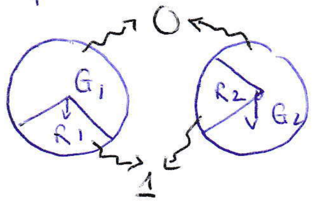
$\sum_{x \in Supp[X]} p(x) = 1$, $\sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x}$

$(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^i b^{n-i}$ binomial Theorem. Let $a = p$, $b = 1-p$, $i = x$ $\Rightarrow (p+(1-p))^n = \sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x}$

* X_1 and X_2 are independent if
$$P(X_1 = x_1 \mid X_2 = x_2) = P(X_1 = x_1)$$
$$P(X_2 = x_2 \mid X_1 = x_1) = P(X_2 = x_2)$$
$\forall x_1 \in Supp[X], \forall x_2 \in Supp[X]$
$$P(X_1 = x_1, X_2 = x_2) = P(X_1 = x_1) P(X_2 = x_2)$$

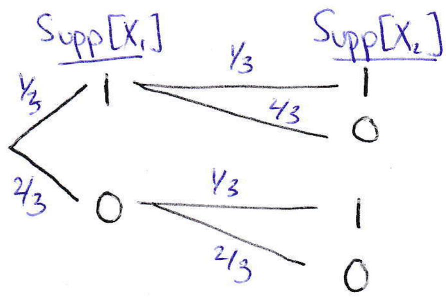
* X_1 and X_2 are independent and identically distributed (iid) if X_1, X_2 are independent and $X_1 \overset{d}{=} X_2$ and denoted $X_1, X_2 \overset{iid}{\sim}$



$X_1, X_2 \overset{iid}{\sim} Bern(\frac{1}{3})$ $Supp[T] = \{0, 1, 2\}$

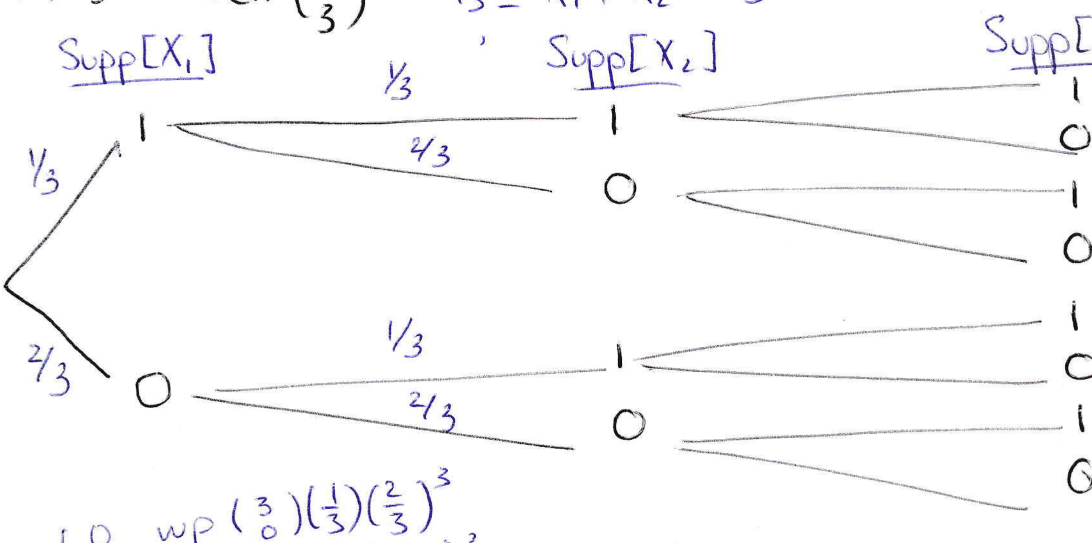
$T_2 = X_1 + X_2 = g(X_1, X_2)$

$$T \sim \begin{cases} 0 & wp\ 4/9 \\ 1 & wp\ 4/9 \\ 2 & wp\ 1/9 \end{cases}$$



	T
$P(X_1 = 1, X_2 = 1) = \frac{1}{9}$	2
$P(X_1 = 1, X_2 = 0) = \frac{2}{9}$	1
$P(X_1 = 0, X_2 = 1) = \frac{2}{9}$	1
$P(X_1 = 0, X_2 = 0) = \frac{4}{9}$	0

$X_1, X_2, X_3 \overset{iid}{\sim} Bern(\frac{1}{3})$ $T_3 = X_1 + X_2 + X_3$



$Supp[X_3]$		T
1	$(\frac{1}{3})^3(\frac{2}{3})^0$	3
0	$(\frac{1}{3})^2(\frac{2}{3})$	2
1	$(\frac{1}{3})^2(\frac{2}{3})$	2
0	$(\frac{1}{3})^1(\frac{2}{3})^2$	1
1	$(\frac{1}{3})^2(\frac{2}{3})$	2
0	$(\frac{1}{3})(\frac{2}{3})^2$	1
1	$(\frac{1}{3})(\frac{2}{3})^2$	1
0	$(\frac{1}{3})^0(\frac{2}{3})^3$	0

$$T \sim \begin{cases} 0 & wp\ \binom{3}{0}(\frac{1}{3})(\frac{2}{3})^3 \\ 1 & wp\ \binom{3}{1}(\frac{1}{3})^1(\frac{2}{3})^2 \\ 2 & wp\ \binom{3}{2}(\frac{1}{3})^2(\frac{2}{3})^1 \\ 3 & wp\ \binom{3}{3}(\frac{1}{3})^3(\frac{2}{3})^0 \end{cases}$$

$X_1, X_2, \dots X_n \overset{iid}{\sim} Bern(\frac{1}{3}) \Rightarrow$

$$T \begin{cases} 0 & wp\ \binom{n}{0}(\frac{1}{3})^0(\frac{2}{3})^n \\ 1 & wp\ \binom{n}{1}(\frac{1}{3})(\frac{2}{3})^{n-1} \\ 2 & wp\ \binom{n}{2}(\frac{1}{3})^2(\frac{2}{3})^{n-2} \\ \vdots & \\ n-2 & wp\ \binom{n}{n-2}(\frac{1}{3})^{n-2}(\frac{2}{3})^2 \\ n-1 & wp\ \binom{n}{n-1}(\frac{1}{3})^{n-1}(\frac{2}{3})^1 \\ n & wp\ \binom{n}{n}(\frac{1}{3})^n(\frac{2}{3})^0 \end{cases}$$

$T_n = \sum_{i=1}^{n} X_i$, $Supp[X] = \{0, 1, \dots, n\}$

$\Rightarrow T \sim Bino(n,p)$

2 concepts for binomial

$\lim_{n\to\infty} Hyper(n,p,N$ OR $X_1, X_2, \ldots, X_n \overset{iid}{\sim} Bern(p)$ $X_1 + \ldots + X_n \sim Bino(n,p)$

1000 coin flips $X \sim Bino(1000, \frac{1}{2})$ $P(600H) = P(X=600) = \binom{1000}{600}\left(\frac{1}{2}\right)^{600}\left(\frac{1}{2}\right)^{400}$

$F(x) = P(X \le x)$ CDF of Binomial $F(x) = \sum_{i=0}^{x} p(i) = \sum_{i=0}^{x} \binom{n}{i} p^i (1-p)^{n-i}$

$= I_{1-p}(n-k, 1+k) := (n-k)\binom{n}{k} \int_0^{1-p} t^{n-k-1}(1-t)\,dt$

$X_1, X_2, \ldots \overset{iid}{\sim} Bern(p)$ infinite sequence of iid r.v's

$T = \min\{t : X_t = 1\}$ $\underline{0}\ \underline{0}\ \underline{1}\ \underline{0}\ \underline{1}\ \underline{0}\ \underline{0}\ \underline{1}$

$\{3, 5, 6, \ldots\}$

$P(1) = P(T=1) = P(X_1 = 1) = p$

$P(2) = P(T=2) = P(X_1 = 0, X_2 = 1) = P(X_1=0) \cdot P(X_2=1) = (1-p) \cdot p$

$P(3) = P(T=3) = P(X_1=0, X_2=0, X_3=1) = (1-p)^2 \cdot p$

$\vdots$

$P(x) = P(T=x) = (1-p)^{x-1} \cdot p$

Parameter Space : $p \in (0,1)$ $Supp[X] = \mathbb{N}$

$X \sim Geometric := (1-p)^{x-1} \cdot p$

$q = 1-p$

$\sum_{x \in Supp[X]} p(x) = 1$ $\sum_{x=1}^{\infty} (1-p)^{x-1} \frac{p}{p} = \frac{1}{p}$ $\Rightarrow \sum_{x=1}^{\infty} q^x = \frac{1}{1-q}$

$S = \sum_{x=0}^{\infty} q^x = q^0 + q^1 + q^2 + q^3 + \ldots = 1 + q + q^2 + q^3 + \ldots = 1 + q(1 + q + q^2 + \ldots)$

$= 1 + q \cdot S \rightarrow S - qS = 1$

$(1-q)S = 1 \Rightarrow S = \frac{1}{1-q}$

$\sum_{i=0}^{\infty} q^i = \frac{1}{1-q}$ Geometric series

FACT: $\sum_{x=1}^{\infty} (1-p)^{x-1} \cdot p = 1$

$F(x) = P(X \le x) = 1 - p(X > x) = 1 - (1-p)^x$

$P(X > x) = P(X = x+1) + P(X = x+2) + \ldots$

$= \sum_{i=x+1}^{\infty} (1-p)^{i-1} p = \sum_{i=1}^{\infty} (1-p)^{x+i-1} p$

$= (1-p)^x \sum_{i=1}^{\infty} (1-p)^{i-1} p$