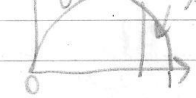
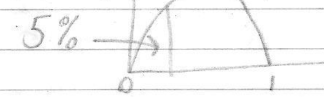
$\underbrace{CR_{\theta, 1-\alpha}}_{\text{two-sided credible region}} := \left[\text{Quantile}[\theta | X, \tfrac{\alpha}{2}], \text{Quantile}[\theta | X, 1 - \tfrac{\alpha}{2}] \right]$

One-sided CR's:

left side



right side



$P[\theta \in CR_{\theta, 1-\alpha}] = 1 - \alpha$

$CR_{L, \theta, 1-\alpha} = \left[0, \text{Quantile}[\theta | X, 1-\alpha]\right] = [0, \text{qbeta}(.95, 2, 2)]$
$= [0, .865]$

$CR_{R, \theta, 1-\alpha} = \left[\text{Quantile}[\theta | X, \alpha], 1\right]$
$= [\text{qbeta}(.05, 2, 2), 1] = [.135, 1]$

Hypothesis Testing AKA Theory Testing

theory (received hypothesis AKA alternate hypothesis)

Assume the theory's opposite (AKA null hypothesis) and reject null (hypothesis (escape your theory)) if "overwhelming" evidence

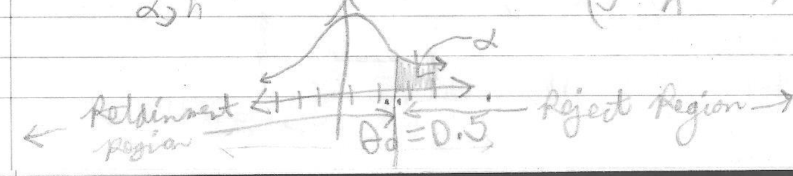
Def. of Overwhelming is "level" α that you decide.

data $\longrightarrow$ sufficient? $\xrightarrow{}$ yes? Reject H_0 (Accept H_a)
$\xrightarrow{\text{no?}}$ Retain H_0

One-sided hypothesis test

$H_0: \theta \leq \theta_0 = 0.5$
$H_a: \theta > \theta_0 = 0.5$
α, n

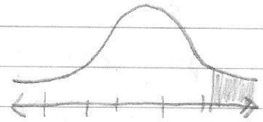
$\hat{P} \sim N\left(\theta_0, \left(\sqrt{\frac{\theta_0(1-\theta_0)}{n}}\right)^2\right)$



$\hat{\theta} \in$ Retainment Region $\Rightarrow$ Retain H_0
$\hat{\theta} \notin$ Retainment Region $\Rightarrow$ Reject H_0

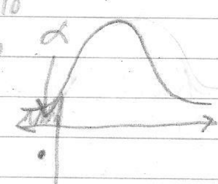
p-val $:= P\left(\begin{matrix}\text{seeing this data} \\ \text{or more "extreme"}\end{matrix} \,\middle|\, H_0 \text{ true}\right)$

$= \underset{\alpha}{\operatorname{argmin}} \{ \hat{\theta} \in \text{Ret. Region} \}$



$\theta_0 = 0.5$

p-value $< \alpha \Rightarrow$ Reject H_0
p-value $\geq \alpha \Rightarrow$ Retain H_0

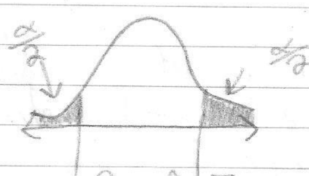


one sided hypothesis test
$H_0 : \theta \geq \theta_0 = 0.5$
$H_a : \theta < \theta_0 = 0.5$

$\theta_0 = 0.5$

two-sided test

$\overbrace{H_0 : \theta = \theta_0 = 0.5}$
$H_a : \theta \neq \theta_0 = 0.5$
$\begin{cases} \theta > 0.5 \\ \theta < 0.5 \end{cases}$?



$\theta_0 = 0.5$

p val $\overset{?}{=} P(H_0)$ NO
p val $\overset{?}{=} P(H_a)$ NO
p val $\overset{?}{=} \cancel{P(H_0|X)}$ NO
p val $\overset{?}{=} P(H_a|X)$ No

If big, have not proven; If small, proven

$H_0 : \theta \leq \theta_0 = 0.5$
$H_a : \theta > \theta_0 = 0.5$

$\alpha = 5\%$ $\quad n = 2$ $\quad X = 1$

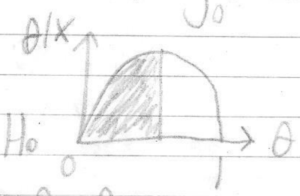
Posterior

$\theta \sim U(0,1) \Rightarrow \theta|X \sim Beta(2,2)$

Bayesian p-val $\rightarrow$ p-val $:= P(H_0|X) = P(\theta \leq 0.5|X) = \int_0^{0.5} Beta(2,2)\,d\theta$

$= pbeta(.5, 2, 2)$

$= 0.5 \not< \alpha \Rightarrow$ Retain H_0



$P(H_0|X) = \frac{P(X|H_0)P(H_0)}{P(X)} = \theta \leq \theta_0$

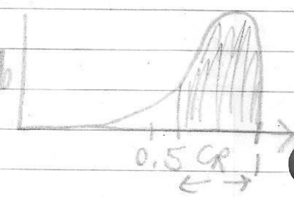
$H_0 : .1$

$H_a : .9$

$P(X) = P(X|H_0)P(H_0) + P(X|H_a)P(H_a)$

Role of $\hat{\theta} = \bar{x}$?

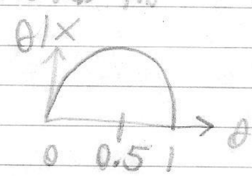
p-val $:= P(H_0|X) < \alpha \Rightarrow$ Reject H_0



$\theta_0 \in CR_{R,\theta,1-\alpha} \Rightarrow$ Retain H_0

$H_0 : \theta = \theta_0 = 0.5$

$H_a : \theta \neq \theta_0 = 0.5$



Bayesian p-val

$\rightarrow$ p-val $:= P(H_0|X) = P(\theta = 0.5|X) = \int_{0.5}^{0.5} Beta(2,2)\,d\theta = 0$

"point null", "prior null"

$0.5 = 0.50001$ (statisticians say)

I $H_0 : \theta \in [\theta_0 \pm \delta]$

$H_a : \theta \notin [\theta_0 \pm \delta]$ ← Margin of equivalence

II $H_0 : \theta = \theta_0 = 0.5$

$H_a : \theta \neq \theta_0 = 0.5$

$\theta_0 \in CR_{\theta,1-\alpha} \Rightarrow$ Retain H_0

CLT for Beta $\alpha=\beta>>0$

$\alpha = 5\%$ $\quad n=100 \quad x=61$

Frequentist

Rej. Region $= \left[\theta_0 \pm z_{\frac{\alpha}{2}}\sqrt{\frac{\theta_0(1-\theta_0)}{n}}\right]$

$= \left[.5 \pm 2\sqrt{\left(\frac{(.5)(.5)}{100}\right)}\right] = [.4, .6]$

$.61 \notin$ Rej. Reg. $\Rightarrow$ Reject

p-val $:= P\left(|Z| > \frac{.61-.5}{.05}\right) = 2P(Z>2.2)$

$= 2(1-\text{pnorm}(2.2)) = .0278 < \alpha = 5\%$

$\Rightarrow$ Reject H_0

Need $\theta \sim U(0,1)$

$\delta = .01 \Rightarrow H_0: \theta \in [.49, .51]$

$H_a: \theta \notin [.49, .51]$

$\theta | x \sim \text{Beta}(\overset{61+1}{62}, \overset{100-61+1}{40})$

P-val $:= P(H_0|X) = P(\theta \in [.49,.51] | X)$

$= \int_{.49}^{.51} \text{Beta}(62,40)\,d\theta \longrightarrow$ pbeta(.51, 62, 40) $-$ pbeta(.49, 62, 40)

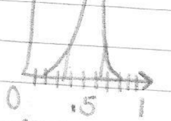
$= .0147 < \alpha = 5\%$

$\Rightarrow$ Reject H_0

$CR_{\theta, 95\%} = [\text{qbeta}(.025, 62, 40),$ $\text{qbeta}(.975, 62, 40)]$

$= [.511, .700]$

$\theta_0 = 0.5 \notin CR \Rightarrow$ Reject H_0



$H_a: \underbrace{\theta \neq \theta_0}_{\theta \sim U(0,1)}$

Define $B := \frac{P_{H_a}(X)}{P_{H_0}(X)} = \frac{\int_{\Theta_{H_a}} P(X|\theta) P_{H_a}(\theta)\,d\theta}{\int_{\Theta_{H_0}} P(X|\theta) P_{H_0}(\theta)\,d\theta}$

"Bayes Factor"

$\frac{\int_0^1 \binom{n}{x}\theta^x(1-\theta)^{n-x}(1)\,d\theta}{\int_{\{.5\}} \binom{n}{x}\theta^x(1-\theta)^{n-x}(1)\,d\theta} =$

$$= \frac{\int_0^1 \theta^{61}(1-\theta)^{39}\,d\theta}{\underbrace{.5^{61}(1-.5)^{39}}_{.5^{100}}} = \frac{B(62, 40)}{.5^{100}} = 1.39$$