May 12 Lecture 22 Math 201

2-sided 1-prop CI's

Set $\alpha = P(\text{not cover})$

$$CI_{p,1-\alpha} = \left[\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]$$

Interpret! Trying to give range of values for p

$p \in CI_{p,1-\alpha}$?

Don't know..

But

$$P\left(p \in \left[\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]\right)_{\text{or } 1-\alpha}$$

or

$$P\left(p \in \hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right) \begin{matrix} = \\ \geq \end{matrix} \begin{matrix} \text{0 or 1 depending} \\ \text{on sample} \\ 1-\alpha \end{matrix}$$

p's $\hat{p}$ → Single iid Bern(p) case

2-Sided 1-prop Hypothesis test

$H_0: p = p_0$, $H_1: p \neq p_0$, α set is $P(\text{type I error})$

$$\text{Acc Region}_{1-\alpha} = \left[p_0 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p_0(1-p_0)}{n}}\right]$$

Assume we know p! Does the sample statistic?

$$\hat{p} \overset{?}{\in} \left[p_0 \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p_0(1-p_0)}{n}}\right]$$

$\Rightarrow$ Yes $\Rightarrow$ Retain H_0

$\Rightarrow$ No $\Rightarrow$ Reject H_0

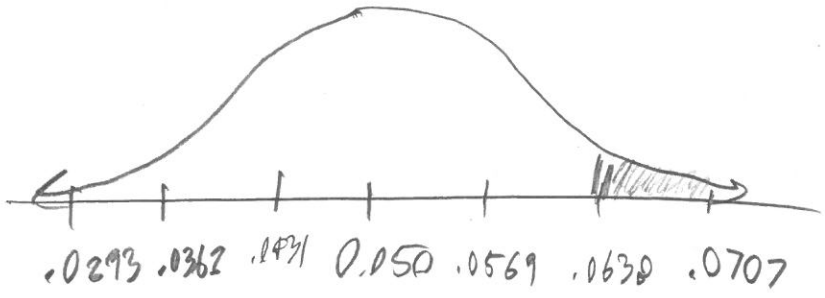
different depending on sample

different mathematically

Another wrinkle. Uber... if more than 5% of customers don't like a driver, the driver should be fired.

After 1,000 rides, Uber makes a decision.

$$\hat{p} \sim N\left(0.05, \left(\sqrt{\frac{0.05(1-0.05)}{1000}}\right)^2\right) = N(0.05, .0069^2)$$



• What's weird?

weird

weird but do I care? NO

$H_a: p > 0.05$ ← this what we're trying to fail

$H_0: p \leq 0.05$ (Null) let $\alpha = 2.5\%$

$$2.5\% = \alpha = P(Z > 2) = P\left(\frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}} > 2\right) = P\left(\hat{p} > p + 2\sqrt{\frac{p(1-p)}{n}}\right)$$

(Z > 2 is the rejection region)

Acc region: $\left(-\infty, p + 2\sqrt{\frac{p(1-p)}{n}}\right]$

One-sided One-proportion Hypothesis Test

Drivers: 71 Acc Rgn $= \left(-\infty, .05 + 2\sqrt{\frac{.05 \cdot .95}{1000}}\right]$

$\hat{p} = \frac{71}{1000} = .071$ $= \left(-\infty, .0630\right]$

$\hat{p} \notin$ Acc Region $\Rightarrow$ Reject $H_0 \Rightarrow$ Fire the driver!

But $P(\text{Type I Error}) = 2.5\%$

Type I error: Reject H_0 when H_0 is true

Fire someone who doesn't deserve to be fired

Type II error: Retain H_0 when H_0 is false

Let a bad driver keep driving

END of MATH 241

So we rejected... but how "strong" is the rejection?

Goal: $P(\text{We see this evidence or more extreme} \mid H_0 \text{ is true})$

$P(\hat{p} > .071 \mid p = .05)$

$= P(\hat{p} > .071 \mid \hat{p} \sim N(.05, .0069^2))$

$= P\left(\frac{\hat{p} - .05}{.0069} > \frac{.071 - .05}{.0069}\right) = P(Z > 3.04) = .0011\mathcal{B} = 0.1\%$

$\Rightarrow$ Low α level! $p\text{-val} < \alpha$! Reject...