

Lecture 16 Math 241 4/2/15 (see p5 Lec 15 for)

if X_1, \dots, X_n ind... not necessarily indep

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In gen $Var[X_1 + \dots + X_n] = Var[X_1] + \dots + Var[X_n]$

or $Var(\sum X_i) = \sum Var(X_i)$

$$Var(\bar{X}_n) = Var\left(\frac{X_1 + \dots + X_n}{n}\right) = Var\left(\frac{1}{n}(X_1 + \dots + X_n)\right) = \frac{1}{n^2} Var(X_1 + \dots + X_n) =$$

$$= \frac{1}{n^2} (\underbrace{\sigma^2 + \dots + \sigma^2}_n) = \frac{1}{n^2} n \sigma^2 = \frac{\sigma^2}{n}$$

$\Rightarrow SD(\bar{X}_n) = \frac{\sigma}{\sqrt{n}}$

$\lim_{n \rightarrow \infty} SD(\bar{X}_n) = 0$

$\Rightarrow \lim_{n \rightarrow \infty} Var(\bar{X}_n) = 0$

given r.v X

IF $SD(X) = 0 \Rightarrow X = \text{const}$

$SD(X) = 0 \Rightarrow Var(X) = 0$

$\Rightarrow \sum_{x \in \text{supp}(X)} (x - \mu)^2 p(x) \downarrow \geq 0$

$0 \text{ only if } x = \mu$

if $x = \mu$

$\Rightarrow \text{supp}(X) = \{\mu\} \Rightarrow X = \text{const}$
"proof" of LLN
 $\bar{X} \rightarrow \mu$

Back to Urn Example...

$T = 3 + 0.40W$, $E(T) = 3 + 0.40E(W) = ?$

$Var(T) = (0.40)^2 \sigma_w^2$, $SD(T) = 0.40 \sigma_w = ?$

Calculate

X_1, \dots, X_n iid Bern(p)

$T_n = X_1 + \dots + X_n$

$E(T_n) = E(X_1 + \dots + X_n) = \underbrace{p + \dots + p}_n = np$ much easier!

$Var(T_n) = Var(X_1 + \dots + X_n) = \underbrace{p(1-p) + \dots + p(1-p)}_n = np(1-p)$ easier than...

$Var(T_n) = E(T_n^2) - (E(T_n))^2 = E(T_n^2) - n^2 p^2 = \dots = np(1-p)$ HARDER!!!

$E(T_n^2) = \sum_{x=0}^n x^2 \binom{n}{x} p^x (1-p)^{n-x} = \sum_{x=1}^n x \cdot x \cdot \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$

$x=1$ why?

$= np \sum_{x=1}^n x \binom{n-1}{x-1} p^{x-1} (1-p)^{n-x}$

let $y = x-1 \Rightarrow x = y+1$

$= np \sum_{y=0}^{n-1} \binom{n-1}{y} p^y (1-p)^{n-y-1}$

let $m = n-1 \Rightarrow n = m+1$

$= np \sum_{y=0}^m \binom{m}{y} p^y (1-p)^{m-y}$

$= np \left(\sum_{y=0}^m \binom{m}{y} p^y (1-p)^{m-y} + \sum_{y=0}^m \binom{m}{y} p^y (1-p)^{m-y} \right) = np((1-p) + 1)$

$Y \sim \text{Bin}(m, p) \Rightarrow E(Y) = mp = (n-1)p$

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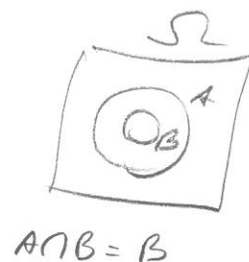
$= np(np - p + 1) = n^2 p^2 - np^2 + np$

$$X \sim \text{Geom}(p) \quad E(X) = \frac{1}{p}, \quad \text{Var}(X) = \frac{1-p}{p^2} \quad (\text{Annoying prof})$$

$$i = (1-p)^{x-1} p$$

$$\text{What about } P(X=17 | X > 10) = \frac{P(X=17 \& X > 10)}{P(X > 10)}$$

$$= \frac{(1-p)^{16} p}{(1-p)^{10}} = (1-p)^6 p = P(X=7)$$



$$P(X = b+x | X > b) = \frac{(1-p)^{b+x-1} p}{(1-p)^b} = (1-p)^{x-1} p = P(X=x)$$

"memorylessness" comes from independence

Midsem 27

Final ↓

REVIEW