MATH 214 Fall 2014 Homework #1

Professor Adam Kapelner

Due 11:59PM in my office, Tuesday, Feb 10, 2015

(this document last updated Monday 2$^{\text{nd}}$ February, 2015 at 4:41pm)

Instructions and Philosophy

The path to success in this class is to do many problems. Unlike other courses, exclusively doing reading(s) will not help. Coming to lecture is akin to watching workout videos; thinking about and solving problems on your own is the actual "working out". Feel free to "work out" with others; **I want you to work on this in groups.**

Reading is still *required*. For this homework set, read ... in Ross 7th edition.

The problems below are color coded: green problems are considered *easy* and marked "[easy]"; yellow problems are considered *intermediate* and marked "[harder]", red problems are considered *difficult* and marked "[difficult]". There are no E.C. problems on this homework. The *easy* problems are intended to be "giveaways" if you went to class. Do as much as you can of the others; I expect you to at least attempt the *difficult* problems.

This homework is worth 100 points but the point distribution will not be determined until after the due date. Late homework will be penalized 10 points per day.

15 points are given as a bonus if the homework is typed using LaTeX. Links to instaling LaTeX and program for compiling LaTeX is found on the syllabus. You are encouraged to use `overleaf.com`. If you are handing in homework this way, read the comments in the code; there are two lines to comment out and you should replace my name with yours and write your section. If you are asked to make drawings, you can take a picture of your handwritten drawing and insert them as figures or leave space using the "\vspace" command and draw them in after printing or attach them stapled.

The document is available with spaces for you to write your answers. If not using LaTeX, print this document and write in your answers. **Handing it in without the printout incurs a penalty of 20 points.** Keep this page printed for your records. Write your name and section below where section A is if you're registered for the 9:15AM–10:30AM lecture and section B is if you're in the 12:15PM-1:30PM lecture.

NAME: ________________________________ SECTION (A or B): ______

Set Theory Problems below are related to set theory. The sets we talk about in class are composed of outcomes in a universe that are events. Some of the problems below will be about abstract sets that are divorced from the sets used in probability.

Problem 1

These are questions on abstract set theory. Assume capital letters are arbitrary sets and Ω is the universe for all the following questions. Answer as succinctly as possible. Some of this will be review.

(a) [easy] Answer the following as best as possible.
$A \cup A =$
$A \cap A =$
$A \cap \varnothing =$
$A \cup \Omega =$
$A \cap \Omega =$
$A \cup A^C =$
$\varnothing^C =$
$\Omega^C =$
$A \backslash A =$
$A \backslash \Omega =$
$A \backslash \varnothing =$

(b) [easy] Are the following true (T) or false (F) for arbitrary sets $A,\ B,\ C$?
$A \subseteq \Omega$
$A \subset \Omega$
$\varnothing \subseteq A$ and $A \subseteq \Omega$
$A \subseteq A \cup B$
$A \subseteq A \cap B$

(c) [harder] Are the following true (T) or false (F) for the arbitrary set A?
$A \subseteq A$
$A \subset A$
$\varnothing \subseteq A$
$\varnothing \subset A$
$\varnothing \subseteq \varnothing$
$\varnothing \subset \varnothing$

(d) [harder] Are the following true (T) or false (F)? The symbol "$\Rightarrow$" denotes logical implication *i.e.* if the conditions on the l.h.s are met, the statement on the r.h.s is always true. Commas should be interpreted to mean "and."
$A \subseteq B \Rightarrow A \cap B = A$
$A \subseteq B,\ B \subseteq C \Rightarrow A \subseteq C$
$A \subseteq B,\ B \subseteq C \Rightarrow A \subset C$
$A \subseteq B,\ A \subseteq C \Rightarrow A \subset B \cap C$
$A \subset A \cup B$

(e) [harder] Express $A \cap B$ only in terms of set subtraction (by using the symbol "$\setminus$").

(f) [easy] Explain why $A \cup B = B \cup A$ *in English*.

(g) [harder] Draw three Venn diagrams illustrating the distributive law $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ one for each of three configurations of A, B, C that you decide. You must draw A, B, C as circles. "Different configurations" means that the circles overlap differently.

Problem 2

Consider the sample space Ω where you flip a fair coin and roll a fair die.

(a) [easy] Draw this event space in a box similar to how we did in class and indicate $|\Omega|$.

(b) [easy] Are singleton sets of the events in Ω mutually exclusive? collectively exhaustive?

(c) [easy] Does it matter if the coin is flipped before the die, after the die, or simultaneously with the die? Explain.

(d) [easy] Consider the set T which represents all events where the coin was flipped tails and E which represents the set of events where the die rolled an even number. Draw a Venn diagram and list the elements of the sets $T \cap E$ and $E \backslash T$ and mark them on the diagram.

(e) [harder] Describe fully the set $2^{(E \cup T)^C}$ *i.e.* list all its elements.