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$\boxed{\text{241 Review}}$

Let ~~X~~ be a random variable (r.v.) and x be a realization. X is a process, x is a result of the process. x is data.

$x \in Supp(X)$ "x belongs to the support of X which is the set of all possible unique values of x"

X can be referred to as a "data generating process"

(I) Discrete

$|Supp(X)| \leq |\mathbb{N}|$ the num of unique values of x is at most countably infinite $\aleph_0$

~~p(x) = probability~~

$p(x) := P(X = x)$ probability mass function (PMF)

tells you how likely a state is

$p: Supp[X] \longmapsto (0, 1]$ b/c if $p(x) = 0$ then $x \notin Supp[X]$

$F(x) := P(X \leq x)$ Cumulative Distribution Function (CDF)

$= \sum_{\{y \in supp[X] \text{ \& } y \leq x\}} p(y)$

$\sum_{x \in Supp[X]} p(x) = 1$ basically definitional but technically a Theorem

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II Continous R.V.

$|Supp[X]| = |\mathbb{R}| = \aleph_1$ uncountably infinite

$F(x) :=$ still the same w/ a CDF

but now $\boxed{f(x) = F'(x) = \frac{d}{dx}(F)}$

PROBABILITY DENSITY FUNCTION

$$P(x \in [a,b]) = P(x \le b) - P(x \le a)$$
$$= F(b) - F(a)$$
$$= \int_a^b f(x)\,dx$$

So what is

$$\int_{-\infty}^{\infty} f(x)\,dx = F(\infty) - F(-\infty)$$
$$= P(x \le \infty) - P(x \le -\infty)$$
$$= P(x \in (-\infty, \infty))$$
$$= 1$$

pretty obvious...

now, what is domain & range?

$$f : Supp[x] \mapsto [0, \infty)$$

PDF is a vacuum of CDF; always gets bigger never smaller, it is continuously adding and so $f(x)$ is not a probability at all... otherwise would be bounded by 1.

$f(x)$ is not $P(X=x) = p(x)$ rather $Supp[X] = \{x : f(x) > 0\}$

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$\boxed{\text{EX:}}$

Discrete
① $X \sim Bernoulli(p) = \underbrace{p^x(1-p)^{1-x}}_{PMF} = p(x)$ Supp[x] = {1, 0}
② $X \sim Binomial(\cancel{p},\ n, p) = \binom{n}{x} p^x (1-p)^{n-x}$ Supp[x] = {0, 1, ... n}
} PMF

Continuous
③ $X \sim Exp(\lambda) := \lambda e^{-\lambda x}$
④ $X \sim \underset{normal}{N}(\mu, \sigma^2) := \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$
} PDF

① $p(X=1) = p(1) = p$
$P(X=0) = p(0) = 1-p$

~~{p is ... a pdf}~~

Values in () are parameters; ie tuning knob on the R.V.

→ w/ p as a parameter, what are its allowable values? ~~p ∈ 0, 1~~
Since p=0, p=1 are degenerate cases (ie cases when X always spits out the same value) that is not ~~interesting~~ interesting. Let's say that $\underbrace{p \in (0,1)}_{\text{parameter space}}$

def: Parameter Space: a set of values that yield non-degenerate r.v's

... leaving 241 for a moment...
Let θ denote an unknown parameter
" $\vec{\theta}$ " " " parameters
" Θ denote a parameter space
cap theta

$X \sim Bernoulli(\theta) = \theta^x(1-\theta)^{1-x}$ is where I don't know the param

$\underbrace{X \sim Binom(n, \theta) \quad \text{n is known or fixed} \quad \theta \text{ is unknown}}$

↗ $X \sim Binom(\theta_2, \theta_1) = \binom{\theta_2}{x} \theta_1^x (1-\theta_1)^{\theta_2 - x}$ ↙ here I don't know either ... so super hard

won't be touching this semester. "(capture, recapture them)"

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Can also do $X \sim Exp(\theta) = \theta e^{\theta x}$
same w/ N ...

Parametric Model

PMF or PDF... $p(x) = f(x)$ for rest of 341

$\mathcal{F} := \{p(x;\theta) : \theta \in \Theta$ s.t. $\dim[\Theta] < \infty\}$

; → imagine $f(x) = \sin(x)$ we can say $f(x;c) = \sin(cx)$
c is something I need to know to calculate the answer.

$\mathcal{F}_{Bern} := \{\theta^x(1-\theta)^{1-x} : \theta \in (0,1)\}$

$p(x_1, x_2 \dots x_n; \theta)$ is Joint Mass Function (jmf)
or
Joint Density Function (jdf)

"what's the probability of $x_1 \dots x_n$ knowing θ"
If $X_1, X_2 \dots X_n \overset{iid}{\sim}$ (independently and identically distributed)
then $p(x_1 \dots x_n; \theta) = \underbrace{\prod_{i=1}^{n} p_i(x_i;\theta)}_{\text{if independent}} = \underbrace{\prod_{i=1}^{n} p(x_i;\theta)}_{\text{if identically distributed}}$

In the real world, we observe data ex: $\vec{x} = \langle 0, 0, 1, 0, 1, 0$
we say $n = 6$ (num. observations).
We have no idea where the numbers came from.

#1 Pick or assume a parametric model $\mathcal{F}$ (Beyond 341 scope)
→ we will pretend $\mathcal{F}$ = Bernoulli family

Purpose of class is to Infer the value of θ.