Lecture 5; September 8, 2016

- 5 card draw poker ← [for pedagogical reasons, won't be asked poker on exams.]

52 cards

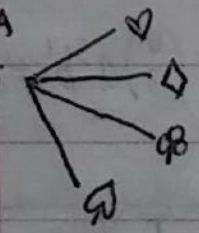
hand: 5 cards

order doesn't matter

$$P(\text{win}) = \frac{|\text{win}|}{|\Omega|} = \frac{|\text{win}|}{\binom{52}{5}}$$

* $P(\text{Royal Flush}) = \frac{\binom{4}{1}\binom{1}{1}}{\binom{52}{5}}$

10 J Q K A one suit



4 suits, need one of them. 9 possibilities choose 1

* $P(\text{Straight flush}) = \frac{\binom{4}{1}\binom{9}{1}}{\binom{52}{5}}$

All same suit

A2345

23456

34567

⋮

910JQK

~~10JQKA~~

$$\binom{4}{1}\left(\binom{10}{1} - \binom{1}{1}\right)$$

including royal flush

subtracting it out.

* $P(\text{4 of a kind}) = \frac{\binom{13}{1}\binom{48}{1}}{\binom{52}{5}} = \frac{\binom{13}{1}\binom{4}{1}}{\binom{52}{5}}$

choose 1 rank out of 13

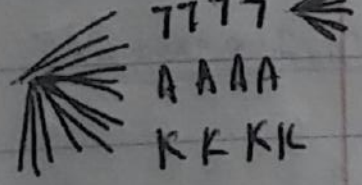
1 want to choose one of other ranks

choose one suit of 4.

7777

AAAA

KKKK



Have to choose 1 out of 13 ranks for it to be 3 of a kind.

only 12 ranks to choose from after 9.

* $P(\text{full house}) = \frac{\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}}{\binom{52}{5}}$

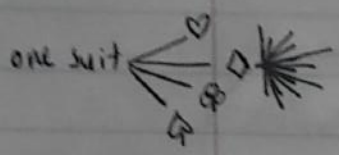
4 suits of Q, need 2.

4 9s, but need only 3, different suits.

99QQQ

* $P(\text{flush}) = \dfrac{\binom{4}{1}\left(\binom{13}{5} - \binom{9}{1} - \binom{1}{1}\right)}{\binom{52}{5}}$

choose 1 suit. | 13 ranks, choose 5. | take out royal & straight flushes.

one suit 

* $P(\text{straight}) = \dfrac{\binom{10}{1}\binom{4}{1}^5 - \binom{4}{1}\binom{4}{1} - \binom{4}{1}\binom{1}{1}}{\binom{52}{5}}$

10 straights, choose 1.

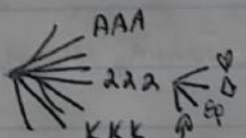
A2345 

There are 5 cards in a straight, 4 suits for each.

* $P(\text{3-of-a-kind}) = \dfrac{\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}\binom{4}{1}}{\binom{52}{5}}$

choose rank to make triple | choose 3 suits of rank. | 12 other ranks, choose 2, order doesn't matter.

KKK 3♡9♤

AAA 
222
KKK

* $P(\text{2 pair}) = \dfrac{\binom{13}{2}\binom{4}{2}^2\binom{11}{1}\binom{4}{1}}{\binom{52}{5}}$

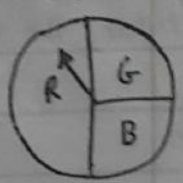
picking 2 9s out 4 9s & 2 Qs out of 4 Qs

13 ranks, choose 2 | 11 ranks left, choose 1 → pick a suit.

order doesn't matter { 99QQ7, QQ997 }

* Is $\binom{13}{2} \stackrel{?}{=} \binom{13}{1}\binom{12}{1}$

$\left[\dfrac{13 \cdot 12}{2} \stackrel{?}{=} 13 \cdot 12 = 13P_2\right]$

- 

$\Omega = \{R, G, B\}$ $\quad P(\{R\}) = \dfrac{|\{R\}|}{|\Omega|} = \dfrac{1}{3}$ → wrong b/c assumed that all 3 on spinner equal. Assumed equally likely.

- $\Omega = \{\text{sun, cloud, rain, snow}\}$ $\quad P(\{\text{snow}\}) = \dfrac{1}{4}$ → more days have non-extreme weather than extreme weather

• $P(\text{Trump}) \stackrel{?}{=} \frac{1}{2}$ ← This is just silly.

(Limiting frequency)

★ Long Run Frequency Definition (I)

$$\mathbb{1}_{\omega \in A} := \begin{cases} 1 \text{ if } \omega \in A \\ 0 \text{ if } \omega \notin A \end{cases}$$

↑ indicator function: good for counting things.

$$\sum_{i=1}^{n} \mathbb{1}_{\omega_i \in A}$$

↑ counting # of times happened out of n.

These two definitions are called OBJECTIVE bits of the physical world independent of humans.

$$P(A) := \frac{1}{n} \sum_{i=1}^{n} \mathbb{1}_{\omega_i \in A}$$

$$\frac{1}{n} \sum_{i=1}^{n} \mathbb{1}_{\omega_i \in A}$$

↑ now proportioned

Problems

① $n \neq \infty$ (can't test forever)

meaning can never actually calculate probability.

② Precise replications (Is ω drawn same way each time)

③ non-general

(Ex) P(Trump) l.r.f (long run frequency) → induces probability

★(II) Propensity Definition

→ There is a inherent propensity (probability) which induces a l.r.f (long run frequency)

→ Objects have inherent property to go one way or another, this the probability which induces the l.r.f.

U238 = unstable
- inherent property that will blow up.
- half life 4.5 billion years, 50-50 chance blow up after 4.5 billion years.

Problems

① Uncomputable

② Non-general

(★Objective Theory: Properties of physical world not of human knowledge)

History of Probability

→ 1654, start of probability. began with gambling game.

$P\left(\left\{\begin{array}{l}\geq 1 \text{ double 6 in} \\ 24 \text{ rolls of 2 die}\end{array}\right\}\right) = .4914 < \frac{1}{2}$

started with this thing ← M. de Chevalier de Mere wrote to Pascal, Fermat this thing.

(III) Logical Theory

- given ~~evidence~~ same evidence all people would agree on the probability
  → wrong, Math 341 discusses this

(IV) Subjective Theory

given same evidence all people do not have to agree.

Ex) P(Trump)

Problems

① No ground truth - everyone has different ~~opinions~~. opinions.