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Lecture 1 Math 281 January 29, 2015

- syllabus

Basic Set Theory & Vocabulary

Mathematics builds on the foundation of sets. Prob & Stats grounded in sets. Began in 1870's, formalized in the 20th century.

Definition Set: a collection of distinct objects without order

none, one, or many

all unique/ no duplicates

anything you can think of, #'s, elephants, ideas, anything

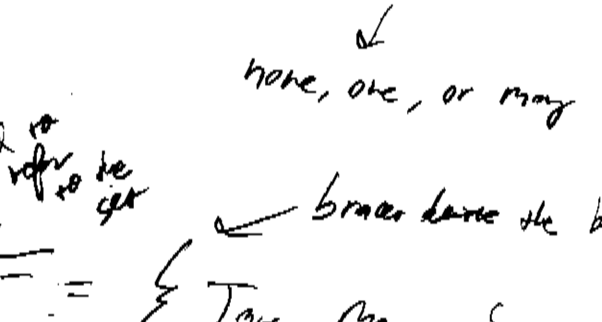
standard to refer to the set

braces denote the beginning and end of the enumeration

$F = \{$Jane, Mary, Susan, Dana$\}$

$M = \{$Bob, Joe, Max, Dana$\}$

They can also be illustrated graphically "Venn Diagram"



Set equality: all elements shared in common.

$M \neq F$ since Jane $\in F$, Jane $\notin M$

"Inclusion" $\quad \text{Jane} \in F$

$$\text{Jane} \in \{\text{Jane}, \text{Mary}, \text{Susan}, \text{Dana}\}$$

=

this element has to be equal to one of the elements in the set

Sets can have any # of elements even infinite elements

natural #'s → $\mathbb{N} := \{1, 2, 3, \ldots\}$, $\mathbb{N}_0 := \{0, 1, 2, 3, \ldots\}$

integers → $\mathbb{Z} := \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

Not inclusion: $\quad \text{Jane} \notin M$

Subsets are denoted with "$\subset$" or "$\subseteq$"

$$\{\text{Jane}, \text{Mary}\} \subset F$$

$\subset$ means "proper subset" which means the set on the l.h.s. $\neq$ set on the r.h.s.

$\subseteq$ means "subset" and it allows for the option for the set on the l.h.s. to be equal to the set on the r.h.s.

$\{\text{Jane}\} \in F$? $\qquad \text{Jane} \subset F$?

We combine sets using the "union" operator "∪" ("cup" symbol)

$\{Jane\} \cup \{Mary, Susan, Dana\} = F$

what is $\{Jane\} \cup \{Jane\}$? $= \{Jane\}$

no duplicates allowed

Union is like addition but doesn't "double-count"

It is also known as "non-exclusive or" operator

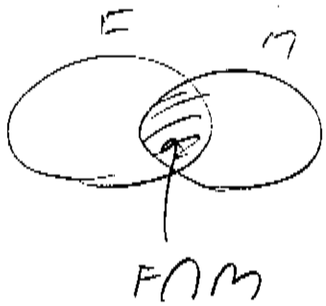
$F \cup M$ means female name, male name (or both)!

$= \{Jane, Mary, Susan, Dana, Bob, Joe, Max\}$

$F \cup M$

An "intersection" finds common elements as a set

$F \cap M = \{Dana\} \neq Dana$



Intersection is the "And" operation

What is a female name and a male name at the same time?

$F \cap \{Bob, Joe\} = \{\}$, $\phi := \{\}$

special symbol: the "empty set" or the "null set"

if $A \cap B = \phi$

$\phi \subset F$? $\phi \in F$?

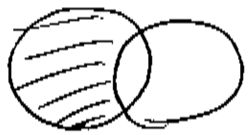
null set is a subset of all sets

they share no elements in common

they are called "mutually exclusive" or "disjoint" (vacuously true)

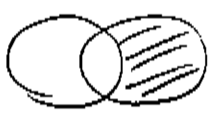
$odd \#'s \cap even \#'s = \phi$

We can also subtract sets

$F \setminus M$  all Females minus all males

Who is $F\setminus M \cup M \setminus F$?

$F\setminus M \cap M \setminus F$

$M \setminus F$  $F \setminus M \neq M \setminus F$ generally

if $A \setminus B = A$ what does that mean about $A \cap B$? $(= \phi)$

We can also take the powerset: expand the set into all subsets

$A = \{1,2,3\}$

$2^A = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

each element $\subseteq A$

We can also take the size of the set. (or "cardinality")

$|F| = 4$ (since there are 4 elements)

$|M| = 4$

$|M \cup F| = 7$, $|F \cap M| = 1$ ← no rules, you have to take the operation and then count

$|M \cup F| \neq |M| + |F|$

$|2^F| = ?$ each subset $A \subseteq 2^F$ has an element or does not

$\underbrace{\{a_1, a_2, a_3, a_4\}}_{A}$ $2^4 = 16$

A special set of items is called Ω (the "universe" or the "sample space" or the "universe of discourse") which is all elements our current scope is limited to. You decide it but it is usually obvious.

Here, let $\Omega = F \cup M$ (only those 7 names)

micro"scope"
tele"scope"
("what you see")

Note that $F \subseteq \Omega$, $M \subseteq \Omega$

all sets $A \subseteq \Omega$ (by definition) if A has elements in the scope.