways to choose the "other" 3 people

Recall set A , let $|A|=n, n \in \mathbb{N}$

and $|2^A| = 2^{|A|} = 2^n$
subsets of A (order doesn't matter)

$$2^A := \{B: B \subseteq A\}$$
$$= \{B: B \subseteq A \ \& \ |B|=1\} \cup \{B: B \subseteq A \ \& \ |B|=2\} \cup \dots$$

Why?
mostly additive?
collectively additive? $\Rightarrow \dots$

$$2^A = \bigcup_{i=0}^n \{B: B \subseteq A \ \& \ |B|=i\}$$

$$n = |2^A| = \sum_{i=0}^n |\{B: B \subseteq A \ \& \ |B|=i\}|$$

How many subsets of n with size i s.t.
order doesn't matter

$$\Rightarrow 2^n = \sum_{i=0}^n \binom{n}{i} \quad \text{Cool...}$$

$$(a+b)^2 = (a+b)(a+b) = \overset{2 \times 2 \text{ terms}}{a^2 + ab + ba + b^2} = \overset{4 \text{ terms}}{a^2 + 2ab + b^2}$$

$$(a+b)^3 = (a+b)(a+b)(a+b) = \overset{8 \text{ terms}}{a^3 + \dots} = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a+b)^4 = (a+b)(a+b)(a+b)(a+b) = \overset{4a^3s}{\binom{4}{1}a^4} + \overset{4a^2s}{\binom{4}{3}a^3b} + \binom{4}{2}a^2b^2 + \binom{4}{1}ab^3 + \binom{4}{0}b^4$$

\uparrow take 4 of a \uparrow take 3 of a

$$(a+b)^n = \binom{n}{n}a^n + \binom{n}{n-1}a^{n-1}b + \binom{n}{n-2}a^{n-2}b^2 + \dots + \binom{n}{2}a^2b^{n-2} + \binom{n}{1}ab^{n-1} + \binom{n}{0}b^n$$

$$= \sum_{i=0}^n \binom{n}{i} a^i b^{n-i} = \sum_{i=0}^n \binom{n}{i} a^{n-i} b^i \quad \text{Binomial Thm.}$$

$$(1+x)^n = \sum_{i=0}^n \binom{n}{i} 1^{n-i} x^i = \sum_{i=0}^n \binom{n}{i} x^i = \binom{n}{0}x^0 + \binom{n}{1}x^1 + \binom{n}{2}x^2 + \dots + \binom{n}{n-1}x^{n-1} + \binom{n}{n}x^n$$

$$= 1 + \sum_{i=1}^{n-1} \binom{n}{i} x^i + x^n$$

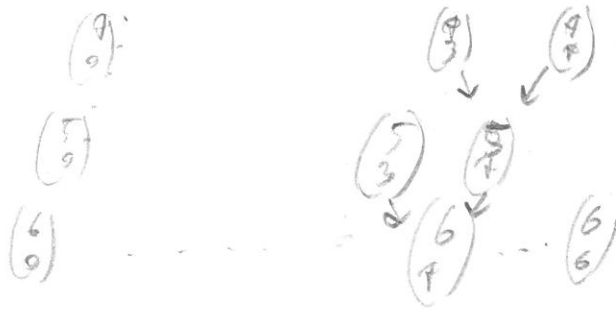
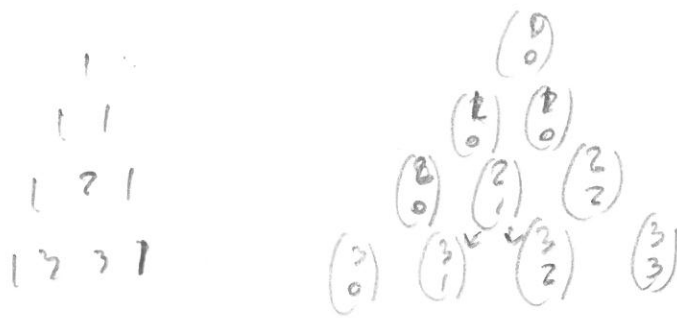
$$= (1+x)(1+x)^{n-1} = (1+x) \sum_{i=0}^{n-1} \binom{n-1}{i} x^i = \sum_{i=0}^{n-1} \binom{n-1}{i} x^i + \sum_{i=0}^{n-1} \binom{n-1}{i} x^{i+1}$$

\uparrow reindexing trick

$$= \sum_{i=0}^{n-1} \binom{n-1}{i} x^i + \sum_{i=1}^n \binom{n-1}{i-1} x^i$$

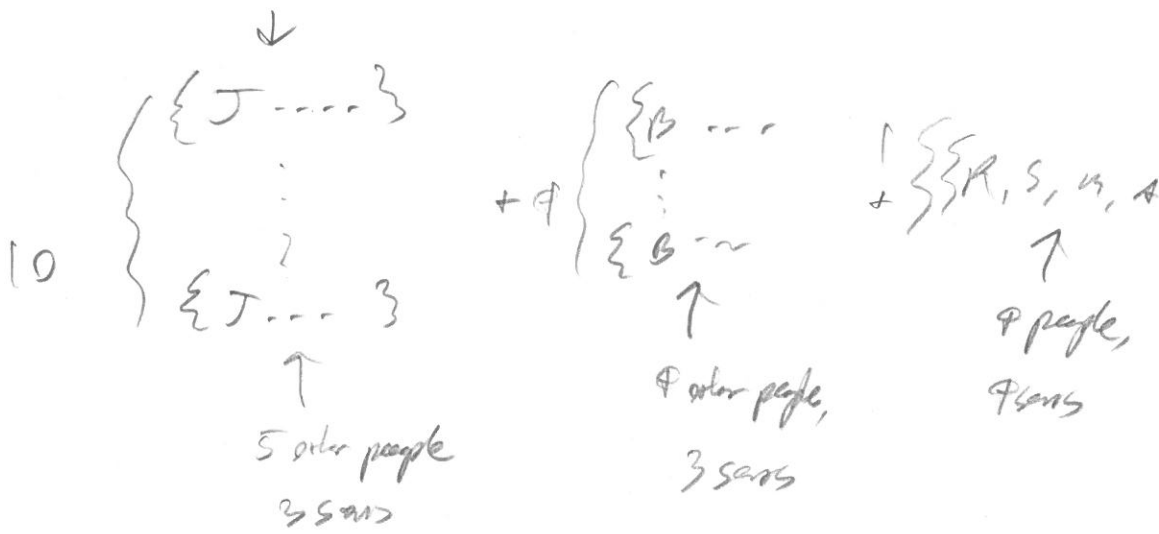
$$= \underbrace{\binom{n-1}{0}}_1 x^0 + \sum_{i=1}^{n-1} \left(\binom{n-1}{i} + \binom{n-1}{i-1} \right) x^i + \underbrace{\binom{n-1}{n-1}}_{x^n} x^n$$

$$\Rightarrow \binom{n}{i} = \binom{n-1}{i} + \binom{n-1}{i-1} \quad \text{"Pascal's Rule"}$$



$$\binom{6}{4} = \binom{5}{3} + \binom{4}{3} + \binom{4}{2}$$

$$= 10 + 4 + 1 = 15$$



$$D = R \times S, \quad R = \{A, 2, 3, \dots, 10, J, Q, K\} \quad |R| = 13$$

$$S = \{A, \heartsuit, \diamondsuit, \clubsuit\}$$

$$|S| = 4$$

Solution: $D = R \times S \cup \{Jok1, Jok2\}$

but we don't care...

$$|D| = 52$$

$$= \{ \langle 2, \heartsuit \rangle, \langle A, \heartsuit \rangle, \dots \}$$