09/29/2016

$$X \sim Hyper(\underset{n}{1}, K, N) = Bern\left(\frac{K}{N}\right)$$

$$= \frac{\binom{K}{x}\binom{N-K}{1-x}}{\binom{N}{1}}$$

Parameter space: $N \in \mathbb{N} \setminus \{1\}$

$K \in \{1, 2, \ldots, N-1\}$

$n \in \{1, 2, \ldots, N-1\}$

$Supp[X] = \{0, 1\}$

$P(X=1) = \frac{K}{N}$; $P(X=0) = \frac{N-K}{N} = 1 - \frac{N}{K} = 1 - p$

$X \sim Hyper(2, 4, 10)$ $n < K, n < N-K \Rightarrow Supp[X] = \{0, \ldots, n\}$

$Supp[X] = \{0, 1, 2\}$

$X \sim Hyper(5, 4, 10)$ $n \geq K, n < N-K \Rightarrow Supp[X] = \{0, \ldots, K\}$

$Supp[X] = \{0, 1, 2, 3, 4\}$

$X \sim Hyper(8, 4, 10)$ $n \geq K, n \geq N-K \Rightarrow Supp[X] = \{n-(N-K), \ldots, K\}$

$Supp[X] = \{2, 3, 4\}$

$X \sim Hyper(5, 7, 10)$ $n < K, n \geq N-K \Rightarrow Supp[X] = \{n-(N-K), \ldots, n\}$

$Supp[X] = \{2, 3, 4, 5\}$

	$n<K$	$n \geq K$
$n<N-K$	$\{0, \ldots, n\}$	$\{0, \ldots, K\}$
$n \geq N-K$	$\{n-(N-K), \ldots, n\}$	$\{n-(N-K), \ldots, K\}$

$Supp[X] = \{\max(0, n-(N-K)), \ldots, \min(n, K)\}$

$$\sum_{x \in Supp[X]} P(x) = 1$$

n: sample size

N: population size

K: # success

p: proportion of success

$$X \sim Hyper(n, p, N) = p(x) = \frac{\binom{pN}{x}\binom{(1-p)N}{n-x}}{\binom{N}{n}}$$

$p = 0.5$
$n = 6$
$N = 100$

$$P(X=3) = \frac{\binom{50}{3}\binom{50}{3}}{\binom{100}{6}} = 0.3223$$

$p = 0.5$
$N = 1000$
$n = 6$

$$P(X=3) = \frac{\binom{500}{3}\binom{500}{3}}{\binom{1000}{6}} = 0.3134$$

$p = 0.5$
$n = 6$
$N = 10000$

$$P(X=3) = \frac{\binom{5000}{3}\binom{5000}{3}}{\binom{10000}{6}} = 0.3126$$

$$\lim_{N\to\infty} \frac{\frac{(pN)!}{x!(pN-x)!} \cdot \frac{((1-p)N)!}{(n-x)!((1-p)N-n+x)!}}{\frac{N!}{n!(N-n)!}} = \frac{\frac{1}{x!(n-x)!}}{\frac{1}{n!}} = \frac{n!}{x!(n-x)!}$$

$$\lim_{x\to\infty} a f(x) = a \lim_{x\to\infty} f(x)$$

$$\lim \frac{\frac{(pN)!}{x!(pN-x)!} \cdot \frac{((1-p)N)!}{(n-x)!((1-p)N-n+x)!}}{\frac{N!}{n!(N-n)!}} = \binom{n}{x} \lim_{N\to\infty} \frac{\overbrace{((pN)(pN-1)\ldots p(N-x+1))}^{x}}{\underbrace{(N)(N-1)\ldots\ldots(N-n+1)}_{n}}$$

$$\lim f(x) g(x) = \lim f(x) \lim g(x)$$

$$= \binom{n}{x} \lim \underbrace{\frac{pN}{N}}_{p} \cdot \lim \underbrace{\frac{pN-1}{N-1}}_{p} \cdot \ldots \cdot \Big\{ \lim \underbrace{\frac{pN-x+1}{N-x+1}}_{p} \Big\} p^x$$

$$\lim \underbrace{\frac{(1-p)N}{N-x}}_{1-p} \cdot \lim \underbrace{\frac{(1-p)N-1}{N-x+1}}_{1-p} \cdot \ldots \cdot \Big\{ \lim \underbrace{\frac{(1-p)N-n+x+1}{N-n+1}}_{1-p} \Big\} (1-p)^{n-x}$$

$$= \binom{n}{x} p^x (1-p)^{n-x}$$

$X \sim \text{Binomial}(n, p) = p(x) = \binom{n}{x} p^x (1-p)^{n-x}$ $\quad$ $\text{Supp}[X] = \{0, 1, \ldots, n\}$

$X \sim \text{Binomial}(n, 0)$ $\quad$ $p = 0 \Rightarrow X \sim \text{Deg}(0)$ $\quad$ $\binom{n}{x} 0^x 1^{n-x}$

$p = 1 \Rightarrow X \sim \text{Deg}(n)$ $\quad$ $\binom{n}{x} 1^x 0^{n-x}$

Parameter Space: $n \in \mathbb{N}$, $p \in (0, 1)$

$$\lim_{x\to 0} x^x = 1$$

$X \sim \text{Binomial}(1, p) = \binom{1}{x} p^x (1-p)^{1-x} = \text{Bern}(p)$

$\text{Supp}[X] = \{0, 1\}$ $\quad$ $\binom{1}{0} = 1$

$\binom{1}{1} = 1$