Math 341 - Lecture 1 01/29/19

data generating process

Let X be a random variable (r.v.)

Let x be a realization (data)

$$x \in \underbrace{\text{Supp}(X)}_{\text{support of } X}$$

set of all possible unique/different values of X.

2 Types of Random Variables

| | = cardinality

I Discrete

$|\text{Supp}[X]| \leq |\mathbb{N}|$

The number of unique values of x is at most countably infinite.

$p(x) := P(X = x)$

↑ probability mass function (pmf)

$p : \text{Supp}[X] \to (0, 1]$ (* if its possible it has a nonzero probability so its noninclusive. 😊)

$F(x) := P(X \leq x)$

↑ cumulative distinction function (CDF)

$$= \sum_{\{y \in \text{Supp}(X) \,\&\, y \leq x\}} p(y) \quad \text{if } X \text{ discrete}$$

$$\sum_{x \in \text{Supp}(X)} p(x) = 1$$

Humpty Dumpty Theorem

II Continuous r.v.'s

$|Supp[X]| = |\mathbb{R}|$ ← uncountably ∞

F(x) is the same

probability density function (PDF) — $f(x) := F'(x) = \frac{d}{dx}(F)$

$$P(X \in [a,b]) = P(X \le b) - P(X \le a)$$
$$= F(b) - F(a)$$
$$= \int_a^b f(x)\,dx$$

probabilit... is in some... you can integrat...

$$\int_{-\infty}^{\infty} f(x)dx = F(\infty) - F(-\infty)$$
$$= P(X \le \infty) - P(X \le -\infty)$$
$$= P(X \in (-\infty, \infty))$$
$$= 1$$

whats the probabili... that x is anything

$f : Supp(X) \to [0, \infty]$

f(x) is not a probability.

f(x) is not $P(X = x) = p(x)$

$Supp(X) = \{x : f(x) > 0\}$

PDF is defin... as the derivat... of the CD... as x gets bigger th... CDF doesnt.

Discrete $\begin{cases} X \underset{\text{Distributed as}}{\sim} Bernouille(p) = p^x(1-p)^{1-x} \to P(X=1) = p(1) = p \\ X \sim Binomial(n \cdot p) = \binom{n}{x} p^x (1-p)^{n-x} \to P(X=0) = p(0) = 1-p \end{cases}$

Continuous $\begin{cases} X \sim \underset{\text{Exponential}}{Exp}(\lambda) := \overbrace{\lambda e^{-\lambda x}}^{f(x)} \\ X \sim \underset{\text{Normal}}{N}(m, \sigma^2) := \frac{1}{\sqrt{}} e^{-\frac{1}{2}(x-m)^2} \end{cases}$

P is a parameter which is a sort of "turning knob".
What are the allowable values of p? $p \in [0,1]$ ✓

$p = 0, \quad X \sim \text{Bern}(0) = 0^x 1^1 = 1$

$$p(1) = 0$$
$$p(0) = 1$$
$$\text{Supp}(X) = \{1\}$$

if you remove 0 and 1 you're removing the degenerative cases.

$p \in \overbrace{(0,1)}$ → parameter space: a set of values of the parameter that yield non-degenerative r.v.

Let θ denote an unknown parameter
Let $\vec{\theta}$ denote an unknown parameter discrete
Θ denote the parameters space.

$$X \sim \text{Bern}(\theta) = \theta^x (1-\theta)^{1-x}$$
$$X \sim \text{Bin}(n, \theta) = \binom{n}{x} \theta^x (1-\theta)^{n-x}$$ — $\text{Supp}(X) = \{0, 1, \ldots$

fixed or unknown (n) — unknown (θ)

counting succe

$$X \sim \text{Binomial}(\theta_2, \theta_1) = \binom{\theta_2}{x} \theta_1^x (1-\theta_1)^{\theta_2 - x}$$

Parametric Model

$\mathcal{F} := \{ p(x;\theta) : \theta \in \Theta \}$

s.t. $\dim[\Theta] < \infty$.

$\mathcal{F}_{Bern} = \{ \theta^x (1-\theta)^{1-x} : \theta \in (0,1) \}$

$P(X_1, X_2, \ldots, X_n; \theta) \overset{\text{if independent}}{=} \prod_{i=1}^{n} P_i(X_i;\theta) \overset{\text{if iid}}{=} \prod_{i=1}^{n} P(X_i;\theta)$

(jmf) joint mass function -or-
joint density function (jdf)

If $X_1, X_2, \ldots, X_n \overset{iid}{\sim}$ → independent & identically distribute

In the real world, you observe data eg <0, 0, 1, 0, 1, 0>
we say $n = 6$ (# of observation)

First assumption: pick a parametric model, $\mathcal{F}$
Beyond the scope of Math 341. Pretend $\mathcal{F}$ = Bernoull function

infer the value of θ => Raison D'être