lecture 7 - September 15, 2016

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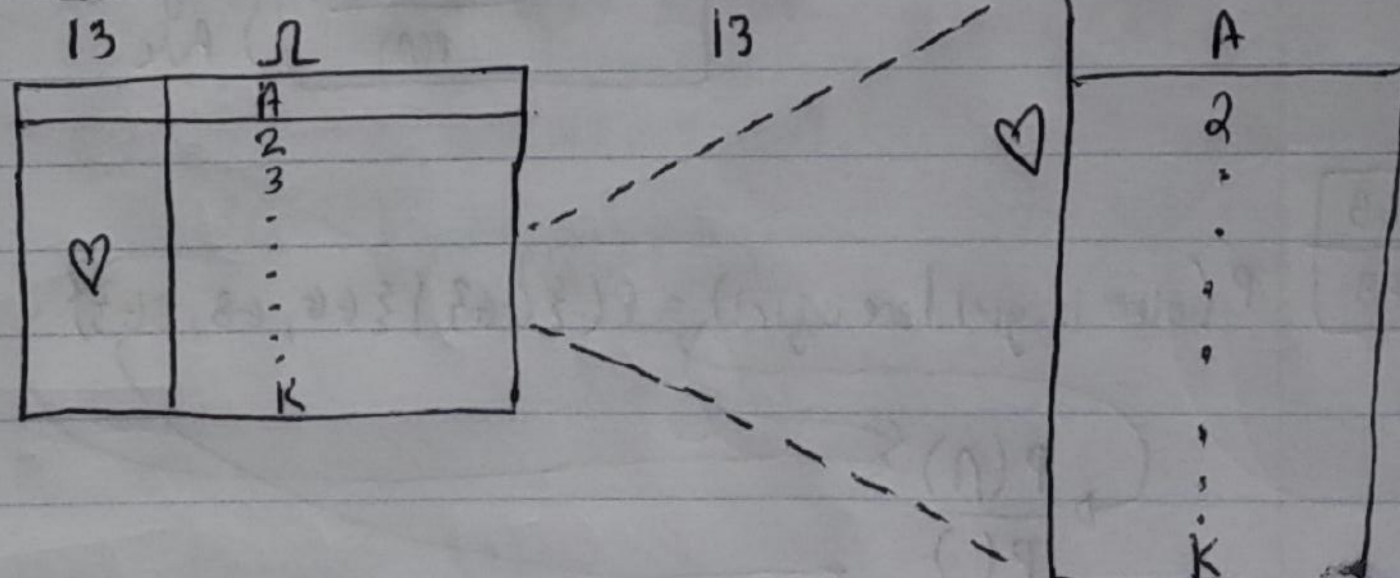
HW: read 3.4

Test #1 on 1.3-1.5, 1.7, 2.1-2.4, 3.2

• $P(A) = \dfrac{4}{52} = \dfrac{1}{13}$

↑ probability of Ace

• $P(A \mid ♡) = \dfrac{1}{13}$



• P(IBM stock ↑ in a day) = P(IBM stock ↑ in a day | rains in Buenos Aires)

informationally irrelevant

★ **Definition**: A, B are independent events if –

$$\left[\begin{array}{c} P(A|B) = P(A) \\ \text{or} \\ P(B|A) = P(B) \end{array}\right]$$

$P(A) = P(A|B)$

$P(A) = \dfrac{P(AB)}{P(B)}$

$\Rightarrow P(AB) = P(A)\,P(B)$

Multiplication Rule

→ $A_1, A_2, \ldots$ independent events

$$P\left(\bigcap_{i=1}^{\infty} A_i\right) = \prod_{i=1}^{\infty} P(A_i)$$

• $P(H_1 H_2 H_3 H_4 H_5) = \dfrac{1}{|\Omega|^5} = \dfrac{1}{2^5}$

$= P(H_1) * P(H_2) \ldots * P(H_5)$

$= \left(\frac{1}{2}\right)^5$

Numerically the same, but conceptually different ways to approach problem

• $P(\{\geq 1 \text{ 6-6 in 24 rolls}\}) < \frac{1}{2}$

$= P(1 \text{ 6-6 in } 24) + P(2 \text{ 6-6 in } 24) + \ldots + P(24 \text{ 6-6 in } 24)$

$= 1 - P(\text{zero 6-6 in } 24)$

"and"

$= 1 - P(\text{not 6-6 in roll 1} \cap \text{not 6-6 roll 2} \cap \ldots \cap \text{not 6-6 roll 24})$

$= 1 - P(\text{not 6-6})^{24}$

$= 1 - (1 - P(6\text{-}6))^{24}$

$= 1 - (1 - P(6)P(6))^{24}$

$= 1 - \left(1 - \left(\frac{1}{6}\right)^2\right)^{24} = \boxed{.4914039}$

★ **Definition**: A, B are dependent if –

$$\left[\begin{array}{c} P(A|B) \neq P(A) \\ \text{or} \\ P(B|A) \neq P(B) \\ P(AB) \neq P(A)P(B) \end{array}\right]$$

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marginal probability → conditional probability

- P(Q64 is late) < P(Q64 is late | raining & traffic)

because knowing there is rain & traffic, way more higher / likely chance bus will be late.

- P(Q64 is late) > P(Q64 is late | sunny & no traffic)

becausing knowing sunny & no traffic, bus is more likely to be on time.

- P(lung cancer) < P(lung cancer | smoking)

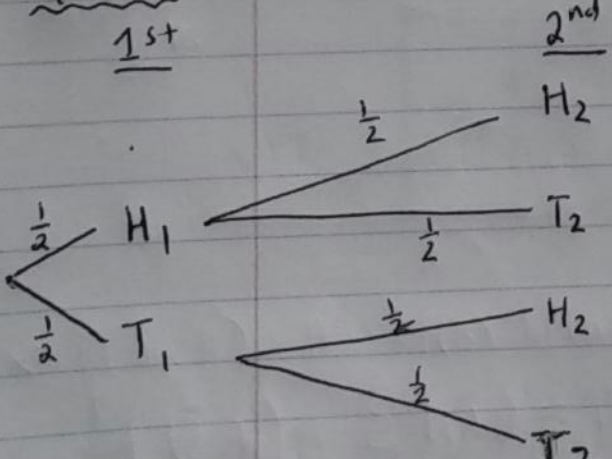
more likely to get lung cancer if smoking

- $\underbrace{\text{A, B are disjoint}}_{\text{A and B}} \overset{?}{\Rightarrow} \underbrace{\text{A, B are independent}}_{\text{A \& B are independent}}$

$\not\Leftarrow$

$$\left[\begin{array}{l} \bullet\ P(A|B) \overset{?}{=} P(A) \\ \quad\Downarrow \\ *\ 0 = P(H|T) \neq P(H) = \tfrac{1}{2} \end{array}\right]$$

- 2 coins

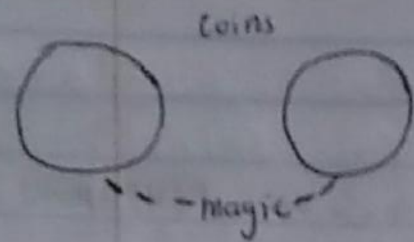


$P(HH) = \frac{1}{4}$

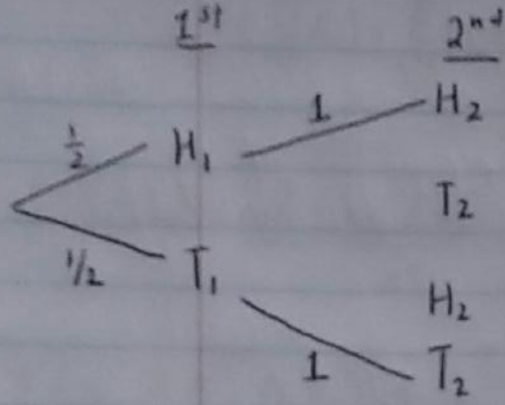
$P(HT) = \frac{1}{4}$

$P(TH) = \frac{1}{4}$

$P(TT) = \frac{1}{4}$

- 

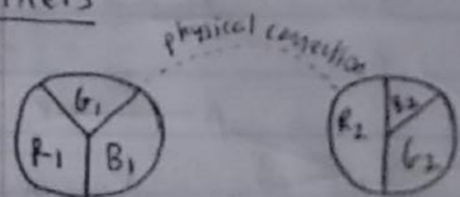
$$1 = P(H_2 | H_1) \neq P(H_2) = \tfrac{1}{2}$$



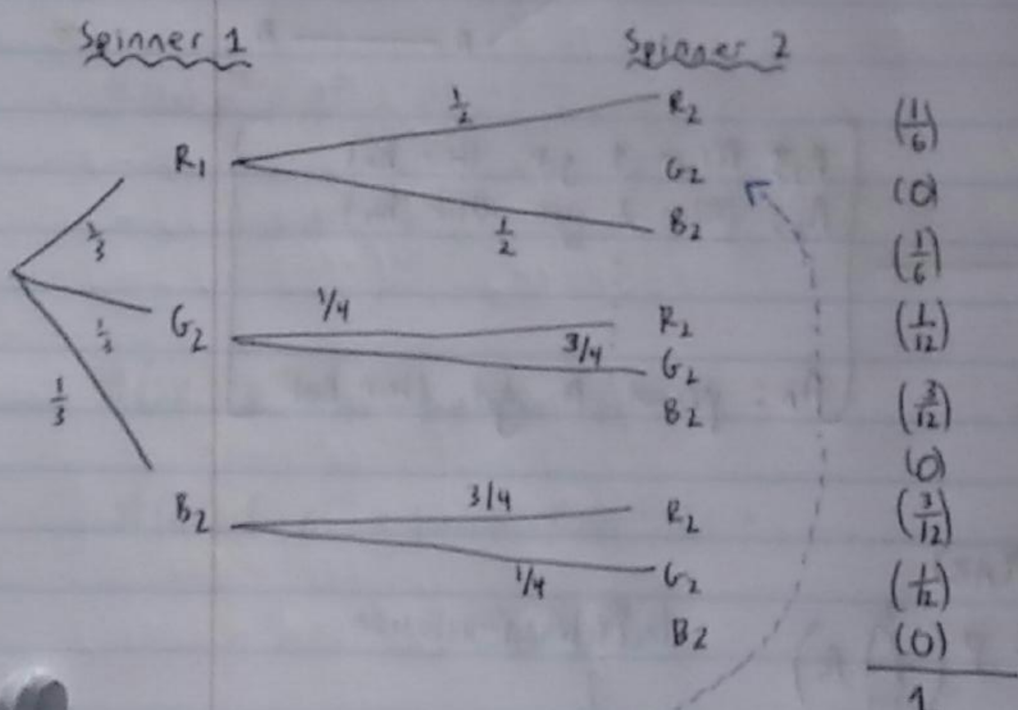
$P(HH) = \frac{1}{2}$

$P(TT) = \frac{1}{2}$

- 2 Spinners



$P(R_1) = P(G_1) = P(B_1) = \frac{1}{3}$ $\quad$ $P(R_2) = \frac{1}{2}$, $P(G_2) = \frac{1}{3}$, $P(B_2) = \frac{1}{6}$



(Are 2 events dependent?)

Are these two spinners dependent or independent?

Not independent because not all events in spinner are dependent on each other.

* Consider R_1, R_2.

$\frac{1}{2} = P(R_2 | R_1) \stackrel{?}{=} P(R_2) = \frac{1}{2}$

* Consider R_1, G_2

$0 = P(G_2 | R_1) \stackrel{?}{=} P(G_2) = \frac{1}{3}$

• $P(\text{shared bday}) = P(\geq 1 \text{ shared b-day among 49 people})$

$$= P(1 \text{ shared bday}) + P(2 \text{ shared bday}) + \ldots\ldots P(49 \text{ shared bday})$$
$$= 1 - P(\text{no shared bday})$$

$$= 1 - \frac{\underline{365}\ \underline{364}\ \underline{363} \ldots \underline{365-49+1}}{365^{49}}$$

$$= 1 - \frac{{}_{365}P_{49}}{365^{49}} = \boxed{\sim 97\%}$$

• $P(\text{no one gets their hat})$

$= 1 - P(\text{someone got their hat})$

$= 1 - (P(\text{one person hat}) + P(2 \text{ people}) + \ldots P(n \text{ people}))$

$= 1 - P(A_1 \cup A_2 \cup \ldots A_n)$

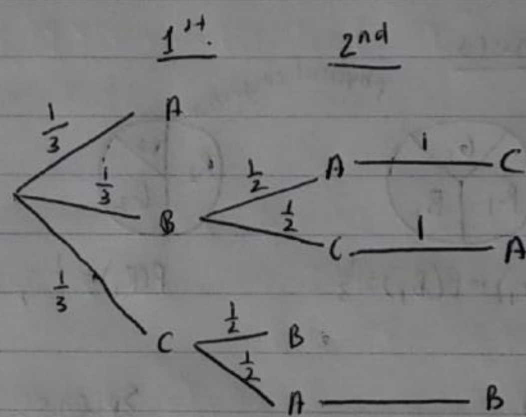
$= 1 - P(\bigcup_{i=1}^{n} A_i)$

$= 1 - (1 - e^{-1})$

$= e^{-1} \approx 0.360$

$\boxed{\approx \frac{1}{3}}$

This problem is independent of n.



$\frac{1}{3} + \frac{1}{6} + \frac{1}{6} = \boxed{\frac{2}{3}}$

A_1: Person 1 got their hat
A_2: person 2 got their hat
⋮
A_n: person n got their hat

$\bigstar$ $P(A \cup B) = P(A) + P(B) - P(AB)$

$$P\left(\bigcup_{i=1}^{2} A_i\right) = \sum_{i=1}^{2} P(A_i) - P\left(\bigcap_{i=1}^{2} A_i\right)$$

Inclusion & Exclusion

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) - \sum_{i \neq j} P(A_i \cap A_j) + \sum_{i \neq j \neq k} P(A_i \cap A_j \cap A_k) - + - + - + (-1)^{n+1} P\left(\bigcap_{i=1}^{n} A_i\right)$$

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$$- \frac{1}{2!} \qquad + \frac{1}{3!} \quad \cdots\cdots = \underline{\underline{1-e^{-1}}}$$

$$\| \qquad\qquad \|$$

$$\frac{\cancel{n!}}{(n-2)!\,2!} \cdot \frac{(n-2)!}{\cancel{n!}} \qquad \frac{\cancel{n!}}{(n-3)!\,3!} \cdot \frac{(n-3)!}{\cancel{n!}}$$

$$\frac{1}{1} \qquad \| \qquad\qquad \|$$

$P(A_1) = \frac{1}{n}$

$P(A_2) = \frac{1}{n}$

$\vdots$

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \underbrace{\sum_{i=1}^{n} P(A_i)}_{\sum_{i=1}^{n} \frac{1}{n}} - \underbrace{\sum_{i \neq j} P(A_i \cap A_j)}_{\binom{n}{2}\frac{(n-2)!}{n!}} + \underbrace{\sum_{i \neq j \neq k} P(A_i \cap A_j \cap A_k)}_{\binom{n}{3}\frac{(n-3)!}{n!}} - + - + - + (-1)^{n+1} P\left(\bigcap_{i=1}^{n} A_i\right)$$

$$P(A_2 \cap _4) = \frac{\underline{n-2}\ \underline{1}\ \underline{n-3}\ \underline{1}\ \underline{n-4} \cdots \underline{\ \ }}{n!} = \frac{(n-2)!}{n!}$$

$$P(A_2 \cap A_4 \cap A_5) = \frac{\underline{1}\ \underline{n-3}\ \underline{n-4}\ \underline{1}\ \underline{1}\ \underline{n-5} \cdots \underline{\ \ }}{n!} = \frac{(n-3)!}{n!}$$

$$\star \boxed{f(x) = \sum_{i=0}^{\infty} \frac{f^{(i)}(c)}{i!}(x-c)^i \quad \forall c \in \mathbb{R}} \star$$

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✿ (Ex) $c=0 \;=\; \sum_{i=0}^{\infty} \frac{f^{(i)}(0)}{i!} x^i \approx f(0) + f'(0)x \ \frac{f''(0)x^2}{2!}$ [3 terms is usually good enough]

✿ (Ex) $e^x = e^0 + e^0 x + \frac{e^0}{2!}x^2 + \frac{e^0}{3!}x^3 + \cdots$

$= 1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots = \sum_{i=0}^{\infty} \frac{1}{i!}$

✿ (Ex) $e^{-1} = 1 - 1 + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}$

✿ (ex) $1 - e^{-1} = 1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \cdots$