9/8 - 52 cards (order doesn't matter)

Continue... 5 Decks. "5 cards draw poker"

$$P(\text{win}) = \frac{|\text{win}|}{\binom{52}{5}}$$

10 J Q K A
same suit

$$P(\text{Royal Flush}) = \frac{\binom{4}{1}}{\binom{52}{5}} = \frac{4}{} = \frac{\binom{4}{1}\binom{5}{5}}{}$$

A 2 3 4 5
2 3 4 5 6
3 4 5 6 7
⋮
9 10 J Q K
~~10 J Q K A~~

$$P(\text{str. flush}) = \frac{\binom{9}{1}\binom{4}{1}}{\binom{52}{5}}$$

" (same suit)

Ex: 7777 (|R|=13)

$$P(4 \text{ of a kind}) = \frac{\binom{13}{1}\binom{48}{1}}{\binom{52}{5}}$$

7777 K♡

$$P(\text{full house}) = \frac{\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}}{\binom{52}{5}}$$

999 JJ

← pick one suit

str flush royal flush

or $\binom{9}{1} - \binom{1}{1}$

$$P(\text{flush}) = \frac{\binom{4}{1}\left(\binom{13}{5} - \binom{10}{1}\right)}{\binom{52}{5}}$$

" all the same suit

♡ ♡ ♡ ♡ ♡

this type is not on exam.

$$P(\text{stright}) = \frac{\binom{10}{1}\binom{4}{1}^5 - \overset{\text{str}}{\binom{9}{1}}\binom{4}{1} - \overset{\text{royal}}{\binom{1}{1}}\binom{4}{1}}{\binom{52}{5}}$$

1^{st} pick one # — pick the other 3 from 4 suits. — p b/c no replacement. (must be 2 different ranks, otherwise = full house)

- $P(3 \text{ of a kind}) = \dfrac{\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}^2}{\binom{52}{5}}$

666 K♡ 3♧

13 ranks choose 2 kinds. — $\overbrace{\quad}^{(44)}$ → the rest of the 11 ranks pick 1. can't be 9/J again (= full house otherwise).

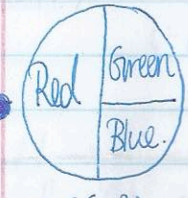
- $P(2 \text{ pair}) = \dfrac{\binom{13}{2}\binom{4}{2}^2\binom{11}{1}\binom{4}{1}}{\binom{52}{5}}$

99 JJ 3

Note: $\binom{13}{1}\binom{12}{1} \overset{?}{\neq} \binom{13}{2}$

$13 \cdot 12 \quad = \dfrac{13 \cdot 12}{2}$

In 2 pair case 99 JJ 3 = JJ 99 3
but
In full house case 999 JJ ≠ JJJ 99



$\Omega = \{R, G, B\}$ $\quad\quad$ $\Omega = \{\text{sunny, cloud, snow, rain}\}$

- $P(\{R\}) = \dfrac{|\{R\}|}{|\Omega|} = \dfrac{1}{3}$ $\quad\quad$ - $P(\{\text{snow}\}) \overset{?}{=} \dfrac{1}{4}$ YES.

Definition: (I) Long Run Freq. Def.

$\mathbb{1}_{\omega \in A} = \begin{cases} 1 & \text{if } \omega \in A \\ 0 & \text{if } \omega \notin A \end{cases}$

$$P(A) := \lim_{n \to \infty} \frac{\sum_{i=1}^{n} \mathbb{1}_{\omega_i \in A}}{n}$$

→ Problems

① $n \neq \infty$, $n < \infty$ (can't test it forever in a finite world!)
② Repeatability ω is closer from Ω same exact way each time.
③ Converge
④ Non-general

$P(A) \approx \frac{1}{n} \sum_{i=1}^{n} \mathbb{1}_{\omega_i \in A}$ for big n.

(II) Propensity Theory 1950's → Problems

- Inherit property to produce outcome prob → LRF
① Can't calculate probability's

Half Life. U 238 4.5 Billion years.

Objective Theory

Propertes of physical world not of human knowledge.

1654 $P(\{\geq$ one double-6 in 24 rolls of 2 dice$\}) \approx .4914 < \frac{1}{2}$

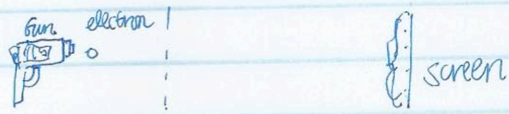
Not on any exams — Ⓘ III) Logical Theory.

Give the same evidence, everyone agrees a prob.

IV) Subjective Theory.

Give the same evidence, everyone does not have to agree.

Problem: no good truth, everything is 50% $P(F=MA)$

1920 

- randomness is embedded in the reality.

- "P" is a set function.
probability. Assume $\Omega \neq \emptyset$.

ⓐ $P(\Omega)=1$
ⓑ $P(A) \geq 0 \ \forall A$
ⓒ $A_1, A_2, A_3, \ldots\ldots$ disjoint $\Rightarrow P(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$

Thm I

$\cdot P(A) = 1 - P(A^c)$

$\Omega = A \cup A^c$

$P(\Omega) = P(A \cup A^c)$

$1 = P(A \cup A^c)$ by condition ⓐ

$1 = P(A) + P(A^c)$ by condition ⓒ A, A^c disjoint.

$\Rightarrow P(A) = 1 - (PA^c)$.