12/06/16

Statistical Inference

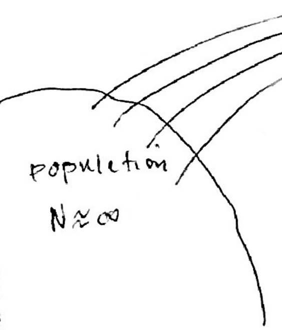
sample

$x_1, \ldots, x_n \overset{iid}{\sim} Bern(p)$

size $n << \infty$ ← way less!

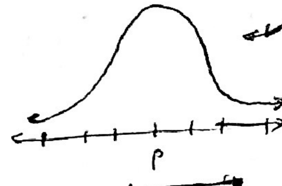
population

$N \approx \infty$



① Best guess
pt. estimation

② Interval constructing
(range of values for p)



$I_p,\ 1-\alpha = \left[\hat{p} = \frac{z_\alpha}{2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]$

$1-\alpha$ is average probability

(the wider it is, more useless it gets)

$\hat{p}$ ← all you see!

Interpretation of Confidence Interval

① if you do many experiments,

$\frac{\#\{p \in CI\}}{n} \to 1-\alpha$

not relevant!

② Before the experiment, $P(p \in CI) = 1-\alpha$

NOT RELEVANT!

③ You want to say $P(p \in CI) = 1-\alpha$ after the experiment!

Not possible unless you believe in subjective probability and assume prior information on p.

③ Testing theories about a parameter
(hypothesis testing)

Human gender ratio:

I believe $p := P(Male)$ is not 50%

CRAZY? → Yes.

DEFAULT position is that

$p = 0.5 \ldots$

we call this the "null hypothesis."

Denoted:

$H_0: p = 0.5$ the alternative hypothesis is

denoted $H_a = p \neq 0.5$

Take a sample of size n. You assume H_0 is true.

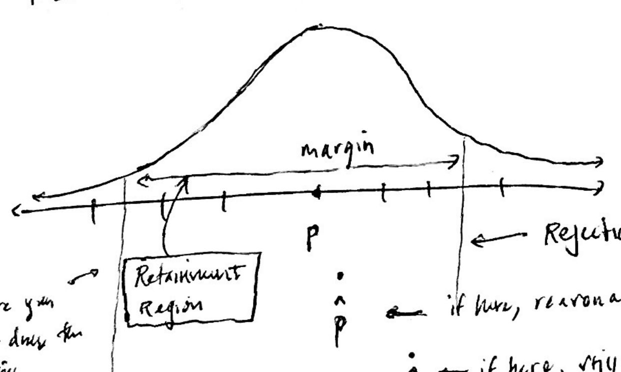
$\alpha := \mathbb{P}(\text{too rare})$

$\hat{p} \approx N\left(p, \left(\sqrt{\frac{p(1-p)}{n}}\right)^2\right)$ $1-\alpha := \mathbb{P}(\text{not too rare})$

P (retainment)

$= P(\hat{p} \in [p \pm \text{margin}])$

$P\left(\hat{p} \in \left[p \pm \frac{z_\alpha}{2}\sqrt{\frac{p(1-p)}{n}}\right]\right)$



$\hat{p}$ ← not reasonable

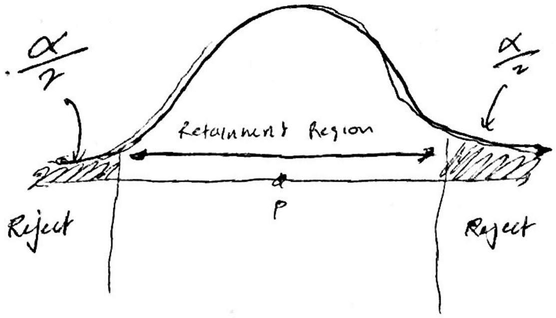
$\hat{p}$ ← if here, reasonable

$\hat{p}$ ← if here, very reasonable

$\hat{p}$ ← ~~still reasonable~~ not reasonable

Retainment Region $= \left[p \pm z_{\alpha/2} \sqrt{\frac{p(1-p)}{n}} \right]$

Now do the experiment. Calculate $\hat{p}$. If $\hat{p} \in$ Retainment Region $\Rightarrow$ Retaint H_0. We do not have enough evidence to reject the null hypothesis.



If $\hat{p} \notin$ Retainment Region $\Rightarrow$ Reject H_0 (accept H_a).
· We have enough evidence to reject null hypothesis.

EX: $n = 345$ births, $\alpha = 5\% \Rightarrow z_{\alpha/2} = 2$

Retain region $= \left[0.5 \pm 2 \cdot \sqrt{\frac{0.5(1-0.5)}{345}}\right] = [.446, .554]$

Do experiment, 169 males $\hat{p} = \frac{169}{345} = .48$ ← in the retain region!

$\Longrightarrow$ Retain H_0. We do not have enough evidence to reject null hypothesis.
↳ equal human gender proportions.

EX: Flip a coin 100 times. Want to test a theory that the coin is unfair. Fair is $p = P(H) = .5$

Scenario I. You get 51 H $\Rightarrow \hat{p} = .51$. fair? yes.

Scenario II. You get 98 H $\Rightarrow \hat{p} = .98$. fair? No.

Scenario III. You get 61 H $\Rightarrow \hat{p} = .61$. fair? No!

$n = 100$, $\alpha = 0.05\ (5\%)$,
$H_0: p = 0.5$, $H_a: p \neq 0.5$

Retainment region $= \left[0.5 \pm 2\sqrt{\frac{0.5(1-.5)}{100}}\right] = [0.4, 0.6]$

$\hat{p} \notin$ Retain region $\Rightarrow$ Reject H_0. We have enough evidence to reject the theory that the coin is fair.

Ex 3: MARS (candy company) claims the ~~number~~ prop. of blue is 20%

Let $p := P(BLUE)$

$\alpha := 1\%$

$H_0 := p = 0.2$

$H_a := p \neq 0.2$

$n = 636$ (M&M's)

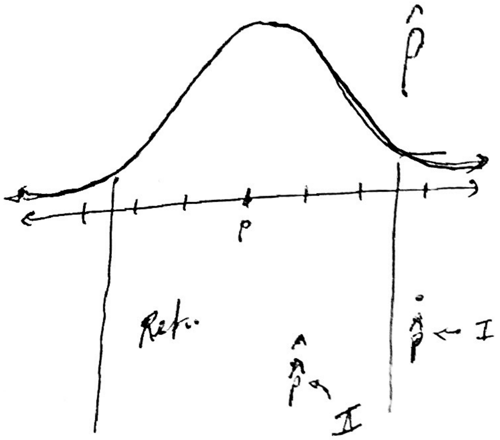
Retainment Region $= \left[p \pm \frac{Z\alpha}{2} \sqrt{\frac{p(1-p)}{n}} \right]$

$= \left[0.2 \pm 2.84 \cdot \sqrt{\frac{0.2(1-0.2)}{636}} \right] =$

$= [.155, .245]$

$\hat{p} = \frac{168}{636} = .264 \notin$ Retain. Region $\Rightarrow$ Reject H_0.

- We have enough evidence to reject the claim made by the MARS candy company that proportion of blue M&M's is 20%.



α controls what happens!

why you need to pick α beforehand.

truth	Retain H_0	Reject H_0
H_0 true	✓	Type I ERROR
H_0 false	Type II error	✓

$P(\text{Type I error}) = P(\text{Reject} \mid H_0 \text{ true}) = \alpha \leftarrow$ You picked this.

$P(\text{Type II error})$ ... not covered in this class

$P(\text{Reject } H_0 \mid H_0 \text{ false}) =$ POWER

↳ NOT COVERED!

$\alpha \uparrow \Rightarrow P(\text{Type I error}) \uparrow \Rightarrow P(\text{Type II error}) \downarrow$

$\alpha \downarrow \Rightarrow P(\text{Type I error}) \downarrow \Rightarrow P(\text{Type II error}) \uparrow$

Court case

H_0 : Innocent

H_a : Guilty

Decision: punish or not.

Type I error: punish an innocent person. Cost?

Type II error: Guilty person goes free. Cost?