* Bayesian Statistics

- Review of Probability Theory:

X - data-generating process (random variable), x - realization/data (outcome
where $x \in \mathrm{SUPP}(X)$

- The support of X contains all possible outcomes of X.

- Discrete case: The support of X is at most countably infinite, $|\mathrm{SUPP}(X)| \le |\mathbb{N}|$.
- The probability mass function $P(x)$ maps $P: \mathrm{SUPP}(X) \to (0,1]$.
- The cumulative distribution function (CDF) is given by $F(x) = P(X \le x)$,

$$F(x) = \sum_{\substack{y \in \mathrm{SUPP}(X) \\ \& \, y \le x}} P(y)$$

- Continuous case: The support of X is uncountably infinite, $|\mathrm{SUPP}(X)| = |\mathbb{R}$
- The CDF is given by $F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\, dx$.
- $P(X \in [a,b]) = P(X \le b) - P(X \le a) = F(b) - F(a) = \int_a^b f(x)\, dx$.
- The probability density function maps $P: \mathrm{SUPP}(X) \to (0, \infty)$.

- Examples of distributions:

$X \sim \mathrm{Bernoulli}(P) \Rightarrow P(x) = P^x (1-P)^{1-x}$, where $x \in \{0,1\}$

$X \sim \mathrm{Binomial}(n,P) \Rightarrow \binom{n}{x} P^x (1-P)^{n-x}$

$X \sim \mathrm{Exponential}(\lambda) \Rightarrow \lambda e^{-\lambda x}$

$X \sim N(\mu, \sigma^2) \Rightarrow \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-1}{2\sigma^2}(x-\mu)^2}$

- We say that a distribution is degenerate when $P(x) = 0$ or 1, for some x (In other words, the process is not random).
- The parameter space Θ of a random variable is the set of paramete values for which the R.V. is non-degenerate.
- A parametric model is given by $F = \{P(x;\theta) : \theta \in \Theta\}$, s.t. $\dim(\Theta) < \infty$

ex: $F_{\mathrm{bern}} = \{\theta^x (1-\theta)^{1-x} : \theta \in (0,1)\}$

* Bayesian Statistics

- Review of Probability Theory:

X - data-generating process (random variable), x - realization/data (outcome) where $x \in SUPP(X)$

- The support of X contains all possible outcomes of X.

- Discrete case: The support of X is at most countably infinite, $|SUPP(X)| \leq |\mathbb{N}|$.
- The probability mass function $P(x)$ maps $P: SUPP(X) \to (0,1]$.
- The cumulative distribution function (CDF) is given by $F(x) = P(X \leq x)$,

$$F(x) = \sum_{\substack{y \in SUPP(X) \\ \& \, y \leq x}} P(y)$$

- Continuous case: The support of X is uncountably infinite, $|SUPP(X)| = |\mathbb{R}|$
- The CDF is given by $F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\, dx$.
- $P(X \in [a,b]) = P(X \leq b) - P(X \leq a) = F(b) - F(a) = \int_a^b f(x)\, dx$.
- The probability density function maps $P: SUPP(X) \to (0,\infty)$.

- Examples of distributions:

$X \sim Bernoulli(P) \Rightarrow P(x) = P^x (1-P)^{1-x}$, where $x \in \{0,1\}$

$X \sim Binomial(n,P) \Rightarrow \binom{n}{x} P^x (1-P)^{n-x}$

$X \sim Exponential(\lambda) \Rightarrow \lambda e^{-\lambda x}$

$X \sim N(\mu, \sigma^2) \Rightarrow \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-1}{2\sigma^2}(x-\mu)^2}$

- We say that a distribution is degenerate when $P(x) = 0$ or 1, for some x (In other words, the process is not random).
- The parameter space Θ of a random variable is the set of parameter values for which the R.V. is non-degenerate.
- A parametric model is given by $\mathcal{F} = \{P(x;\theta) : \theta \in \Theta\}$, s.t. $\dim(\Theta) < \infty$

ex: $\mathcal{F}_{Bern} = \{\theta^x (1-\theta)^{1-x} : \theta \in (0,1)\}$