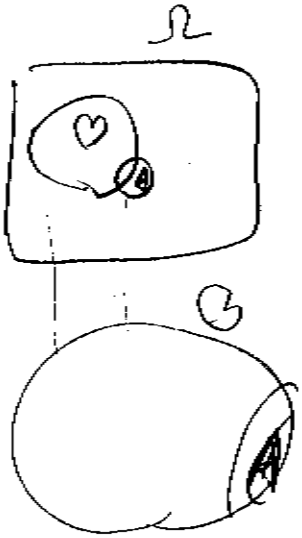
Lecture 7 2/27/15



$P(A) = \frac{4}{52} = \frac{1}{13}$

$P(A|♡) = \frac{1}{13}$

conditional prob

after the "bar" is no longer "random"
it is as if those ω's already occurred.

All prob's are "conditional"

$P(A) := P(A|\Omega)$ ← always assume the universe which we get to choose.

the unconditional prob

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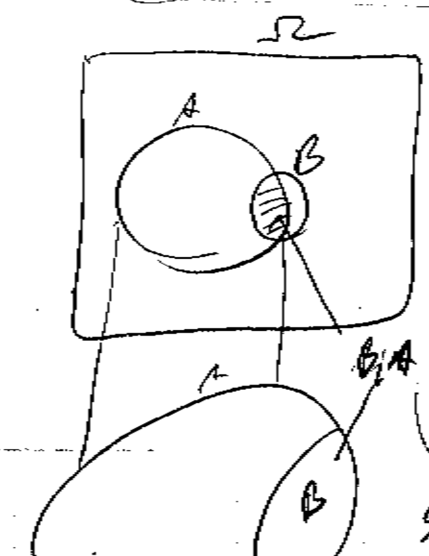
n = 1000, 200 smokers, 60 lung cancer, 36 S & LC
(A) (B) (A, B) ← joint event

$P(A) \approx \frac{200}{1000} = 0.2$, $P(B) \approx \frac{60}{1000} = .06$, $P(A,B) \approx \frac{36}{1000} = .036$ trying to learn from A, B? l.r. def!



unconditional prob's. Best you can do given no information.

Now let's say we have information. Prob's change!

$P(l.c. | s)$ $\Omega' = A \subset \Omega$ zoom factor

$P(B|A) = P(B,A) \cdot \frac{P(\Omega)}{P(\Omega')} = P(B,A) \cdot \frac{1}{P(A)} = \frac{P(B,A)}{P(A)}$ $P(A|B) = \frac{P(A,B)}{P(B)}$

same shape but... bigger

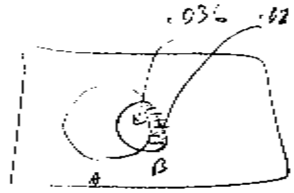
def of cond prob

Why so low? 12

$P(\text{smoking} \mid \text{l.c}) = P(A \mid B) = \frac{.036}{.06} = .6$ good chance he smoked but not 99%.

$P(\text{l.c.} \mid \text{no smoke}) = P(B \mid A^c) = \frac{P(B, A^c)}{P(A^c)} = \frac{.024}{.8} = .03$ still happen

litmus? low

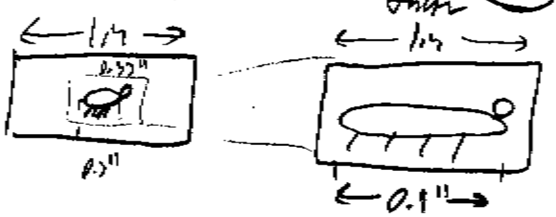


$P(B) = P(B, A) + P(B, A^c)$ $\forall A$ see this later

$.06 = .036 + x \Rightarrow .024$

gonna see this again...

Risk of getting l.c if you smoke $\frac{P(B \mid A)}{P(B \mid A^c)} = \frac{.18}{.03} = \boxed{6x}$



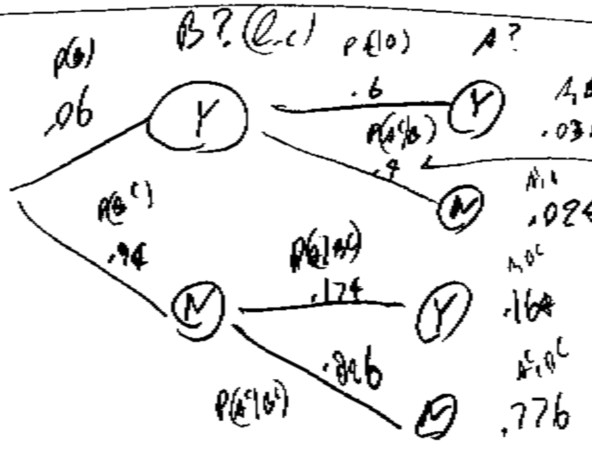
Zoom $3x = \frac{1}{0.33"} = \frac{1}{\text{prop of orig zoomed}} \Rightarrow \frac{1}{P(A)}$ is zoom factor

$P(A \mid B) = \frac{P(A,B)}{P(B)} \Rightarrow P(A, B) = P(A \mid B) P(B) = P(A) P(B) = P(A, B)$ if ind.

corollary of def of ind.

~~Also~~ $P(A \mid B) = \frac{P(A,B)}{P(B)} = \frac{P(B \mid A) P(A)}{P(B)}$ "Bayes Rule" $P(A \mid B) \propto P(B \mid A)$

same shape, diff zoom ratio.



$P(A) = 1 - P(A^c)$ $P(A \mid B) = 1 - P(A^c \mid B)$

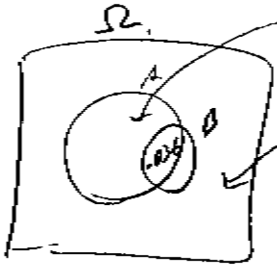
pf $\frac{P(A,B)}{P(B)} = 1 - \frac{P(A^c, B)}{P(B)}$

can't get any other way

Many strategies here

$\nearrow$ Principle: Axiom (C)

split universe into mutually exclusive collectively exhaustive



$P(A,B^c) = P(A)-P(A,B) = .164$

$P(A^c,B^c) = 1-(P(A)+P(B)-P(A,B)) = 1-(.2-.06+.036) = .776$

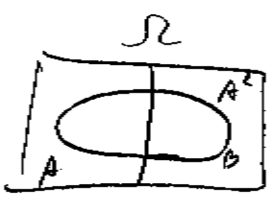
Can we do this all with Bayes Rule?

we solved $P(A|B)$. Can we solve $P(A^c|B)$?

Remember.. $P(A) = 1-P(A^c)$

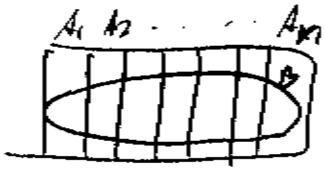
Possible $P(A|B) = 1-P(A^c|B) \quad \forall B \neq \phi$? Yes

$$\Rightarrow \frac{P(A,B)}{P(B)} = 1-\frac{P(A^c,B)}{P(B)} \Rightarrow P(A,B) = P(B)-P(A^c,B) \Rightarrow P(B) = P(A,B)+P(A^c,B)$$



Seems like this is true.

Moreover... what about



s.t. $A_i \cap A_j = \phi$ and $\bigcup A_i = \Omega$

$A_1,\dots,A_n$ are a set of mutually exclusive collectively exhaustive sets $n \leq \infty$

Is $P(B) = \sum_{i=1}^{n} P(B,A_i)$? Yes. Proof.

Law of total prob.

$P(B\cap A_1)+\dots+P(B\cap A_n)$ Are $B\cap A_1,\dots,B\cap A_n$ disjoint?

$(B\cap A_i)\cap(B\cap A_j) = \underbrace{B\cap B}\cap\underbrace{A_i\cap A_j} = B\cap\phi = \phi$ Yes

$\Rightarrow\ = P((B\cap A_1)\cup\dots\cup(B\cap A_n))$ by axiom (C)

$= P(B)$ ∎

$(B\cap A_1)\cup\dots\cup(B\cap A_n) = B\cap(A_1\cup\dots\cup A_n)$ fact from set thy

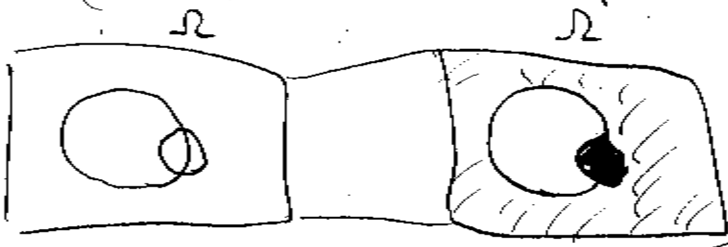
$= B\cap\Omega$

$= B$.

Just proved complement Rule

Prob(not smoke | no lung cancer)

$P(A^c | B^c)$



$P(A^c) = P(A^c, B) + P(A^c, B^c)$

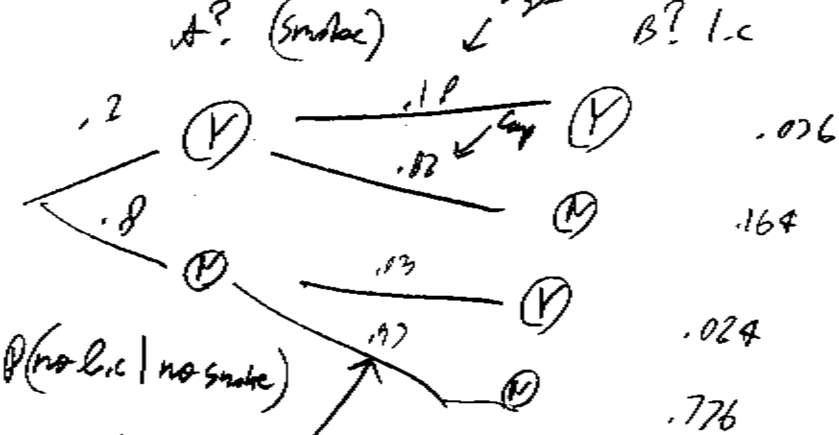
$P(B) = P(B, A) + P(B, A^c)$

$$P(A^c|B^c) = \frac{P(A^c,B^c)}{P(B^c)} = \frac{P(A^c) - P(A^c,B)}{1-P(B)} = \frac{(1-P(A)) - (P(B) - P(A,B))}{1-P(B)}$$

$$= \frac{.8 - (.06 - .036)}{.94} = \frac{.776}{.94} = .826 \ ■$$

This tree is done... are all trees done? How many cond. prob's are there? $\underline{2 \cdot 2} = 4 \cdot 2! = 8$

Invert the tree..



P(no l.c | no smoke)

$P(B^c|A^c) \approx 1$ Good place to be!

8 cond. prob's and all 8 are necessary!

Law of total prob $P(A) = \sum_{i=1}^{n} P(A, B_i)$ for $B_1, B_2, \dots, B_n$, $n \le \infty$ disjoint + coll. exhaustive

$\Rightarrow P(A) = \sum_{i=1}^{n} P(A|B_i)\,P(B_i)$ Bayes Rule n times

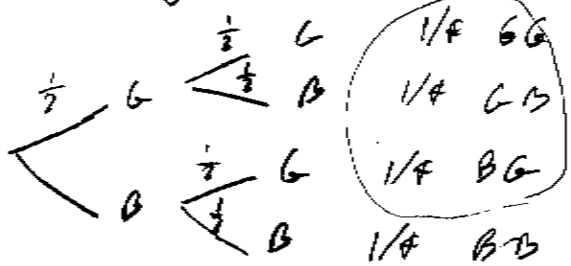
$$P(B_\ell|A) = \frac{P(A|B_\ell)\,P(B_\ell)}{\sum_{i=1}^{n} P(A|B_i)\,P(B_i)}$$

Given that A happened, how likely did this piece of Ω occur

Bayes Thm = Bayes Rules + Law of Tot Prob.

Cond. Prob is really weird! e.g. Children paradox.

I know you have two kids. One of them is a girl. What's the prob the other is a girl? You want to say 50%... but...



$P(GG \mid \{GG\} \cup \{GB\} \cup \{BG\}) = \boxed{\frac{1}{3}}$

Bayes Rule:

$$P(GG \mid \quad) = \frac{P(\{GG\} \cap (\{GG\} \cup \{GB\} \cup \{BG\}))}{P(GG \quad)} = \frac{P(GG)}{\_\_} = \frac{1}{3}$$

Monte Hall Game

STOP

9 VOTE

If you switch