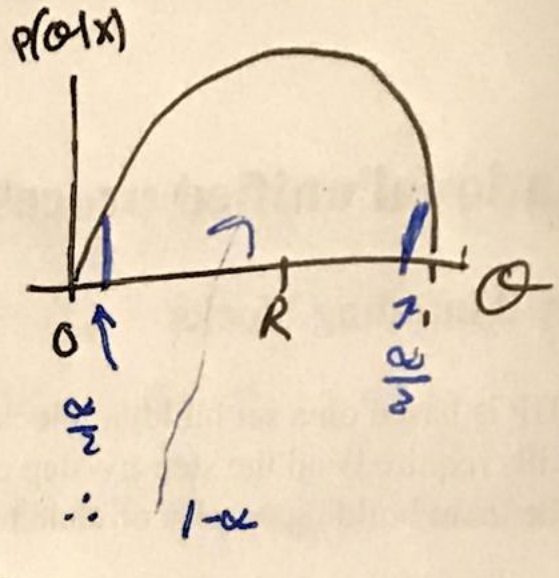
# Lecture 7 : 2/21/19

Recall $P(\theta) = U(0,1) = \text{Beta}(1,1)$

for $X=1,\ n=2 \Rightarrow P(\theta|X) = \text{Beta}(x+\alpha, n-x+\beta) = \text{Beta}(2,2)$ $\Rightarrow$




How can we make a set $R$ s.t. $P(\theta \in R | X) \geq 1-\alpha$
where $R$ is in the "middle" of $\theta|X$
ie $\int_R P(\theta|X)\,d\theta = \cancel{1}-\alpha$

So lets define $CR_{\theta,1-\alpha} := \left[\text{Quantile}\left[\theta|X, \frac{\alpha}{2}\right], \text{Quantile}\left[\theta|X, 1-\frac{\alpha}{2}\right]\right]$

here $CR_{\theta,95\%} = \left[\underset{2.5\%}{\text{qbeta}(.025,2,2)},\ \underset{97.5\%}{\text{qbeta}(.975,2,2)}\right] = [.094, .906]$

1) 95% chance $\theta \in CR$, given prior & data
2)

If we did this the frequentist way w/ $\hat{\theta}_{MLE} = 0.5$

~~$CI_{\theta,1-\alpha}$~~ $CI_{\theta,95\%} = \left[0.5 \pm 1.96\sqrt{\frac{0.5(1-0.5)}{2}}\right] = [-0.2, 1.2]$ which means nothing! ~~θ ∈~~ $\theta \in (0,1)$!

This ~~does~~ doesn't have a big enough $n$ for a normal distribution

$\rightarrow$ at least Baysian tells us SOMETHING

Above was a Baysian 2-Sided Credible Region. Let's try 1-Sided:

LEFT: $CR_{L,\theta,1-\alpha} := \left[\begin{matrix}\text{smallest value} \\ \text{in } \Theta \\ \text{or } -\infty\end{matrix},\ Q[\theta|X, 1-\alpha]\right]$

in our example: $= [0, \text{qbeta}(.95,2,2)] = [0, .864]$

"95% chance $\theta$ is at most .864"

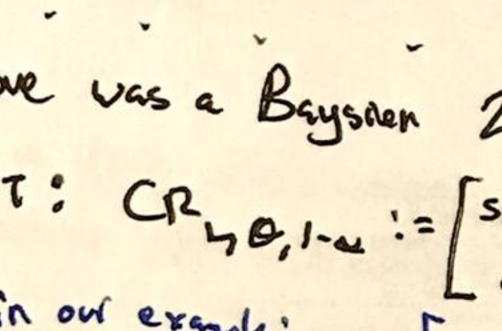
RIGHT: $CR_{R,\theta,1-\alpha} := \left[Q[\theta|X, 1-\alpha],\ \begin{matrix}\text{largest value} \\ \text{in } \Theta \text{ or} \\ \infty\end{matrix}\right]$

$= [\text{qbeta}(.95,2,2), 1] = [.135, 1]$

"95% chance $\theta$ is greater that .135"

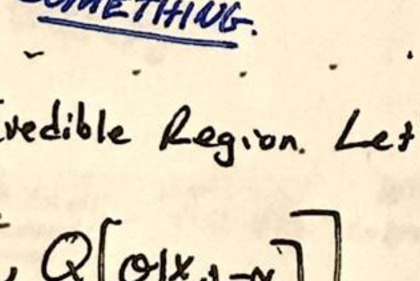
HDR is nicer b/c its smaller. Though it is more difficult to calculate, making it ① computationally expensive. Also, it is ② fragmented which is annoying.
$\rightarrow$ is this even useful...?

BUT WHAT IF MY POSTERIOR LOOKS LIKE



$CR_{\theta,1-\alpha}$ is not so bueno

$\longrightarrow$




HDR (High Density Region)

More bueno $\longrightarrow$ HDR = $A \cup B \cup C$

$P(\theta \in HDR | X) = 1-\alpha$

Onto theory testing $\longrightarrow$

Review for prev. Students...

# THEORY TESTING

You wish to convince people of $H_a$: Alternate Hypothesis when they all believe $H_0$: null hypothesis

(I) Assume $H_a$ and wait for others to disprove based on evidence (wierd placing of Burden of Proof)

(II) Assume $H_0$ and demonstrate contradictory evidence until others reject $H_0$

II is intellectually honest, which forces you to provide evidence for your claim ~~to get~~ to convince beyond a reasonable doubt. := $\alpha$ level

## ARE UFO's AND ALIENS REAL?

~~$H_0$: They~~ ~~$H_a$~~

(I) $H_0$: They Exist
$H_a$: They don't exist

if I have a high $\alpha$ maybe you can convince me of $H_a$... if low $\alpha$ good luck

(II) $H_0$: Don't exist
$H_a$: Exist

now I'll be nicer, join yourside and show you that aliens exist, and the higher your $\alpha$-level the easier I can convince you of $H_a$.

In our application, we look at common $H_a$'s

(I) $H_a: \theta \neq \theta_0 \longrightarrow H_0: \theta = \theta_0$

(II) $H_a: \theta < \theta_0 \longrightarrow H_0: \theta \geq \theta_0$

(III) $H_a: \theta > \theta_0 \longrightarrow H_0: \theta \leq \theta_0$

For II, III we calculate a Baysian $P_{val}$

$$P_{val} := P(H_0|X) < \alpha$$

lets standardize a default $\alpha$ $\alpha = 5\%$

Flip a coin 100 times, get 61 heads. Is the coin wieghted?
assume $P(\theta) = U(0,1)$, $\alpha = 5\%$.

$P(\theta|X) = Beta(61+1, 39+1)$

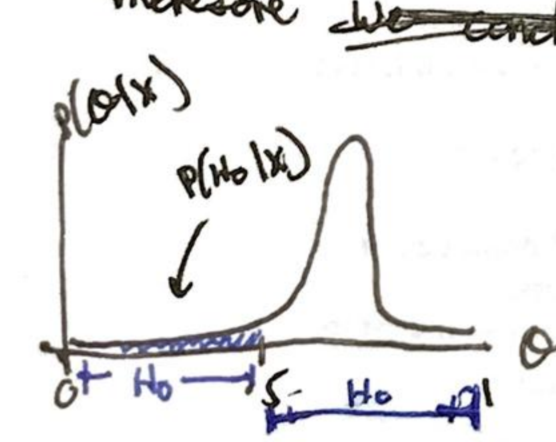
$H_0: \theta \leq .5$
$H_a: \theta > .5$

(p = probability is CDF
q = quantile

CDF of Beta $F_{\theta|X}(0.5)$

$$P_{val} = P(H_0|X) = P(\theta \leq .5 / X) = \int_0^{0.5} \frac{1}{B(62,40)} \theta^{61} (1-\theta)^{39} d\theta \xrightarrow{\text{computer}} pbeta(0.5, 62, 40) = 0.014 < \alpha = 0.05$$

therefore ~~we conclude~~ we reject $H_0$ and conclude that the coin is unfairly wieghted




Uber Driver did 200 rides w/ 37 5-stars reviews
If $\theta < 0.25\%$ then Uber fires the driver
Prob. of 5★ rating
assume $P(\theta) = U(0,1)$, $\alpha = 5\%$.

$\Rightarrow P(\theta|X) = Beta(38, 164)$

$H_0: \theta \geq .25$
$H_a: \theta < .25$

$$P_{val} = P(H_0|X) = P(\theta \geq 0.25 / X) = \int_{.25}^{1} \frac{1}{B(38,164)} \theta^{37} (1-\theta)^{163} d\theta$$

$= 1 - pbeta(0.25, 36, 164) = .017 < \alpha$ $\therefore$ reject $H_0$ and fire driver

qbeta does integral up to point K
pbeta does integral to specified point
then qbeta spits out K, pbeta take % and spits out area
inverse operations

If we ask the question "is coin fair" we will end up integrating .5 → .5, which gives $P_{val} = 0$ b/c $P(X=x)$ for a continous RV $= 0$ always. ~~So, we modify $H_0$~~ Can do 2 things
↳ next pg