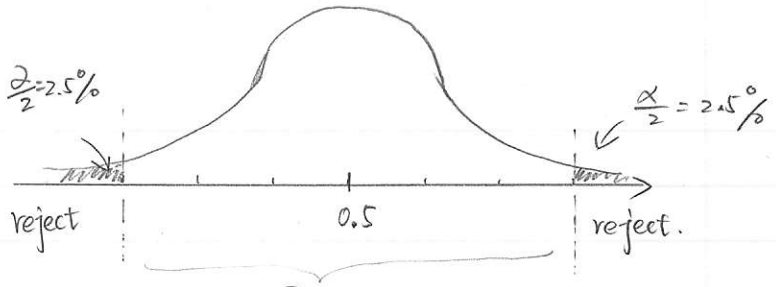
12/08/2016

$P := P(\text{male})$, $H_0: P = 0.5$, $H_a: P \neq 0.5$, we choose $\rightarrow \alpha = 0.5$, $n = 345$



$$\hat{P} \mid H_0 \text{ true} \sim N\left(p, \sqrt{\frac{P(1-P)}{n}}^2\right) = N(0.5, 0.269^2)$$

$$\left[P \pm \frac{\alpha}{2}\sqrt{\frac{(1-P)P}{n}}\right] = [0.5 \pm 2 \times 0.26] = [0.446,\ 0.554]$$

"Two side test for one proportion"

Ex: collect male. 169 males

$\Rightarrow \hat{P} = \frac{169}{345} = 0.48$ ← Retain Regin ↳ Retain H_0

	Retain H_0	Reject H_0
H_0	✓	Type I error
H_a	Type II error	✓

$\alpha \downarrow \Rightarrow P(\text{Type II error}) \uparrow$

$\alpha \uparrow \Rightarrow P(\text{Type II error}) \downarrow$

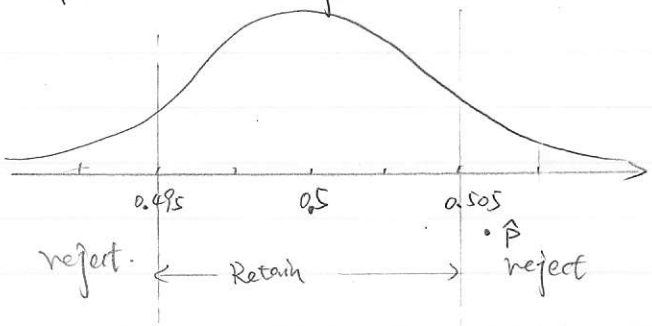
☆ I get more data, $n = 4{,}427{,}000$ (2008 baby born in USA)

$SE(\hat{p}) = \sqrt{\frac{.5(1-0.5)}{4{,}427{,}000}}$; $\text{RetainRegin} = \left[P \pm \frac{\alpha}{2}\sqrt{\frac{P(1-P)}{n}}\right] = \left[0.5 \pm 2\sqrt{\frac{0.5 \times 0.5}{4{,}427{,}000}}\right] = [0.495, 0.505]$

male = 2173000, $\Rightarrow \hat{P} = \frac{2173000}{4{,}427{,}000} \approx 0.51165$ & Retain $\Rightarrow$ Reject H_0.

* $n \uparrow \Rightarrow \alpha$ Not change $\Rightarrow$ Type II error ↓

(α is Type I error, is always same value.)



— Retain H_0 ≠ Accepting H_0

$H_0: P = 0.5$
$H_a: P \neq 0.5$
$n = 345$
$\alpha = 5\%$
$\hat{P} = 0.48 \Rightarrow$ Retain H_0

$H_0: P = 0.50001$
$H_a: P \neq 0.50001$
$n = 345$
$\alpha = 5\%$
$\hat{P} = 0.48 \Rightarrow$ Reject H_0
???

Skeptical, will not allow any arguments to sway his opinion

↓

Ex: Ho: UFO's & Aliens do not exist
Ha: do exist.
α low.

Gullible will allow any argument to sway his opinion.

↓

Ho = UFO & Aliens do not exist
Ha = do exist.
α high.

Ho = Aliens do not exist
Ha = don't
α = low

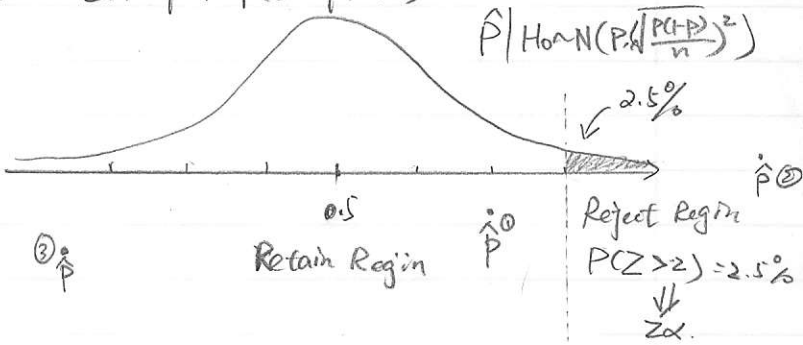
You won't convince this guy that Aliens don't exist.

Ho = Aliens do exist
Ha = don't
α = high.

You can easily convince him that aliens don't exist.

Ex: If more than 5% of passage complain. the Uber driver is fired.. Uber make diction (keep/fire) after 1000 ride. Let p := P(complain)

Ho : good driver $p \le 0.05$
Ha : bad driver $p > 0.05$



Let $\alpha = 2.5\%$ $z\alpha := F_Z^{-1}(1-\alpha)$

Type I. fire a good driver with too much complain by bad ass.
Type II. don't fire a bad driver with no complain.

— "One-Side or right side" test of One proportion.

$P(\text{Reject}) = \alpha$.

$\Rightarrow P(\text{Retain}) = (1-\alpha) \Rightarrow P(Z \le z\alpha) = 1-\alpha \Rightarrow P(\sqrt{\frac{p(1-p)}{n}} Z \le \sqrt{\frac{p(1-p)}{n}} z\alpha) = 1-\alpha$

$P(p + \sqrt{\frac{p(1-p)}{n}} Z \le p + \sqrt{\frac{p(1-p)}{n}} z\alpha) = 1-\alpha$

$P(\hat{p} \le \underbrace{p + z\alpha \sqrt{\frac{p(1-p)}{n}}}_{\text{Retain Regin}}) = 1-\alpha$

$(-\infty, p + z_\alpha \sqrt{\frac{p(1-p)}{n}}] = (-\infty, 0.5 + 2\sqrt{\frac{0.5(1-0.5)}{1000}}] = (-\infty, 0.0638)$

experment: # of complain = 71. $\hat{p} = \frac{71}{1000} = 0.071 \notin$ Retain Regin $\Rightarrow$ Reject Ho. Fire driver.

2 side $\alpha = 5\%$ $Z = 2$.
1% $Z = 2.84$

one side $\alpha = 2.5\%$ $Z = 2$
$= 0.5\%$ $Z = 2.84$

- Change to: H_0: bad drive. H_a: good drive.
$p \geq 0.05$ $p < 0.05$